Course Name:	Vibration		اهتزازات		اسم المقرر:
Course Code:	ME/ 0893		همك/0893		رمز المقرر:
Units:	4		4		الوحدات:
Hours per Week			الساعات الأسبوعية		
Theoretical	Experimental	Tutorial	مناقشة	عملي	نظري
3	2	-	-	2	3

Week	Contents	المحتويات	الاسبوع
1	**Basic concepts of vibration** • Degree of freedom • General concepts of the importance of the vibration and its application • Definitions and calculation for the degree of freedom for different system	**مبادئ عامة في الاهتزازات** • درجة حرية الطلاقة • استعراض المبادىء العامة لأهمية دراسة الاهتزازات وتطبيقاتها العملية • تعريف وتطبيقات لاحتساب درجة حرية الطلاقة	1
2	**Introduction to oscillatory motion** • Simple harmonic motion • Amplitude of vibration, velocity and acceleration relation • Presentation of the oscillatory motion and the simple harmonic motion & its conditions also presenting the relation between displacement, velocity and acceleration and the phase difference between them	**مقدمة في الحركة التذبذبية** • الحركة التوافقية البسيطة • علاقات الإزاحة السرعة التعجيل • تمثيل الحركة التذبذبية وعرض الحركة التوافقية البسيطة وشروطها واستعراض علاقات الإزاحة والسرعة والتعجيل وفرق الطور بينها	2
3	**Free vibration of an undamped singledegree of freedom** • Examples • Formulation for the eq. of motion for system with single degree of freedom without damping and solving the eq. of motion finding the natural frequency. Different examples are presented	**الاهتزاز الحر غير المخمد لنظام أحادي درجة الحرية** • أمثلة • اشتقاق المعادلة الأساسية للحركة لنظام أحادي الحرية بدون تخميد وحل المعادلة وإيجاد التردد الطبيعي لها وإعطاء أمثلة متفرقة عنها	3
4	**Simple energy method (Rayleigh principle)** • Presenting the comparison between the conservative and non- conservative system and applyig the simple energy method for different system to find eq. of motion and natural frequency	**طريقة الطاقة (مبدأ رايلي)** • استعراض مقارنة لمنظومات محفوظة الطاقة وغير محفوظة الطاقة وتطبيق طريقة الطاقة المبسطة على عدد من المنظومات لاستخراج معادلة الحركة والتردد الطبيعي الأول	4

5	**Free vibration viscous damped single degree of freedom system.** • Types of damping. • Examples. • Studying the free damped system single degree of freedom , presenting the types of damping in application and formulation and solving the equation of such system for different damping ratio	**الاهتزاز الحر المخمد لنظام أحادي درجة الحرية** • أنواع التخميد • أمثلة • دراسة نظام الاهتزاز الحر المخمد لمنظومة أحادية درجة الحرية واستعراض أنواع التخميد التي تتعرض لها المنظومات في التطبيق واشتقاق وحل المعادلات الخاصة بالتخميد الحر لمختلف نسب التخميد	5
6	**Equivalent springs and dampers** • In series and parallel. • Examples. • Studying the application and types of equivalent sprirg and damping for parallel and seriese connection with their application	**الصلابة المكافئة والتخميد المكافئ** • التكافؤ لحالة الربط التوازي والتوالي • أمثلة • دراسة تطبيقات الصلابة المكافئة والتخميد المكافيء لحالات التوازي والتوالي واشتقاق المعادلات المتعلقة بكل حالة وربطها بأمثلة تخص الواقع العملي	6
7	**Logarithmic decrement** • Derivation • Examples • Formulation of the original eq. of logarithmic decrement and studying the importance of this subject, calculation the time required for the decay of the signal for no. of cycles with different applicable example	**التناقص اللوغارتمي** • اشتقاق • أمثلة • اشتقاق المعادلة الأساسية للتناقض اللوغارتمي ودراسة أهمية هذا الموضوع واحتساب الزمن الذي تتنافص فيه الإشارة بعد عدد من الدورات مع أمثلة عملية حول الموضوع	7
8	**Forced vibration of single degree of freedom** • Forced damped vibration • Formulation of the original eq. of motion for damped and undamped forced vib for different excitation forces and studying the behaviors of the amplitude with w/wn and formulating the necessary eq. of resonance	**الاهتزاز ألقسري لنظام أحادي درجة الحرية** • الاهتزاز ألقسري المخمد • اشتقاق المعادلة الخاصة للاهتزاز القسري لنظام أحادي درجة الحرية ولقوى استثارة مختلفة بوجود التخميد وعدم وجوده ودراسة سلوكية المتغيرات الخاصة بالسعة مع نسبة التردد الطبيعي الى القسري وتوضيح أماكن الرنين واشتقاق المعادلات الخاصة بذلك	8
9	**Forced vibration for constant force** • Examples • Studying the behavior of the system with constant excitation force formulating the study state and transient solution and solving such system with some examples **Forced Vibration for sinusoidal force** • Resonance conditions	**الاهتزاز ألقسري لقوة ثابتة** • أمثلة • دراسة سلوكية المنظومة المعرضة لقوة قسرية ثابتة واشتقاق المعادلات الخاصة بذلك وحلها المكون من الحل المستقر والانتقالي مع أمثلة حول الموضوع **الاهتزاز ألقسري لقوة جيبية** • شروط الرنين • دراسة سلوكية المنظومة المعرضة لقوة قسرية	9

	• Studying the behavior of the system with sinusoidal force obtaining the necessary eq. for resonance with some examples	جيبية واشتقاق المعادلات الخاصة بها وحلها واستخراج معادلة الرنين مع الأمثلة	
10	**Vibration isolation** • Transmissibility • Discussion of transmissibility behavior • Examples • Explaining the application of Vib. Isolation and diffinition of transmissibility and the behaviors of it with w/ wn for different damping ratio. Solving some examples on the subject	**عزل الاهتزازات** • الانتقالية • مناقشة منحني الانتقالية • أمثلة • شرح لتطبيقات عزل الاهتزازات وتعريف الانتقالية وكيفية احتسابها والسيطرة على القوة المستقلة الى الارض واشتقاق المعادلة الخاصة بذلك ومناقشة سلوكية انتقالية الاهتزازات وتقنية الانتقالية الى الارض مع التغيير لنسبة التردد الطبيعي الى التردد القسري لنسب تخميد مختلفة مع الامثلة	10
11	**Two degree of freedom** • Coordinate couplings • Semi definite system	**النظام الثنائي لدرجة الحرية** • المزدوج الاحداثي • المنظومة شبه المعرفة	11
12	• Study and analyze the equation of motion for 2- Degree system. Estimating the natural frequencies and their mode shapes, Also studying the coordinal coupling and semi definite system with some examples	• دراسة معادلات الحركة لنظام ثنائي درجة الحرية باستخراج الترددات الطبيعية واشكال الاطوار الاهتزازية مع دراسة المزدوج الاحداثي والمنظومات الشبه معرفة مع الامثلة	12
13	**Mode shapes** • Study the mode shapes fore different system of two Degree of freedom with examples	**نسق الاهتزازات** • دراسة الاطوار الاهتزازية المختلفة لمنظومات من الدرجة الثانية مع الامثلة	13
14	**Lagrange equation** • Examples • Study Lagrange, eq. for damped & undamped system free and forced Vib. and applying it for several times according to the coordinate under consideration with examples	**معادلة لاكرانج** • أمثلة • دراسة معادلا لاكرانج في حالة الانظمة المحفوظة وغير المحفوظة الطاقة بوجود القوة القسرية وعدم وجودها وتطبيقها لعدد من المرات وفقا لدرجة حرية الطلاقة مع الامثلة	14
15	**Dynamic absorber (undamped)** • Study and formulate the eq. of dynamic absorber and its characteristic without damping in addition to some examples	ماص الاهتزازات غير المخمد • دراسة واشتقاق المعادلات الخاصة بماص الاهتزازات غير المخمد مع الامثلة	15

References

1. Theory of Vibration with Application, by: William T. Thomson, 1997.
2. Mechanical Vibrations, by: William Wiseto (Schaum's outline series).

Vibration

محاضرة
(1)

book:

- Mechanical vibration 8th eddition by RAo
- Theory of Machine J.K. GupTa.
- Mechanical vibration & application William Thoms

* 1-1 Basic Concepts of Vibration.

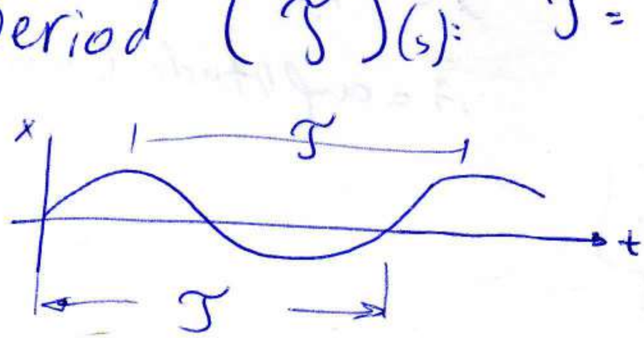
a- frequency ω (rad/s) (rpm) , F (HZ)

$$\omega = 2\pi f$$

b- natural frequency ω_n (rad/s), (rpm)
This is the properties of the elastic body, it is constant no. depending on the stiffness (K)(N/m) and mass (m)(Kg).

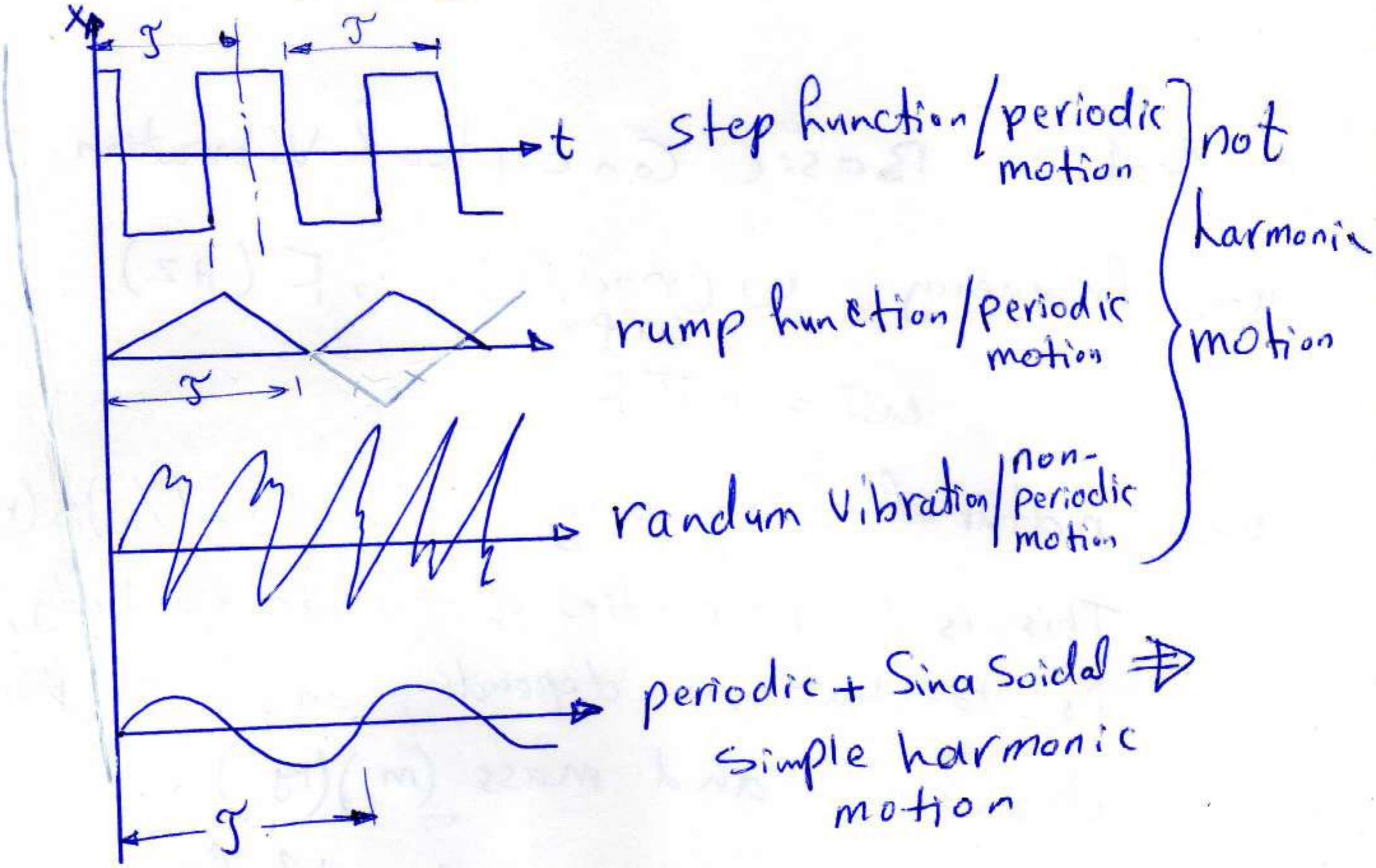
c- Resonance occures when the frequency of the body equal the natural frequeny of the system if the system is undamped.

d- period (τ)(s): $\tau = \frac{1}{f} = \frac{2\pi}{\omega}$



1-2 Simple Harmonic Motion

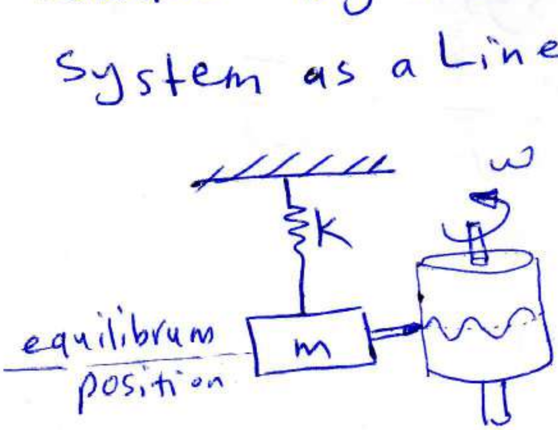
periodic motion is said to be aperiodic when it repeats itself after a constant interval of time called the period (τ) and its reciprocal called frequency (Hz)



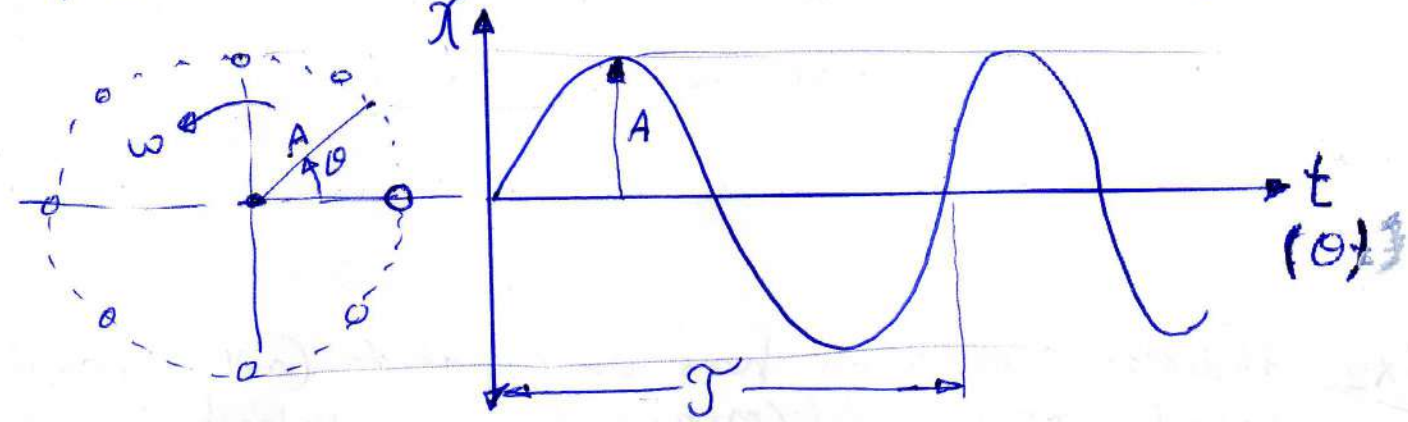
* The Simple harmonic Motion (SHM) may be represented by the motion of Simple spring mass System as a Linear motion

$$x = A \sin \theta$$

A = amplitude (mm)



* the Simple harmonic motion Can be represented by the motion of a partical rotate on a circular path with constant angular Velocity ω



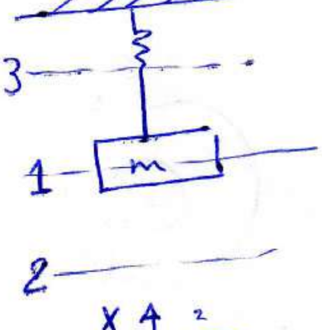
Angular displacement $\theta = \omega t$ $\quad \omega = \frac{\theta}{t} = \frac{2\pi}{\mathcal{T}}$

$X = A \sin \theta$ $\quad f = \frac{1}{\mathcal{T}} = \frac{\omega}{2\pi}$

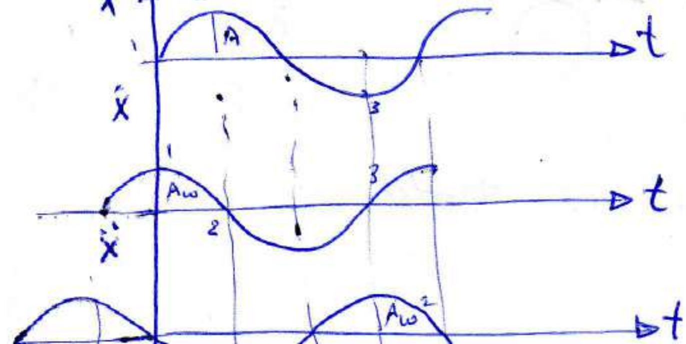
$X = A \sin \omega t$

$\frac{dX}{dt} = \dot{X} = A\omega \cos\omega t = A\omega \sin(\omega t + \frac{\pi}{2})$

$\frac{d^2X}{dt^2} = \ddot{X} = -A\omega^2 \sin\omega t = A\omega^2 \sin(\omega t + \pi)$



$X = \min$	$\dot{X} = 0$	$\ddot{X} = \max$
$X = 0$	$\dot{X} = \max$	$\ddot{X} = 0$
$X = \max$	$\dot{X} = 0$	$\ddot{X} = \min$



$X = A \sin \omega$

$\dot{X} = A\omega \cos \omega t$

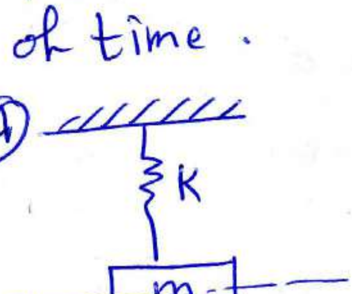
$\ddot{X} = -A\omega^2 \sin \omega t$

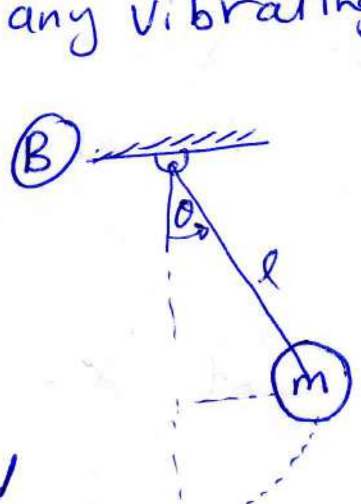
Ex1 A harmonic motion has frequency of (10) Hz and it's max. velocity is 400 cm/s. Determine it's amplitude, periode & max. acceleration.

