الجامعة التكنولوجية
قسم هندسة المكائن و المعدات

Fluid Mechanics II
ميكانيك الموائع(2)
(ملخص)

3rd year Mechanical Engineering
المرحلة الثالثة ـ عام/تكييف/سيارات
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مدرسو المادة

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Syllabus		مفردات المـادة (فرع الميكانيك العام)

Subject No.: ME\653 Subject: Fluid Mechanics II Units:5 Weekly Hours: Theoretical:2 　　　　　　　　Tutorial:1 　　　　　　　　Practical:1	رقم الموضوع: همك/ 653 الموضوع: ميكانيك الموائع II الوحدات: 5 الساعات الإسبوعية: نظري:2 مناقشة:1 عملي:1

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2	- Laminar flow between parallel 　　plates 　- Couette flow	ـ الجريان الطباقي بين صفيحتين متوازيتين ـ جريان كوتا	2
3	- Hydrodynamic lubrication	ـ التزييت الهيدروديناميكي	3
4	- Sliding bearing 　- Collar bearing	ـ المحمل الانزلاقي ـ المحمل ذو أطواق	4
5	- Laminar flow between coaxial 　　rotating cylinders	ـ الجريان الطباقي بين اسطوانتين متمركزتين دوارتين	5
6	**Boundary layer theory** 　- Introduction 　- Displacement, Momentum, and 　　Energy thicknesses.	نظرية الطبقة المتاخمة ـ مقدمة ـ سمك الإزاحة ، سمك الزخم ، سمك الطاقة	6
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10	- Effect of pressure gradient 　- Separation and pressure drag	ـ تأثير انحدار الضغط ـ الانفصال و الكبح الناشئ عن الضغط	10
11	**Hydraulic machines** 　- General introduction 　- Classification	المكائن الهيدروليكية ـ مقدمة عامة ـ التصنيف	11
12	**Centrifugal pump** 　- The moment of momentum 　　equation 　- Velocity diagrams 　- Efficiencies	المضخة الطاردة المركزية ـ معادلة عزم الزخم ـ مثلثات السرعة ـ الكفاءات	12
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Syllabus		مفردات المادة (فرعي التكييف و السيارات)

Subject No.: ME\663	رقم الموضوع: همك/ 663
Subject: Fluid Mechanics II	الموضوع: ميكانيك الموائع II
Units:5	الوحدات :5
Weekly Hours: Theoretical:2	الساعات الإسبوعية: نظري:2
Tutorial:1	مناقشة:1
Practical:1	عملي:1

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2	- Laminar flow between parallel plates - Couette flow	ـ الجريان الطباقي بين صفيحتين متوازيتين ـ جريان كوتا	2
3	- Hydrodynamic lubrication	ـ التزييت الهيدروديناميكي	3
4	- Sliding bearing - Collar bearing	ـ المحمل الانزلاقي ـ المحمل ذو أطواق	4
5	- Laminar flow between coaxial rotating cylinders	ـ الجريان الطباقي بين اسطوانتين متمركزتين دوارتين	5
6	**Boundary layer theory** - Introduction - Displacement, Momentum, and Energy thicknesses.	نظرية الطبقة المتاخمة ـ مقدمة ـ سمك الإزاحة ، سمك الزخم ، سمك الطاقة	6
7	- Momentum equation for the boundary layer.	ـ معادلة الزخم للطبقة المتاخمة	7
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9	- Turbulent boundary layer - Transition from laminar to turbulent flow	ـ الطبقة المتاخمة الاضطرابية ـ التحول من الجريان الطباقي إلى الاضطرابي	9
10	- Effect of pressure gradient - Separation and pressure drag	ـ تأثير انحدار الضغط ـ الانفصال و الكبح الناشئ عن الضغط	10
11	- Separation of flow inside pipes and ducts	ـ انفصال الجريان في الأنابيب و مجاري الهواء	11
12	**Pumps** - General introduction - Classification	المضخات ـ مقدمة عامة ـ التصنيف	12
13	**Centrifugal pump** - The moment of momentum equation - Velocity diagrams - Efficiencies	المضخة الطاردة المركزية ـ معادلة عزم الزخم ـ مثلثات السرعة ـ الكفاءات	13
14	- Types of impellers - Performance curves	ـ أنواع الدفاعات ـ منحنيات الأداء	14
15	- Operating point - Pump connection (series / parallel)	ـ نقطة الاشتغال ـ ربط المضخات (توالي / توازي)	15

References   المصادر

1- Fluid Mechanics   By Streeter & Wylie

2- Fluid Mechanics with Engineering Applications
                        By Daugherty & Franzini

3- Introduction to Fluid Mechanics
                        By Fox & McDonald

4- Engineering Fluid Mechanics
                        By Roberson & Crowe

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ملحوظة:
هذه المحاضرات تتضمن ملخص لمادة ميكانيك الموائع **II** للمرحلة الثالثة
وهي لا تغني الطالب عن حضور الساعات النظرية المقررة في جدول
الدروس الأسبوعي.

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Chapter One
Navier – Stokes Equations

Contents

1- Navier-Stokes equations:

The general equations of motion for viscous incompressible, Newtonian fluids may be written in the following form:

x- direction:

$$\rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right] = \rho g_x - \frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right] \text{------(1)}$$

y- direction:

$$\rho\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right] = \rho g_y - \frac{\partial p}{\partial y} + \mu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right] \text{------(2)}$$

Equations (1) and (2) are called: Navier Stokes equations.
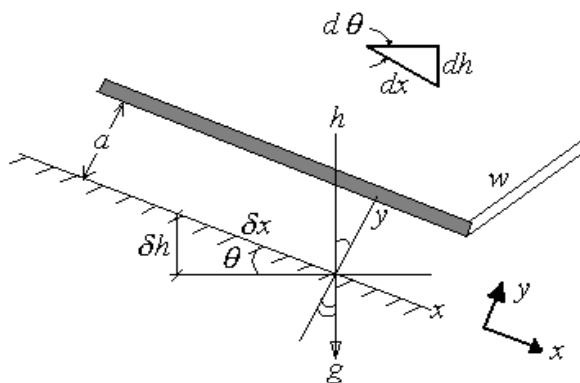
2- Steady laminar flow between parallel flat plates:

Fig.25

The fluid moves in the x- direction without acceleration.

v= 0 , w= 0 , $\frac{\partial}{\partial t} = 0$

the Navier-Stokes equation in the x- direction (eq. 1) reduces to:

$$-\rho g_x + \frac{dp}{dx} = \mu \frac{d^2u}{dy^2} \quad \text{---------(3)}$$

$$g_x = g.\sin\theta = -g\frac{dh}{dx}$$

eq. 3 will be

$$\frac{d^2u}{dy^2} = \frac{1}{\mu}\frac{d(p+\gamma h)}{dx} \quad \text{---------(4)}$$

Integration of eq. 4:

$$\frac{du}{dy} = \frac{1}{\mu}\frac{d(p+\gamma h)}{dx}y + A$$

$$u = \frac{1}{2\mu}\frac{d(p+\gamma h)}{dx}y^2 + Ay + B \quad \text{----------(5)}$$

B.C (Two fixed parallel plates)

$y = 0 \qquad u = 0 \quad \Rightarrow \quad B = 0$

$y = a \qquad u = 0 \quad \Rightarrow \quad A = \dfrac{-a}{2\mu}\dfrac{d(p+\gamma h)}{dx}$

eq. 5 will be

$$u = \frac{1}{2\mu}\frac{d(p+\gamma h)}{dx}(y^2 - ay) \quad \text{-----------(6)}$$

B.C (One plate is fixed and the other plate moves with a constant velocity U) (Couette flow)

$y = 0 \qquad u = 0 \quad \Rightarrow \quad B = 0$

$y = a \qquad u = U \quad \Rightarrow \quad A = \dfrac{U}{a} - \dfrac{a}{2\mu}\dfrac{d(p+\gamma h)}{dx}$

eq. 5 will be

$$u = \frac{1}{2\mu}\frac{d(p+\gamma h)}{dx}(y^2 - ay) + \frac{Uy}{a} \quad \text{-----------(7)}$$

For the case of horizontal parallel plates:

$$\frac{dh}{dx} = 0$$

eq. 7 will be

$$u = \frac{1}{2\mu}\frac{dp}{dx}(y^2 - ay) + \frac{Uy}{a} \quad \text{----------(8)}$$

The location of maximum velocity u_{max} may be found by evaluating $\dfrac{du}{dy}$ and setting it to zero.

The volume flow rate is

$$Q = w\int_0^a u.dy \quad ----------(9)$$

3- Hydrodynamic lubrication:

Sliding bearing

Large forces are developed in small clearance when the surfaces are slightly inclined and one is in motion so that fluid is wedged into the decreasing space. Usually the oils employed for lubrication are highly viscous and the flow is of laminar nature.
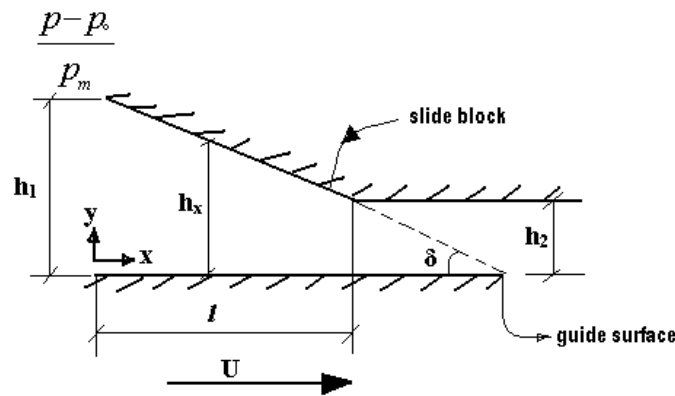
Fig.32

Assumptions:

The acceleration is zero.

The body force is small and can be neglected.

Also $\dfrac{\partial^2 u}{\partial y^2} \gg \dfrac{\partial^2 u}{\partial x^2}$ and $\dfrac{\partial^2 u}{\partial y^2} \gg \dfrac{\partial^2 u}{\partial z^2}$

The Navier-Stokes equation in the x-direction (eq. 1) reduces to:

$$\frac{d^2 u}{dy^2} = \frac{1}{\mu}\frac{dp}{dx}$$

Integration:

$$u = \frac{1}{2\mu}\left(\frac{dp}{dx}\right)y^2 + Ay + B$$

B.C

$y = 0 \qquad u = U \quad \Rightarrow \quad B = U$

$y = h_x \qquad u = 0 \quad \Rightarrow \quad A = \dfrac{-h_x}{2\mu}\dfrac{dp}{dx} - \dfrac{U}{h_x}$

$$u = \frac{1}{2\mu}\frac{dp}{dx}\left(y^2 - h_x y\right) + U\left(1 - \frac{y}{h_x}\right)$$

The volume flow rate in every section will be constant.

$$Q = w\int_0^{h_x} u.dy \qquad \text{assume } w = 1$$

$$\therefore \quad Q = \frac{Uh_x}{2} - \frac{h_x^3}{12\mu}\frac{dp}{dx} \quad \text{--------(*)}$$

** For a constant taper bearing:

$$\delta = \frac{h_1 - h_2}{l}$$

$$\therefore \quad h_x = (h_1 - \delta x)$$

Sub in eq.(*) and solving for $\frac{dp}{dx}$ produces:

$$\frac{dp}{dx} = \frac{6\mu U}{(h_1 - \delta x)^2} - \frac{12\mu Q}{(h_1 - \delta x)^3}$$

Integration gives:

$$p(x) = \frac{6\mu U}{\delta(h_1 - \delta x)} - \frac{6\mu Q}{\delta(h_1 - \delta x)^2} + C \quad \text{---------(**)}$$

B.C
$$x = 0 \qquad p = p_o = 0$$
$$x = l \qquad p = p_o = 0$$

$$\Rightarrow \quad Q = \frac{Uh_1 h_2}{h_1 + h_2} \quad \text{and} \quad C = \frac{-6\mu U}{\delta(h_1 + h_2)}$$

With these values inserted in eq.(**) we obtain the pressure distribution inside the bearing.

$$p(x) = \frac{6\mu U x(h_x - h_2)}{h_x^2(h_1 + h_2)}$$

The load that the bearing will support per unit width is:
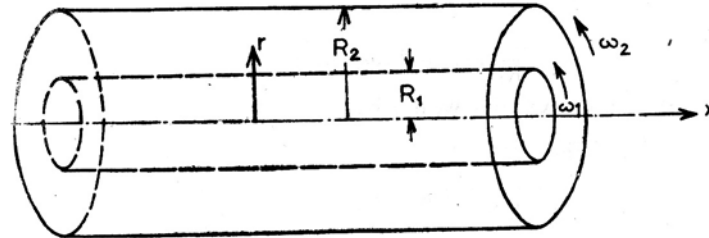
$$F = \int_0^l p(x).dx$$

$$F = \frac{6\mu U l^2}{(h_1 - h_2)^2}\left[\ln k - \frac{2(k-1)}{k+1}\right]$$
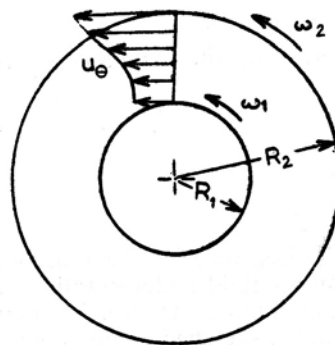
where
$$k = \frac{h_1}{h_2}$$

4- Laminar flow between concentric rotating cylinders:

Consider the purely circulatory flow of a fluid contained between two long concentric rotating cylinders of radius R_1 and R_2 at angular velocities ω_1 and ω_2.

(a) PICTORIAL REPRESENTATION

(b) VELOCITY PROFILE

In this case the Navier-Stokes equations in cylindrical coordinates are used.

r- direction:
$$\frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta^2}{r} + w\frac{\partial u_r}{\partial z} = \frac{-1}{\rho}\frac{\partial p}{\partial r} + \frac{\mu}{\rho}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_r}{\partial r}\right) - \frac{u_r}{r^2} + \frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial^2 u_r}{\partial z^2}\right] + g_r$$

θ- direction:
$$\frac{\partial u_\theta}{\partial t} + u_r\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r u_\theta}{r} + w\frac{\partial u_\theta}{\partial z} = \frac{-1}{\rho r}\frac{\partial p}{\partial \theta} + \frac{\mu}{\rho}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_\theta}{\partial r}\right) - \frac{u_\theta}{r^2} + \frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial u_r}{\partial \theta} + \frac{\partial^2 u_\theta}{\partial z^2}\right] + g_\theta$$

In the above equations:
$u_r = 0$
$w = 0$
$\frac{\partial}{\partial t} = 0$, $\frac{\partial u_\theta}{\partial \theta} = 0$, $\frac{\partial p}{\partial \theta} = 0$
body force = 0

The equation in θ- direction reduces to:
$$\frac{d^2 u_\theta}{dr^2} + \frac{d}{dr}\left(\frac{u_\theta}{r}\right) = 0$$

Integration:

$$\frac{1}{r}\frac{d}{dr}(ru_\theta) = A$$

$$u_\theta = Ar + \frac{B}{r} \quad \text{---------(i)}$$

B.C

$r = R_1 \qquad u_\theta = R_1\omega_1$

$r = R_2 \qquad u_\theta = R_2\omega_2$

$$\Rightarrow \quad A = \omega_1 + \frac{R_2^2}{R_2^2 - R_1^2}(\omega_2 - \omega_1)$$

$$B = -\frac{R_1^2 R_2^2}{R_2^2 - R_1^2}(\omega_2 - \omega_1)$$

Sub. in eq.(i) yields:

$$u_\theta = \frac{1}{R_2^2 - R_1^2}\left[(\omega_2 R_2^2 - \omega_1 R_1^2)r - \frac{R_1^2 R_2^2}{r}(\omega_2 - \omega_1)\right] \quad \text{-------(ii)}$$

The shear stress may be evaluated by the equation:

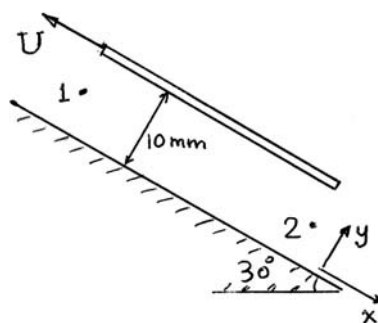$$\tau = \mu\left[r\frac{d}{dr}\left(\frac{u_\theta}{r}\right)\right]$$

By using eq.(ii):

$$\tau = \frac{2\mu}{R_2^2 - R_1^2}\frac{R_1^2 R_2^2}{r^2}(\omega_2 - \omega_1)$$

5- *Examples:*

1- Using the Navier-Stokes equation in the flow direction, calculate the power required to pull (1m × 1m) flat plate at speed (1 m/s) over an inclined surface. The oil between the surfaces has ($\rho = 900$ kg/m^3, $\mu = 0.06$ Pa.s).The pressure difference between points 1 and 2 is (100 kN/m^2) .

Solution:

The Navier-Stokes equation in x- direction

$$\rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right] = \rho g_x - \frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right]$$

We have: Acceleration =0 , v=0 , w=0 , $\frac{\partial^2 u}{\partial x^2} = 0$, $\frac{\partial^2 u}{\partial z^2}$

The equation reduces to:

$$\frac{d^2 u}{dy^2} = \frac{1}{\mu}\frac{dp}{dx} - \frac{\rho}{\mu}g_x$$

Integration

$$\frac{du}{dy} = \frac{1}{\mu}\frac{dp}{dx}y - \frac{\rho}{\mu}g_x y + A$$

$$u = \frac{1}{2\mu}\frac{dp}{dx}y^2 - \frac{\rho}{2\mu}g_x y^2 + Ay + B$$

B.C   (b=10 mm)

$y = 0$ $u = 0$ $\Rightarrow$ $B = 0$

$y = b$ $u = -U$ $\Rightarrow$ $A = \frac{-U}{b} - \frac{b}{2\mu}\frac{dp}{dx} + \frac{\rho b}{2\mu}g_x$

$$\therefore \quad \frac{du}{dy} = \frac{1}{\mu}\frac{dp}{dx}y - \frac{\rho}{\mu}g_x y - \frac{U}{b} - \frac{b}{2\mu}\frac{dp}{dx} + \frac{\rho b}{2\mu}g_x$$

The shearing force on the moving plate:

$$F = \tau_o \times area$$

$$F = \mu \cdot \frac{du}{dy}\bigg|_{y=b} \times area$$

area=1 m^2

$$F = -\frac{\mu U}{b} + \frac{b}{2}\frac{dp}{dx} - \frac{b}{2}\rho g_x$$

We have $g_x = g \cdot \sin\theta$, $\frac{dp}{dx} = \frac{-\Delta p}{l}$

$$F = \frac{-0.06 \times 1}{0.01} - \frac{0.01}{2}\left(\frac{100 \times 10^3}{1}\right) - \frac{0.01}{2} \times 900 \times 9.81 \times \sin 30$$

$$F = -528\,N$$

$$Power = F \cdot U$$

$$Power = 528 \times 1 = 528\,W \quad (Ans)$$

2- The external and internal diameters of a **collar bearing** are (20 cm and 15 cm) respectively. An oil film (0.3 mm) thick is maintained between the collar surface and the bearing. Find the power lost in overcoming the viscous resistance of the oil when the shaft is running at (240 rpm). Take μ=0.1 Pa.s .

Solution:

The figure shows two views of a collar bearing.
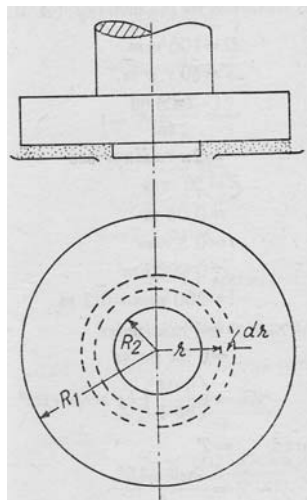
Consider an elemental angular circular strip of the bearing surface of radius r and width dr. Circumferential resistance offered by the oil film over the strip

$$= \tau \times area$$

$$= \mu \frac{\omega r}{h} \times 2\pi r dr = \frac{2\pi \mu \omega r^2 dr}{h}$$

Torque required to overcome this resistance:

$$T = \frac{2\pi \mu \omega r^2 dr}{h} \times r = \frac{2\pi \mu \omega r^3 dr}{h}$$

Total torque required $T = \int_{R_2}^{R_1} \frac{2\pi \mu \omega r^3 dr}{h} = \frac{\pi \mu \omega}{2h}\left(R_1^4 - R_2^4\right)$

Power absorbed by the bearing $= T\omega = \frac{\pi \mu \omega^2}{2h}\left(R_1^4 - R_2^4\right)$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 240}{60} = 8\pi \text{ rad/s}$$

$$\text{Power} = \frac{\pi \times 0.1(8\pi)^2}{2 \times 0.0003}\left(0.1^4 - 0.075^4\right) = 22.6 \text{ W} \quad \text{(Ans)}$$

1- Using the Navier-Stokes equations, determine the pressure gradient along flow, the average velocity, and the discharge for an oil of viscosity 0.02 $N.s/m^2$ flowing between two stationary parallel plates 1 m wide maintained 10 mm apart. The velocity midway between the plates is 2 m/s.   [-3200 N/m^2 per m ; 1.33 m/s ; 0.0133 m^3/s]

2- An incompressible, viscous fluid is placed between horizontal, infinite, parallel plates as shown in figure. The two plates move in opposite directions with constant velocities U_1 and U_2. The pressure gradient in the x-direction is zero. Use the Navier-Stokes equations to derive expression for the velocity distribution between the plates. Assume laminar flow.

$$[u = \frac{y}{b}(U_1 + U_2) - U_2]$$

3- Two parallel plates are spaced 2 mm apart, and oil ($\mu = 0.1$ $N.s/m^2$, S = 0.8) flows at a rate of 24×10^{-4} m^3/s per m of width between the plates. What is the pressure gradient in the direction of flow if the plates are inclined at 60° with the horizontal and if the flow is downward between the plates?   [-353.2 kPa/m]

4- Using the Navier-Stokes equations, find the velocity profile for fully developed flow of water ($\mu = 1.14 \times 10^{-3}$ Pa.s) between parallel plates with the upper plate moving as shown in figure. Assume the volume flow rate per unit depth for zero pressure gradient between the plates is 3.75×10^{-3} m^3/s. Determine:
a- the velocity of the moving plate.
b- the shear stress on the lower plate.
c- the pressure gradient that will give zero shear stress at y = 0.25b. (b = 2.5 mm)
d- the adverse pressure gradient that will give zero volume flow rate between the plates.
          [3 m/s ; 1.37 N/m^2 ; 2.19 kN/m^2 per m ; -3.28 kN/m^2 per m]

5- A vertical shaft passes through a bearing and is lubricated with an oil ($\mu = 0.2$ Pa.s) as shown in figure. Estimate the torque required to overcome viscous resistance when the shaft is turning at 80 rpm. (Hint: The flow between the shaft and bearing can be treated as laminar flow between two flat plates with zero pressure gradient).   [0.355 N.m]

6- Determine the force on the piston of the figure due to shear, and the leakage from the pressure chamber for U = 0.   [295.1 N ; 1.636×10^{-8} m^3/s]

7- A layer of viscous liquid of thickness b flows steadily down an inclined plane. Show that, by using the Navier-Stokes equations that velocity distribution is: $u = \frac{\gamma}{2\mu}(2by - y^2)\sin\theta$ and that the discharge per unit width is: $Q = \frac{\gamma}{3\mu}b^3 \sin\theta$

8- A wide moving belt passes through a container of a viscous liquid. The belt moving vertically upward with a constant velocity V_o, as illustrated in figure. Because of viscous forces the belt picks up a film of fluid of thickness h. Gravity tends to make the fluid drain down the belt. Use the Navier-Stokes equations to determine an expression for the average velocity v_{av} of the fluid film as it is dragged up the belt. Assume the flow is laminar, steady, and uniform. $[v_{av} = V_o - \frac{\gamma h^2}{3\mu}]$

9- Determine the formulas for shear stress on each plate and for the velocity distribution for flow in the figure when an adverse pressure gradient exists such that Q = 0.
$$[\tau_{y=0} = \frac{-2\mu U}{b}; \tau_{y=b} = \frac{4\mu U}{b}; u = 3U\frac{y^2}{b^2} - 2U\frac{y}{b}]$$

10- A plate 2 mm thick and 1 m wide is pulled between the walls shown in figure at speed of 0.4 m/s. The space over and below the plate is filled with glycerin ($\mu = 0.62$ N.s/m^2). The plate is positioned midway between the walls. Using the Navier-Stokes equations, determine the force required to pull the plate at the speed given for zero pressure gradient; and the pressure gradient that will give zero volume flow rate.
[496 N ; 372 kN/m^2.m]

11- A slider plate 0.5 m wide constitutes a bearing as shown in figure. Estimate:
a- the load carrying capacity.
b- the drag.
c- the power lost in the bearing.
d- the maximum pressure in the oil and its location.
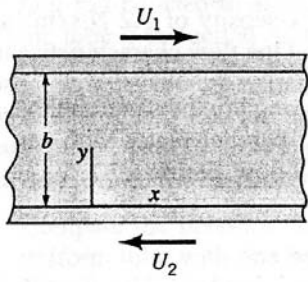[739.6 kN ; 348.6 N ; 348.6 W ; 12500 kN/m^2 ; 150 mm]

