UNIVERSITY OF TECHNOLOGY
MECHANICAL ENG. DEPT

THIRD YEAR

LECTURES OF HEAT TRANSFER

Heat Transfer Rate Processes

Mode	*Transfer Mechanism*	*Rate of heat transfer (W)*
Conduction	Diffusion of energy due to random molecular motion	$q = -kA\frac{dT}{dx}$
Convection	Diffusion of energy due to random molecular motion plus bulk motion	$q = hA(T_s - T_\infty)$
Radiation	Energy transfer by electromagnetic Waves	$q = \sigma\varepsilon A(T_s^4 - T_{sur}^4)$

By

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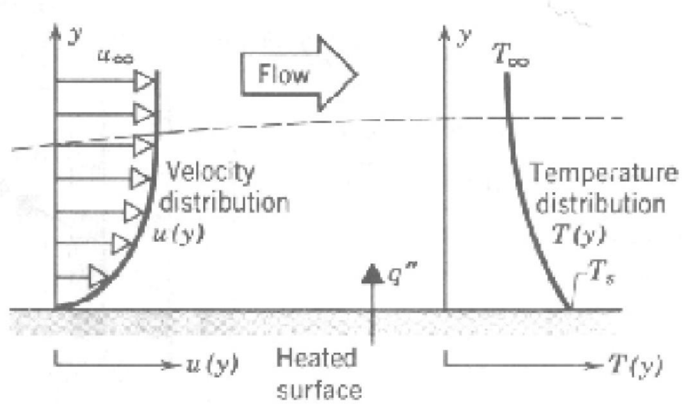

Course Name:	Heat transfer I		انتقال حرارة	اسم المقرر:
Course Code:	ME392-1		392-1همك	رمز المقرر:
Units:	2		2	الوحدات:
Hours per Week				الساعات الأسبوعية
Theoretical	Experimental	Tutorial	مناقشة	عملي	نظري
2	-	-	-	-	2

Week	Contents	المحتويات	الاسبوع
1	**Introduction** - General concepts and definition - Modes and basic laws of heat transfer - Heat transfer by conduction - Convective heat transfer - Thermal radiation	**مقدمة** مبادئ وتعاريف عامة طرق وقوانين انتقال الحرارة انتقال الحرارة بالتوصيل انتقال الحرارة بالحمل الاشعاع الحراري	1
2	**Steady state conduction** - Conduction through a plan wall - Conduction through composite wall - Overall heat transfer coefficient	**التوصيل المستقر** توصيل خلال جدار مستوي توصيل خلال جدار مركب معامل انتقال الحرارة الاجمالي	2
3	**Steady state conduction** - Conduction through solid cylinder - Conduction through hollow cylindrical and composite wall - Overall heat transfer coefficient - Critical thickness of insulation	**التوصيل المستقر** التوصيل خلال اسطوانة صلدة التوصيل خلال اسطوانة مجوفة وجدار مركب معامل انتقال الحرارة الاجمالي السمك الحرج للعازل الحراري	3
4	**Conduction with heat generation** - Plan wall - Solid cylinder - Hollow cylinder	**التوصيل مع توليد حراري** جدار مستوي اسطوانة صلبة اسطوانة مجوفة	4
5	**Heat transfer with finned surface** - General equation for temperature distribution - Long fin - Short fin - Fins with isolated tip	**انتقال الحرارة في السطوح المزعنفة** المعادلة العامة لتوزيع درجات الحرارة زعنفة طويلة زعنفة قصيرة زعنفة معزولة النهاية	5
6	**Heat transfer with finned surface** - Fins with convection at tip - Fin efficiency - Design of fins	**انتقال الحرارة في السطوح المزعنفة** زعنفة مع انتقال الحرارة بالحمل عند الطرف كفاءة الزعنفة تصميم الزعانف	6
7	**Applications and examples (case study)**	**تطبيقات وتمارين**	7
8	**Principle of convection heat transfer** - Free and forced convection - Laminar boundary layer on flat plate - Energy equation and boundary layer	**مبادئ انتقال الحرارة بالحمل** الحمل الحر والقسري الطبقة المتاخمة الطباقية على صفيحة مستوية معادلة الطاقة والطبقة المتاخمة	8

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11	**Empirical correlation for free and force convection** - Empirical relation for pipe and tube flow	**العلاقات التجريبية للحمل الحراري القسري** العلاقات التجريبية للجريان في الانابيب	11
12	**Empirical correlation for free and force convection** - Flow across cylinders and spheres - Flow across tube bank - Flow across finned tube bank	**العلاقات التجريبية للحمل الحر والقسري** الجريان عبر سطح اسطواني وكروي الجريان عبر حزمة من الانابيب الجريان عبر أنبوب مزعنف	12
13	**Natural convection** - Free convection heat transfer on vertical flat plate - Empirical relation for free convection	**الحمل الحر** انتقال الحرارة بالحمل الحر لصفيحة مستوية عمودية العلاقات التجريبية للحمل الحر	13
14	**Natural convection system** - Vertical plane and cylinder - Horizontal cylinder - Horizontal plates - Inclined surface - Compact heat exchanger design elements	**الحمل الحر** الاسطوانة والمستوى العمودي الاسطوانات الافقية الصفائح الافقية السطوح المائلة تصميم مقاطع المبادلات الحرارية المدمجة	14
15	**Applications and examples (case study)**	**تطبيقات وتمارين**	15

Dimensionless Group

1	*Biot number*	$Bi = \frac{h\,s}{k}$	$\frac{\text{Internal resistance to heat conduction}}{\text{External resistance to heat conduction}}$	s = characteristic dimension s = ½ t (thickness) for plate s = r (for circle and sphere)
2	*Brinkman number*	$Br = \frac{\mu\,V^2}{k(T_w - T_\infty)} = \Pr \mathrm{Ec}$	$\frac{\text{Viscosity} \times \text{Inertia}}{\text{heat transfer rate}}$	
3	*Drag coefficient*	$C_D = \frac{D\,g}{\frac{1}{2}\rho\,V^2\,D^2}$	$\frac{\text{Drag force}}{\text{kinetic energy or inertia of the flow}}$	D = drag
4	*Coefficient of friction*	$C_f = \frac{\tau_w\,g}{\frac{1}{2}\rho\,V^2}$	$\frac{\text{Wall shear stress}}{\text{kinetic energy of the flow}}$	
5	*Force coefficient*	$C_F = \frac{F\,g}{\frac{1}{2}\rho\,V^2\,D^2}$	$\frac{\text{Force}}{\text{kinetic energy or inertia of the flow}}$	
6	*Lift coefficient*	$C_L = \frac{L\,g}{\frac{1}{2}\rho\,V^2\,D^2}$	$\frac{\text{Lift force}}{\text{kinetic energy of the flow}}$	
7	*Pressure coefficient*	$C_p = \frac{\Delta p\,g}{\frac{1}{2}\rho\,V^2}$	$\frac{\text{Pressure forces}}{\text{Inertia of the flow}}$	
8	*Eckert number*	$Ec = \frac{u_\infty^2}{c_p(T_\infty - T_w)}$	$\frac{\text{Kinetic energy}}{\text{Thermal energy}}$	
9	*Euler number*	$Eu = \frac{p}{\rho\,u_\infty^2}$	$\frac{\text{Pressure forces}}{\text{Inertia forces}}$	
10	*Friction factor*	$f = \frac{\Delta p\,D}{\frac{1}{2}\rho\,V^2\,s}$	$\frac{\text{Pressure Drop}}{\text{Kinetic energy of the flow}}$	s = characteristic dimension

11	*Fourier modulus*	$Fo = \frac{\alpha\,\tau}{s^2}$	Dimensionless time for transient conduction	$\alpha = \frac{k}{\rho\,c}$ s = characteristic dimension
12	*Froude number*	$Fr = \frac{u_\infty^2}{s\,g}$	$\frac{\text{Inertia forces}}{\text{gravity forces}}$	
13	*Grashof number*	$Gr = \frac{g\beta(T_w - T_\infty)x^3}{\nu^2}$	$\frac{\text{Buoyancy force}}{\text{viscous force}}$	In free convection system
14	*Graetz number*	$Gz = \text{Re}\,\text{Pr}\,\frac{d}{L}$	Combined free and forced convection	
15	*Knudsen number*	$Kn = \frac{\lambda}{L} = \sqrt{\frac{\pi\,\gamma}{2}}\,\frac{\text{M}}{\text{Re}}$	$\frac{\text{Mean free path}}{\text{characteristic body dimension}}$	M = mach number
16	*Lewis Number*	$Le = \frac{\alpha}{D}$	$\frac{\text{Thermal difusivity}}{\text{Mass difusivity}}$	D = diffusion
17	*Mach Number*	$M = \frac{u}{a}$	$\frac{\text{Velocity}}{\text{sonic velocity}}$	a = speed of sound
18	*Magnetic influence Number*	$N = \frac{\sigma\,B_y^2\,\text{x}}{\rho\,\text{u}_\infty}$		σ = electrical conductivity B_y = magnetic field strength in y- dir.
19	*Nusselt Number*	$Nu = \frac{h\,\text{x}}{k}$	$\frac{\text{Convection heat transfer}}{\text{Conduction heat transfer}}$	
20	*Peclet Number*	$Pe = \text{Re}\,\text{Pr}$	Forced convection heat transfer	
21	*Pranrdtl Number*	$\text{Pr} = \frac{\mu\,c_p}{k}$	$\frac{\text{Momentum difusivity}}{\text{Thermal difusivity}}$	
22	*Rayleigh Number*	$Ra = Gr\,\text{Pr}$	$\frac{\text{Forces duo to buoyancy and inertia}}{\text{Forces duo to viscosity and thermal diffusion}}$	
23	*Reynolds Number*	$\text{Re} = \frac{\rho\,u\,\text{x}}{\mu}$	$\frac{\text{Inertia force}}{\text{Viscous force}}$	

24	*Schmidt Number*	$Sc = \frac{\nu}{D}$	$\frac{\text{Momentum diffusivity}}{\text{Mass diffusivity}}$	D = diffusion Sc ≡ Pr in convection heat transfer problems
25	*Sherwood Number*	$Sh = \frac{h_D\ x}{D}$	Ratio of concentration gradients	h_D = mass transfer coefficient D = diffusion
26	*Stanton Number*	$St = \frac{h}{\rho\, c_p\ u} = \frac{\text{Nu}}{\text{Re Pr}}$	$\frac{\text{Heat transfer at wall}}{\text{convection heat transfer}}$	
27	*Weber Number*	$We = \frac{\rho\, V^2\ s}{\sigma\ g}$	$\frac{\text{Inertia of flow}}{\text{Surface tension forces}}$	

Chapter One
Introduction

1

1. Introduction

A consider the cooling of a hot steal rod which is place in a cold water ***Thermodynamics*** may be used to predict the final equilibrium temperature of the rod-water combination. It will not tell us how long it takes to reach this equilibrium condition. ***Heat Transfer*** may be used to predict the temperature of the rod and the water as a function of time.

1.1 Definition:

Heat: is the energy transit as a result of the temperature difference.
Heat transfer: is that science which seeks to predict the energy transfer that may take place between materials bodies as a result of a temperature difference.
Thermodynamics: is the state science of energy, the transformation of energy and the change in the state of matter. (Thermodynamics can be able to determination of heat and work requirements for chemical and physical process and the equilibrium conditions).
Heat flux: heat transfer flow in the direction per unit area (q”).
Steady state: Temperature is very does not very with time (dT/dt) =0.
Unsteady state: temperature is depending on time

1.2 Modes of Heat Transfer

The engineering area frequently referred to as thermal science includes *thermodynamics* and *heat transfer.* The role of heat transfer is to supplement thermodynamic analyses, which consider only systems in equilibrium, with additional laws that allow prediction of time rates of energy transfer. These supplemental laws are based upon the three fundamental modes of heat transfer *conduction, convection,* and *radiation.*

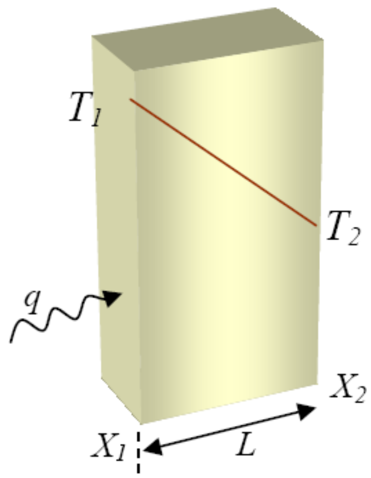
1.3 Conduction Heat Transfer

Conduction may be viewed as the transfer of energy from the more energetic to the less energetic particles of a substance due to interactions between the particles. A temperature gradient within a homogeneous substance results in an energy transfer rate within the medium which can be calculated by ***Fourier's law***

$$q = -kA\frac{dT}{dx} \qquad (1.1)$$

Where $\boldsymbol{q}$ is the heat transfer rate (*W or J/s*) and $\boldsymbol{k}$ thermal conductivity ***(****W/m K*) is an experimental constant for the medium involved, and it may depend upon other properties, such as temperature and pressure. $\frac{dT}{dx}$ Is the temperature gradient in the direction normal to the area ***A.***

$$\frac{dT}{dx} = \frac{\Delta T}{\Delta \chi} = \frac{T_2 - T_1}{\chi_2 - \chi_1}$$



$T_1 > T_2$

Figure 1.1 Temperature distributions for steady state conduction. Through a plate wall

The minus sign in Fourier's Law (1.1) is required by the second law of thermodynamics: thermal energy transfer resulting from a thermal gradient must be from a warmer to a colder region. If the temperature profile within the medium is linear Fig. 1.1 it is permissible to replace the temperature gradient (partial derivative) with

$$q = -kA\frac{T_2 - T_1}{L} \qquad (1.2)$$

The quantity (L/kA) is equivalent to a thermal resistance R_k (K/W) which is equal to the reciprocal of the conductance. As:

$$q = \frac{T_2 - T_1}{R_k} \qquad , \qquad R_k = \frac{L}{kA} \qquad (1.3)$$

Such linearity always exists in a homogeneous medium of fixed k during steady state heat transfer occurs whenever the temperature at every point within the body, including the surfaces, is independent of time.

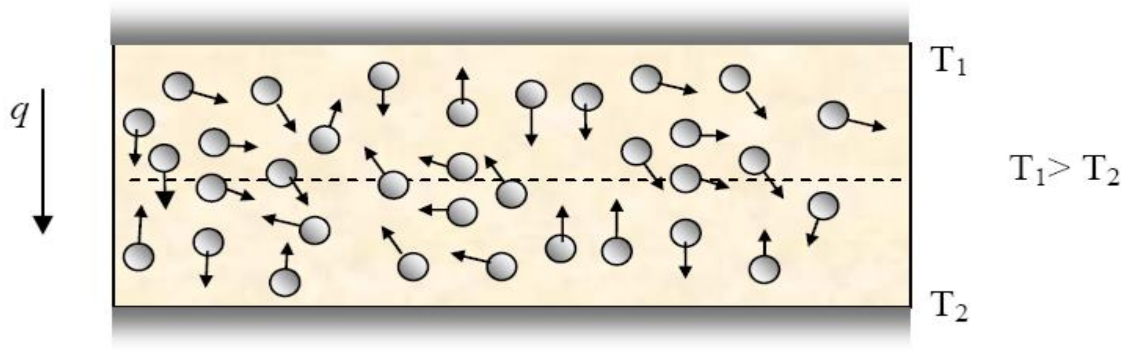


Figure 1.2 Association of conduction heat transfer with diffusion of energy due to molecular activity.

If the temperature changes with time $\frac{dT}{dt}$, energy is either being stored in or removed from the body. This storage rate is

$$q_{stored} = m\,c_p \frac{dT}{dt} \qquad (1.4)$$

Where m is the mass of substance and Cp is specific heat capacity.

1.3.1 Thermal Conductivity

The thermal conductivity of a material is a measure of the ability of the material to conduct heat.

I. **Thermal Conductivity of Solids**: In general, k for a pure metal decreases with temperature; alloying elements tend to reverse this trend. The thermal conductivity of a metal can usually be represented over a wide range of temperature by

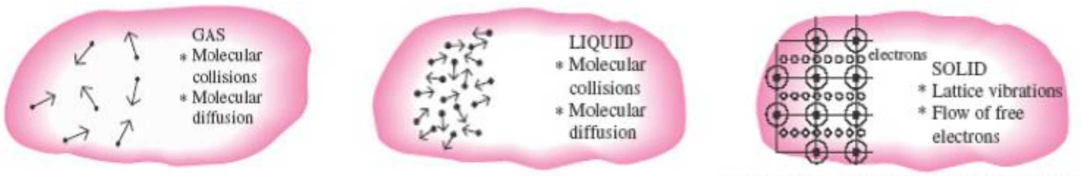
$$k = k_o(a + b\theta + b\theta^2) \qquad (1.5)$$

Where $\theta = T - T_{ref}$ and k_o is the conductivity at the reference temperature T_{ref}

The thermal conductivity of a non-homogeneous material is usually markedly dependent upon the apparent bulk density, As a general rule, k for a no homogeneous material increases both with increasing temperature and increasing apparent bulk density.

II. **Thermal Conductivity of Liquids:** Thermal conductivities of most liquids decrease with increasing temperature. But insensitive to pressure the exception is water, which exhibits increasing k up to about 150°C and decreasing k there after. Water has the highest thermal conductivity of all common liquids except the so-called liquid metals.

III. **Thermal Conductivity of Gases:** The thermal conductivity of a gas increases with increasing temperature, but is essentially independent of pressure for pressures close to atmospheric. For high pressure (i.e., pressure of the order of the critical pressure or greater), the effect of pressure may be significant.



Fig(1.3) The mechanism of heat conduction of different phases of a substance.

1.4 Convection Heat Transfer

Whenever a solid body is exposed to a moving fluid having a temperature different from that of the body, energy is carried or ***convected*** from or to the body by the fluid If the upstream temperature of the fluid is ***T ,*** and the surface temperature of the solid is T_s the heat transfer per unit time is given by ***Newton s Law*** of cooling:

$$q = hA(T_s - T_\infty) \tag{1.6}$$

Where h is Convective Heat transfer coefficient (W/m2 K) as the constant of proportionality relating the heat transfer per unit time and area to the overall temperature difference. It is important to keep in mind that the fundamental energy exchange at a solid-fluid boundary is by conduction, and that this energy is then converted away by the fluid flow.

The thermal resistance to convection heat transfer Rc, as:

$$R_c = \frac{1}{h\,A} \quad , \quad q = \frac{T_s - T_\infty}{R_c} \tag{1.7}$$

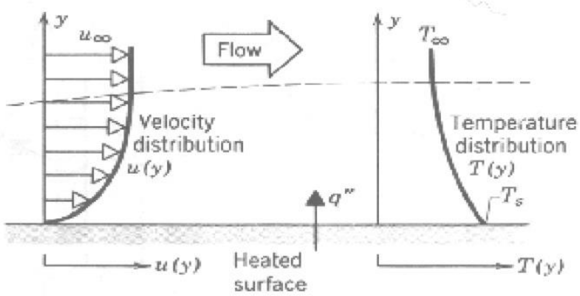


Fig (1.4) Velocity and temperature distribution on flat plate

1.5 Radiation Heat Transfer

The third mode of heat transmission is due to electromagnetic wave propagation, which can occur in a total vacuum as well as in a medium. Experimental evidence indicates that radiant heat transfer is proportional to the fourth power of the absolute temperature, whereas conduction and convection are proportional to a linear temperature difference. The fundamental Stefan-Boltzmann Law is:

$$q = \sigma \varepsilon T^4 \tag{1.8}$$

Where T is the absolute temperature, σ is Boltzmann constant independent of surface, medium, and temperature; its value is $5.6697 \times 10^{-8}\ W/m^2.K^4$., the thermal emission from many surfaces **(gray bodies)** can be well represented by:

$$q = \sigma \varepsilon A(T_s^4 - T_{sur}^4) \tag{1.9}$$

Where ε, the emissivity of the surface, ranges (0-1). The ideal emitter or blackbody is one, All other surfaces emit some what less than one. Ts and Tsur The temperature of surface and surroundings respectively. Similarly, the thermal resistance to radiation heat transfer R_r, as:

$$R_r = \frac{T_s - T_{sur}}{\sigma \varepsilon A(T_s^4 - T_{sur}^4)}, q = \frac{T_s - T_{sur}}{R_r} \tag{1.10}$$

Table 1. Summary of heat transfer rate processes

Mode	Transfer Mechanism	Rate of heat transfer(W)	Thermal Resistance (K/W)
Conduction	Diffusion of energy due to random molecular motion	$q = -kA\frac{dT}{dx}$	$R_k = \frac{L}{kA}$
Convection	Diffusion of energy due to random molecular motion plus bulk motion	$q = hA(T_s - T_\infty)$	$R_c = \frac{1}{h\,A}$
Radiation	Energy transfer by electromagnetic Waves	$q = \sigma \varepsilon A(T_s^4 - T_{sur}^4)$	$R_r = \frac{T_s - T_{sur}}{\sigma \varepsilon A(T_s^4 - T_{sur}^4)}$

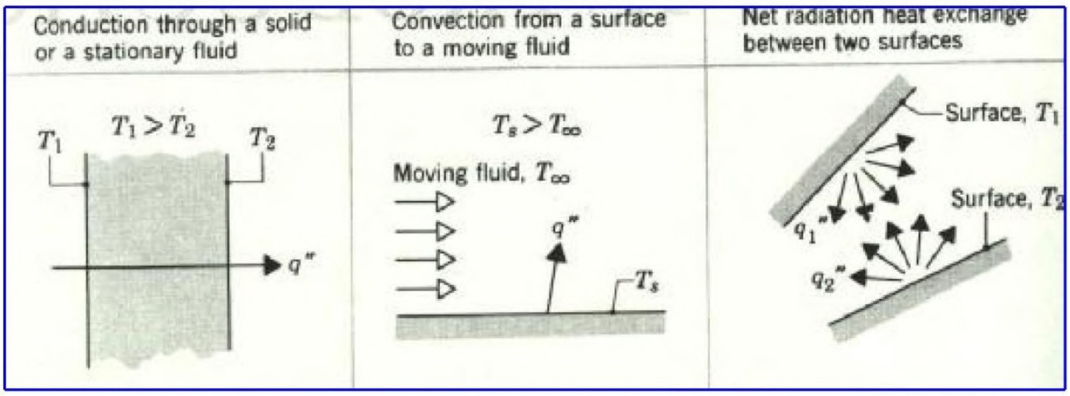


Figure (1.5) Conduction, Convection and Radiation Heat transfer Modes

The concept of thermal resistance (analogous to electrical resistance) is introduced as an aid to solving conduction heat transfer problems.