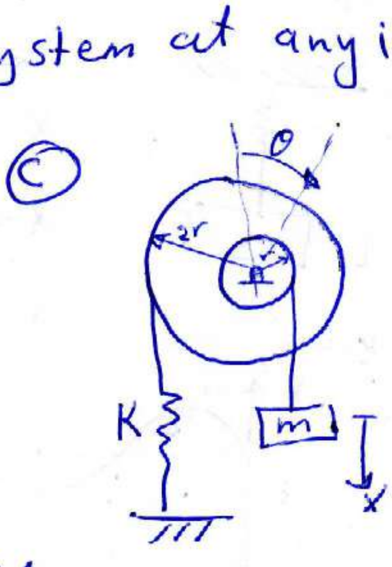
Ex2 A harmonic motion has an amplitude (0.4 cm) and a period (0.15 s). determined the max. velocity & max. acceleration.

1-3 Degree of Freedom:

It is the no. of undependent axes which specify the Location of any vibrating system at any instant of time.







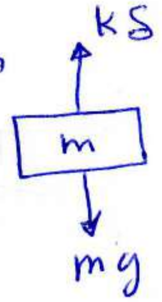
one axis needed
Single degree of Freedom
S.D.F.

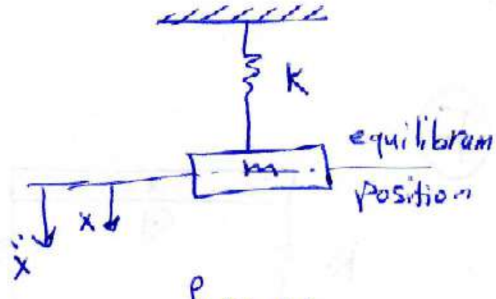
one axis needed

1-4 Single degree of freedom (SDF) Free Vibration without damping

1-4-1 for static consideration

fig(a)
free body diagram
for mass m





fig(b)

fig(a)

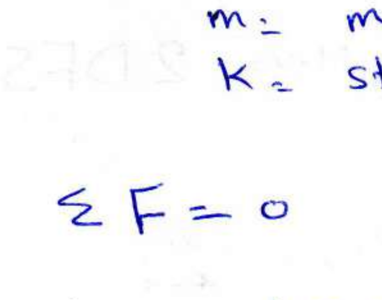
where δ: initial displacement (initial tension spring)

m = mass

K = stiffness

$\Sigma F = 0$ for static

$mg - K\delta = 0 \Rightarrow mg = K\delta \rightarrow 1$



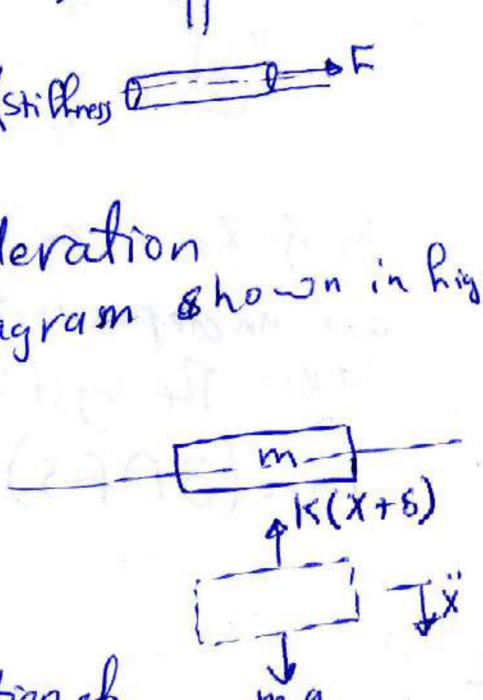
$$slop = \frac{\Delta F}{\Delta x} = K \ (N/m)$$

$$\delta = \frac{Pl}{AE} \Rightarrow \frac{P}{\delta} = \frac{AE}{L} = K \text{ (stiffness)}$$

1-4-2 for dynamic consideration

from fig(b) the free body diagram shown in fig(c)

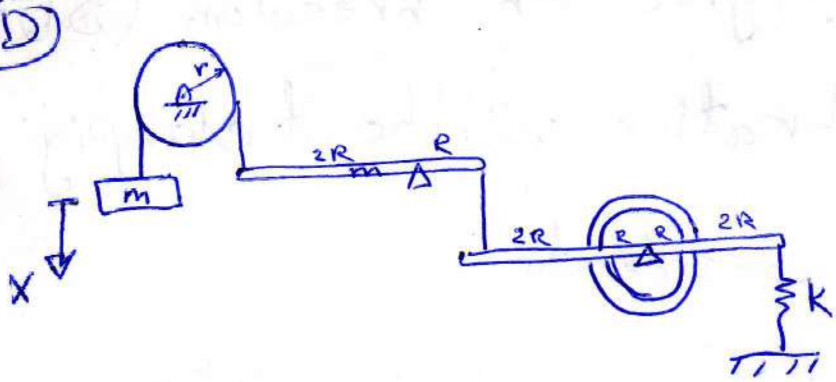
$\Sigma F = ma \Rightarrow \Sigma F = m\ddot{x}$



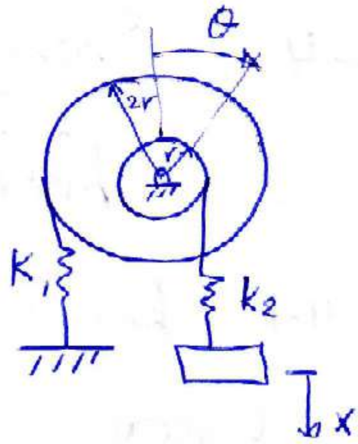
$mg - K(x+\delta) = m\ddot{x}$

$m\ddot{x} + Kx + \cancel{K\delta} - \cancel{mg} = 0$ ... tion of

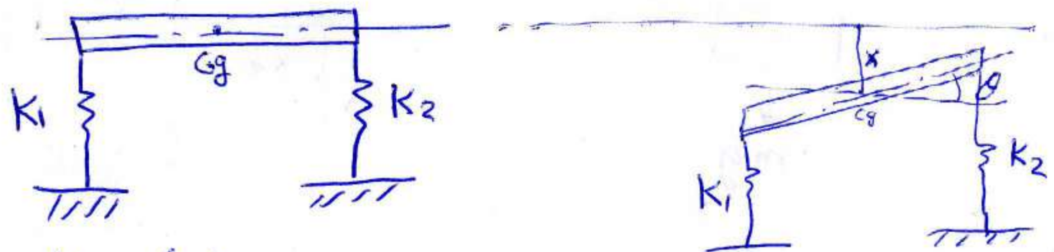
D



E



F

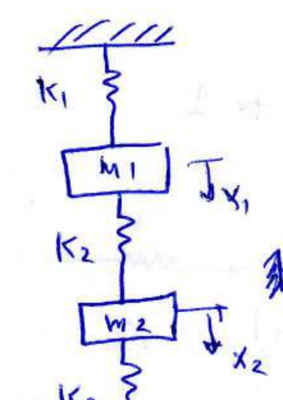


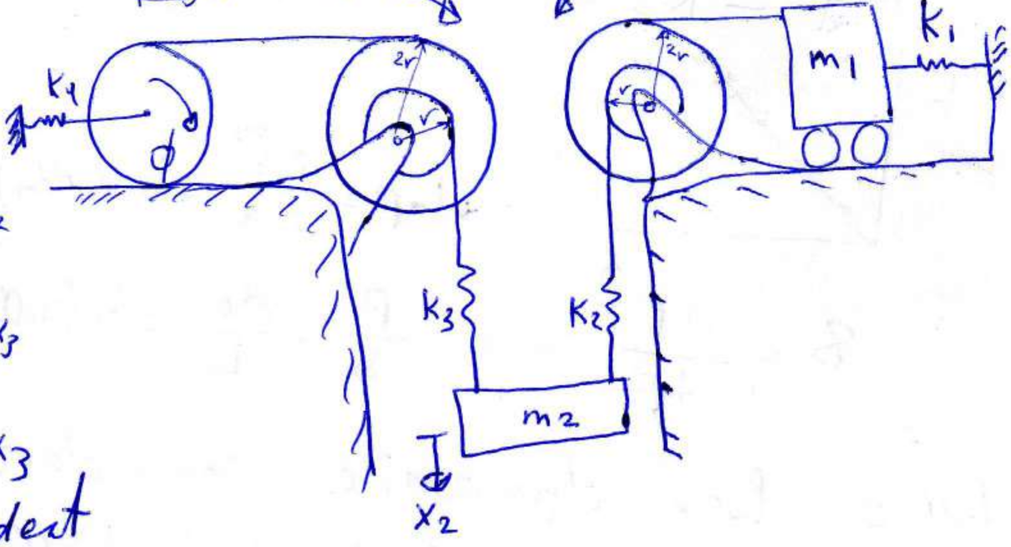
$K_1 \neq K_2$

X & θ are two undependent axis

∴ two degree of Freedom System 2DFS

G





X_1 & X_2 & X_3 are undependent axis, The system have (3DFS)

Apply I.C on the general solution

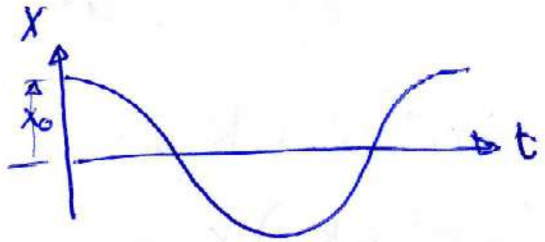
$$x_0 = A\cos(0) + B\sin(0) \Rightarrow \boxed{x_0 = A}$$

$$\dot{x} = -A\omega_n \sin\omega_n t + B\omega_n \cos\omega_n t$$

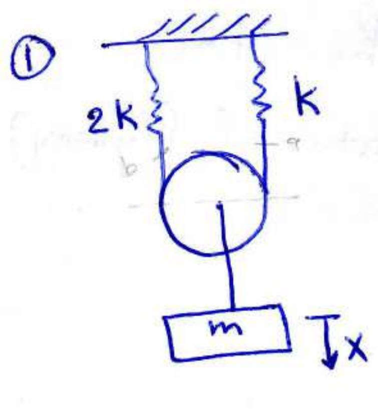
$$0 = -A\omega_n \sin(0) + B\omega_n \cos(0) \Rightarrow \boxed{B = 0}$$

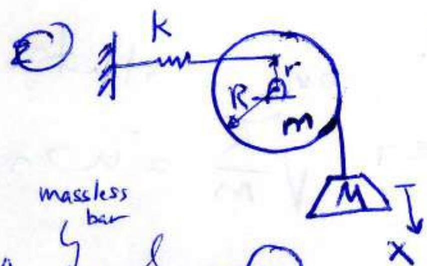
$\therefore$ general solution

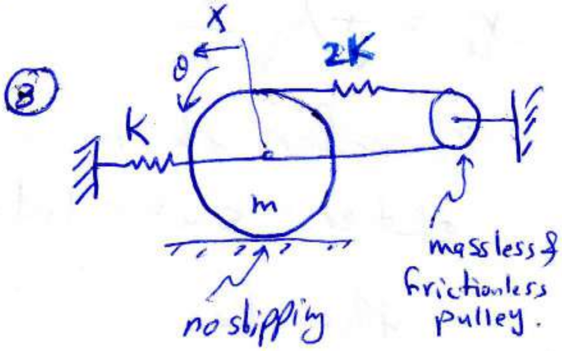
$$X = X_0 \cos\omega_n t$$

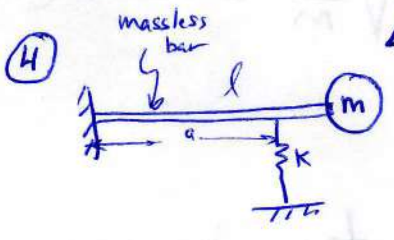


Ex: Find the equation of motion and natural frequency of the system shown in figures?









to find the solution of the above eq.(1)

$$m\frac{d^2x}{dt^2}+kx=0$$

let $r=\frac{d}{dt}$ $\quad \frac{d^2}{dt^2}(x)=r^2x$ subsituted in eq(1)

$$mr^2x+kx=0$$

$$(mr^2+k)x=0$$

either $x=0$ or $mr^2+k=0 \Rightarrow r^2=\frac{-k}{m}$

$r_{12}=\mp i\sqrt{\frac{k}{m}}$ $\quad r=\alpha\mp i\beta$

therefor the homoginous solution for this after subsituted $\sqrt{\frac{k}{m}}=\omega_n$ (natural frequency)

therefor

$$r=\mp i\omega_n$$

The general solution becoms:

$$x=e^{-\alpha t}(A\cos\beta t+B\sin\beta t)$$

$$\therefore x=e^{-0t}(A\cos\omega_n t+B\sin\omega_n t)$$

$$\therefore \boxed{x=A\cos\omega_n t+B\sin\omega_n t}$$ general solution

where A & B are constant can be found from Initial conditions:

<u>I.C</u> $\quad t=0 \quad x(0)=x_0 \quad \dot{x}(0)=0$

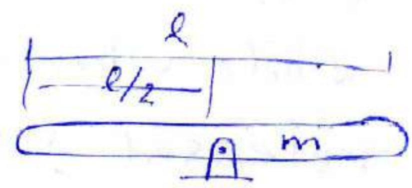
Not: 1

$$I_{cylinder} = \frac{1}{2} m r^2$$
$$I_{ring} = m r^2$$

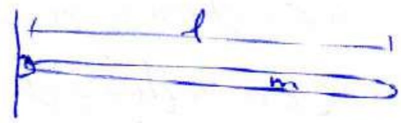
about it's centre

mass moment of Inertia ($kg.m^2$)

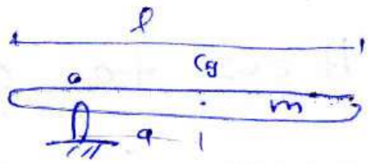
$$I_{bar} = \frac{1}{12} m \ell^2$$



$$I_{bar} = \frac{1}{3} m \ell^2$$



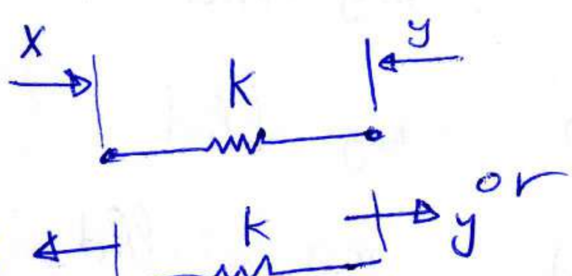
$$I_o = I_c + I_a$$
$$= \frac{1}{12} m \ell^2 + m a^2$$



Not 2

$$F = K(y - x)$$

$$F = k(x + y)$$



Not 3

To find the angular natural frequency for the system from equation of motion. The following must be considered.

1- make the coefficient of $\ddot{x}$ & $\ddot{\theta}$ unity.

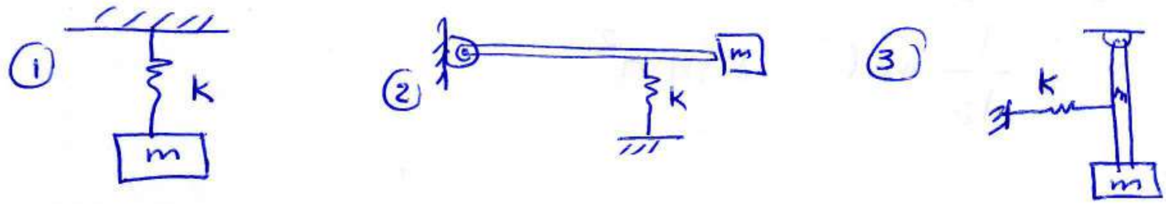
2- The coefficient of x or θ under the square root will be the angular natural frequency for the vibrating system.

$m\ddot{x} + Kx = 0 \quad \ddot{x} + \frac{K}{m} x = 0 \quad \omega_n = \sqrt{\frac{K}{m}} \left(\frac{rad}{s}\right)$

Not 4 when the connection of the system are released & the system moves or rotates the (mg) is not considered.

while when the connection of the system are released & the system does not moves or does not rotate then (mg) must be considered.

How to consider "mg"

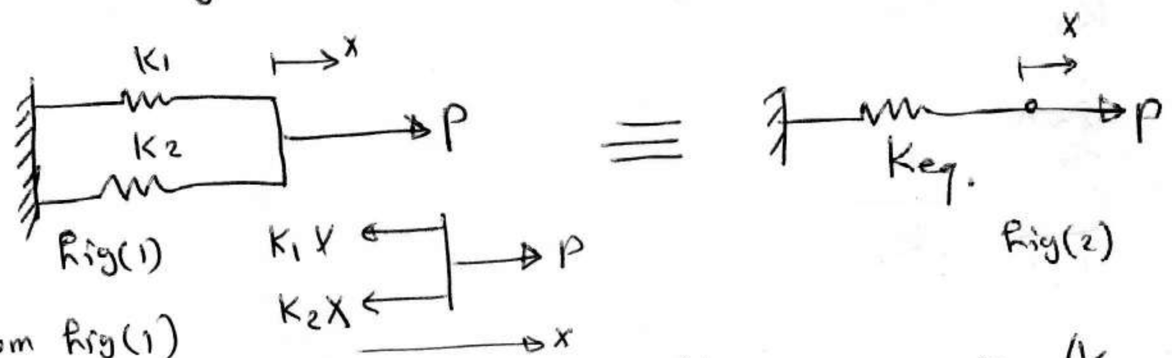


	static	dynamic	Result
fig (1)	mg effect	mg effect	No (mg) considered
fig (2)	mg effect	mg effect	No (mg) considered
fig (3)	mg not effect	mg effect	(mg) is considered

محض ٨ (2)
ثالث معدات

1-5 Equivalent springs :

a) springs in parallel



from fig(1)

$\Sigma F = 0$, $P = K_1 X + K_2 X$, $P = (K_1 + K_2) X$ —①

from fig(2) $P = K_{eq} \cdot X$ —②

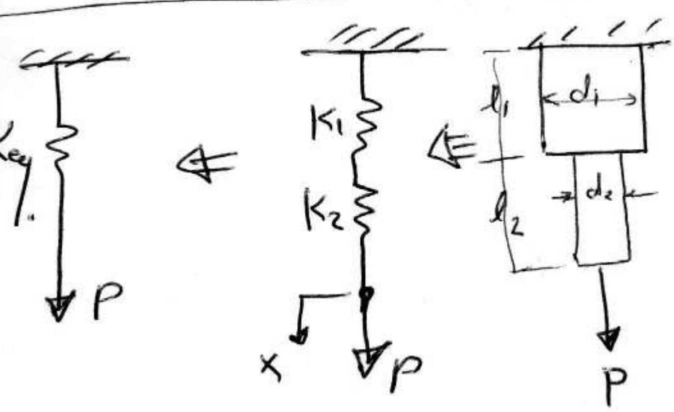
X = displacement
P = Force
K_{eq} : equivalent springs

from eq. 1 & 2

$$K_{eq} = K_1 + K_2$$

in general for springs in parallel

$$\boxed{K_{eq} = \sum_{i=1}^{i=n} K_i}$$



b) Spring in series

$\delta = \dfrac{P\,l}{A\,E}$ $\quad K = \dfrac{P}{\delta} = \dfrac{A E}{l}$

$\therefore K_1 = \dfrac{A_1 E_1}{l_1}$, $K_2 = \dfrac{A_2 E_2}{l_2}$

X at free end $= X_1 + X_2$

$X = X_1 + X_2$

$X = \dfrac{P}{K_1} + \dfrac{P}{K_2} \Rightarrow \dfrac{X}{P} = \dfrac{1}{K_1} + \dfrac{1}{K_2}$ —①

for equivalent spring, $P = K_{eq} \cdot X \Rightarrow \dfrac{X}{P} = \dfrac{1}{K_{eq}}$ —②

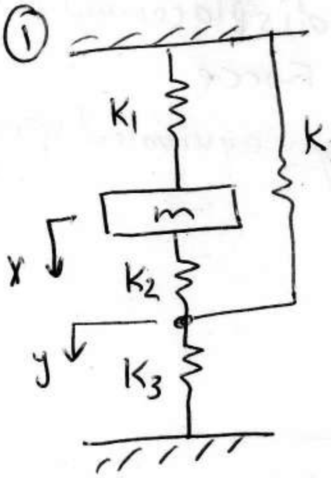
From eq. 1 & 2

$$\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2}$$ in general for spring in series

$$\frac{1}{K_{eq}} = \sum_{i=1}^{i=n} \frac{1}{K_i}$$

Ex: find the equivalent springs.