12- Consider a shaft that turns inside a stationary cylinder, with a lubricating fluid in the annular region. Using the Navier-Stokes equation in θ-direction, show that the torque per unit length acting on the shaft is given by: $T = \dfrac{4\pi\mu\omega R_1^2}{\left(\dfrac{R_1}{R_2}\right)^2 - 1}$
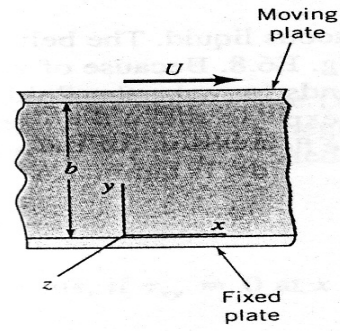
Where: ω = angular velocity of the shaft.
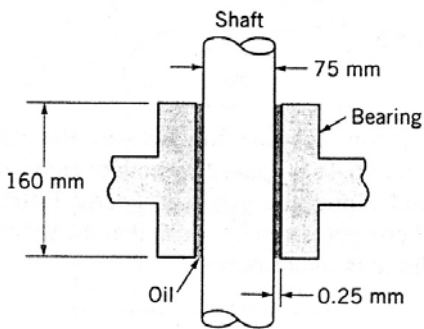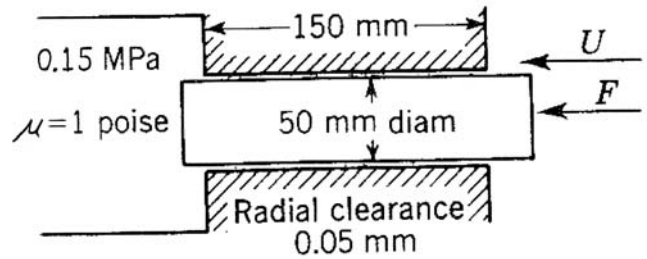 R_1 = radius of the shaft.
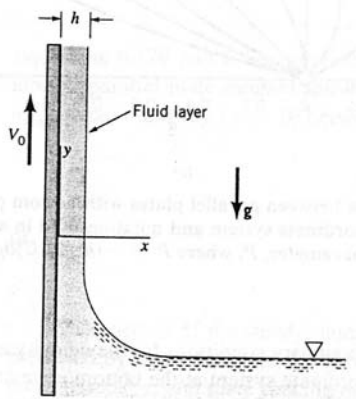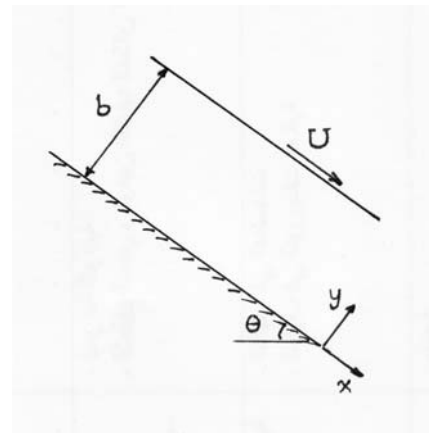 R_2 = radius of the cylinder.
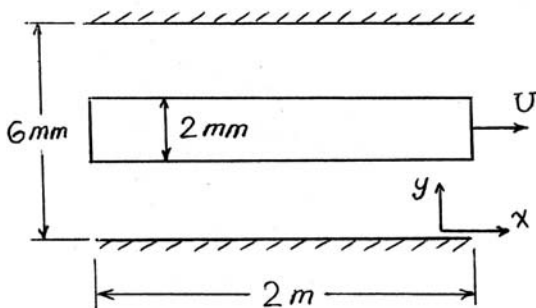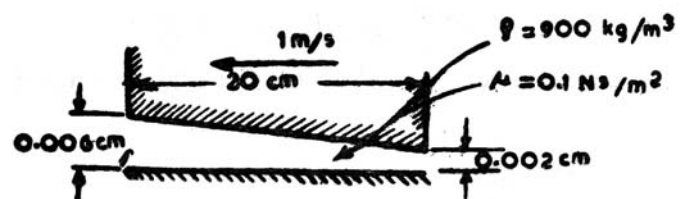
Problem No. 2

Problem No. 4

Problem No. 5

Problem No. 6

Problem No. 8

Problem No. 9

Problem No. 10

Problem No. 11

Chapter Two
Boundary Layer Theory

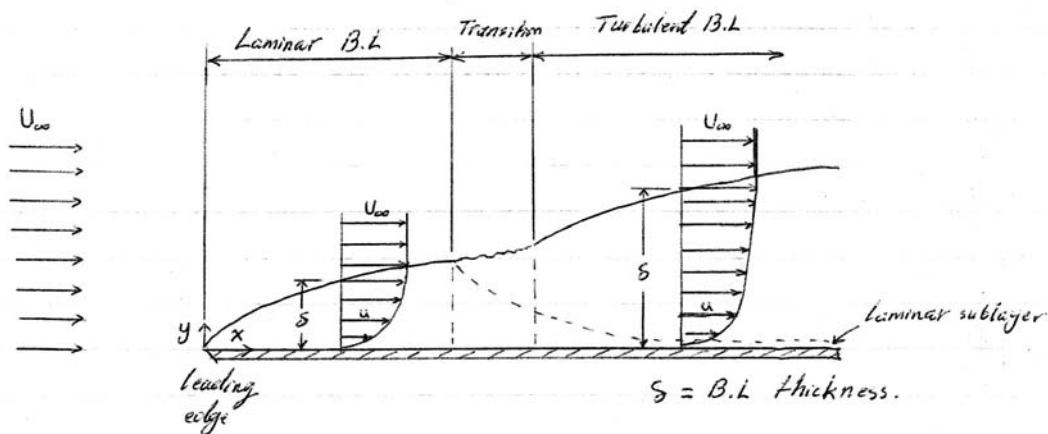## Contents

1- Introduction:

Development of boundary layer on a flat plate

The flow of a viscous fluid on a solid surface represents a region in which velocity increases from zero at the surface and approaches the velocity of the main stream. This region is known as *the boundary layer*.

The figure shows the development of a boundary layer on one side of a long flat plate held parallel to the flow direction.

Velocity distribution in boundary layer

Laminar B.L   Turbulent B.L
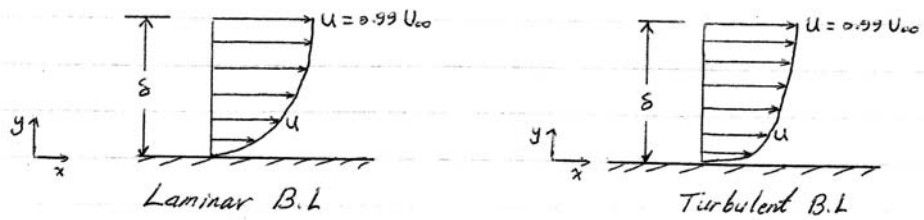
The velocity gradient will give rise to a large shear stress at the wall τ_o (or τ_w).

$$\tau_o = \mu\left(\frac{du}{dy}\right)_{y=0} \qquad \qquad \tau_o \equiv \tau_w$$

As shown in figure:
the velocity gradient in the turbulent boundary layer is larger than that in the laminar boundary layer.

$$\left(\frac{du}{dy}\right)_{y=0} \text{(in turbulent BL)} \rangle \left(\frac{du}{dy}\right)_{y=0} \text{(in laminar BL)}$$

$$\therefore \qquad \tau_o \text{ (in turbulent BL) } \rangle \tau_o \text{ (in laminar BL)}$$

The shear stress for a turbulent boundary layer is greater than the shear stress for a laminar boundary layer.

Boundary layer thickness (δ)

Boundary layer thickness is the distance from the solid surface to the point in the flow where $u = 0.99U_\infty$.

Displacement thickness (δ*)

Displacement thickness represents the outward displacement of the streamlines caused by the viscous effects on the solid surface.

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy$$

Or

$$\delta^* = \delta \int_0^1 (1 - f(\eta)) d\eta \qquad \text{Where } \eta = \frac{y}{\delta} \quad \text{and} \quad f(\eta) = \frac{u}{U_\infty}$$

Momentum thickness (θ)

Momentum thickness, is defined as the thickness of a layer of fluid , with velocity U_∞ , for which the momentum flux is equal to the deficit of momentum flux through the boundary layer.

$$\theta = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right)dy$$
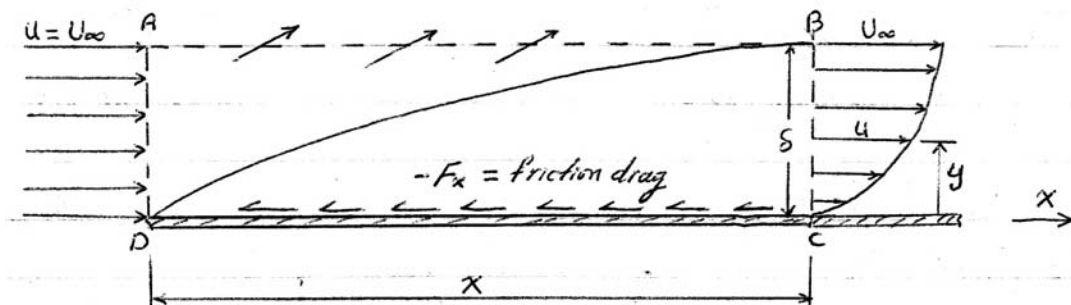
Or

$$\theta = \delta \int_0^1 f(\eta)(1 - f(\eta))d\eta$$

Shape factor (H)

H is a velocity profile shape factor.

$$H = \frac{\delta^*}{\theta}$$

2- Momentum equation for boundary layer:

Consider the control volume for flow over one side of a flat plate of width b.

$$-F_x = \rho b \int_0^\delta u^2 dy + \rho\left(U_\infty b\delta - b\int_0^\delta udy\right)U_\infty - \rho(U_\infty b\delta)U_\infty$$

$$F_x = \rho b \int_0^\delta u(U_\infty - u)dy \qquad ------(*)$$

F_x is the total friction drag on the plate from the leading edge up to x.

Assuming that the velocity profiles at various distances along the plate are similar to each other.

$$\frac{u}{U_\infty} = f\left(\frac{y}{\delta}\right) = f(\eta)$$

where

$$\eta = \frac{y}{\delta}$$

Equation (*) may be written as:

$$F_x = \rho b U_\infty^2 \delta \alpha \qquad ------(1)$$

where

$$\alpha = \int_0^1 f(\eta)(1 - f(\eta))d\eta$$

The local wall shear stress is :

$$\tau_o = \rho U_\infty^2 \alpha \frac{d\delta}{dx} \qquad ------(2)$$

Equations (1) and (2) are valid for either laminar or turbulent flow in the boundary layer.

3- Laminar boundary layer:

The wall shear stress:

$$\tau_o = \mu\left(\frac{du}{dy}\right)_{y=0}$$

Let $\quad \beta = \left[\frac{df(\eta)}{d\eta}\right]_{\eta=0}$

$$\Rightarrow \quad \tau_o = \frac{\mu U_\infty \beta}{\delta}$$

Another expression for shear stress:

$$\tau_o = \frac{1}{2}\rho U_\infty^2 c_f$$

Where c_f = local friction coefficient.

The total friction drag is:

$$F_f = b\int_0^L \tau_o \, dx$$

also

$$F_f = \frac{1}{2}\rho U_\infty^2 (b.L) C_f$$

Where C_f = total friction coefficient

The boundary layer thickness:

$$\delta = \sqrt{\frac{2\beta}{\alpha}} \frac{x}{\sqrt{\mathrm{Re}_x}}$$

Where Re_x = local Reynolds number

$$\mathrm{Re}_x = \frac{\rho U_\infty x}{\mu}$$

<u>Blasius solution:</u>

$(\alpha = 0.135 \; ; \; \beta = 1.63)$

$$\frac{\delta}{x} = \frac{4.91}{\sqrt{\mathrm{Re}_x}}$$

$$\frac{\delta^*}{x} = \frac{1.721}{\sqrt{\mathrm{Re}_x}}$$

$$c_f = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$$

$$C_f = \frac{1.328}{\sqrt{\mathrm{Re}_L}}$$

Where Re_L = total Reynolds number

$$\mathrm{Re}_L = \frac{\rho U_\infty L}{\mu}$$

* The laminar boundary layer will remain laminar up to a value of
$\mathrm{Re}_x = 500000$

4- Turbulent boundary layer:

Velocity profile in turbulent boundary layer:

$$\frac{u}{U_\infty} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}} = \eta^{\frac{1}{7}}$$

$$\therefore \ \alpha = \int_0^1 f(\eta)(1 - f(\eta))d\eta = \int_0^1 \eta^{\frac{1}{7}}\left(1 - \eta^{\frac{1}{7}}\right)d\eta = \frac{7}{72}$$

The wall shear stress for the turbulent boundary layer on smooth plate is:

$$\tau_o = 0.023\rho U_\infty^2 \left(\frac{\mu}{\rho U_\infty \delta}\right)^{\frac{1}{4}} \quad --------(**)$$

$$\frac{\delta}{x} = \frac{0.377}{(Re_x)^{\frac{1}{5}}}$$

$$c_f = \frac{0.0587}{(Re_x)^{\frac{1}{5}}}$$

The total friction coefficient is calculated form the following relations:

$$C_f = \frac{0.0735}{(Re_L)^{\frac{1}{5}}} \qquad \text{for } Re_L \langle 10^7$$

$$C_f = \frac{0.455}{(\log_{10} Re_L)^{2.58}} \qquad \text{for } Re_L \rangle 10^7$$

and

$$F_f = \frac{1}{2}\rho U_\infty^2 (b.L)C_f$$

Note: equation (**) was obtained from the following pipe equations:

$$\tau_o = \frac{1}{8}\rho f U_{av}^2 \qquad (f = 4c_f)$$

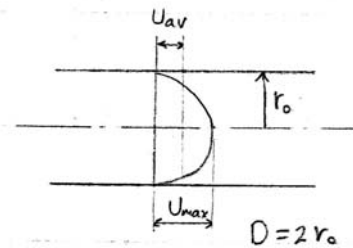$$f = \frac{0.316}{(\mathrm{Re})^{\frac{1}{4}}} \quad ; \quad \mathrm{Re} = \frac{\rho D U_{av}}{\mu}$$

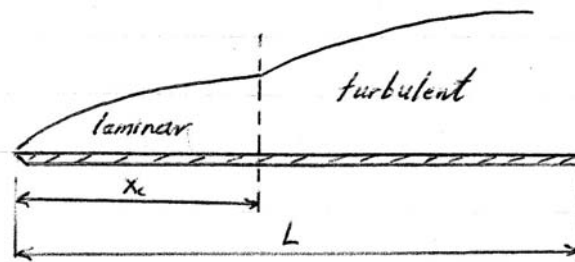$$U_{av} = \frac{U_{max}}{1.235}$$

To transfer to the flat plate :

$$r_o \approx \delta \qquad , \qquad U_{max} \approx U_\infty$$

$$\therefore \quad \tau_o = \frac{1}{8}\rho \frac{0.316}{\left[\dfrac{\rho(2\delta)\dfrac{U_\infty}{1.235}}{\mu}\right]^{\frac{1}{4}}}\left(\frac{U_\infty}{1.235}\right)^2$$

$$\Rightarrow \quad \tau_o = 0.023\rho U_\infty^2\left(\frac{\mu}{\rho U_\infty \delta}\right)^{\frac{1}{4}}$$
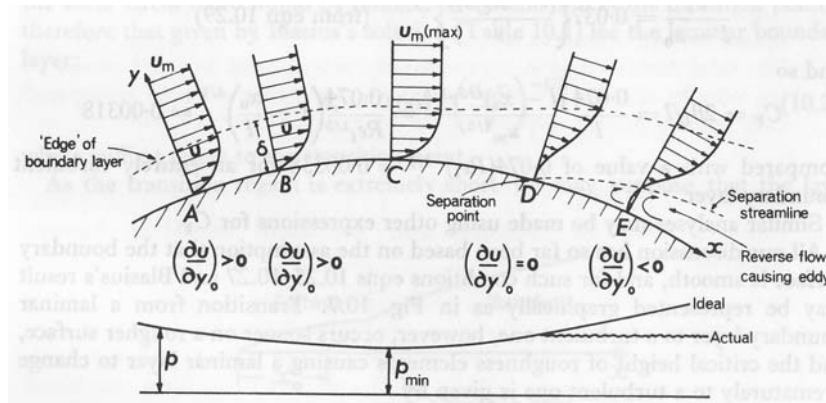

5- Friction drag in transition region:

$F_f = F_{laminar} + F_{turbulent}$

$$F_f = \frac{1}{2}\rho U_\infty^2 (b.L)C_f$$

$$C_f = \left[1.328\frac{\sqrt{\mathrm{Re}_{xc}}}{\mathrm{Re}_L} + \frac{0.455}{(\log_{10}\mathrm{Re}_L)^{2.58}} - 0.0735\frac{\mathrm{Re}_{xc}^{\frac{4}{5}}}{\mathrm{Re}_L}\right]$$

6- Effect of pressure gradient:

Consider the flow over a curved surface as shown in figure. As the fluid is deflected round the surface it is accelerated over the left-hand section (points A and B) until at point C, the velocity just outside the boundary layer is a maximum and the pressure is a minimum.

Beyond C, the velocity outside the boundary layer decreases, resulting in an increase in pressure. The velocity of the layer close to the wall is reduced and finally brought to a stop at D. Now the increasing pressure calls for further retardation so the boundary layer separates from the wall. At E there is a backflow (reverse flow) next to the wall, driven in the direction of decreasing pressure.

Down stream from the separation point the flow is characterized by irregular turbulent eddies. This disturbed region is called the wake of the body. The pressure within the wake remains close to that at the separation point. The pressure is always less than the pressure at the forward stagnation point.

An additional drag force is resulted from differences of pressure. This force is known as *the pressure drag (or form drag)*

** The total drag on a body is the sum of the friction drag and the pressure drag.

$$F_D = F_f + F_P$$

$$F_D = \frac{1}{2} \rho U_\infty^2 A C_D$$

Where   A = projected area of the body perpendicular to the oncoming
            flow.
        C_D = total drag coefficient.

Values of C_D for two- and three-dimensional bodies are given in figures (1) and (2) respectively.
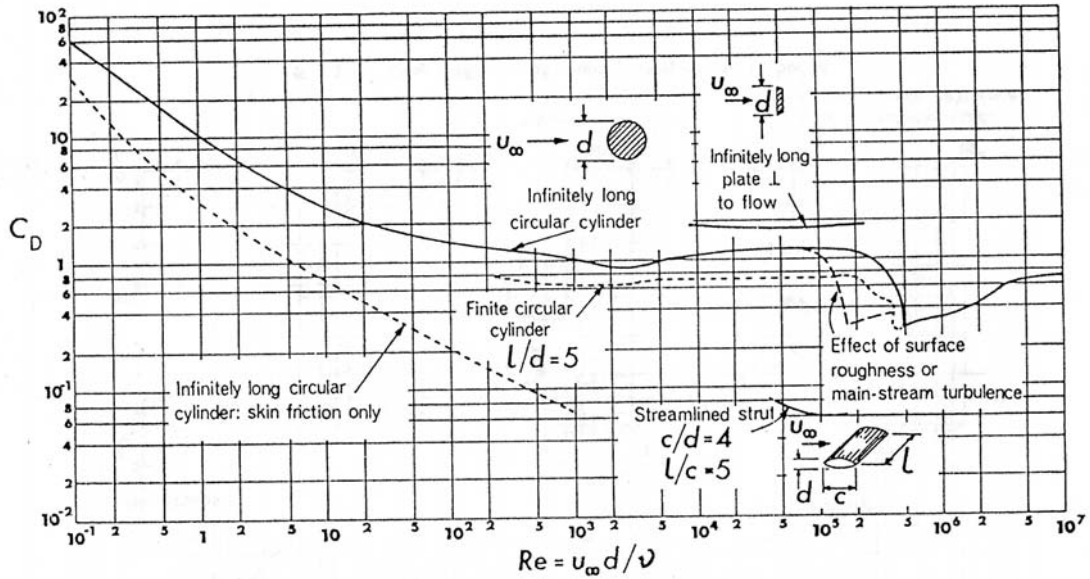

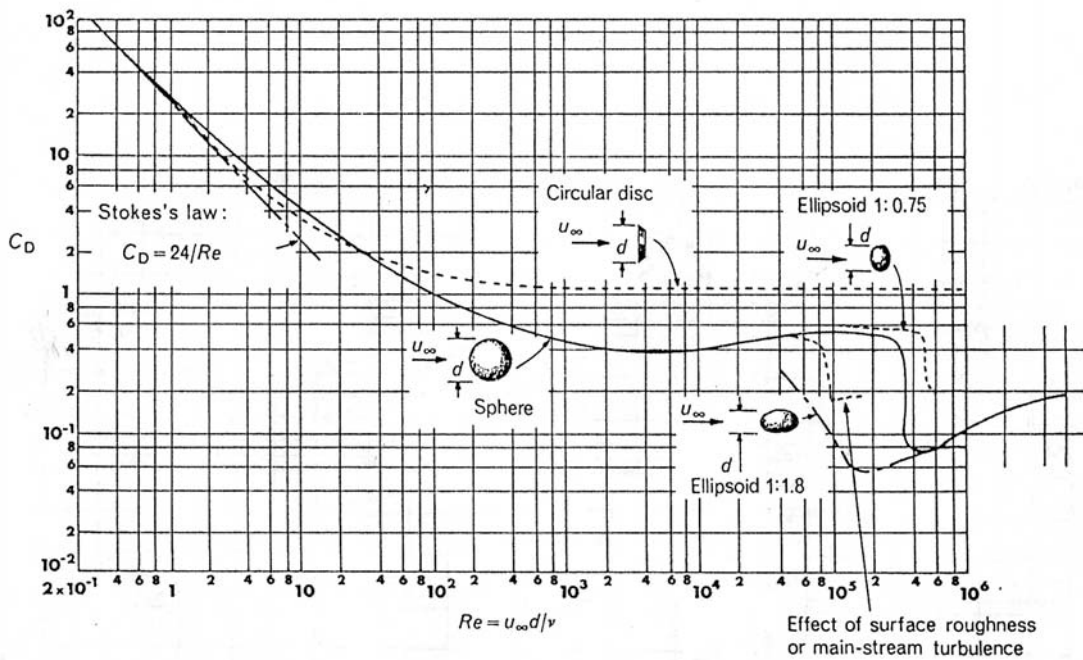FIG. 1   Drag coefficient for two-dimensional bodies

FIG. 1   Drag coefficients of smooth, axially-symmetric bodies

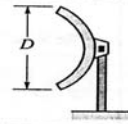Typical drag coefficients for objects of interest are given in table (1).

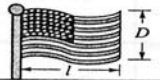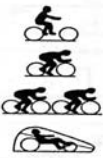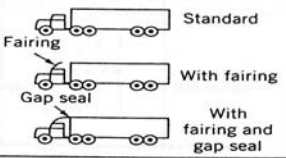| Shape	Reference area	Drag coefficient C_D
Parachute	Frontal area $A = \frac{\pi}{4}D^2$	1.4
Porous parabolic dish	Frontal area $A = \frac{\pi}{4}D^2$	Porosity: 0 / 0.2 / 0.5 → 1.42 / 1.20 / 0.82 ← 0.95 / 0.90 / 0.80 Porosity = open area/total area
Average person	Standing Sitting Crouching	$C_DA = 9$ ft^2 $C_DA = 6$ ft^2 $C_DA = 2.5$ ft^2
Fluttering flag	$A = \ell D$	ℓ/D : C_D 1 : 0.07 2 : 0.12 3 : 0.15
Empire State Building	Frontal area	1.4
Six-car passenger train	Frontal area	1.8
Bikes — Upright commuter	$A = 5.5$ ft^2	1.1
Racing	$A = 3.9$ ft^2	0.88
Drafting	$A = 3.9$ ft^2	0.50
Streamlined	$A = 5.0$ ft^2	0.12
Tractor-trailor tucks — Standard	Frontal area	0.96
With fairing	Frontal area	0.76
With fairing and gap seal	Frontal area	0.70
Tree $U = 10$ m/s $U = 20$ m/s $U = 30$ m/s	Frontal area	0.43 0.26 0.20
Dolphin	Wetted area	0.0036 at Re = 6×10^6 (flat plate has $C_{Df} = 0.0031$)
Large birds	Frontal area	0.40

Table (1)

- **Streamlined body:** the pressure drag on streamlined body is small and the friction drag is the major part of the total drag.

- **Bluff body:** the pressure drag on bluff body is much greater than the friction drag.

Relative comparison between skin friction drag and pressure drag for various aerodynamic shapes
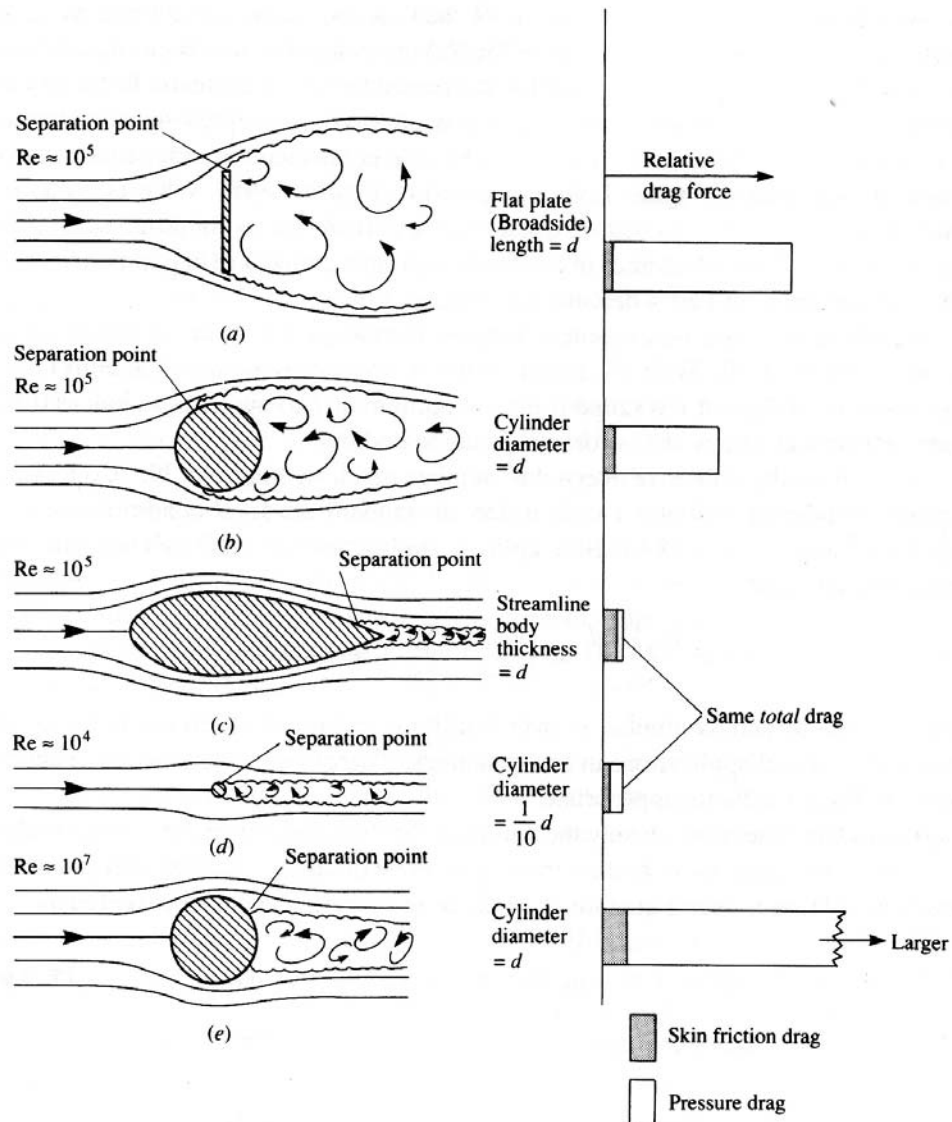
7- Separation of flow inside duct systems:

Most duct systems consist of more than straight duct. These additional components produce head losses (termed minor losses).