Example 1.1

Calculate the rate of heat transfer by natural convection between a shed roof of area 20 m x 20 m and ambient air, if the roof surface temperature is 27°C, the air temperature 3°C, and the average convection heat transfer coefficient 10 W/m2 K.

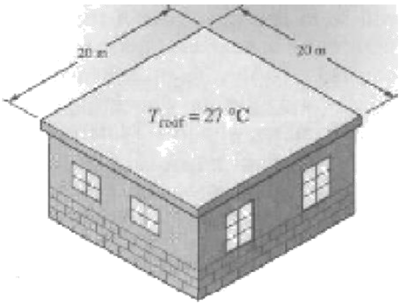


Figure 1.6 Schematic Sketch of Shed for Analysis of Roof Temperature.

Solution

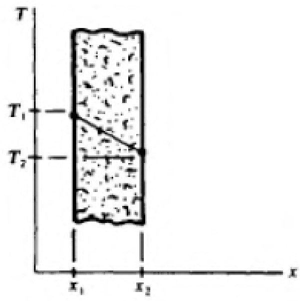
Assume that steady state exists and the direction of heat flow is from the air to the roof. The rate of heat transfer by convection from the air to the roof is then given by Eq:

$$q_c = hA(T_{air} - T_{roof}) = 10 \times 400 \times (-3 - 27) = -120{,}000\ W$$

Note we initially assumed that the heat transfer would be from the air to the roof. But since the heat flow under this assumption turns out to be a negative quantity the direction of heat flow is actually from the roof to the air.

Example 1.2

Determine the steady state rate of heat transfer per unit area through a 4.0cm thick homogeneous slab with its two faces maintained at uniform temperatures of 38oC and 21 oC. The thermal conductivity of the material is 0.19 W/m K.



$$\frac{q}{A} = -k\frac{(T2 - T1)}{(X2 - X1)} = -0.19 \times \frac{(38 - 21)}{(0.04)} = +80.75\frac{W}{m^2}$$

Example 1.3

The forced convective heat transfer coefficient for a hot fluid x1 x2 flowing over a cool surface is 225 W/m2.oC for a particular problem. The fluid temperature upstream of the cool surface is 120 oC, and the surface is held at 10 oC. Determine the heat transfer rate per unit surface area from the fluid to the surface.

q = h A (T_s-T_∞)
q/A= 225(120-10)=24750 W/m2

Example 1.4

After sunset, radiant energy can be sensed by a person standing near a brick wall. Such walls frequently have surface temperatures around 44 oC, and typical brick emissivity values are on the order of 0.92. What would be the radiant thermal flux per square foot from a brick wall at this temperature?

$$q = \sigma \varepsilon T^4 = 0.92 \times 5.6697 \times 10^{-8} \times (44 + 273) = 527 \frac{W}{m^2}.$$

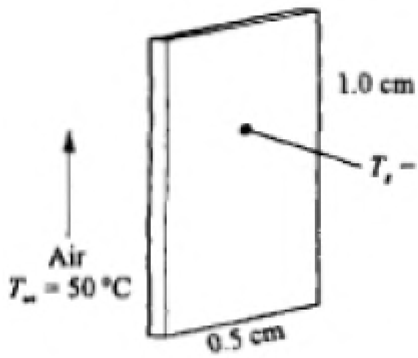
Example 1.5

In the summer, parked automobile surfaces frequently average 40-50 °C. Assuming 45°C and surface emissivity of 0.9, determine the radiant thermal flux emitted by a car roof

$$q = \sigma \varepsilon T^4 = 0.9 \times 5.6697 \times 10^{-8} \times (318) = 522 \frac{W}{m^2}.$$

Example 1.6

The air inside an electronics package housing has a temperature of 50°C. A "chip" in this housing has internal thermal power generation (heating) rate of 3 X 10 -3 W. This chip is subjected to an air flow resulting in a convective coefficient h of 9 W/m2.oC over its two main surfaces which are 0.5 cm X 1.0 cm. Determine the chip surface temperature neglecting radiation and heat transfer from the edges.



$$q = hA(T_s - T_\infty)$$

In this case q is known ***3 X 10 $^{-3}$ W,*** and this is from two surfaces having total area $A = 2 \times \frac{0.5}{100} \times \frac{1}{100} = 10^{-4}\, m^2$

$$T_s = T_\infty + \frac{q}{hA} = 50 + \frac{\mathbf{0.003}}{9 \times 10^{-4}} = 53.33\ ^{o}\mathrm{C}$$

Example 1.7

Calculate the thermal resistance and the rate of heat transfer through a pane of window glass (k = 0.78 W/m K) 1 m high, 0.5 m wide, and 0.5 cm thick, if the outer-surface temperature is 24°C and the inner-surface temperature is 24.5°C

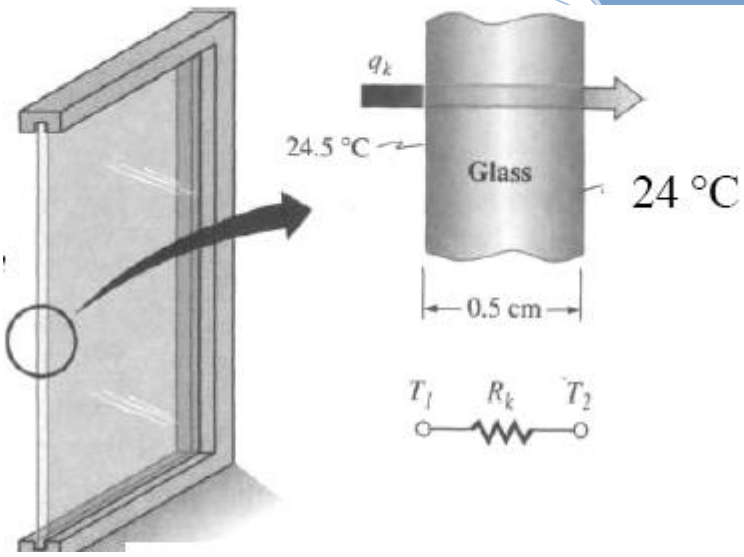
Solution

Assume that steady state exists and that the temperature is uniform over the inner and outer surfaces. The thermal resistance to conduction Rk is from Eq.

$$R_k = \frac{L}{kA} = \frac{0.005}{0.78 \times 1 \times 0.5} = 0.0128 \frac{K}{W}$$

The rate of heat loss from the interior to the exterior surface is:

$$q = \frac{\Delta T}{R_k} = \frac{24.5 - 24}{0.0128} = 39.1\, W$$



Example 1.8

A long, cylindrical electrically heated rod, 2 cm in diameter, is installed in a vacuum furnace as shown in Fig.1.8. The surface of the heating rod has an emissivity of 0.9 and is maintained at 1000 K, while the interior walls of the furnace are black and are at 800 K. Calculate the net rate at which heat is lost from the rod per unit length and the radiation heat transfer coefficient.

Figure 1.7 Schematic Diagram of Vacuum Furnace with Heating Rod

Solution

Assume that steady state has been reached.

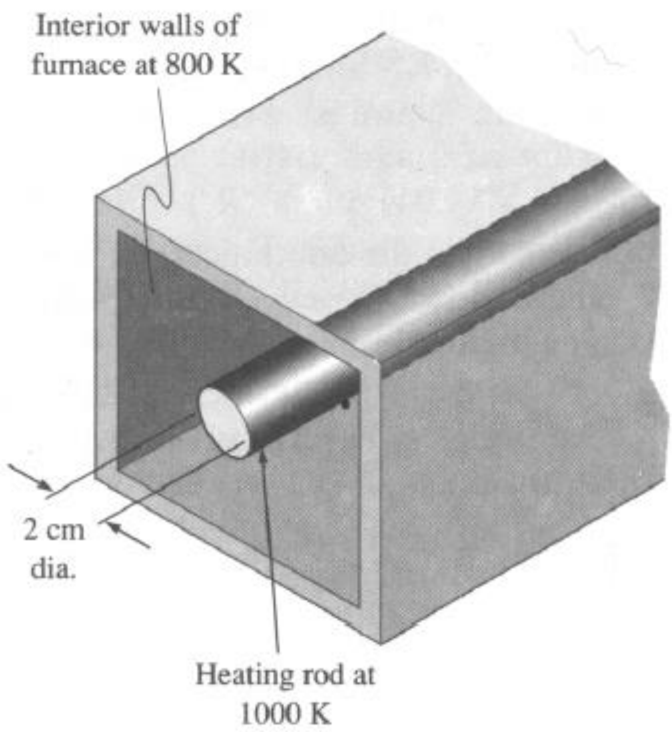
Moreover, note that since the walls of the

furnace completely enclose the heating rod, all the radiant energy emitted by the surface of the rod is intercepted by the furnace walls. Thus, for a black enclosure, Eq. (1.9) applies and the net heat loss from the rod of surface A1 is

$$q = \sigma \varepsilon A(T_s^4 - T_{sur}^4)$$
$$= \sigma \varepsilon \pi D L(T_s^4 - T_{sur}^4)$$

$$= 5.67 \times 10^{-8} \times 0.9 \times \pi \times 0.02 \times 1 \times (1000^4 - 800^4)$$

=1893 W



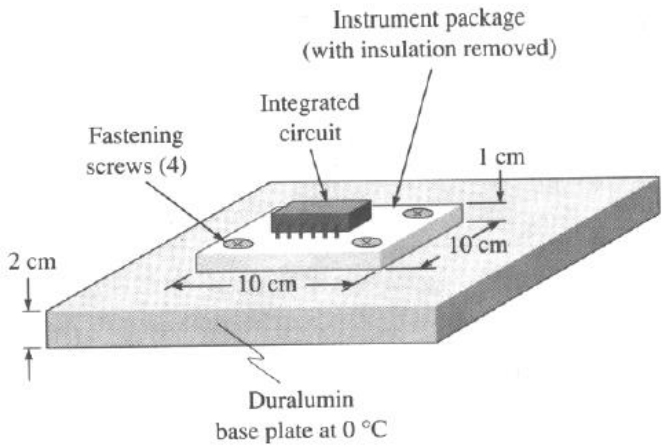
Note that in order for steady state to exist, the heating rod must dissipate electrical energy at the rate of 1893 W and the rate of heat loss through the furnace walls must equal the rate of electric input to the system, that is, to the rod.

$$h_r = \frac{\sigma\varepsilon(T_1^4 - T_2^4)}{T_1 - T_2} = 151\,\frac{W}{m^2.K}$$

Example 1.9

An instrument used to study the Ozone depletion near the poles is placed on a large 2-cm-thick duralumin plate. To simplify this analysis the instrument can be thought of as a stainless steel plate 1 cm tall with a 10 cm x 10 cm square base, as shown in Fig. 1.6. The interface roughness of the steel and the duralumin is between 20 and 30 rms (μm) the contact resistance is 0.05 k/w. Four screws at the corners. The top and sides of the instrument are thermally insulated. An integrated circuit placed between the insulation and the upper surface of the stainless steel plate generates heat. If this heat is to be transferred to the lower surface of the duralumin, estimated to be at a temperature of 0°C, determine the maximum allowable dissipation rate from the circuit if its temperature is not to exceed 40°C.

***Figure 1.8** Schematic Sketch of Instrument for Ozone Measurement.*



Solution

Since the top and the sides of the instrument are insulated, all the heat generated by the circuit must flow downward. The thermal circuit will have three resistances the stainless steel, the contact, and the duralumin. Using thermal conductivities kss = 14.4 W/m K, kM = 164 W/m K the thermal resistances of the metal plates are calculated from Equations:

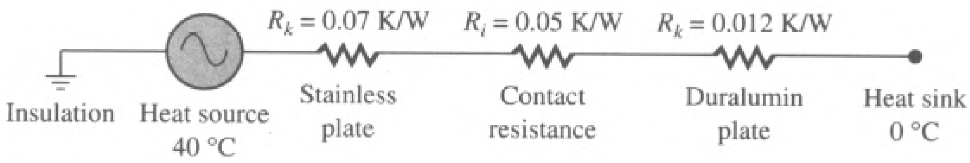
Stainless:

$$R_k = \frac{L_{ss}}{Ak_{ss}} = \frac{0.01 \text{ m}}{0.01 \text{ m}^2 \times 14.4 \text{ W/m K}} = 0.07\,\frac{\text{K}}{\text{W}}$$

Duralumin:

$$R_k = \frac{L_{Al}}{Ak_{Al}} = \frac{0.02 \text{ m}}{0.01 \text{ m}^2 \times 164 \text{ W/m K}} = 0.012\,\frac{\text{K}}{\text{W}}$$

THERMAL CIRCUIT



The total resistance is 0.132 K/W, and the maximum allowable rate of heat dissipation is therefore

$$q_{max} = \frac{\Delta T}{R_{total}} = \frac{40 \text{ K}}{0.132 \text{ K/W}} = 303 \text{ W}$$

1.6 The Energy Balance

In this special case the control surface includes no mass or volume and appears as shown in Figure 1.8.Accordingly, the generation and storage terms of the Energy expression,

Ein –Eout -Est + Eg= 0

Consequently, there can be no generation and storage. The conservation requirement then becomes

Ein –Eout = 0

In Figure 1.8 three heat transfer terms are shown for the control surface. On a unit area basis they are conduction from the medium to the control surface q"cond convection from the surface to a fluid q"conv, and net radiation exchange from the surface to the surroundings q"rad. The energy balance then takes the Form and we can express each of the terms according to the appropriate rate equations.

$$q''_{cond} = q''_{conv} + q''_{rad}$$

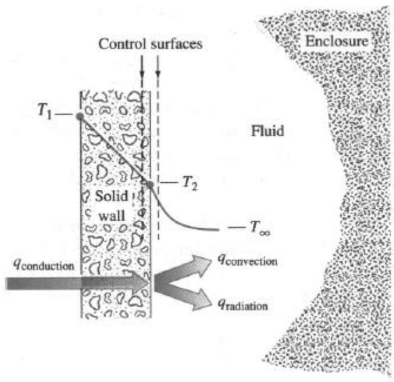


FIGURE 1.34 Application of Conservation of Energy Law at the Surface of a System.

1.7 Combined heat transfer systems

Summarizes the basic relations for the rate equation of each of the three basic heat transfer mechanisms to aid in setting up the thermal circuits for solving combined heat transfer problems.

1.7.1 Plane Walls in Series

In Fig. 1.15 for a three-layer system, the temperature gradients in the layers are different. The rate of heat conduction through each layer is qk, and from Eq. (1.1) we get

$$q_k = \left(\frac{kA}{L}\right)_A (T_1 - T_2) = \left(\frac{kA}{L}\right)_B (T_2 - T_3) = \left(\frac{kA}{L}\right)_C (T_3 - T_4)$$

Eliminating the intermediate temperatures T2 and T3 in Eq. qk can be expressed in the form

$$q_k = \frac{T_1 - T_4}{(L/kA)_A + (L/kA)_B + (L/kA)_C}$$

Similarly, for N layers in series we have

$$q_k = \frac{T_1 - T_{N+1}}{\sum_{n=1}^{n=N} (L/kA)_n}$$

$$q_k = \frac{T_1 - T_{N+1}}{\sum_{n=1}^{n=N} R_{k,n}} = \frac{\Delta T}{\sum_{n=1}^{n=N} R_{k,n}}$$

where T1 is the outer-surface temperature of layer 1 and TN+1 is the outer-surface temperature of layer N. and ΔT is the overall temperature difference, often called the temperature potential.

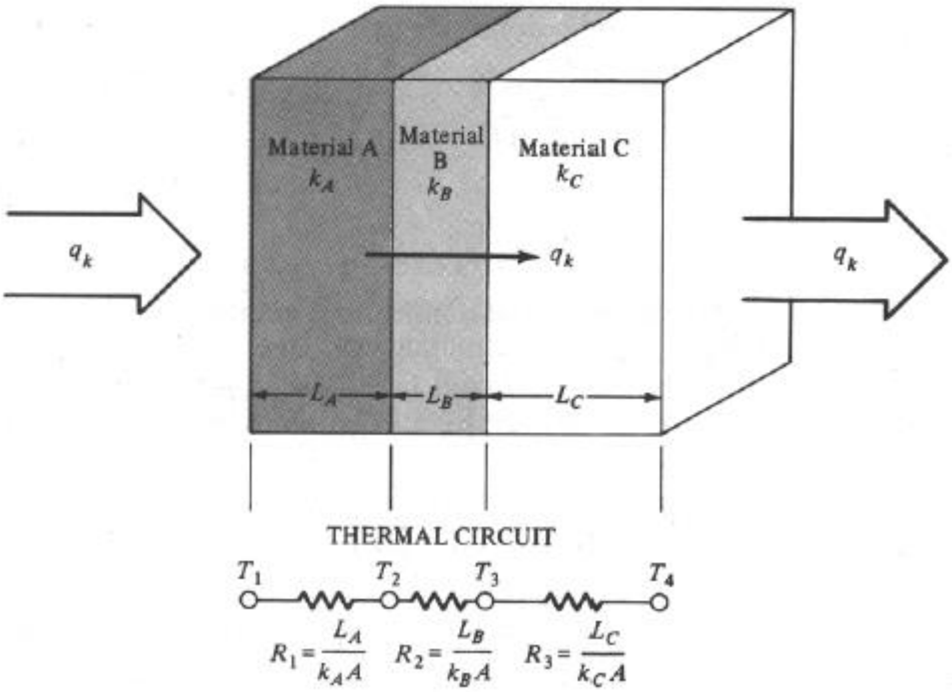


***Figure 1.9** Conduction Through a Three-Layer System in Series.*

Example 1. 6

Calculate the rate of heat loss from a furnace wall per unit area. The wall is constructed from an inner layer of 0.5 cm thick steel (k : 40 W/m K) and an outer layer of 10 cm zirconium brick (k = 2.5 W/m K) as shown in Fig. The inner-surface temperature is 900 K and the outside surface temperature is 460 K. What is the temperature at the interface?

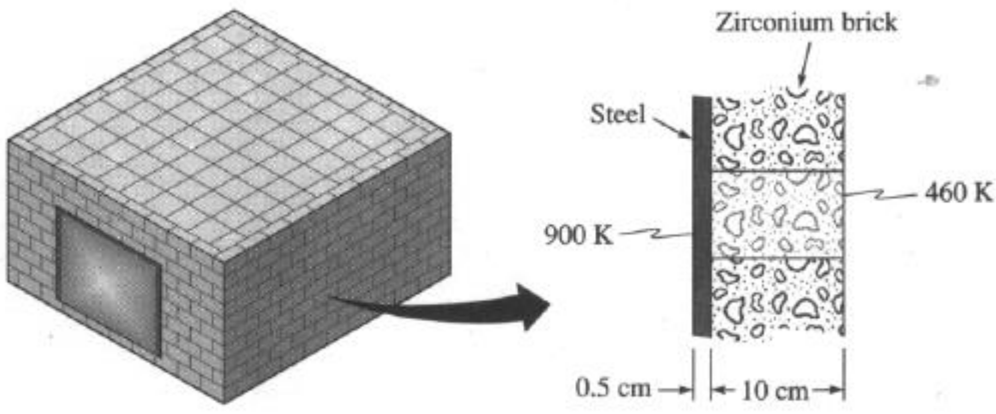


Figure 1.10 *Schematic Diagram of Furnace Wall.*

Solution

Assumptions:

- Assume that steady state exists,
- Neglect effects at the corners and edges of the wall,
- The surface temperatures are uniform.

The rate of heat loss per unit area can be calculated from Eq:

$$\frac{q_k}{A} = \frac{(900 - 460)\text{K}}{(0.005\ \text{m})/(40\ \text{W/m K}) + (0.1\ \text{m})/(2.5\ \text{W/m K})}$$

$$= \frac{440\ \text{K}}{(0.000125 + 0.04)(\text{m}^2\ \text{K/W})} = 10{,}965\ \text{W/m}^2$$

The interface temperature T2 is obtained from $\frac{q}{A} = \frac{T_1 - T_2}{R_1}$

Solving for T2 gives

$$T_2 = T_1 - \frac{q_k}{A_1}\frac{L_1}{k_1} = 900\ \text{K} - \left(10{,}965\ \frac{\text{W}}{\text{m}^2}\right)\left(0.000125\ \frac{\text{m}^2\ \text{K}}{\text{W}}\right)$$

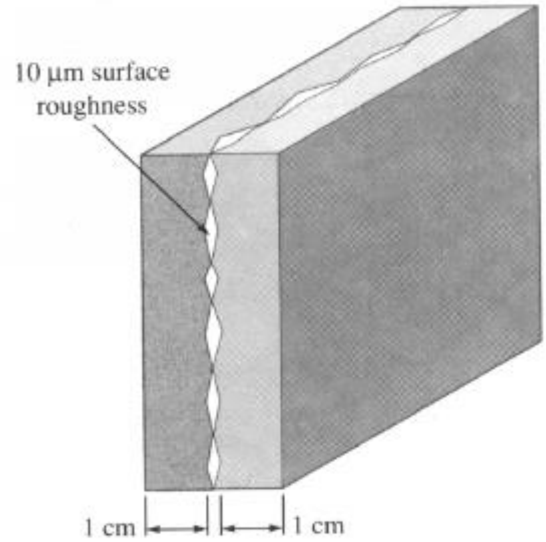
$$= 898.6\ \text{K}$$

Note that the temperature drop across the steel interior wall is only 1.4 K because the thermal resistance of the wall is small compared to the resistance of the brick.