1



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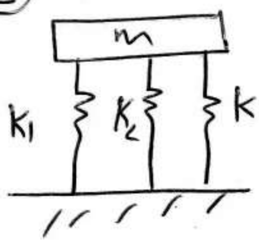

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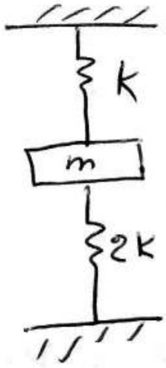

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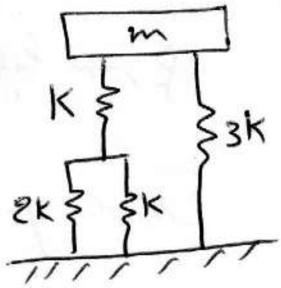


1-6 Simple Energy Method (SEM) Rayleigh Principle:

For conservative system. The sum of the K.E system and The P.E system at any instent of time must equal constant.

$$K.E_s + P.E_s = \text{const.}$$

It is found that the derivation above eq. with respect to time yeild in to the equation of motion for vibrating system...

$$\frac{d}{dt}\left(K.E_s + P.E_s\right) = 0$$

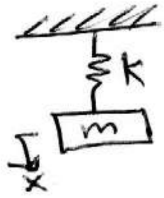
while equating the max. (K.E) & the max (P.E) Leads to having the angular natural frequency (fundamental frequency) of the system

التردد الاساسي

$$K.E_{max} = P.E_{max}$$

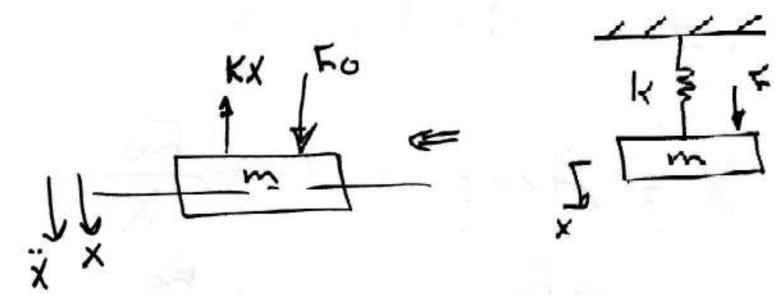
The resulting above eq. is ω_n

Ex For The system shown in fig. Find the eq. of motion and its angular natural frequency by using SEM.



1-7 Forced vibration without damping

1-7-1 a- If a force applied to the system was constant



$\Sigma F = m\ddot{x}$

$m\ddot{x} = F_o - Kx$

$\boxed{m\ddot{x} + kx = F_o}$

equation of motion

complementry solution for $m\ddot{x} + kx = 0$

The solution is already obtained.

$X_h = A\cos\omega_n t + B\sin\omega_n t$

For particular Integral solution assum

$X_p = H$ (H constant for F_o constant)

$\dot{X}_p = 0 \quad \ddot{x}_p = 0$ subsitited in eq. of motion

$m(0) + KH = F_o$

$\therefore H = \frac{F_o}{K} \qquad \therefore X_p = \frac{F_o}{K}$

$\therefore X_{general} = X_h + X_p$

$\therefore$ General Solution $X = A\cos\omega_n t + B\sin\omega_n t + \frac{F_o}{K}$

while A & B are constant can be found from I.C

$x = 0 \quad \dot{x} = 0$ at $t = 0$

Apply I.C to the G.S gives

$A\cos 0 + B\sin 0 + F_o/k \Rightarrow \therefore A = -F_o$

$$\dot{X} = -A\omega_n \sin\omega_n t + B\omega_n \cos\omega_n t + 0$$

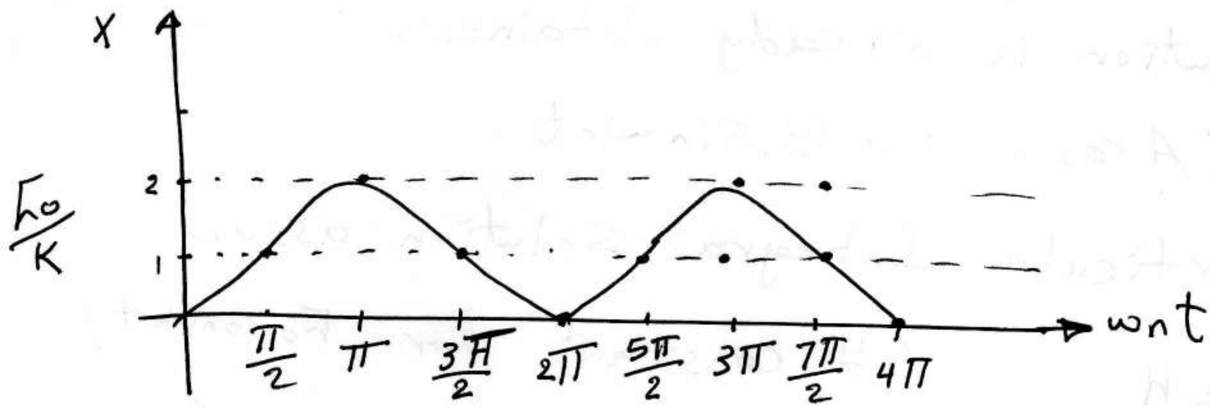
$$0 = -A\omega_n(0) + B\omega_n$$

$$\omega_n \neq 0 \quad \therefore B = 0$$

$$\therefore \quad X = A\cos\omega_n t + \frac{F_o}{K}$$

$$= \frac{-F_o}{K}\cos\omega_n t + \frac{F_o}{K}$$

$$X = \frac{F_o}{K}\left(1 - \cos\omega_n t\right)$$



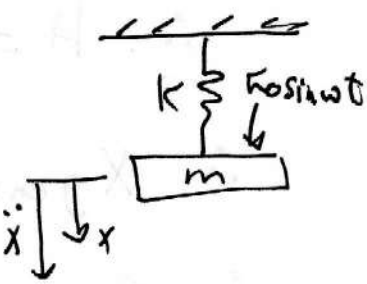
1-7-2 (b) at $F(t) = F_o \sin\omega t$ (Force depend on time)

ω = natural force frequeny caused by ~~F_o~~ unbalance Force.

$$\Sigma F = m\ddot{X}$$

$$m\ddot{X} = -KX + F_o\sin\omega t$$

$$\boxed{m\ddot{X} + KX = F_o\sin\omega t}$$ equation of motion



$m\ddot{X} + KX = 0$ The complementry solution of this equation is $X_{c \text{ or } h} = A\cos\omega_n t + B\sin\omega_n t$

particular solution

$$X = H\sin\omega t + M\cos\omega t$$

$$\dot{X} = H\omega\cos\omega t - M\omega\sin\omega t$$

$$\ddot{X} = -H\omega^2\sin\omega t - M\omega^2\cos\omega t$$

Subsituted X, $\dot{X}$ & $\ddot{X}$ in eq. of motion

$$m\left(-H\omega^2\sin\omega t - M\omega^2\cos\omega t\right) + K\left(H\sin\omega t + M\cos\omega t\right) = F_o\sin\omega t$$

$$\left(KH - mH\omega^2\right)\sin\omega t + \left(KM - mM\omega^2\right)\cos\omega t = F_o\sin\omega t$$

$$KH - mH\omega^2 = F_o \Rightarrow H = \frac{F_o}{K - m\omega^2}$$

$$KM - mM\omega^2 = 0 \Rightarrow M = 0$$

$$\therefore X_p = \frac{F_o}{K - m\omega^2}\sin\omega t$$

For non-dimensional form take $\omega_n^2 = \frac{K}{m}$

$$X_p = \frac{F_o/K}{1 - \left(\frac{\omega}{\omega_n}\right)^2}\sin\omega t$$

ω = angular Force frequency
ω_n = angular natural frequency

$$\therefore X = X_c + X_p$$

$$X = \underbrace{A\cos\omega_n t + B\sin\omega_n t}_{\text{transient Solution}} + \underbrace{\frac{F_o/k}{1 - \left(\frac{\omega}{\omega_n}\right)^2}\sin\omega t}_{\text{Steady State}}$$

Consider steady state Solution

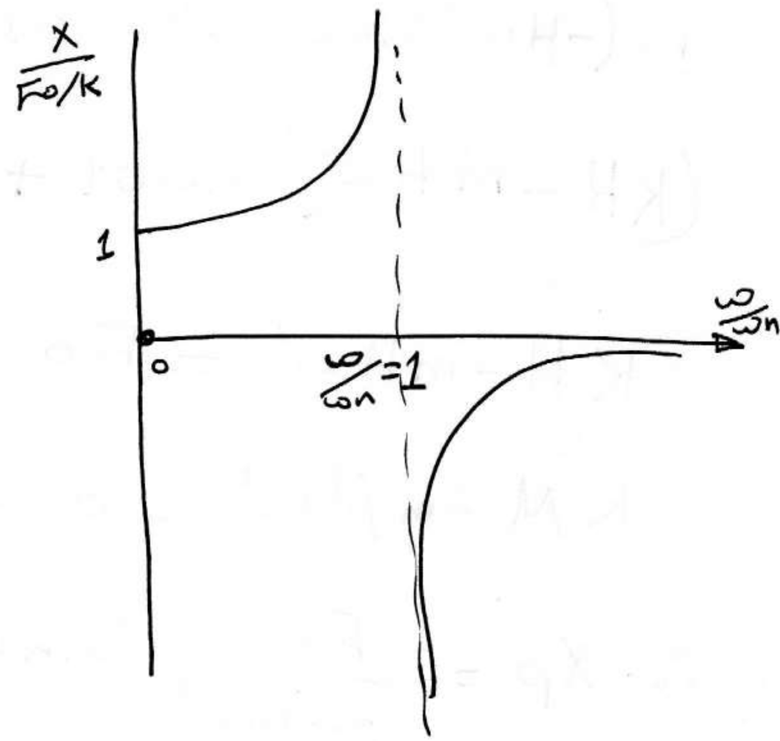
$$X_s = \frac{Fo/K}{1-\left(\frac{\omega}{\omega n}\right)^2} \sin \omega t$$

at max. X $\qquad \sin \omega t = 1$

$$\frac{X}{Fo/K} = \frac{1}{1-\left(\frac{\omega}{\omega n}\right)^2}$$

$\frac{\omega}{\omega n}$	$X/Fo/K$
0	
0.25	
0.5	
0.75	
1	
1.5	

$\Longrightarrow$

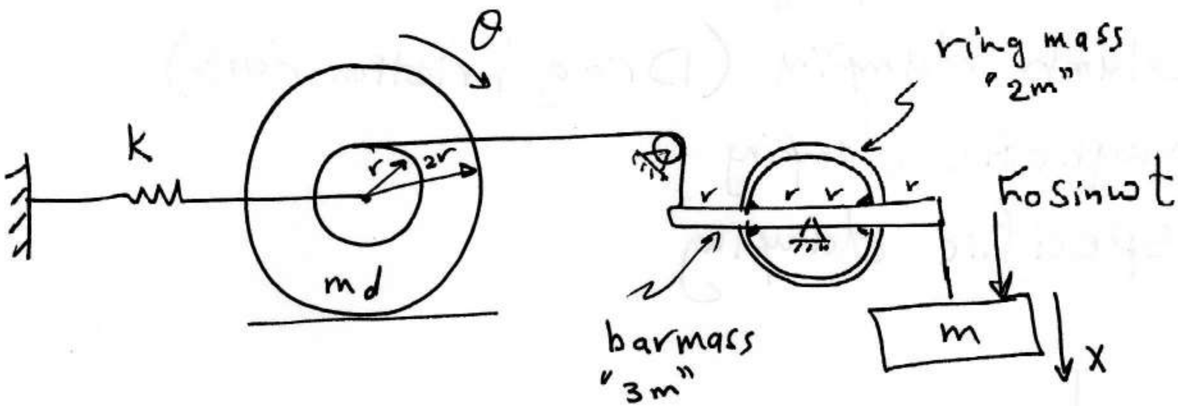


In vibrating system, Resonance happed when

$\boxed{\frac{\omega}{\omega n} = 1}$ For undamped system

For damped system
the resonance happend at $\boxed{\frac{\omega}{\omega n} < 1}$

Ex: find the steady state solution of the vibrating system shown in fig



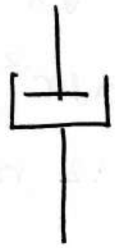
Given that : $m_d = 2$ Kg , I_d about it's centre $= 0.5\, m_d r^2$, $K = 1200$ N/m

$r = 20$ cm , $m = 2$ Kg & $\omega = 12$ rad/s

1-8 Damping

type of damping

1. Viscous damping
2. Columb damping (Drag friction Force)
3. magnatic damping
4. Specific damping

damper  C

C = damping coefficient Factor $\left(\frac{N \cdot s}{m}\right)$

$$F_d \propto \dot{x}$$

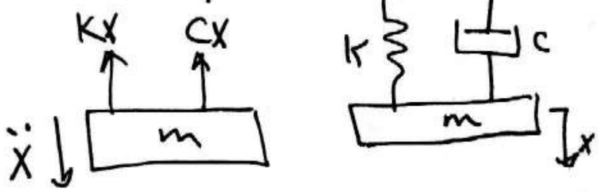
$$F_d = c\dot{x}$$

Not: * It is not conservative system
* Simple energy method can not be applied on it
* $\omega / \omega_n < 1$

1-8-1 Damped Free Vibration

$$\Sigma F = m\ddot{x}$$

$$-Kx - c\dot{x} = m\ddot{x}$$



$$m\ddot{x} + Kx + c\dot{x} = 0$$

$$\therefore\ m\ddot{x} + c\dot{x} + Kx = 0 \quad \text{(1) eq. of motion}$$

Assume $x = Ae^{rt}$ $\dot{x} = Are^{rt}$ $\ddot{x} = Ar^2e^{rt}$

Apply x, $\dot{x}$, & $\ddot{x}$ in eq. 1 give:

$$mAr^2e^{rt} + cAre^{rt} + kAe^{rt} = 0$$

$$\underbrace{Ae^{rt}}_{\neq 0}(\underbrace{mr^2 + cr}_{=0} + K) = 0$$

$$mr^2 + cr + K = 0$$

$$r^2 + \frac{c}{m}r + \frac{K}{m} = 0$$

$$r_{1,2} = \frac{-\frac{c}{m} \mp \sqrt{\left(\frac{c}{m}\right)^2 - 4\frac{K}{m}}}{2}$$

$$r_{12} = \frac{-c}{2m} \mp \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{K}{m}}$$ the root of eq.

$\therefore$ general Solution is $X = Ae^{r_1t} + Be^{r_2t}$

The motion dependes on the square root of $\sqrt{\left(\frac{c}{2m}\right)^2 - \frac{K}{m}}$ if it is equal (+ve) or (−ve) or (0)

$\boxed{1-8-1-1}$ Critical damping

$$\sqrt{\left(\frac{c}{2m}\right)^2 - \frac{K}{m}} = 0$$ when the square root = 0

$$\frac{K}{m} = \omega_n^2$$

$$\left(\frac{C_c}{2m}\right)^2 = \omega_n^2$$

$$C_c = 2m\omega_n$$

$$C_c = 2\sqrt{Km}$$

$$\xi = \frac{c}{C_c} = \text{damping ratio} = \frac{\text{real damping}}{\text{critical damping}}$$

$\therefore \quad C = \xi c$

$$\boxed{C = 2m\omega_n \xi} \quad \frac{N \cdot s}{m}$$

c = damping Coefficient Factor Can be found from eq. of motion

$$\ddot{X} + \frac{c}{m}X + \frac{k}{m} = 0$$

$$\left.\frac{c}{m}\right|_{\substack{\text{from} \\ \text{eq. of motion}}} = 2\xi\omega_n$$

$\xi = 0$ no damping $\Rightarrow m\ddot{X} + KX = 0$

$\xi = 1$ critical damping

$\xi < 1$ under damping (viscous damping)

$\xi > 1$ over damping.

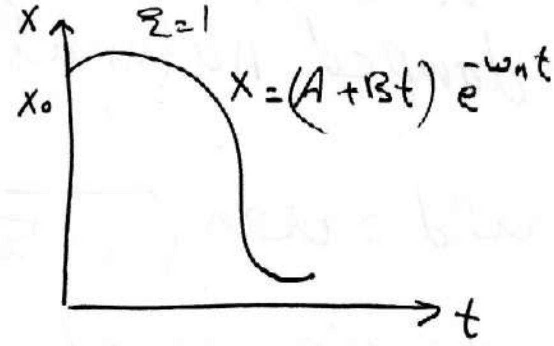
The roots of eq. 1 becoms

$$r_{12} = -\xi\omega_n \mp \sqrt{(\xi\omega_n)^2 - \omega_n^2}$$

$$= \omega_n \left[-\xi \mp \sqrt{\xi^2 - 1} \right.$$

$* \text{ at } \xi = 1 \Rightarrow \sqrt{\xi^2 - 1} = 0$ critical damping

$\therefore r_{12} = \omega_n \left[-1 \mp \sqrt{0} \right]$

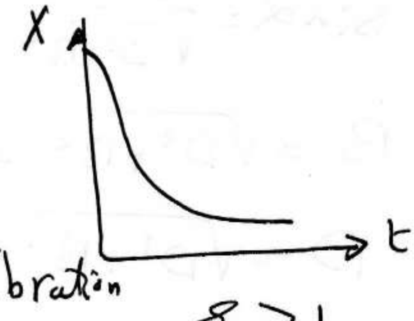


$r_1 = r_2 = -\omega_n$

$\therefore x = (A + Bt) e^{-\omega_n t}$

$* \text{ at } \xi > 1 \Rightarrow \sqrt{\xi^2 - 1} > 0 \ (+)$

$r_1 = (-ve) , \ r_2 = (-ve)$



$\therefore x = A e^{r_1 t} + B e^{r_2 t}$

there is no damping of vibration in this case.

