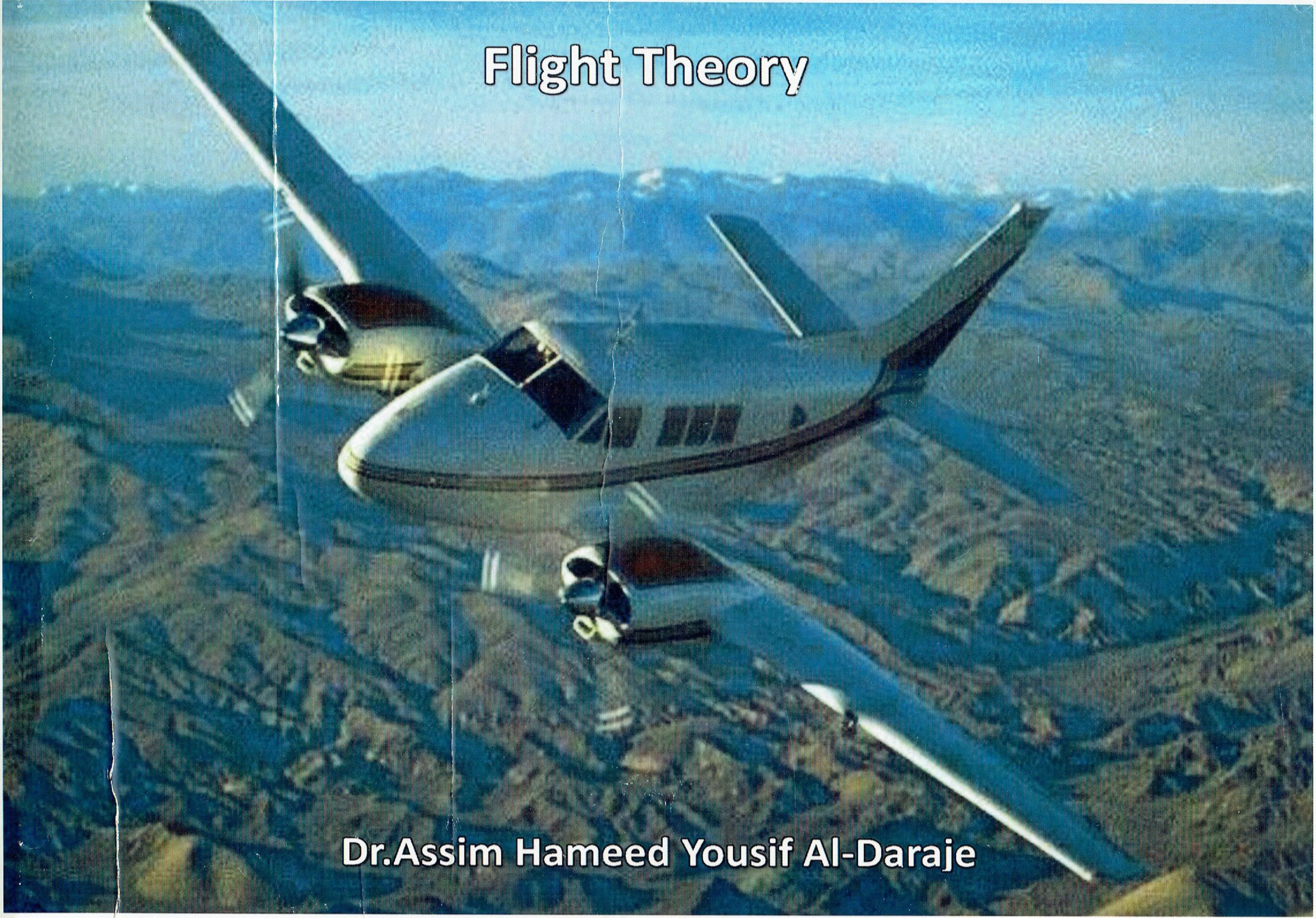
Flight Theory

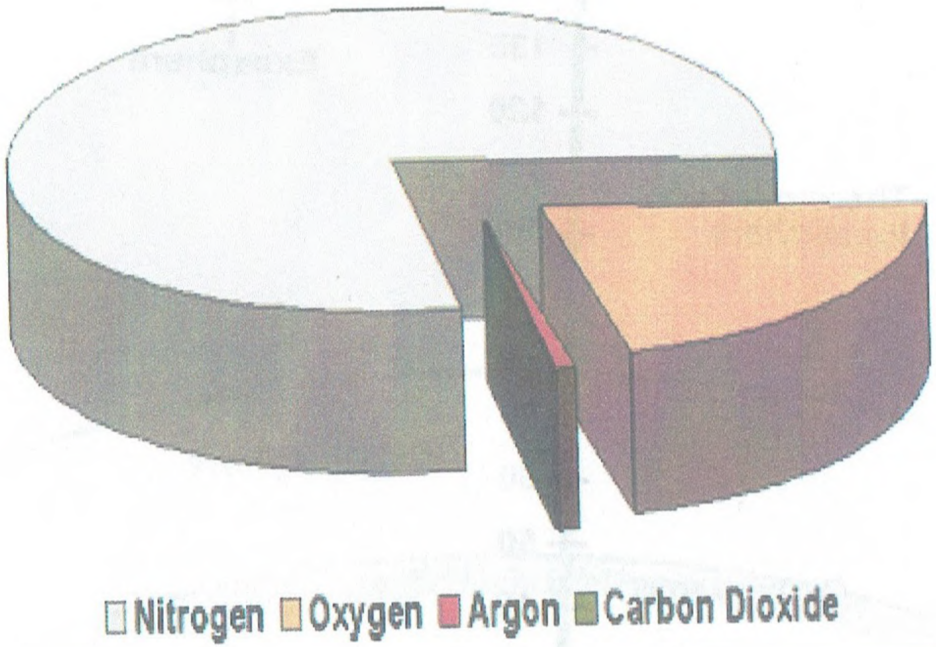


Dr.Assim Hameed Yousif Al-Daraje

Part I: Aerodynamics concept

Chapter (1)
The Atmosphere

Composition of Air

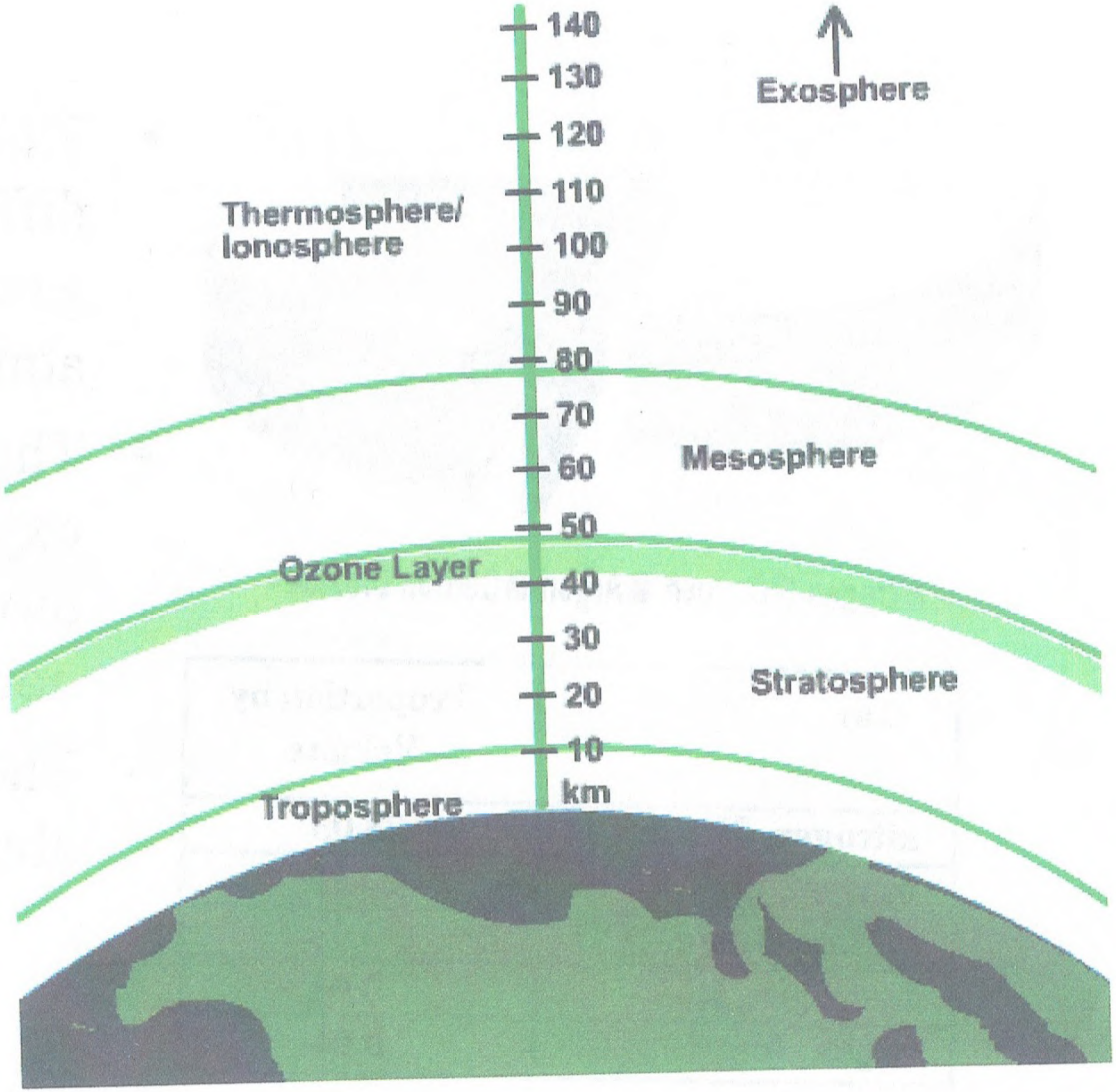


Gas	Proportion by Volume
nitrogen, N	78.03
oxygen, O	20.99
carbon dioxide, CO	0.03
hydrogen, H	0.01
argon, Ar	0.94

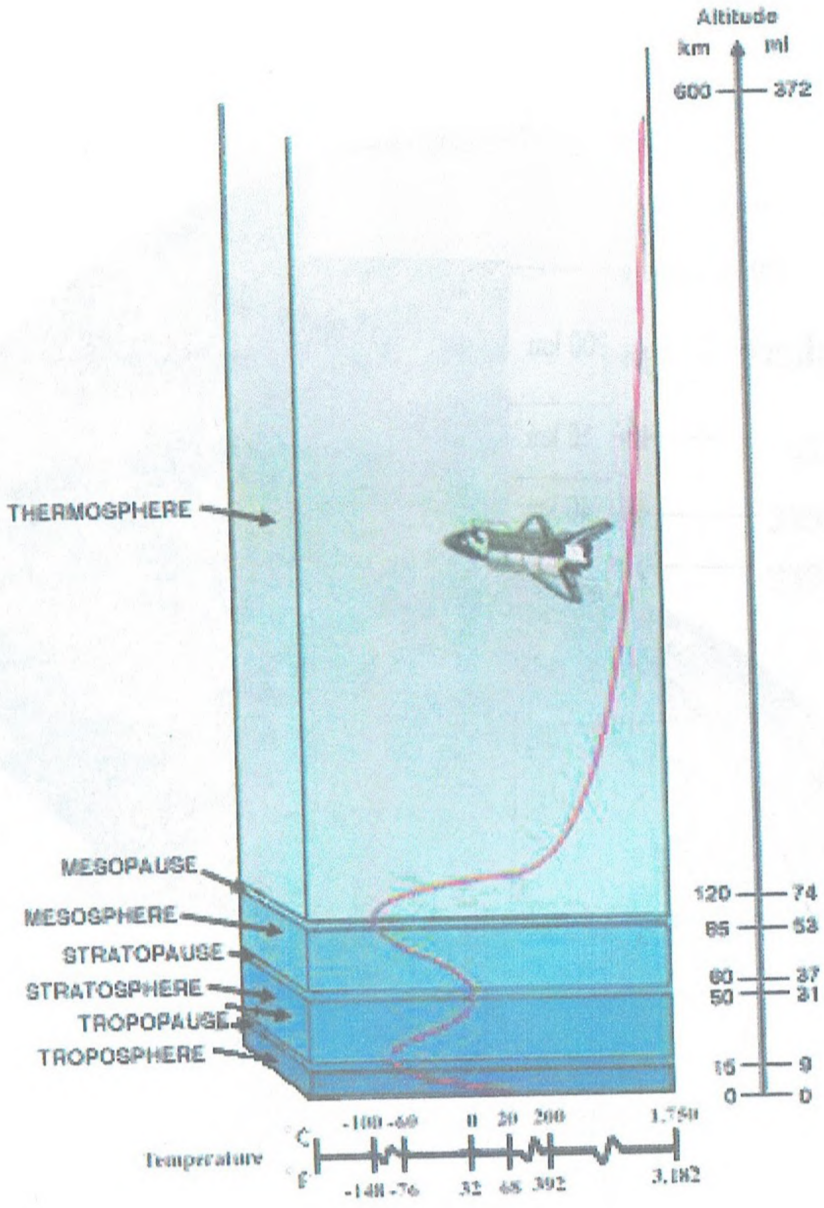
- There are many different types of gasses in the atmosphere
- They include nitrogen, oxygen, argon, carbon dioxide and other noble gasses
- The gas that is most abundant is nitrogen

Atmosphere

- There are 4 layers in the atmosphere
- They are the troposphere, mesosphere, thermosphere, and stratosphere



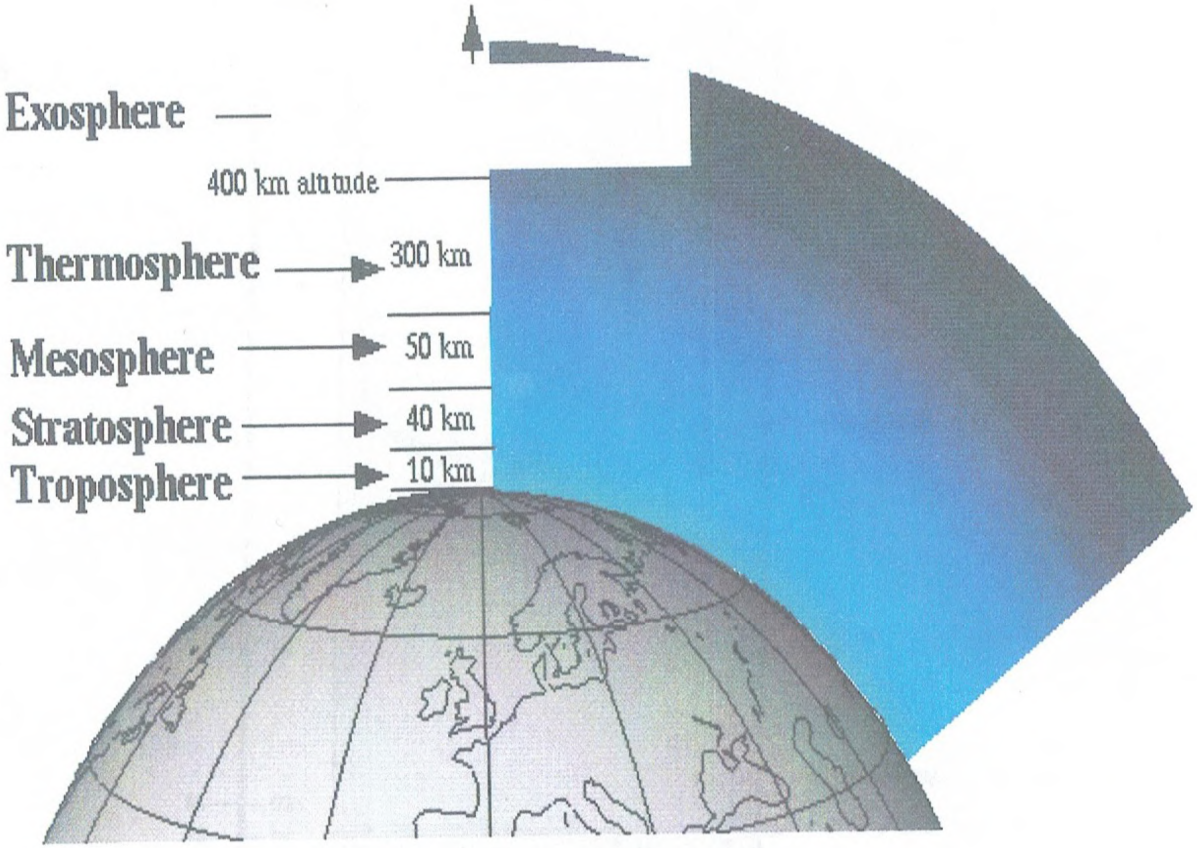
Troposphere



- This is the layer that is closest to the surface of the earth
- It's elevation ranges from 0 to 10 km

Stratosphere

- This layer sits on top of the troposphere
- It's elevation ranges from 10 km to around 25 km
- This layer contains the ozone layer, which protects us from harmful sunlight



Earth Atmosphere Model

Metric Units

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For h > 25000 (Upper Stratosphere)

$$T = -131.21 + .00299\, h$$

$$p = 2.488 * \left[\frac{T + 273.1}{216.6} \right]^{-11.388}$$

For 11000 < h < 25000 (Lower Stratosphere)

$$T = -56.46$$

$$p = 22.65 * e^{(1.73 - .000157\, h)}$$

For h < 11000 (Troposphere)

$$T = 15.04 - .00649\, h$$

$$p = 101.29 * \left[\frac{T + 273.1}{288.08} \right]^{5.256}$$

ρ = density (kg/cu m)

p = pressure (K-Pa)

$$\rho = p / (.2869 * (T + 273.1))$$

T = temperature (^{0}C)

h = altitude (m)

Air Properties Definitions

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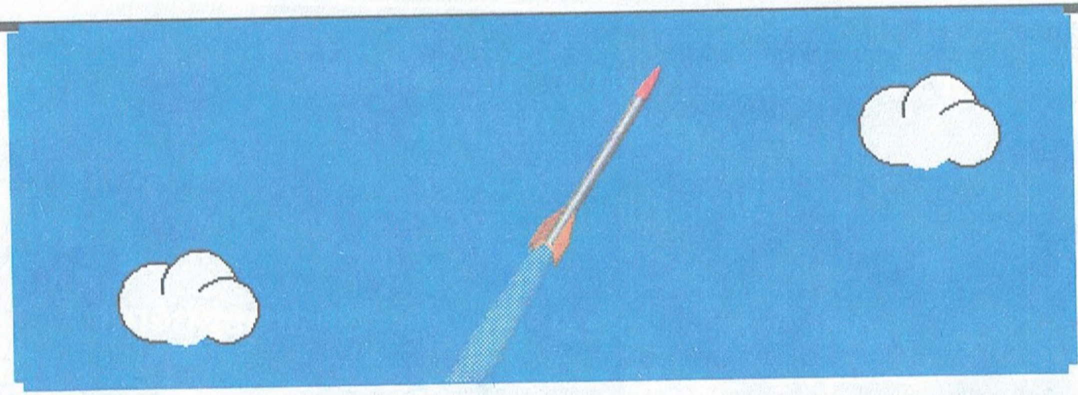

Air is a Gas. 78% Nitrogen, 21% Oxygen, traces H_2O, CO_2, Ar, ..