The minor losses are due to the separation region of flow inside duct as shown in the following figures:

Flow pattern
in 90° bend.

separation zone

Q

separated
flow
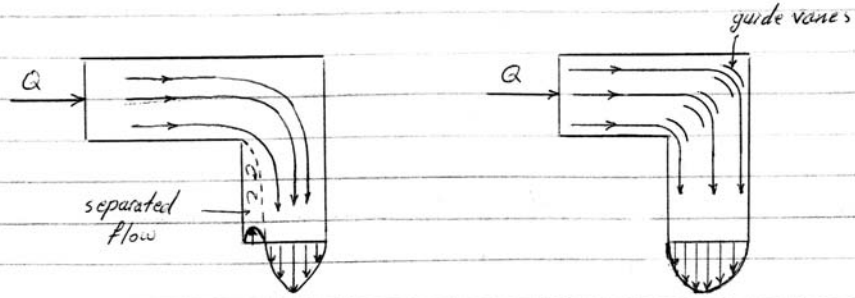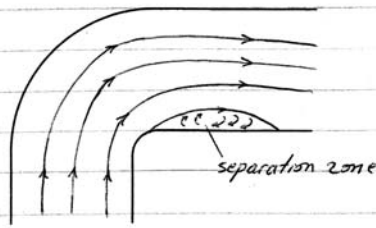
Flow in a 90° bend
without guide vanes

guide vanes

Q

Flow in a 90° bend
with guide vanes

large
separation
zone

small
separation
zone
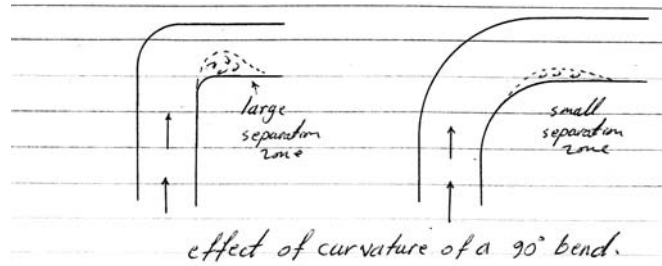
effect of curvature of a 90° bend.

airflow in a supply
branch piece

airflow in a suction
branch piece

abrupt reducer gradual reducer

abrupt expansion gradual expander
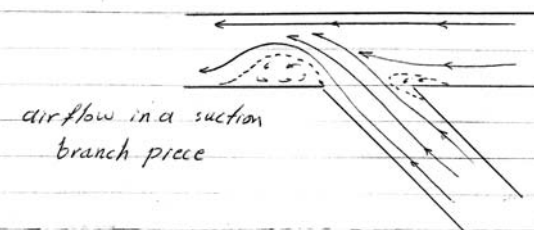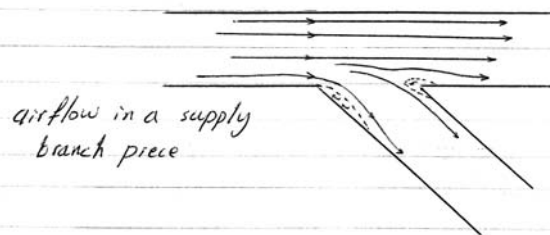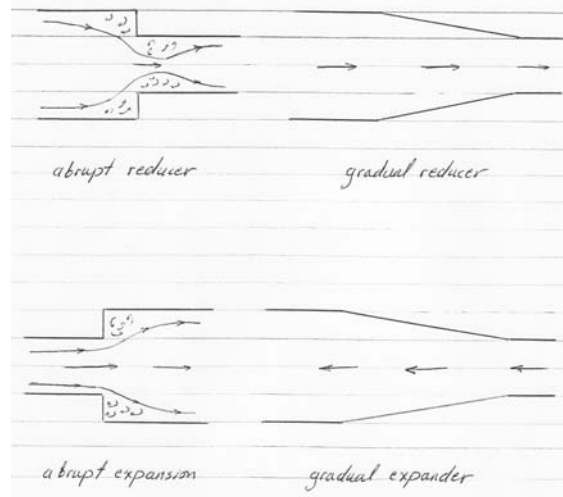
8- Examples:

1- A smooth flat plate 3 m wide and 30 m long is towed through still water (ρ = 998 kg/m^3 , ν = 1.007×10^{-6} m^2/s) with speed of 6 m/s. Determine the friction drag on one side of the plate and on the first 3 m of the plate.

Solution:

For the whole plate:-

$$Re_L = \frac{U_\infty L}{\nu} = \frac{6 \times 30}{1.007 \times 10^{-6}} = 1.787 \times 10^8 > 5 \times 10^5$$

the B.L is turbulent.

$$C_f = \frac{0.455}{(\log_{10} Re_L)^{2.58}} = 0.00196$$

The drag on one side is:-

$$F_f = \frac{1}{2} \rho U_\infty^2 A \, C_f$$

$$F_f = \frac{1}{2} \times 998 \times 6^2 \times (3 \times 30) \times 0.00196 = \underline{3169 \ N} \quad (Ans)$$

For the first 3m of the plate: $Re_x = 1.787 \times 10^7 > 10^7$

$$F_f = \frac{1}{2} \times 998 \times 6^2 \times (3 \times 3) \times \frac{0.455}{[\log_{10}(1.787 \times 10^7)]^{2.58}}$$

$$F_f = \underline{443 \ N} \quad (Ans)$$

2- Calculate the diameter of a parachute (in the form of a hemispherical shell) to be used for dropping a small object of mass 90 kg so that it touches the earth at a velocity no greater than 6 m/s. The drag coefficient for a hemispherical shell with its concave side upstream is approximately1.32 for Re > 10^3, (ρ = 1.22 kg/m^3).

Solution:

$$F_D = \frac{1}{2}\rho U_\infty^2 A \, C_D \quad —①$$
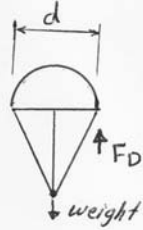
also

$$F_D = weight = m.g \quad —②$$

$$= 90 * 9.81 = 882.9 N$$

equ ① = equ ②

$$882.9 = \frac{1}{2} * 1.22 * 6^2 \; \frac{\pi d^2}{4} * 1.32$$

$$\implies \underline{d = 6.23 \, m} \qquad (Ans)$$

3- If $\dfrac{u}{U_\infty} = 2\dfrac{y}{\delta} - \left(\dfrac{y}{\delta}\right)^2$, find the thickness of the boundary layer, the shear stress at the trailing edge, and the drag force on one side of the plate 1 m long, if it is immersed in water flowing with a velocity of 0.3 m/s ($\rho = 1000$ kg/m^3 , $\mu = 0.001$ Pa.s)

Solution:

$$Re_L = \frac{\rho L U_\infty}{\mu} = \frac{1000 * 1 * 0.3}{0.001} = 3 \times 10^5 < 5 \times 10^5$$

The flow is laminar ; assume the width of the plate = 1m

Velocity profile: $f(\eta) = 2\eta - \eta^2$

$$\alpha = \int_0^1 f(\eta)(1 - f(\eta))d\eta = \int_0^1 2\eta - \eta^2(1 - 2\eta + \eta^2)d\eta = \frac{2}{15}$$
$$= 0.133$$

$$\beta = \left[\frac{d f(\eta)}{d\eta}\right]_{\eta=0} = 2$$

Boundary layer thickness at the trailing edge :- (x = L)

$$\delta = \sqrt{\frac{2\beta}{\alpha}} \; \frac{L}{\sqrt{Re_L}} = 0.01 m$$
$$= \underline{10 \, mm} \qquad (Ans)$$

Shear stress at the trailing edge :-

$$\tau_0 = \frac{\mu U_\infty \beta}{\delta} = \frac{0.001 * 0.3 * 2}{0.01} = \underline{0.06 \, N/m^2}$$
$$\qquad\qquad (Ans)$$

$$F_f = \rho b U_\infty^2 \delta \alpha$$
$$F_f = 1000 * 1 * (0.3)^2 * 0.01 * 0.133$$
$$= \underline{0.12 \, N} \qquad (Ans)$$

1- Calculate the displacement thickness and momentum thickness for the following velocity profiles in the boundary layer:

a- $\dfrac{u}{U_\infty} = 2\dfrac{y}{\delta} - \left(\dfrac{y}{\delta}\right)^2$;   b- $\dfrac{u}{U_\infty} = \left(\dfrac{y}{\delta}\right)^{\frac{1}{9}}$ $[\dfrac{1}{3}\delta\,;\dfrac{2}{15}\delta\,;0.1\delta\,;\dfrac{9}{110}\delta\,]$

2- Air ($\nu = 1.8\times10^{-5}$ m^2/s) flows along a flat plate with a velocity of 150 km/hr. How long does the plate have to be to obtain a laminar boundary layer thickness of 8 mm.

[6.146 m]

3- Assuming that the velocity distribution in the laminar boundary layer: $\dfrac{u}{U_\infty} = \sin\left(\dfrac{\pi y}{2\delta}\right)$.

Determine the total friction coefficient in terms of the Reynolds number. $[1.31/\sqrt{\text{Re}_L}\,]$

4- A thin plate 2 m wide is placed in a uniform air stream of velocity 100 m/s, ($\rho = 1.2$ kg/m^3). If the skin friction drag force is 60 N, calculate the displacement thickness of the boundary layer at trailing edge of the plate. Assume that the velocity profile at all points in the boundary layer is: f(η) = $\eta^{1/6}$.   [3.3 mm]

5- A river barge which is 50 m long and 12 m wide has flat bottom; therefore, its resistance is similar to one side of a flat plate. If the barge is towed at speed of 3 m/s through still water, what towing force is required to overcome viscous resistance and what is the boundary layer thickness at mid length? Assume the boundary layer is turbulent for the entire length. ($\rho = 1000$ kg/m^3 ; $\nu = 1.21\times10^{-6}$ m^2/s)   [5.57 kN ; 0.26 m]

6- A uniform free stream of air at 0.8 m/s flows over a flat plate (4 m long × 1 m wide). Assuming the boundary layer to be laminar on the plate and the velocity profile is:

$\dfrac{u}{U_\infty} = \dfrac{3}{2}\left(\dfrac{y}{\delta}\right) - \dfrac{1}{2}\left(\dfrac{y}{\delta}\right)^3$. Find the ratio of the drag force on the front half portion to the drag force on the rear half portion of the plate. ($\rho = 1.2$ kg/m^3 ; $\nu = 1.51\times10^{-5}$ m^2/s)   [2.42]

7- Air flows over a horizontal smooth flat plate at speed 14.5 m/s. The plate length is 1.5 m and its width is 0.8 m. The boundary layer is turbulent from the leading edge. The

velocity profile is: $\dfrac{u}{U_\infty} = \eta^{\frac{1}{6}}$ where $\eta = \dfrac{y}{\delta}$. Evaluate the boundary layer thickness and the

wall shear stress at the trailing edge of the plate. ($\rho = 1.21$ kg/m^3 ; $\nu = 1.5\times10^{-5}$ m^2/s)

[30.75 mm ; 0.447 N/m^2]

8- Air (ρ = 1.21 kg/m^3) flows over a thin flat plate 1 m long and 0.3 m wide. The flow is uniform at the leading edge of the plate. Assume the velocity profile in the boundary layer is linear, and the free stream velocity is 2.7 m/s. Using control volume (abcd) shown in figure, compute the mass flow rate across surface (ab). Determine the magnitude and direction of the x- component of the force required to hold plate stationary.

[3.9×10^{-3} kg/s ; -3.5×10^{-3} N]

9- Estimate the power required to move a flat plate 9 m long and 3 m wide in oil (ρ = 920 kg/m^3 ; μ = 0.067 Pa.s) at 8 m/s. For the following cases:
a- the boundary layer is laminar over the surface of the plate.
b- the boundary layer is turbulent over the surface of the plate from the leading edge.
c- transition from laminar to turbulent at $Re_c = 5\times10^5$.
(Assume the velocity profile for the turbulent boundary layer is $f(\eta) = \eta^{1/9}$).

[8.5 kW ; 28.55 kW ; 18.05 kW]

10- For the velocity profile: $\dfrac{u}{U_\infty} = \dfrac{3}{2}\left(\dfrac{y}{\delta}\right) - \dfrac{1}{2}\left(\dfrac{y}{\delta}\right)^3$, determine whether the flow has separated

or not separated or will attach with the surface after separation.

11- A honeycomb type of flow straightener is formed from perpendicular flat metal strips to give 25 mm square passages, 150 mm long. Water of kinematic viscosity 1.21 mm^2/s approaches the straightener at 1.8 m/s. Calculate the displacement thickness of the boundary layer and the velocity of the main stream at the outlet end of the straightener. Applying Bernoulli's equation to the main stream, deduce the pressure drop through the straightener. [0.546 mm ; 1.968 m/s ; 316.5 Pa]

12- Air of kinematic viscosity 15 mm^2/s and density 1.21 kg/m^3 flows past a smooth 150 mm diameter sphere at 60 m/s. Determine the drag force. What would be the drag force on a 150 mm diameter circular disc held perpendicular to this air stream.

[3 N ; 42 N]

13- The chimney of a boiler house is 50 m tall and has an outside diameter of 3 m.
Compute the overturning moment about the base if a 30 m/s wind blows past it at the standard atmospheric conditions. (ρ = 1.21 kg/m^3 ; ν = 15×10^{-6} m^2/s)   [1430 kN.m]
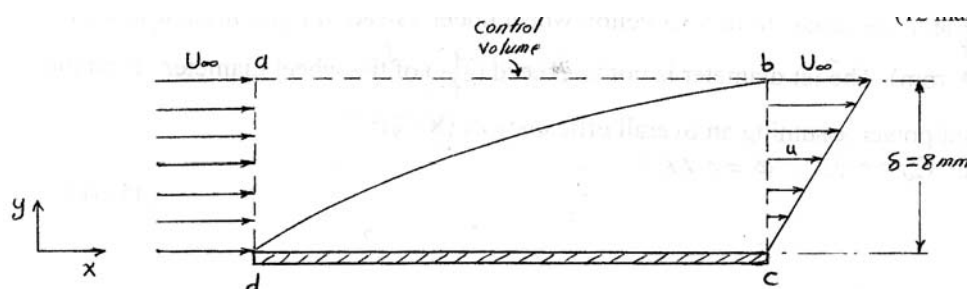
Problem No. 8

Chapter Three
Two-Phase Flow

Contents

1- Introduction:

Background

      Two-phase flow is widely seen in the nature: Rain, snow, smog, dust-flow, etc.

Two-phase Flow:
Is a flow of two-phase mixture of a substance with the same chemical composition.   (e.g., Water-steam flow, water-ice flow, etc.)

Two-Component Flow:
Is a flow of two-component mixture with different chemical composition.
(e.g., air-water flow, oil-water flow, air-dust flow, etc.)

Two- phase flows are related to the phase-change Phenomena.
(e.g., boiling, condensation, freezing, melting, solidification, crystallization).

2- Notation and relations:
Subscripts

f = fluid
g= gas
l= liquid
h= enthalpy

Void fraction α :

$$\alpha = A_g / A$$

Thus

$$1 - \alpha = A_f / A$$

And

$$A = A_f + A_g$$

Mass flow rate W_g, W_f:

$$W_g = u_g \rho_g A_g$$
$$W_f = u_f \rho_f A_f$$

Total mass flow rate W:

$$W = W_g + W_f$$

Volumetric flow rate Q_g, Q_f:

$$Q_f = u_f A_f = W_f / \rho_f$$

$$Q_g = u_g A_g = \frac{W_g}{\rho_g}$$

Total volumetric flow rate Q:

$$Q = Q_g + Q_f$$

Quality (or mass quality) x:

$$x = W_g / W$$

And

$$1 - x = W_f / W$$

And in a thermodynamic equilibrium,

$$x = (i - i_f) / i_{fg}$$

Mass flux (or superficial velocity) j:

$$G = W/A$$

And

$$G_g = \frac{W_g}{A}$$

$$G_f = \frac{W_f}{A}$$

Therefore

$$G = G_f + G_g$$

Volumetric flux (or superficial velocity) j:

$$j = Q/A$$

$$j_g = Q_g/A \quad , \quad j_f = Q_f/A$$

Thus

$$j = j_g + j_f$$

Volumetric quality β:

$$\beta = Q_g/Q$$

$$1 - \beta = Q_f/Q$$

Relative velocity u_{gf}, u_{fg}:

$$u_{gf} = u_g - u_f$$

$$u_{fg} = u_f - u_g$$

3- Flow patterns:

Vertical flow patterns

Bubbly   Slug   Churn   Wispy-annular   Annular

(a) (b) (c)   (d) (e)   (f) (g)   (h)   (i)

Bubbly flow   Slug flow   Churn flow   Annular flow   Annular-mist flow
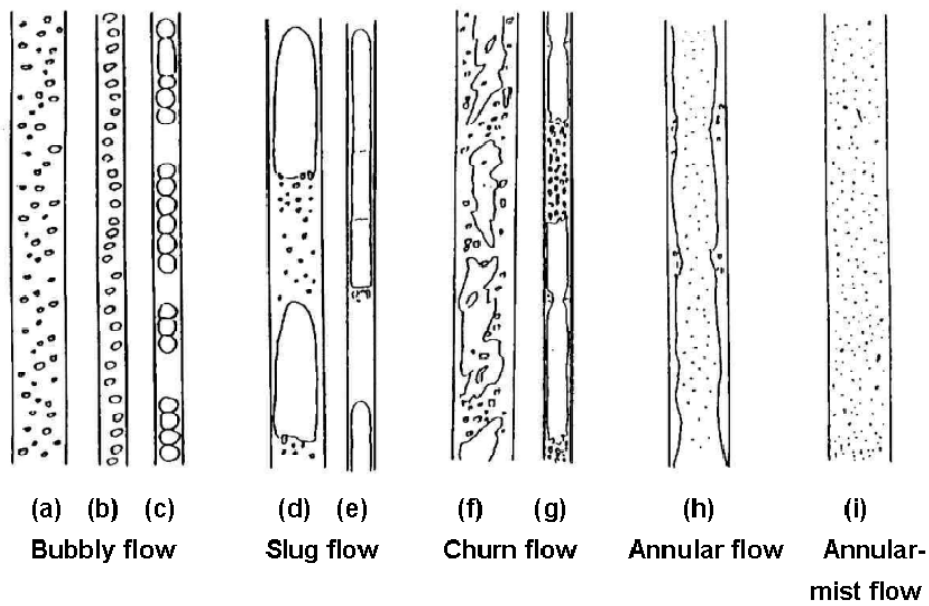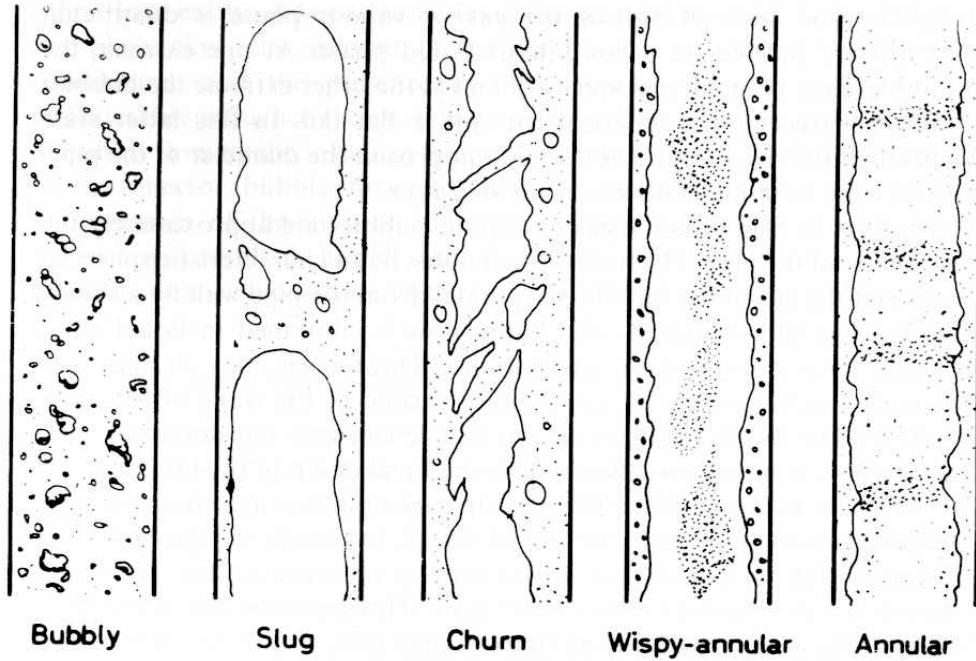
(a), (d), (f), (h) and (i): Flow patterns frequently observed in large diameter tubes.

(b), (c), (e) and (g): Flow patterns specially appear in capillary tubes.

Horizontal flow patterns

Bubbly

Wavy

Plug

Slug

Stratified

Annular

Flow pattern maps

(1) Vertical flow [Hewitt & Roberts, 1969]
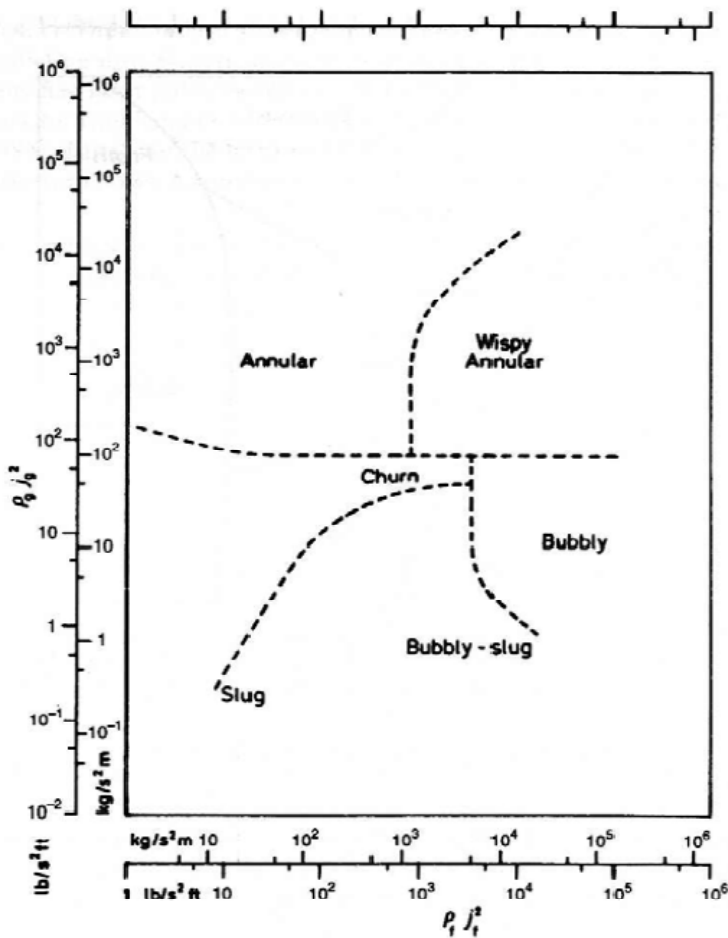
$$\rho_f j_f^2 = \frac{\left(G(1-x)\right)^2}{\rho_f}$$

$$\rho_g j_{fg}^2 = \frac{\left(G(x)\right)^2}{\rho_g}$$

(2) Horizontal flow [Baker, 1954]
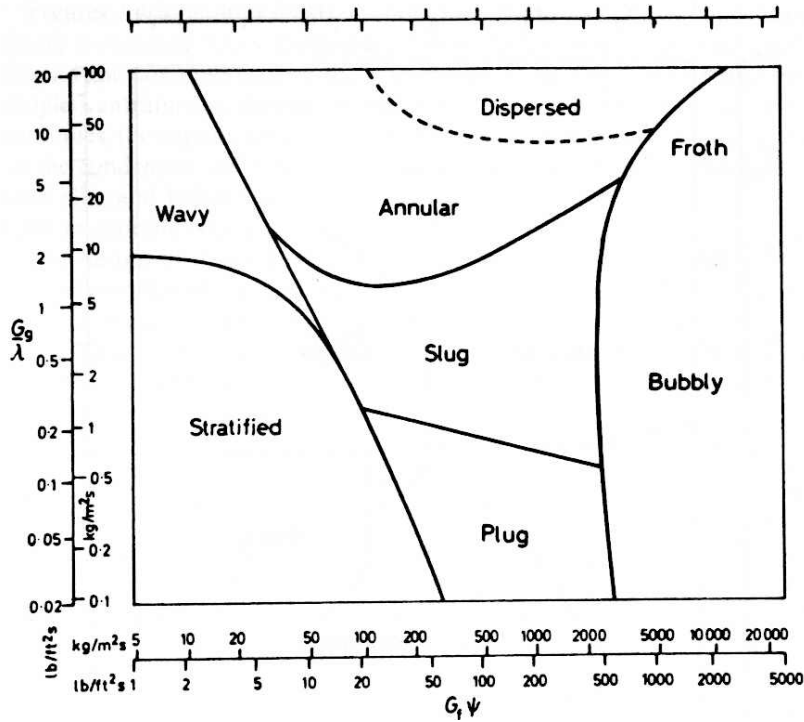
$$\lambda = \left[\left(\frac{\rho_g}{\rho_A} \right) \left(\frac{\rho_f}{\rho_w} \right) \right]^{\frac{1}{2}}$$

$$\psi = \frac{\sigma_w}{\sigma} \left[\left(\frac{\mu_f}{\mu_w} \right) \left(\frac{\rho_w}{\rho_f} \right)^2 \right]^{\frac{1}{3}}$$