Example 1. 7

Two large aluminum plates (k = 240 W/m K), each 1 cm thick, with 10 µm surface roughness the contact resistance Ri = 2.75 x 10-4 m2 K/W. The temperatures at the outside surfaces are 395°C and 405°C. Calculate (a) the heat flux (b) the temperature drop due to the contact resistance.

Figure 1.11 *Schematic Diagram of Interface Between Plates.*



Solution

(a) The rate of heat flow per unit area, q'' through the sandwich wall is

$$q'' = \frac{T_{s1} - T_{s3}}{R_1 + R_2 + R_3} = \frac{\Delta T}{(L/k)_1 + R_i + (L/k)_2}$$

The two resistances is equal to

(L/k) = (0.01 m)/(240 W/m.K) = 4.17 x 10^{-5} m^2 K/W

Hence, the heat flux is

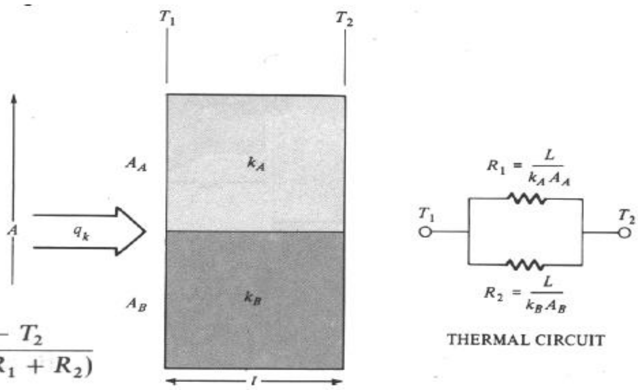
$$q'' = \frac{(405 - 395)°C}{(4.17 \times 10^{-5} + 2.75 \times 10^{-4} + 4.17 \times 10^{-5})\ m^2\ K/W}$$
$$= 2.79 \times 10^4\ W/m^2\ K$$

(b) The temperature drop in each section. The fraction of the contact resistance is

$$R_i \Big/ \sum_{n=1}^{3} R_n = 2.75/3.584 = 0.767$$

Hence 7.67°C of the total temperature drop of 10°C is the result of the contact resistance.

1.7.2 Plane Walls in Parallel



$$q_k = q_1 + q_2$$
$$= \frac{T_1 - T_2}{(L/kA)_A} + \frac{T_1 - T_2}{(L/kA)_B} = \frac{T_1 - T_2}{R_1 R_2/(R_1 + R_2)}$$

Conduction can occur in a section with two different materials in parallel between the same potential. Fig. 1.18 shows a slab with two different materials of areas AA and AB in parallel. If the temperatures over the left and right faces are uniform at T1 and T2, the total rate of heat flow is the sum of the flows through AA and AB:
Note that the total heat transfer area is the sum of AA and AB and that the total resistance equals the product of the individual resistances divided by their sum, as in any parallel circuit. A more complex application of the thermal network approach is illustrated in Fig. 1.19, where heat is transferred through a composite structure involving thermal resistances in series and in parallel. For this system the resistance of the middle layer, R2 becomes and the rate of heat flow is

$$R_2 = \frac{R_B R_C}{R_B + R_C}$$

$$q_k = \frac{\Delta T_{overall}}{\sum_{n=1}^{n=3} R_n}$$

Where N is number of layers in series
R_n : Thermal resistance of n^{th} layer
$\Delta T_{overall}$: temperature difference across two outer surfaces

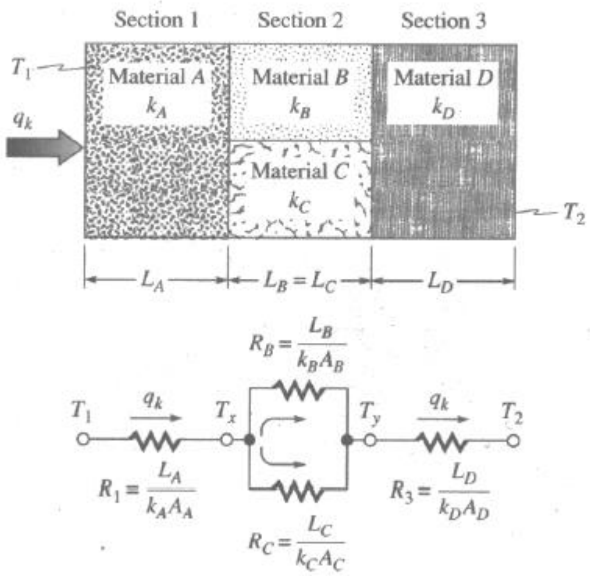
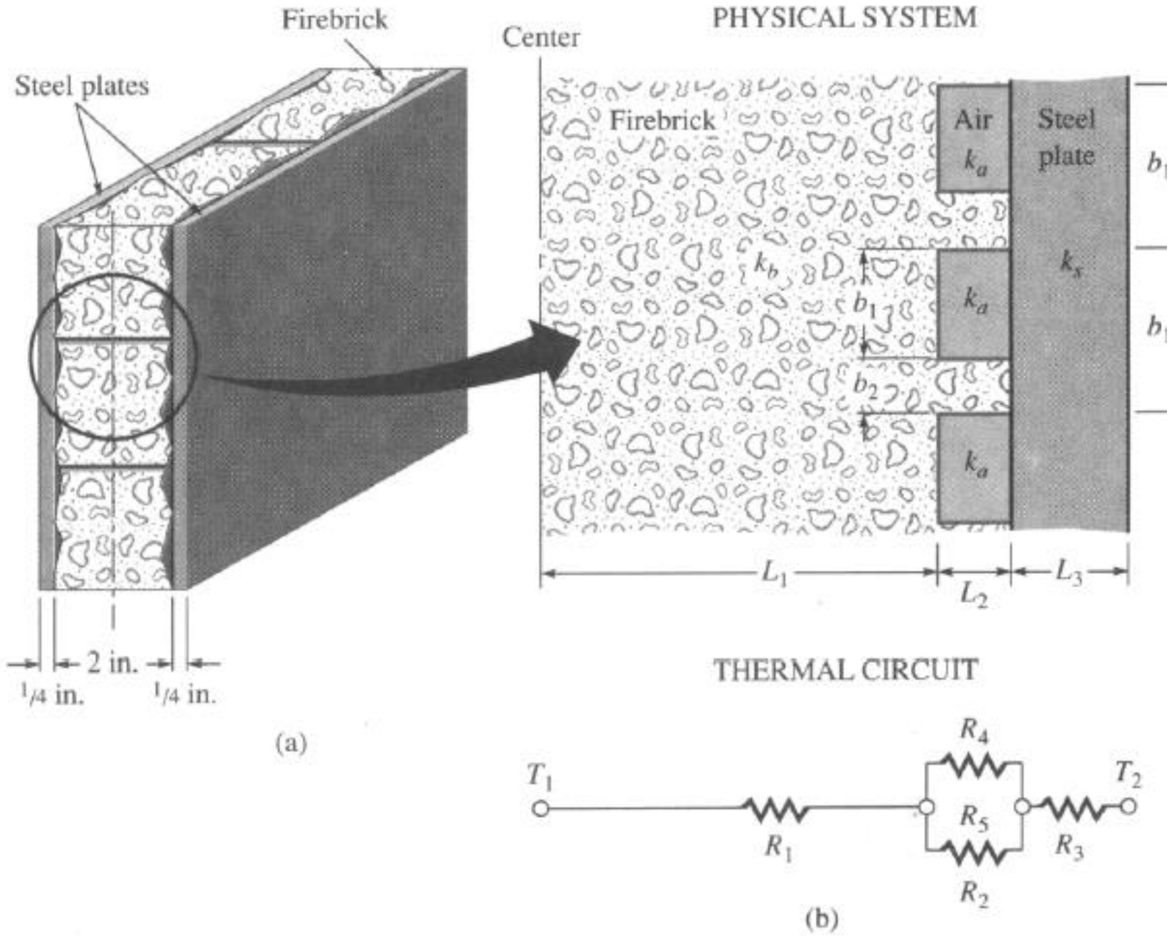


***Figure 1.13** Conduction Through a Wall Consisting of Series and Parallel Thermal Paths.*

<u>Example 1. 8</u>

A layer of 2 in thick firebrick (kb = 1.0 Btu/hr ft °F) is placed between two ¼ in.-thick steel plates (ks = 30 Btu/hr ft °F). The faces of the brick adjacent to the plates are rough, having solid-to-solid contact over only 30 % of the total area, with the average height of asperities being L2=1/32 in. If the surface temperatures of the steel plates are 200° and 800°F, respectively. The conductivity of air ka is 0.02 Btu/hr ft °F, determine the rate of heat flow per unit area.

Figure 1.14 *Thermal Circuit for the Parallel-Series Composite Wall.* $L_1 = 1$ *in.;*$L_2 = 1/32$ *in.;* $L_3 = 1/4$ *in.;* T_1 *is at the center.*



Solution

The overall unit conductance for half the composite wall is then, from an inspection of the thermal circuit

$$K_k = \frac{1}{R_1 + [R_4R_5/(R_4 + R_5)] + R_3}$$

$$R_3 = \frac{L_3}{k_s} = \frac{(1/4 \text{ in.})}{(12 \text{ in./ft})(30 \text{ Btu/hr °F ft})} = 0.694 \times 10^{-3} \text{ (Btu/hr ft}^2 \text{ °F)}^{-1}$$

$$R_4 = \frac{L_2}{0.3k_b} = \frac{(1/32 \text{ in.})}{(12 \text{ in./ft})(0.3)(1 \text{ Btu/hr °F ft})} = 8.68 \times 10^{-3} \text{ (Btu/hr ft}^2 \text{ °F)}^{-1}$$

Since the air is trapped in very small compartments, the effects of convection are small and it will be assumed that heat flows through the air by conduction. At a temperature of 300°F. Then R5 the thermal resistance of the air trapped between the asperities, is, on the basis of a unit area, equal to The factors 0.3 and 0.7 in R4 and R5, respectively, represent the percent of the total area for the two separate heat flow paths. The total thermal resistance for the two paths, R4 and R5 in parallel, is

$$R_5 = \frac{L_2}{0.7k_a} = \frac{(1/32 \text{ in.})}{(12 \text{ in./ft})(0.7)(0.02 \text{ Btu/hr °F ft})} = 186 \times 10^{-3} \text{ (Btu/hr ft}^2 \text{ °F)}^{-1}$$

$$R_2 = \frac{R_4R_5}{R_4 + R_5} = \frac{(8.7)(187) \times 10^{-6}}{(8.7 + 187) \times 10^{-3}} = 8.29 \times 10^{-3} \text{ (Btu/hr ft}^2 \text{ °F)}^{-1}$$

$$R_1 = \frac{L_1}{k_b} = \frac{(1 \text{ in.})}{(12 \text{ in./ft})(1 \text{ Btu/hr °F ft})} = 83.3 \times 10^{-3} \text{ (Btu/hr ft}^2 \text{ °F)}^{-1}$$

The thermal resistance of half of the solid brick, Rl is and the overall unit conductance is

$$K_k = \frac{1/2 \times 10^3}{83.3 + 8.3 + 0.69} = 5.4 \text{ Btu/hr ft}^2 \text{ °F}$$

$$\frac{q}{A} = K_k \Delta T = \left(5.4 \frac{\text{Btu}}{\text{hr ft}^2 \text{ °F}}\right)(800 - 200)(\text{°F}) = 3240 \text{ Btu/hr ft}^2$$

Inspection of the values for the various thermal resistances shows that the steel offers a negligible resistance

1.5.2 Convection and Conduction in Series

Figure (1.15) shows a situation in which heat is transferred between two fluids separated by a wall, the rate of heat transfer from the hot fluid at temperature Thot to the cold fluid at temperature T_{cold} is

$$q = \frac{T_{hot} - T_{cold}}{\sum_{n=1}^{n=3} R_i} = \frac{\Delta T}{R_1 + R_2 + R_3}$$

where $R_1 = \frac{1}{(\bar{h}_c A)_{hot}}$

$R_2 = \frac{L}{kA}$

$R_3 = \frac{1}{(\bar{h}_c A)_{cold}}$

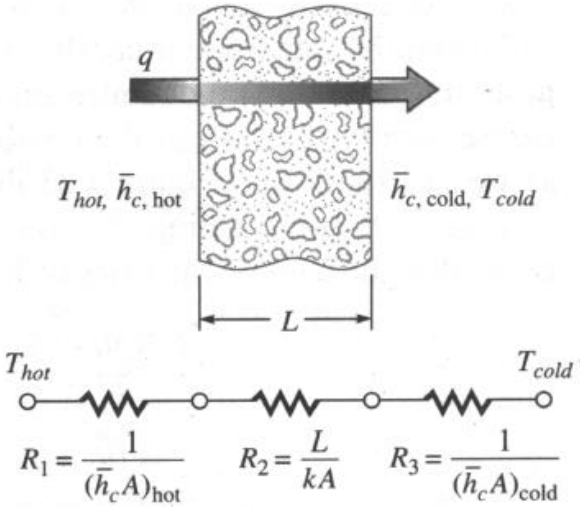


Figure 1.15 *Thermal Circuit with Conduction and Convection in Series.*

Example 1.8

A 0.1 m thick brick wall (k = 0.7 W/m K) is exposed to a cold wind at 270 K through a convection heat transfer coefficient of 40 W/m2 K. On the other side is air at 330 K, with a natural convection heat transfer coefficient of 10 W/m2 K. Calculate the rate of heat transfer per unit area.

Solution

The three resistances are the rate of heat transfer per unit area is :

$$R_1 = \frac{1}{\bar{h}_{c,\,hot} A} = \frac{1}{(10\ \text{W/m}^2\ \text{K})(1\ \text{m}^2)} = 0.10\ \text{K/W}$$

$$R_2 = \frac{L}{kA} = \frac{(0.1\ \text{m})}{(0.7\ \text{W/m K})(1\ \text{m}^2)} = 0.143\ \text{K/W}$$

$$R_3 = \frac{1}{\bar{h}_{c,\,cold} A} = \frac{1}{(40\ \text{W/m}^2\ \text{K})(1\ \text{m}^2)} = 0.025\ \text{K/W}$$

and from Eq. (1.29) the rate of heat transfer per unit area is

$$\frac{q}{A} = \frac{\Delta T}{R_1 + R_2 + R_3} = \frac{(330 - 270)\ \text{K}}{(0.10 + 0.143 + 0.025)\ \text{K/W}} = 223.9\ \text{W}$$

1.5.3 Convection and Radiation in Parallel

In many engineering problems a surface loses or receives thermal energy by convection and radiation simultaneously. Figure 1.23 illustrates the co current heat transfer from a surface to its surroundings by convection and radiation.

$$q = q_c + q_r$$
$$q = h_c A (T_1 - T_2) + h_r A (T_1 - T_2)$$
$$q = (h_c + h_r) A (T_1 - T_2)$$

where hc is the average convection heat transfer coefficient between area A1 and the surroundings air at T2, the radiation heat transfer coefficient

$$h_r = \frac{\epsilon_1 \sigma (T_1^4 - T_2^4)}{T_1 - T_2}$$

The combined heat transfer coefficient is h = hc + hr

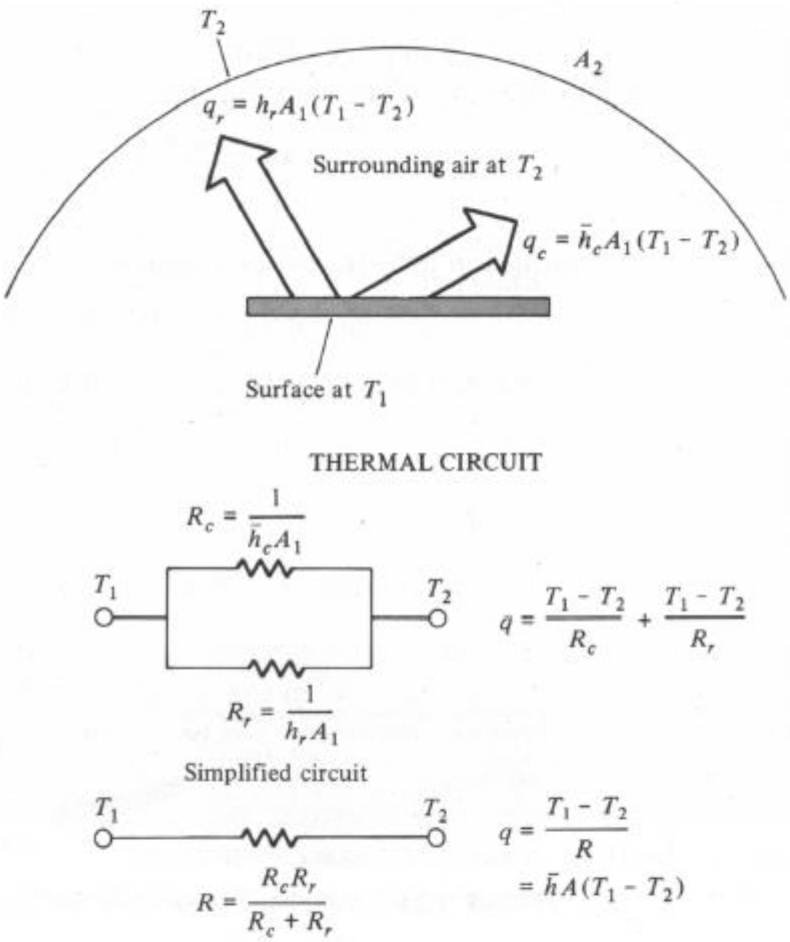


FIGURE 1.23 Thermal Circuit with Convection and Radiation Acting in Parallel.

Example 1.5

Air at 20C blow over a hot plate 50 x 75 cm and thick 2 cm maintained at 250 oC. The convection heat transfer coefficient is 25 W/m2 C. calculate the inside plate temperature if it is mode of carbon steel and that 300 W is lost from the plate surface by radiation. Where thermal conductivity is 43 w/m C.

Solution

$$q_{conv} = h\,A(T_s - T_\infty)$$

$$q_{conv} = 25\ (0.5 * 0.75)\ (250 - 20)$$

$$q_{conv} = 2.156\ KW$$

$$q_{cond} = q_{conv} + q_{rad}$$

$$q_{cond} = 2.156\ + 0.\ 3 = 2.456\ kW$$

$$q_{cond} = kA\frac{T_1 - T_2}{L}$$

$$2.456 = 43\ (0.5 \times 0.75)\frac{T_1 - 250}{0.02}$$

$$T_1 = 253.05\ ^oC$$

Example 1.9

A 0.5 m diameter pipe (ε = 0.9) carrying steam has a surface temperature of 500 K. The pipe is located in a room at 300 K, and the convection heat transfer coefficient between the pipe surface and the air in the room is 20 W/m2 K. Calculate the combined heat transfer coefficient and the rate of heat loss per meter of pipe length.