Property	Dimensions	Value (SLS *) Metric	Value (SLS *) Imperial
Mass, Volume			
Density (r)	mass/volume	1.229 kg/m^3	.00237 slug/ft^3
Specific Volume (v)	volume/mass	.814 m^3/kg	422 ft^3/slug
Pressure (p)	force/area	101.3 kN/m^2	14.7 lb/in^2
Temperature (T)	degrees	15 ^oC	59 ^oF
Viscosity (mu)	force–time/area	1.73×10^{-5} N-s/m^2	3.62×10^{-7} lb-s/ft^2

* Sea Level Static (Standard Day)

Similarity Parameters

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	Viscosity	Compressibility
Characteristic	"Stickiness"	"Springiness"
Parameter	Reynolds (Re)	Mach (M)
Definition	$\frac{\text{density} \times \text{velocity} \times \text{length}}{\text{viscosity coefficient}}$	$\frac{\text{flow velocity}}{\text{speed of sound}}$
Equation	$\frac{\rho \times V \times L}{\mu}$	$\frac{V}{a}$

Aerodynamic Forces depend on Re and M

For a valid experiment, Reynolds Number and Mach Number must match flight conditions.

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Mach Number

$$\text{ratio} = \frac{\text{Object Speed}}{\text{Speed of Sound}} = \text{Mach Number}$$

Hypersonic
Mach > 5.0

Supersonic
Mach > 1.0

Transonic
Mach = 1.0

Subsonic
Mach < 1.0

Airplane Parts Definitions
and Function

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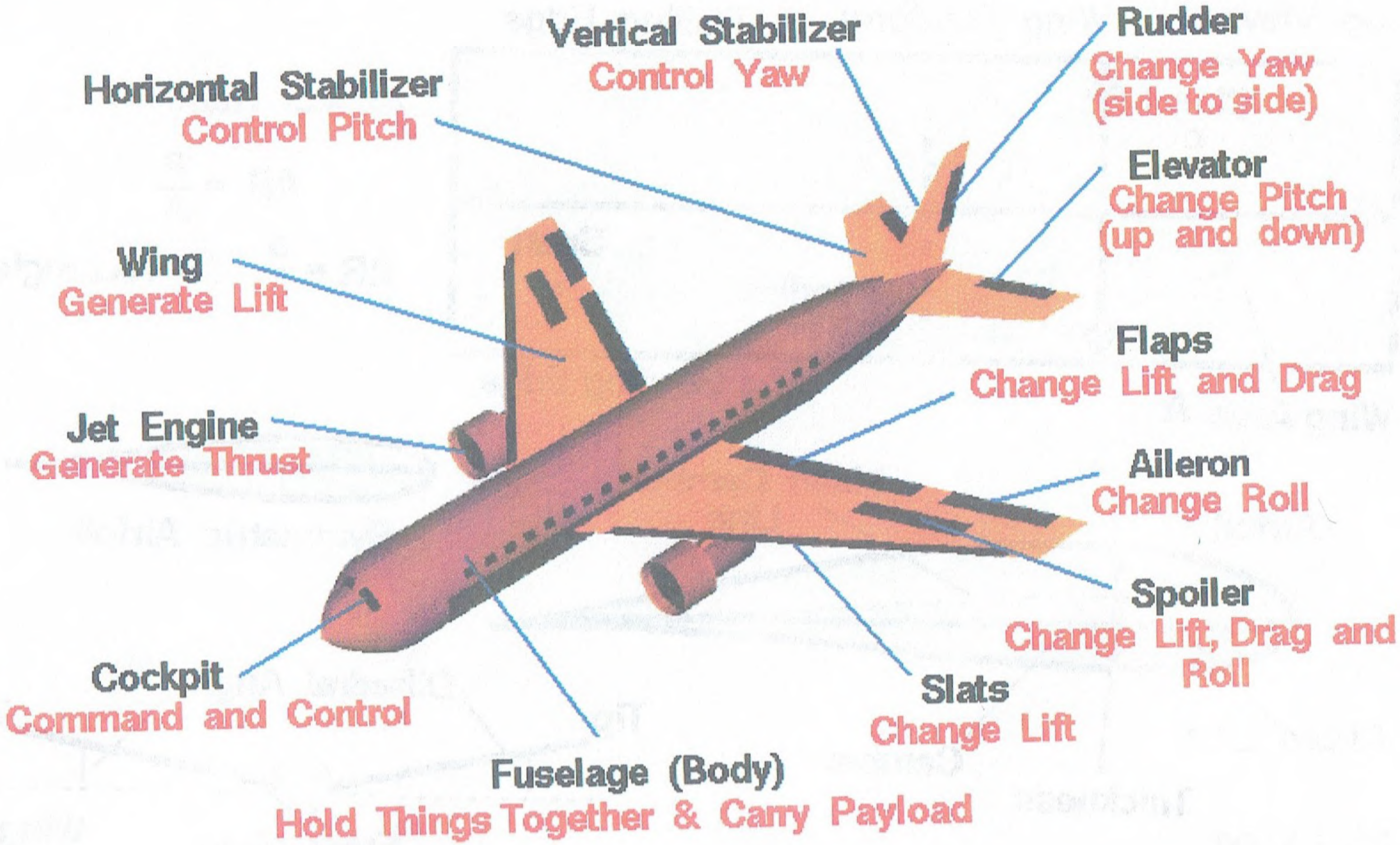

Wing Geometry Definitions

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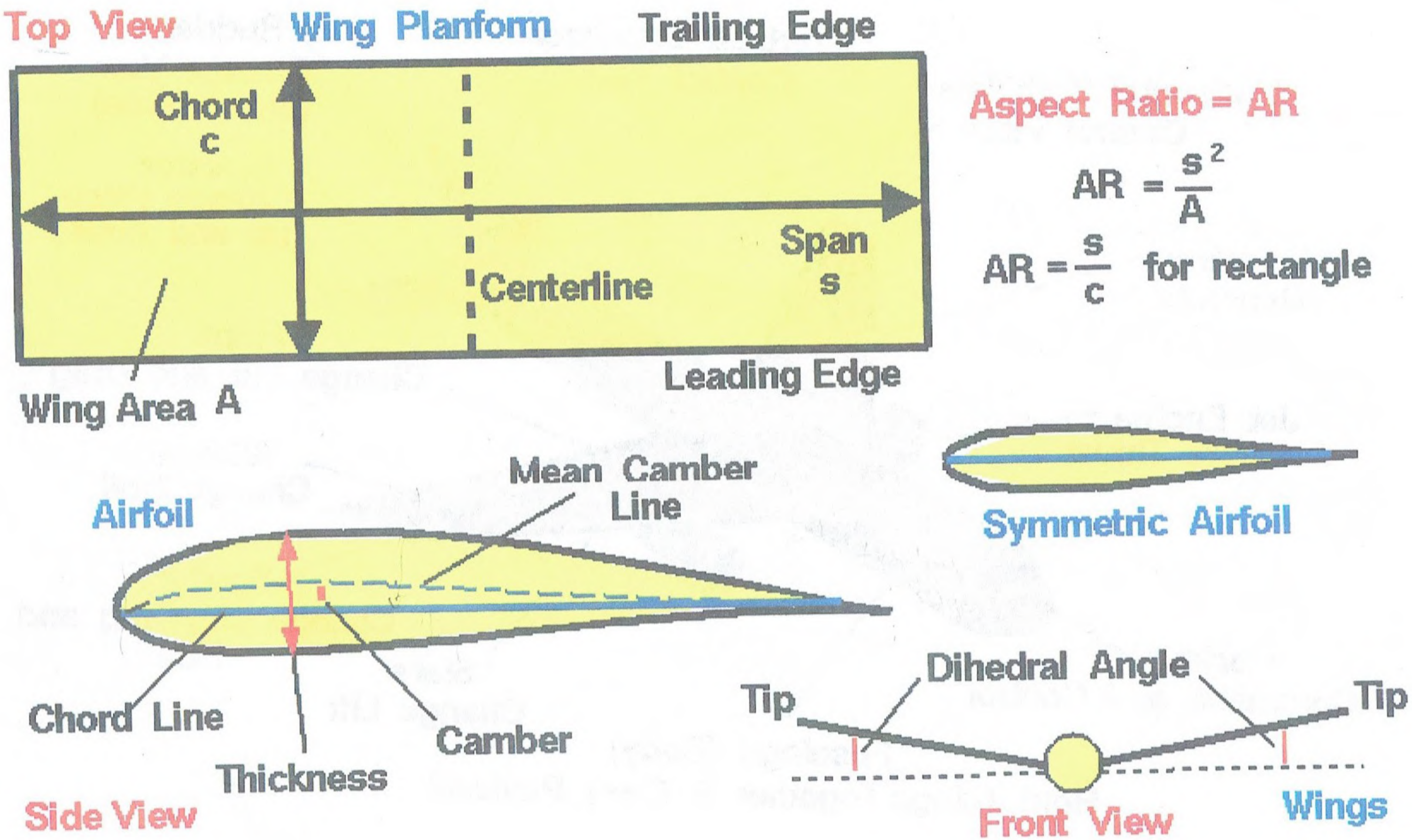
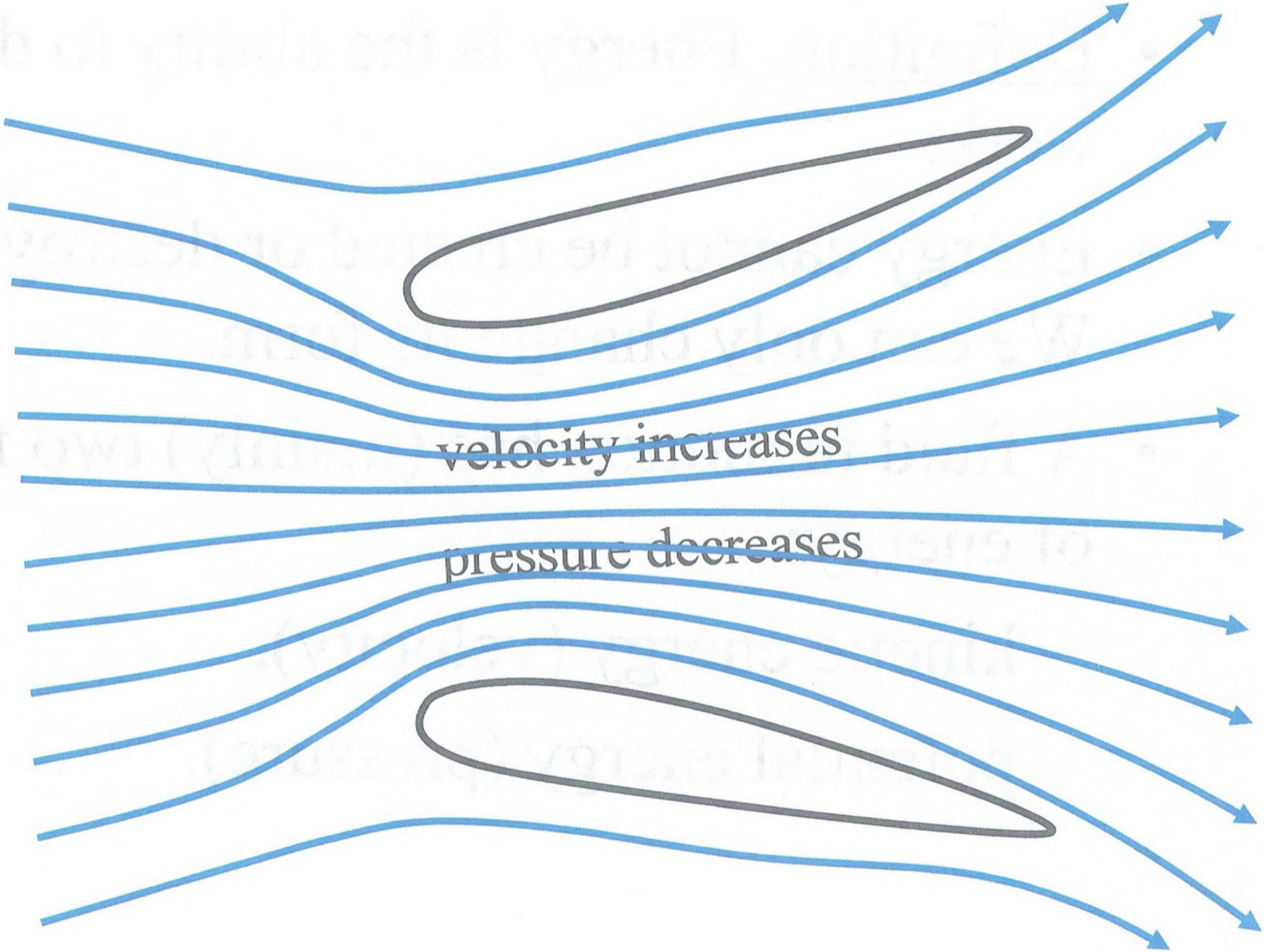


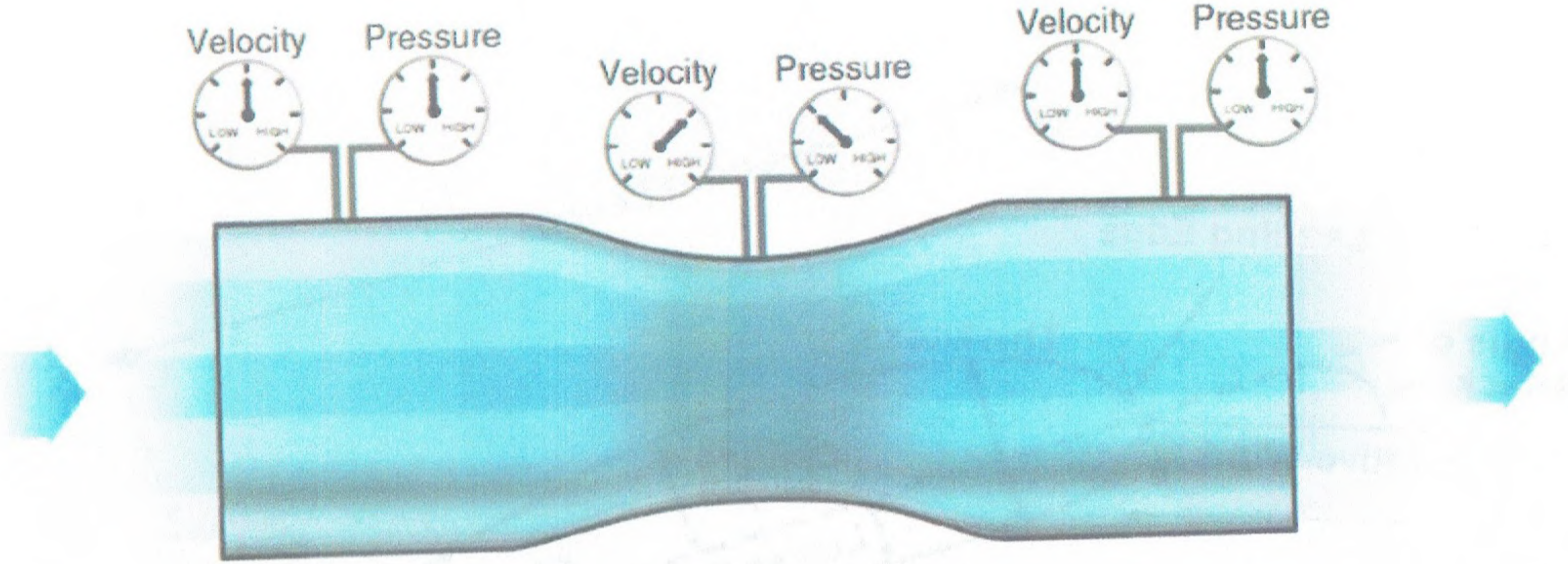
Chapter (3) Energy, Lift and Drag

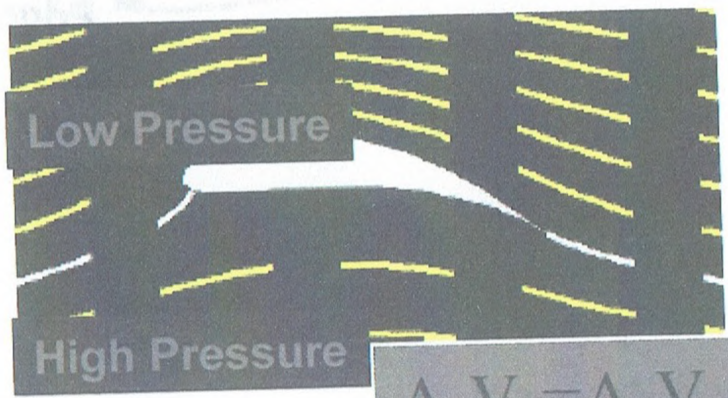
- Definition: Energy is the ability to do work.
- Energy cannot be created or destroyed. We can only change its form.
- A fluid in motion has (mainly) two forms of energy:
  - – kinetic energy (velocity),
  - – potential energy (pressure).