(3) Generalized map [Taitel & Dukler, 1976]
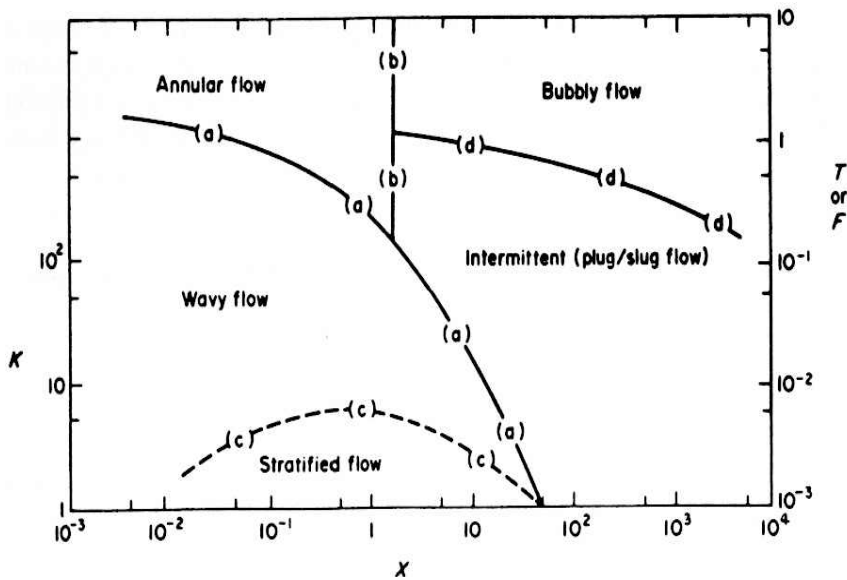
a+b : F vs. X

c : K vs. X

d : T vs. X

$$X = \left(\frac{dp}{dz} \right)_f \Big/ \left(\frac{dp}{dz} \right)_g$$

$$K = \left[\frac{\rho_g j_g^2 j_f}{(\rho_f - \rho_g) g \cos\theta} \right]^{0.5}$$

$$T = \left[\frac{\left(\frac{dp}{dz} \right)_f}{(\rho_f - \rho_g) g \cos\theta} \right]^{0.5}$$

$$F = \left(\frac{\rho_g}{\rho_f - \rho_g} \right)^{0.5} \frac{j_g}{(D_g \cos\theta)^{0.5}}$$
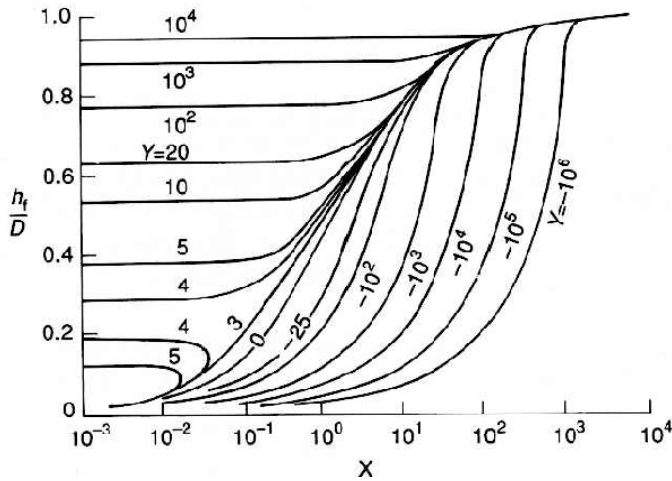
$$Y = \frac{(\rho_f - \rho_g)g\sin\theta}{\left(\frac{dp}{dz}\right)_F}$$

4- Homogeneous flow model:

Continuity equation:

$$A\rho_h u_h = W = cons.$$

Momentum Equation:

$$-\frac{dp}{dz} = -\left(\frac{dp}{dz}\right)_F - \left(\frac{dp}{dz}\right)_Z - \left(\frac{dp}{dz}\right)_a$$

$$-\frac{dp}{dz} = \frac{1}{A}\frac{dF}{dz} + \frac{g\sin\theta}{v_h} + \frac{W}{A}\frac{du_h}{dz}$$

Energy Equation:

$$\frac{di}{dz} + \frac{1}{2}\frac{d}{dz}\left(u_h^2\right) + g\sin\theta = \frac{dq}{dz} - \frac{dw}{dz}$$

For dw=0, the energy equation becomes:

$$-\frac{dp}{dz} = \frac{1}{v_h}\left(\frac{dE}{dZ} + \frac{1}{2}\frac{d}{dz}\left(u_h^2\right) + g\sin\theta\right)$$

$$dF = \tau_w pdZ$$

τ_w : wall shear stress ,p:tube perimeter

$$\tau_w = f_h\left(\frac{1}{2}\rho_h u_h^2\right)$$

f_h: friction factor

For a tube of diameter, D

$$-\left(\frac{dp}{dz}\right)_F = \frac{1}{A}\frac{dF}{dz}$$

$$-\left(\frac{dp}{dz}\right)_F = \frac{1}{A}(\tau_w p) = \frac{p}{A} f_h\left(\frac{1}{2}\rho_h u_h^2\right)$$

$$= \frac{2 f_h G^2 v_h}{D}$$

Get f_h

$$Re_h = \frac{GD}{\mu_h}$$

McAdams

$$\frac{1}{\mu_h} = \frac{x}{\mu_g} + \frac{1-x}{\mu_f}$$

Cicchitti

$$\mu_h = x\mu_g + (1-x)\mu_f$$

Dukler

$$\mu_h = \frac{j_f}{j}\mu_f + \frac{j_g}{f}u_g = \rho_h\left[x\upsilon_g\mu_g + (1-x)v_f\mu_f\right]$$

For laminar flow:

$$f_h = \frac{16}{Re_h}$$

For turbulent flow (Blasius correlation):

$$f_h = 0.079 Re_h^{-1/4}$$

When using the McAdams Correlations,

$$-\left(\frac{dp}{dz}\right)_F = -\left(\frac{dp}{dz}\right)_{fo}\left[1+x\left(\frac{v_{fg}}{v_f}\right)\right]\left[1+x\left(\frac{\mu_{fg}}{\mu_g}\right)\right]^{-1}$$ Laminar flow

$$-\left(\frac{dp}{dz}\right)_F = -\left(\frac{dp}{dz}\right)_{fo}\left[1+x\left(\frac{v_{fg}}{v_f}\right)\right]\left[1+x\left(\frac{\mu_{fg}}{\mu_g}\right)\right]^{-\frac{1}{4}}$$ Turbulent Flow

Thus

$$\phi_{fo}^2 = \left[1+x\left(\frac{v_{fg}}{v_f}\right)\right]\left[1+x\left(\frac{\mu_{fg}}{\mu_g}\right)\right]^{-1}$$ (Laminar Flow)

$$\phi_{fo}^2 = \left[1+x\left(\frac{v_{fg}}{v_f}\right)\right]\left[1+x\left(\frac{\mu_{fg}}{\mu_g}\right)\right]^{-\frac{1}{4}}$$ (Turbulent flow)

Acceleration Pressure Drop

$$-\left(\frac{dp}{dz}\right)_a = G\left(\frac{du_h}{dz}\right) = G\frac{d}{dz}\left(\frac{W}{A}v_h\right)$$

$$= G^2\frac{dv_h}{dz} + Gv_hW\frac{d}{dz}\left(\frac{1}{A}\right)$$

Where

$$\frac{dv_h}{dz} = \frac{d}{dz}\left[xv_g + (1-x)v_f\right]$$

$$= \frac{dp}{dz}\left[x\frac{dv_g}{dp} + (1-x)\frac{dv_f}{dp}\right] + v_{gf}\frac{dx}{dz}$$

And

$$-\left(\frac{dp}{dz}\right)_a = G^2\left\{v_{gf}\frac{dx}{dz} + \frac{dp}{dz}\left[x\frac{dv_g}{dp} + (1-x)\frac{dv_f}{dp}\right] - \left(v_f + xv_{gf}\right)\frac{1}{A}\frac{dA}{dz}\right\}$$

Gravitational Pressure Drop

$$-\left(\frac{dp}{dz}\right)_Z = \frac{g\sin\theta}{v_f + xv_{fg}}$$

Finally

$$-\left(\frac{dp}{dz}\right)_a = \frac{\dfrac{2f_h G^2}{D}\left(v_f + xv_{fg}\right) + G^2 v_{gf}\dfrac{dx}{dz} - G^2\left(v_f + xv_{fg}\right)\dfrac{1}{A}\dfrac{dA}{dz} + \dfrac{g\sin\theta}{v_f + xv_{fg}}}{1 + G^2\left[x\dfrac{dv_g}{dp} + (1-x)\dfrac{dv_f}{dp}\right]}$$

5- Separated flow model:

Pressure Drop Correlations:-

$$-\frac{dp}{dz} = -\left(\frac{dp}{dz}\right)_F - \left(\frac{dp}{dz}\right)_Z - \left(\frac{dp}{dz}\right)_a$$

Where:

$$-\left(\frac{dp}{dz}\right)_F = \frac{1}{A}\frac{dF}{dz}$$

$$-\left(\frac{dp}{dz}\right)_Z = \left[(1-\alpha)\rho_f + \alpha\rho_g\right]g\sin\theta$$

$$-\left(\frac{dp}{dz}\right)_a = \frac{W^2}{A}\frac{d}{dz}\left[\frac{1}{A}\left\{\frac{x^2 v_g}{\alpha} + \frac{(1-x)^2 v_f}{1-\alpha}\right\}\right]$$

For a circular tube of diameter, D

$$-\left(\frac{dp}{dz}\right)_F = -\left((\frac{dp}{dz})_F\right)_{fo} \phi_{fo}^2 = \frac{2 f_{fo} G^2 v_f}{D} \phi_{fo}^2$$

$$-\left(\frac{dp}{dz}\right)_F = -\left((\frac{dp}{dz})_F\right)_{go} \phi_{go}^2 = \frac{2 f_{go} G^2 v_g}{D} \phi_{go}^2$$

$$-\left(\frac{dp}{dz}\right)_F = -\left((\frac{dp}{dz})_F\right)_{f} \phi_{f}^2 = \frac{2 f_{f} G^2 (1-x)^2 v_f}{D} \phi_{f}^2$$

$$-\left(\frac{dp}{dz}\right)_F = -\left((\frac{dp}{dz})_F\right)_{g} \phi_{g}^2 = \frac{2 f_{g} G^2 x^2 v_g}{D} \phi_{g}^2$$

Where $\phi_{fo}, \phi_{go}, \phi_f, \phi_g$ are the two-phase frictional multipliers related to each other as follows:

$$\phi_{fo}^2 = \phi_f^2 (1-x)^2 \frac{f_f}{f_{fo}} = \phi_g^2 x^2 \frac{v_g f_g}{v_f f_{fo}} = \phi_{go}^2 \frac{v_g f_{go}}{v_f f_{fo}}$$

$$-\left(\frac{dp}{dz}\right)_a = \frac{W^2}{A} \frac{d}{dz}\left[\frac{1}{A}\left\{\frac{x^2 v_g}{\alpha} + \frac{(1-x)^2 v_f}{1-\alpha}\right\}\right]$$

$$= G^2 \frac{dx}{dz}\left[\left\{\frac{2 x v_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha}\right\} + \left(\frac{\partial \alpha}{\partial x}\right)_p \cdot \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\}\right]$$

$$+ G^2 \frac{dp}{dz}\left[\left\{\frac{x^2 dv_g}{\alpha dp} + \frac{(1-x)^2}{1-\alpha}\frac{dv_f}{dp}\right\} + \left(\frac{\partial \alpha}{\partial p}\right)_x \cdot \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\}\right]$$

$$- \frac{G^2}{A}\frac{dA}{dz}\left[\left\{\frac{x^2 v_g}{\alpha} + \frac{(1-x)^2 v_f}{1-\alpha}\right\}\right]$$

Finally,

$$-\frac{dp}{dz} = \frac{H_1}{H_2}$$

Where

$$H_1 = \frac{2 f_{fo} G^2 v_f}{D} \phi_{fo}^2 + G^2 \frac{dx}{dz} \left[\left\{ \frac{2 x v_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha} \right\} + \left(\frac{d\alpha}{dx} \right) \cdot \left\{ \frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2} \right\} \right]$$

$$- \frac{G^2}{A} \frac{dA}{dz} \left[\left\{ \frac{x^2 v_g}{\alpha} + \frac{(1-x)^2 v_f}{1-\alpha} \right\} \right] + \left(\rho_g \alpha + (1-\alpha)\rho_f \right) g \sin \theta$$

and

$$H_2 = 1 + G^2 \left[\left\{ \frac{x^2 dv_g}{\alpha dp} + \frac{(1-x)^2}{1-\alpha} \frac{dv_f}{dp} \right\} + \left(\frac{d\alpha}{dp} \right) \cdot \left\{ \frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2} \right\} \right]$$

6- Examples:

1- Using the homogeneous flow pressure drop method, calculate the two phase pressure drop for up flow in vertical tube of 10mm internal diameter that is 2m long .The flow is adiabatic ,the mass flow rate is 0.02 kg/s and the vapor quality is 0.05 .The fluid is R-123 at a saturation temperature of 3°C and saturation pressure of 0.37 bar ,whose physical properties are : ρ_l= 1518 kg/m^3 , ρ_g= 2.6 kg/m^3 , μ_g= 12.6×10^{-6} Pa.s , μ_l= 58.56×10^{-5} Pa.s

Solution:

$$\alpha_h = \frac{1}{1 + \frac{(1-x)\rho_g}{x\rho_l}} = 0.9685$$

$$\rho_h = \rho_l(1 - \alpha_h) + \rho_{g\alpha_h} = 50.3 \frac{kg}{m^3}$$

$$\Delta p_z = \rho_h L g \ sin\theta = 50.3 * 2 * 9.81 * sin90$$
$$\Delta p_z = 987 \quad {N}/{m^2}$$

$$\frac{1}{\mu_h} = \frac{x}{\mu_g} + \frac{1-x}{\mu_l} = 0.000557 \ Pa.s$$

$$Re = \frac{GD}{\mu_h} = 4571$$

$$f = \frac{0.079}{Re^{0.25}} = 0.00961$$

$$\Delta p_F = \frac{2 * f * L * G^2}{D * \rho_h} = 4953 \quad {N}/{m^2}$$

Total pressure drop $=\Delta p_F + \Delta p_z = 5.94 \ kPa.$

1- Find the flow pattern when 4 kg/s of steam –water mixture of quality 20% at 20 bar flows in a circular tube of internal diameter 0.1 m:
  a) When the flow is vertically upward.
  b) When the flow is horizontal.
The physical properties required are:
ρ_l= 850 kg/m^3, ρ_g = 10 kg/m^3 , μ_l= 128*10^{-6} Pa.s , μ_g= 16*10^{-6} Pa.s

2- A steam –water mixture of quality 0.1 at 5 bar flow through a smooth vertical round tube of diameter 0.05m.The total flow rate is 0.6 kg/s .Calculate:
  a) The homogeneous void fraction.
  b) Homogeneous gravitational pressure gradient.
  c) Homogeneous frictional pressure gradient.
The physical properties required are:
ρ_l= 915 kg/m^3, ρ_g = 2.67 kg/m^3 , μ_l=180*10^{-6} Pa.s , μ_g= 14*10^{-6} Pa.s
                                            [0.97 ; 255 N/m^3 ; 547 N/m^3]

3- Consider mixture of water (W_l=0.42 kg/s) and air (W_g=0.01 kg/s) flowing upward in a vertical pipe (D=25mm ,L=45cm). Given friction factor of (f=0.079/Re$^{0.25}$).Find the total pressure drop ,the volumetric flow rates ,void fraction ,and mean water and air velocity. Using homogeneous model.
the physical properties required are:
ρ_l= 915 kg/m^3, ρ_g = 1.2 kg/m^3 ,μ_l= 1*10^{-3} Pa.s , μ_g= 1.8*10^{-5} Pa.s
                          [3169 N/m^2 ; 0.00833m^3/s ; 0.0042m^3/s ; 0.952 ; 17.65m/s]

Chapter Five
Fans

Contents

1- Types of fans.
2- Centrifugal fan.
3- Axial flow fan.
4- Fan and system characteristics.
5- Example.
6- Problems;   sheet No. 5

1- Types of fans:

1- Centrifugal fan, (fig.1)
   - backward curved impeller
   - forward curved impeller
   - radial impeller
2- Axial flow fan, (fig.2)
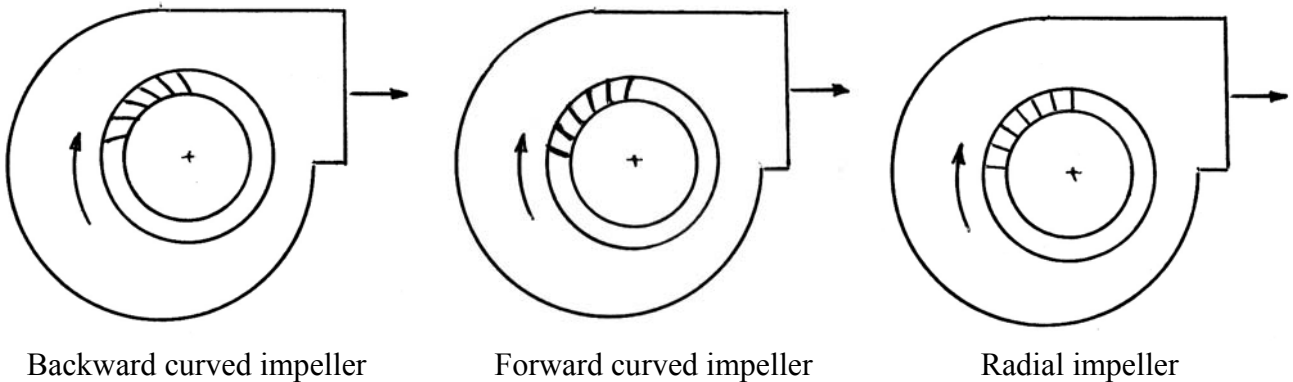   - ducted fan
   - propeller fan
3- Mixed flow fan

Backward curved impeller   Forward curved impeller   Radial impeller
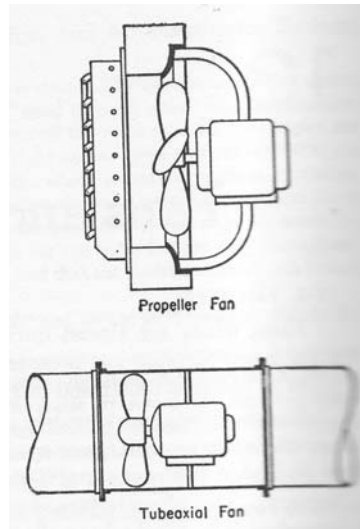
Fig.(1) Centrifugal fan

Fig.(2) Axial flow fan

2- Centrifugal fan:

Centrifugal fan consists of an impeller running in a casing normally of volute shape, fig.(1). The air enters axially and is discharged at the periphery.

Advantages: 1- quiet.
2- efficient operation at high pressures.
3- fan inlet can be connected to larger apparatus sections and its outlet to smaller supply duct sections.

Types:
- *Backward curved impeller ($\beta_2 < 90^o$)*
  This type is employed for lower pressures and lower flow rates. It has higher efficiencies than that for forward type. It must run at higher speed to develop the same pressure rise as that for forward type.

- *Forward curved impeller ($\beta_2 > 90^o$)*
  This type is employed for higher stage pressure rise. They are also available as high volume flow fan. Forward fans are smaller and slower thus they tend to be quieter and cheaper. They are used in small air conditioners with drum-type construction.
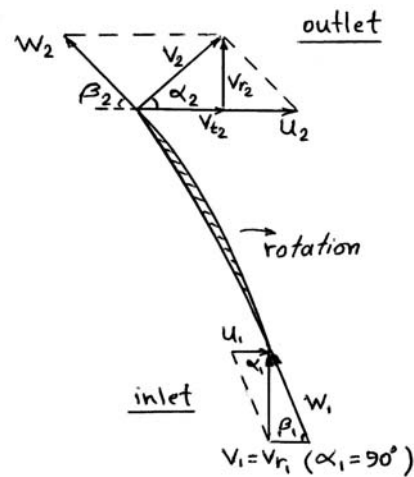
- *Radial impeller ($\beta_2 = 90^o$)*
  This type is ideal for handling dust-laden air because it is less prone to blockage, dust erosion and failure

Theory:
Total pressure developed by fan, p_t
(Δp_t) or $p_t = \gamma H$
where H is the theoretical total head.

$$H = \frac{(u_2 V_2 \cos\alpha_2 - u_1 V_1 \cos\alpha_1)}{g}$$

for entry of air to the blade passage being radial without pre whirl ($\alpha_1 = 90^\circ$) we have:

$$H = \frac{(u_2 V_2 \cos\alpha_2)}{g}$$

and

$$p_t = \rho u_2 V_2 \cos\alpha_2$$

Then fan static pressure, p_s

$$p_s = p_t - p_d$$

P_d is the dynamic pressure.

Form the continuity equation:

$$Q = \pi d_1 b_1 V_{r1} = \pi d_2 b_2 V_{r2}$$

where d_1 = impeller diameter at inlet.
       d_2 = impeller diameter at outlet.
       b_1 = width of blade at inlet.
       b_2 = width of blade at outlet.

Fan total efficiency: $\eta = \dfrac{\gamma Q H}{\text{shaft power}} = \dfrac{Q p_t}{\text{shaft power}}$

The following **dimensionless design parameters** are used for fans:

Flow coefficient: $\phi = \dfrac{Q}{\dfrac{\pi d_2^2}{4} u_2}$

Pressure coefficient: $\psi = \dfrac{p_t}{\dfrac{1}{2}\rho u_2^2}$

Power coefficient: $\lambda = \dfrac{\phi\psi}{\eta}$

Similarity laws:

For any series of geometrically similar fans and for any point on the characteristic curves, the following relationships hold:

$\dfrac{Q}{ND^3} = \text{constant}$ $(Q \propto N)$ and $(Q \propto D^3)$

$\dfrac{gH}{N^2 D^2} = \text{constant}$

$\dfrac{p_t}{\rho N^2 D^3} = \text{constant}$

$\dfrac{\text{power}}{\rho N^3 D^5} = \text{constant}$
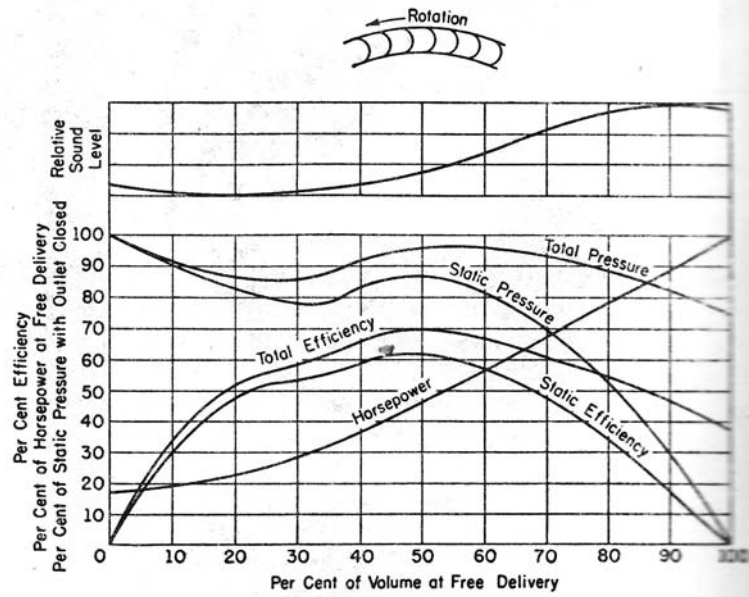
Characteristics of centrifugal fan:

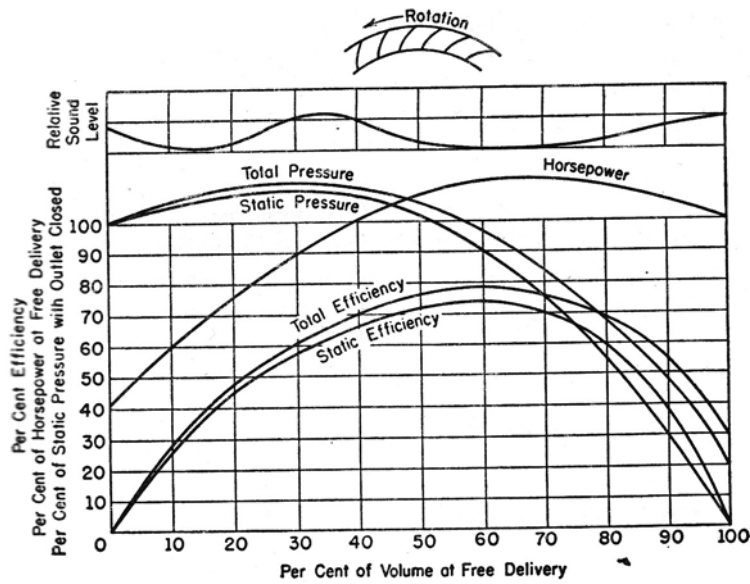FIG. 13–4. Forward-tip-fan characteristics.
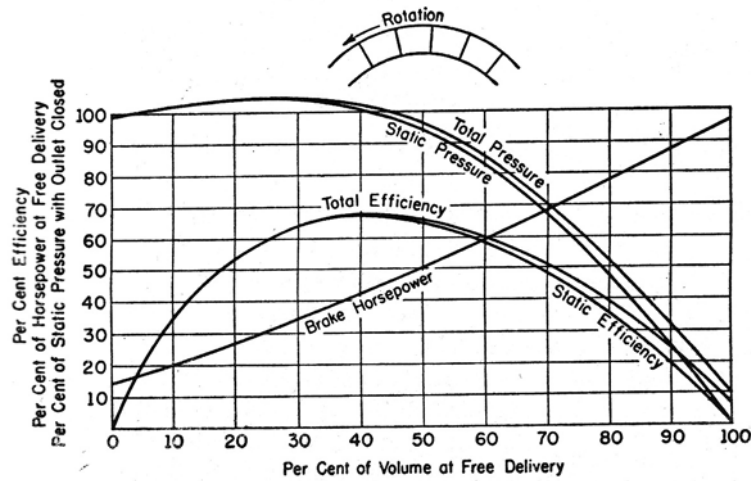

FIG. 13–5. Backward-tip-fan characteristics.