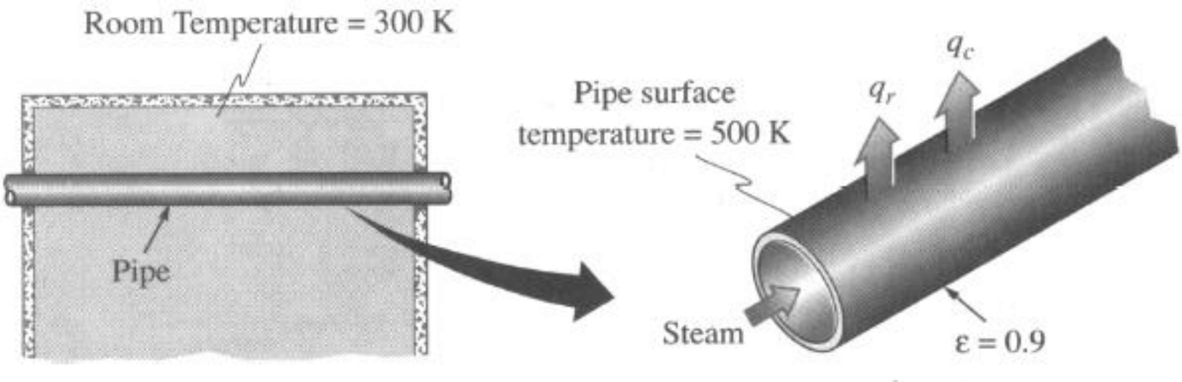


Figure 1.17 *Schematic Diagram of Steam Pipe*

Solution

$$h_r = \frac{\epsilon_1 \sigma (T_1^4 - T_2^4)}{T_1 - T_2}$$

$$h_r = 13.9\ W/m^2 K$$

The combined heat transfer coefficient is h = hc + hr = 20 + 13.9 = 33.9 W/m2 K
and the rate of heat loss per meter is

$$q = \pi D L \bar{h}(T_{pipe} - T_{air}) = (\pi)(0.5\ m)(1\ m)(33.9\ W/m^2\ K)(200\ K) = 10{,}650\ W$$

1.5.4 Overall Heat Transfer Coefficient

We noted previously that a common heat transfer problem is to determine the rate of heat flow between two fluids, gaseous or liquid, separated by a wall. If the wall is plane and heat is transferred only by convection on both sides, the rate of heat transfer in terms of the two fluid temperatures is given by:

$$q = \frac{T_{hot} - T_{cold}}{(1/h_c A)_{hot} + (L/kA) + (1/h_c A)_{cold}} = \frac{\Delta T}{R_1 + R_2 + R_3}$$

the rate of heat flow is expressed only in terms of an overall temperature potential and the heat transfer characteristics of individual sections in the heat flow path., the overall transmittance, or the overall coefficient of heat transfer U Writing Eq. (1.29) in terms of an overall coefficient gives

$$q = UA\,\Delta T_{total}$$

where

$$UA = \frac{1}{R_1 + R_2 + R_3} = \frac{1}{R_{total}}$$

An overall heat transfer coefficient U can be based on any chosen area

Example 1.10

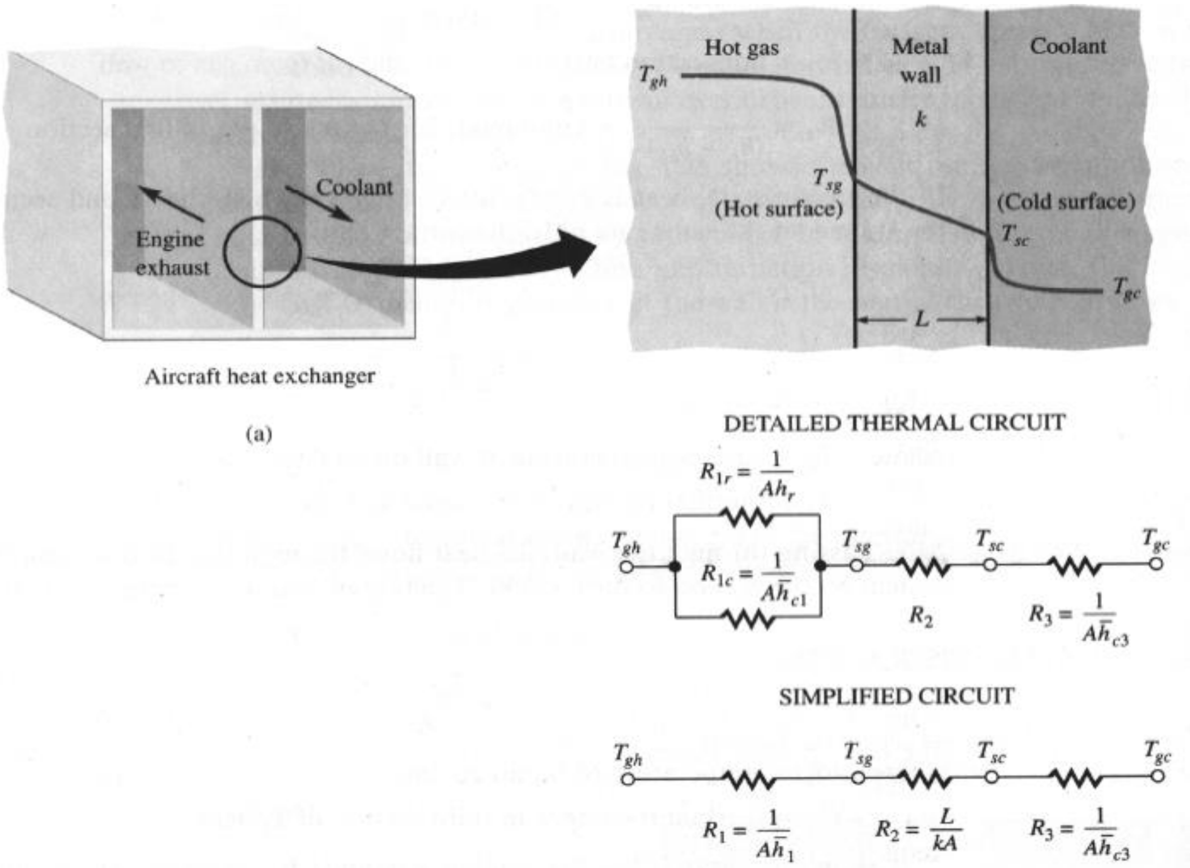


Figure 1.18 Physical System and Thermal Circuit.

In the design of a heat exchanger for aircraft application, the maximum wall temperature in steady state is not to exceed 800 k. For the conditions tabulated above, determine the maximum permissible unit thermal resistance per square meter of the metal wall that separates the hot gas Tgh = 1300 K from the cold gas Tgc = 300 K. Combined heat transfer coefficient on hot side h1= 200 W/m2 K Combined heat transfer coefficient on cold side h3 = 400 W/m2 K

Solution

In the steady state we can write

$$\frac{q}{A} = \frac{T_{gh} - T_{sg}}{R_1} = \frac{T_{gh} - T_{gc}}{R_1 + R_2 + R_3}$$

$$\frac{1300 - 800}{1/200} = \frac{1300 - 300}{1/200 + R_2 + 1/400}$$

$$\frac{1300 - 800}{0.005} = \frac{1300 - 300}{R_2 + 0.0075}$$

Solving for R2 gives R2 = 0.0025 m^2 K/W

Example 1.11

The door for an industrial gas furnace is 2 m x 4 m in surface area and is to be insulated to reduce heat loss to no more than 1200 W/m2. The interior surface is a 3/8-in.-thick Inconel 600 sheet (K= 25 W/m K), and the outer surface is a l/4 in.-thick sheet of Stainless steel 316. Between these metal sheets a suitable thickness of insulators material is to be placed.

The effective gas temperature inside the furnace is 1200°C, and the overall heat transfer coefficient between the gas and the door is Ui = 20 W/m2 K. The heat transfer coefficient between the outer surface of the door and the surroundings at 20°C is hc= 5 W/m2 K. calculate the thickness of insulated should be use

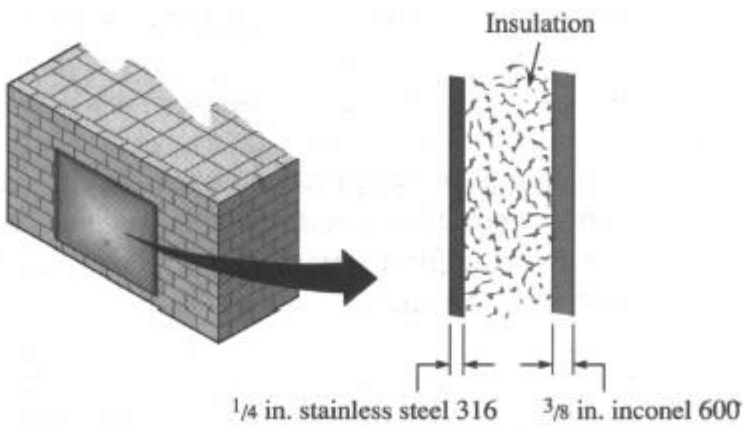


Figure 1.19 *Cross section of composite wall of gas furnace door*

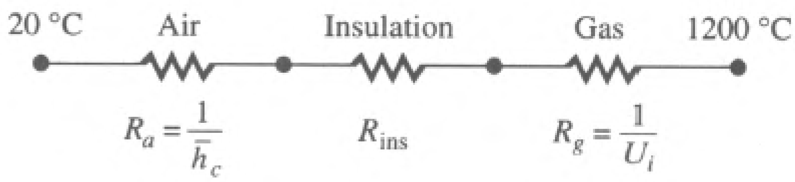
Solution

The thermal resistance of the two metal sheets are approximately 25 W/m K the thermal resistance of the two metal sheets are approximately:

$$L_1+L_2=0.25+0.375=0.625\ in$$

$$R = L/k \sim \frac{0.625\ \text{in.}}{25\ \text{W/m K}} \times \frac{1\ \text{m}}{39.4\ \text{in.}} \sim 6 \times 10^{-4}\ \text{m}^2\ \text{K/W}$$

These resistances are negligible compared to the other three resistances shown in the simplified thermal circuit below;



The temperature drop between the gas and the interior surface of the door at the

$$Q=AU\Delta T$$

$$\Delta T = \frac{q/A}{U} = \frac{1200\ \text{W/m}^2}{20\ \text{W/m}^2\ \text{K}} = 60\ \text{K}$$

Specified heat flux is:

Hence, the temperature of the In cornel will be about (1200-60)=1140°C. This is acceptable since no appreciable load is applied. The temperature drop at the outer surface is

$$\Delta T = \frac{1200(\text{W/m}^2)}{5(\text{W/m}^2\ \text{K})} = 240°\text{C}$$

$$L = \frac{k\,\Delta T}{q/A} = \frac{0.27\ \text{X}\ (1140\text{-}240\ )}{1200\ \text{W/m}^2} = 0.2\ \text{m}$$

The insulation thickness for k = 0.27 W/m K is:

Chapter Two
Heat Conduction

2.1 Introduction

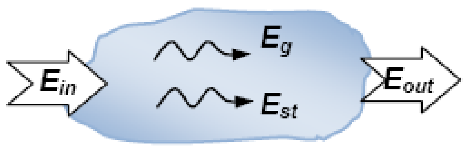
A major objective in a conduction analysis is to determine the temperature field in a medium (***Temperature Distribution***), which represents how temperature varies with position in the medium. knowledge of the temperature distribution:

• Determination of thermal stresses, It could be used to ascertain structural integrity through
• To determine the optimize thickness of an insulating material
• To determine the compatibility of special coatings or adhesives used with the material.

2.2 Conservation of Energy

Applying energy conservation to the control volume. At an instant, these include the rate at which thermal and mechanical energy enter E_{in} and leave E_{out}. through the control surface, Is additional to the rate of change of energy generation E_g and stored E_{st}. A general form of the energy conservation requirement may then be expressed on rate basis as:

$$E_{in} + E_g - E_{out} = E_{st} \qquad 2.1$$



2.3 The Conduction Equation of Rectangular Coordinate

Consider the energy processes that are relevant to this control volume. If there are temperature gradients, conduction heat transfer will occur across each of the control surfaces at the x, y, and z coordinate. The conduction heat rates at the opposite surfaces can then be expressed as a Taylor series expansion where, neglecting higher order terms,

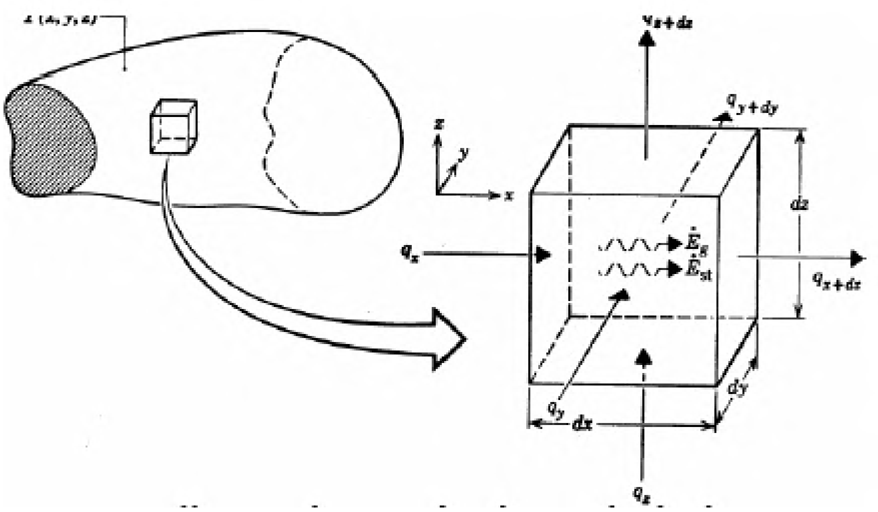
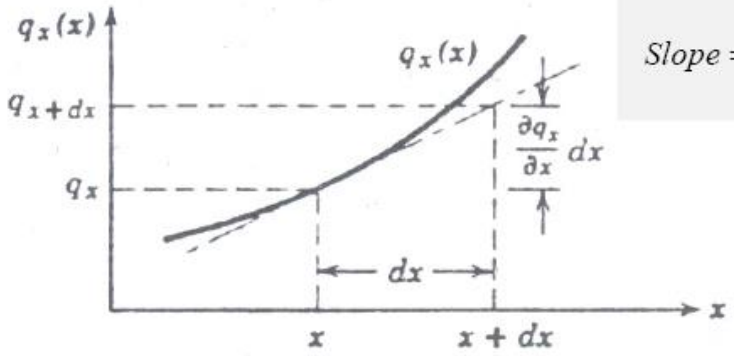


Figure 2.1 *Differential control volume, dx dy dz.*

$$q_{x+dx} = q_x + \frac{dq_x}{dx}dx$$
$$q_{y+dy} = q_y + \frac{dq_y}{dy}dy$$
$$q_{z+dz} = q_z + \frac{dq_z}{dz}dz$$



$$Slope = \frac{dq_x}{dx}$$

The rate of change of energy generation E_g and stored E_{st}

$$E_g = \dot{q}V = \dot{q}dxdydz$$

$$E_{st} = mC_p\frac{dT}{dt} = \rho VC_p\frac{dT}{dt} = \rho(dxdydz)C_p\frac{dT}{dt}$$

where $\dot{q}$ is the rate at which energy is generated per unit volume (W/m3) and to express conservation of energy using the foregoing rate equation

$$E_{in} + E_g - E_{out} = E_{st}$$

and, substituting equations, we obtain

$$\cancel{q_x} + \cancel{q_y} + \cancel{q_z} - (\cancel{q_x} + \frac{dq_x}{dx}dx) - (\cancel{q_y} + \frac{dq_y}{dy}dy) - (\cancel{q_z} + \frac{dq_z}{dz}dz) + \dot{q}dxdydz = \rho dxdydzC_p\frac{dT}{dt} \qquad \mathbf{2.2}$$

The conduction heat rates may be evaluated from Fourier's law,

$$\left.\begin{aligned} q_x &= -kA\frac{dT}{dx} = -kdzdy\frac{dT}{dx} \\ q_y &= -kA\frac{dT}{dy} = -kdzdx\frac{dT}{dy} \\ q_z &= -kA\frac{dT}{dz} = -kdzdx\frac{dT}{dz} \end{aligned}\right\} \qquad (2.3)$$

Substituting Equations 2.3 into Equation 2.2 and dividing out the dimensions of the control volume (*dx dy dz*), we obtain

$$\frac{\partial}{\partial x}(k\frac{\partial T}{\partial x}) + \frac{\partial}{\partial y}(k\frac{\partial T}{\partial y}) + \frac{\partial}{\partial z}(k\frac{\partial T}{\partial z}) + \dot{q} = \rho C_p\frac{\partial T}{\partial t} \qquad (2.4)$$

It is often possible to work with simplified versions of ***Heat Equation*** *(k=Const) is*

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad (2.5)$$

where $\alpha = k/\rho C_p$ (m2/s) is the thermal diffusivity.

2.3.1 One Dimension Steady State Conduction

A plane wall separates two fluids of different temperatures. Heat transfer occurs by convection from the hot fluid at $T_{\infty,1}$ to one surface of the wall at T_{s1}, by conduction through the wall, and by convection from the other surface of the wall at T_{s2} to the cold fluid at $T_{\infty,2}$
If the heat transfer one dimensional and under steady-state conditions (there can be no change in the amount of energy storage and generation; hence ***Heat Equation*** reduces to

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)=0 \qquad (2.4)$$

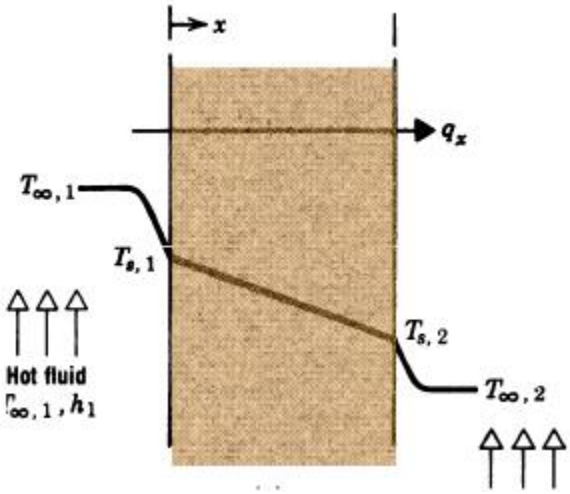


Figure 2.2 *Heat transfer through a plane wall.*

If the thermal conductivity is assumed to be constant ($k=Const$), the equation may be integrated twice to obtain the general solution $T(x)=C_1 x+C_2$
To obtain the constants of integration, C_1 and C_2 boundary conditions must be introduced. Applying the conditions

B.C.1	$x = 0$	*at*	$T=T_{s1}$	-----------→	$C_2=T_{s1}$
B.C.2	$x = L$	*at*	$T=T_{s2}$		

$$T_{s2}=C_1L+C_2=C_1L+T_{s1} \quad \text{-----------}\rightarrow \quad C_1=\frac{T_{s2}-T_{s1}}{L}$$

Substituting into the general solution, the ***Temperature Distribution*** is then

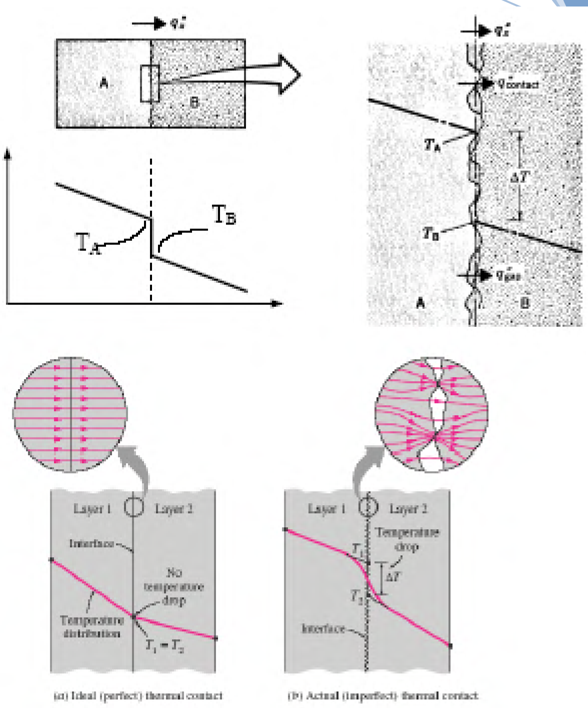
$$T(x)=(T_{S2}-T_{S1})\frac{x}{L}+T_{s1} \qquad \textit{Linearly equation.} \qquad (2.7)$$

2.3.2 Contact Resistance

The existence of a finite contact resistance is due principally to surface roughness effects. Contact spots are interspersed with gaps that are, in most instances, air filled. Heat transfer is therefore due to conduction across the actual contact area and to conduction and/or radiation across the gaps. The contact resistance may be viewed as two parallel resistances: that due to:
(1) The contact spots
(2) That due to the gaps (the major contribution to the resistance).
The resistance is defined as

$$R''_{tc}=\frac{T_A-T_B}{q''_x}$$

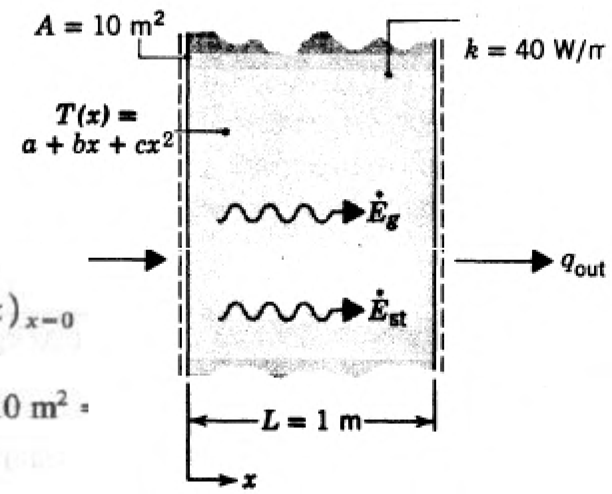
Figure 2.3 *Temperature drop due to thermal contact resistance.*



Example 2.1

The temperature distribution across a wall *1 m* thick at a certain instant of time is given as ($T(x) = a + bx + cx^2$) where T is in degrees Celsius and x is in meters, while $a = 900°\,C$, $b = -300°C/m$, and $c = -50°C/m^2$. A uniform heat generation $q = 1000\ W/m^3$, is present in the wall of area $10\ m^2$ having the properties $\rho = 1600\ kg/m^3$, $k = 40\ W/m\ K$, and $C_p = 4\ kJ/kg\ K$.

1. Determine the rate of heat transfer entering (x = 0) and leaving the wall (x = 1 m).
2. Determine the rate of change of energy storage in the wall.
3. Determine the time rate of temperature change *at* $x = 0$, 0.25 and $0.5\ m$.