The Venturi Tube and Bernoulli's Principle



Bernoulli's Principle - Lift

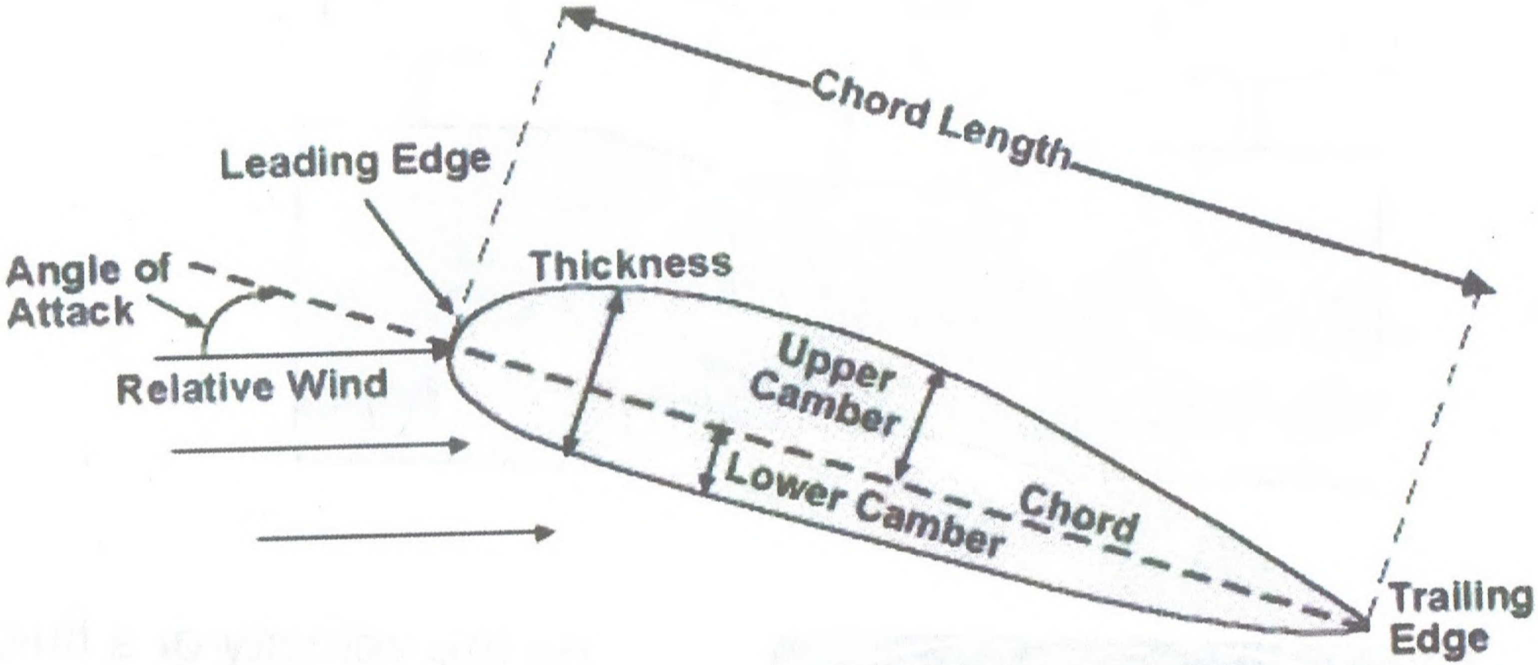




$$A_1V_1=A_2V_2$$

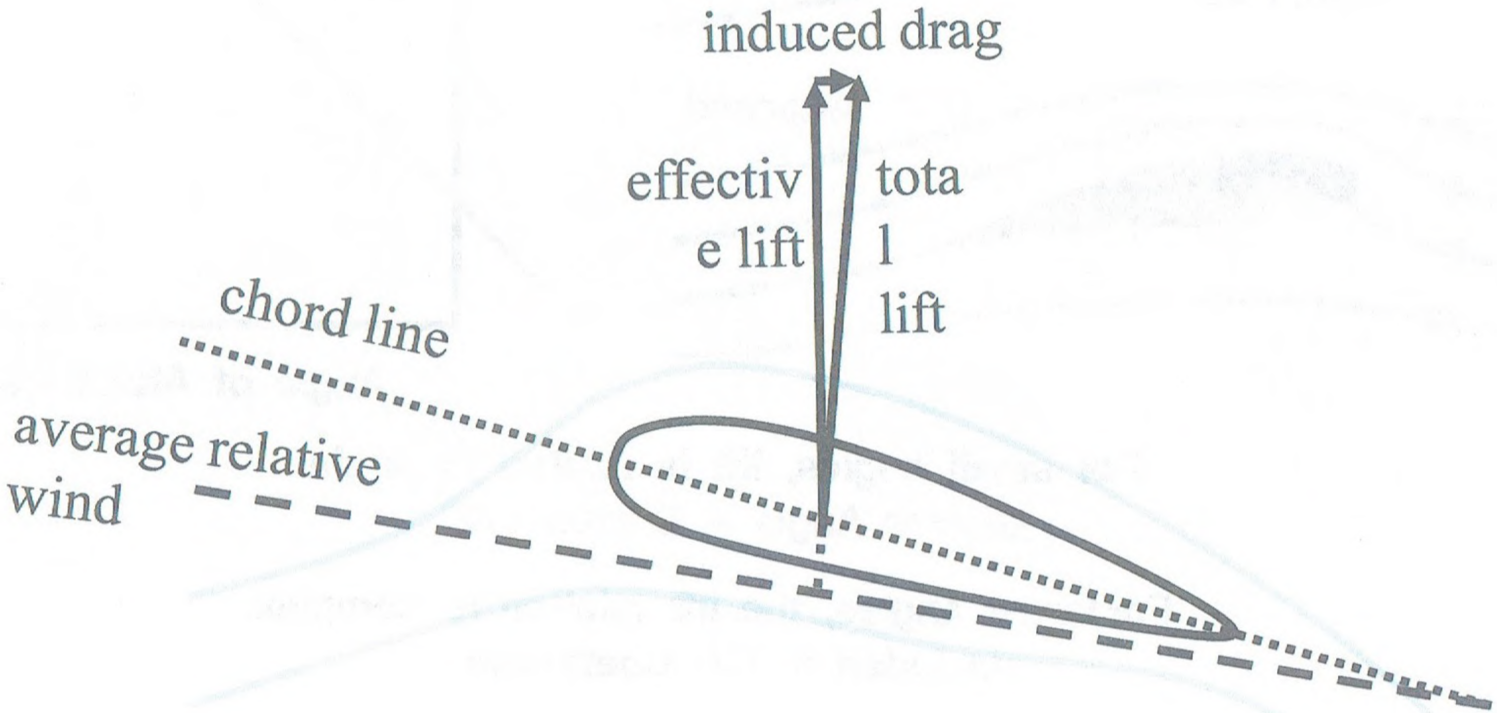
- "As the velocity of a fluid increases, its internal pressure decreases."
  - From Newton's 2nd (F=ma)
  - Shown by Venturi tube

Airfoil Nomenclature

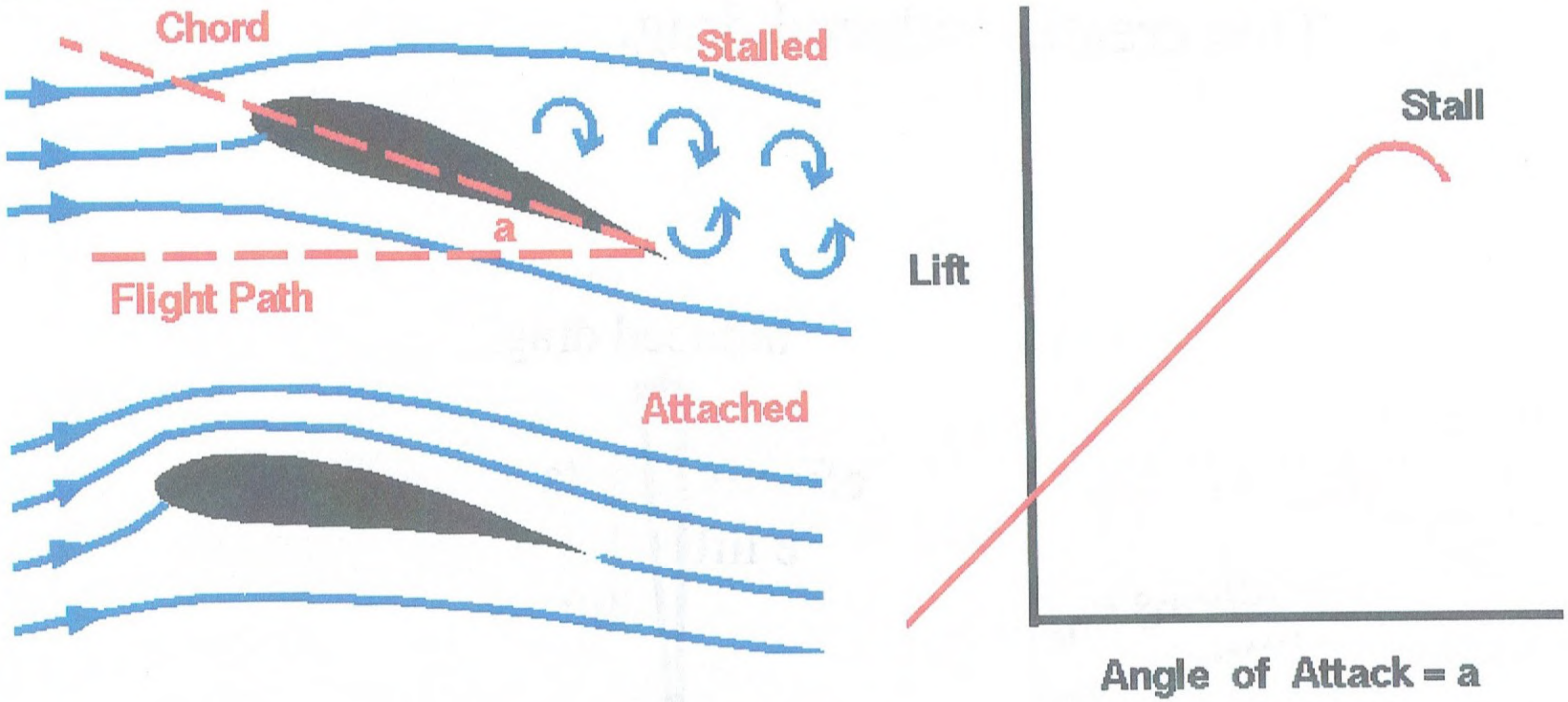


Lift and Induced Drag

- Lift acts through the center of pressure, and perpendicular to the relative wind.
- This creates induced drag.



Inclination Effects on Lift



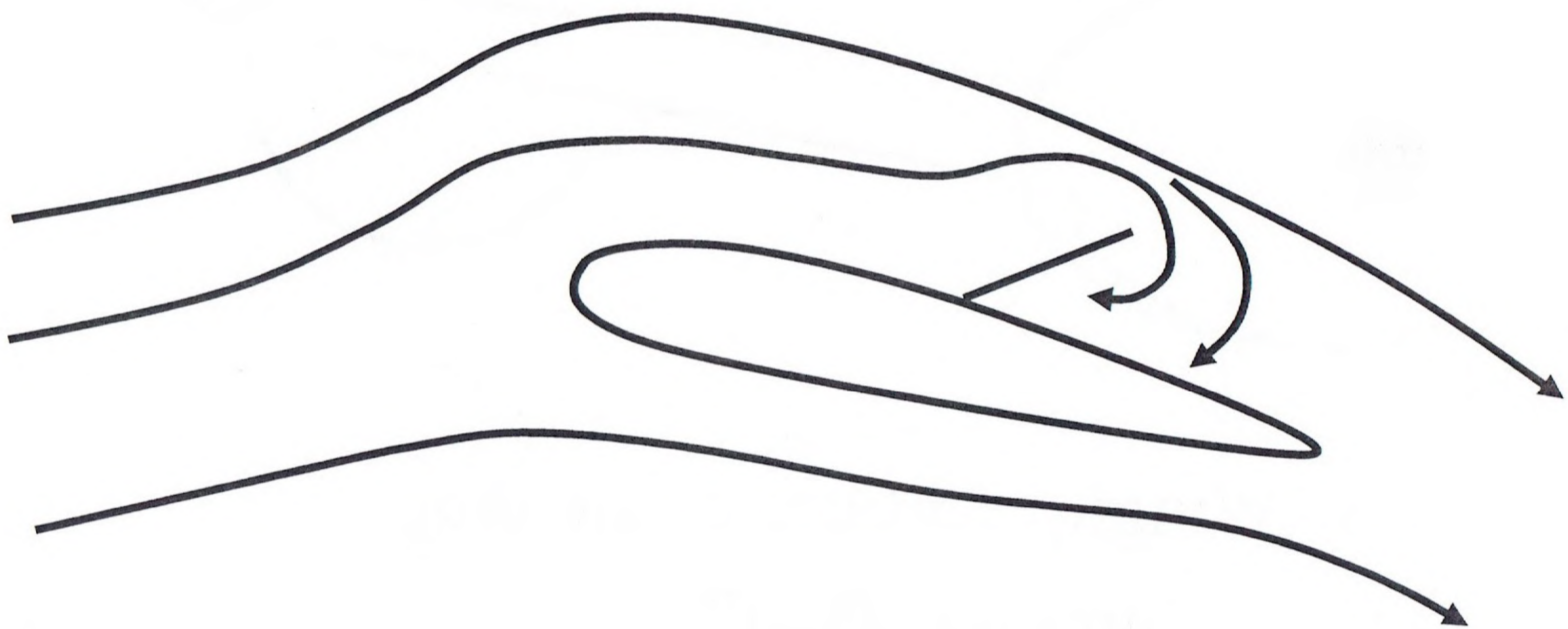
For small angles, lift is related to angle.
Greater Angle = Greater Lift

For larger angles, the lift relation is complex.
Included in Lift Coefficient

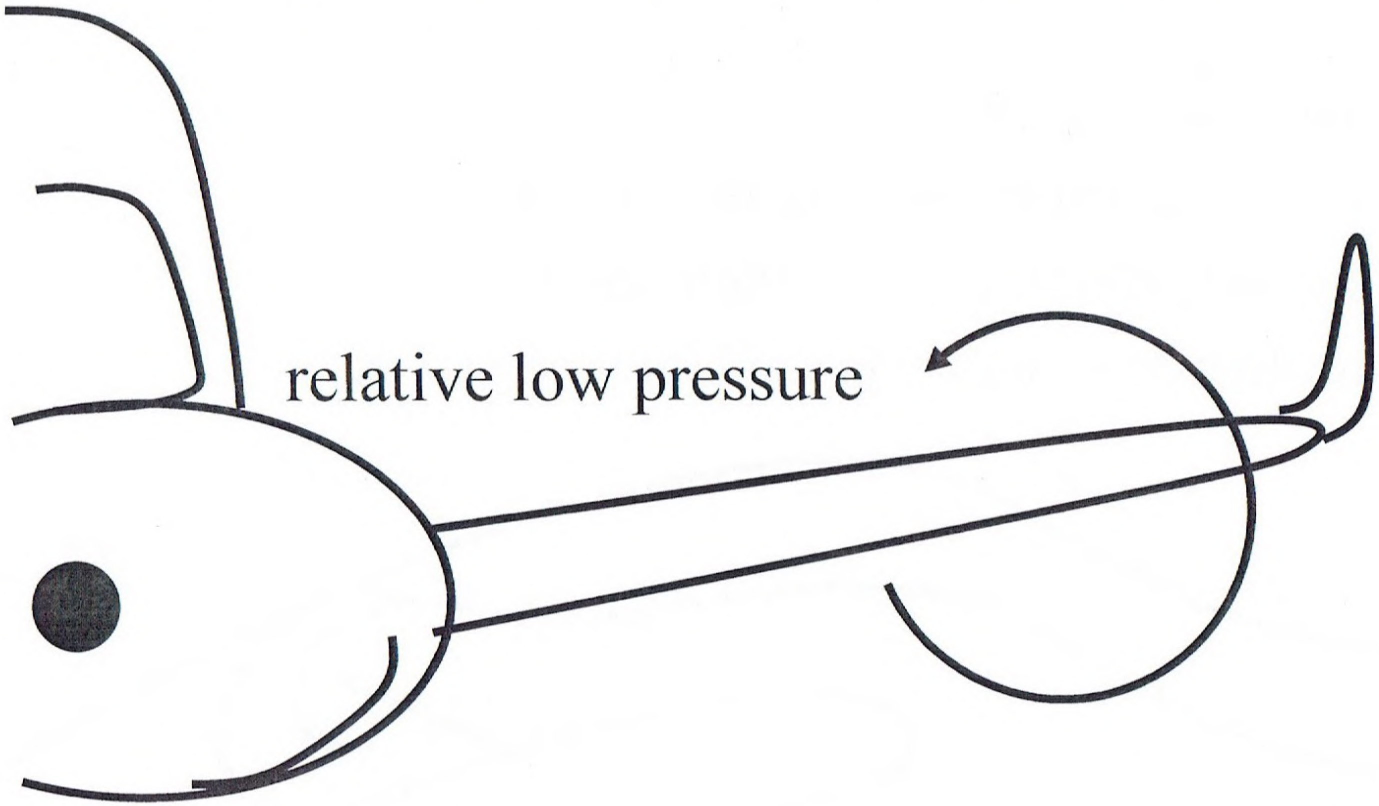
Too Much Lift? Spoilers

Spoilers destroy lift:

- to slow down in flight (flight spoilers);
- for roll control in flight (flight spoilers);
- to slow down on the ground (ground spoilers).