$* \text{ at } \xi < 1$ under damping

$$r_{12} = \omega_n \left[-\xi \mp \sqrt{\xi^2 - 1} \right] = \omega_n \left[-\xi \mp \sqrt{-(1-\xi^2)} \right]$$

$$= \omega_n \left[-\xi \mp i \sqrt{1-\xi^2} \right]$$

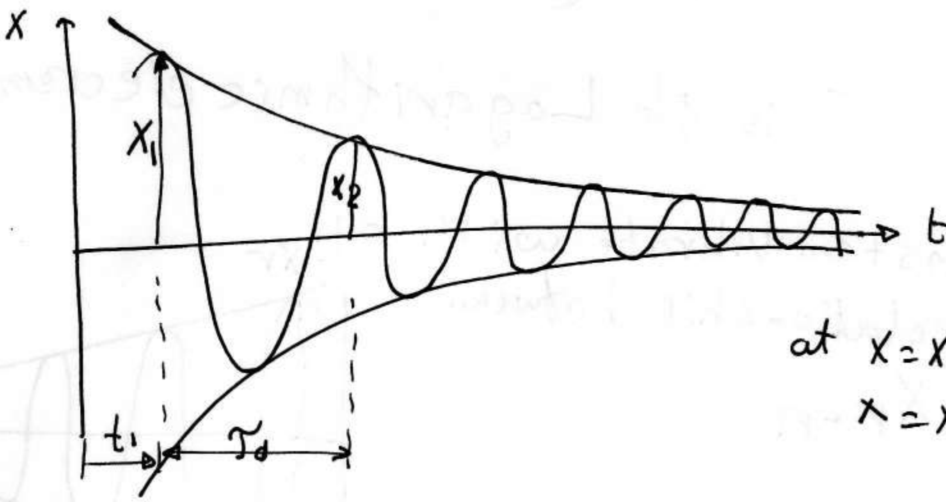
$$= \underbrace{-\xi \omega_n}_{\text{real part}} \mp i \underbrace{\omega_n \sqrt{1-\xi^2}}_{\text{Imaginary part}} = \alpha \mp i\beta$$

$\therefore$ the G-S become

$$x = A e^{\alpha t} \left(B \cos \beta t + C \sin \beta t \right)$$

1-9 Logarithmic decrement For free Vibration

$$x = X e^{-\xi \omega_n t} \sin(\omega_d t + \phi)$$



at $x = X_1$ $t = t_1$
$x = X_2$ $t = t_1 + \tau_d$

at $t = t_1$ $x = X_1$

$$X_1 = X e^{-\xi \omega_n t_1} \sin(\omega_d t_1 + \phi)$$

at $x = X_2$ $t = t_1 + \tau_d$

$$X_2 = X e^{-\xi \omega_n (t_1 + \tau_d)} \sin(\omega_d (t_1 + \tau_d) + \phi)$$

$\therefore$ Sin( ) at X_1 & X_2 = 1 at max.

$$\therefore \frac{X_1}{X_2} = \frac{e^{-\xi \omega_n t_1}}{e^{-\xi \omega_n (t_1 + \tau_d)}}$$

$$\frac{X_1}{X_2} = \frac{e^{-\xi \omega_n t_1}}{e^{-\xi \omega_n t_1} \cdot e^{-\xi \omega_n \tau_d}} = e^{\xi \omega_n \tau_d}$$

$\omega_d = \omega_n \sqrt{1 - \xi^2}$

$$\therefore \ln \frac{X_1}{X_2} = \xi \omega_n \tau_d \Rightarrow \ln \frac{X_1}{X_2} = \xi \omega_n \frac{2\pi}{\omega_d}$$

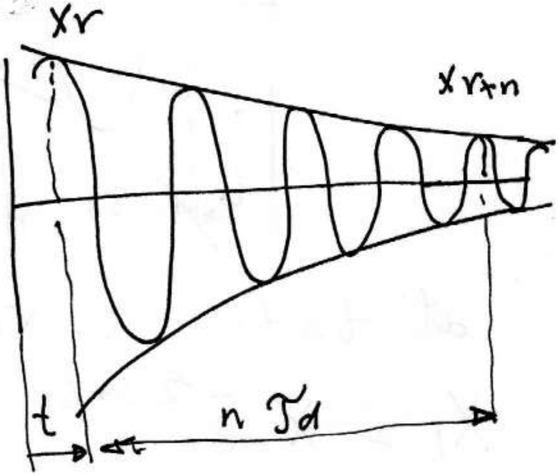
$$\ln \frac{X_1}{X_2} = 2\pi\xi / \sqrt{1 - \xi^2}$$

$$\ln \frac{X_1}{X_2} = \frac{2\pi\xi}{\sqrt{1-\xi^2}} \quad \text{or} \quad \xi = \frac{1}{\sqrt{1+\left(\frac{2\pi}{\delta}\right)^2}}$$

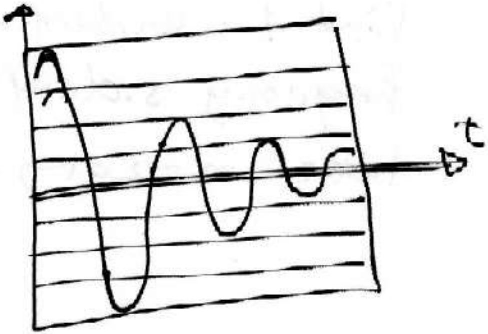
$$\delta = \ln \frac{X_1}{X_2} = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

where δ is the Logarithmic decrement

x:
Find the relationship between X_r & X_{r+n}



Ex: If the recorded signal of vibration is shown in fig(a)

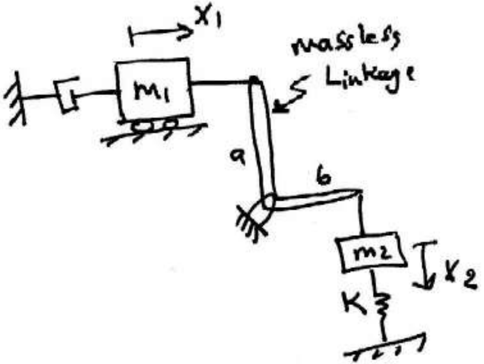


a) Find the damping coefficient "c"

b) If the mass m_2 is displaced by 22 mm and released how many cycles will be executed befor the amplitude is released to 0.8mm.

Given that $m_1 = 10$ kg, $m_2 = 25$ kg

$a/b = 0.5$ $K = 10^5$ N/m



محاضرة (3)
الثالث عشر

1-10 Lagrange equation:

For conservative or non-conservative system Lagrang equation may be written as:

$$\frac{d}{dt}\left(\frac{\partial K.E}{\partial \dot{q}_r}\right) - \frac{\partial K.E}{\partial q_r} + \frac{\partial P.E}{\partial q_r} + \frac{\partial D.E}{\partial \dot{q}_r} = Q_r$$

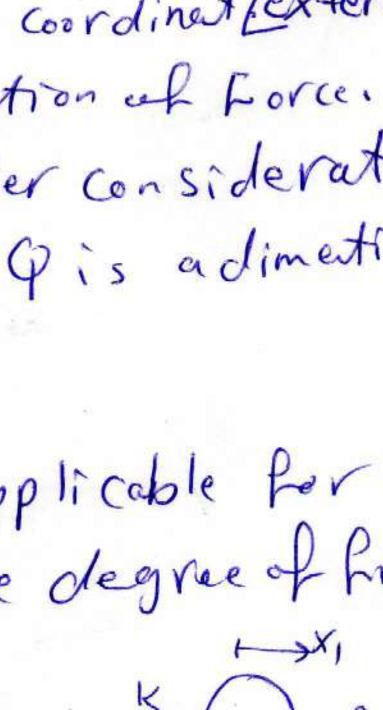
wher q_r: is the coordinate under consideration

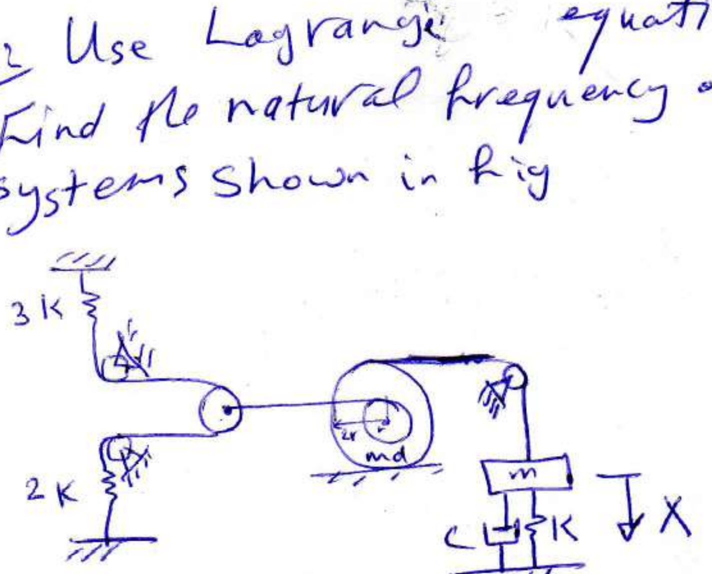
D.E: is the damping Energy $= \frac{1}{2} C\dot{x}^2$ (dissipation energy)

Q_r: generalized Force and its dimention depends on the dimention of the coordinat [external Force] is mean (X) is the dimation of Force. while if the coordinate under consideration is (θ) then the dimension of Q is adimention of Torque.

Hence, Lagrange eq. is applicable for any system, Single or Multiple degree of freedom

Ex: Use Lagrange equation to Find the natural frequency of the systems shown in fig



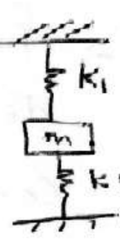


تمارين

Sheet 1

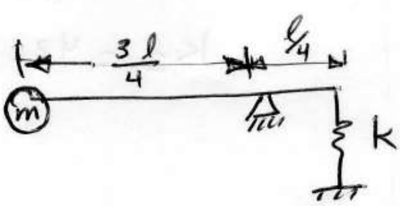
قسم الهندسة الميكانيكية
المرحلة الثالثة
محطات

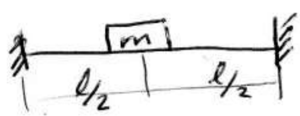
1. a) find the natural frequency of the system shown
b) what is the displacement of the mass from the equilibrium position at a time = 3 sec. take the initial conditions
$X(0) = 0.003$ m , $\dot{X}(0) = 0$



$K_1 = 5$ N/m
$K_2 = 3$ N/m
$m = 2$ kg.

2. Drive an expression for the natural frequency of each of the following system

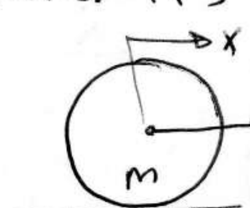




tak $\delta = \frac{Wl^3}{192EI}$

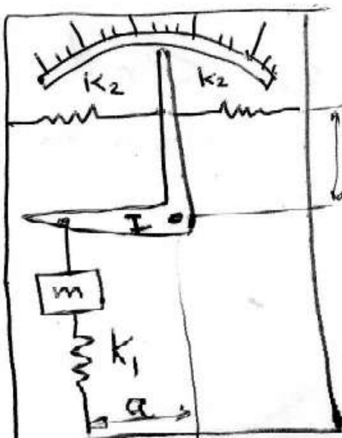
3. A circular cylinder of mass m and radius r is connected by a spring of modulus K as shown. If it is free to roll on the rough horizontal surface without slipping. find it's frequency.

ans. $\omega_n \sqrt{\frac{2K}{3m}}$



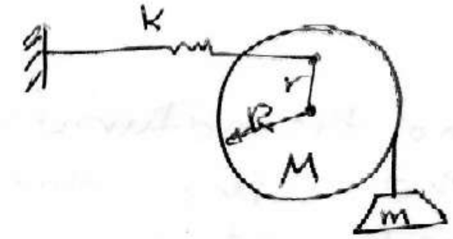
4. An amplitude meter consists of a seismic mass suspended as shown. Determine the natural frequency (f_n) of the meter in terms of K_1, K_2, m and I of the pointer

$$f_n = \frac{1}{2\pi}\sqrt{\frac{2K_2 b^2 + K_1 a^2}{I + m a^2}}$$

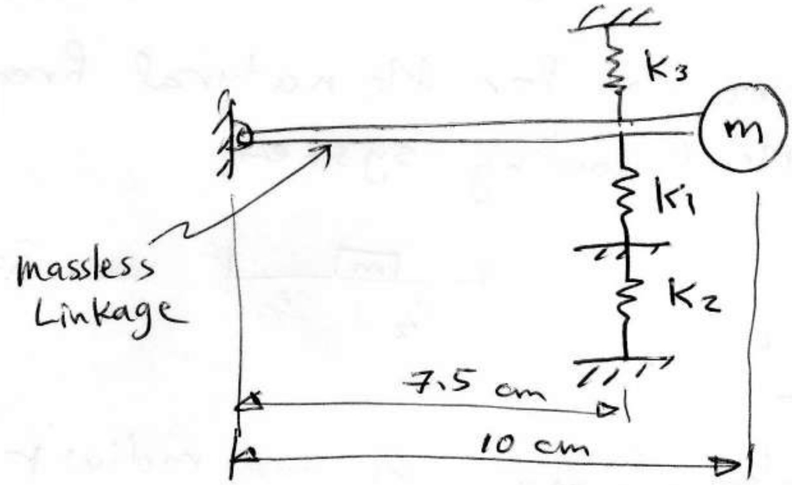


5- Find the natural frequency of system

Ans. $\omega_n = \sqrt{\dfrac{kr^2}{R^2(m+\frac{M}{2})}}$



6- For the system shown. derive and solve the differential equation of motion with initial condition $\theta_0 = 0.1$ rad & $\dot{\theta} = 0.6$ rad/s at $t=0$



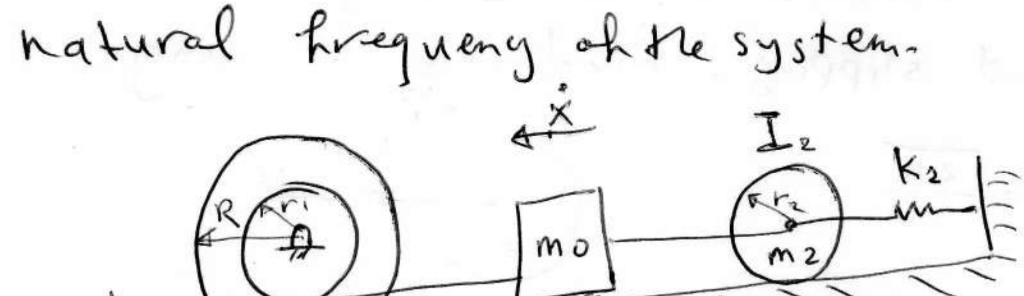
$m = 3.65$ kg

$k_3 = 1022$ N/m

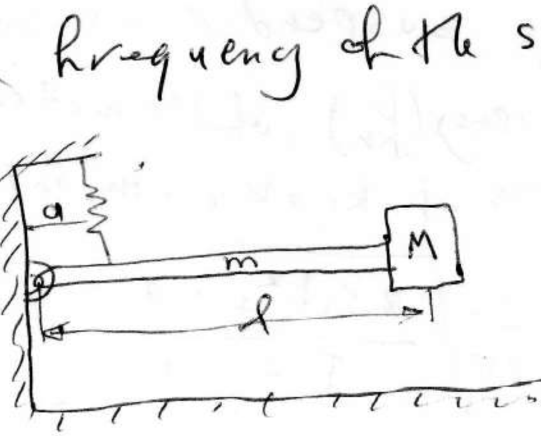
$K_1 = 87.6$ N/m

$K_2 = 43.8$ N/m

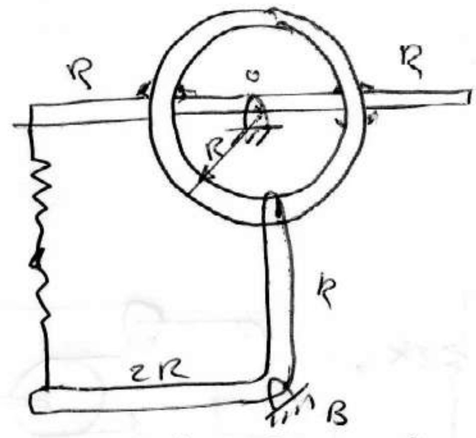
7- Determine the Kinetic energy of the system shown in the fig. in terms of $\dot{x}$, then find the natural frequency of the system.



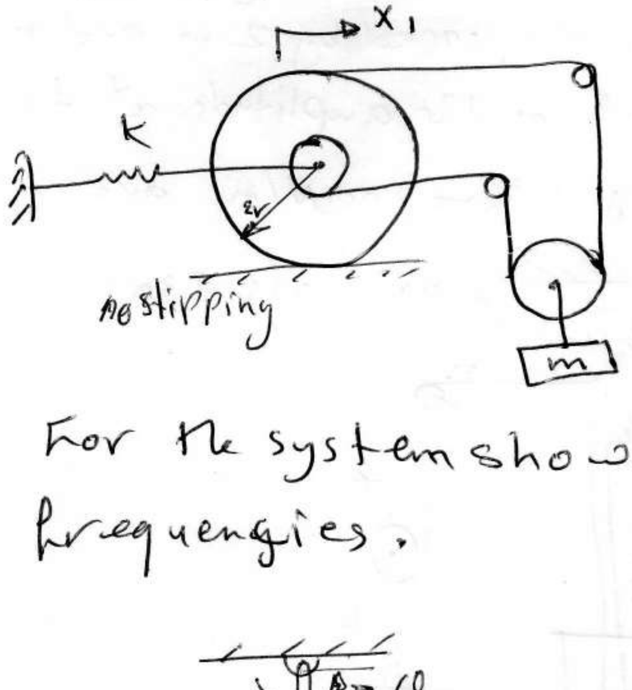
8. Find the natural frequency of the system shown in fig.