FIG. 13–6. Radial-tip-fan characteristics.

3- Axial flow fan:

Axial flow fan consists of a rotor made up of a number of blades fitted to the hub, fig.(2).
In the ducted type fan, a cylindrical casing encloses the rotor. Unenclosed or open fans are
called propeller fan. The flow of air is substantially parallel to the axis of the impeller.

Advantages: 1- suitable for handling large volume..
    2- used in industrial ventilation and air conditioning systems.

Types:
- *ducted fan*
- *propeller fan*

Some designs have guide vanes down stream which remove the rotational components
converting some excess velocity into more useful static pressure. Another designs have up
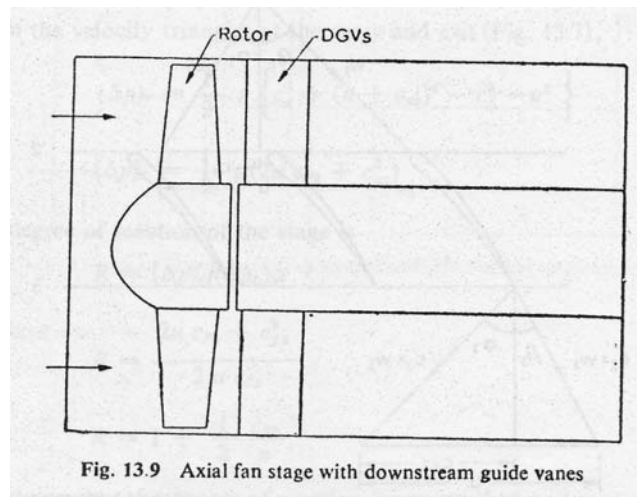stream guide vanes, as shown in the following figures.

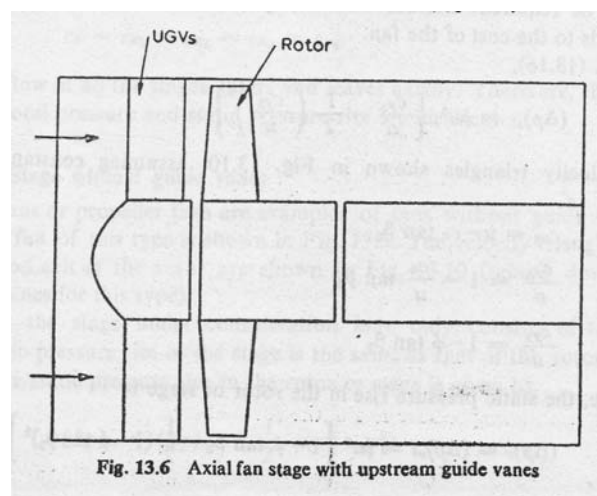Fig. 13.9   Axial fan stage with downstream guide vanes

Fig. 13.6   Axial fan stage with upstream guide vanes

4- Fan and system characteristics:

Effect of change in system resistance:
Point (A) represent the point of operation. If the system resistance change say due to unclean filters, the system curve will move and the operating point will then shift to (B) resulting in decreased flow.
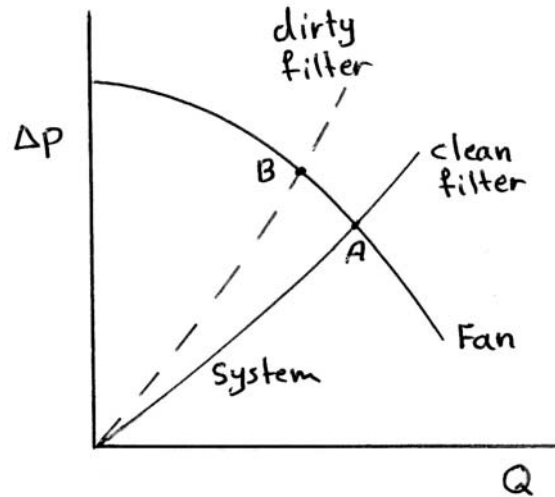
Effect of change in fan speed:
If the fan speed increase the operating point will shift from point (1) to point (2).
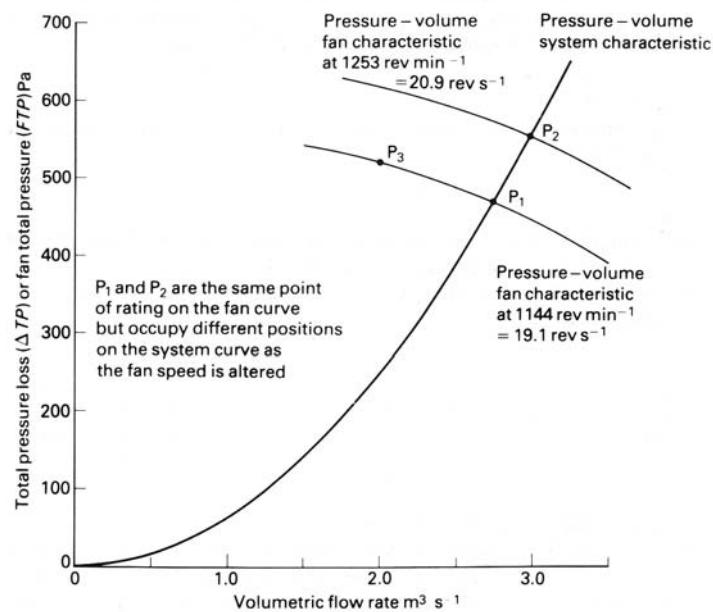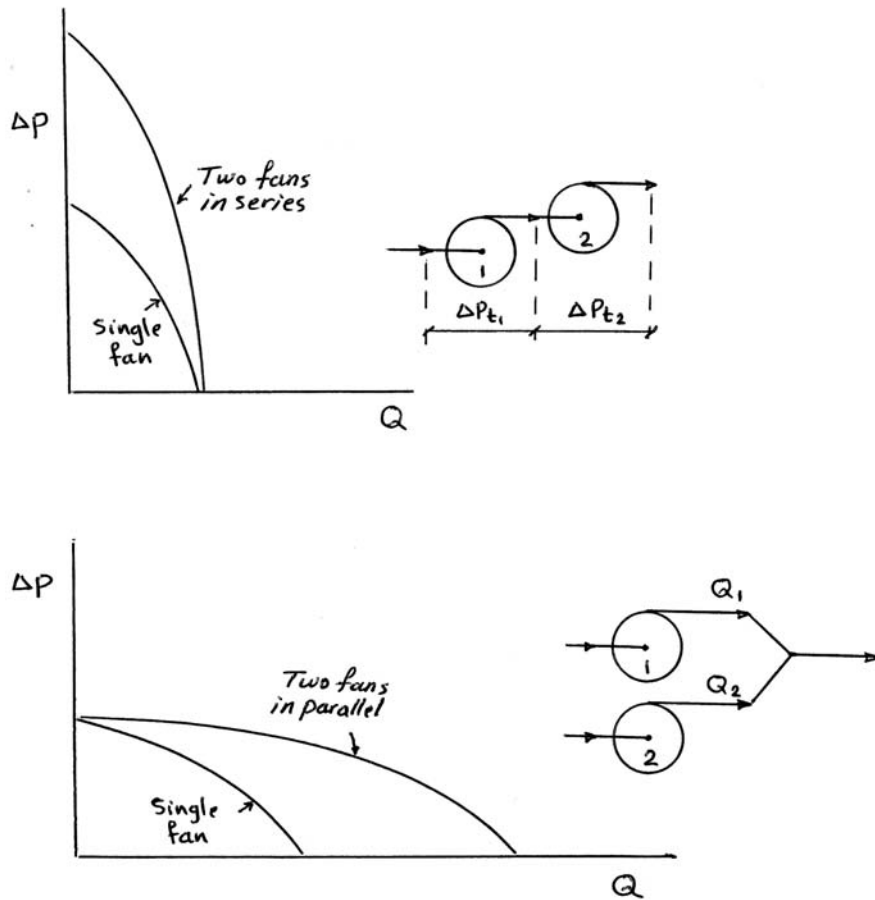

Figure 15.20 The point of rating on a fan characteristic curve is fixed but it slides down the system characteristic curve as the fan speed is reduced

Fan Arrangements

The figures show the characteristic curves for two identical fans in series and for two identical fans in parallel.

- fans in series tend to increase the total pressure.

$$\Delta p_{t\,\text{double}} = \Delta p_{t1} + \Delta p_{t2}$$

- fans in parallel tend to increase the volume flow rate.

$$Q_{\text{double}} = Q_1 + Q_2$$

5- Example:

A backward-swept centrifugal fan develops a pressure of (75 mm H$_2$O). It has an impeller diameter of (89 cm) and runs at (720 rpm). The blade air angle at tip is (39°) and the width of the impeller (10 cm). Assuming a constant radial velocity of (9.15 m/s) and density of (1.2 kg/m^3), determine the fan efficiency, discharge, power required and pressure coefficient.

Solution:

$$Q = \pi d_2 b_2 V_{r2}$$
$$= \pi \times 0.89 \times 0.1 \times 9.15$$
$$= 2.558 \, \text{m}^3/\text{s} \qquad \text{(Ans)}$$

$$u_2 = \frac{\pi d_2 N}{60} = \frac{\pi \times 0.89 \times 720}{60} = 33.55 \, \text{m/s}$$

$$\text{input power} = \rho Q u_2 V_2 \cos \alpha_2$$
$$= \rho Q u_2 V_{t2}$$
$$= \rho Q u_2 \left(u_2 - \frac{V_{r2}}{\tan \beta_2} \right)$$
$$= 1.2 \times 2.558 \times 33.55 \left(33.55 - \frac{9.15}{\tan 39} \right)$$
$$= 2291.5 \, \text{W} \qquad \text{(Ans)}$$

$$\text{output power} = \gamma Q H$$
$$= 9810 \times 2.558 \times 0.075$$
$$= 1882.05 \, \text{W}$$

Fan efficiency: $\eta = \dfrac{\text{output power}}{\text{input power}} = \dfrac{1882.05}{2291.5} \times 100 = 82\%$ \qquad (Ans)

Pressure coefficient: $\psi = \dfrac{p_t}{\frac{1}{2} \rho u_2^2} = \dfrac{\gamma H}{\frac{1}{2} \rho u_2^2}$

$$\psi = \frac{9810 \times 0.075}{\frac{1}{2} \times 1.2 \times 33.55^2} = 1.09 \qquad \text{(Ans)}$$

1- A centrifugal fan (blower) with a radial impeller produces a pressure equivalent to 100 cm column of water. The pressure and temperature at its entry are (0.98 bar, 310 K). The electric motor driving the blower runs at 3000 rpm. The efficiency of the fan and drive are 82% and 88% respectively. The velocity at the inlet eye is 0.4 u_2. If the blower handles 200 m^3/min of air at the entry conditions; determine the power required by the electric motor, the impeller diameter, and the inner diameter of the blade ring.

[45.31 kW; 66.4 cm ; 31.9 cm]

2- A fan running at 1480 rpm, takes in 6 m^3/min of air at inlet conditions of (p=950 mbar, t=15°C). If the fan impeller diameter is 40 cm and the blade tip air angle 20°; determine the fan total pressure, and the width of the impeller at exit. Take the radial velocity at the exit as 0.2 times the impeller tip speed.   [497.9 N/m^2 ;1.28 cm]

3- A backward-swept (β_2=30°, d_2=46.6 cm) centrifugal fan is required to deliver 3.82 m^3/s (4.29 kg/s) of air at a total pressure of 618.05 N/m^2. The flow coefficient at the impeller exit is 0.25 and the power supplied to the impeller is 3 kW. Determine the fan efficiency, pressure coefficient, and rotational speed.   [78.7% ; 0.895 ; 1440 rpm]

4- A fan has a pressure-volume characteristic given by the following information obtained from a test at (t=0°C and p_{atm}=101.325 kPa), when running at 1144 rpm.

Q (m^3/s)	1.5	2	2.5	3	3.5
P_t (N/m^2)	542	520	488	443	388

a- determine the quantity of air handled if it is installed in a system of the following data:

Q (m^3/s)	0.5	1	1.5	2	2.5	2.75	3	3.25
ΔP_t (N/m^2)	15	61	138	245	383	464	552	648

b- determine the fan input power, if the fan total efficiency is 84%.
c- determine the air quantity handled, the fan total pressure and the fan input power, if the air temperature is 35°C and the barometric pressure is 85 kPa.
d- at what speed must the fan run to deliver 3 m^3/s.

[2.74 m^3/s ; 1.504 kW ; 2.74 m^3/s ; 343 N/m^2 ; 1.118 kW ; 1253 rpm]

Chapter Six
Ideal Fluid Flow

Contents

1- Introduction.

2- Requirements for ideal fluid flow.

3- Relationships between stream function (ψ), potential function (φ) and velocity component.

4- Basic flow patterns.

5- Combination of basic flows.   (لفرع الميكانيك العام فقط)

6- Examples.

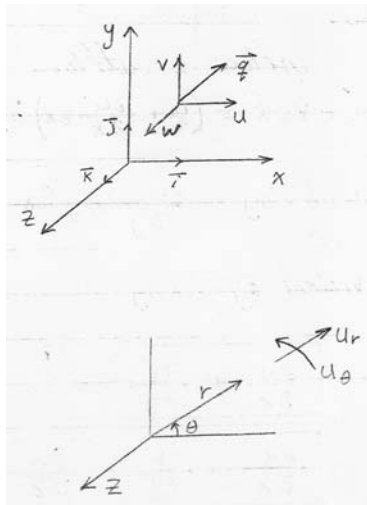7- Problems sheet; No. 6

1- Introduction

Velocity vector

$\vec{q} = u\vec{i} + v\vec{j} + w\vec{k}$ In Cartesian coordinates

$\vec{q} = u_r\vec{r} + u_\theta\vec{\theta} + w\vec{k}$ In Polar coordinates

Divergence of $\vec{q} = \nabla \cdot \vec{q}$

$$\nabla \cdot \vec{q} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$
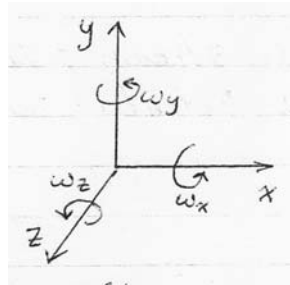
Continuity equation

$\nabla \cdot \vec{q} = 0$

Or

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Curl of $\bar{q} = \nabla \times \bar{q}$

Vorticity equation

$$\nabla \times \bar{q} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right)\vec{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right)\vec{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)\vec{k}$$

$$\nabla \times \bar{q} = \omega_x \vec{i} + \omega_y \vec{j} + \omega_z \vec{k}$$

If $\nabla \times \bar{q} \neq 0$ the flow is called rotational
If $\nabla \times \bar{q} = 0$ the flow is called irrotational

2- Requirements for ideal- fluid flow
  1- non viscous.
  2- incompressible.
  3- $\nabla \cdot \bar{q} = 0$
  4- $\nabla \times \bar{q} = 0$

3- Relationships between stream function (ψ), potential function (φ) and velocity component

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial \phi}{\partial x}$$

$$v = -\frac{\partial \psi}{\partial x} = \frac{\partial \phi}{\partial y}$$
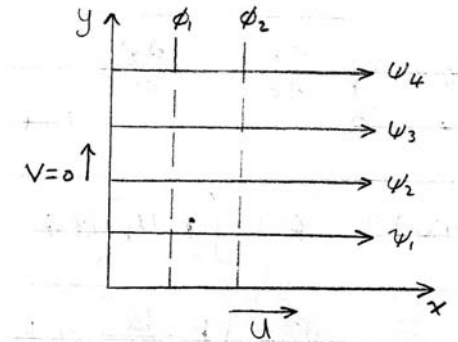
In cylindrical coordinates :

$$u_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = \frac{\partial \phi}{\partial r}$$

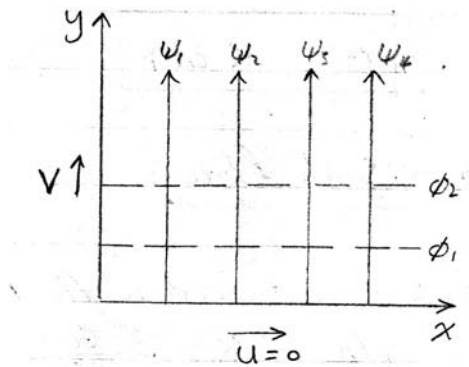$$u_\theta = -\frac{\partial \psi}{\partial r} = \frac{1}{r}\frac{\partial \phi}{\partial \theta}$$

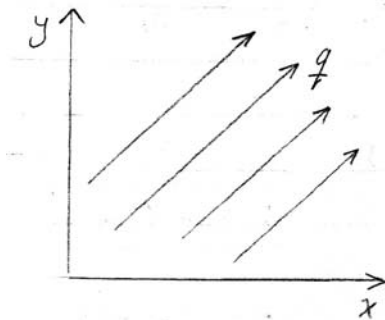# 4- Basic flow patterns:

1- Uniform flow
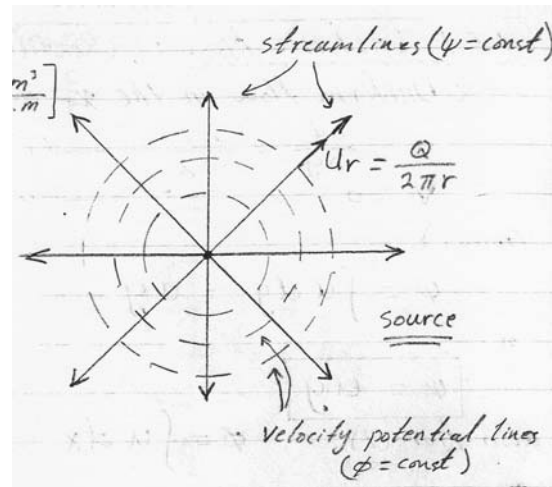a- Uniform flow in the x- direction
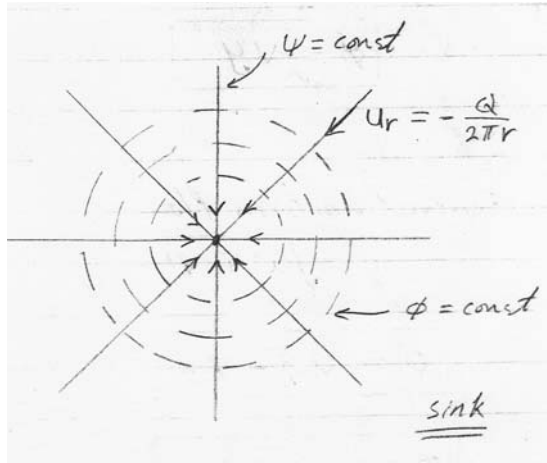

b- Uniform flow in the y- direction
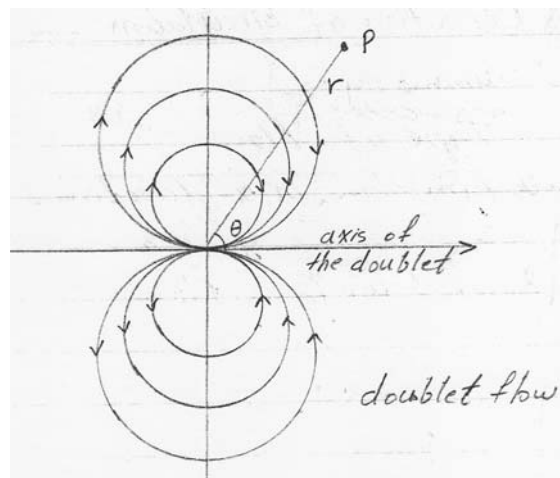

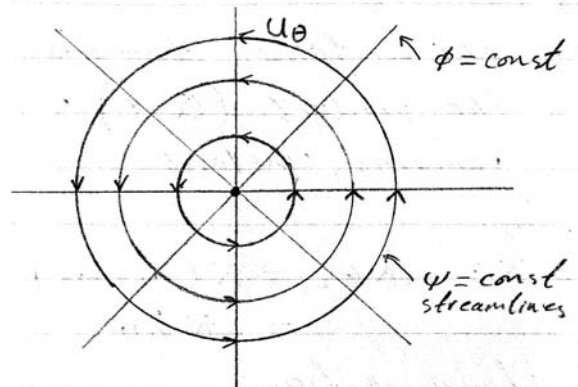c- General uniform flow

2- Source flow

3- Sink flow

4- Doublet flow

5- Free vortex flow

Stream function and Potential function for Basic flow patterns:

Type of flow	ψ	φ
Uniform flow in the x- direction	uy	ux
Uniform flow in the y- direction	-vx	vy
General uniform flow	uy-vx	ux+vy
Source	kθ	k ln r
Sink	-kθ	-k ln r
Doublet	$-\dfrac{\mu}{2\pi}\dfrac{\sin\theta}{r}$	$\dfrac{\mu}{2\pi}\dfrac{\cos\theta}{r}$
Free vortex	$-\dfrac{\Gamma}{2\pi}\ln r$	$\dfrac{\Gamma}{2\pi}\theta$

<u>Note:</u> k = strength of the source = $\dfrac{Q}{2\pi}$ or $(=\dfrac{m}{2\pi})$
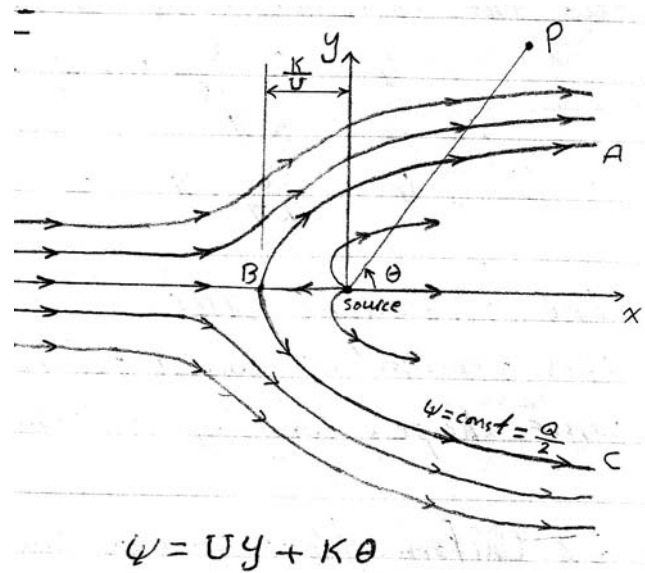
Definition of circulation (Γ):

$$\Gamma = \oint_c q_s\, ds$$

Circulation = vorticity×area

$$\Gamma \quad = \quad \omega_z \quad \times \quad A$$

5- *Combination of basic flows:*

1- Uniform flow and a source.

$$\psi = Uy + k\theta$$

The stream function:
$$\psi = Uy + k\theta$$
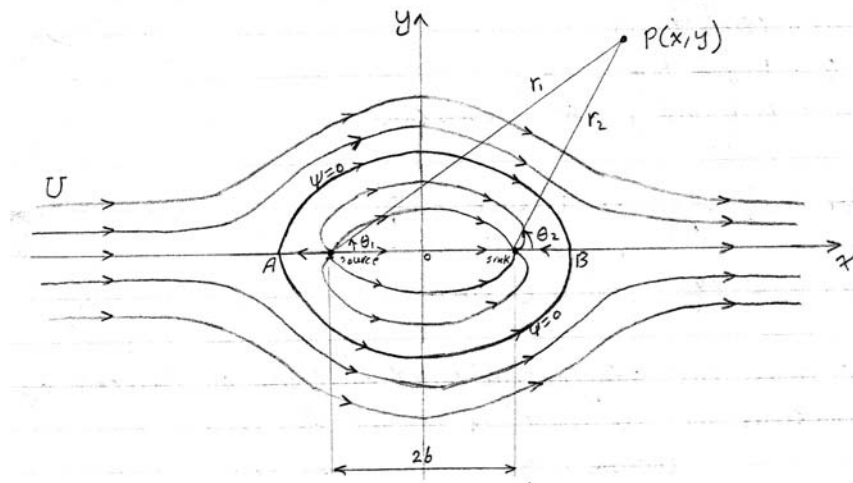$$\psi = U.r\sin\theta + k\theta$$

The velocity components:
$$u_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = U.\cos\theta + \frac{k}{r}$$
and
$$u_\theta = -\frac{\partial\psi}{\partial r} = -U.\sin\theta$$

The dividing streamline ($\psi = \dfrac{Q}{2}$) could be replaced by a solid surface of the same shape, forming a semi-infinite body (half-body).

2- Uniform flow and a source-sink pair.

The stream function:

$$\psi = Uy + k\theta_1 - k\theta_2$$

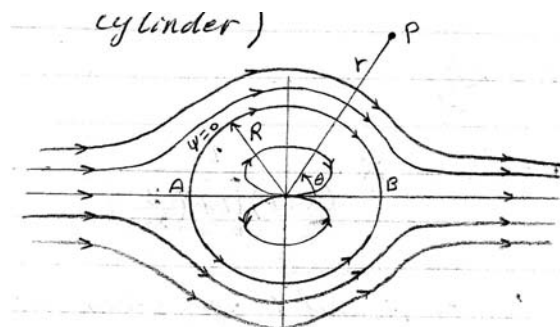$$\psi = Uy + k \tan^{-1} \frac{y}{x+b} - k \tan^{-1} \frac{y}{x-b}$$

The velocity component:

$$u = \frac{\partial \psi}{\partial y} = U + \frac{k}{(x+b)\left[1+\left(\dfrac{y}{x+b}\right)^2\right]} - \frac{k}{(x-b)\left[1+\left(\dfrac{y}{x-b}\right)^2\right]}$$

The dividing streamline ($\psi = 0$) could be replaced by a solid surface of the same shape, forming an oval called a Rankine oval.