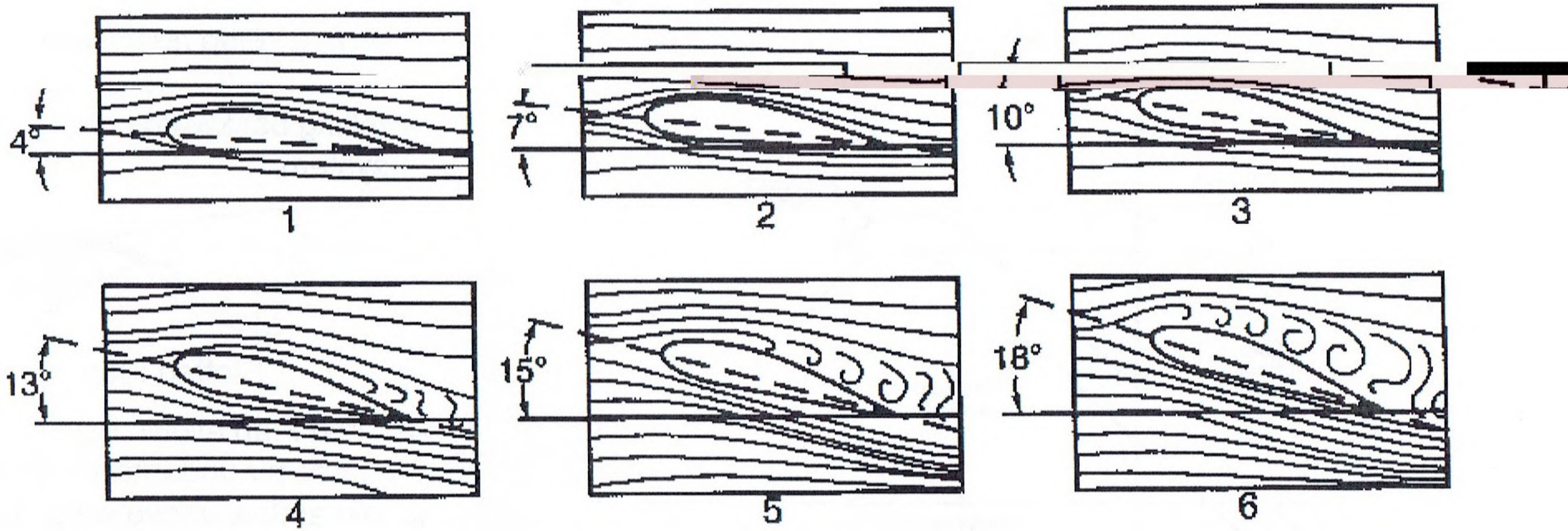
Wingtip Vortices and Wake Turbulence



- Wingtip vortices create drag:
  - "ground effect";
  - tip tanks, drooped wings, "winglets".

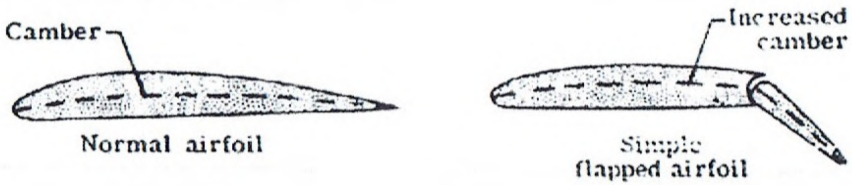
Stalls

Effect of increasing the airfoil (wing) angles of attack are shown in the Figures attached bellow, for some angle-of-attack, Figs. 4 & 5 called the ***stall angle-of-attack,*** the lift reaches a maximum value

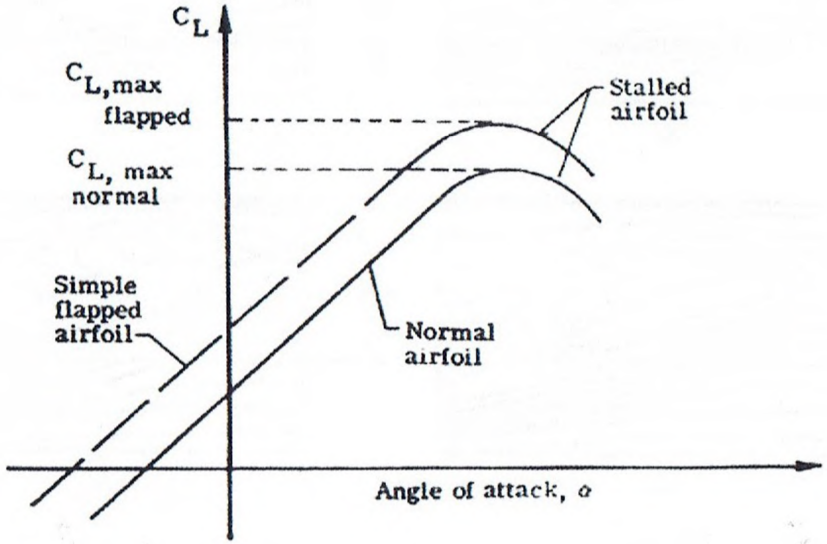


Flaps

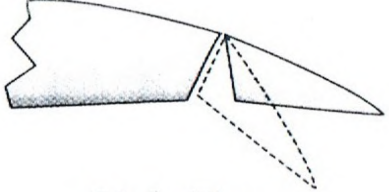
- Flaps increase lift and decrease stall speed
- Flaps allow steep rate of descent for approaches without increasing airspeed



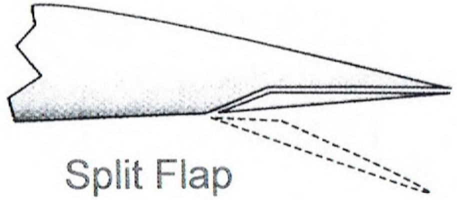
(a) Flap.



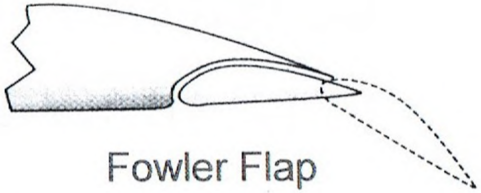
(b) Flap aerodynamic effects.



Plain Flap

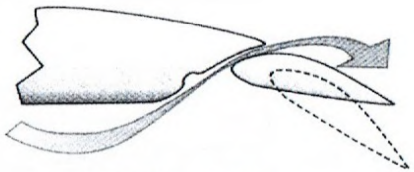


Split Flap



Fowler Flap

-Fowler Flap effectively increases the wing area by rolling backwards on a roller system.



Slotted Flap

-Slotted Flap allows high pressure air underneath wing to join airflow above wing. This effectively increases velocity of top airflow and thus increases lift.

Drag and Lift

- **Drag**
- Types of drag:
- **Induced drag**
- A function of the lift
- Increases with increased angle of attack
- **Parasite drag**
- Form drag. Drag because of the shape of the object, i.e. the more aerodynamically the form, the less the form drag.
- Interference drag. Surfaces on the airplane that meet at sharp angles form a vortex that increases drag.
- Skin friction. Drag resulting from the friction between the air and the surface of the object. The smoother the surface, the less skin friction.
- The induced drag and the parasite drag of the wing amounts for about half of the aircraft's total drag.
- **Lift**

Factors that influence the lift

Angle of attack. The higher the angle of attack , the more lift.

Speed. Lift varies with the square of the speed, i.e. double speed equals four times the lift.

Air density. Lift increases with increased density.

Wing shape. A wing with higher cord creates more lift.

Wing area. Lift increases linearly with the area.

Aspect ratio. The longer and narrower the wings, the more lift.

Flaps

increases lift by increasing the camber of the wing

also increases drag

moves the center of pressure

Center of Pressure – cp

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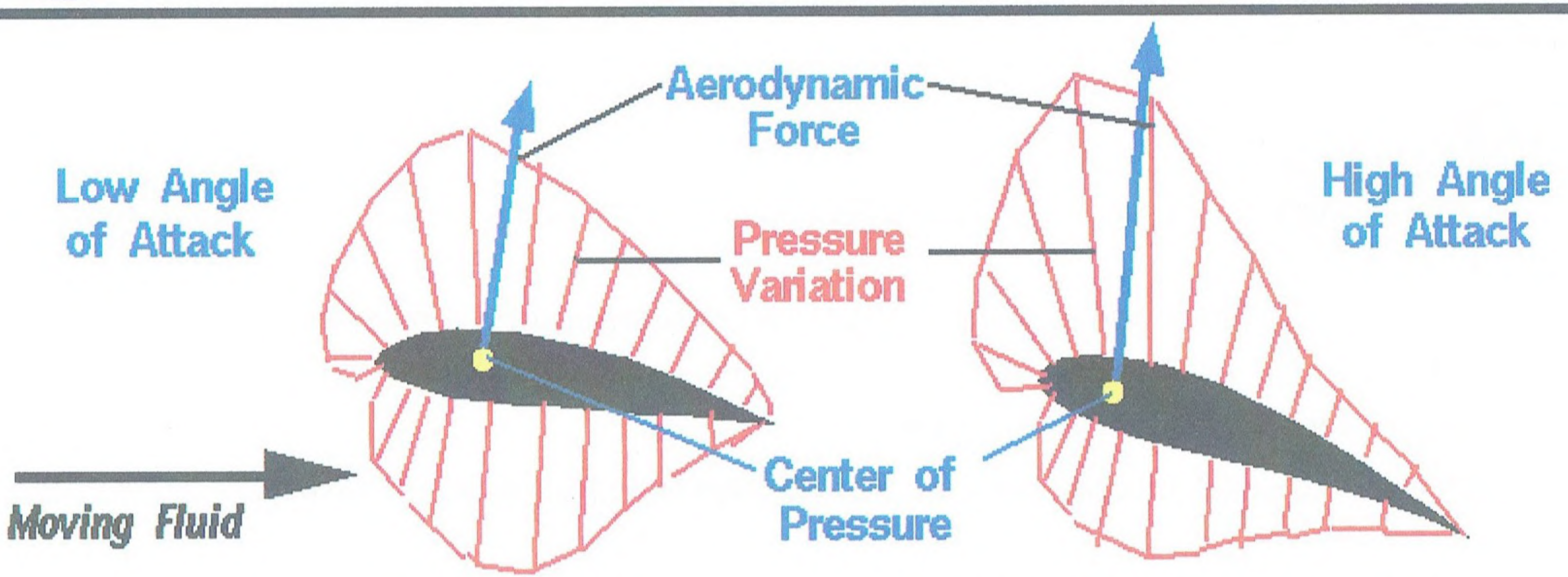

Center of Pressure is the average location of the pressure.

Pressure varies around the surface of an object. $P = P(x)$

$$cp = \frac{\int x\, p(x)\, dx}{\int p(x)\, dx}$$

Aerodynamic force acts through the center of pressure.

Center of pressure moves with angle of attack.

Aerodynamic Center – ac

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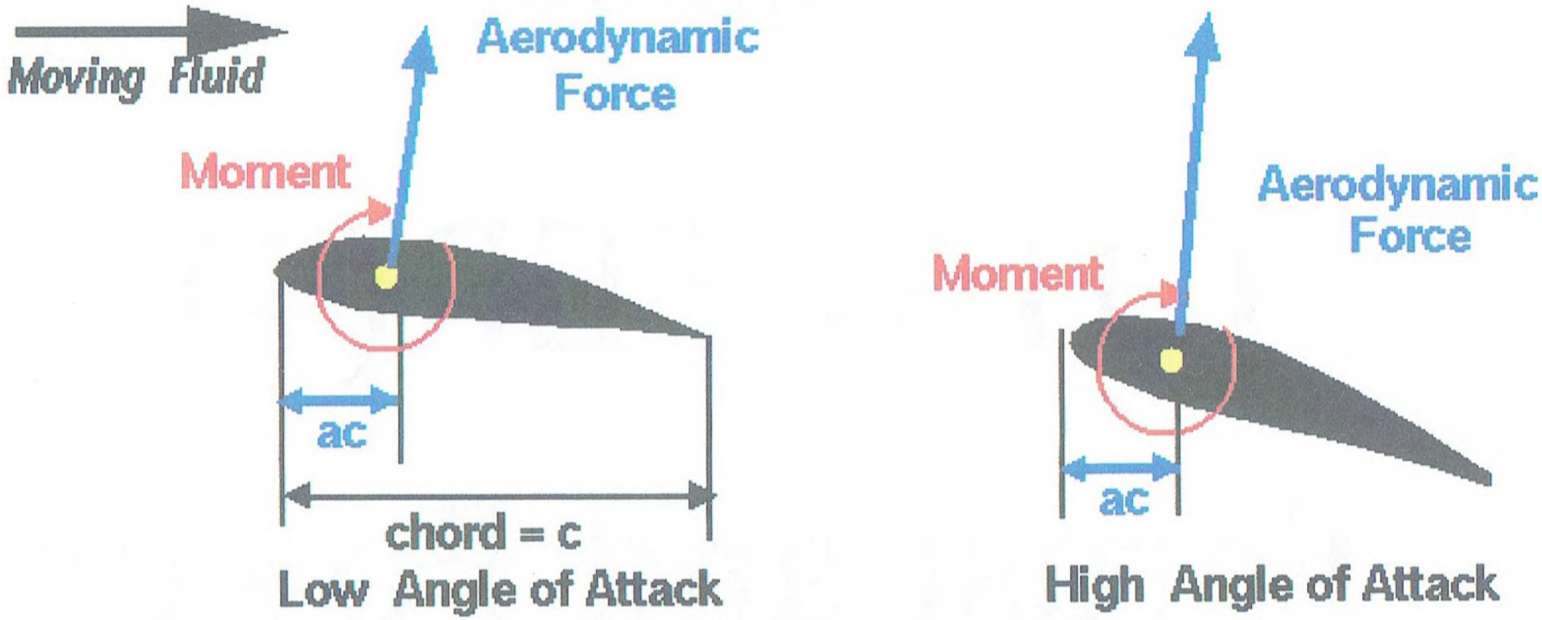


Aerodynamic Center

For low speed, thin airfoils (flat plate) :

$$ac = \frac{c}{4}$$

Moment about the aerodynamic center is constant with angle.

Aerodynamic center does not move with angle.

Theory of Flight – Part II, FLIGHT MECHANICS

CHAPTER(4)

Thrust and Power Requirements

EXAMPLE: BEECHCRAFT QUEEN AIR

- The results we have developed so far for lift and drag for a finite wing may also be applied to a complete airplane. In such relations:
  - C_D is drag coefficient for complete airplane
  - $C_{D,0}$ is parasitic drag coefficient, which contains not only profile drag of wing (c_d) but also friction and pressure drag of tail surfaces, fuselage, engine nacelles, landing gear and any other components of airplane exposed to air flow
  - C_L is total lift coefficient, including small contributions from horizontal tail and fuselage
  - Span efficiency for finite wing replaced with Oswald efficiency factor for entire airplane
- Example: To see how this works, assume the aerodynamicists have provided all the information needed about the complete airplane shown below

Beechcraft Queen Air Aircraft Data

$W = 38{,}220$ N
$S = 27.3\ m^2$
$AR = 7.5$
e (complete airplane) = 0.9
$C_{D,0}$ (complete airplane) = 0.03

What thrust and power levels are required of engines to cruise at 220 MPH at sea-level?
How would these results change at 15,000 ft

OVERALL AIRPLANE DRAG

- No longer concerned with aerodynamic details
- Drag for complete airplane, not just wing

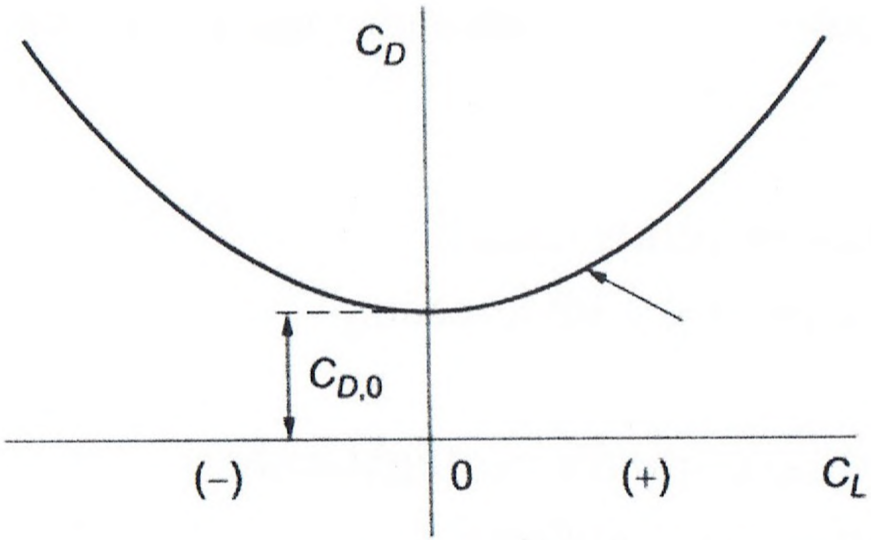
$$C_D = C_d + \frac{C_L^2}{\pi e AR} \quad \longrightarrow \quad C_D = C_{D,0} + \frac{C_L^2}{\pi e AR}$$