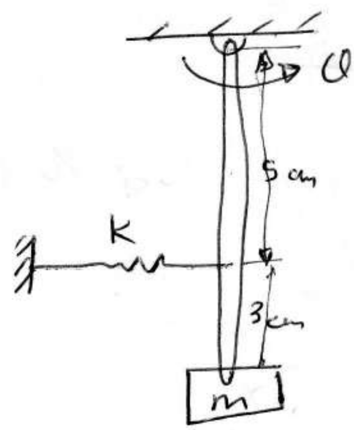
9. Find the natural frequency for the system shown given that the Ring mass is m & its radius R. The Length of the bar DOE is 4R and its mass m. neglect the mass of the rigid arm CBA

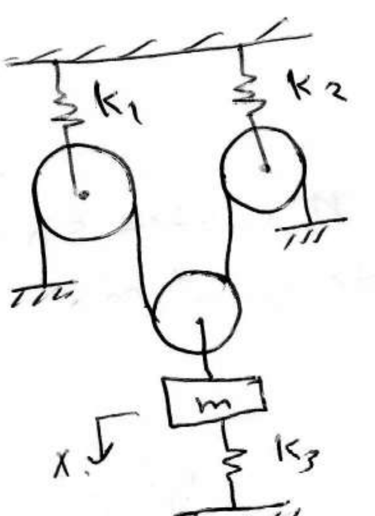


10. For the system shown in fig find it's max. amplitude of vibration. Given that : I_d about it's centre $= 0.4mr^2$, drum mass $= 4m$

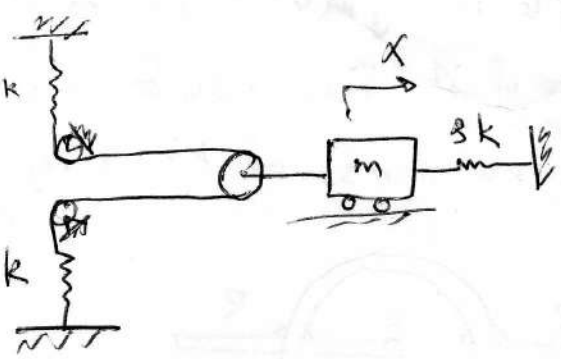


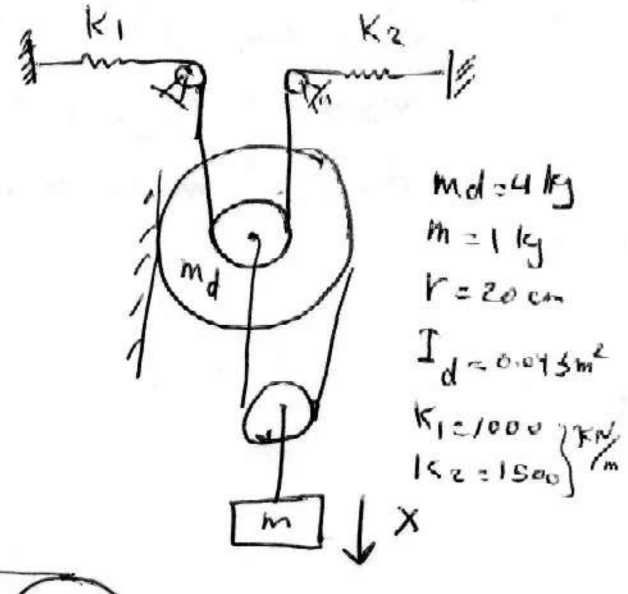
11. For the system shown a & b find the natural frequencies.

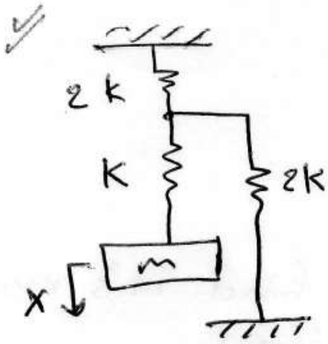


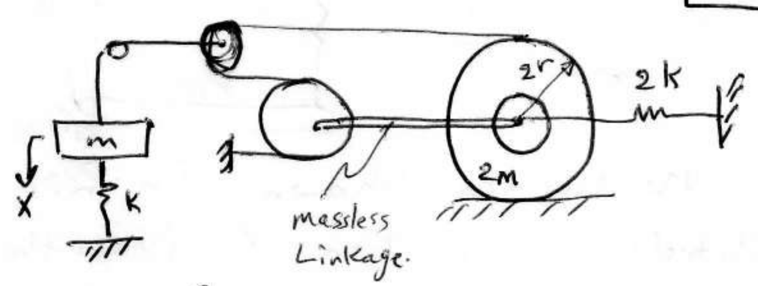


12. For the system a & b find the natural frequencies.



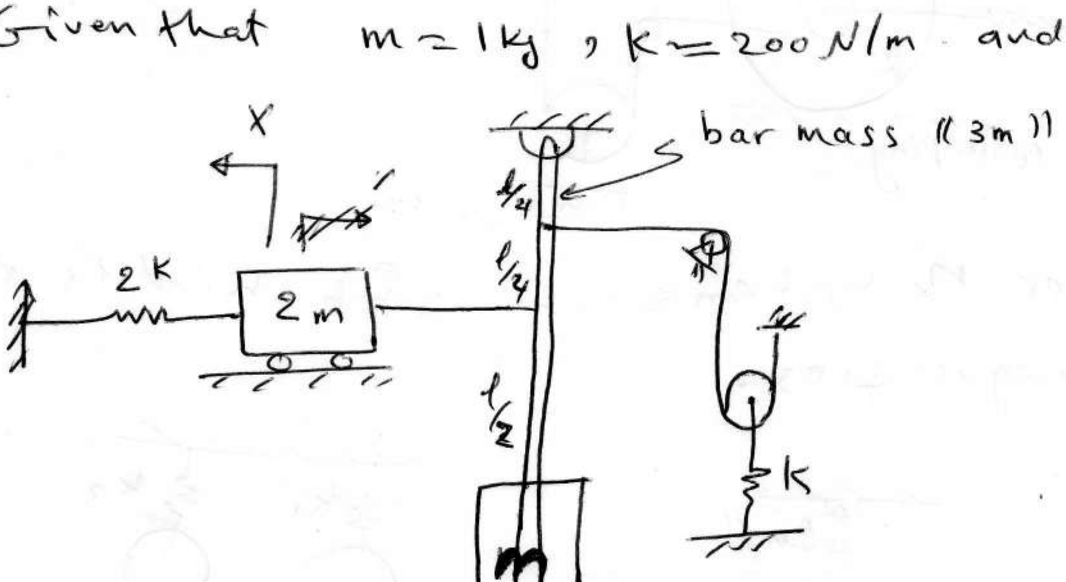




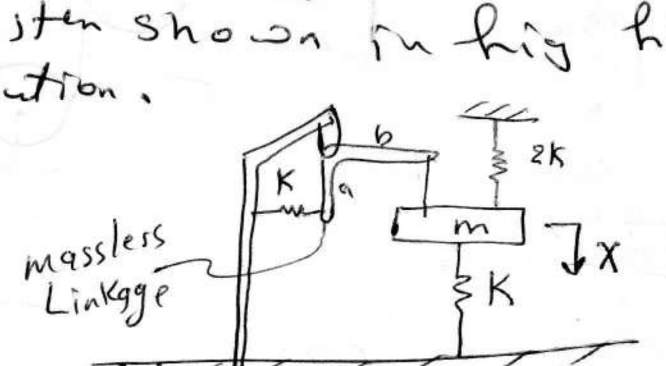


13. a) Find the natural frequency for the system shown

b) If the mass "2m" is displaced by 2 m and released from rest what will be the amplitude at time 2 sec. Given that $m = 1$ kg, $k = 200$ N/m and $l = 60$ cm.



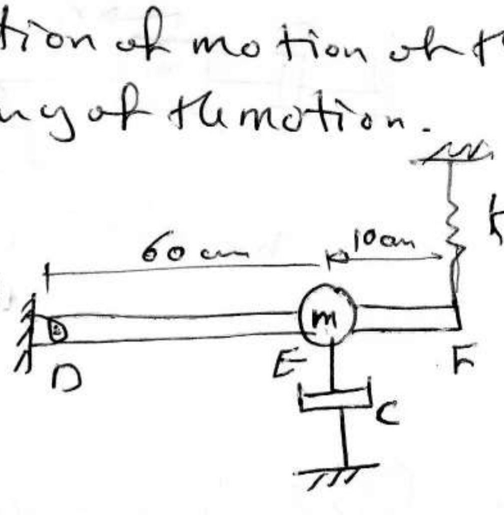
14. For the system shown in fig find the steady state solution.



Sheet 2

الجامعة التكنولوجية
قسم الهندسة الميكانيكية
ثالث معدات

1. The 4 kg mass E is fastened to the rod DF whose weight can be neglected. The spring has stiffness 1.5 N/m and the coefficient of the dashpot is 0.5 N.s/m. The system is in equilibrium when DF is horizontal. The rod is displaced (0.1) rad clockwise & released from rest when t = 0. Determine a) equation of motion of the rod b) damped circular frequency of the motion. c) damping ratio ξ.



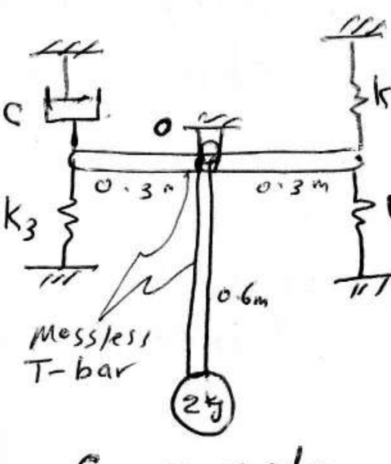
Ans. $\theta = e^{-0.0625\,t}\,(0.1\cos\omega_n t + 0.00878\sin\omega_n t)$

$\omega_n = 0.71169$ rad/s

$f_d = 0.11326$

$\xi = 0.087$

2- The riged T-bar rotates in a vertical plane about axis (O). The bar is caused to vibrate by displacing the weigh & releasing it from rest by 0.5 cm. Determine the damped frequency of the vibration & the ratio of the positive aplitudes of the third and fourth cycles & general solution of the vibrating system.



$C = 11$ N.s/m

$K_1 = 15$, $K_2 = 13$, $K_3 = 5$ N/m

mg is considered.

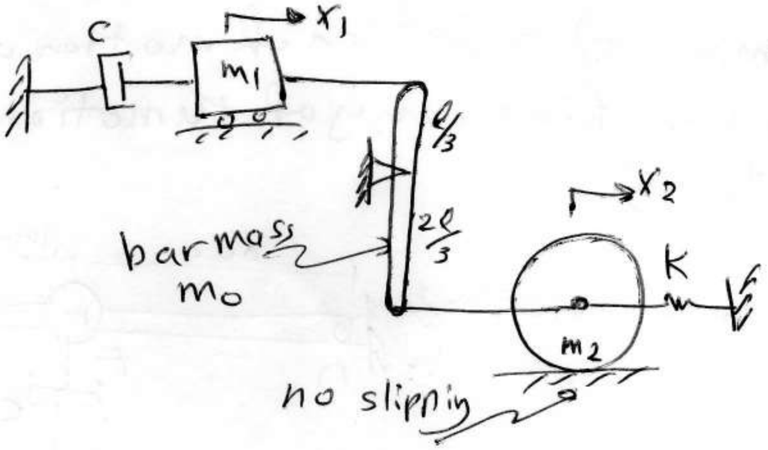
$f_d = 0.304$ Hz, $\ln \frac{X_3}{X_4} = 2$,

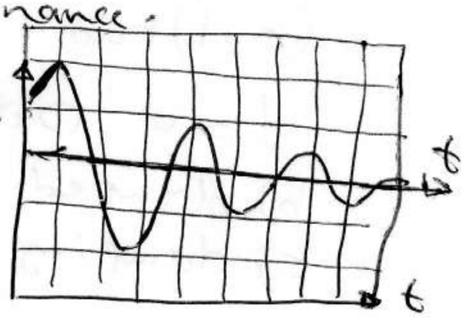
$X = e^{-0.6875t}\,(0.005\cos 1.94t + 0.00179\sin 1.911t)$

3. If the recorded signal of vibration is shown in fig (a). find the damping coefficient "c"

b) If the mass m_1 is displaced by 10 mm and released, how many cycles will be executed befor the amplitude is reduced to 0.2 mm. Given that $m_1 = 2kg$, $m_2 = 8 kg$, $K = 10^5 N/m$, $r = 20 cm$ $m_o = 1 kg$ and $l = 60 cm$.

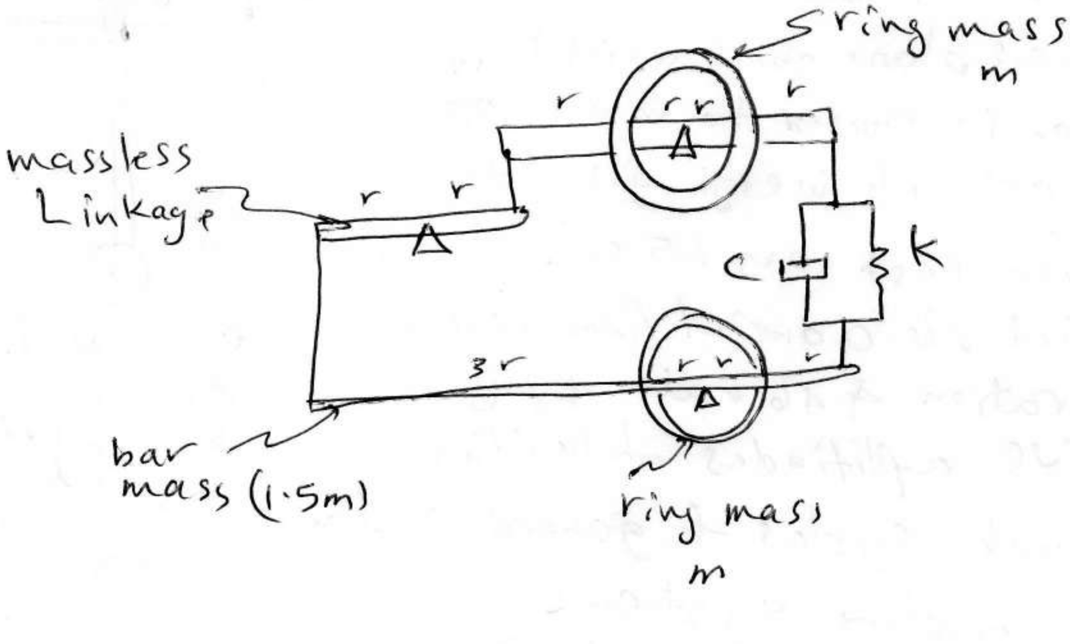
c) if a force $= 0.1 \sin \omega t$ is applied on m_1, what will be it's amplitude at resonance.





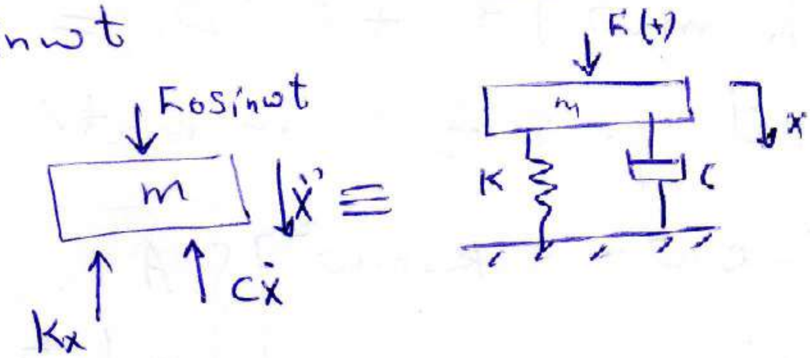
(a)

4. find the natural frequeny of the system shown and find it's damping ratio



1-11 Forced Vibration with Damping:

when $F(t) = F_o \sin\omega t$



$\Sigma F = m\ddot{x}$

$m\ddot{x} = -Kx - c\dot{x} + F_o \sin\omega t$

$m\ddot{x} + c\dot{x} + Kx = F_o \sin\omega t$ — ① eq. of motion

- Complementry Solution gives

$m\ddot{x} + c\dot{x} + Kx = 0.$

the amplitude is $X = X_o e^{-\zeta\omega_n t} \sin(\omega_d t + \phi)$

- the particular integral for all equation

$m\ddot{x} + c\dot{x} + Kx = F_o \sin\omega t$

Assume $\quad X_p = A\cos\omega t + B\sin\omega t$

therefore $\quad \dot{x} = -A\omega \sin\omega t + B\omega\cos\omega t$

$\qquad \ddot{x} = -A\omega^2 \cos\omega t + B\omega^2 \sin\omega t$

Subsituted x_p, $\dot{x}_p$ & $\ddot{x}_p$ in eq 1

$-mA\omega^2 \cos\omega t - Bm\omega^2 \sin\omega t - cA\omega \sin\omega t + cB\omega\cos\omega t +$

$KA\cos\omega t + KB\sin\omega t = F_o \sin\omega t$

put the function of sin( ) in one term. Leads to

$-mB\omega^2 - cA\omega + KB = F_o$ — ⊛

$(K - m\omega^2)B - cA\omega = F_o$ — ②

while $-mA\omega^2 + cB\omega + KA = 0$

$$(K - m\omega^2)B - cA\omega = F_o \quad (2)$$

$$(k - m\omega^2)A + c\omega B = 0 \quad (3)$$

put eq. 2 & 3 in matrix form

$$\begin{bmatrix} -c\omega & K - m\omega^2 \\ K - m\omega^2 & c\omega \end{bmatrix} \begin{Bmatrix} A \\ B \end{Bmatrix} = \begin{Bmatrix} F_o \\ 0 \end{Bmatrix}$$

$$\Delta\omega = \begin{vmatrix} -c\omega & K - m\omega^2 \\ K - m\omega^2 & c\omega \end{vmatrix} = -(c\omega)^2 - (K - m\omega^2)^2$$
$$= \cancel{-} \left[(K - m\omega^2) + (c\omega)^2 \right] = +$$

to find the constant A & B use Grumers' rule

$$A = \frac{\begin{vmatrix} F_o & K - m\omega^2 \\ 0 & c\omega \end{vmatrix}}{\Delta\omega} = \frac{F_o c\omega}{\Delta\omega}$$

$$B = \frac{\begin{vmatrix} -c\omega & F_o \\ K - m\omega^2 & 0 \end{vmatrix}}{\Delta\omega} = \frac{-F_o(K - m\omega^2)}{\Delta\omega}$$

$$\therefore\ x_p = A\cos\omega t + B\sin\omega t$$

$$\therefore\ x_p = \frac{F_o c\omega}{\Delta\omega}\cos\omega t - \frac{F_o(K - m\omega^2)}{\Delta\omega}\sin\omega t$$

$\therefore$ the G. Solution

$$X = X_h + X_p$$

$$= \underbrace{X_o e^{-\zeta\omega_n t}\sin(\omega_{d}t + \phi)}_{\text{transient}} + \frac{F_o c\omega}{\Delta\omega}\cos\omega t - \frac{F_o(K - m\omega^2)}{\Delta\omega}\sin\omega t$$

take $X_p = \frac{F_o c\omega}{D\omega}\cos\omega t - \frac{F_o(k-m\omega^2)}{D\omega}\sin\omega t$

$$X_p = \frac{F_o}{D\omega}\left[c\omega\cos\omega t - (k-m\omega^2)\sin\omega t\right] \quad (**)$$