3- Uniform flow and a doublet:
   (Non lifting flow over a cylinder)

The stream function:

$$\psi = Uy - \frac{\mu}{2\pi}\frac{\sin\theta}{r}$$

$$\psi = U.r\sin\theta\left(1 - \frac{R^2}{r^2}\right)$$

where

$$R^2 = \frac{\mu}{2\pi U}$$

The velocity components:

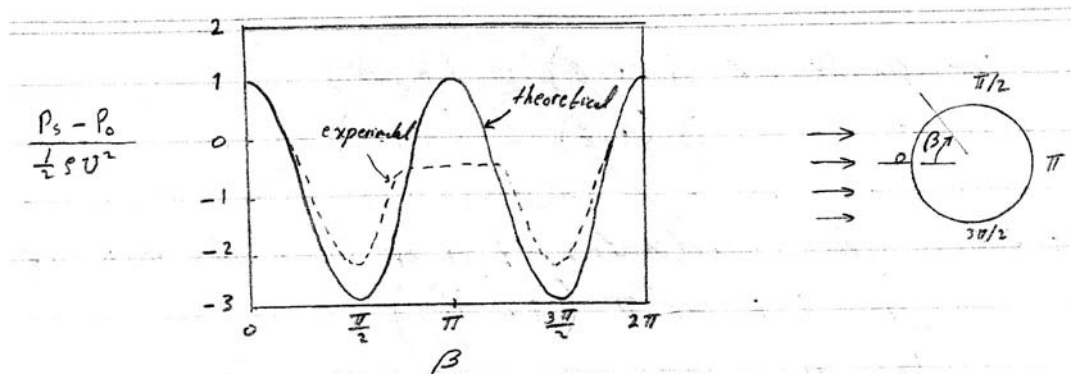$$u_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = U.\cos\theta\left(1 - \frac{R^2}{r^2}\right)$$

and

$$u_\theta = -\frac{\partial\psi}{\partial r} = -U.\sin\theta\left(1 + \frac{R^2}{r^2}\right)$$

The dividing streamline ($\psi = 0$) could be replaced by a solid surface of the same shape, forming a circular cylinder with radius $R = \sqrt{\dfrac{\mu}{2\pi U}}$.

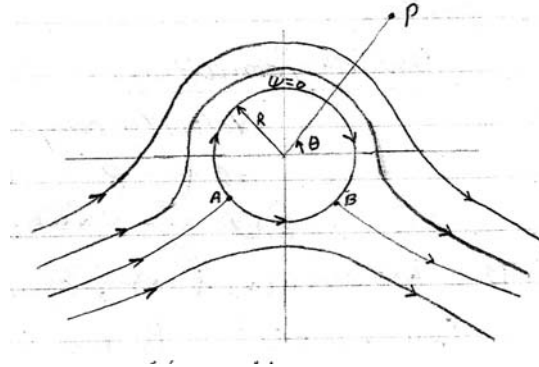The pressure distribution on the cylinder surface is obtained from:

$$P_s = P_o + \frac{1}{2}\rho U^2\left(1 - 4\sin^2\theta\right)$$

The pressure distribution is symmetrical around the cylinder and the resultant force developed on the cylinder = zero.

4- Doublet and free vortex in a uniform flow:
(Lifting flow over a cylinder)

The stream function:
$$\psi = U.r\sin\theta\left(1 - \frac{R^2}{r^2}\right) + \frac{\Gamma}{2\pi}\ln r$$

The velocity components:
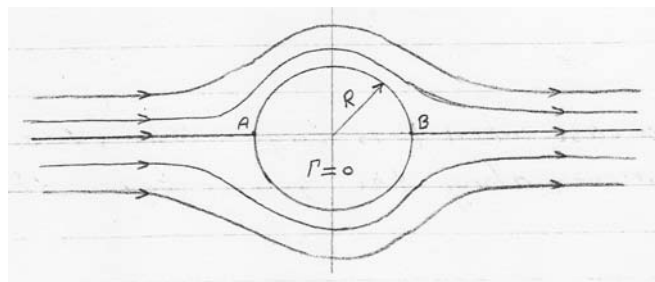$$u_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = U.\cos\theta\left(1 - \frac{R^2}{r^2}\right)$$

and

$$u_\theta = -\frac{\partial\psi}{\partial r} = -U.\sin\theta\left(1 + \frac{R^2}{r^2}\right) - \frac{\Gamma}{2\pi r}$$

The location of the stagnation points is given by:
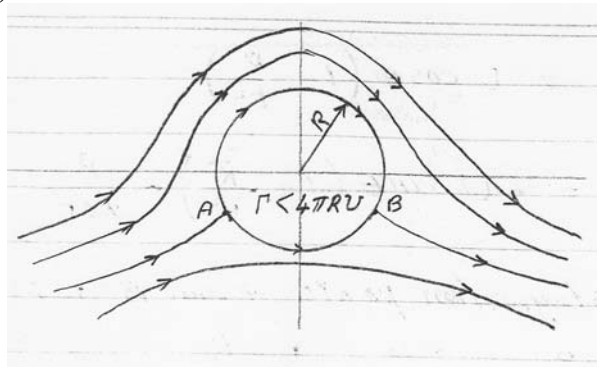$$r = R \; ; \; \sin\theta = \left(\frac{-\Gamma}{4\pi RU}\right)$$

There are four possible cases:
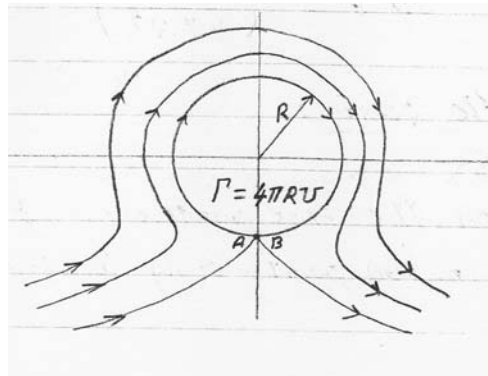a- ($\Gamma = 0$)

b- ($\Gamma < 4\pi RU$)

c- ($\Gamma = 4\pi RU$)
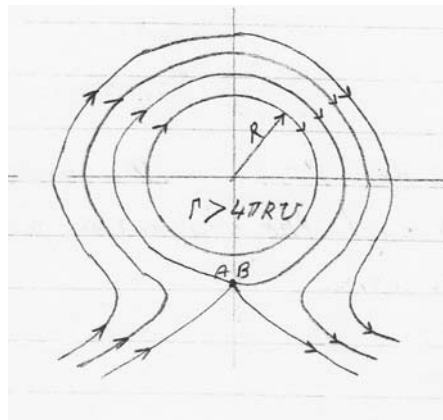

d- ($\Gamma > 4\pi RU$)

The pressure distribution on the cylinder surface is obtained from:

$$P_s = P_o + \frac{1}{2}\rho U^2 - \frac{1}{2}\rho\left(-2U.\sin\theta - \frac{\Gamma}{2\pi R}\right)^2$$

The lift force on the cylinder is

Lift = ρUΓL   where L = length of the cylinder

Lift = ρUΓ
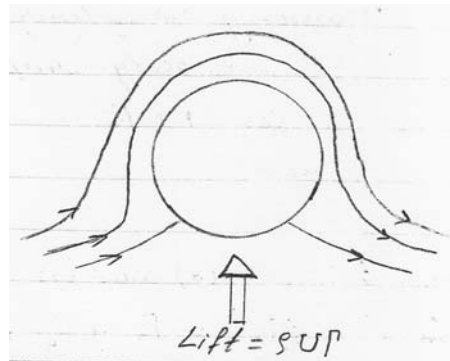
6- Examples:

1- Does the stream function ($\psi = xy$) represent a physically possible flow? If so, determine the velocity at a point (2,3).

Solution:

To prove that the flow is possible, the continuity equation must be satisfied.

$$\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial y \partial x} = 0$$

$$1 - 1 = 0 \quad \checkmark$$

$$u = \frac{\partial \psi}{\partial y} = x \qquad ; \qquad v = -\frac{\partial \psi}{\partial x} = -y$$
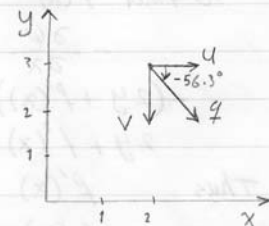
at point (2, 3) $u = x = 2$

$$v = -y = -3$$

$$q = \sqrt{u^2 + v^2} = \sqrt{4 + 9} = 3.6 \text{ units} \qquad (Ans)$$

$$\theta = \tan^{-1} \frac{v}{u} = \frac{-3}{2} \implies \theta = -56.3° \quad (Ans)$$

2- A velocity potential in two-dimensional flow is given by ($\varphi = y+x^2-y^2$); find the stream function for this flow.

Solution:

a)- $\phi = y + x^2 - y^2$

$\dfrac{\partial \psi}{\partial y} = \dfrac{\partial \phi}{\partial x} = 2x$

$\partial \psi = 2x \, \partial y$

$\therefore \quad \psi = 2xy + f(x) \qquad \qquad —(*) \qquad f(x) = $ constant of integration.

To find $f(x)$

$-\dfrac{\partial \psi}{\partial x} = \dfrac{\partial \phi}{\partial y}$

$-(2y + f'(x)) = 1 - 2y$

$2y + f'(x) = -1 + 2y$

Thus $\quad f'(x) = -1$

$f(x) = -x + C$

sub in eqn (*) $\implies \qquad \psi = 2xy - x + C \qquad$ (Ans)

3- A stream function in two-dimensional flow is ($\psi = 9+6x-4y+7xy$); find the velocity potential for this flow.

Solution:

$\psi = 9 + 6x - 4y + 7xy$

$\dfrac{\partial \phi}{\partial x} = \dfrac{\partial \psi}{\partial y} = -4 + 7x$

$\therefore \quad \phi = -4x + \dfrac{7}{2} x^2 + f(y) \qquad \qquad$ where $f(y) = $ constant of integration.

to find $f(y)$

$\dfrac{\partial \phi}{\partial y} = -\dfrac{\partial \psi}{\partial x}$

$f'(y) = -(6 + 7y) = -6 - 7y$

$\therefore \quad f(y) = -6y - \dfrac{7}{2} y^2 + C$

$\therefore \quad \phi = -4x - 6y + \dfrac{7}{2}(x^2 - y^2) + C \qquad$ (Ans)

1- Show that the two-dimensional flow described by the equation $\psi = x + 2x^2 - 2y^2$ is irrotational. Find the velocity potential for this flow. $[\phi = -y - 4xy + c\,]$

2- A certain flow field is described by the velocity potential $\phi = A \ln r + B r \cos\theta$ where A and B are positive constants. Determine the corresponding stream function and locate any stagnation points in this flow field. $[\psi = A\theta + B r \sin\theta + c\ ;\ \left(\dfrac{A}{B},0\right)\ \left(\dfrac{A}{B},\pi\right)]$

3- The velocity components in a two-dimensional flow field for an incompressible fluid are expressed as: $u = \dfrac{y^3}{3} + 2x - x^2 y\ ;\ v = xy^2 - 2y - \dfrac{x^3}{3}$.

a) show that these functions represents a possible case of irrotational flow.
b) obtain expressions for the stream function and velocity potential.

4- The formula $\phi = 0.04 x^3 + axy^2 + by^3$ represent the velocity potential of a two-dimensional ideal flow. Evaluate the constants a and b, and calculate the pressure difference between the points (0,0) and (3,4)m, if the fluid has density of 1300 kg/m^3.

$[a = -0.12,\ b = 0\ ;\ 5.85\ \text{kN/m}^2]$

5- The two-dimensional flow of a non-viscous, incompressible fluid in the vicinity of the 90° corner of figure is described by the stream function $\psi = 2r^2 \sin 2\theta$.
a) determine the corresponding velocity potential.
b) if the pressure at point(1,0) on the wall is 30kPa, what is the pressure at point (0,0.5) , assume $\rho = 1000$ kg/m^3 , and x-y plane is horizontal. $[\phi = 2r^2 \cos 2\theta + c\ ;\ 36\ \text{kPa}]$

6- The stream function for an incompressible flow filed is given by the equation $\psi = 3tx^2 y - ty^3$. Find the potential function and determine the flow rates across the faces of the triangular prism OAB shown in figure having a thickness of 5 units in the z-direction at time t = 1. $[\phi = tx^3 - 3txy^2 + c\ ;\ 40;\ 0;\ 40]$
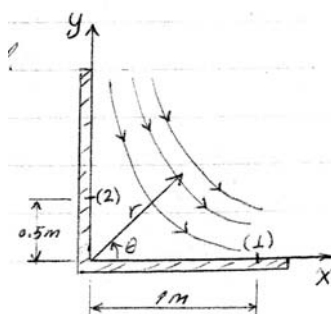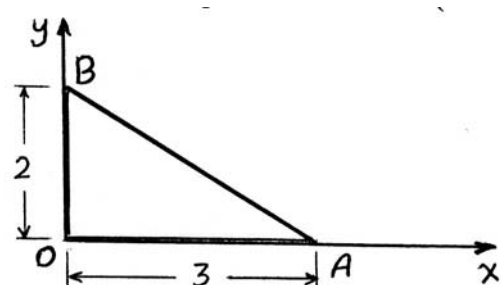
Problem No. 5   Problem No.6

7- Prove that for a two-dimensional flow, the vorticity at a point is twice the rotation (angular velocity).

8- The pressure far from an irrotational vortex in the atmosphere is zero gage. If the velocity at r = 20 m is 20 m/s, find the velocity and pressure at r = 2 m. (ρ = 1.2 kg/m^3)

[200 m/s ; -23.76 kPa]

9- A non viscous incompressible fluid flow between wedge shaped-wall into small opening as shown in figure. The velocity potential which described the flow is $\phi = -2 \ln r$. Determine the volume rate of flow (per unit length) in the opening.   [-π/3 m^3/s per m]

10- A source with strength 0.2/2π m^3/s.m and a vortex with strength 1/2π m^2/s are located at the origin. Determine the equations for velocity potential and stream function. What are the velocity components at x = 1 m , y = 0.5 m?   [0.0285 m/s ; 0.143 m/s]

11- In an infinite two-dimensional flow filed, a sink of strength 3/2π m^3/s.m is located at the origin, and another of strength 4/2π m^3/s.m at (2 , 0). What is the magnitude and direction of the velocity at point (0 , 2).   [0.429 m/s ; -68.22°]

12- Flow over a plane half-body is studied by utilizing a free-stream at 5 m/s superimposed on a source at the origin. The body has a maximum width 2 m. Calculate:
a) the coordinates of the stagnation point.
b) the width of the body at the origin.
c) the velocity at a point (0.5 , π/2).   [(0.32 , π) ; 1 m ; 5.93 m/s]

13- The shape of a hill arising from a plain can be approximated with the top section of a half-body as is shown in figure. The height of the hill approaches 61 m. When a 18 m/s wind blows toward the hill, what is the magnitude of the air velocity at point (2) above the origin. What is the elevation of point (2) and what is the difference in pressure between point (1) and point (2). (ρ_{air} = 1.23 kg/m^3)   [21.34 m/s ; 30.5 m ; 448.83 Pa]

14- A circular cylinder 0.5 m diameter rotates at 600 rpm in a uniform stream of 15 m/s. Locate the stagnation points. Calculate the minimum rotational speed for detached stagnation point in the same uniform flow.   [-31.6° and -148.4° ; 1146 rpm]

15- A circular cylinder 20 m long is placed in a uniform stream of 100 m/s (ρ= 0.7 kg/m^3). The lift force generated by the cylinder is 2100 kN. The stagnation points are at (-60° and -120°). Derive a relationship between the locations of the stagnation points and the circulation around the cylinder. Calculate the diameter of the cylinder.   [2.75 m]
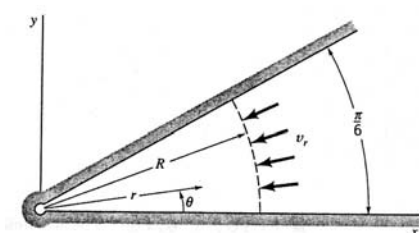
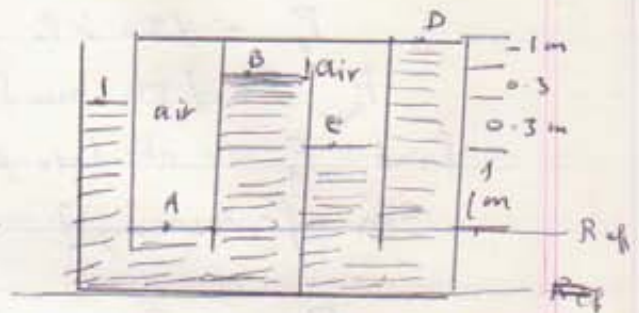Problem No. 9   Problem No. 13

6
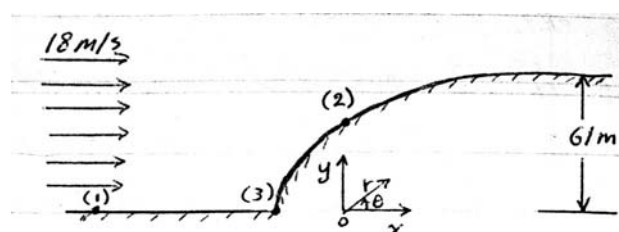
2.2

$P_1 + \gamma h_1 = P_A$

$0 + 9810 \times 1.3 = P_A$

$\therefore P_A = 12.753 \ kPa$

$P_1 + \gamma_1 h_1 - \gamma_2 h_2 = P_B$ $\therefore \ 0 + 9810 \times 1.3 - 9810 \times 1.6 = P_B$

$\therefore P_B = -2.943 \ kPa = P_c$ $P_c + \gamma_1 h_1 - \gamma_2 h_2 = P_D$

A

$\therefore P_D = -18.639$
kPa.

2.3

find the pressure at A, B
in meter of water ?

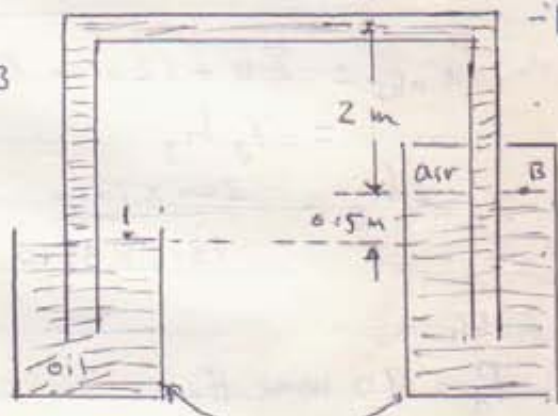$P_1 - \gamma h_1 + \gamma h_2 = P_B$

$0 - 0.85 \times 2.5 \times 9810$

$+ 0.85 \times 9810 \times 2 = P_B$

$P_B = 0.85 \times 9810 \times (2 - 2.5)$ Relative density

$= -0.425 \times 9810 = -4169.25 \ N/m^2$ 0.85

$P_B = \gamma_{oil} \ h_{oil} = \gamma_\omega \ h_\omega$

$h_\omega = \dfrac{\gamma_{oil} \ h_{oil}}{\gamma_\omega} = \dfrac{-0.425 \times 9810}{9810} = -0.425 \ m$

also   of H_2O

$0 - \gamma h_1 = P_A = -20846 \ N/m^2$

$\therefore P_A = -\gamma_{oil} \ h_{oil} = \gamma_\omega h_\omega$ $\therefore h_\omega = \dfrac{-0.85 \times 9810 \times 2.5}{9810}$
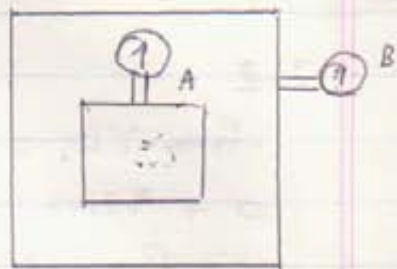
$h_\omega = -2.125 \ m \ of \ water.$

Q.13 $P_A = 80$ kPa gage

$P_B = 120$ kPa gage

$P_{bar} = 750$ mm Hg

find P_A in absolute pressure in cm of mercury?

$$P_{abs} = P_{gage} + P_{bar}$$

$$P_{bar} = 13.6 \times 9810 \times 0.75 = 100 \text{ KPa}$$

$$\therefore P_{A\,abs} = 80 + 120 + 100 = 300 \text{ KPa}$$

$$= \gamma_g h_g$$

$$\therefore h_g = \frac{300 \times 1000}{13.6 \times 9810} = 2.25 \text{ m of Hg}$$
$$= 225 \text{ cm of Hg}$$

Q.2.24

a) $P_A = 90$ mm H_{2O}

$= \gamma_w h_w = 9810 \times 0.09 = Pa$

$P_A + \gamma_w h_1 - \gamma_s R = 0$

$9810 \times 0.09 + 9810 \times 0.6 -$
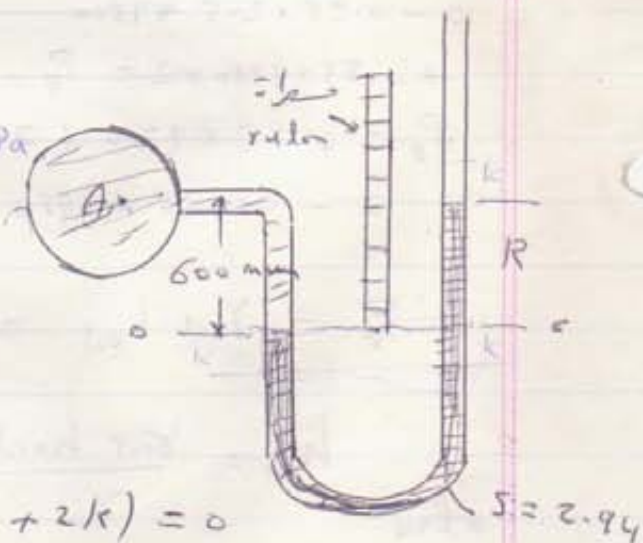$\qquad 2.94 \times 9810 R = 0$

$R = 0.2347$ m

b) $P_A = 8$ kPa

$P_A + \gamma_w (h_1 + k) - \gamma_s (R + 2k) = 0$

$8000 + 9810 (0.6 + k) - 2.94 \times 9810 (0.2347 + 2k) = 0$

$\therefore k = 0.148$ m

The total Reading without adjustment $= k + R = 0.148 + 0.23$
$$= 0.383 \text{ m}.$$

1

Q 2-4 $P_a + \gamma h_1 - \gamma h_2 = 0$

$\therefore P_a = \gamma h_2 - \gamma h_1 = 9810 (1.6 - 2.2)$

$= -5.88$ KPa.

$P_b + 9810 \times 1 - 1.6 \times 9810 = 0$

$P_b = 5.88$ kPa

$P_c = P_b = 5.88$ مقدار فشار در نقطه

$P_D = P_c + 1.4 \times 0.9 \times 9810 = 22.54$ KPa.

Q.2.14

a) Water $\gamma_w h_w = \gamma_g h_g$

$9810 \times h_w = 13.6 \times 9810 \times 0.2$

$h_w = 2.72$ m

b) Kerosene $\gamma_k h_k = 13.6 \times 9810 \times 0.2$

$h_k = \dfrac{13.6 \times 9810 \times 0.2}{0.83 \times 9810} = 3.277$ m

c) acetylene

$h_{as} = \dfrac{13.6 \times 9810 \times 0.2}{2.94 \times 9810} = 0.925$ m

Q. 2.16

$P_a + \gamma h = 0$ $P_a = -9810 \times 0.83 \cdot h = 3000$

$\therefore h = 3.688$ m

Q. 2.19 $P_A = \gamma h_w$

$\gamma_g h_g = \gamma_w h_w$ $\therefore h_g = \dfrac{9810 \times 0.075}{9810 \times 13.6} \times 1000$

$= 5.5$ mm Hg.

Q. 2-20 $S_1 = 1.0$ $S_2 = 0.95$ $S_3 = 1.0$

$h_1 = h_2 = 280$ mm $h_3 = 1m$

$P_A - P_B = ?$ in meter of water.

$P_A - \gamma_1 h_1 - \gamma_2 h_2 + \gamma_3 h_3 = P_B$

$P_A - P_B = \gamma_1 h_1 + \gamma_2 h_2 - \gamma_3 h_3$

$= 9810 \times 0.28 + 9810 \times 0.95 \times 0.28 - 9810 \times 1 \times 1$

$= -4.45$ kN/m^2

$\therefore h_A - h_B = \dfrac{-4.45 \times 1000}{9810} = 0.454$ m H$_2$O

$= 454$ mm H$_2$O

Q. 2-22

a) $P_A + S_1 \gamma_w h_1 - S_2 \gamma_w h_2 - S_3 \gamma_3 h_3 = P_B$

$P_A + 0.83 \times 9810 \times 0.15 - 13.6 \times 9810 \times 0.07 - 0.83 \times 9810 \times 0.12$

$= 70000$

$P_A = 79.09$ kPa.

b)

$P_{abs} = P_{gy} + P_{atm}$ $\therefore P_g = 140000 - 13.6 \times 9810 \times 0.72$

$= 43.9$ kPa

$43900 + 0.83 \times 9810 \times 0.15 - 13.6 \times 9810 \times 0.07 - 0.83 \times 9810 \times 0.12$

$= P_B$

$\therefore P_B = 34.82$ kPa

$\therefore h_B = \dfrac{P_B}{\gamma_w} = 3.55$ m of water.