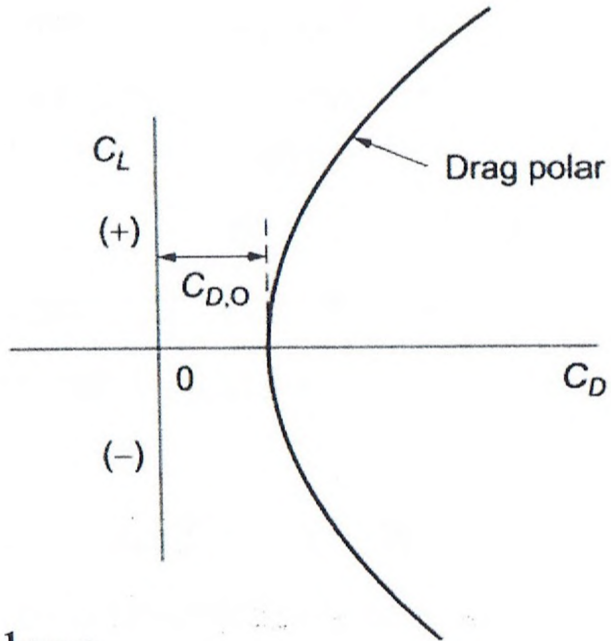
Wing or airfoil — Entire Airplane

DRAG POLAR

- $C_{D,0}$ is parasite drag coefficient at zero lift (α_L=0)
- $C_{D,i}$ drag coefficient due to lift (induced drag)
- Oswald efficiency factor, e, includes all effects from airplane
- $C_{D,0}$ and e are known aerodynamics quantities of airplane

$$C_D = C_{D,0} + \frac{C_L^2}{\pi e AR} = C_{D,0} + C_{D,i}$$





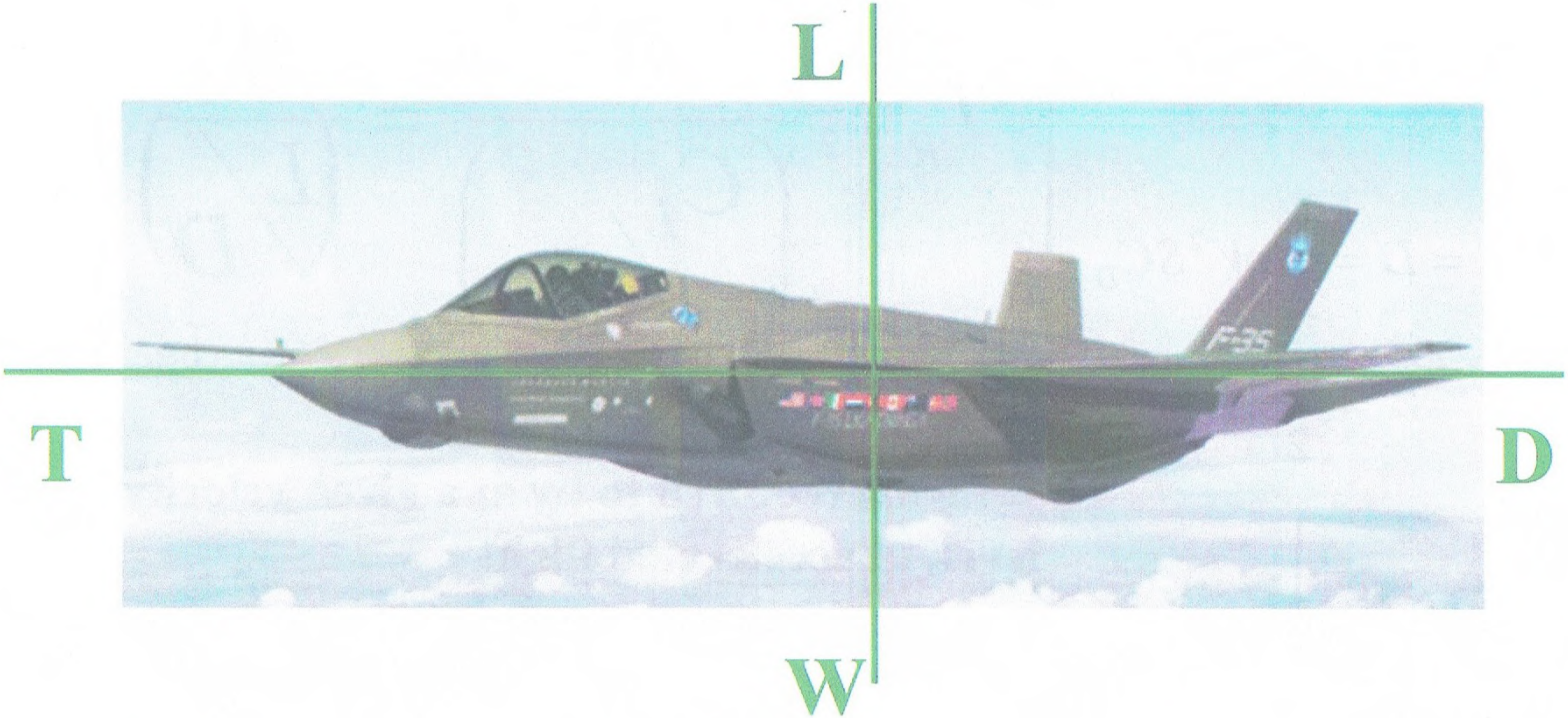
Example of *Drag Polar* for complete airplane

4 FORCES ACTING ON AIRPLANE

- Model airplane as rigid body with four natural forces acting on it
  1. **Lift, L**
     - Acts perpendicular to flight path (always perpendicular to relative wind)
  2. **Drag, D**
     - Acts parallel to flight path direction (parallel to ***incoming*** relative wind)
  3. **Propulsive Thrust, T**
     - For most airplanes propulsive thrust acts in flight path direction
     - May be inclined with respect to flight path angle, α_T, usually small angle
  4. **Weight, W**
     - Always acts vertically toward center of earth
     - Inclined at angle, θ, with respect to lift direction

- Apply Newton's Second Law (**F**=m**a**) to curvilinear flight path
  - Force balance in direction parallel to flight path
  - Force balance in direction perpendicular to flight path

LEVEL, UNACCELERATED FLIGHT



- JSF is flying at constant speed (no accelerations)
- Sum of forces = 0 in two perpendicular directions
- Entire weight of airplane is perfectly balanced by lift (L = W)
- Engines produce just enough thrust to balance total drag at this airspeed (T = D)
- For most conventional airplanes α_T is small enough such that $\cos(\alpha_T) \sim 1$

LEVEL, UNACCELERATED FLIGHT

$$T = D$$

$$L = W$$

$$T = D = \frac{1}{2}\rho V^2 S C_D$$

$$L = W = \frac{1}{2}\rho V^2 S C_L$$

$$\boxed{\frac{T}{W} = \frac{C_D}{C_L}}$$

$$\boxed{T_R = \frac{W}{\left(C_L / C_D\right)} = \frac{W}{\left(L / D\right)}}$$

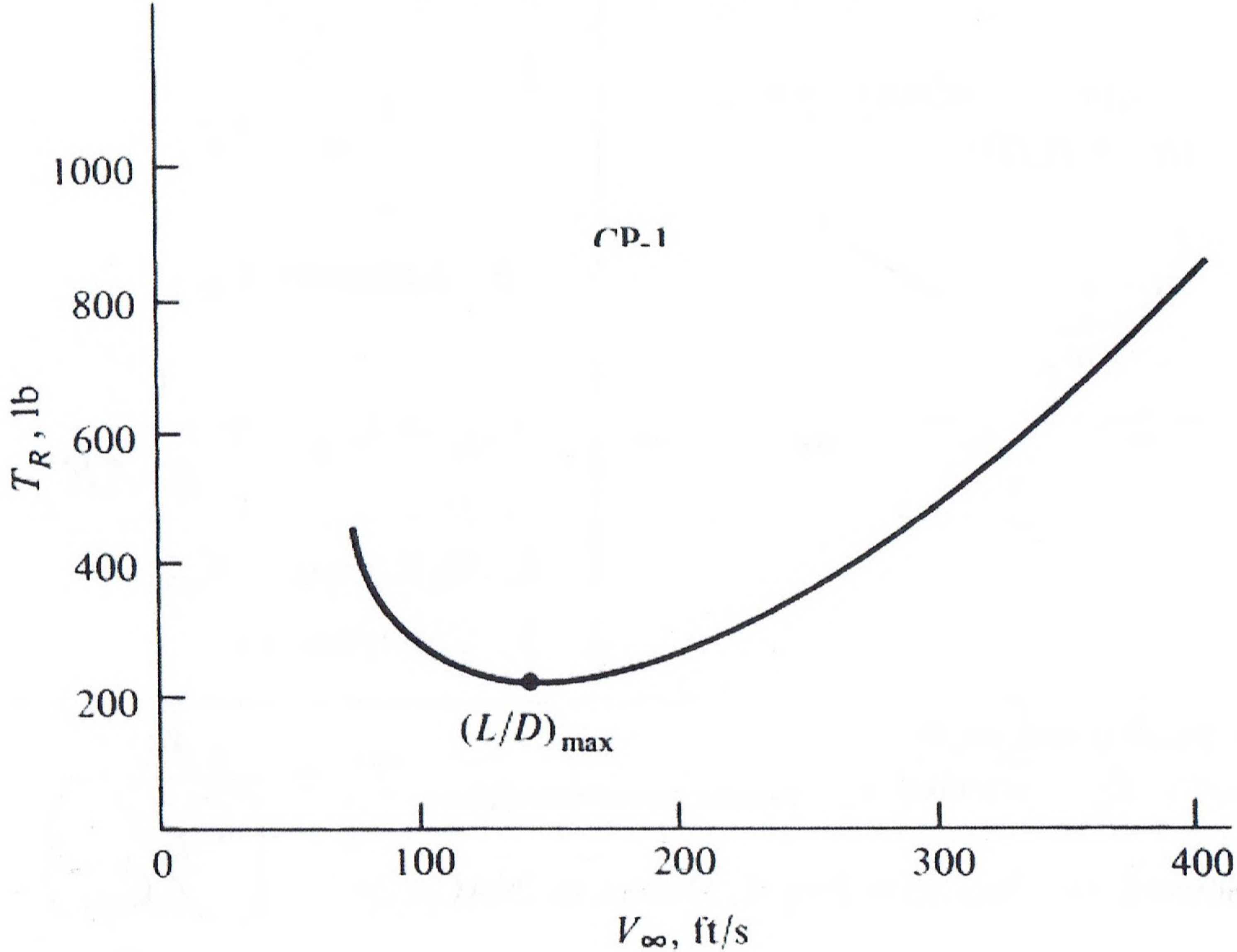
T_R is thrust required to fly at a given velocity in level, unaccelerated flight

Notice that **minimum** T_R is when airplane is at **maximum** L/D

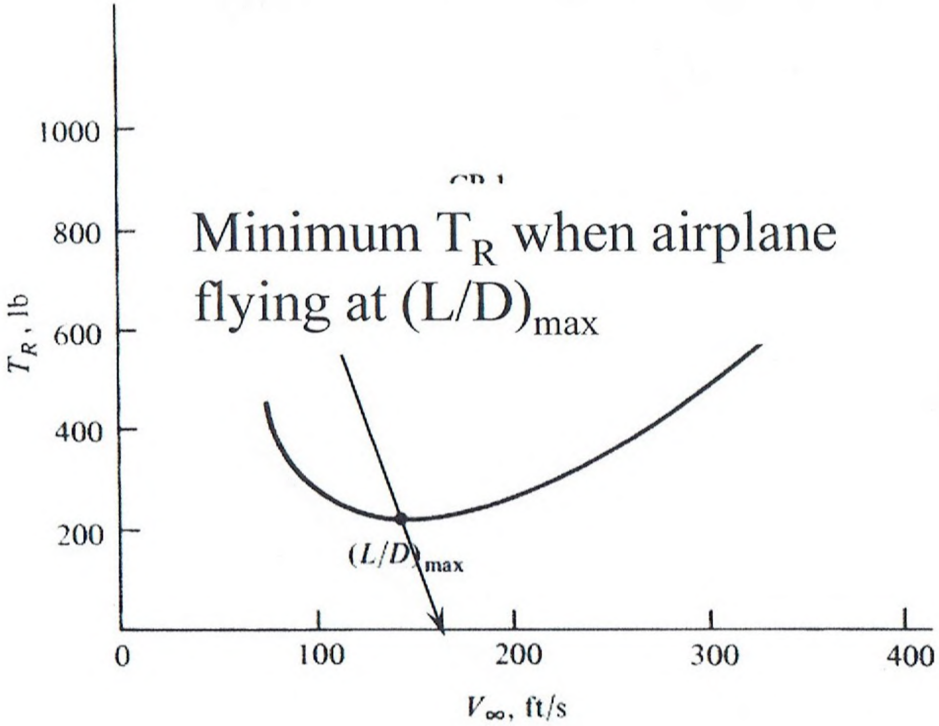
- L/D is an important aero-performance quantity

THRUST REQUIREMENT (6.3)

- T_R for airplane at given altitude varies with velocity
- Thrust required curve: T_R vs. V_∞



PROCEDURE: THRUST REQUIREMENT



1. Select a flight speed, V_∞
2. Calculate C_L

$$C_L = \frac{W}{\frac{1}{2}\rho_\infty V_\infty^2 S}$$

3. Calculate C_D

$$C_D = C_{D,0} + \frac{C_L^2}{\pi e AR}$$

4. Calculate C_L/C_D
5. Calculate T_R

This is how much thrust engine ***must*** produce to fly at selected V_∞

$$T_R = \frac{W}{\left(C_L/C_D\right)}$$

Recall Homework Problem #5.6, find (L/D)max for NACA 2412 airfoil

THRUST REQUIREMENT (6.3)

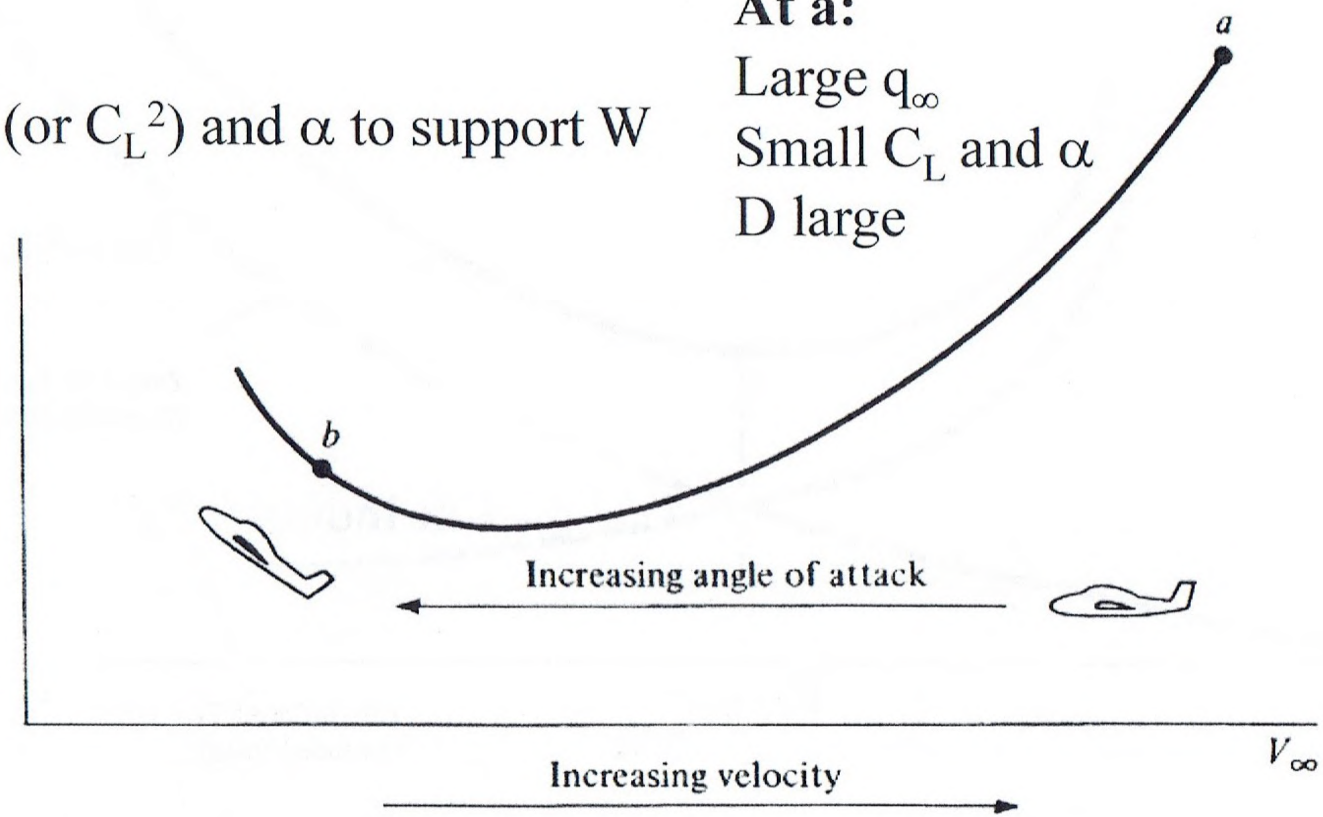
- Different points on T_R curve correspond to different angles of attack