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\sin\alpha = \frac{c\omega}{\sqrt{(c\omega)^2+(k-m\omega^2)}}$$

so $c\omega = \sqrt{(k-m\omega^2)+(c\omega)^2}\,\sin\alpha$

$$k-m\omega^2 = \sqrt{(k-m\omega^2)^2+(c\omega)^2}\,\cos\alpha$$

substituted in (**) get

$$x = \frac{F_o}{D\omega}\left[\sqrt{}\,\sin\alpha\cos\omega t - \sqrt{}\,\cos\alpha\sin\omega t\right]$$

$$= \frac{F_o\sqrt{}}{D\omega}\Big[\underbrace{\sin\alpha\cos\omega t - \cos\alpha\sin\omega t}_{\sin(\alpha-\beta)}\Big]$$

$$= \frac{F_o\sqrt{(k-m\omega^2)^2+(c\omega)^2}}{\left[(k-m\omega^2)^2+(c\omega)^2\right]}\sin(\omega t-\alpha)$$

$$X_p = \frac{F_o}{\quad}\sin(\omega t-\alpha)$$

$$X_p = \frac{F_0/K}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}} \sin(\omega t - \alpha)$$

$$\alpha = \tan^{-1} \frac{c\omega}{K - m\omega^2}$$

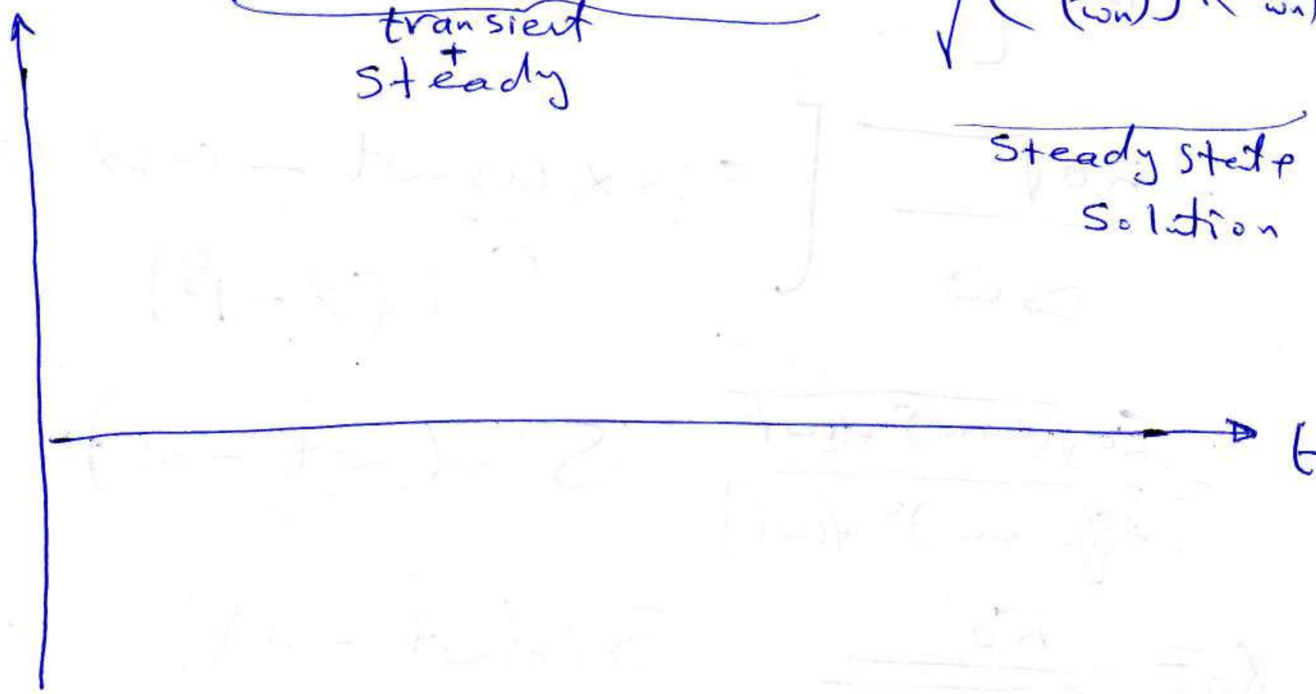
For non-dimentiond relationship

$$\alpha = \tan^{-1} \frac{2\xi\frac{\omega}{\omega_n}}{1 - \left(\frac{\omega}{\omega_n}\right)^2}$$

$$X(t) = X_h + X_p$$

$$= \underbrace{X_0 e^{-\xi\omega_n t} \sin(\omega_d t + \alpha)}_{\text{transient}} + \underbrace{\frac{F_0/k \sin(\omega t - \phi)}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(\frac{2\xi\omega}{\omega_n}\right)^2}}}_{\text{Steady state Solution}}$$

transient + steady



$$X_s = \frac{F_o/K}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}} \sin(\omega t-\phi)$$

if $f(t) = F_o = \text{constant}$

$$X_{st} = \frac{F_o}{K}$$

therefor $\frac{X}{X_{st}} > 1$ ther is amaser in velocity

التضخيم الديناميكي

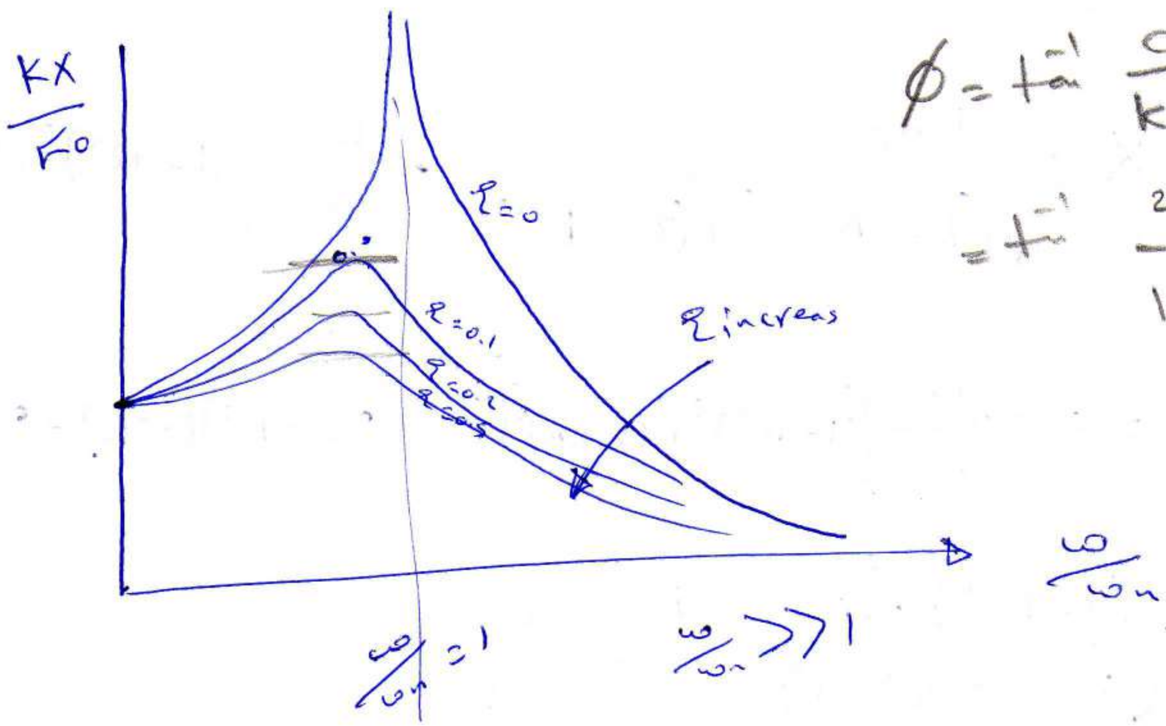
$\frac{X}{F_o/K}$ = amplification in dynamic capacity

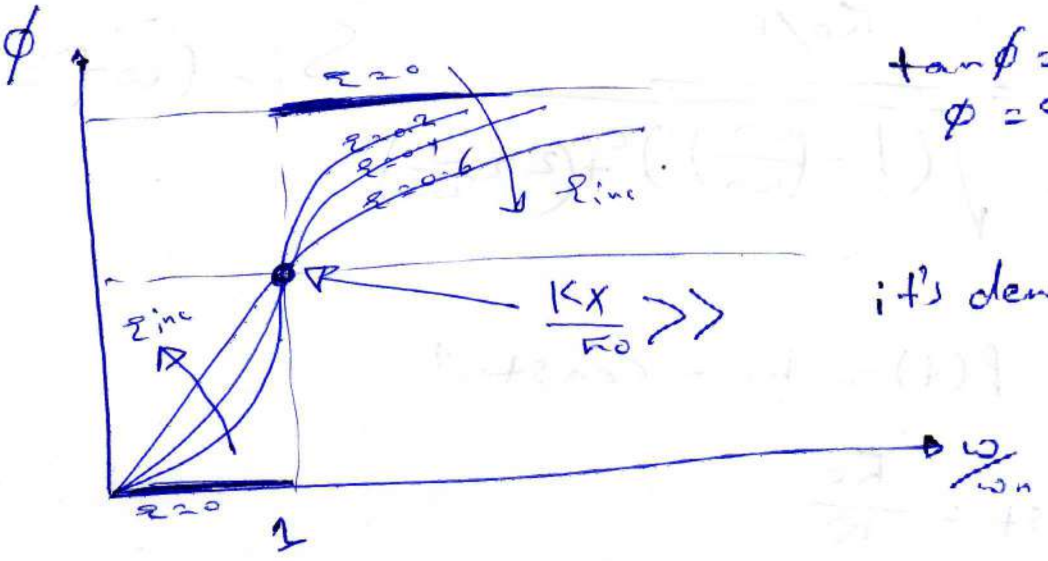
the inertia force increased when $\frac{\omega}{\omega_n} >> 1$. $\xi \uparrow$ $\downarrow x$

because it depends on ω^2

$$\phi = \tan^{-1}\frac{c\omega}{k-m\omega^2}$$

$$= \tan^{-1}\frac{2\xi\frac{\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}$$





$\tan\phi=\infty$
$\phi=90^\circ$

it's dengerous.

for resonance. Condition.

$$X=\frac{F_o/K}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}\quad \sin(\omega t-\phi)$$

let $\frac{\omega}{\omega_n}=R$ at max X $\sin(\omega t-\phi)=1$

$$\therefore\ X=\frac{F_o/K}{\sqrt{(1-R^2)^2+(2\xi R)^2}}=\frac{F_o}{K}\left[(1-R^2)^2+(2\xi R)^2\right]^{-\frac{1}{2}}$$

$$\frac{dX}{dR}=0=\frac{-F_o}{2K}\left[(1-R^2)^2+(2\xi R)^2\right]^{-\frac{3}{2}}\left[2(1-R^2)(-2R)+8\xi^2R\right]$$

$-4R(1-R^2)+8\xi^2R=0$

$4R(R^2+2\xi^2-1)=0\Longrightarrow R=0$

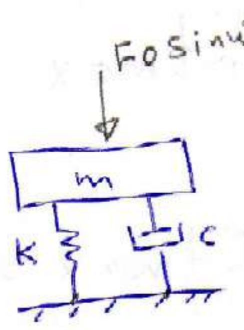
or $R^2+2\xi^2-1=0$

$R^2=1-2\xi^2$

$\therefore \frac{\omega}{\omega_n}=\sqrt{1-2\xi^2}$ at resonance.

1-12 Transmissibility Ratio TR

It is the ratio of max. transmitted force to the floor to the max. impressed force.



the force transmitted to the floor is

$$F_t = c\dot{X} + KX \quad \text{--- (1)}$$

$$X \text{ for this system} = \frac{F_o/K}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}} \sin(\omega t - \phi) \quad \text{-- (*)}$$

let $A = \dfrac{F_o/K}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$

$$X_s = A \sin(\omega t - \phi)$$

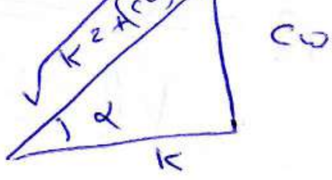
$$\dot{X} = A\omega \cos(\omega t - \phi)$$

substituted $x, \dot{x}$ in eq. 1 get

$$F_t = c\left(A\omega\cos(\omega t - \phi)\right) + k\left(A\sin(\omega t - \phi)\right)$$

$$= A\left[c\omega\cos(\omega t - \phi) + K\sin(\omega t - \phi)\right]$$

$$\sin(\alpha + \beta) = \underline{\sin\alpha}\cos\beta + \underline{\cos\beta}\sin\alpha$$



$$c\omega = \sqrt{K^2 + (c\omega)^2}\sin\alpha$$

$$K = \sqrt{K^2 + (c\omega)^2}\cos\alpha$$

$$F_t = A\left[\sqrt{K^2+(c\omega)^2}\sin\alpha\cos(\omega t-\phi) + \sqrt{K^2+(c\omega)^2}\cos\alpha\sin(\omega t-\phi)\right]$$

$$= A\sqrt{K^2+(c\omega)^2}\left[\sin\alpha\cos(\omega t-\phi) + \cos\alpha\sin(\omega t-\phi)\right.$$

$$F_t = A\sqrt{K^2+(c\omega)^2}\ \sin(\omega t-\phi+\alpha)$$

$$\therefore F_{t\,max} = A\sqrt{K^2+(c\omega)^2} \quad \text{at } \sin(\omega t-\phi+\alpha)=1$$

$$F_{t\,max} = \frac{F_o/k \quad \sqrt{K^2+(c\omega)^2}}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}} \qquad \begin{array}{l} K\sqrt{1+\left(\frac{c\omega}{K}\right)^2} \\ K\sqrt{1+\left(2\xi\frac{\omega}{\omega_n}\right)^2} \end{array}$$

$$F_{t\,max} = \frac{F_o\sqrt{1+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

$$\frac{F_{t\,max}}{F_o} = \frac{\sqrt{1+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

$$\frac{F_{t\,max}}{F_o} = \text{Transmissibility ratio} = TR$$

$$\therefore TR = \frac{\sqrt{1+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

If all the Force was transmitted to the floor (Earth) Then $TR = 1$

if $T_R = 1$ let $\frac{\omega}{\omega_n} = R$

$$1 = \frac{\sqrt{1+(2\xi R)^2}}{\sqrt{(1-R^2)^2+(2\xi R)^2}}$$

$$\sqrt{(1-R^2)^2+(2\xi R)^2} = \sqrt{1+(2\xi R)^2}$$

$$(1-R^2)^2+(2\xi R)^2 = 1+(2\xi R)^2$$

$$(1-R^2)^2 = 1$$

$$1-2R^2+R^4 = 1$$

$$R^2(R^2-2) = 0 \longrightarrow \text{either } R=0$$

$$\text{or } R = \mp\sqrt{2}$$

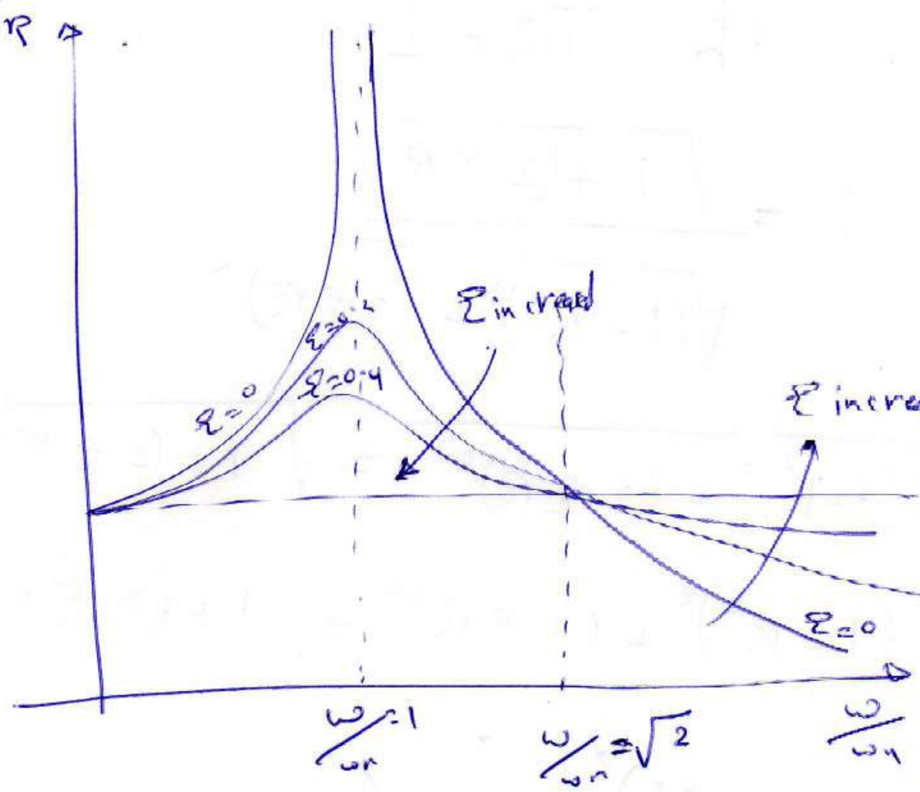
$$\therefore T_R = 1 \Longrightarrow \omega/\omega_n = \sqrt{2}$$

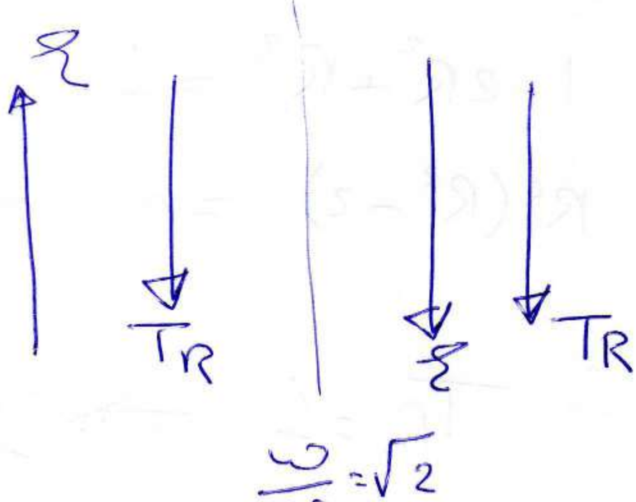
$$T_R = \frac{\sqrt{1+(2\xi\frac{\omega}{\omega_n})^2}}{\sqrt{(1-(\frac{\omega}{\omega_n})^2)^2+(2\xi\frac{\omega}{\omega_n})^2}}$$

for Less T_R

$\frac{\omega}{\omega_n} < \sqrt{2}$ $\quad \xi \uparrow$

$\frac{\omega}{\omega_n} > \sqrt{2}$ $\quad \xi \downarrow$

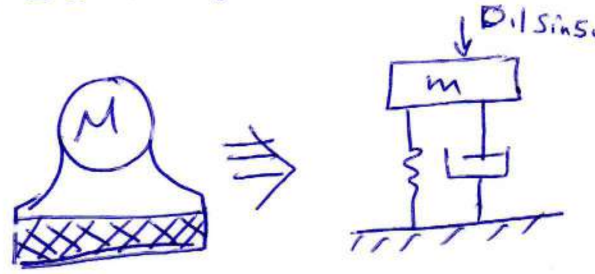




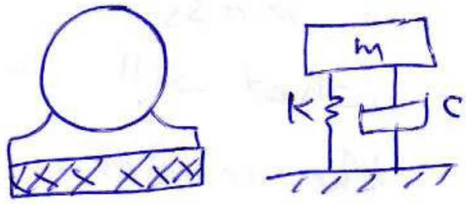
Ex:- An electric motor is replaced on isolator as shown in fig. The isolator is represented by spring & damper of damped ratio $\xi = 0.2$. It is found that all the max impressed force is transmitted to the floor.