2.42 $F = P_G A = \gamma \bar{h} A$

$\qquad = 9810 \times 2 \, (\pi r_1^2 - \pi r_2^2)$

$\qquad = 46.2 \, kN$

Q.43

$\qquad F = P_G A = \gamma \bar{h} A$

$\qquad = 9810 \times 2 \times 2 \times 1.3$

$\qquad = 51.012 \, kN$

$y_p = \bar{y} + \dfrac{I_G}{\hat{y} A} = 2 + \dfrac{1.3 \times 2^3}{12 \times 2 \times 2 \times 1.3}$

$\qquad = 2.166 \, m$

The moment about A

$M_A = F \times (y_p - 1) = 51.012 \times (2.166 - 1)$

$\qquad = 59.8 \, kN \cdot m$

Q.44

$\qquad F_1 = P_G A = \gamma \bar{h} A$
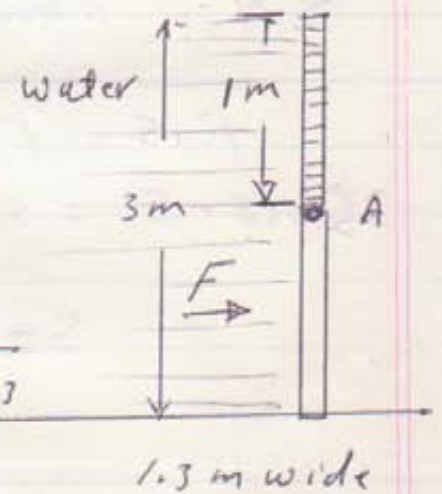
$\qquad = 9810 \times 2 \times 2 \times 1.3 =$

$\qquad = 51.012 \, kN$

$y_{P_1} = 2 + \dfrac{1.3 \times 2^3}{12 \times 2 \times 2 \times 1.3} = 2.166 \, m$

$F_2 = 9810 \times 1 \times 2 \times 1.3 =$

$\qquad = 25.506 \, kN$
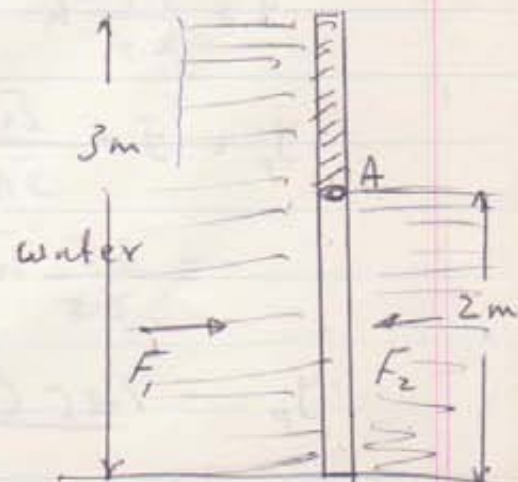
$$y_{P_2} = 1 + \frac{1.3 \times 2^3}{12 \times 4 \times 2 \times 1.3} = 1.333 \text{ m}$$

$$R = F_1 - F_2 = 25.506 \text{ kN} \longrightarrow$$

$\sum M = 0$ about A جميل

$$R \times S = F_1 (y_{P_1} - 1) - F_2 \times y_{P_2}$$

$$25.066 \times S = 51.032 (1.166) - 25.506 \times 1.333$$

$S \cong 1$ m   the line of action of the resultant force from A

Q. 2.45

$$F = \gamma \bar{h} A$$
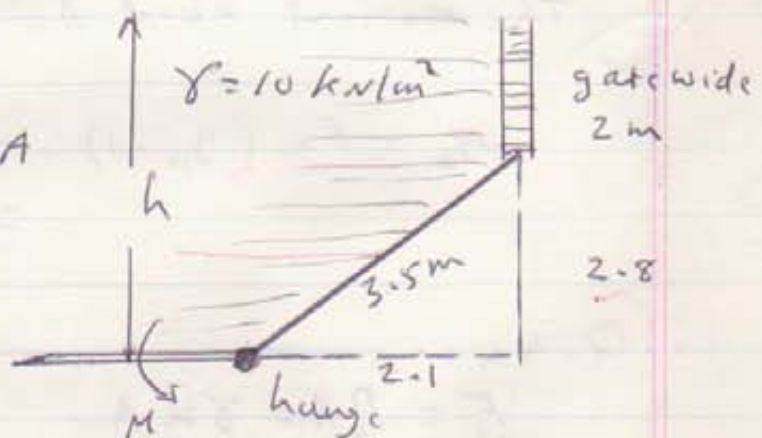$$= \gamma (h - 1.4) A$$

when $\bar{h} = h - 1.4$

$$\bar{h} = \bar{y} \sin \theta$$
$$\bar{y} = \frac{3.5}{2.8} \bar{h}$$

$\gamma = 10 \text{ kN/m}^2$ gate wide 2 m

h

3.5m   2.8

2.1

M hinge

$$y_p = \bar{y} + \frac{I_q}{\bar{y} A} = \frac{3.5 \bar{h}}{2.8} + \frac{2 \times 3.5^3}{12 \times \frac{3.5}{2.8} \bar{h} \times 3.5 \times 2}$$

$$= \frac{3.5}{2.8} \bar{h} + \frac{3.5^3 \times 2.8}{12 \bar{h}} = 1.25 \bar{h} + \frac{0.82}{\bar{h}}$$

$$y_p = \frac{1.25 (h - 1.4)^2 + 0.82}{(h - 1.4)}$$

$$\Sigma M = 145 \, k\overline{N} \cdot m \quad F(L - y_p)$$

$$L \sin \theta = h \qquad \therefore L = \frac{3.5}{2.8} h$$

$$\therefore 14,500 = \gamma A (h - 1.4) \left(\frac{3.5}{2.8} h - \left(1.25 (h - 1.4) + \frac{0.82}{h - 1.4} \right) \right)$$

$$14,5000 = 10000 \times 2 \times 3.5 \, (h - 1.4) \left(1.25 h - 1.25 h + 1.75 - \frac{0.82}{h - 1.4} \right)$$

$$= 70000 \, (h - 1.4) \left(1.75 - \frac{0.82}{h - 1.4} \right)$$

$$\frac{145000}{70000} = 1.75 (h - 1.4) - 0.82$$

$$= 1.75 h - 2.45 - 0.82$$

$$\therefore h = 3.05 \, m$$

Q.46

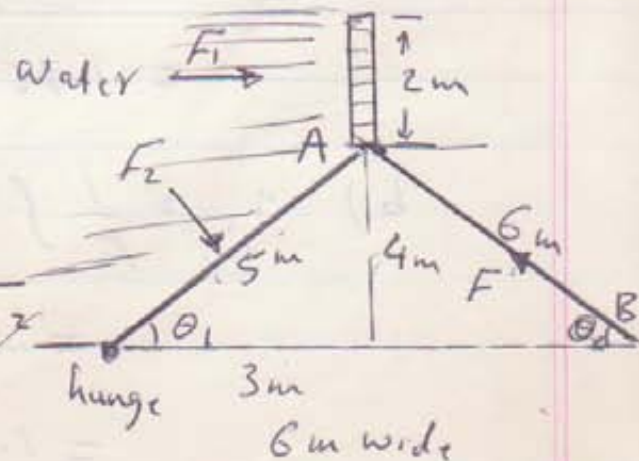$$F_1 = \gamma \overline{h} A$$
$$= 9810 \times 1 \times 2 \times 6$$
$$= 117.72 \, kN$$

$$y_{p_1} = \overline{y} + \frac{I_c}{\overline{y} A} = 1 + \frac{6 \times 2^3}{12 \times 1 \times 6 \times 2}$$

$$= 1.333 \, m$$

$F_2 = \gamma \bar{h} . A = 9810 \times 4 \times 5 \times 6$
$$= 1177.2 \; kw$$

$y_{P_2} = \bar{y} + \dfrac{I_G}{\bar{y} A}$ 　　　　　$\bar{h} = \bar{y} \sin \theta, \quad \therefore \bar{y} = \dfrac{4}{\sin \theta} = 5 \, m$

$$= 5 + \dfrac{6 \times 5^3}{12 \times 5 \wedge 6 \times 5} = 5.41 \, m$$

$\Sigma M = 0$ about the hinge.

$$F_1 (6 - y_{P_1}) + F_2 (L - y_{P_2}) - F \cos \theta_2 \times 4 - F \sin \theta_2 \times 3 = 0$$

$$L \sin \theta_1 = h = 6 \qquad \therefore L = 7.5$$

$$\sin \theta_2 = \dfrac{4}{6} \qquad \cos \theta_2 = \dfrac{4.47}{6}$$

$$\therefore \quad F = 602.37 \; kN.$$

Q. 47

a) $y_P = \bar{y} + \dfrac{I_G}{\bar{y} A} = \left(\dfrac{0.48}{3} + 1.6 \right) + \dfrac{bh^3}{36 \times 1.76 \times \dfrac{1 \times 0.48}{2}}$

$$= 1.76 + \dfrac{1 \times 0.48^2 \times 2}{36 \times 1.76 \times 0.48} \qquad = 1.76727 \, m$$

b) $y_P = \dfrac{1}{F} \int y P \, dA = \dfrac{\int \bar{y} \, dA}{\bar{y} A}$ 　　　　$dA = l \, dy$

$$= \dfrac{\int_{1.6}^{2.08} (2.08 y^2 - y^3) \, dy}{\bar{y} A \times 0.48} = \dfrac{\dfrac{2.08 y^3}{3} - \dfrac{y^4}{4} \Big]_{1.6}^{2.08}}{0.48 \times 1.76 \times \dfrac{0.48}{2}}$$

$$= 1.76727 \, m$$

Q.62 $\quad y_p = \bar{y} + \dfrac{I_a}{\bar{y}A} = 0.5 + \dfrac{1\times1^3}{12\times0.5\times1\times1} = 0.666\,m$

$\therefore y = 1 - y_p = 0.333$ $\quad$ The flashboard will tumble.

Q.63 $\quad$ The gate opened when y at pressure center

i.e

$$y_p = \bar{y} + \dfrac{I_a}{\bar{y}A} = 1.5 + \dfrac{1\times1^3}{12\times1.5\times1} = 1.55\,m$$

$\therefore y = 2 - 1.55 = 0.45\,m$

Q.65 $\quad$ mass = 2 Mg

$\quad F_1 = \gamma \bar{h}_1 A_1$ $\qquad\qquad\qquad\qquad \gamma = 8500\ N/m^3$

$\qquad = 8500 \times 3 \times 3.33 \times 2$

$\qquad = 170.0136\ kN$

$F_2 = 8500 \times 1.7 \times 3.33 \times 2$

$\qquad = 96.32\ kN$

$\bar{y}_1 = 3 \times \dfrac{1}{\sin\theta} = 3 \times \dfrac{3.333}{2} = 5\,m$

$y_{P_1} = \bar{y} + \dfrac{I_a}{\bar{y}A}$

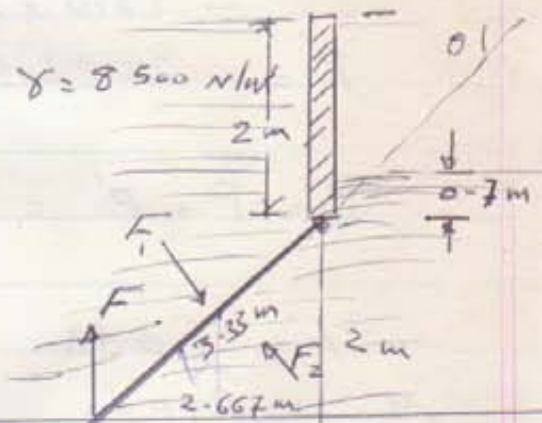$\qquad = 5 + \dfrac{2\times 3.33^3}{12\times 5\times 2\times 3.33} = 5.185\,m$

$\bar{y}_2 = 1.7 \times \dfrac{3.333}{2} = 2.83\,m$

$y_{P_2} = 2.83 + \dfrac{2\times 3.33^3}{12\times 2.83\times 2\times 3.33} = 3.157\,m$

$\Sigma M = 0$

$F_1\left(y_{P_1} - \dfrac{2}{\sin\theta}\right) - F_2\left(y_{P_2} - \dfrac{0.7}{\sin\theta}\right) - F\times 2.667\,m +$

$W \times \dfrac{2.667}{2} = 0$

Gate Wide 2 m.

$W = 2000 \times 9.81$

$R = 74\ kN$

$\cancel{a_J = 1.66\ m}$ }

$-170 \times 1.855 + 76 \times 1.166 \cancel{+} R \cdot a_J = 0$ $\overset{74}{}$ $a_J = 2.74$

$170.0136 \times 1.852 - 96.52 \times \cancel{1.166} - F \times 2.667 \cancel{=}$

$+ W \dfrac{2.667}{2} = \quad \therefore F = \cancel{99\ kN} \cdot 85.84\ kN$

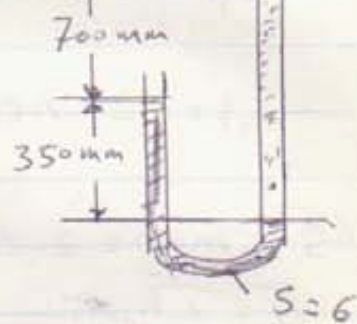Gr $\quad F \times 2.667 - R \cdot a_J - W \times \dfrac{2.641}{2} = 0$

$\cdot 2.68$
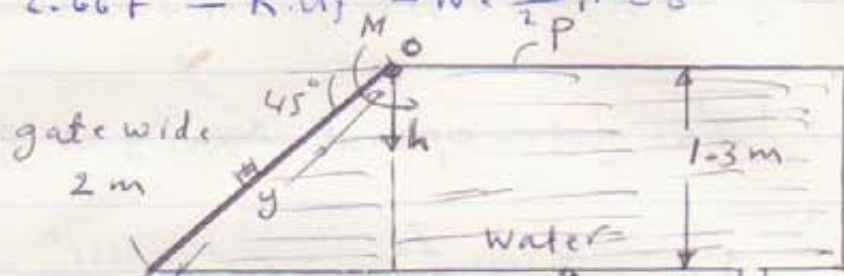
from the manometer

$0 + \gamma_s R - \gamma R - \gamma \times 2 = P'$

$0 + 6 \times 9810 \times 0.35 - 9810 \times 0.35$

$\qquad - 9810 \times 2 = P'$

$P' = -2452.5\ N/m^2$

$P_a = P + P' = \gamma h + P' = \gamma \dfrac{y}{\sqrt{2}} - 2452.5$

$\qquad =$

$F = \int P\, dA$

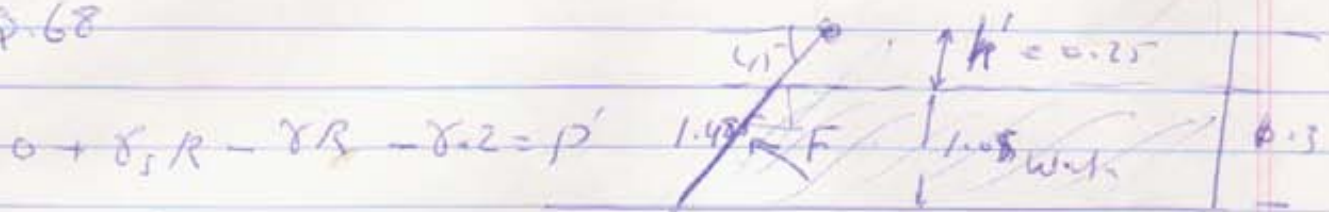$M = \int_A y P\, dA = \int y \left(\gamma \dfrac{y}{\sqrt{2}} - 2452.5 \right) dA$

$dA = 2\, dy$

$\therefore M = \int_0^{1.3\sqrt{2}} 2y \left(\gamma \dfrac{y}{\sqrt{2}} - 2452.5 \right) dy$

$\qquad = \int_0^{1.3\sqrt{2}} \left(\dfrac{2 \times 9810}{\sqrt{2}} y^2 - 4905\, y \right) dy$

$\qquad = \left[\dfrac{2 \times 9810}{3\sqrt{2}} y^3 - \dfrac{4905}{2} y^2 \right]_0^{1.3\sqrt{2}} = 20.447\ kN \cdot m$

Q.68

$0 + \gamma_s R - \gamma R - \gamma \cdot 2 = p'$

$p' = -2452.5 \ N/m^2$

$h' = -0.25 \ m \quad vaccume.$

$F = \gamma \bar{h} A = 9810 \times \dfrac{1.05}{2} \times (1.485 \times 2)$

$\quad = 15.3 \ kN$

$y_p = \bar{y} + \dfrac{I_G}{\bar{y}A}$

$\bar{y} \sin\theta = \bar{h}$

$\therefore \bar{y} = \dfrac{1.05}{2\sin\theta} = 0.742 \ m$

$\therefore y_p = 0.742 + \dfrac{2 \times (1.485)^3}{12 \times 0.742 \times 1.485 \times 2} = 0.99 \ m$

$M = F \left(y_p + \dfrac{0.25}{\sin 45} \right)$

$a = 1.34 \ m$

$\quad = 15.3 \times \left(0.99 + \dfrac{0.25}{\sin 45} \right)$

$\quad = 26.5 \ kN$

2-59

$m = 450 \, kg$

$W = 450 \times 9.81 \, N$
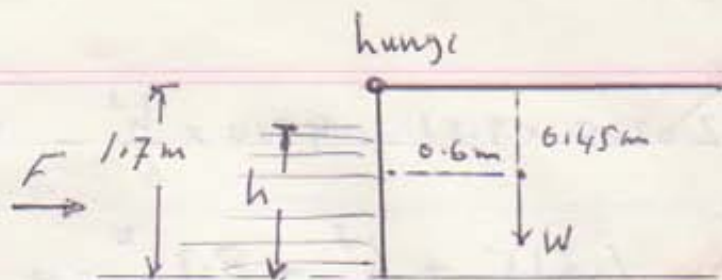
$F = \gamma \bar{h} A = 9810 \times \frac{h}{2} \times h \times 1 = \frac{9810}{2} h^2$

$y_p = \bar{y} + \frac{I_G}{\bar{y} A} = \frac{h}{2} + \frac{1 \times h^3}{\frac{h}{2} \times h \times 1 \times 12} = \frac{2}{3} h$

$\Sigma M = 0$ about the hinge?

$\quad F \times arm = W \times 0.6$

$\quad arm = 1.7 - \frac{1}{3} h$

$\therefore \quad 9810 \cdot \frac{h^2}{2} \left(1.7 - \frac{1}{3} h\right) = 450 \times 9.81 \times 0.6$

$\quad\quad h^3 - 5.1 h^2 + 1.62 = 0$
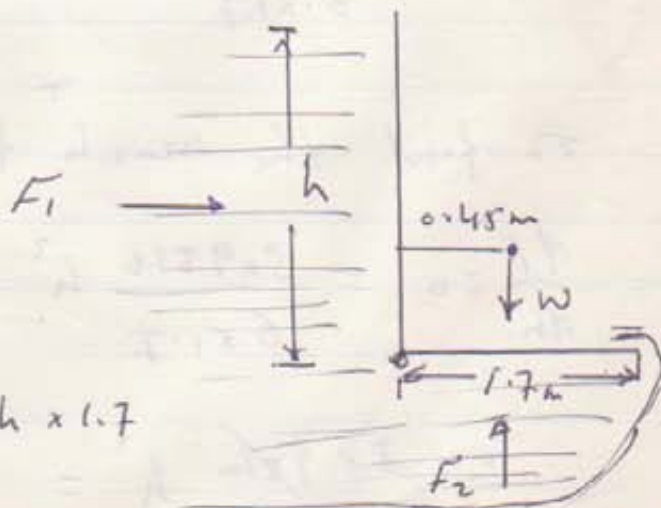
$\quad\quad h = 0.6 \, m$

2-60

$\quad W = 450 \times 9.81$

$F_1 = \gamma \bar{h} A = \gamma \frac{h}{2} \times h \times 1$

$\quad = \gamma \frac{h^2}{2}$

$y_{P_1} = \frac{2}{3} h$

$F_2 = \gamma h \times A \times 1 = \cancel{\frac{2}{3 h}} \, \gamma h \times 1.7$

$\Sigma M = 0$ about the hinge

$\quad W \times 0.45 + F_1 \left(h - y_p\right) - F_2 \times \frac{1.7}{2} = 0$

$450 \times 9.81 \times 0.45 + \gamma \frac{h^2}{2} \left(h - \frac{2}{3} h\right) - \gamma h \times \frac{1.7^2}{2} = 0$

$$2 \, \text{o2.5} \times 9.81 + 9810 \times \frac{h^3}{6} - 9810 \, h \times 1.445 = 0$$

$$1.215 + h^3 - 8.7 h^2 = 0$$
$$h^3 - 8.7 h^2 + 1.215 = 0$$

$$\therefore h = 2.\overset{0.14}{\cancel{85}} \, m \qquad \text{أو } 2.85 \, m \quad \text{أيضاً}$$

. 2-61

$$F_1 \times \frac{1}{3} h + W \times 0.45 + F \times 1.7 - F_2 \times \frac{1.7}{2} = 0$$

$$\frac{9810 \, h^3}{6} + 202.5 \times 9.81 \qquad\qquad \underset{\longrightarrow}{F_1}$$

$$+ F \times 1.7 - 9810 \times 1.445 \, h = 0$$

$$\therefore F = \frac{9810}{6 \cdot \times 1.7} h^3 - 9810 \times \frac{1.445}{1.7} h$$

$$+ \; 202.5 \times 9.81$$

To find the max h for max. F

$$\frac{dF}{dh} = 0 = \frac{3 \times 9810}{6 \times 1.7} h^2 - \frac{9810 \times 1.445}{1.7} + 0 = 0$$

$$\therefore \frac{3 \times 9810}{2 \cancel{6} \times 1.\cancel{7}} h^2 = \frac{9810 \times 1.445}{1.\cancel{7}}$$

$$\boxed{h = 1.7 \, m}$$

2-71 $D = 700$ mm

$W_{gate} = 1800$ N

$F_1 = \gamma \bar{h} A = 2 \times 9810 \times \left(\dfrac{0.7 \sin 45}{2} + 1.5 \right) \times \dfrac{\pi}{4} D^2$ $S = 2$

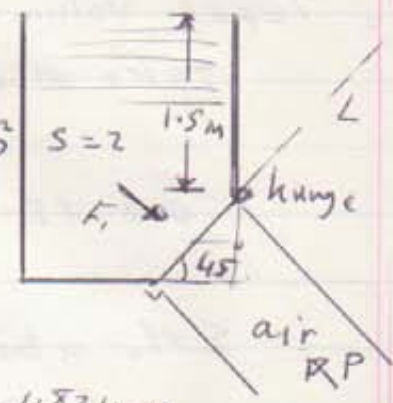$= 13.195$ kN

$y_p = \bar{y} + \dfrac{I_s}{\bar{y} A}$ $\bar{h} = \bar{y} \sin \theta$

$\bar{y} = 2.47$ m

$= 2.47 + \dfrac{\not{\pi} r^4}{\not{4} \times 2.47 \times \not{\dfrac{\pi}{4}} \times 0.7^2} = 2.4824$ m

$F_1 \times (y_p - L) + W_r \dfrac{0.7}{2} \cos 45° + PA \times 0.35 = 0$

$L \sin \theta = 1.5$ $\therefore$ $L = 2.12$

$\therefore 13.195 \times 0.3624 + 1.8 \times 0.247$

$5.23 = P \times \dfrac{\pi}{4} \times 0.7^2 \times 0.35$

$\therefore P = 38.82$ kPa

? . 2-78

$F_H = P_G A = \gamma \bar{h} A$

$= 9810 \times 4 \times 2 \times 2$

$= 156.96$ kN

$F_v = \gamma V$ water

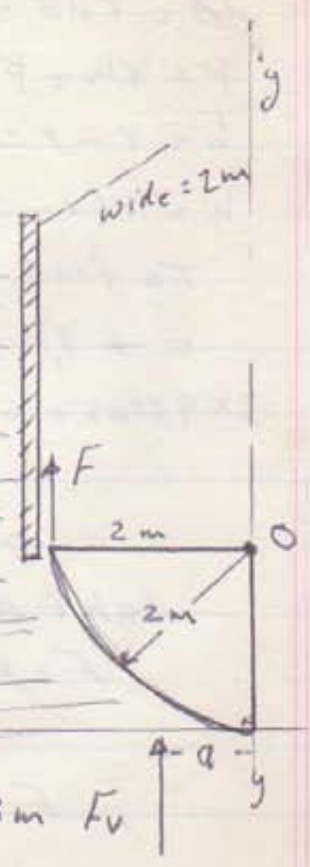$= 9.81 \left[2 \times \dfrac{\pi r^2}{4} + r \times 2 \times 3 \right]$

$= 179.358$ kN

To calculate F it need the c.p of the forces.