1.

$$q_{in} = q_x(0) = -kA\left.\frac{\partial T}{\partial x}\right|_{x=0} = -kA(b + 2cx)_{x=0}$$

$$q_{in} = -bkA = 300°C/m \times 40\ W/m\cdot K \times 10\ m^2$$

Similarly,

$$q_{out} = q_x(L) = -kA\left.\frac{\partial T}{\partial x}\right|_{x=L} = -kA(b + 2cx)_{x=L}$$

$$q_{out} = -(b + 2cL)kA = -[-300°C/m$$

$$+2(-50°C/m^2)\times 1\ m] \times 40\ W/m\cdot K \times 10\ m^2 = 160\ kW$$

2. $$E_{in} + E_g - E_{out} = E_{st} \qquad 2.1$$

where $\dot{E}_g = \dot{q}AL$, it follows that

$$\dot{E}_{st} = \dot{E}_{in} + \dot{E}_g - \dot{E}_{out} = q_{in} + \dot{q}AL - q_{out}$$

$$\dot{E}_{st} = 120\ kW + 1000\ W/m^3 \times 10\ m^2 \times 1\ m - 160\ kW$$

$$\dot{E}_{st} = -30\ kW$$

3. The time rate of change of the temperature at any point in the medium may be determined from the heat equation, Equation 2.15, as

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{\rho c_p}$$

From the prescribed temperature distribution, it follows that

$$\frac{\partial^2 T}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) = \frac{\partial}{\partial x}(b + 2cx) = 2c = 2(-50°C/m^2) = -100°C/m^2$$

$$\frac{\partial T}{\partial t} = \frac{40\ W/m\cdot K}{1600\ kg/m^3 \times 4\ kJ/kg\cdot K} \times (-100°C/m^2) + \frac{1000\ W/m^3}{1600\ kg/m^3 \times 4\ kJ/kg\cdot K}$$

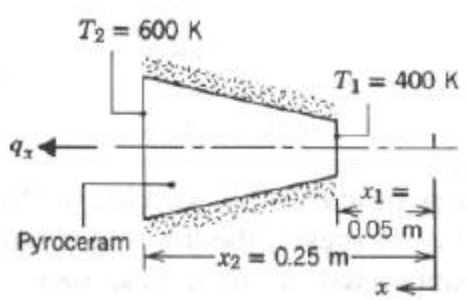
$$\frac{\partial T}{\partial t} = -6.25 \times 10^{-4}°C/s + 1.56 \times 10^{-4}°C/s = -4.69 \times 10^{-4}°C/s$$

Example 2.2

The diagram shows a conical section from pyroceram ($k = 3.46\ W/m\ K$). It is of circular cross section with the diameter $D = ax$. The small end is at $x_1 = 50\ mm$ and the large end at $x_2 = 250\ mm$. The end temperatures are $T_1 = 400\ K$ and $T_2 = 600\ K$, while the lateral surface is well insulated and $a=0.25$.

1. Derive an expression for the temperature distribution $T(x)$ in symbolic form, assuming one-dimensional conditions.
2. Sketch the temperature distribution.
3. Calculate the heat rate through the cone.

Schematic:



Solution

Assumptions:

1. Steady-state conditions.
2. One-dimensional conduction in the x direction.
3. No internal heat generation.
4. Constant properties.

$$q_x = -kA\frac{dT}{dx}$$

With $A=\pi D^2/4=\pi a^2 x^2/4$ and separating variables

$$\frac{4q_x dx}{\pi a^2 x^2} = -kdT$$

Integrating from x1 to any x within the, it follows that

$$\frac{4q_x}{\pi a^2}\int_{x1}^{x}\frac{dx}{x^2} = -k\int_{T1}^{T} dT \qquad (k = const)$$

Hence

$$\frac{4q_x}{\pi a^2}(-\frac{1}{x}+\frac{1}{x_1}) = -k(T - T_1)$$

and solving for q

$$q_x = \frac{\pi a^2 k(T_1 - T)}{4[((1/x_1)-(1/x))]}$$

or solving for T

$$T(x) = T_1 - \frac{4q_x}{\pi a^2 k}(-\frac{1}{x}+\frac{1}{x_1})$$

B.C.2 $\quad T=T_{s2} \quad at \quad x=x_2$

$$q_x = \frac{\pi a^2 k(T_1 - T_2)}{4[((1/x_1)-(1/x_2))]}$$

$$\frac{4q_x}{\pi a^2 k} = \frac{(T_1 - T_2)}{[((1/x_1)-(1/x_2))]}$$

Substituting for q into the expression for T(x), the temperature distribution becomes

$$T(x) = T_1 + (T_1 - T_2)\left[\frac{(1/x)-(1/x_1))}{(1/x_1)-(1/x_2)}\right]$$

Substituting numerical values into the foregoing result for the heat transfer rate

$$q_x = \frac{\pi(0.25)^2 \times 3.46\ \text{W/m}\cdot\text{K}\ (400 - 600)\ \text{K}}{4\left(\frac{1}{0.05\ \text{m}} - \frac{1}{0.25\ \text{m}}\right)} = -2.12\ \text{W}$$

2.4 The Conduction Equation of Cylindrical Coordinate

A common example is the hollow cylinder, whose inner and outer surfaces are exposed to fluids at different temperatures. For a general transient three-dimensional in the cylindrical coordinates *T= T(r, φ ,z, t)*, the general form of the conduction equation in cylindrical coordinates becomes

$$\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial T}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad (2.8)$$

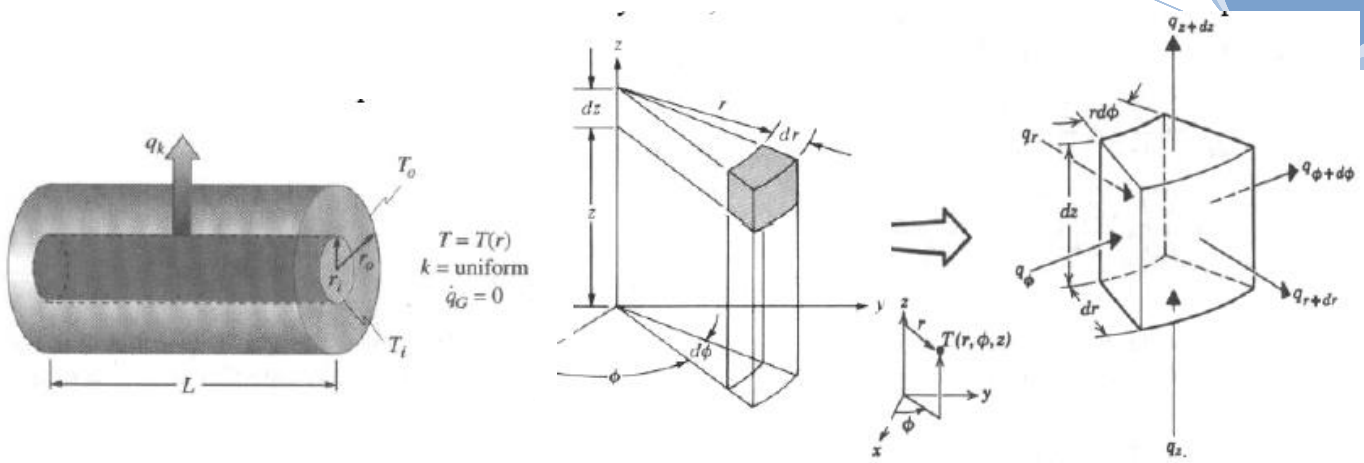


Figure 2.4 *Hollow cylinder with convective surface conditions.*

For a general transient three-dimensional in the cylindrical coordinates $T= T(r, \varphi ,z, t)$,the general form of the conduction equation in cylindrical coordinates becomes

$$\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial T}{\partial r})+\frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\dot{q}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad (2.8)$$

If the heat flow in a cylindrical shape is only in the radial direction and for steady-state conditions with no heat generation, the conduction equation reduces to

$$\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial T}{\partial r})=0$$

Integrating once with respect to radius gives

$$r\frac{\partial T}{\partial r}=C_1 \qquad \text{and} \qquad \frac{\partial T}{\partial r}=\frac{C_1}{r}$$

A second integration gives $T = C_1 \ln r + C_2$. **2.9**

To obtain the constants (C_1 and C_2), we introduce the following boundary conditions

B.C.1 $T=T_i$ *at* $r=r_i$ $T_i = C_1 \ln r_i + C_2$.

B.C.2 $T=T_o$ *at* $r=r_o$ $T_o = C_1 \ln r_o + C_2$.

Solving for C_1 and C_2 and substituting into the general solution, we then obtain

$$T_o - T_i = C_1 \ln\frac{r_o}{r_i}$$

$$C_1 = \frac{T_o - T_i}{\ln(r_o / r_i)} \qquad C_2 = T_o - \frac{T_o - T_i}{\ln(r_o / r_i)}\ln r_o$$

$$T(r) = \frac{T_o - T_i}{\ln(r_o / r_i)}\ln(\frac{r}{r_i}) + T_i \qquad \mathbf{2.10}$$

we obtain the following expression for the heat transfer rate

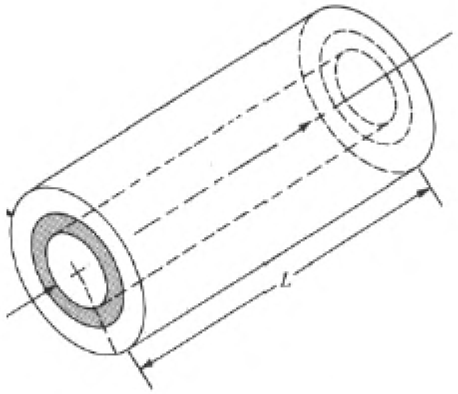
$$q_r = -kA\frac{dT}{dr} = -(2\pi rLk)\frac{C_1}{r} = \frac{2\pi Lk(T_i - T_o)}{\ln(r_o / r_i)} \qquad \mathbf{2.11}$$

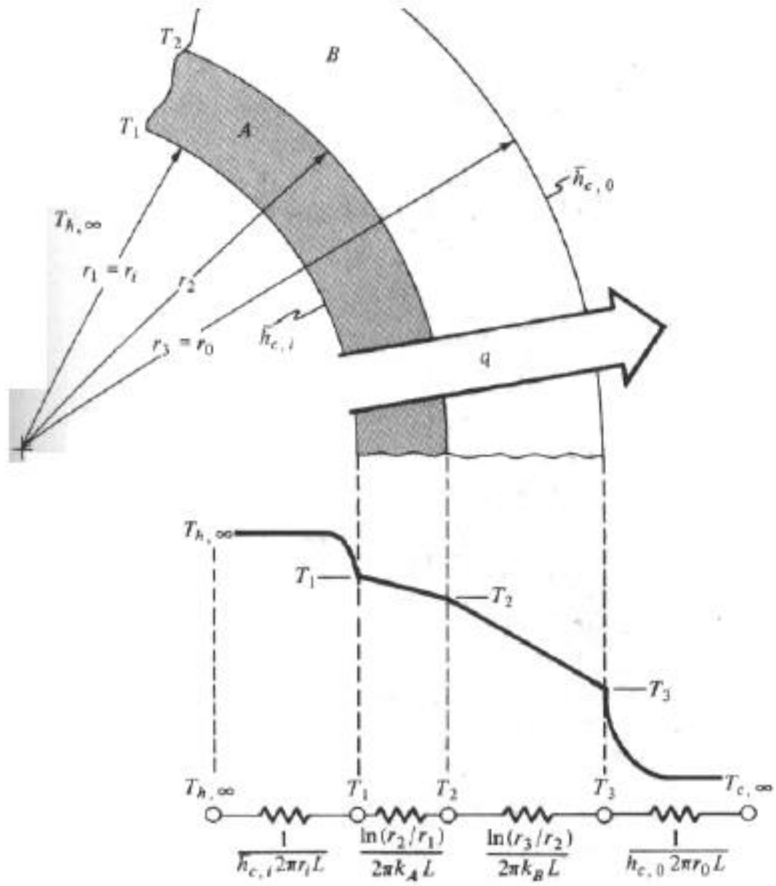
$$q_r = \frac{(T_i - T_o)}{R} \qquad R = \frac{\ln(r_o / r_i)}{2\pi Lk} \qquad \mathbf{2.12}$$

2.4.1 Overall Heat Transfer Coefficient

A hot fluid flows through a tube that is covered by an insulating material. The system loses heat to the surrounding air through an average heat transfer coefficient $h_{c,o}$. the thermal resistance of the two cylinders at the inside of the tube and the outside of the insulation gives the thermal network shown below the physical system

where $T_{h,\infty}$ hot fluid temperature
and
$T_{c,\infty}$ the environmental air temperature





the rate of heat flow is

$$q = \frac{\Delta T}{\sum_1^4 R_{th}} = \frac{T_{h,\infty} - T_{c,\infty}}{\frac{1}{\bar{h}_{c,i}2\pi r_1 L} + \frac{\ln(r_2/r_1)}{2\pi k_A L} + \frac{\ln(r_3/r_2)}{2\pi k_B L} + \frac{1}{\bar{h}_{c,o}2\pi r_3 L}} \quad (2.13)$$

it is often convenient to define an overall heat transfer coefficient by the equation

$$q = UA_o\,(T_{hot} - T_{cold})$$

The area varies with radial distance. Thus, the numerical value of U will depend on the area selected. Since the outermost diameter is the easiest to measure in practice, $A_o = 2\pi\, r_3 L$ is usually chosen as the base area. Comparing between above Equations. we see that

$$UA = \frac{1}{\sum_1^4 R_{th}} = \frac{1}{\frac{1}{\bar{h}_{c,i}A_i} + \frac{\ln(r_2/r_1)}{2\pi k_A L} + \frac{\ln(r_3/r_2)}{2\pi k_B L} + \frac{1}{\bar{h}_{c,o}A_o}}$$

Note that

$$UA = U_i A_i = U_o A_o \quad (2.14)$$

$A_o = 2\pi r_o^{\,3} L$ and the overall coefficient becomes

$$U = \frac{1}{\dfrac{r_3}{r_1 \bar{h}_{c,i}} + \dfrac{r_3 \ln(r_2/r_1)}{k_A} + \dfrac{r_3 \ln(r_3/r_2)}{k_B} + \dfrac{1}{\bar{h}_{c,o}}} \qquad 2.15$$

Example 2.3

Compare the heat loss from an **insulated** and an **un-insulated** copper pipe *(k = 400 W/m K)* has an internal diameter of *10 cm* and an external diameter of *12 cm*. Saturated steam flows inside the pipe at *110°C* (h_{ci} = *10,000 W/m2 K*). The pipe is located in a space at 30°C and the heat transfer coefficient on its outer surface is estimated to be *15 W/m2 K*. The insulation available to reduce heat losses is 5 cm thick and its thermal conductivity is *0.20 W/m K*

Solution

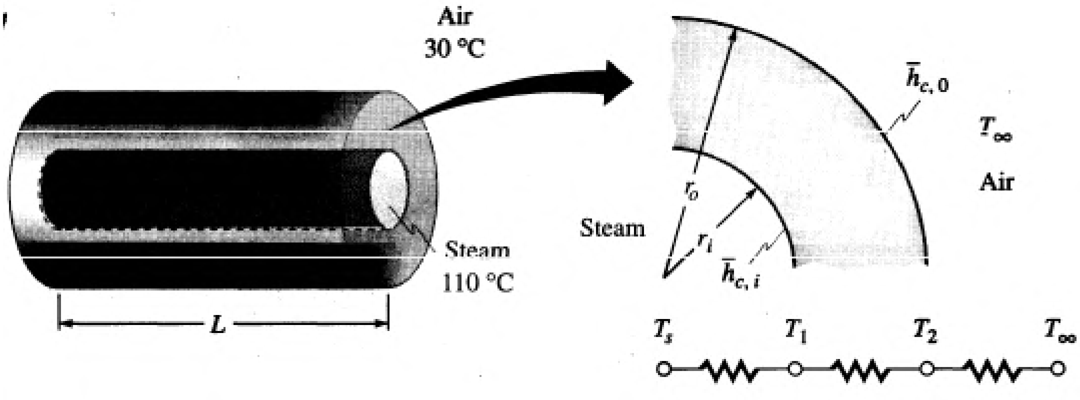


Figure 2.5 *Schematic Diagram and Thermal Circuit for a Hollow Cylinder with Convection Surface Conditions*

The heat loss per unit length is

$$\frac{q}{L} = \frac{T_s - T_\infty}{R_1 + R_2 + R_3}$$

Hence we get

$$R_1 = R_i = \frac{1}{2\pi r_i \bar{h}_{c,i}} \simeq \frac{1}{(2\pi)(0.05\text{ m})(10{,}000\text{ W/m}^2\text{ K})} = 0.000318\text{ m K/W}$$

$$R_2 = \frac{\ln(r_o/r_i)}{2\pi k_{\text{pipe}}} = \frac{0.182}{(2\pi)(400\text{ W/m K})} = 0.00007\text{ m K/W}$$

$$R_3 = R_o = \frac{1}{2\pi r_o \bar{h}_{c,o}} = \frac{1}{(2\pi)(0.06\text{ m})(15\text{ W/m}^2\text{ K})} = 0.177\text{ m K/W}$$

Since R_1 and R_2 are negligibly small compared to R_3

For the un-insulated pipe. *q/L = 80/0.177 =* ***452 W/m***

For the insulated pipe, we must add a fourth resistance between r_1 *and* r_3.

$$R_4 = \frac{\ln(r_i / r_o)}{2\pi k} = \frac{\ln(11/6)}{2\pi(0.2W/mK)} = 0.482 mK/W$$

Also, the outer convection resistance changes to

$$R_o = \frac{1}{2\pi(0.11\times 15)} = 0.096 mK/W$$

The total thermal resistance per meter length (*RTotal=R4+Ro= 0.578 m K/W)*
q/L = 80/0.578 = ***138 W/m***.
Adding insulation will reduce the heat loss from the steam by 70%.

Example 2.4

A hot fluid at an average temperature of 200oC flows through a plastic pipe of 4 cm OD and 3 cm ID. The thermal conductivity of the plastic is 0.5 W/m K, and the heat transfer coefficient at the inside is 300 W/m2 K. The pipe is located in a room at 30°C, and the heat transfer coefficient at the outer surface is 10 W/m2 K, Calculate the overall heat transfer coefficient and the heat loss per unit length of pipe.

Solution

The overall heat transfer coefficient is based on the outside area of the pipe

$$U = \frac{1}{\dfrac{r_o}{r_i \bar{h}_{c,i}} + \dfrac{r_o \ln(r_o/r_i)}{k} + \dfrac{1}{\bar{h}_{c,o}}}$$

$$= \frac{1}{\dfrac{0.02}{0.015\times 300} + \dfrac{0.02\ln(2/1.5)}{0.5} + \dfrac{1}{10}} = 8.62\ \text{W/m}^2\ \text{K}$$

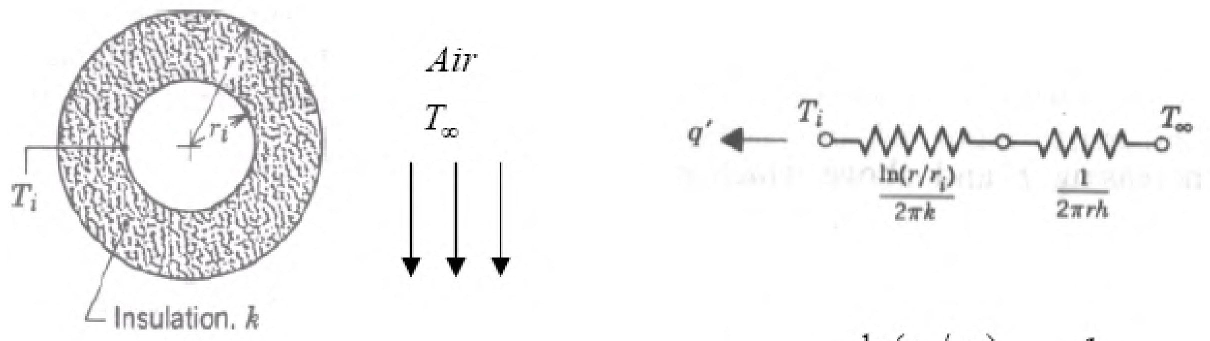
The heat toss per unit length is

$$\frac{q}{L} = UA_o(T_{hot} - T_{cold})$$

$$= (8.62\ \text{W/m}^2\ \text{K})(\pi)(0.04\ \text{m})(200 - 30)(\text{K}) = 184\ \text{W/m}$$

2.4.2 Critical Radius of Insulation

Although the conduction resistance increases with the addition of insulation, the convection resistance decreases due to increasing outer surface area. Hence there may exist an insulation thickness that minimizes heat loss by maximizing the total resistance to heat transfer.



$$R_{Total} = \frac{\ln(r/r_i)}{2\pi k} + \frac{1}{2\pi rh}$$

An optimum insulation thickness would be associated with the value of r that minimized

q_r or maximized R_{Total}. Such a value could be obtained from the requirement that

$$\frac{dq}{dr_c}=0 \qquad at \qquad r=r_{\ Critical}$$

$$\frac{dq_r}{dr_c}=\frac{-2\pi L(T_i-T_o)((1/Kr_c)-(1/hr_c^{\ 2}))}{\left[\frac{\ln(r_c/r_i)}{k}+\frac{1}{rh}\right]^2}=0$$

$$\frac{1}{kr_c}-\frac{1}{r_c^{\ 2}h}=0 \qquad r_c=\frac{k}{h} \qquad \textbf{2.16}$$

For spherical shape:

$$r_c=\frac{2k}{h}$$

Example 2.5

Calculate the total thermal resistance per unit length of tube for a 10 mm diameter tube having the following insulation thicknesses: 0, 2, 5, 10, 20 and 40 mm. The insulation is composed of *Cellular Glass* (*k=0.055 w/m K*), and the outer surface convection coefficient is *5 W/m2 K.*

Solution $$r_c=\frac{k}{h}=\frac{0.055}{5}=0.011m$$

Hence $r_c > r$, and heat transfer will increase with the addition of insulation up to a thickness of $r_c\text{-}r_i=(0.011\text{-}0.005)=0.006m$

The thermal resistances corresponding to the prescribed insulation thicknesses may be calculated and are summarized as follows.