ⓐ what will be the stiffness of the spring when unbalance of force $0.1 \sin 50t$ is exerted on the mass

ⓑ what will be the transmitted force when the unbalance force is $0.2 \sin \omega t$. Given that $m = 10\,kg$.



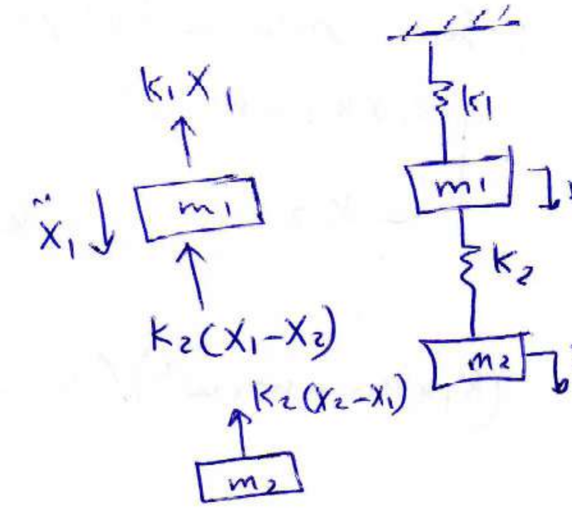
EX2/ A compressor of (9 kg) is mounted on isolator which represented by spring of stiffness 9 kN/m and damper. It is found that the compressor suffers from vibration due to the unbalance of 0.4 kg.m The available isolator have damping ratio (0.3, 0.4, 0.5, 0.7, 0.85). Choose an isolator damping from those stated above. Find the max compressor speed such that max. amplitude of vibration is not exceeding 0.002 m and minimum transmitted force to the floor.



$\boxed{1-13}$ Two degree of freedom

- have two coordinate of motion x_1, x_2
- have two natural frequencies ω_1, ω_2

$\boxed{a}$ free Vibration



$$\Sigma F = m_1 \ddot{x}_1$$

$$-k_1 x_1 - k_2(x_1 - x_2) = m_1 \ddot{x}_1$$

$$m_1 \ddot{x}_1 + (k_1 + k_2)x_1 - k_2 x_2 = 0 \quad \text{—} (1)$$

eq. of motion of mass "m_1"

$$\Sigma F = m_2 \ddot{x}_2$$

$$-k_2(x_2 - x_1) = m_2 \ddot{x}_2$$

$$m_2 \ddot{x}_2 + k_2 x_2 - k_2 x_1 = 0 \quad \text{—} (2)$$ eq. of motion of mass "m_2"

Assume $x_1 = A\sin(\omega t + \psi)$, $x_2 = B\sin(\omega t + \psi)$

$\dot{x}_1 = A\omega\cos(\omega t + \psi)$ $\quad \dot{x}_2 = B\omega\cos(\omega t + \psi)$

$\ddot{x}_1 = -A\omega^2 \sin(\omega t + \psi)$ $\quad \ddot{x}_2 = -B\omega^2 \sin(\omega t + \psi)$

where ψ = phase angle

Substituted for $x_1, x_2, \ddot{x}_1$ & $\ddot{x}_2$ in eq. 1 & 2

Then simplification we endup with

$$\left[-m_1 A\omega^2 + (k_1 + k_2)A - k_2 B\right] \sin(\omega t + \psi) = 0$$

$$(k_1 + k_2 - m_1\omega^2)A - k_2 B = 0 \quad \text{—} (3)$$

$$\left[-m_2 B\omega^2 + k_2 B - k_2 A\right] \sin(\omega t + \psi) = 0$$

$$(k_2 - m_2\omega^2)B - k_2 A = 0 \quad \text{—} (4)$$

put eq. 3 & 4 in matrix form:

$$\begin{bmatrix} k_1+k_2-m_1\omega^2 & -k_2 \\ -k_2 & k_2-m_2\omega^2 \end{bmatrix} \begin{Bmatrix} A \\ B \end{Bmatrix} = 0$$

For non-trival solution

$$\begin{vmatrix} k_1+k_2-m_1\omega^2 & -k_2 \\ -k_2 & k_2-m_2\omega^2 \end{vmatrix} = 0$$ frequeny determinant.

$$(k_1+k_2-m_1\omega^2)(k_2-m_2\omega^2)-k_2^2 = 0$$ frequency equation.

$$k_1k_2 - k_1m_2\omega^2 + k_2^2 - k_2m_2\omega^2 - k_2m_1\omega^2 + m_1m_2\omega^4 - k_2^2$$

$$m_1m_2\omega^4 - \left[k_1m_2 + k_2m_2 + k_2m_1\right]\omega^2 + k_1k_2 = 0$$

$$\underbrace{\ }_{a}\omega^4 - \underbrace{\left[\frac{k_1}{m_1} + \frac{k_2}{m_1} + \frac{k_2}{m_2}\right]}_{b}\omega^2 + \underbrace{\frac{k_1k_2}{m_1m_2}}_{c} = 0$$

$$\omega_{1,2}^2 = \frac{+b \mp \sqrt{b^2-4c}}{2} \Rightarrow \omega_1^2 = \quad \omega_2^2 =$$

let $k_1 = k_2 = k$, $m_1 = m_2 = m$

$$\therefore \omega_{12} = \frac{3k}{2m} \mp \sqrt{\left(\frac{3k}{2m}\right)^2 - \left(\frac{k}{m}\right)^2} = \frac{3}{2}\frac{k}{m} \mp \frac{\sqrt{5}}{2}\frac{k}{m}$$

$$\omega_{12}^2 = \frac{3\mp\sqrt{5}}{2}\ \frac{k}{m}$$

$\therefore \omega_1^2 = \frac{3-\sqrt{5}}{2} = 0.3825\frac{k}{m}$ fundimantal natural freq.

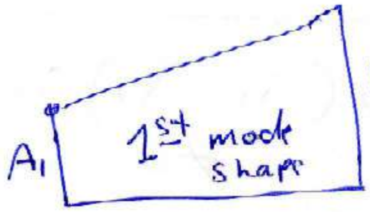
$$\omega_2 = \frac{3+\sqrt{5}}{2} = 2.6175\frac{k}{m}$$

Consider eq. 3 or eq. 4

$$(K_1+K_2-m_1\omega^2)A - K_2 B = 0$$

$$(2K - m\omega^2)A - KB = 0$$

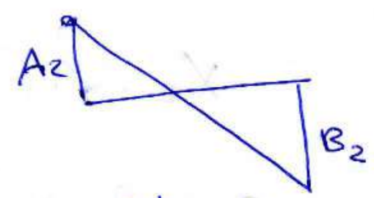
$$\frac{B}{A} = \frac{2K - m\omega^2}{K}$$



at $\omega = \omega_1$

if $\omega = \omega_1 = \sqrt{0.3825 \frac{K}{m}}$

$$\therefore \frac{B_1}{A_1} = \lambda_1 = \frac{2K - m\left(0.3825\frac{K}{m}\right)}{K} = 1.618$$



2nd mode shape

at $\omega = \omega_2$

if $\omega = \omega_2 = \sqrt{2.6175 \frac{K}{m}}$

$$\frac{B_2}{A_2} = \lambda_2 = \frac{2K - m\left(2.6175\frac{K}{m}\right)}{K}$$

$$\left.\frac{B}{A}\right)_2 = -0.618$$

The solution becomes

$$X_1 = A_1 \sin(\omega_1 t + \psi_1) + A_2 \sin(\omega_2 t + \psi_2) \quad (5)$$

$$X_2 = B_1 \sin(\omega_1 t + \psi_1) + B_2 \sin(\omega_2 t + \psi_2) \quad (6)$$

general solution of harmonic motion

substituted B_1, B_2 in eq. 6 Equation (6) becomes

$$X_2 = A_1\lambda_1 \sin(\omega_1 t + \psi_1) + A_2\lambda_2 \sin(\omega_2 t + \psi_2)$$

ψ_1, ψ_2 is the phase angle.

substituted I.C to find A_1, A_2, B_1, B_2

$X(0) = 1 \quad \dot{X}(0) = 0 \quad X_2(0) = \lambda_1 \quad \dot{X}(0) = 0$

$$[1 = A_1 \sin\psi_1 + A_2 \sin\psi_2 \quad]\,\lambda_1$$

$$\lambda_1 = A_1\lambda_1 \sin\psi_1 + A_2\lambda_1 \sin\psi_2$$

$$\lambda_1 = A_1\lambda_1 \sin\psi_1 + A_2\lambda_1 \sin\psi_2$$
$$\mp \lambda_1 = \mp A_1\lambda_1 \sin\psi_1 + A_2\lambda_2 \sin\psi_2$$

$$A_2\underbrace{(\lambda_1-\lambda_2)}_{\neq 0}\sin\psi_2 = 0 \qquad \text{either } A_2 = 0$$
$$\sin\psi_2 = 0 \Rightarrow \psi_2 = 0, \pi$$

$$\lambda_1 = A_1\lambda_1 \sin\psi_1$$

$$A_1 = \frac{1}{\sin\psi_1}$$

$$\therefore \quad X_1 = A_1 \sin(\omega_1 t + \psi_1) \qquad \dot{X}_1 = A_1\omega_1 \cos(\omega_1 t + \psi_1)$$
$$X_2 = A_1\lambda_1 \sin(\omega_1 t + \psi_1) \qquad \dot{X}_2 = A_1\lambda_1\omega_1 \cos(\omega_1 t + \psi_1)$$

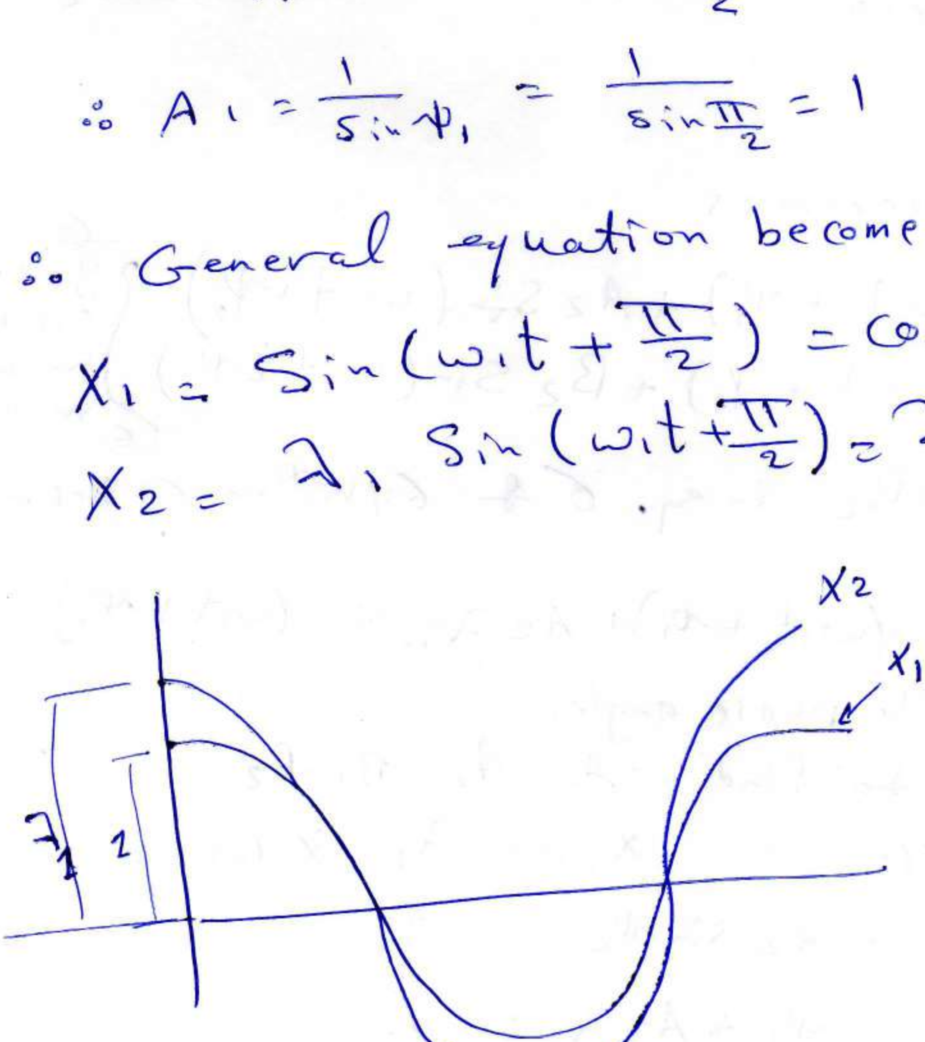
$$0 = A_1\omega_1 \cos\psi_1$$

$$\cos\psi_1 = 0 \qquad \psi_1 = \frac{\pi}{2}$$

$$\therefore A_1 = \frac{1}{\sin\psi_1} = \frac{1}{\sin\frac{\pi}{2}} = 1$$

$\therefore$ General equation become

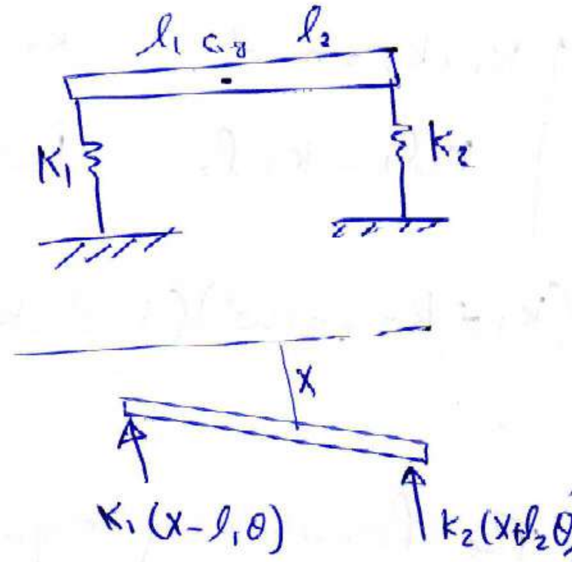
$$X_1 = \sin(\omega_1 t + \tfrac{\pi}{2}) = \cos\omega_1 t$$
$$X_2 = \lambda_1 \sin(\omega_1 t + \tfrac{\pi}{2}) = \lambda_1 \cos\omega_1 t$$



Coordinate couplings:

$\Sigma F = m\ddot{x}$

$-k_1(x - l_1\theta) - k_2(x + l_2\theta) = m\ddot{x}$



$m\ddot{x} + (k_1+k_2)x + (k_2 l_2 - k_1 l_1)\theta = 0$ — (1)

eq. of motio

$\Sigma M = I\ddot{\theta}$

$-k_2(x + l_2\theta)\cdot l_2 + k_1(x - l_1\theta)\cdot l_1 = I\ddot{\theta}$

$I\ddot{\theta} + k_2 l_2 x + k_2 l_2^2\theta - k_1 l_1 x + k_1 l_1^2\theta = 0$

$I\ddot{\theta} + (k_2 l_2 - k_1 l)x + (k_2 l^2 + k_1 l_1^2)\theta = 0$ — (2)

if $k_1 = k_2$ it's vibrate as a one degree of freedom

Assume

$x = A\sin(\omega t + \psi)$, $\ddot{x} = -A\omega^2\sin(\omega t + \psi)$

$\theta = B\sin(\omega t + \psi)$ $\ddot{\theta} = -B\omega^2\sin(\omega t + \psi)$

subsituted the above equation into 1 & 2 then Simplification we end up with:

$$\Big[\underbrace{-mA\omega^2 + (K_1+K_2)A + (K_2 l_2 - K_1 l_1)B}_{=0}\Big]\underbrace{\sin(\omega t+\psi)}_{\neq 0} = 0$$

$(K_1+K_2)A - m\omega^2 A + (K_2 l_2 - K_1 l_1)B = 0$ — (3)

$-I\omega^2 B + (K_2 l_2 - K_1 l_1)A + (K_2 l_2^2 + k_1 l_1^2)B = 0$

$\left[I\omega^2 - (K_2 l_2^2 + K_1 l_1^2)\right]B - (K_2 l_2 - K_1 l_1)A = 0$ — (4)

$$\begin{bmatrix} (K_1+K_2-m\omega^2) & K_2 l_2 - K_1 l_1 \\ -(K_2 l_2 - k_1 l_1) & I\omega^2 - (K_2 l_2^2 + K_1 l_1^2) \end{bmatrix}\begin{Bmatrix} A \\ B \end{Bmatrix} = 0$$

for non-trivial Solution

$$\begin{vmatrix} K_1+K_2-m\omega^2 & K_2 l_2-K_1 l_1 \\ K_1 l_1-K_2 l_2 & I\omega^2-K_2 l_2-K_1 l_1^2 \end{vmatrix} = 0$$ frequency determinant.

$$\underbrace{(K_1+K_2-m\omega^2)(I\omega^2-K_2 l_2^2-K_1 l_1^2)-(K_2 l_2-K_1 l_1)(K_1 l_1-K_2 l_2)=0}_{\text{frequency equation.}}$$

The frequeny equation will gives ω_1 & ω_2

$\therefore$ General Solution becoms

$$X = A_1 \sin(\omega_1 t+\psi_1) + A_2 \sin(\omega_2 t+\psi_2)$$
$$\theta = B_1 \sin(\omega_1 t+\psi_1) + B_2 \sin(\omega_2 t+\psi_2)$$

Apply I.C gives A_1, A_2, ψ_1 & ψ_2

<u>mode Shapes:-</u>

Reconsiders eq. 3 & 4

$$A\left[K_1+K_2-m\omega^2\right]+B(K_2 l_2-K_1 l_1)=0$$

$$\frac{B}{A} = \frac{K_1+K_2-m\omega^2}{K_1 l_1-K_2 l_2} = \lambda_1$$

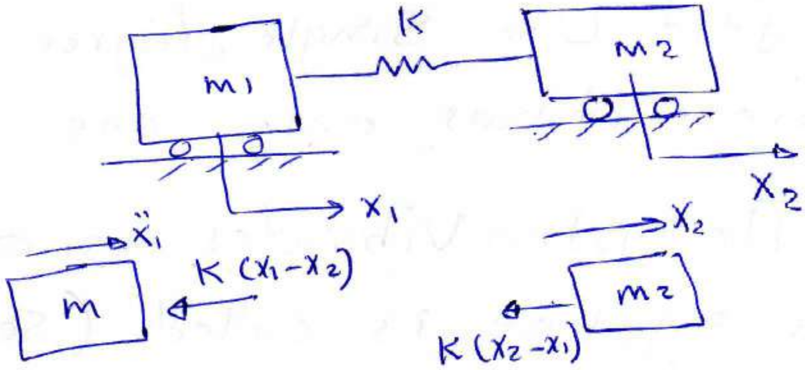
put $\omega = \omega_1$ (small one)

$\therefore \frac{B_1}{A_1} = \lambda_1$ (+ve) 1st mode shape

put $\omega = \omega_2$

$\frac{B_2}{A_2} = \lambda_2$ (-ve) 2nd mode shape.