$y_p = \bar{y} + \dfrac{I_G}{\bar{y} A} = 4 + \dfrac{2 \times 2^3}{12 \times 4 \times 2 \times 2} = 4.083$ m F_v

Total volume $= 2 \times 3 \times 2 + \pi r^2 \times 2 \times \frac{1}{4} = 18.28 \, m^3$

take moments about $y-y$

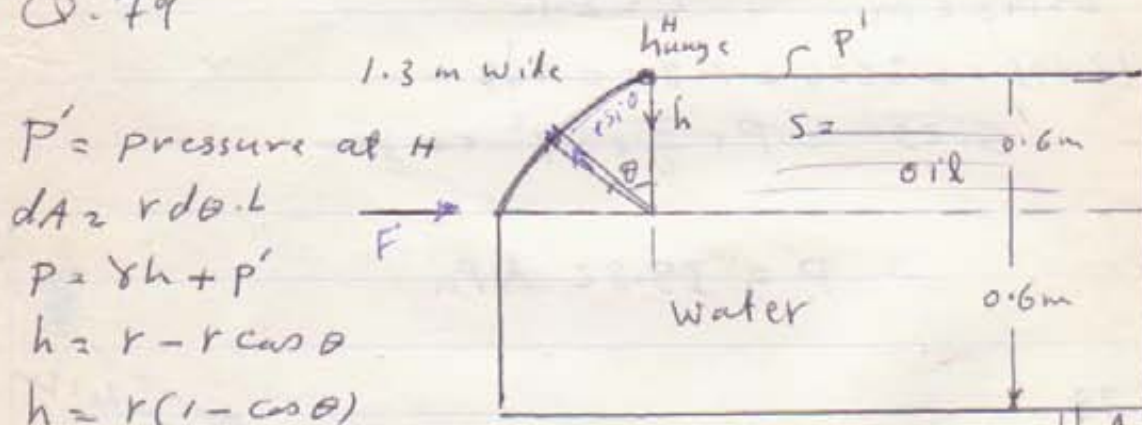$$a \times 18.28 = 12 \times 1 + 2\pi \times \frac{4r}{5\pi}$$

$$a = 0.948 \, m$$

ΣM_o about o

$$F \times 2 + F_v \times a = F_H (y_p - 3)$$

$$F \simeq 0$$

Q.79

P' = pressure at H

$dA = r \, d\theta \cdot L$

$P = \gamma h + P'$

$h = r - r \cos\theta$

$h = r(1 - \cos\theta)$

To find P' from the manometer

$$0 + \gamma_s R - \gamma \times 0.2 - 0.9 \times 9810 \times 0.6 = \acute{P} \, \acute{R}$$

$$3 \times 9810 \times 0.6 - 9810 \times 1.2 - 0.9 \times 9810 \times 0.6 = \acute{P}$$

$$\therefore P' = 588.6 \, N/m^2$$

$$\therefore P = \gamma r (1 - \cos\theta) + 588.6$$

take a moment about H

$$F \times \gamma = \int_0^{\pi/2} (0.9 \times 9810 \times 0.6 (1 - \cos\theta) + 588.6) r \sin\theta \, \gamma \, d\theta \cdot L$$

$$F = 1.3 \int_0^{\pi/2} (0.9 \times 9810 \times 0.6 (1 - \cos\theta) + 588.6)' \sin\theta \, d\theta$$

since $\sin\theta\cos\theta = \dfrac{\sin 2\theta}{2}$

$\therefore F = 1.3 \times 0.6 \left[\dfrac{0.9 \times 9810 \times 0.6}{2} + 588.6 \right]$

$F = 2.5251$ kN.

2.80 $R = 4.5$ cm

$\therefore P' = -3825.9$ N/m^2

from qu. 79

$\therefore F = Lr \left[\dfrac{\gamma r}{2} + P' \right)$

$= -918.2$ N

2.81   from Q. 79

$F = Lr \left[\dfrac{\gamma r}{2} + P' \right]$

$P' = 0 + 3 \times 9810 R - 9810 \times 1.2 - 0.9 \times 9810 \times 0.6$
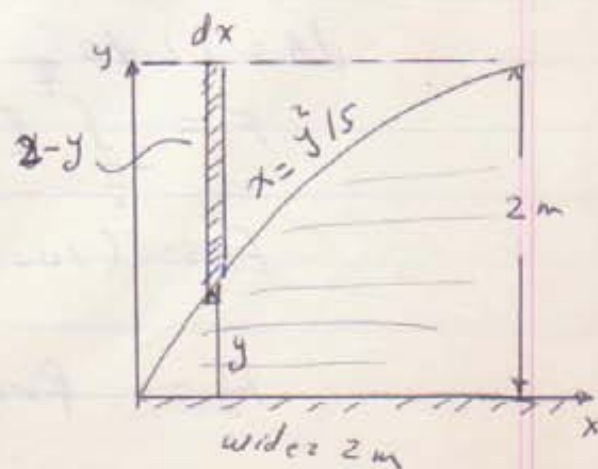
$= 29430 R - 17069.4$

$\therefore F = 1.3 \times 0.6 \left[\dfrac{0.9 \times 9810 \times 0.6}{2} + 29430 R - 17069.4 \right]$

$= 0$

$\therefore R = 0.49$ m

Q.2.82 $x = \dfrac{y^2}{5}$ $L = 2$

for $y = 2$ $x = 0.8$

$F_v = \gamma L \displaystyle\int_0^{x_o} (2 - y) \, dx$

wide = 2 m

$$F_v = 9 \times 2 \int_0^{0.8} (2 - \sqrt{5}\, x^{1/2})\, dx$$

$$F_v = 9.8 \, kN$$

$$F_v \, \bar{x} = \gamma L \int_0^{0.8} (2 - \sqrt{5}\, x^{1/2})\, x\, dx$$

$$\bar{x} = \frac{1}{9.8} \times 9 \times 2 \left[x^2 - \sqrt{5} \times \frac{2}{5}\, x^{5/2} \right]_0^{0.8}$$

$$\bar{x} = 0.245 \, m$$

X Q. 2.83

1-y

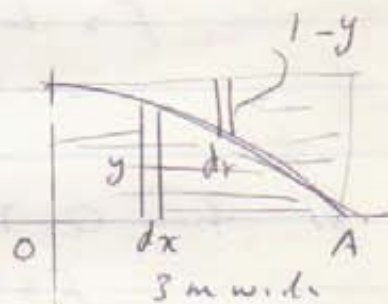3 m wide

$y = \frac{x^2}{8}$ at $y=1$ $x = 2\sqrt{2}$

$$F = \gamma L \int_0^A (1-y)\, dx$$

$$= 9 \times L \int_0^{2\sqrt{2}} \left(1 - \frac{x^2}{8}\right) dx$$

$$F = 9 \times L \left[x - \frac{x^3}{24} \right]_0^{2\sqrt{2}} = 50.92 \, kN$$

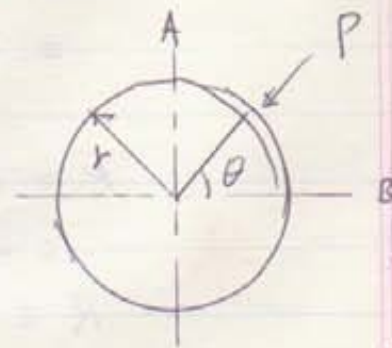$$F = 9 \times 3 \left[\frac{x}{24} \right]_0 = 25.45 \, kN.$$

2.84

$$F_x = \int_A P \cos\theta \, dA$$

$$dA = r L\, d\theta$$

$$F = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[2P(1 - 4\sin^2\theta) + 10 \right] \cos\theta \, r L\, d\theta$$

$$F_x = \left(100 - \frac{20}{3} P\right) r$$

$F_y = 0$ from symetrical عند العمود في النتصف

الزاوية يتج = 0

2.98

$$T + F_{v_1} = W_1$$

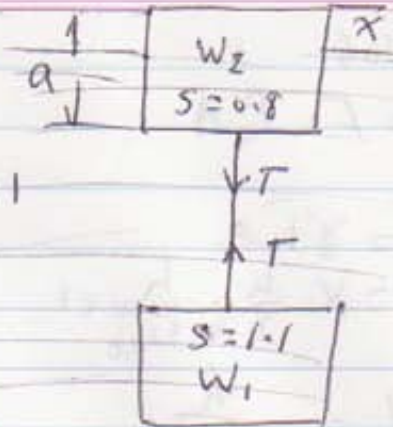$$T + 1 \cdot 0 \times 9810 \times 1 = 1 \cdot 1 \times 9810 \times 1$$

$$T = 980 \, N$$

also. $T + W_2 = F_{v_2}$

$$980 + 0 \cdot 8 \times 9810 \times 1 = 9810 \times a^2 (a - x)$$

since, $a = 1$

$$\therefore \quad x = 0 \cdot 1 \, m = 100 \, mm$$
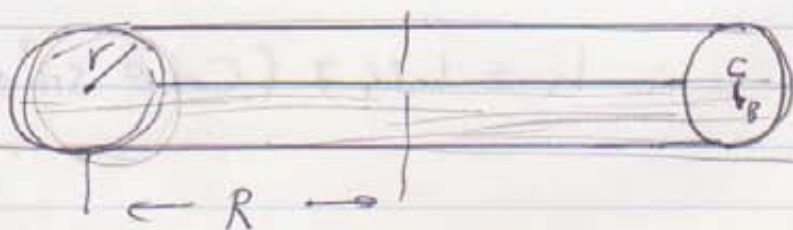

2.105

$$GB = \frac{4r}{3\pi} = \frac{4 \times 0.3}{3\pi} = 0.127 \, m$$

$$MB = \frac{I}{V} = \frac{\pi (R+r)^4 / 4 - \pi (R-r)^4 / 4}{\frac{1}{2} \pi r^2 \cdot 2\pi R}$$

$$= \frac{(R+r)^2 [(R+r)^2 -}{4 \pi R r^2}$$

$$= \frac{(1.3)^4 - (0.7)^4}{4 \times \pi \times 1 \times (0.3)^2} = 2.313$$

$$\therefore MG = MB - GB = 2.186 \, m$$

2.106

a) $F = P_G A$

$= \gamma \bar{h} A$

$= \gamma \dfrac{h}{2} \cdot \dfrac{h}{\sin\theta} \times 1$

$L = \dfrac{h}{\sin\theta}$

طول اللوح المغمور

$y_p = \bar{y} + \dfrac{I_G}{\bar{y}A}$ $\qquad \bar{y} = \dfrac{1}{2}\dfrac{h}{\sin\theta}$ $\qquad I_G = \dfrac{bh^3}{12}$

$\therefore y_p = \dfrac{h}{2\sin\theta} + \dfrac{1\times\left(h/\sin\theta\right)^3}{6\cancel{12}\times \dfrac{h}{2\sin\theta}\times\dfrac{h}{\sin\theta}\times 1}$

$= \dfrac{2}{3}\dfrac{h}{\sin\theta}$

The arream take from $O = \dfrac{1}{3}\dfrac{h}{\sin\theta}$

$\therefore \sum M$ about $O = 0 \Big)_+$

$F * \dfrac{1}{3}\dfrac{h}{\sin\theta} = 2\,000 \times 2 \times \cos\theta$

$\dfrac{\gamma h^2}{2\sin\theta} \times \dfrac{1}{3}\dfrac{h}{\sin\theta} = 4000\,\cos\theta$

$\therefore h = 1.347 \left(\cos\theta\cdot\sin^2\theta\right)^{1/3}$
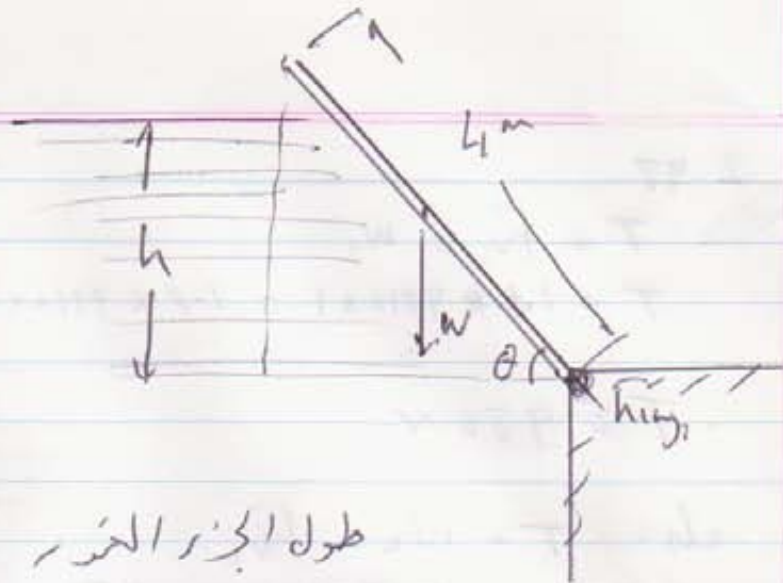
b) from (a) $\sum M = \dfrac{\gamma h^3}{6 \sin^2 \theta} - 4000 \cos\theta$

$\dfrac{\partial M}{\partial \theta} = -\dfrac{\gamma h^3}{3} \csc^3 \cos\theta + 4000 \sin\theta$

Substituting for h

$\dfrac{\partial M}{\partial \theta} = -\dfrac{\gamma}{3} (2.45 \cos\theta \sin^2\theta) \csc^3\theta \cos\theta$

$+ 4000 \sin\theta$

$= -\dfrac{\gamma}{3} \times 2.45 \cos\theta \sin^2\theta \times \dfrac{1}{\sin^3\theta} \cos\theta +$

$4000 \sin\theta$

$= - 8000 \dfrac{\cos^2\theta}{\sin\theta} + 4000 \sin\theta$

$= 4000 \left(\sin\theta - \dfrac{2\cos^2\theta}{\sin\theta} \right)$

$\dfrac{\partial M}{\partial \theta} < 0$ Stable

$\therefore \sin\theta < \dfrac{2\cos^2\theta}{\sin\theta}$

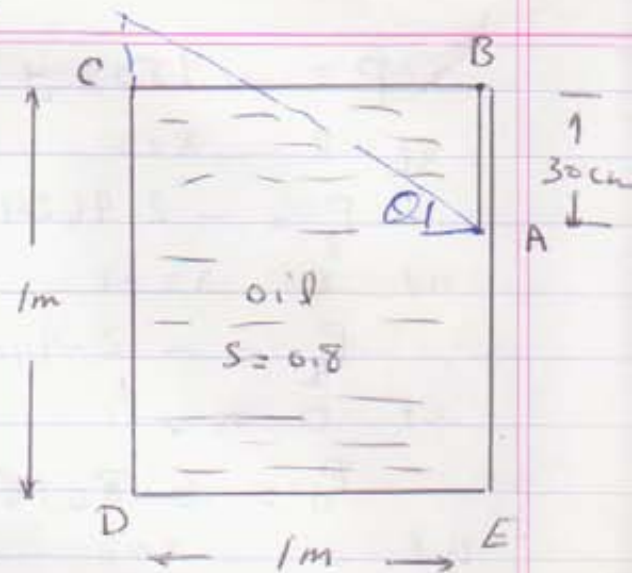or $\tan^{-1} \sqrt{2} > \theta$ $\therefore \theta \geq 54.4°$

3

Q.110

find P_B, P_c, P_D, P_E

$a_x = 3.9 \ m/s^2 \qquad a_y = 0$

since the tanke filled and

$\qquad P_A = P_{atm}$

$P = P_o - \gamma \dfrac{a_x}{g} x - \gamma (1 + \dfrac{a_y}{g}) y$

$\therefore P = 0 - 0.8 \times 9810 \times \dfrac{3.9}{9.81} x - 9.8 \times 9810 (1 + \dfrac{0}{g}) y$

$P = -3120 \, x - 7848 \, y$

at point B $\quad x = 0 \quad , y = 0.3$

$\therefore P_B = 0 - 7848 \times 0.3 = -2.354 \ kN/m^2$

at C $\qquad x = -1 \qquad y = 0.3$

$\therefore P_c = 0.7656 \ kPa$

at D $\qquad x = -1 \qquad y = -0.7$

$\qquad P_D = 8.614 \ kPa$

at E $\qquad x = 0 \qquad y = -0.7$

$\qquad P_E = 5.494 \ kPa$

Q 2.111 $\quad a_x = 0 \qquad a_y = 2.45 \ m/s^2 \quad$ fid P_B, P_c, P_D, P_E

$\therefore P = 0 - 0.8 \times 9810 \times \dfrac{0}{9.81} x - 0.8 \times 9810 (1 + \dfrac{2.45}{9.81}) y$

$$\therefore P = -9808 \, y$$

at B $x = 0$ $y = 0.3$

    $P_B = -2.9424 \, KPa$

at C $x = -1$ $y = 0.3$

    $P_C = -2.9424 \, kPa$

at D $x = -1$ $y = -0.7$

    $P_D = 6.8656 \, kPa$

at E $x = 0$ $y = -0.7$

    $P_E = 6.8656 \, kPa.$

Q.112 $a_x = 2.45 \, m/s^2$ $a_y = 4.902 \, m/s^2$

      $\rho \cdot g_x$

$$P = 0 - 9810 \times \frac{2.45}{9.81} \cdot X - 0.8 \times 9810 \left(1 + \frac{4.902}{9.81} \right) y$$

$$P = -1960 \, X - 11772 \, y$$

at B $x = 0$ $y = 0.3$

    $P_B = -3.532 \, KPa$

at C $x = -1$ $y = 0.3$ $P_c = -1.572 \, kPa$

at D $x = -1$ $y = -0.7$ $P_D = 10.2 \, kPa$

at E $x = 0$ $y = -0.7$ $P_E = 8.24 \, kPa$

Q. 2.113 find P_A, P_B, P_C
let the origins at E

$\tan\theta = \dfrac{a_x}{a_g + y} = \dfrac{9.806}{0 + 9.806} = 1$ the water pass
                                     Point A $\theta > 28°$

$\theta = 45°$ $\tan\theta = \dfrac{h}{x}$ $\therefore x\tan\theta = h$

Volume of space = Volume of new space
    Initial   final

$0.3 \times 1.3 \times 1$ $=$ $x \cdot \dfrac{h}{2}$ $= x \cdot \dfrac{x\tan\theta}{2}$

$\therefore x = 0.883176$ m
    $h = 0.883176$ m

also $\tan\theta = \dfrac{h+s}{1.3} = 1$

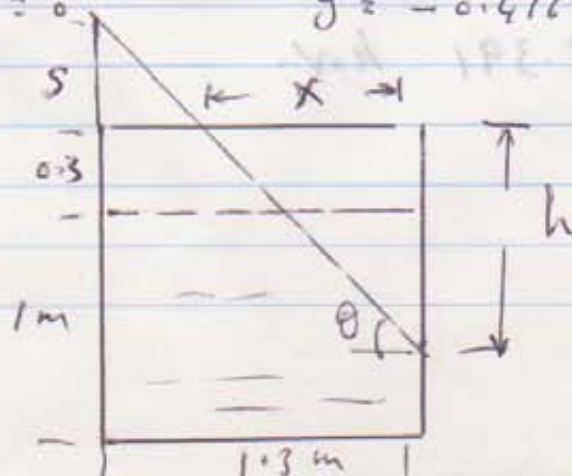$\therefore s = 1.3 - 0.883176 = 0.416824$ m

$\therefore P = P_0 - \gamma \dfrac{a_x}{g} x - \gamma \left(1 + \dfrac{a_y}{g}\right) y$

$P = -9810\, x - 9810\, y$

at A $x = -1.3$ $y = 0.883176$ $\therefore P_A = 4.09$ kPa
at B $x = -1.3$ $y = -0.416824$ $\therefore P_B = 16.842$ kPa
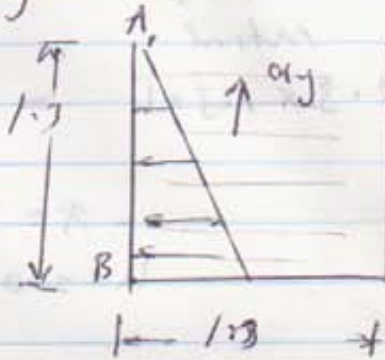at C $x = 0$ $y = -0.416824$ $P_C = 4.09$ kPa

2.135

$$P = \gamma \left(1 + \frac{a_y}{g}\right) h \qquad\qquad h = -y$$

$$P = 9810 \left(1 + \frac{2.45}{9.81}\right) 1.3 = 15.94 \ kp_a.$$

$$F = \gamma \bar{h} . A$$

$$F = 9810 \left(\frac{P_B + P_A}{2}\right) 1.3 \times 1.3$$

$$= 13.47 \ kN.$$

2.136 $a_y = 2.45 \ m/s^2 \downarrow$

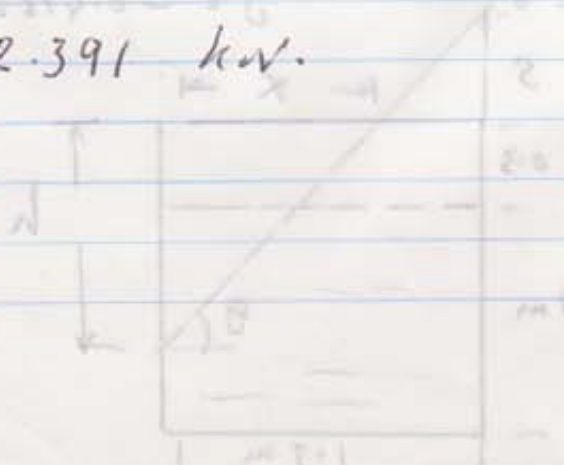$$P = 0.65 \times 9810 \left(1 - \frac{2.45}{9.81}\right) \times 1$$

$$P = 4.7823 \ kPa$$

$$F = \left(\frac{P_A + P_A}{2}\right) 1 \times 1$$

or

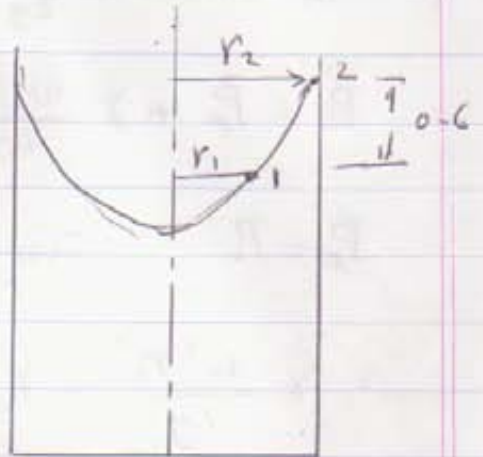$$F = \gamma \bar{h} \ A \qquad\qquad \bar{h} = \frac{P}{\gamma} \times \frac{1}{2}$$

$$F = 2.391 \ kN.$$

Q.2.119 $P_x = P_y = P_z$ استعمال المعادلة لتخفيذ

120, 121   لسم 2

Q.122

$r_1 = 0.6$ m $r_2 = 1.2$ m

$P_1 = P_o + \gamma \dfrac{\omega^2 r_1^2}{2g} - \gamma y$

$P_2 = P_o + \gamma \dfrac{\omega^2 r_2^2}{2g} - \gamma(y + 0.6)$

since $P_1 = P_2$

$\therefore \gamma \dfrac{\omega^2 r_1^2}{2g} - \gamma y = \gamma \dfrac{\omega^2 r_2^2}{2g} - \gamma y - 0.6\gamma$

$0.6 = \dfrac{\omega^2}{2g}(r_2^2 - r_1^2) = \dfrac{\omega^2}{2 \times 9.81}\left((1.2)^2 - (0.6)^2\right)$

$\omega = 3.3$ rad/s $= \dfrac{2\pi N}{60}$

$\therefore N = 31.5$ rpm

يمكن also

$h = \dfrac{\omega^2 r^2}{2g}$ $\therefore h_1 = \dfrac{\omega^2 r_1^2}{2g}$ $h_2 = \dfrac{\omega^2 r_2^2}{2g}$

$h_2 = h_1 + 0.6$

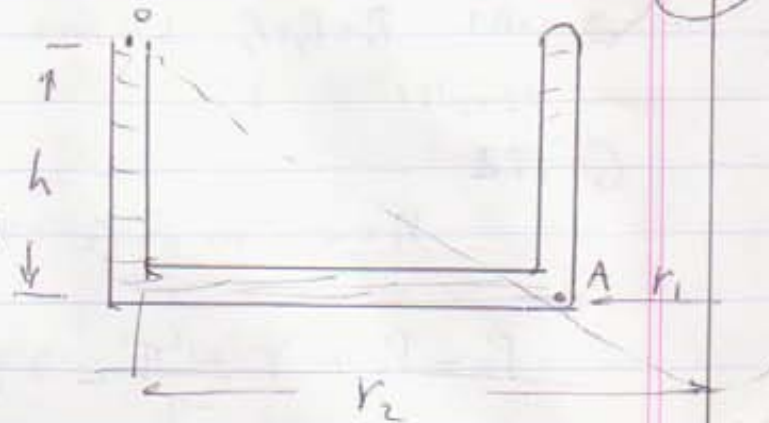$\therefore 0.6 = \dfrac{\omega^2}{2g}(r_2^2 - r_1^2)$ $\Rightarrow$ $\omega = 3.3$ rad/s

$N = 31.5$ rpm

2-123

$$P_A = P_0 + \gamma \frac{\omega^2 r_1^2}{2g} - \gamma y$$

$$P_0 = P_0 + \gamma \frac{\omega^2 r_2^2}{2g} - \gamma(y+h)$$

$$P_A = P_0 \qquad \text{Zero gage صفر الضغط}$$

$$\therefore \gamma \frac{\omega^2 r_1^2}{2g} - \gamma y = \gamma \frac{\omega^2 r_2^2}{2g} - \gamma y - \gamma h$$

$$h = \frac{\omega^2}{2g}(r_2^2 - r_1^2) = \frac{\omega^2}{2 \times 9.81}((0.75)^2 - (0.15)^2)$$

$$\therefore \omega = \sqrt{\frac{0.3 \times 2 \times 9.81}{0.54}} = 3.3 \quad \text{rad/s}$$

$$= \frac{2\pi N}{60} \qquad \therefore N = 31.5 \quad \text{rpm}$$

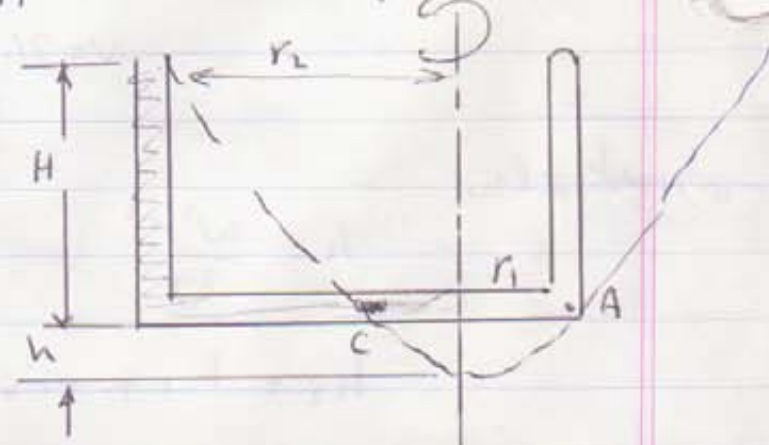
2.124 from symmetrical $r_1 = 0.15\,m$ $\therefore r_2 = 0.45\,m$

$$H + h = \frac{\omega^2 r_2^2}{2g} \qquad \text{---①}$$

$$h = \frac{\omega^2 r_1^2}{2g} \qquad \text{---②}$$

①-②

$$H = \frac{\omega^2(r_2^2 - r_1^2)}{2g}$$

$$\therefore \omega = \sqrt{\frac{0.3 \times 2 \times 9.81}{(0.45)^2 - (0.15)^2}} = 5.7 \quad \text{rad/s}$$