$$L = W = \frac{1}{2}\rho_\infty V_\infty^2 S C_L = q_\infty S C_L$$

$$D = q_\infty S C_D = q_\infty S\left(C_{D,0} + \frac{C_L^2}{\pi e AR}\right)$$

At b:
Small q_∞
Large C_L (or C_L^2) and α to support W
D large

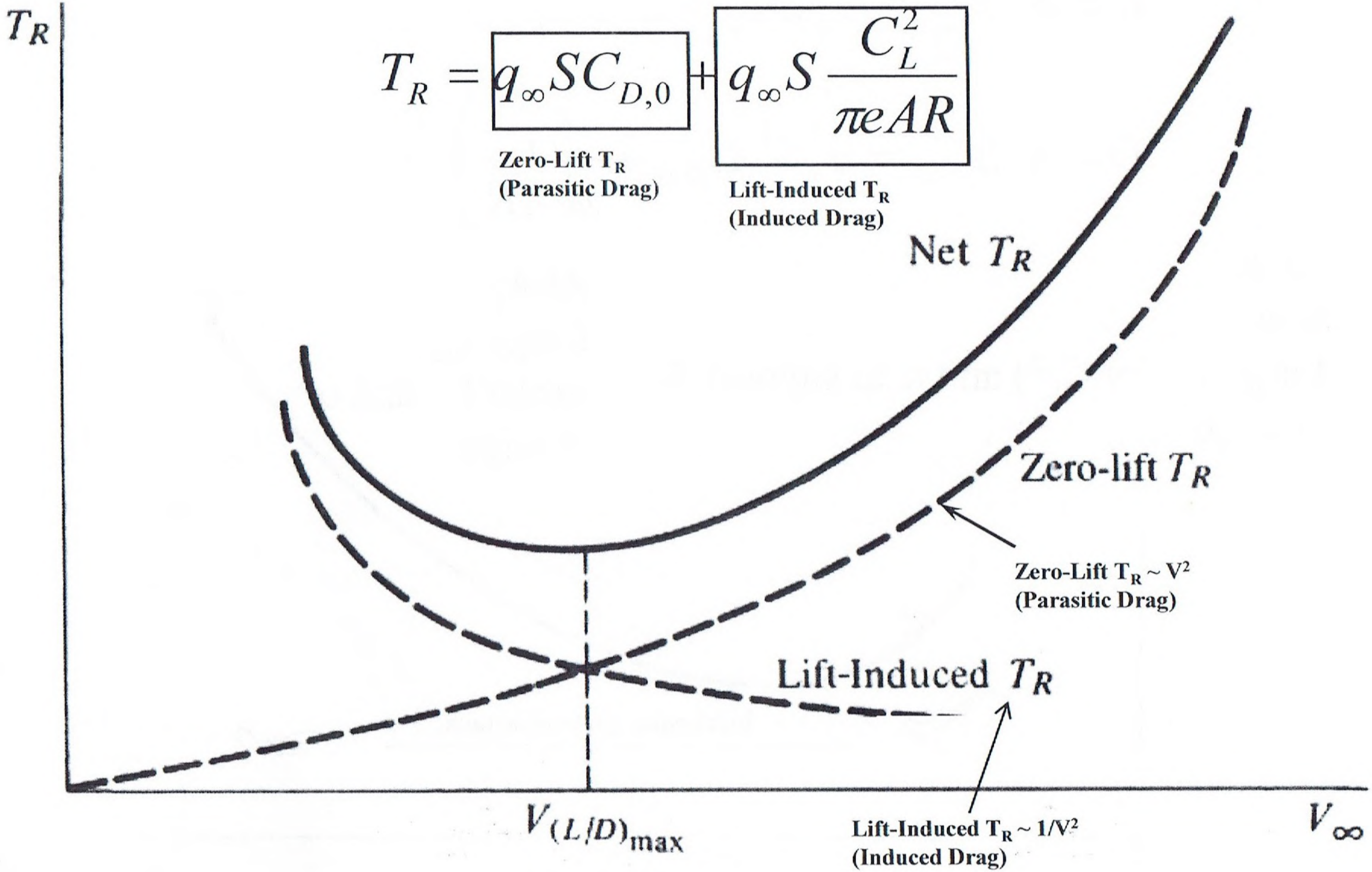
At a:
Large q_∞
Small C_L and α
D large



THRUST REQUIRED VS. FLIGHT VELOCITY

$$T_R = D = q_\infty S C_D = q_\infty S\left(C_{D,0} + C_{D,i}\right)$$

$$T_R = \boxed{q_\infty S C_{D,0}} + \boxed{q_\infty S \frac{C_L^2}{\pi e AR}}$$



THRUST REQUIRED VS. FLIGHT VELOCITY

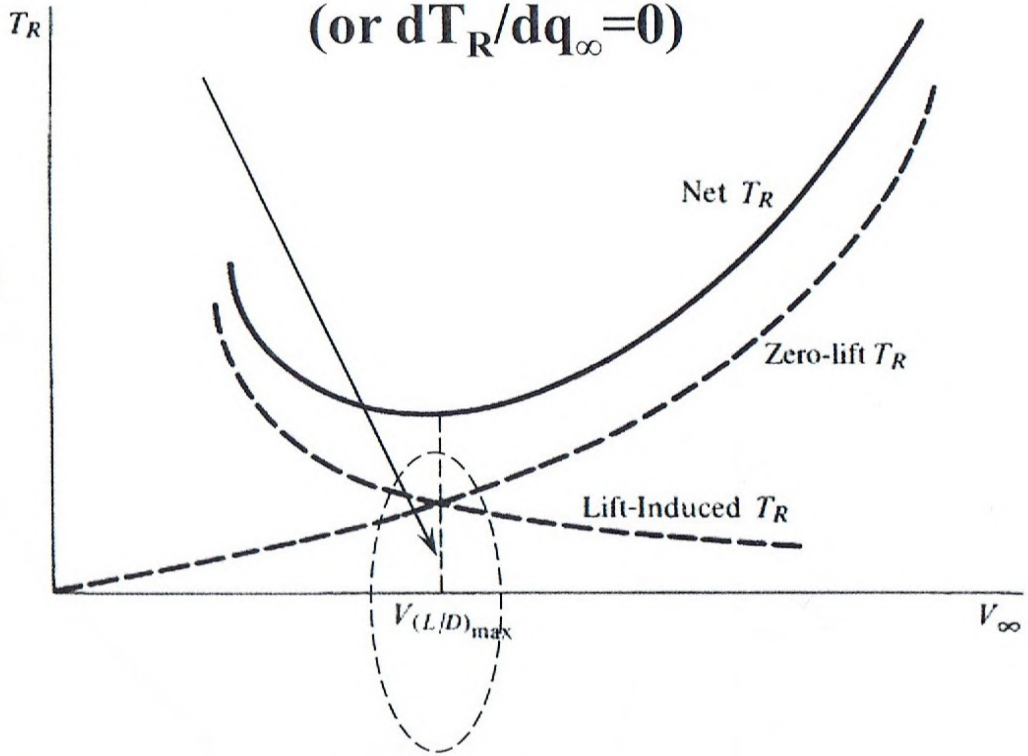
$$T_R = q_\infty S C_{D,0} + \frac{W^2}{q_\infty S \pi e AR}$$

$$\frac{dT_R}{dq_\infty} = \left(\frac{dT_R}{dV_\infty}\right)\left(\frac{dV_\infty}{dq_\infty}\right)$$

$$\frac{dT_R}{dq_\infty} = SC_{D,0} - \frac{W^2}{q_\infty^2 S \pi e AR} = 0$$

$$C_{D,0} = \frac{C_L^2}{\pi e AR} = C_{D,i}$$

At point of minimum T_R, $dT_R/dV_\infty=0$ (or $dT_R/dq_\infty=0$)

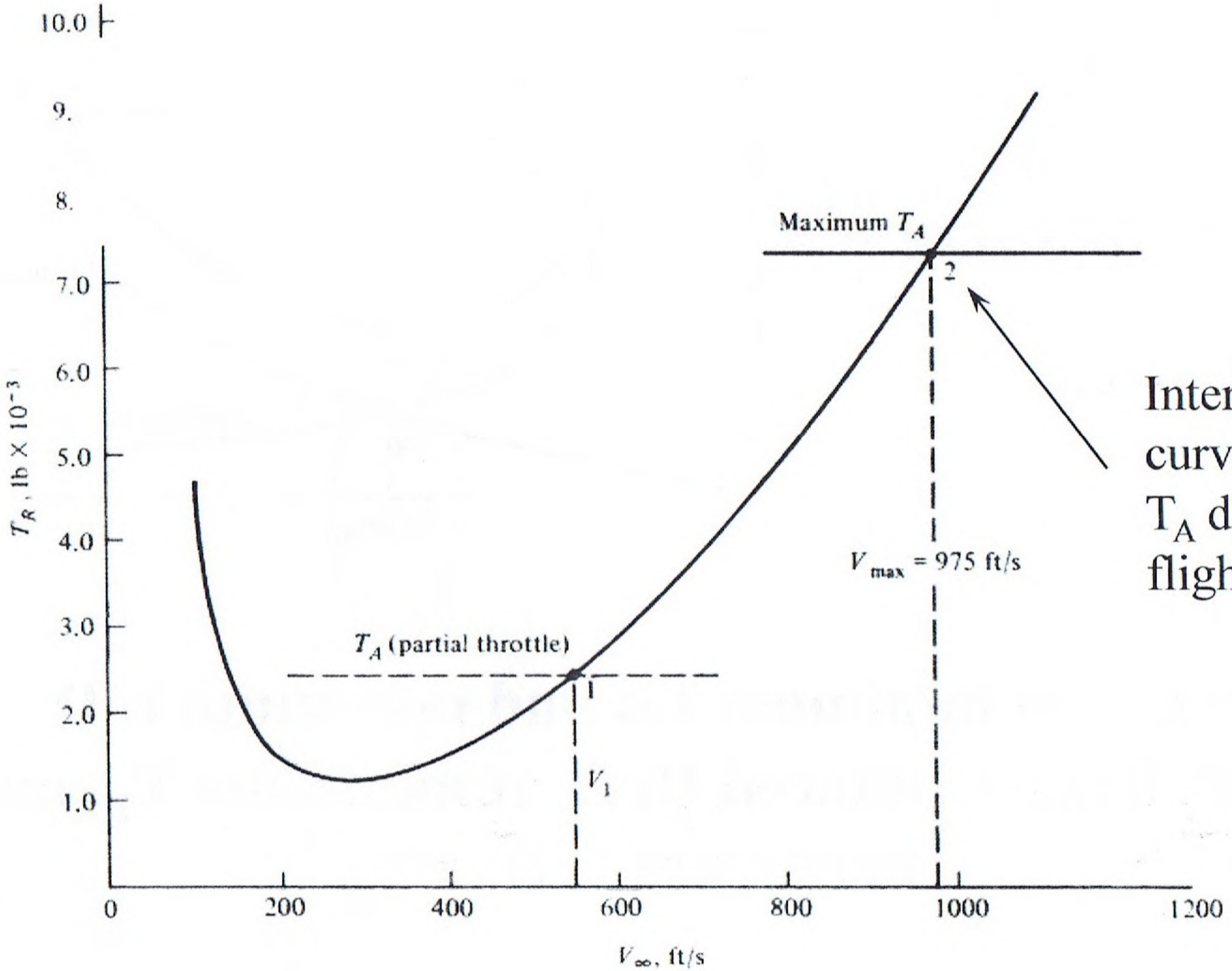


$C_{D,0} = C_{D,i}$ at minimum TR and maximum L/D
Zero-Lift Drag = Induced Drag at minimum T_R and maximum L/D

HOW FAST CAN YOU FLY?

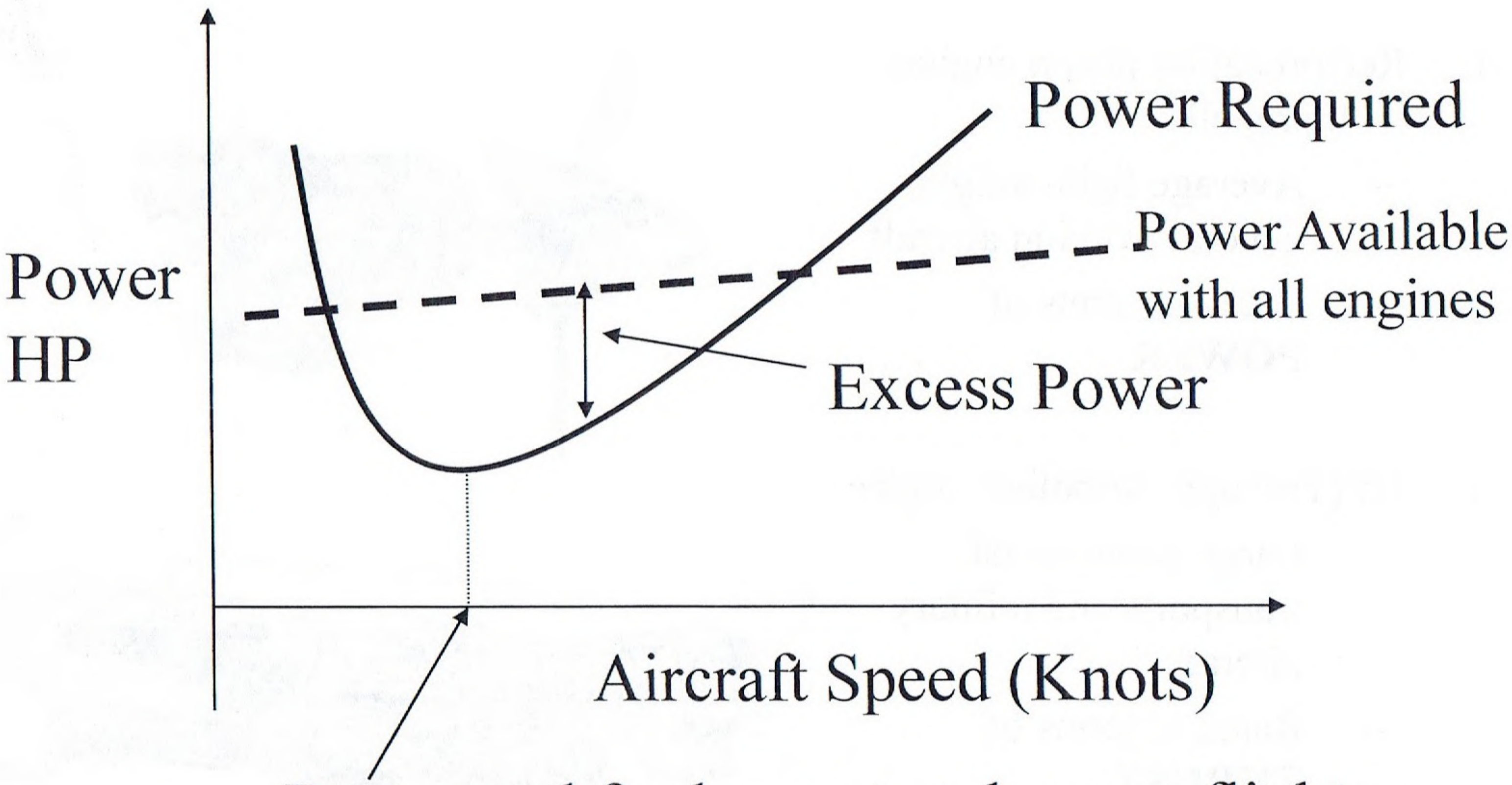
Maximum flight speed occurs when thrust available, $T_A = T_R$

- Reduced throttle settings, $T_R < T_A$
- Cannot physically achieve more thrust than T_A which engine can provide



Intersection of T_R curve and maximum T_A defined maximum flight speed of airplane

Level Flight Performance



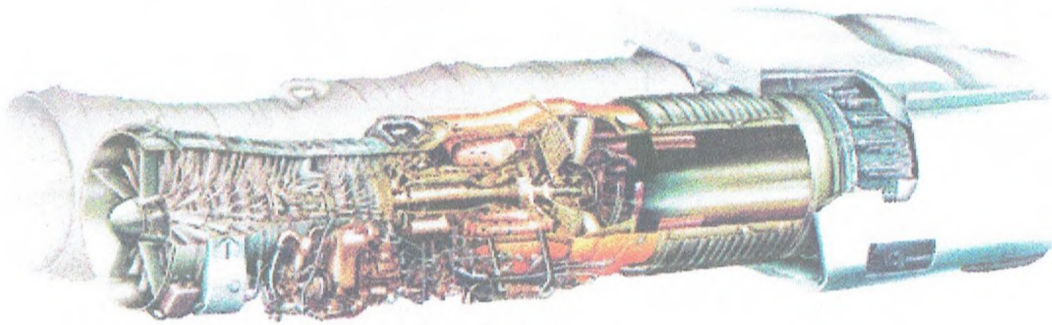
Best speed for longest endurance flights
since the least amount of fuel is burned

CHAPTER 5: AIRPLANE POWER PLANTS

Two types of engines common in aviation today

1. Reciprocating piston engine with propeller
   - Average light-weight, general aviation aircraft
   - Rated in terms of **POWER**

2. Jet (Turbojet, turbofan) engine
   - Large commercial transports and military aircraft
   - Rated in terms of **THRUST**



THRUST VS. POWER

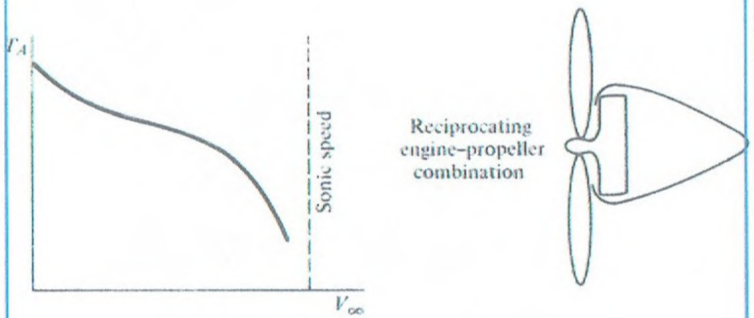
- **Jets Engines** (turbojets, turbofans for military and commercial applications) are usually rate in **Thrust**
  - Thrust is a Force with units (**N** = kg m/s^2)
  - For example, the PW4000-112 is rated at 98,000 lb of thrust