Semi-definit system:



$\Sigma F = m_1 \ddot{x}$

$-K(x_1 - x_2) = m_1 \ddot{x}$

$m_1\ddot{x} + Kx_1 - Kx_2 = 0$ —— (1) eq. of motion of mass m_1

$\Sigma F = m_2 \ddot{x}_2$

$-K(x_2 - x_1) = m_2 \ddot{x}_2$

$m_2\ddot{x}_2 + Kx_2 - Kx_1 = 0$ —— (2) eq. of motion of mass m_2

let $x_1 = A\sin(\omega t + \psi)$ $\quad \ddot{x}_1 = -A\omega^2 \sin(\omega t + \psi)$

$x_2 = B\sin(\omega t + \psi)$ $\quad \ddot{x}_2 = -B\omega^2 \sin(\omega t + \psi)$

substituted x_1, x_2, $\ddot{x}_1$ & $\ddot{x}_2$ in eq. 1 & 2 then after simplification we end up with:

$(K - m_1\omega^2)A - KB = 0$ —— (3)

$(K - m_2\omega^2)B - KA = 0$ —— (4)

$$\begin{bmatrix} K - m_1\omega^2 & -K \\ -K & K - m_2\omega^2 \end{bmatrix} \begin{Bmatrix} A \\ B \end{Bmatrix} = 0$$

$$\begin{vmatrix} K - m_1\omega^2 & -K \\ -K & K - m_2\omega^2 \end{vmatrix} = 0 \qquad (K - m_1\omega^2)(K - m_2\omega^2) - K^2 = 0$$

$$\cancel{K^2} - Km_2\omega^2 - Km_1\omega^2 + m_1 m_2\omega^4 - \cancel{K^2} = 0$$

$\therefore \omega^2\left(m_1 m_2\omega^2 - (Km_2 + Km_1)\right) = 0$

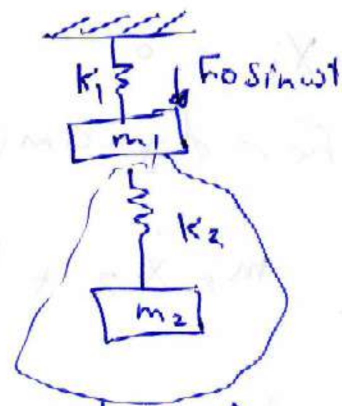
either $\omega_1 = 0$

$\omega_2 = \sqrt{Km_2 + Km_1}$

Even the system is two degree of freedom it behave just Like Single degree of freedom system. Since it has only one natural frequeny. The system Vibrates as one Complete unit such system is called (Semi-definit system).

Dynamic absorber

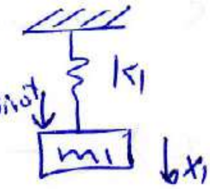
befor dynamic absorber is used



dynamic absorber

$$\Sigma F = m_1 \ddot{x}_1$$

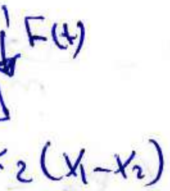
$$m_1 \ddot{x}_1 + k_1 x_1 = F_o \sin \omega t$$



$$\omega_{n_1} = \sqrt{\frac{k_1}{m_1}}$$

if we use dynamic absorber

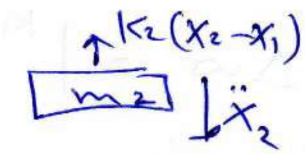
$$\Sigma F = m_1 \ddot{x}_1$$



$$m_1 \ddot{x}_1 = F_o \sin \omega t - k_1 x_1 - k_2(x_1 - x_2)$$

$$m_1 \ddot{x}_1 + (k_1 + k_2) x_1 - k_2 x_2 = F_o \sin \omega t \quad — (1)$$

$$\Sigma F = m_2 \ddot{x}_2$$



$$m_2 \ddot{x}_2 + k_2 x_2 - k_2 x_1 = 0 \quad — (2)$$

Assum $x_1 = A \sin \omega t$ $\qquad x_2 = B \sin \omega t$

$\ddot{x}_1 = -A\omega^2 \sin \omega t \qquad \ddot{x}_2 = -B\omega^2 \sin \omega t$

substituted the above eq. in eq 1 & 2

then take only the coefficiont of $\sin \omega t$

$$-m_1 \omega^2 A + (k_1 + k_2) A - k_2 B = F_o \quad — (3)$$

$$-k_2 A + (k_2 - m_2 \omega^2) B = 0 \quad — (4)$$

use Grummerl's rule to find A & B

$$A = \frac{\begin{vmatrix} F_o & k_2 \\ 0 & k_2 - m_2 \omega^2 \end{vmatrix}}{\begin{vmatrix} k_1 + k_2 - m_1 \omega^2 & -k_2 \\ -k_2 & k_2 - m_2 \omega^2 \end{vmatrix}} = \frac{F_o (k_2 - m_2 \omega^2)}{D\omega}$$

$$B = \frac{\begin{vmatrix} k_1 + k_2 - m_1 \omega^2 & F_o \\ -k_2 & 0 \end{vmatrix}}{\quad} = \frac{F_o k_2}{D\omega}$$

if the mass m_1 stopped of moving then

$X_1 = 0$

For dynamic absorber eq. 2 becoms

$$m_2 \ddot{X}_2 + K_2 X_2 - \underbrace{\boxed{K_2 X_1}}_{=0} = 0$$

$$\therefore \omega_n = \frac{K_2}{m_2}$$

subs $\omega^2 = \frac{K_2}{m_2}$ to Find A & B

$$\therefore A = \frac{F_o \left(K_2 - m_2 \frac{K_2}{m_2}\right)}{\Delta\omega} = 0$$

$$\Delta\omega = \begin{vmatrix} K_1 + K_2 - m_1\omega^2 & -K_2 \\ -K_2 & K_2 - m_2\omega^2 \end{vmatrix} = -K_2^2$$

$$B = \frac{F_o K_2}{-K_2^2} = \frac{-F_o}{K_2}$$

$$\therefore X_1 = 0 \quad X_2 = B \sin\omega t = \frac{-F_o}{K_2} \sin\omega t$$

Thus we may write for the case of the ~~dyn~~ dynamic absorber $\boxed{\frac{K_1}{m_1} = \frac{K_2}{m_2}}$

$$\begin{vmatrix} K_1 + K_2 - m_1\omega^2 & -K_2 \\ -K_2 & K_2 - m_2\omega^2 \end{vmatrix} = 0 \quad \text{freq. det}$$

$$(K_1 + K_2 - m_1\omega^2)(K_2 - m_2\omega^2) - K_2^2 = 0$$

$$K_1 K_2 + K_2^2 - K_2 m_1 \omega^2 - K_1 m_2 \omega^2 - K_2 m_2 \omega^2 + m_1 m_2 \omega^4 - K_2^2 = 0$$

$$\frac{m_1 m_2 \omega^4}{K_1 K_2} - \left[\frac{m_2}{K_2} + \frac{m_2}{K_1} + \frac{m_1}{K_1}\right]\omega^2 + 1 = 0$$

$$\omega_n^2 = \frac{K_1}{} \quad \text{or} \quad \omega_n^2 = \frac{K_2}{}$$

$$\frac{\omega^4}{\omega_n^4} - \left[\frac{1}{\omega_n^2} + \frac{1}{\omega_n^2}\frac{k_2}{k_1} + \frac{1}{\omega_n^2}\right]\omega^2 + 1 = 0$$

let $\omega / \omega_n = r$

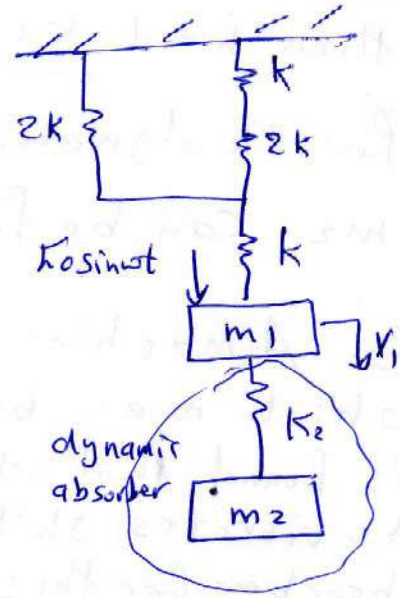
$$\boxed{r^4 - \left(2 + \frac{k_2}{k_1}\right) r^2 + 1 = 0}$$

From this eq. we can find $\frac{k_2}{k_1}$ and then find k_2.

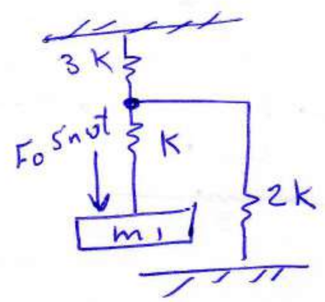
for the dynamic absorber take $\frac{k_1}{m_1} = \frac{k_2}{m_2}$ then m_2 can be found.

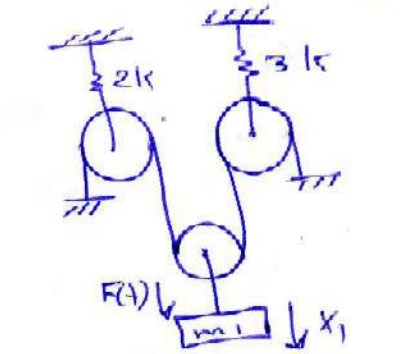
Ex: A machine of (100) kg is replaced on a foundation which may be represented by spring of $k = 100$ kN/m it's found that when the machine runs at (300) rpm its vibrates slolly. It's required to design a dynamic absorber for this requires system. find the characteristic of vibration absorber.

Ex: the modal of reciprocating machin and the supporting fondation shown in fig. of mass m runs at a constant speed of (3000 rpm). After installating forcing freq. found to be to closed to the natural frequency of the ~~mass~~ system. Design a dynamic absorber and find the characteristics of the dynamic absorber.
Given that $K = 6.9 \times 10^6$ N/m $\quad \omega_n = 448.6$ rad/s



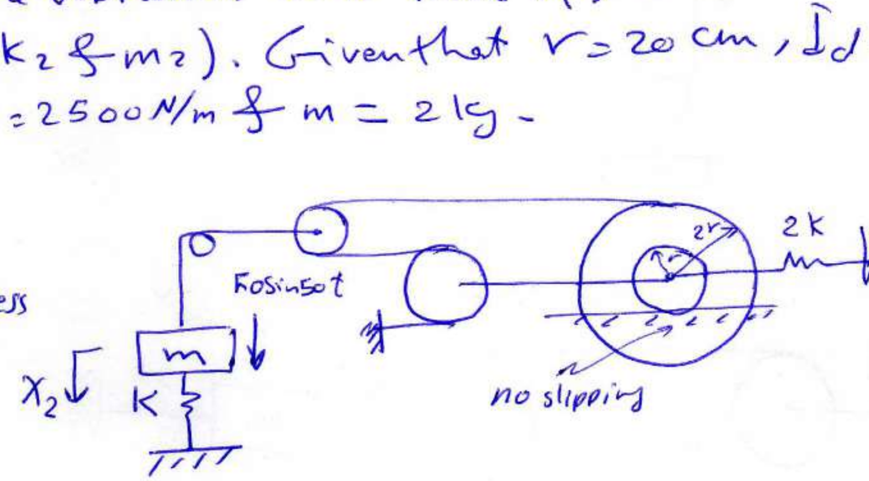
EX3 Design a dynamic absorber to absorb a vibration of fig. Give that $F(t) = 0.1 \sin 75t$, $K = 10 \, kN/m$ & $m_1 = 4$ kg



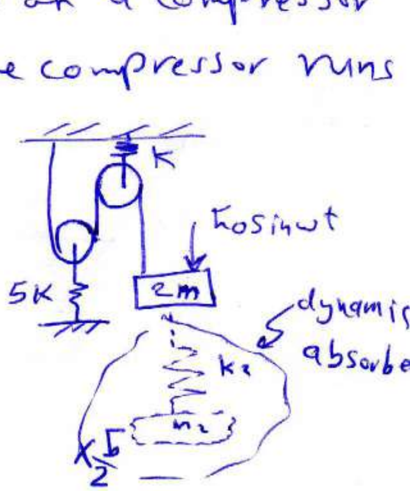


EX4 The system shown vibrat violently due to the force $F_o \sin 50t$ which is acting on mass m. Design a dynamic absorber to be absorbe the vibration and find it's characteristic properties (k_2 & m_2). Given that $r = 20$ cm, $I_d = 4 m_d r^2$ $m_d = 6$ kg, $K = 2500$ N/m & $m = 2$ kg.

All pulleys are massless & frictionless

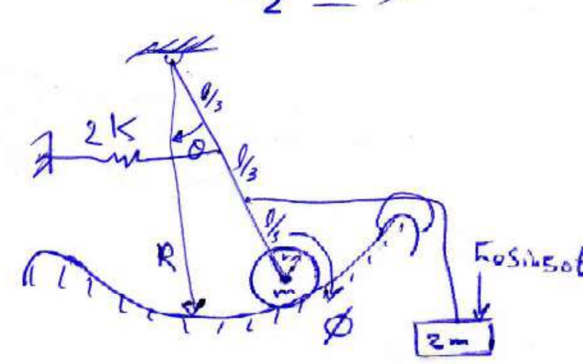


EX5 A figure represented a mathematical model of a compressor of mass (2m) and it's supporting structure. The compressor runs at a constant speed of 1500 r.p.m. after installation the forcing frequency is found too close to the natural frequency of the system. Design a dynamic absorber to absorb the vibration of the compressor. $K = 10^6$ N/m & $m = 5$ kg.



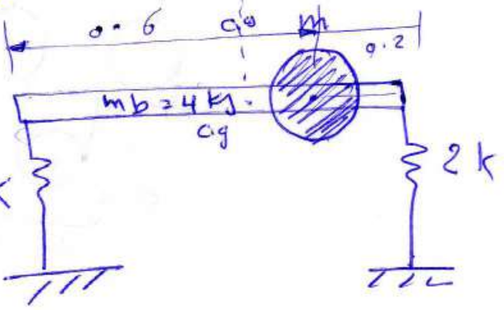
EX6 Design a dynamic absorber in figures

$R\theta = r\phi$
$R = 3r$

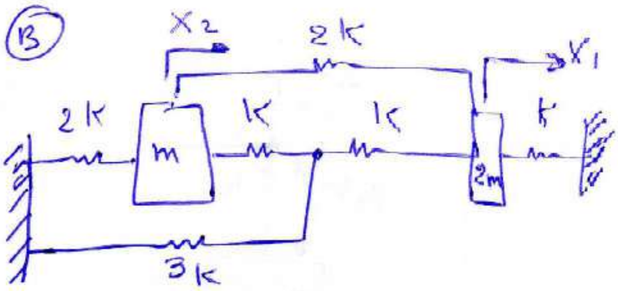


Ex 7 Find the natural frequencies & their associated mode shapes.

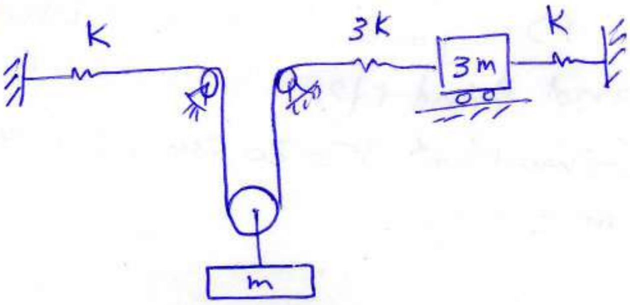
(A)



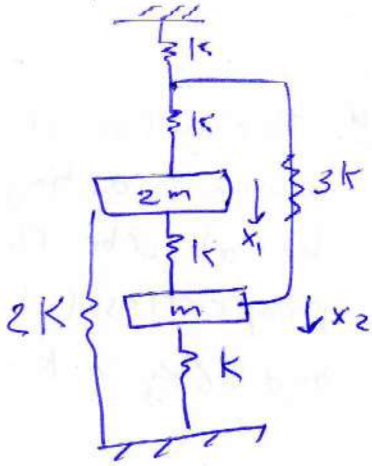
(B)



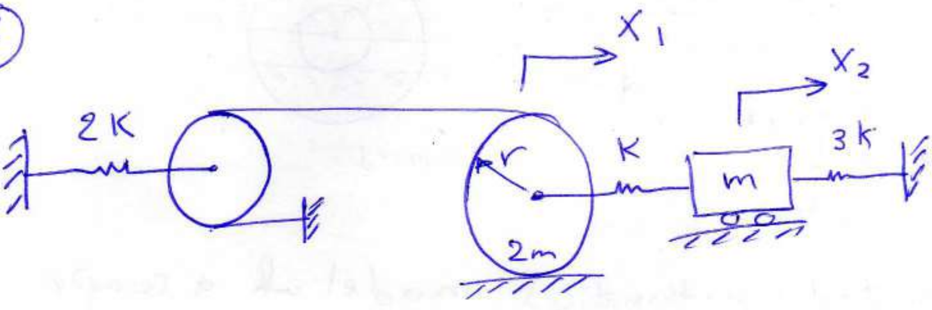
(C)



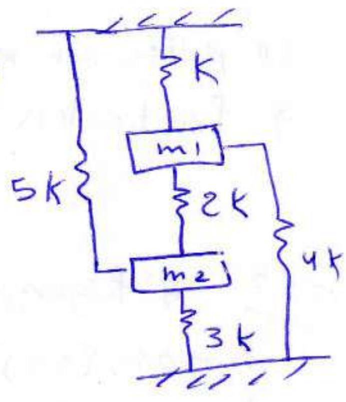
(D)



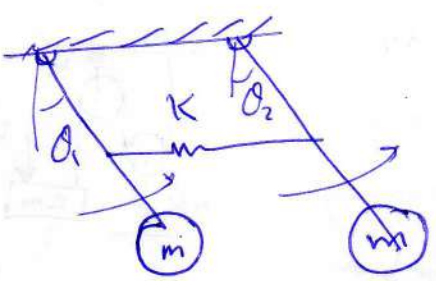
(E)



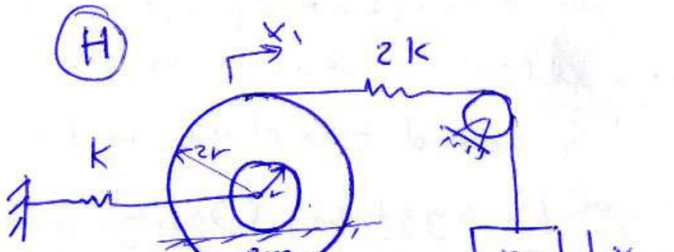
(F)



(G)



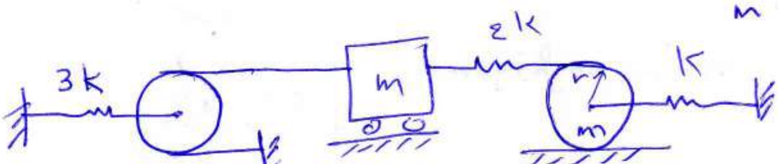
(H)



$r = 20 \quad K = 1\ KN/m \quad I_d = 0.32\ kg \cdot m^2$

$m = 4\ kg$.

(I)



(J)