INSULATION THICKNESS $(r-r_i)$ (mm)	INSULATION RADIUS r (m)	THERMAL RESISTANCES (m · K/W) R'_{cond}	R'_{conv}	R'_{tot}
0	0.005	0	6.37	6.37
2	0.007	0.97	4.55	5.52
5	0.010	2.00	3.18	5.18
6	r_{cr} = 0.011	2.28	2.89	5.17
10	0.015	3.18	2.12	5.30
20	0.025	4.66	1.27	5.93
40	0.045	6.35	0.71	7.06

2.5 The Conduction Equation of Spherical Coordinate

For spherical coordinates, the temperature is a function of the three space coordinates $T(r, \theta, \varphi, t)$. The general form of the conduction equation is then

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial\phi^2}+\frac{\dot{q}_G}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad 2.17$$

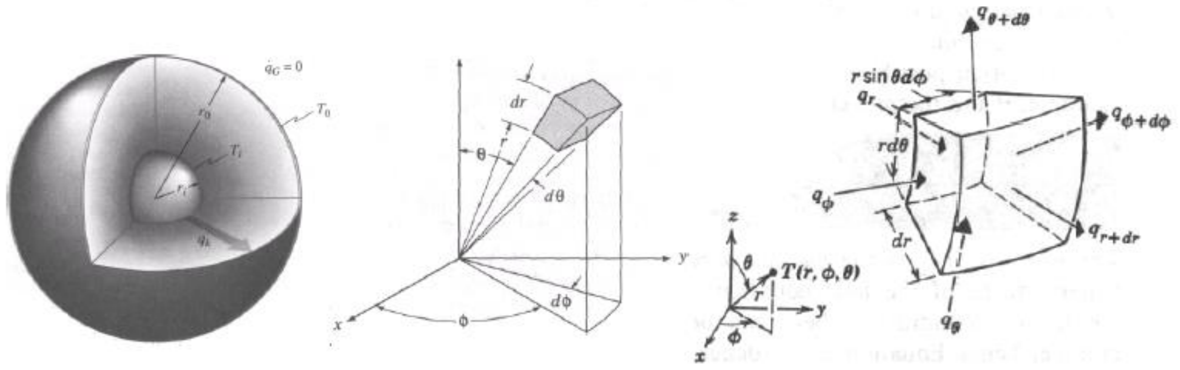


Figure 2.6 *Spherical Coordinate System*

For a hollow sphere with uniform temperatures at the inner and outer surfaces, the temperature distribution without heat generation in the steady state can be obtained by simplifying Eq 2.17. Under these boundary conditions the temperature is only a function of the radius r, and the conduction equation is

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial T}{\partial r})=0$$

$$r^2\frac{\partial T}{\partial r}=C_1 \qquad \partial T=\frac{C_1}{r^2}\partial r$$

$$T(r)=C_2-\frac{C_1}{r}$$

B.C.1 $T=T_i$ at $r=r_i$ $\qquad Ti=C_2-\frac{C_1}{r_i}$

B.C.2 $T=T_o$ at $r=r_o$ $\qquad T_o=C_2-\frac{C_1}{r_o}$

$$C_1=\frac{T_i-T_o}{(\frac{1}{r_o})-(\frac{1}{r_i})} \qquad C_2=T_o+(\frac{T_i-T_o}{((1/r_o)-(1/r_i))})\frac{1}{r_o}$$

The temperature distribution is

$$T(r)=(\frac{T_i-T_o}{\frac{1}{r_i}-\frac{1}{r_o}})(\frac{1}{r}-\frac{1}{r_o})+T_i \qquad (2.18)$$

The rate of heat transfer through the spherical shell is

$$q_r = -kA\frac{dT}{dr} = -k(4\pi r^2)\frac{dT}{dr}$$

$A=4\pi r^2$	مساحة الكرة
$A=\pi D^2$	
$V=4\pi r^2/3$	حجم الكرة

may be expressed in the integral form

$$\frac{1}{4\pi}\int_{ri}^{ro}\frac{q_r dr}{r^2} = -\int_{Ti}^{To} kdT$$

Assuming constant k and q_r, we obtain

$$q_r = \frac{4\pi k r_o r_i (T_i - T_o)}{r_o - r_i} \qquad R = \frac{(r_o - r_i)}{4\pi k r_o r_i} \qquad (2.19)(2.20)$$

Example 2.6

The spherical, thin-walled metallic container is used to store liquid nitrogen at 77 K. The container has a diameter of 0.5 and is covered with an evacuated insulation system composed of silica powder ($k = 0.0017$ *W/m K*). The insulation is 25 mm thick, and its outer surface is exposed to ambient air at 300 K. The latent heat of vaporization h_{fg} of liquid nitrogen is 2×10^5 J/kg. If the convection coefficient is *20* W/m^2 K over the outer surface,

1. Determine the rate of liquid boil-off of nitrogen per hour?
2. Show expiration of critical radius of insulation? Ans: $r_c = 2h/k$

Solution

1. The rate of heat transfer from the ambient air to the nitrogen in the container can be obtained from the thermal circuit. We can neglect the thermal resistances of the metal wall and between the boiling nitrogen and the inner wall because that heat transfer coefficient is large. Hence

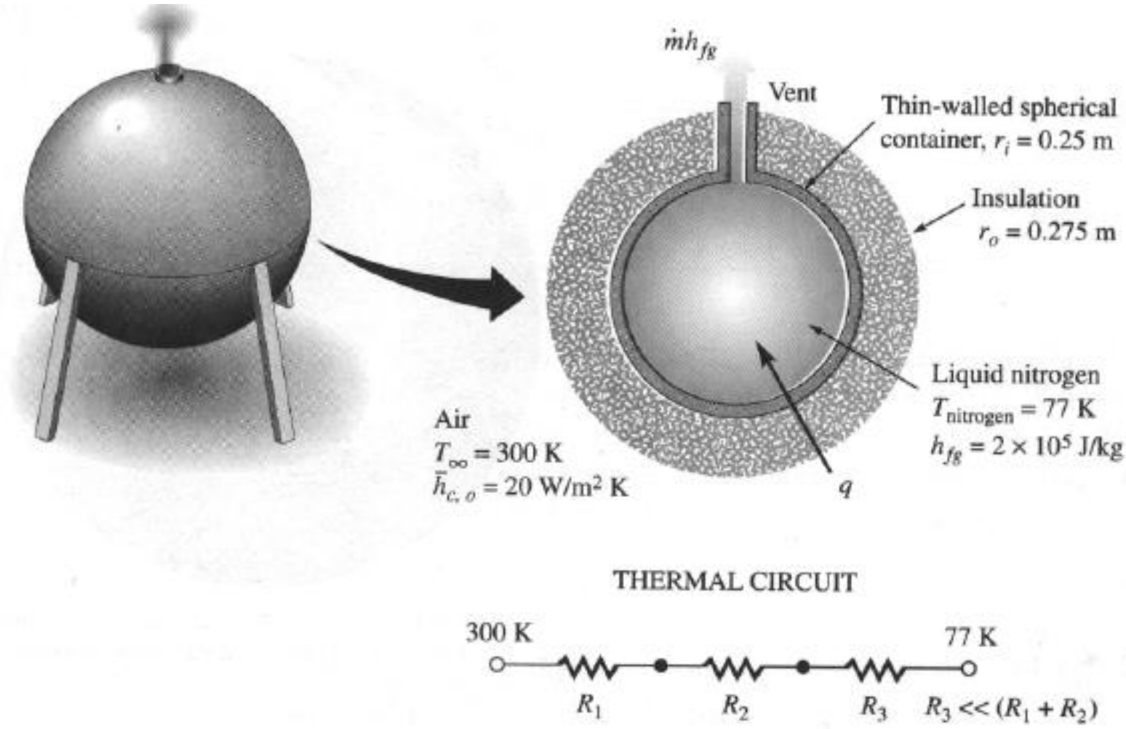


Figure 2.7 *Schematic Diagram of Spherical Container*

$$q = \frac{T_{\infty,\text{air}} - T_{\text{nitrogen}}}{R_1 + R_2} = \frac{(300 - 77)\text{ K}}{\dfrac{1}{\bar{h}_{c,o} 4\pi r_o^2} + \dfrac{r_o - r_i}{4\pi k r_o r_i}}$$

$$= \frac{223\text{ K}}{\dfrac{1}{(20\text{ W/m}^2\text{ K})(4\pi)(0.275\text{ m})^2} + \dfrac{(0.275 - 0.250)\text{ m}}{4\pi(0.0017\text{ W/m K})(0.275\text{ m})(0.250\text{ m})}}$$

$$= \frac{223\text{ K}}{(0.053 + 17.02)\text{ K/W}} = 13.06\text{ W}$$

To determine the rate of boil-off we perform an energy balance

$E_{in} = E_{out}$ $\qquad\qquad \dot{m} h_{fg} = q$

Solving for m gives $\qquad \dot{m} = \dfrac{q}{h_{fg}} = \dfrac{(13.06\text{ J/s})(3600\text{ s/hr})}{2\text{ x }105\text{ J/kg}} = 0.235 kg / hr$

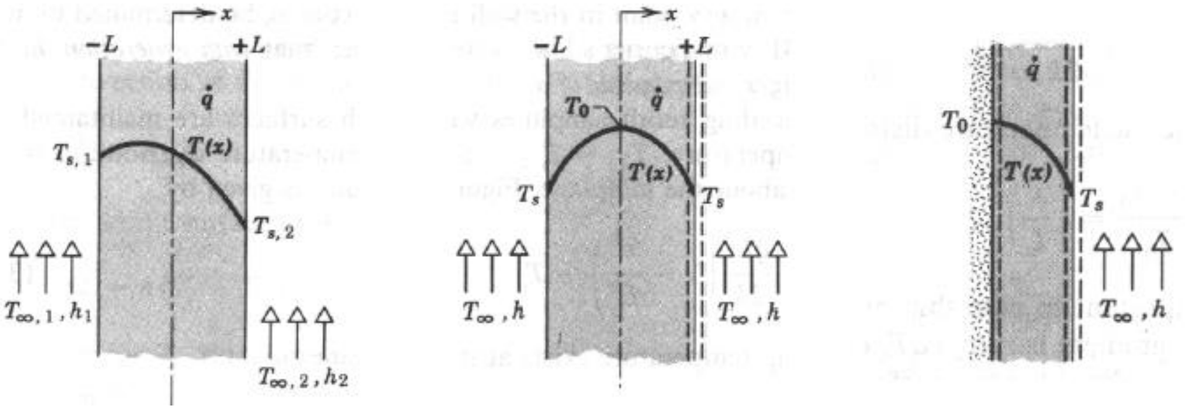
2.6 Conduction with Heat Generation

A common thermal energy generation process involves

- The conversion from electrical to thermal energy in a current-carrying medium $E_g=I^2R$.
- The deceleration and absorption of neutrons in the fuel element of a nuclear reactor
- Exothermic chemical reactions occurring within a medium. Endothermic reactions would, of course, have the inverse effect
- A conversion from electromagnetic to thermal energy may occur due to the absorption of radiation within the medium.

Note: *Remember not to confuse energy generation with energy storage.*

2.6.1 Plane Wall with Heat Generation



(a) *Asymmetrical plane wall* ***(b)****Symmetrical plane wall* ***(c)*** *Adiabatic surface at midline*

Figure 2.8 *Conduction in a with uniform heat generation*

Assumptions

- Uniform heat generation per unit volume q =*Const.*
- For constant thermal conductivity k=*Const.*
- One dimension and steady state heat transfer.

The appropriate form of the heat equation, is

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\dot{q}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad (2.5)$$

The equation may be integrated twice to obtain the general solution

$$T(x) = -\frac{\dot{q}}{2k}x^2 + C_1 x + C_2 \qquad (2.6)$$

To obtain the constants of integration, C_1 and C_2 boundary conditions must be introduced.

B.C.1 $T=T_{s1}$ at $x=L$ $\qquad T_{s1} = -\frac{\dot{q}}{2k}L^2 + C_1 L + C_2$

B.C.2 $T=T_{s2}$ at $x=-L$ $\qquad T_{s2} = -\frac{\dot{q}}{2k}L^2 - C_1 L + C_2$

$$C_1 = \frac{T_{s1} - T_{s2}}{2L} \qquad C_2 = \frac{\dot{q}L^2}{2k} + \frac{T_{s1} + T_{s2}}{2}$$

In which case the Temperature distribution is

$$T(x) = \frac{\dot{q}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + \frac{T_{s2} - T_{s1}}{2}\frac{x}{L} + \frac{T_{s1} + T_{s2}}{2} \qquad 2.22$$

The Symmetrical Plane Wall

when both surfaces are maintained at a common temperature, $T_{s1} = T_{s2} = T_s$. The temperature distribution is given by

$$T(x) = \frac{\dot{q}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + T_s \qquad \mathbf{2.23}$$

The maximum temperature ($T=T_o$) exists at the midline ($x=0$).

$$T_o = \frac{\dot{q}L^2}{2k} + T_s \qquad \text{or} \qquad T_o - T_s = \frac{\dot{q}L^2}{2k} \qquad \mathbf{2.24}$$

which case the temperature distribution, after substitution eq 2.24 into eq 2.23

$$\frac{T(x) - T_s}{T_o - T_s} = 1 - \frac{x^2}{L^2} \qquad \mathbf{2.25}$$

Consider the surface at $x = L$ for (*Fig. 2.8b*) or the insulated plane wall (*Fig. 2.8c*). The energy balance given by

$$E_g = E_{out}$$

$$\dot{q}V = Ah(T_s - T_\infty) \qquad \textit{Neglecting radiation}$$

$$\dot{q}\cancel{A}L = \cancel{A}h(T_s - T_\infty)$$

The surface temperature is

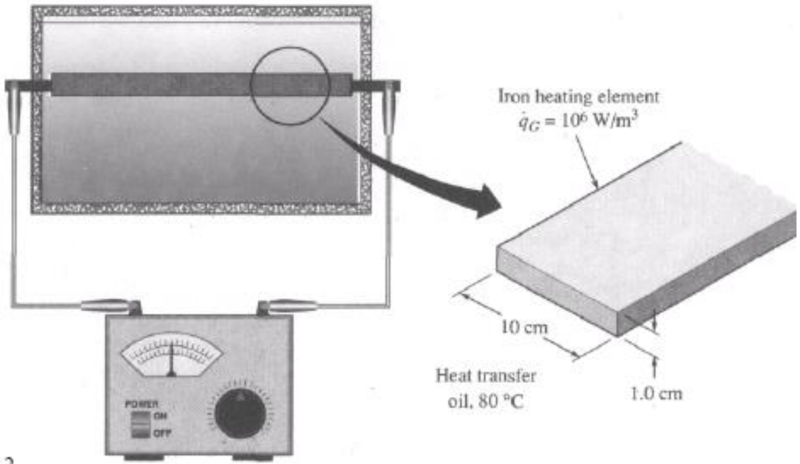
$$T_s = T_\infty + \frac{\dot{q}L}{h} \qquad \mathbf{2.26}$$

***Note** :A heat generation cannot be represented by a thermal circuit element*

<u>Example 2.7</u>

A long electrical heating element made of iron has a cross section of 10 cm x 1.0 cm. It is immersed in a heat transfer oil at 80°C. If heat is generated uniformly at a rate of 10^6 W/m^3

by an electric current, determine the heat transfer coefficient necessary to keep the temperature of the heater below 200°C. The thermal conductivity for iron is 64 W/m K.



Solution

$$T_{max} - T_1 = \frac{\dot{q}L^2}{8k} = \frac{10^6 \times (0.01)^2}{8 \times 64} = 0.2^\circ C$$

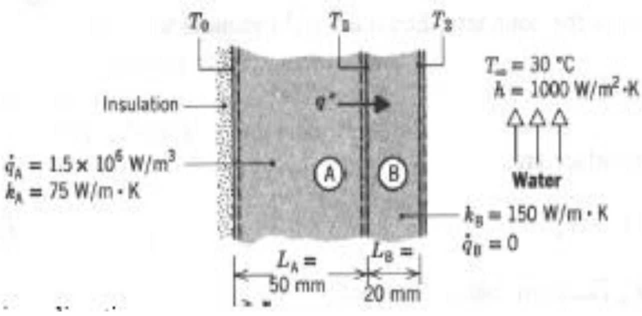
$$\dot{q}V = Ah(T_s - T_\infty) \qquad \dot{q}A\frac{L}{2} = Ah(T_s - T_\infty)$$

$$h = \frac{\dot{q}L}{2(T_s - T_\infty)} = 42W/m^2K$$

Example 2.8

A plane wall is a composite of two materials, A and B. The wall of material A (*k = 75 W/m K*) has uniform heat generation *1.5 X 106 W/m³*, and thickness *50 mm*. The wall material B has no generation with (k = 150 W/m K) and thickness 20 mm. The inner surface of material A is well insulated, while the outer surface of material B is cooled by a water stream with 30°C and heat transfer coefficient *1000 W/m² K*.

1. Sketch the temperature distribution that exists in the composite under steady-state conditions.
2. Determine the maximum temperature T_o of the insulated surface and the temperature of the cooled surface T_s.



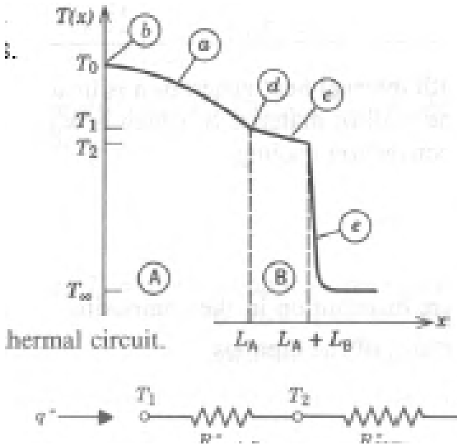
Solution

Assumptions:

1. Steady-state conditions.
2. One-dimensional conduction in x direction.
3. Negligible contact resistance between walls.
4. Inner surface of A adiabatic.
5. Constant properties for materials A and B.

$$T_2 = T_\infty + \frac{qL_A}{h} \qquad 2.26$$

$$T_2 = 30 + \frac{1.5\times10^6 \times 0.05}{1000} = 105^\circ C$$



$$q'' = \frac{T_1 - T_\infty}{R''_{cond} + R''_{conv}}$$

$$T_1 = T_\infty + (R''_{cond} + R''_{conv})q''$$

$$\dot{q} = \frac{q''}{AL_A}$$

where the resistances for a unit surface area are

$$R''_{cond,B} = \frac{L_B}{k_B} \qquad R''_{conv} = \frac{1}{h}$$

Hence

$$T_1 = 30^\circ C + \left(\frac{0.02\ m}{150\ W/m\cdot K} + \frac{1}{1000\ W/m^2\cdot K}\right) \times 1.5\times10^6\ W/m^3 \times 0.05\ m$$

$$T_1 = 115^\circ C$$

From Equation 2.24 the temperature at the insulated surface is

$$T_o = T_1 + \frac{\dot{q}L^2_A}{2k_A}$$

$$T_o = 115 + \frac{1.5\times10^6(0.05)^2}{2\times75} = 140^\circ C$$

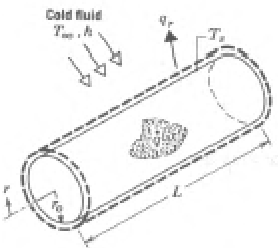
2.6.2 Radial Shapes with Heat Generation

To determine the temperature distribution in the cylinder, we begin with the appropriate form of the heat equation. For constant thermal conductivity is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\dot{q}}{k} = 0$$

$$r\frac{\partial T}{\partial r} = -\frac{\dot{q}}{2k}r^2 + C_1$$

$$T(r) = -\frac{\dot{q}}{4k}r^2 + C_1 \ln r + C_2$$



A. Solid Cylinder

To obtain the constants (C_1 & C_2), we introduce the following boundary conditions

B.C.1	$dT/dr=0$	at $r=0$	$C_1=0$
B.C.2	$T=T_s$	at $r=r_o$	$C_2 = \frac{\dot{q}}{4k}r_o^2 + T_s$

Solving for C_1 and C_2 and substituting into the general solution, we then obtain

$$T(r) = -\frac{\dot{q}r_o^2}{4k}(1-\frac{r^2}{r_o^2}) + T_s \qquad 2.28$$

The maximum temperature $T=T_o$ at $r=0$

$$T_o = -\frac{\dot{q}r_o^2}{4k} + T_s \qquad \frac{\dot{q}r_o^2}{4k} = T_o - T_s \qquad 2.29$$

substitution replace group $\frac{\dot{q}r_o^2}{4k}$ in equation 2.28

$$\frac{T(r)-T_s}{T_o-T_s} = 1-\frac{r^2}{r_o^2} \qquad 2.30$$

The energy balance given by

$$E_g = E_{out}$$

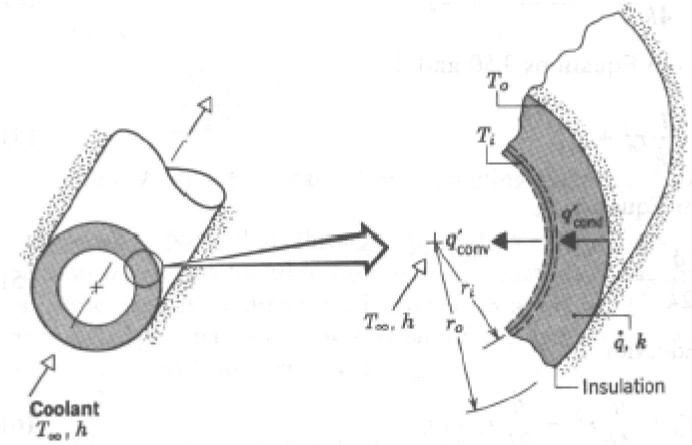
$$\dot{q}V = Ah(T_s - T_\infty)$$

$$\dot{q}\pi r_o^2 L = 2\pi r_o hL(T_s - T_\infty)$$

The surface temperature is

$$T_s = T_\infty + \frac{\dot{q}r_o}{2h} \qquad 2.31$$

B. **For Hollow Cylinder**



$$T(r) = -\frac{\dot{q}}{4k} r^2 + C_1 \ln r + C_2$$

To obtain the constants (C_1 *and* C_2), we introduce the following boundary conditions

B.C.1 $T=T_i$ *at* $r=r_i$ $\quad T_i = -\frac{\dot{q}}{4k} r_i^2 + C_1 \ln r_i + C_2$

B.C.2 $T=T_o$ *at* $r=r_o$ $\quad T_o = -\frac{\dot{q}}{4k} r_o^2 + C_1 \ln r_o + C_2$

Solving for C_1 and C_2 and substituting into the general solution, we then obtain

$$C_1 = \frac{(T_i - T_o) + \dot{q}(r_i^2 - r_o^2)/4k}{\ln(r_i/r_o)}$$

$$C_2 = T_o + \frac{\dot{q}}{4k} r_o^2 - \frac{(T_i - T_o) + \dot{q}(r_i^2 - r_o^2)/4k}{\ln(r_i/r_o)} \times \ln r_o$$

In which case the Temperature distribution is

$$T(r) = T_o + \frac{\dot{q}(r_i^2 - r_o^2)}{4k} + \frac{\ln(r/r_o)}{\ln(r_o/r_i)}\left[\frac{\dot{q}}{4k}(r_o^2 - r_i^2) + (T_o - T_i)\right] \qquad 2.32$$

The energy balance given by

$$E_g = E_{out}$$

$$\dot{q}V = Ah(T_s - T_\infty)$$

$$\dot{q}\pi(r_o^2 - r_i^2)L = 2\pi r_o hL(T_s - T_\infty)$$

The surface temperature is

$$T_s = T_\infty + \frac{\dot{q}(r_o^2 - r_i^2)}{2hr_o} \qquad 2.33$$

اشتقاقات مهمة:

- اشتق علاقة لتوزيع درجة الحرارة للاسطوانه الحلقية المجوفة؟
- اشتق علاقة لتوزيع درجة الحرارة للاسطوانة الحلقية المجوفة اذا كان السطح الخارجي معزول؟
- اشتق علاقة لتوزيع درجة الحرارة للكرة؟

Example 2.9

A graphite-moderated nuclear reactor. Heat is generated uniformly in uranium rods of 0.05 m diameter at the rate of 7.5 x 107 W/m^3. These rods are jacketed by an annulus in which water at an average temperature of 120°C is circulated. The water cools the rods and the average convection heat transfer coefficient is estimated to be 55,000 W/m^2 K. If the thermal conductivity of uranium is 29.5 W/m K, determine the center temperature of the uranium fuel rods.