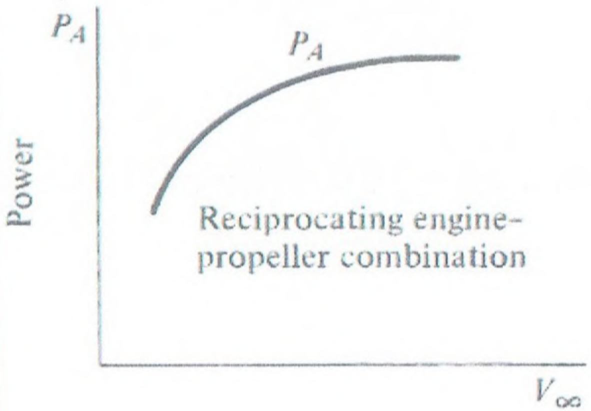
- **Piston-Driven Engines** are usually rated in terms of **Power**
  - Power is a precise term and can be expressed as:
    - Energy / time with units (kg m^2/s^2) / s = kg m^2/s^3 = **Watts**
      - Note that Energy is expressed in Joules = kg m^2/s^2
    - Force * Velocity with units (kg m/s^2) * (m/s) = kg m^2/s^3 = **Watts**
  - Usually rated in terms of horsepower (1 hp = 550 ft lb/s = 746 W)

- Example:
  - Airplane is level, unaccelerated flight at a given altitude with speed V_∞
  - **Power Required, $P_R = T_R * V_\infty$**
  - [W] = [N] * [m/s]

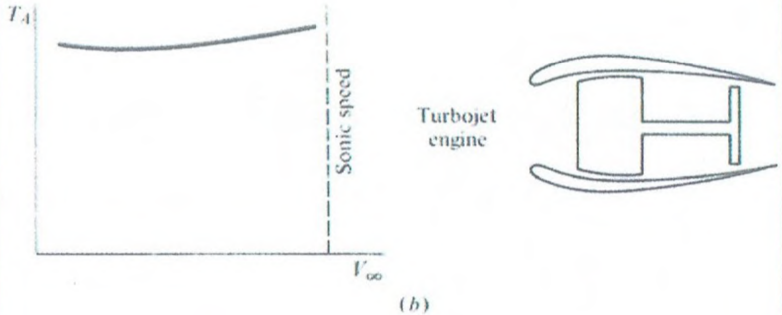
POWER AVAILABLE

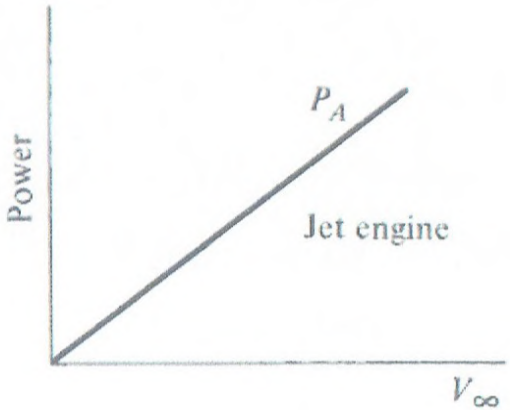
Propeller Drive Engine





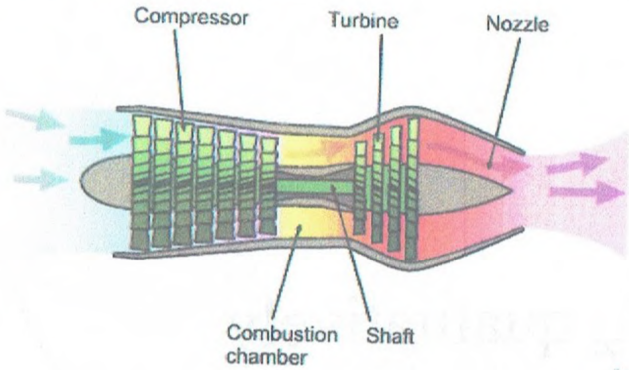
Jet Engine

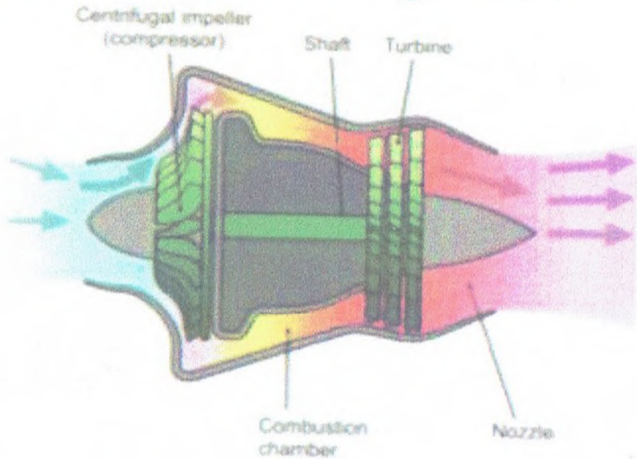




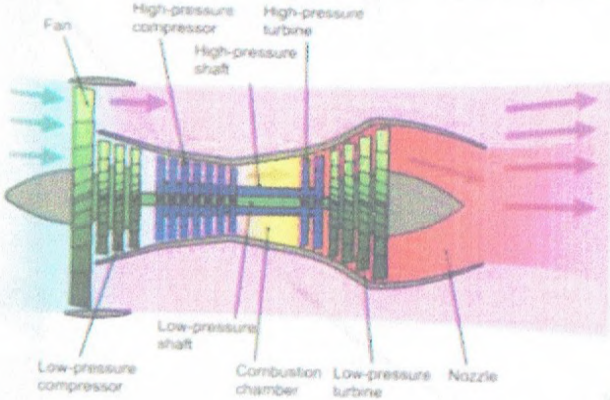
Understand and analyze gas turbine engines:

- Turbojet

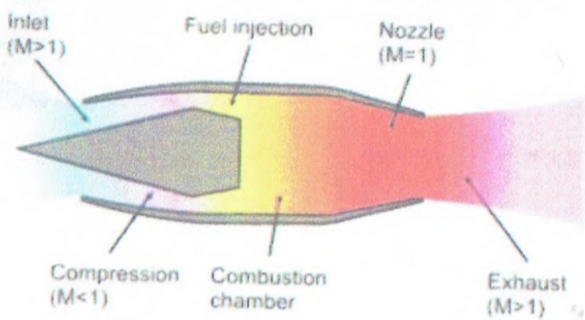




- Turbofan (turbojet + fanned propeller)!



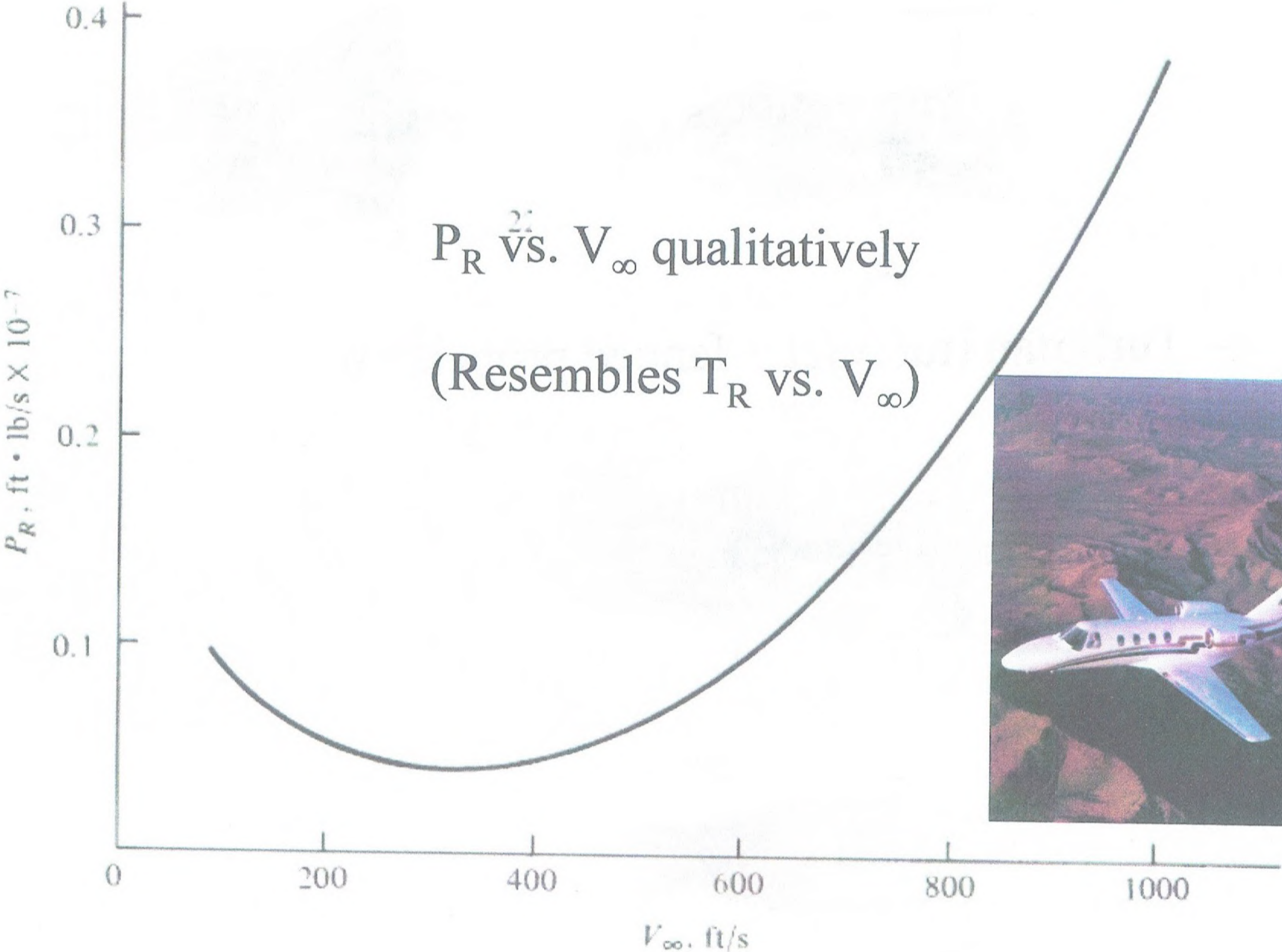
- Ramjet



POWER REQUIRED (6.5)

P_R vs. V_∞ qualitatively

(Resembles T_R vs. V_∞)



POWER REQUIRED (6.5)

$$P_R = T_R V_\infty = \frac{W}{\left(C_L / C_D\right)} V_\infty$$

$$L = W = \frac{1}{2}\rho_\infty V_\infty^2 S C_L \rightarrow V_\infty = \sqrt{\frac{2W}{\rho_\infty S C_L}}$$

$$P_R = \frac{W}{\left(C_L / C_D\right)} \sqrt{\frac{2W}{\rho_\infty S C_L}}$$

$$P_R = \sqrt{\frac{2W^3 C_D^2}{\rho_\infty S C_L^3}} \propto \frac{1}{\left(C_L^{3/2} / C_D\right)}$$

P_R varies inversely as $C_L^{3/2}/C_D$

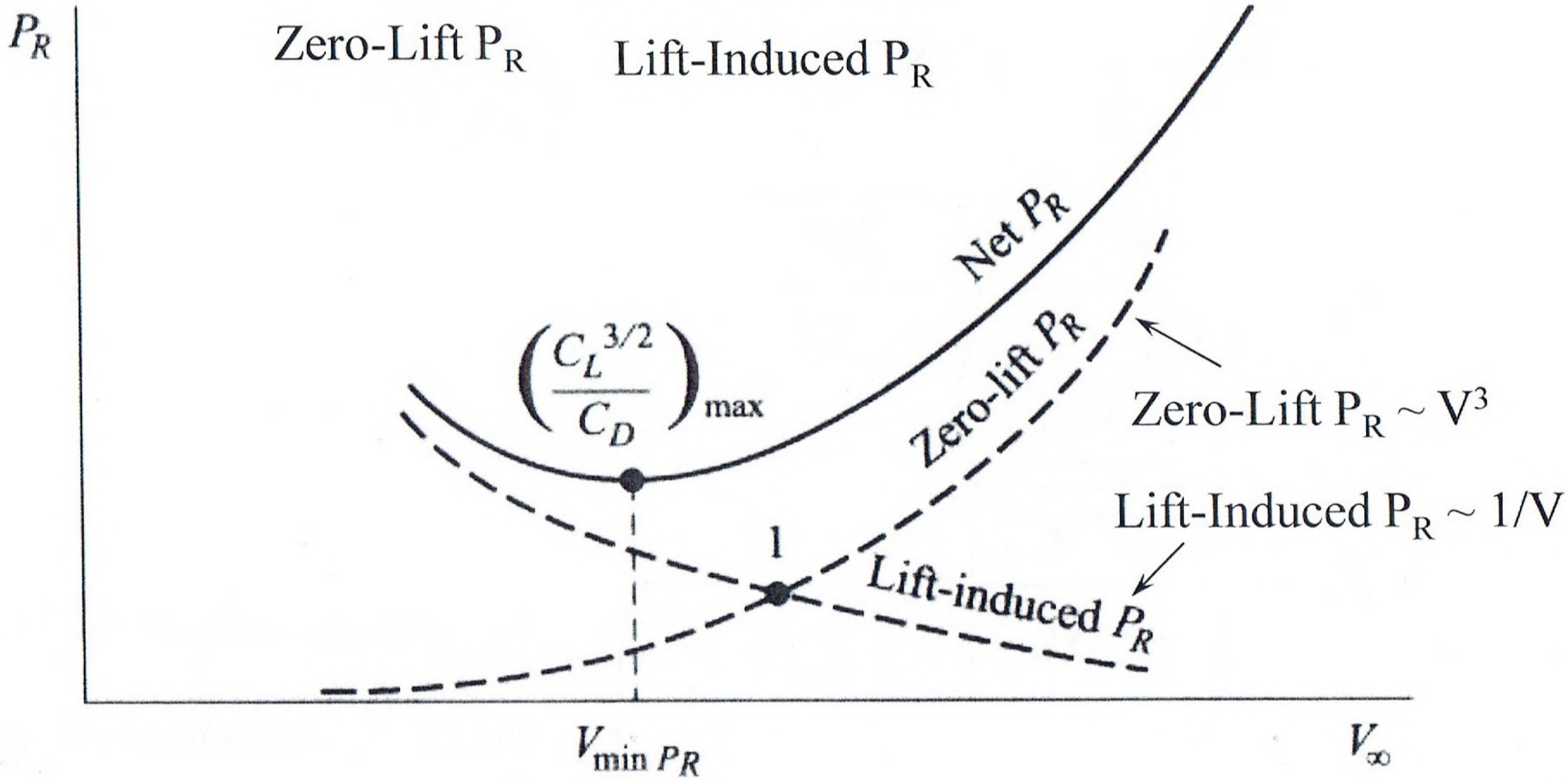
Recall: T_R varies inversely as C_L/C_D

POWER REQUIRED (6.5)

$$P_R = T_R V_\infty = D V_\infty = q_\infty S C_D V_\infty = q_\infty S \left(C_{D,0} + C_{D,i}\right) V_\infty$$

$$P_R = \boxed{q_\infty S C_{D,0} V_\infty} + \boxed{q_\infty S V_\infty \frac{C_L^2}{\pi e AR}}$$

Zero-Lift P_R Lift-Induced P_R

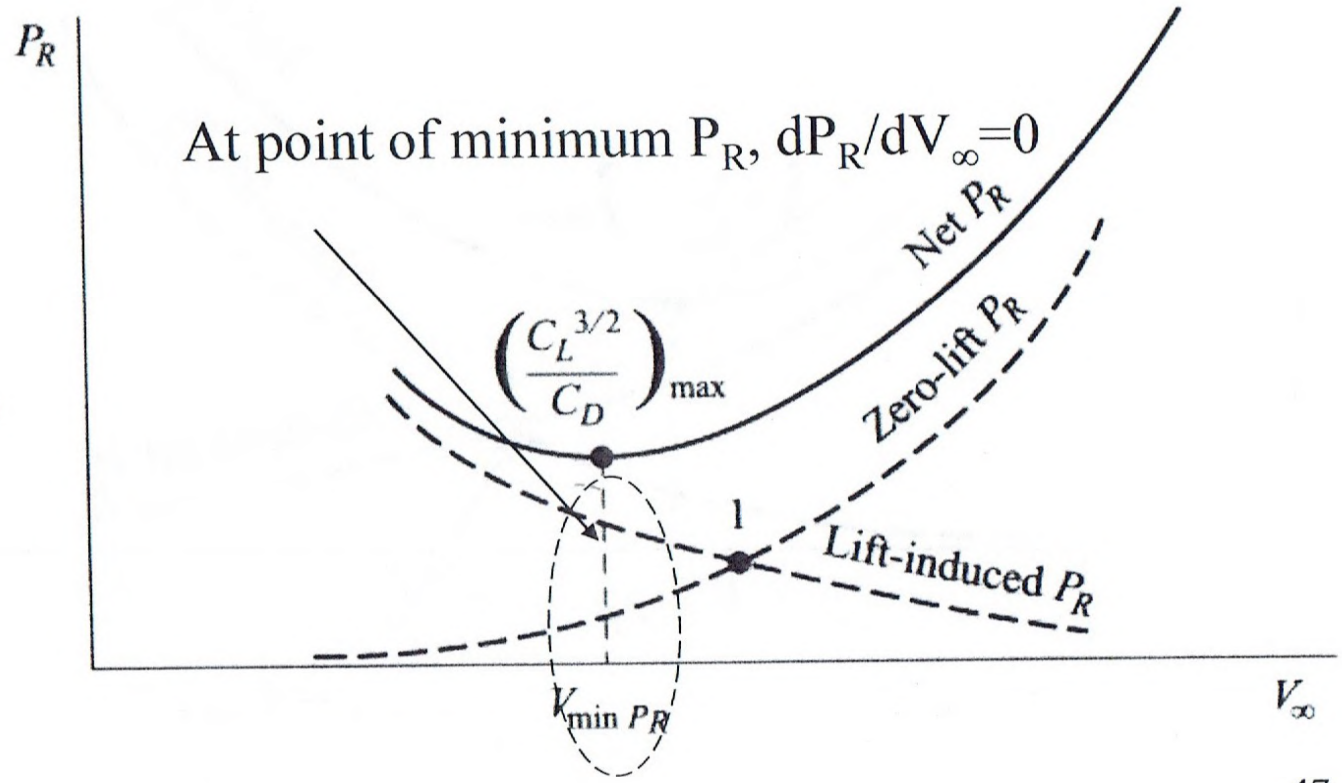


POWER REQUIRED

$$P_R = \frac{1}{2}\rho_\infty V_\infty^3 S C_{D,0} + \frac{W^2}{\frac{1}{2}\rho_\infty V_\infty S \pi e AR}$$