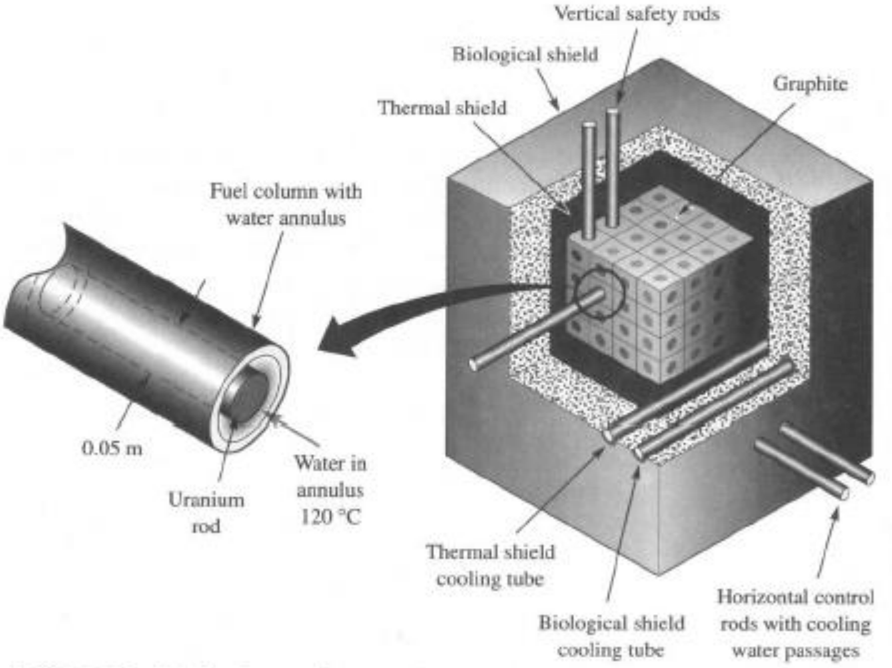


Figure 2.9 *Nuclear Reactor.*

Solution

The rate of heat flow by conduction at the outer surface equals the rate of heat flow by convection from the surface to the water

$$T_s = T_\infty + \frac{\dot{q} r_o}{2h}$$

$$T_s = 120 + \frac{7.5 \times 10^7 \times 0.025}{2 \times 55000} = 137\,^o C$$

The maximum temperature from equation 2.29

$$T_o = \frac{\dot{q} r_o^{\,2}}{4k} + T_s = \frac{7.5 \times 10^7 \times (0.025)^2}{4 \times 29.5} + 137 = 534^o C$$

2.7 Heat Transfer in Extended Surfaces (Finned surface)

Extended surfaces have wide industrial application as fins attached to the walls of heat transfer equipment in order to increase the rate of heating or cooling $q = h A_s (T_s - T_\infty)$. Fins come in many shapes and forms, some of which are shown in Fig 2.11.

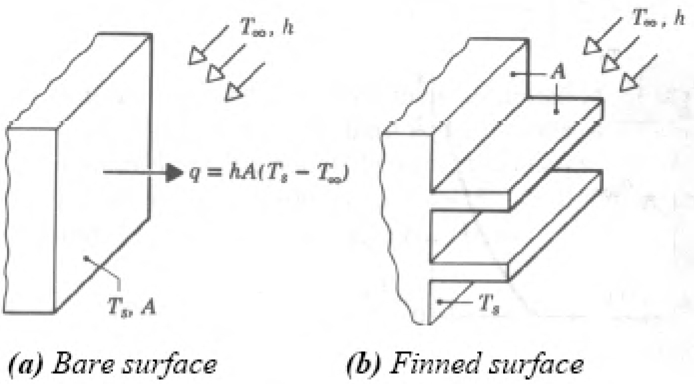


Figure 2.10 *Use of fins to enhance heat transfer from a plane wall.*

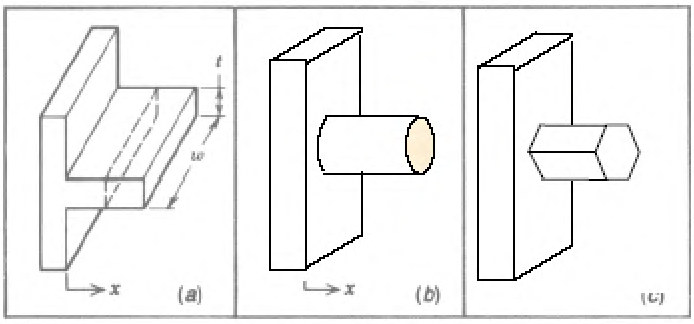


Figure 2.11 *uniform Fin configurations (a) Rectangular Fin, (b)& (c)Pin Fin*

The selection of fins is made on the basis of ***thermal performance*** and ***cost***. the fins is stronger when the fluid is a gas rather than a liquid. The selection of suitable fin geometry requires a compromise among:

- *A cost and weight are available space*
- *Pressure drop of the heat transfer fluid*
- *Heat transfer characteristics of the extended surface.*

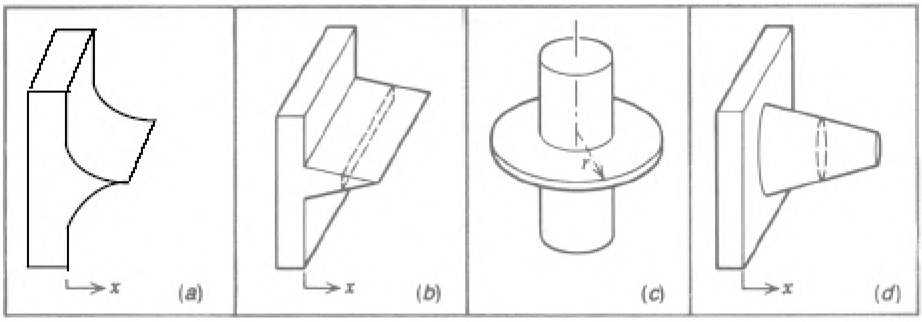


Figure 2.12 *non-uniform Fin configurations*
(a) Parabolic (b) Triangular (c) Annular fin (d) Pin fin.

Consider a pin fin having the shape of a rod whose base is attached to a wall at surface temperature T_s. The fin is cooled along its surface by a fluid at temperature T_∞ To derive an equation for temperature distribution, we make a heat balance for a small element of the fin. Heat flows by conduction into the left face of the element, while heat flows out of the element by conduction through the right face and by convection from the surface.

Assumptions

1. The fin has a uniform cross-sectional area
2. The fin is made of a material having uniform conductivity (k = constant)
3. The heat transfer coefficient between the fin and the fluid is constant (h=constant).
4. One dimensional steady state condition only.
5. Non heat generation(q=0).
6. Radiation is negligible.

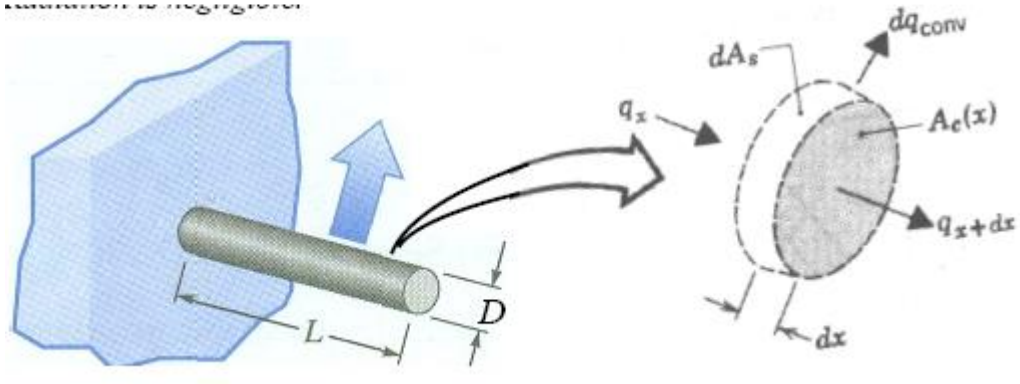


Figure 2.12 *Schematic Diagram of a Pin Fin Protruding from a Wall*

$$E_{in} = E_{out}$$

In symbolic form, this equation becomes

$$q_x = q_{x+dx} + q_{conv}$$

$$q_{x+dx} = q_x + \frac{dq_x}{dx}dx$$

$$-kA\frac{dT(x)}{dx}\bigg|_x = -kA\frac{dT(x)}{dx}\bigg|_{x+dx} + \bar{h}_c dA_s (T(x) - T_\infty) \qquad 2.34$$

$dAs = Pdx$

Where

P is the perimeter of the fin

Pdx is the fin surface area between x and $x+dx$.

A Cross section area of fin

If ***k*** and ***h*** are uniform, Eq. *2.34* simplifies to the form

$$\frac{d^2T(x)}{dx^2} - \frac{hP}{kA}[T(x) - T_\infty] = 0 \qquad 2.35$$

It will be convenient to define an excess temperature of the fin above the environment, $\theta(x) = [T(x) - T_\infty]$, and transform Eq. 2.35 into the form

$$\frac{d^2\theta(x)}{dx^2} - m^2\theta = 0 \qquad 2.36$$

Where $m^2 = hP/kA$.

Last equation is a linear, homogeneous, second-order differential equation whose general solution is of the form

$$\theta(x) = C_1\, e^{mx} + C_2\, e^{-mx} \qquad 2.37$$

To evaluate the constants C_1 and C_2 it is necessary to specify appropriate boundary conditions.

B.C.1 $\theta(0) = (T_s - T_\infty)$ at $x = 0$

$$\theta_s = C_1 + C_2 \qquad 2.38$$

A second boundary condition depends on the physical condition at the end of the fin. we will treat the following ***Four Cases***:

Case1: The fin is very long and the temperature at the end approaches the fluid temperature:

$$\theta(\infty) = (T_\infty - T_\infty) = 0 \qquad at \qquad x=\infty$$

Case2: The end of the fin is insulated:

$$\frac{d\theta(x)}{dx} = 0 \qquad at \qquad x=L$$

Case3: The temperature at the end of the fin is fixed:

$$\theta(L) = (T_L - T_\infty) \qquad at \qquad x=L$$

Case4: The tip loses heat by convection

$$-k\left.\frac{d\theta(x)}{dx}\right|_{x=L} = h\theta(L) \qquad at \qquad x=L$$

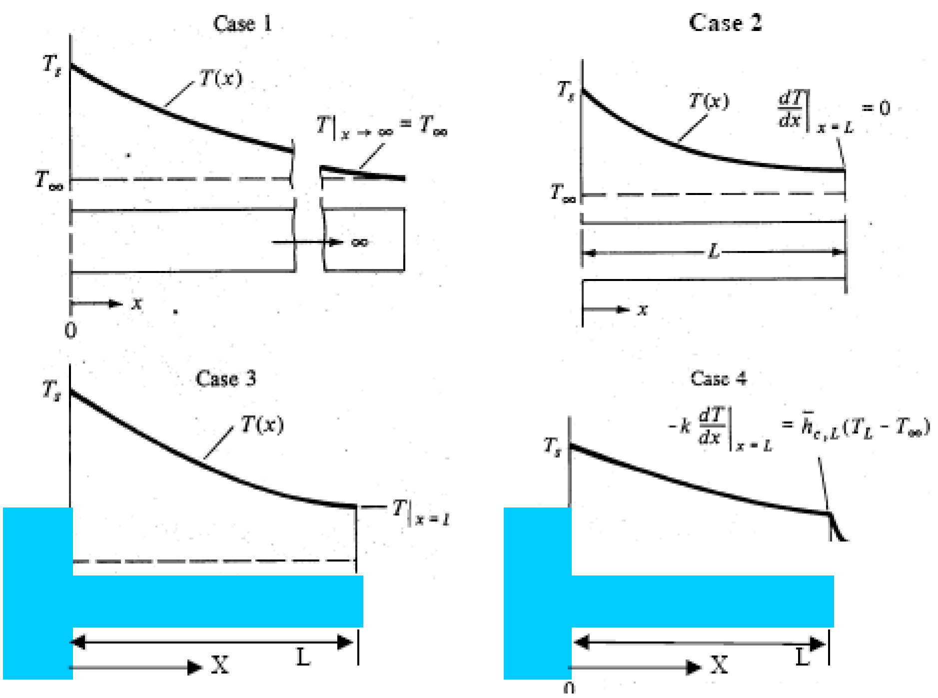


Figure 2.13 *Representations of Four Boundary Conditions at the Tip of a Fin*

Case 1

The second boundary condition is:

B.C.2 $\theta(\infty) = (T_\infty - T_\infty) = 0$ at $x = \infty$

$0 = C_1 e^{m\infty} + C_2 e^{-m\infty}$ $C_1=0$

B.C.1 $\theta(0) = (T_s - T_\infty)$ at $x = 0$

$\theta_s = C_1 + C_2$ $C_2 = \theta_s$

$$\theta(x) = \theta_s e^{-mx} \qquad 2.39$$

Differentiating

$$\frac{\partial\theta}{\partial x} = -m\theta_s e^{-mx} \qquad 2.40$$

Since the heat conducted across the root of the fin must equal the heat transferred by convection from the surface of the rod to the fluid,

$$q_{fin} = -kA\frac{dT}{dx}\bigg|_{x=0} = \bar{h}P(T(x) - T_\infty)dx \qquad 2.41$$

The rate of heat flow can be obtained by ***Two*** different methods.

Method 1. By left term in equation 2.41 substituting Eq. 2.40 for $x=0$ yields

$$q_{fin} = -kA\frac{d\theta}{dx}\bigg|_{x=0} = -kA\left[-m\theta(0)e^{(-m)0}\right] \qquad 2.42$$

$$q_{fin} = kA[m\theta_s] = kA\left[\sqrt{\frac{\bar{h}P}{kA}}\theta_s\right]$$

$$q_{fin} = \sqrt{\bar{h}PAk}\cdot\theta_s$$

Method 2. By right term in equation 2.41

$$q_{fin} = hP(T(x) - T_\infty)dx = hP\int_0^\infty \theta_s e^{-mx}dx$$

$$q_{fin} = \bar{h}P\theta_s \frac{e^{-mx}}{m}\bigg|_0^\infty = \sqrt{\bar{h}PAk}\cdot\theta_s \qquad 2.43$$

Case 2

The second boundary condition is *:*

B.C. 1 $\theta_s = C_1 + C_2$

B.C.2 $\frac{dT}{dx} = 0$ at $x=L$

$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$

$$\frac{d\theta(x)}{dx}\bigg|_{x=L} = mC_1e^{mL} - mC_2e^{-mL} = 0$$

$\cancel{m}C_1e^{mL} = \cancel{m}C_2e^{-mL}$ $C_1 = C_2e^{-2mL}$

Substituting in *B.C.1*

$\theta_s = C_2e^{-2mL} + C_2$ ➔ $C_2 = \frac{\theta_s}{1+e^{-2mL}}$

$$C_1 = \frac{\theta_s}{1+e^{-2mL}} e^{-2mL} \quad \rightarrow \quad C_1 = \frac{\theta_s}{1+e^{2mL}}$$

Substituting the above relations for C_1 and C_2 into Eq.(2.37)

$$\theta(x) = \frac{\theta_s}{1+e^{2mL}} e^{mx} + \frac{\theta_s}{1+e^{-2mL}} e^{-mx}$$

$$\theta(x) = \frac{\theta_s}{1+e^{2mL}} e^{mx}\left(\frac{e^{-mL}}{e^{-mL}}\right) + \frac{\theta_s}{1+e^{-2mL}} e^{-mx}\left(\frac{e^{mL}}{e^{mL}}\right)$$

$$\theta(x) = \frac{\theta_s}{e^{-mL}+e^{mL}} e^{mx} e^{-mL} + \frac{\theta_s}{e^{mL}+e^{-mL}} e^{-mx} e^{mL}$$

$$\theta(x) = \theta_s\left(\frac{e^{-m(L-x)}}{e^{-mL}+e^{mL}} + \frac{e^{m(L-x)}}{e^{mL}+e^{-mL}}\right)$$

$$\theta(x) = \theta_s\left(\frac{e^{-m(L-x)}+e^{m(L-x)}}{e^{-mL}+e^{mL}}\right) = \theta_s\left(\frac{\left(e^{-m(L-x)}+e^{m(L-x)}\right)/2}{\left(e^{-mL}+e^{mL}\right)/2}\right)$$

Noting that $\quad Sinh(mL) = \dfrac{e^{mL}-e^{-mL}}{2} \qquad Cosh(mL) = \dfrac{e^{mL}+e^{-mL}}{2}$

The temperature distribution is:

$$\theta(x) = \theta_s\left(\frac{\cosh m(L-x)}{\cosh(mL)}\right) \qquad 2.44$$

The heat loss from the fin can be found by substituting the temperature gradient at the root into Eq.(2.37), we get

$$\frac{d\theta(x)}{dx} = \theta_s \frac{-m\sinh m(L-x)}{\cosh(mL)}$$

$$\left.\frac{d\theta(x)}{dx}\right|_{x=0} = \theta_s \frac{-m\sinh mL}{\cosh mL} = -\theta_s m \tan mL$$

$$q_{fin} = -kA\left.\frac{d\theta}{dx}\right|_{x=0}$$

$$q_{fin} = \sqrt{hPAk}\cdot\theta_s \tanh(mL) \qquad 2.45$$

Case 3

The second boundary condition is :

B.C. 1 $\theta_s = C_1 + C_2$ $C_2 = \theta_s - C_1$

B.C.2 $\theta(x) = \theta_L$ *at* $x=L$

Substituting in *B.C.2*

$$\theta(x) = C_1\ e^{mx} + C_2\ e^{-mx} \qquad 2.37$$

$$\theta_{(L)} = C_1\ e^{mL} + C_2\ e^{-mL}$$

$$\theta_{(L)} = C_1\ e^{mL} + (\theta_s - C_1)\ e^{-mL}$$

$$C_1 = \frac{\theta_{(L)} - \theta_s \cdot e^{-mL}}{e^{mL} - e^{-mL}}$$

$$C_2 = \theta_s - \frac{\theta_{(L)} - \theta_s \cdot e^{-mL}}{e^{mL} - e^{-mL}} = \frac{\theta_s(e^{mL} - e^{-mL}) - \theta_{(L)} + \theta_s \cdot e^{-mL}}{e^{mL} - e^{-mL}}$$

$$C_2 = \frac{\theta_s \cdot e^{mL} - \theta_{(L)}}{e^{mL} - e^{-mL}}$$

Substituting the above relations for C_1 and C_2 into *Eq.(2.37)*

$$\theta(x) = \frac{\theta_{(L)} - \theta_s \cdot e^{-mL}}{e^{mL} - e^{-mL}}\ e^{mx} + \frac{\theta_s \cdot e^{mL} - \theta_{(L)}}{e^{mL} - e^{-mL}}\ e^{-mx}$$

$$\theta(x) = \theta_s\left[\frac{(\theta_{(L)}/\theta_s)(e^{mx} - e^{-mx}) + e^{m(L-x)} - e^{-m(L-x)}}{e^{mL} - e^{-mL}}\right]$$

$$\theta(x) = \theta_s\left[\frac{\left(\frac{\theta_{(L)}}{\theta_s}\right)\left(\frac{e^{mx} - e^{-mx}}{2}\right) + \left(\frac{e^{m(L-x)} - e^{-m(L-x)}}{2}\right)}{\frac{e^{mL} - e^{-mL}}{2}}\right]$$

The temperature distribution is:

$$\theta(x) = \theta_s\left[\frac{(\theta_{(L)}/\theta_s)\sinh mx + \sinh m(L-x)}{\sinh mL}\right] \qquad 2.46$$

The heat loss from the fin can be found by substituting the temperature gradient at the root into Eq.(2.37), we get

$$\left.\frac{d\theta(x)}{dx}\right|_{x=0} = \theta_s\left[\frac{\{(\theta_{(L)}/\theta_s)m \cdot \cosh mx + (-m)\cosh m(L-x)\}(\sinh mL) - 0}{(\sinh mL)^2}\right]$$

$$\left.\frac{d\theta(x)}{dx}\right|_{x=0} = \theta_s\left[\frac{\{(\theta_{(L)}/\theta_s)m \cdot -m\cosh mL\}(\sinh mL)}{(\sinh mL)^2}\right]$$

$$\left.\frac{d\theta(x)}{dx}\right|_{x=0} = m\theta_s\left[\frac{(\theta_{(L)}/\theta_s) - \cosh mL}{\sinh mL}\right]$$

$$q_{fin} = -kA\left.\frac{d\theta}{dx}\right|_{x=0} = -m\theta_s kA\left[\frac{-(\theta_{(L)}/\theta_s) + \cosh mL}{\sinh mL}\right]$$

$$q_{fin} = \sqrt{\frac{hP}{kA}}\theta_s kA\left[\frac{\cosh mL - (\theta_{(L)}/\theta_s)}{\sinh mL}\right]$$

$$q_{fin} = M\left[\frac{\cosh mL - (\theta_{(L)}/\theta_s)}{\sinh mL}\right] \qquad 2.47$$

Noting that $M = \sqrt{hPAk}\cdot\theta_s$

Case 4

The second boundary condition is:

B.C. 1 $\theta_s = C_1 + C_2$ $\qquad C_2 = \theta_s - C_1$

B.C.2 $-k\left.\dfrac{d\theta(x)}{dx}\right|_{x=L} = h\theta(L)$

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx} \qquad (2.37)$$

$$\theta(L) = C_1 e^{mL} + C_2 e^{-mL}$$

$$\left.\frac{d\theta(x)}{dx}\right|_{x=L} = mC_1e^{mL} - mC_2e^{-mL}$$

Substituting above equations in *B.C.2*

$$-k(mC_1e^{mL} - mC_2e^{-mL}) = h(C_1 e^{mL} + C_2 e^{-mL})$$

Substituting B.C.2

$$-k(mC_1e^{mL} - m(\theta_s - C_1)e^{-mL}) = h(C_1 e^{mL} + (\theta_s - C_1) e^{-mL})$$

$$C_1 = \frac{\theta_s (e^{-2mL} - e^{-2mL}(h/km))}{e^{2mL} + (h/km)e^{2mL} - (h/km) + 1}$$

$$C_1 = \frac{\theta_s (e^{-2mL} - (h/km)e^{-2mL})}{e^{2mL} + (h/km)e^{2mL} + 1 - (h/km)}$$

$$C_1 = \frac{\theta_s (e^{-2mL} - (h/km)e^{-2mL})}{e^{2mL} + (h/km)e^{2mL} + 1 - (h/km)}$$

$$C_2 = \theta_s - \frac{\theta_s e^{-2mL}(1 - (h/km))}{e^{2mL} + (h/km)e^{2mL} + 1 - (h/km)}$$

$$C_2 = \frac{\theta_s(e^{2mL} + (h/km)e^{2mL} + 1 - (h/km)) - \theta_s e^{-2mL}(1 - (h/km))}{e^{2mL} + (h/km)e^{2mL} + 1 - (h/km)}$$

$$C_2 = \theta_s \frac{e^{2mL} + (h/km)e^{2mL} + 1 - (h/km) - e^{-2mL} + e^{-2mL}(h/km))}{e^{2mL} + (h/km)e^{2mL} + 1 - (h/km)}$$

$$C_2 = \theta_s \frac{e^{2mL} - e^{-2mL} + e^{2mL}(h/km) + e^{-2mL}(h/km) + 1 - (h/km))}{e^{2mL} + (h/km)e^{2mL} + 1 - (h/km)}$$

Substituting the above relations for C_1 and C_2 into *Eq.(2.37)*

$$\theta(x) = \frac{\theta_s (e^{-2mL} - (h/km)e^{-2mL})}{e^{2mL} + (h/km)e^{2mL} + 1 - (h/km)} e^{mx} +$$

$$\theta_s \frac{e^{2mL} - e^{-2mL} + e^{2mL}(h/km) + e^{-2mL}(h/km) + 1 - (h/km))}{e^{2mL} + (h/km)e^{2mL} + 1 - (h/km)} e^{-mx}$$

$$\theta(x) = \theta_s \frac{(h/km)e^{m(L-x)} - (h/km)e^{-m(L-x)} + e^{m(L-x)} + e^{-m(L-x)}}{e^{mL} + e^{-mL} + (h/km)e^{mL} - (h/km)e^{-mL}}$$

The temperature distribution is:

$$\theta(x) = \theta_s \frac{(h/km)\sinh m(L-x) + \cosh m(L-x)}{(h/km)\sinh mL + \cosh mL} \qquad 2.48$$

The heat loss from the fin can be found by substituting the temperature gradient at the root into Eq.(2.37), we get

$$\left.\frac{d\theta(x)}{dx}\right|_{x=0} = \theta_s \frac{(-m(h/km)\cosh mL - (-m\sinh mL))(\cosh mL + (h/km)\sinh mL) - 0}{((h/km)\sinh mL + \cosh mL)^2}$$

Table 2

$$\left.\frac{d\theta(x)}{dx}\right|_{x=0} = -m\theta_s \frac{(h/km)\cosh mL - \sinh mL)}{(h/km)\sinh mL + \cosh mL}$$

$$q_{fin} = -kA\left.\frac{d\theta}{dx}\right|_{x=0} = m\theta_s kA_s \frac{(h/km)\cosh mL - \sinh mL)}{(h/km)\sinh mL + \cosh mL}$$

$$q_{fin} = \sqrt{\frac{hP}{kA}}\theta_s kA \frac{(h/km)\cosh mL - \sinh mL)}{(h/km)\sinh mL + \cosh mL}$$

$$q_{fin} = M\frac{(h/km)\cosh mL - \sinh mL)}{(h/km)\sinh mL + \cosh mL} \qquad 2.49$$

Noting that $M = \sqrt{hPAk}\cdot\theta_s$

Temperature distribution and rate of heat transfer for fins

Case	Tip Condition ($x = L$)	Temperature Distribution, θ/θ_s	Fin Heat Transfer Rate, q_{fin}
1	Infinite fin ($L \to \infty$): $\theta(L) = 0$	e^{-mx}	M
2	Adiabatic: $\left.\frac{d\theta}{dx}\right\|_{x=L} = 0$	$\frac{\cosh m(L-x)}{\cosh mL}$	$M \tanh mL$
3	Fixed temperature: $\theta(L) = \theta_L$	$\frac{(\theta_L/\theta_s)\sinh mx + \sinh m(L-x)}{\sinh mL}$	$M\frac{\cosh mL - (\theta_L/\theta_s)}{\sinh mL}$
4	Convection heat transfer: $\bar{h}\theta(L) = -k\left.\frac{d\theta}{dx}\right\|_{x=L}$	$\frac{\cosh m(L-x) + (\bar{h}/mk)\sinh m(L-x)}{\cosh mL + (\bar{h}/mk)\sinh mL}$	$M\frac{\sinh mL + (\bar{h}/mk)\cosh mL}{\cosh mL + (\bar{h}/mk)\sinh mL}$

$\theta = T - T_\infty$ $\qquad \theta_s = \theta(0) = T_s - T_\infty$

$M = \sqrt{hPAk}\cdot\theta_s$ $\qquad m^2 = \frac{\bar{h}P}{kA}$ $\qquad m = \sqrt{\frac{\bar{h}P}{kA}}$

P : Perimeter of the fin
A : Cross section area of fin

2.7.1 Fin Performance

The heat transfer effectiveness of a fin is measured by a parameter called fin effectiveness and the fin efficiency, which is defined as

I. ***Fin Effectiveness ε.*** A ratio of the fin heat transfer rate to the heat transfer rate that would exist without the fin.

$$\varepsilon_f = \frac{q_{fin}}{q_{withou\text{-}fin}} = \frac{q_{fin}}{hA_c(T_s - T_\infty)} \qquad 2.52$$

where A_c is the fin cross-sectional area at the base. the use of fins may rarely be justified unless $\varepsilon >= 2$.

II. ***Fin Efficiency η***

$$\eta_f = \frac{q_{fin}}{q_{\max}} \qquad 2.53$$

$$q_{\max} = hA_f(T_b - T_\infty) = hPL\theta_b \qquad 2.54$$

Where A_f is the surface area of the fin is

$$A_f = 2wL_c \qquad \text{Rectangular}$$

$$A_f = 2w\left[L^2 + (t/2)^2\right]^{1/2} \qquad \text{Triangular}$$

$$A_f = 2.05w\left[L^2 + (t/2)^2\right]^{1/2} \qquad \text{Parabolic}$$

$$A_f = 2\pi\left(r_{2c}^2 - r_1^2\right) \qquad \text{Annular}$$

Where as for a fin of rectangular cross section (*length **L** & thickness **t***) and an adiabatic end (Case 2) is

$$\eta_f = \frac{M \tanh mL}{hPL\theta_b} = \frac{\tanh mL}{mL} \qquad 2.55$$

a corrected fin length of the form $L_c = L + (t/2)$.

$$\eta_f = \frac{\tanh mL_c}{mL_c} \qquad or \qquad \eta_f = \frac{\tanh\sqrt{\bar{h}PL^2/kA}}{\sqrt{\bar{h}PL^2/kA}}$$

A fin efficiency for a circular pin fin (Diameter ***D*** & Length ***L***) and an adiabatic end (Case 2) is

$$\eta_f = \frac{\tanh\sqrt{4L^2\bar{h}/kD}}{\sqrt{4L^2\bar{h}/kD}} \qquad 2.56$$

In Figures *2.14* and *2.15* fin efficiencies are plotted as a function of the parameter $L_c^{3/2}(h/kA_p)^{1/2}$ inferred for the straight and the annular fins. Fin efficiencies obtained from the figures may be used to calculate the actual fin heat transfer rate from the expression

$$q_f = \eta_f q_{\max} = \eta_f hA_f\theta_b \qquad 2.57$$

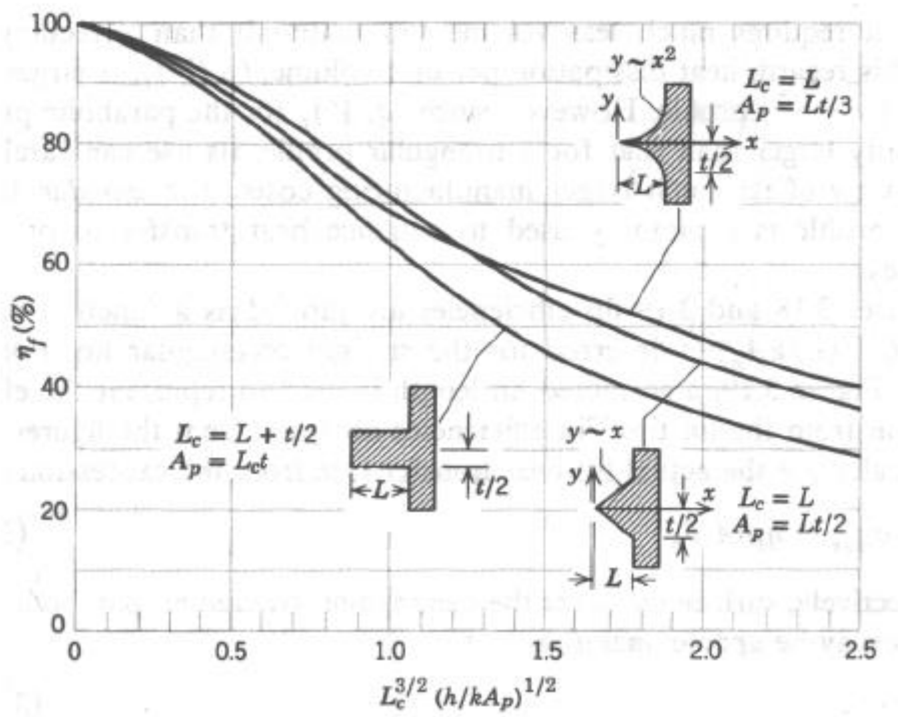


Figure 2.14 *Efficiency of straight fins (rectangular, triangular, and parabolic profiles).*

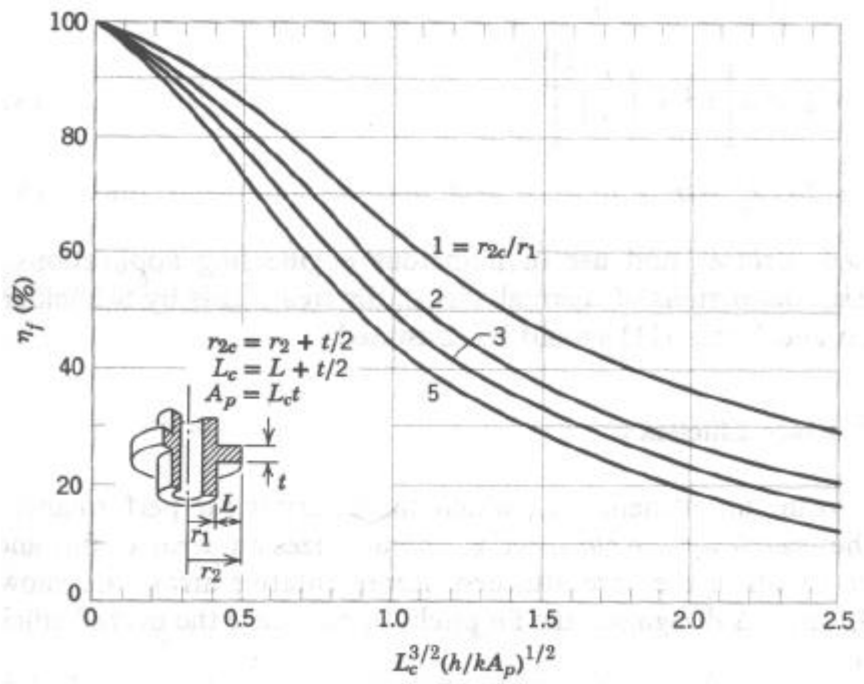


Figure 2.15 *Efficiency of annular fins of rectangular profile.*

Example 2.10

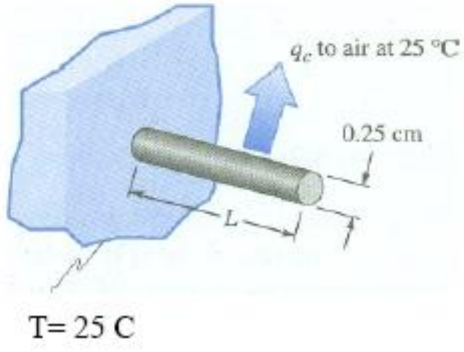
Consider a copper pin fin *0.25 cm* in diameter *k = 396 W/m K* that protrudes from a wall at *95°C* into ambient air at *25°C*. The heat transfer is mainly by natural convection with a coefficient equal to *10 W/m2 K*. Calculate the heat loss, assuming that :

(a) The fin is "*infinitely long*"

(b) The fin is 2.5 cm long and the coefficient at the end is the same as around the circumference.

(c) How long would the fin have to be for the infinitely long solution to be correct within 5%?

Solution



(a) A heat loss for the "*Infinitely long*" fin is

$$q_{fin} = -kA\left(-m\theta(0)e^{(-m)0}\right) = \sqrt{hPAk}\,\theta_s$$

$$q = [(10\ W/m^2\ K)\ \pi(0.0025\ m)(396\ W/m\ K)\ (\pi/4(0.0025\ m)^2]^{0.5}\ (95\text{-}25)°C$$
$$q = 0.865\ W$$

(b) The equation for the heat loss from the finite fin is case 4:

$$q_{fin} = \sqrt{hPAk}\,\theta_s \frac{\sinh mL + (\bar{h}/mk)\cosh mL}{\cosh mL + (\bar{h}/mk)\sinh mL} = 0.140\ W$$

(c) For the two solutions to be within *5%*, it is necessary that

$$\frac{\sinh mL + (\bar{h}/mk)\cosh mL}{\cosh mL + (\bar{h}/mk)\sinh mL} >= 0.95$$

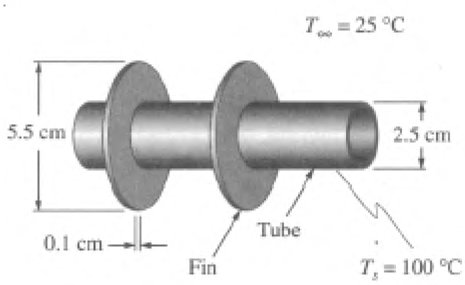
This condition is satisfied when *mL > 1.8 or L > 28.3 cm*.

Example 2.11

To increase the heat dissipation from a 2.5 cm OD tube, circumferential fins made of aluminum (*k = 200 W/m K*) are soldered to the outer surface. The fins are 0.1 cm thick and have an outer diameter of 5.5 cm. If the tube temperature is 100°C, the environmental temperature is 25°C, and the heat transfer coefficient between the fin and the environment is 65 W/m2 K, calculate the rate of heat loss from two fins.

Solution

a parameters required to obtain the fin efficiency curve in Fig. 2.15 are

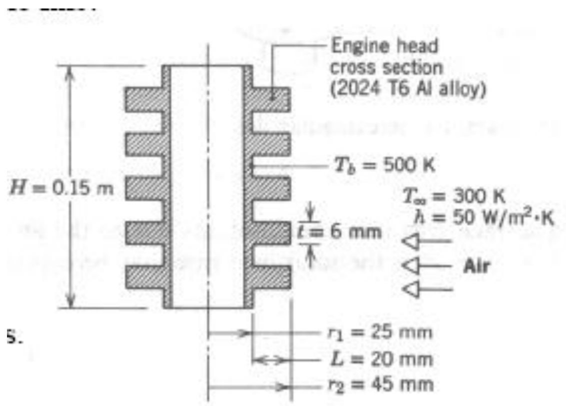


Example 2.12

The cylinder barrel of a motorcycle is constructed of 2024-T6 aluminum alloy (k = 186 W/m K) and is of height H = 0.15 m and OD = 50 mm. Under typical operating conditions the outer surface of the cylinder is at a temperature of 500 K and is exposed to ambient air at 300 K, with a convection coefficient of 50 W/m² K. Annular fins of rectangular profile are typically added to increase heat transfer to the surroundings. Assume that five (*N=5*) such fins, which are of thickness *t = 6 mm*, length *L = 20 mm* and equally spaced, are added. What is the increase in heat transfer due to addition of the fins?

Solution

Assumptions:
1. Steady-state conditions.
2. One-dimensional radial conduction in fins.
3. Constant properties.
4. No internal heat generation.
5. Negligible radiation exchange with surroundings.
6. Uniform convection coefficient over outer surface (with or without fins).



With the fins in place, the heat transfer rate is $q=q_f+q_b$

$$q_f = N\eta_f q_{max} = N\eta_f hA_f\theta_b$$
$$q_f = N\eta_f h2\pi\left(r_{2c}^2 - r_1^2\right)(T_b - T_\infty)$$

Heat. transfer from the exposed cylinder surface is

$$q = hA_b(T_b - T_\infty) \qquad A_b = (H - Nt)2\pi r_1$$

Hence

$$q = N\eta_f h2\pi\left(r_{2c}^2 - r_1^2\right)(T_b - T_\infty) + h(H - Nt)2\pi r(T_b - T_\infty)$$

The fin efficiency may be obtained from Figure 2.19 with

$$r_{2c} = r_2 + \frac{t}{2} = 0.048\text{ m}, \quad L_c = L + \frac{t}{2} = 0.023\text{ m}$$

$$\frac{r_{2c}}{r_1} = 1.92, \quad A_p = L_c t = 1.38 \times 10^{-4}\text{ m}^2, \quad L_c^{3/2}\left(\frac{h}{kA_p}\right)^{1/2} = 0.15$$

Hence from Figure 3.19, $\eta_f \approx 0.95$. It follows that

$$q = 5\{0.95 \times 50\text{ W/m}^2\cdot\text{K} \times 2\pi[(0.048^2 - 0.025^2)\text{ m}^2] \times (500 - 300)\text{ K}\}$$
$$+ 50\text{ W/m}^2\cdot\text{K}\,(0.15 - 5 \times 0.006)(2\pi \times 0.025)\text{ m}^2 \times (500 - 300)\text{ K}$$

Hence *q = 5 (100.22) + 188.5 = 690 W*

Without the fins, the heat transfer rate is

$$q_f = hA_{wo}(T_b - T_\infty) \qquad A_{wo} = H(2\pi r_1)$$

Hence q_{wo} = *50 W/m² K (0.15 x л x 0.025) m² (200 K) = 236 W*