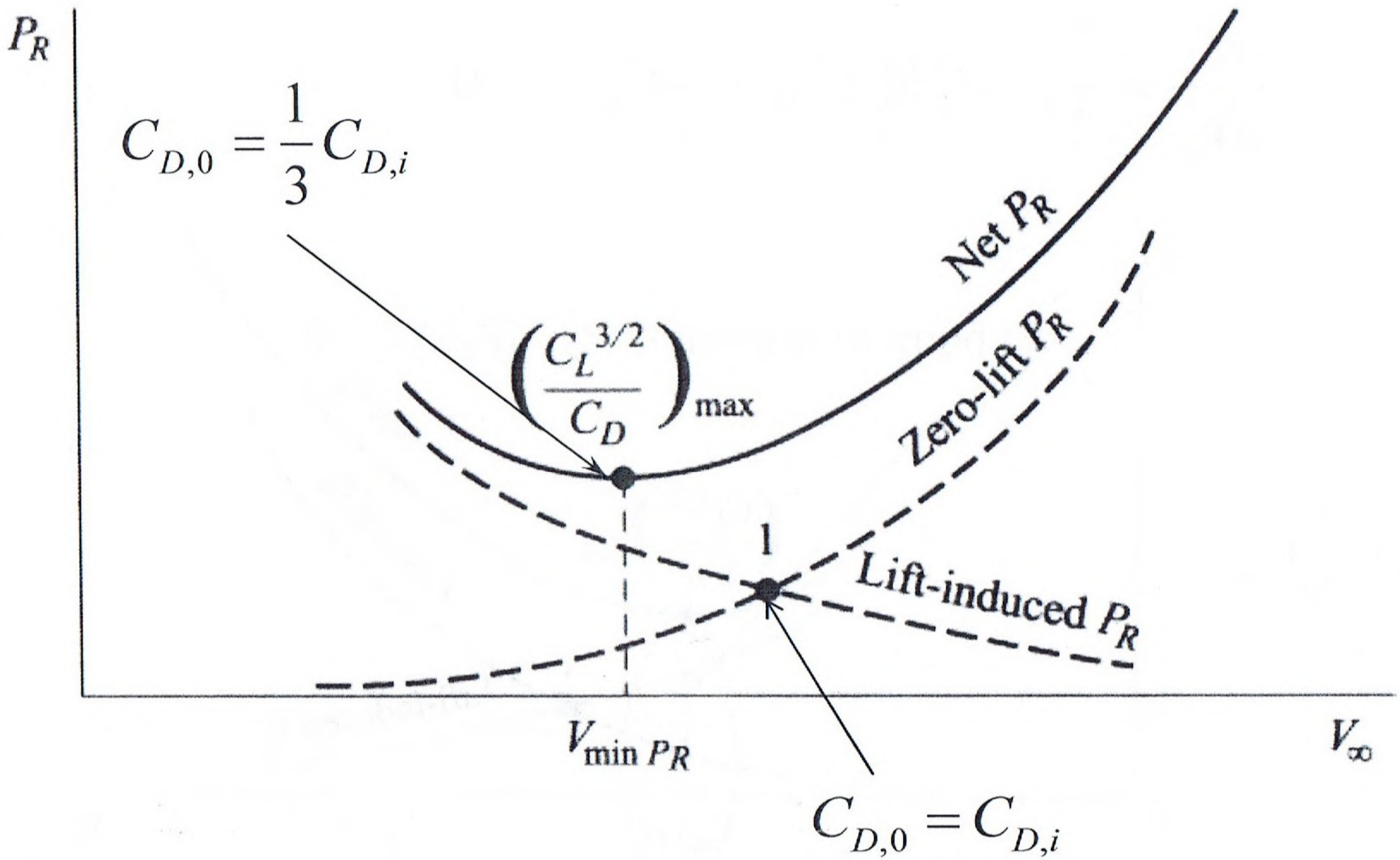
$$\frac{dP_R}{dV_\infty} = \frac{3}{2}\rho_\infty V_\infty^2 S\left(C_{D,0} - \frac{1}{3}C_{D,i}\right) = 0$$

$$C_{D,0} = \frac{1}{3}C_{D,i}$$



POWER REQUIRED

V_∞ for minimum P_R is less than V_∞ for minimum T_R



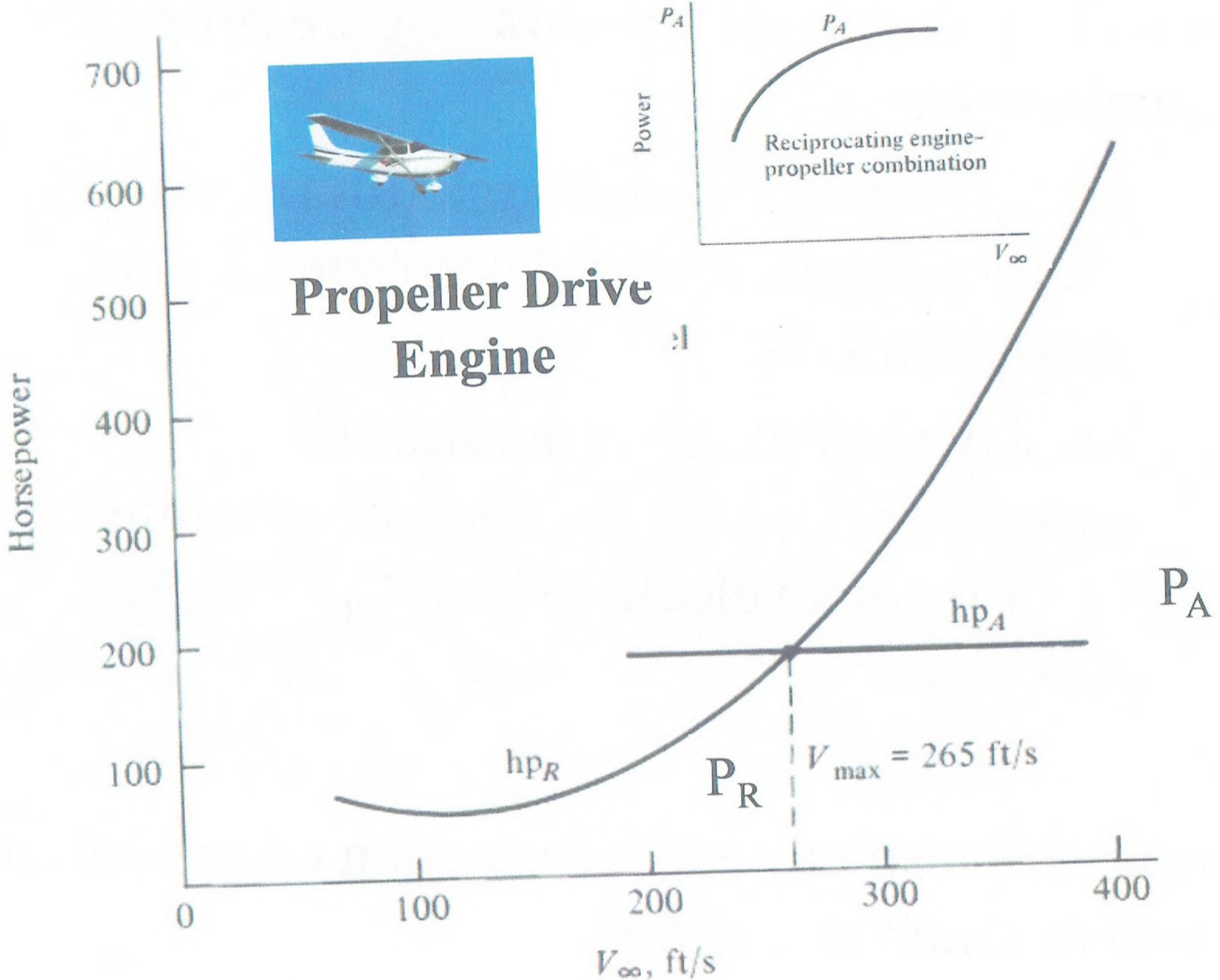
WHY DO WE CARE ABOUT THIS?

We will show that for a **piston-engine propeller** combination

- To fly longest distance (**maximum range**) we fly airplane at speed corresponding to maximum L/D
- To stay aloft longest (**maximum endurance**) we fly the airplane at minimum P_R or fly at a velocity where $C_L^{3/2}/C_D$ is a maximum

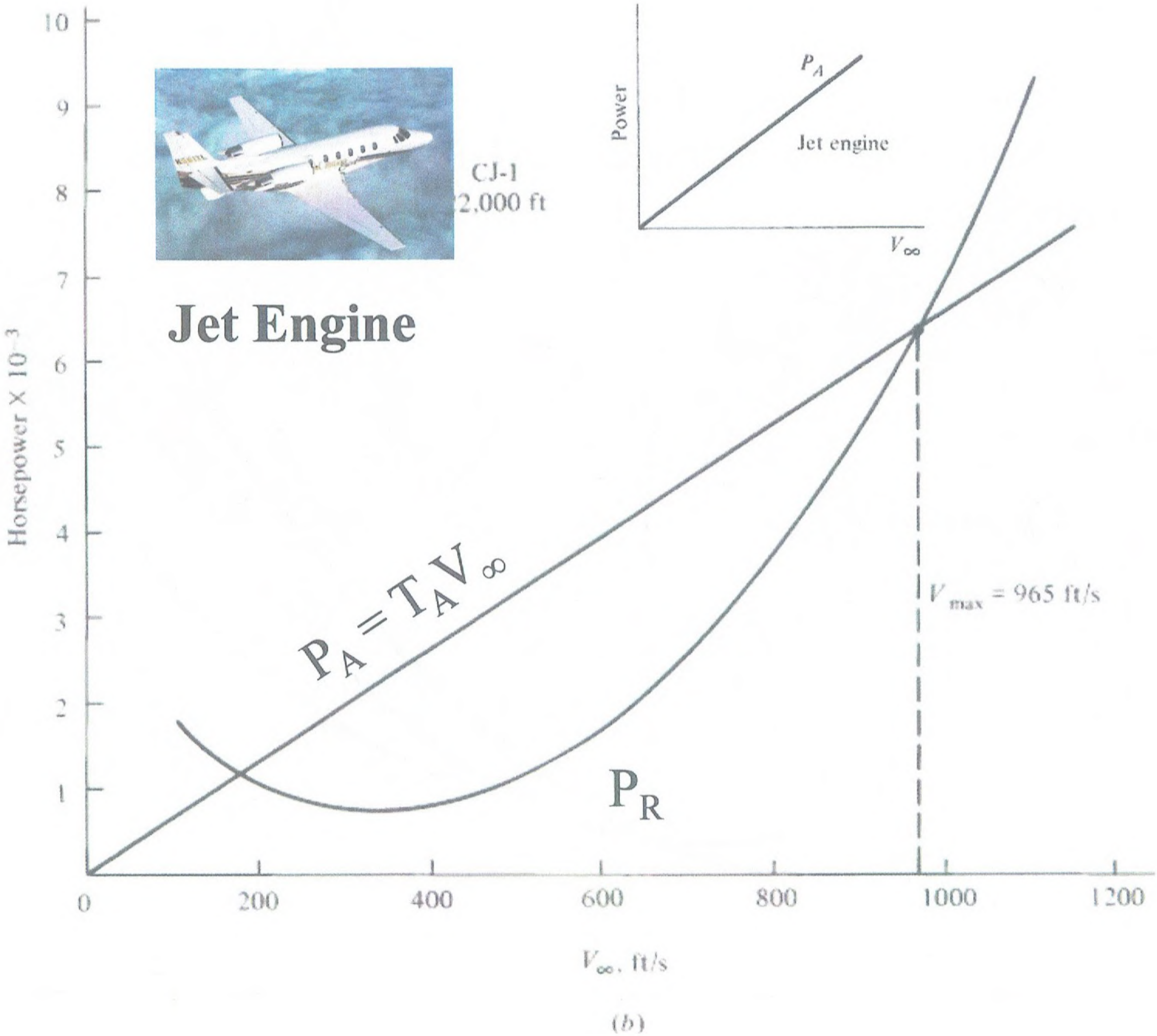
Power will also provide information on maximum rate of climb and altitude

POWER AVAILABLE AND MAXIMUM VELOCITY



$$1 \text{ hp} = 550 \text{ ft lb/s} = 746 \text{ W}$$

POWER AVAILABLE AND MAXIMUM VELOCITY

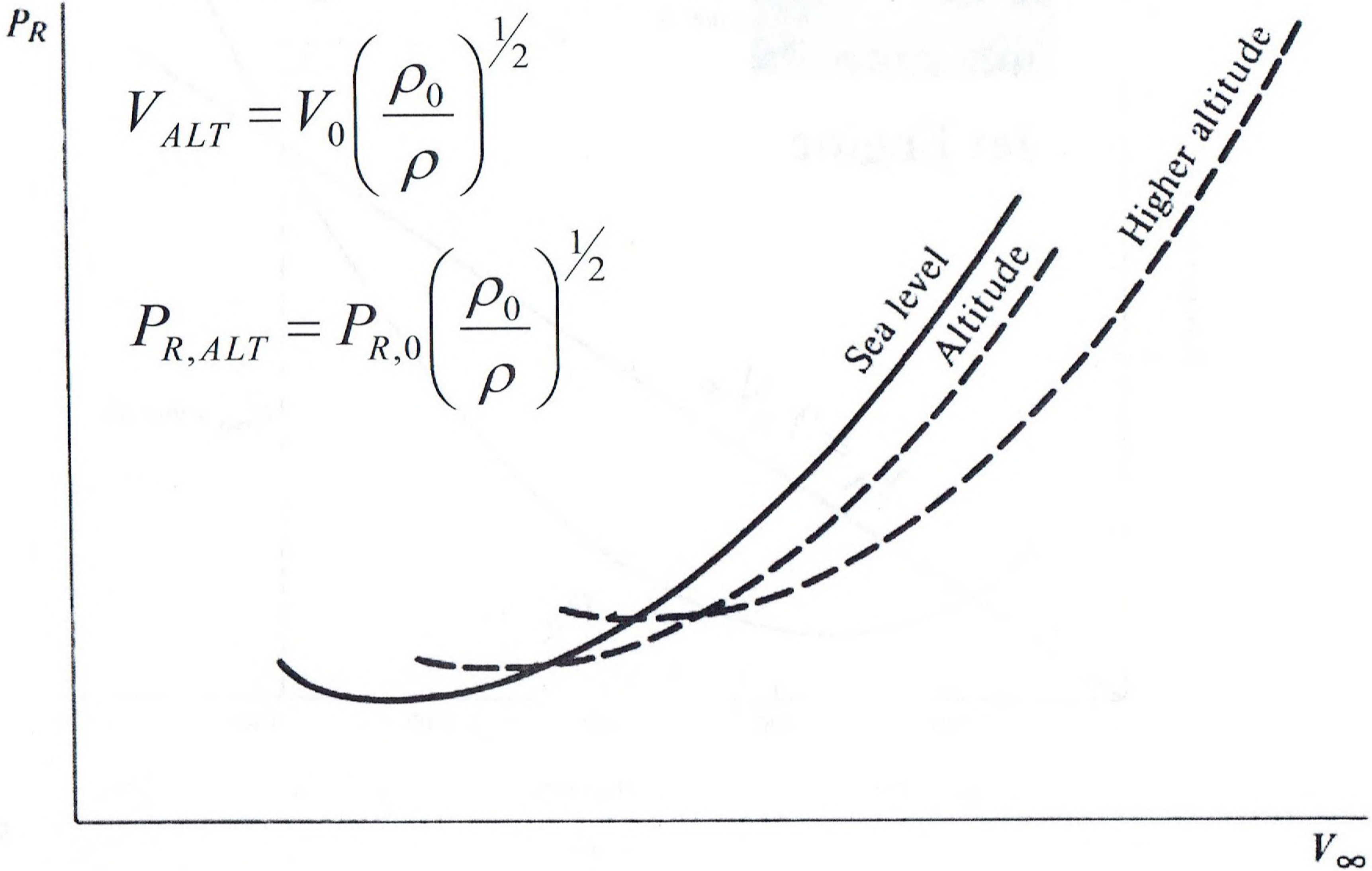


Recall $P_R = f(\rho_\infty)$

Subscript '0' denotes seal-level conditions

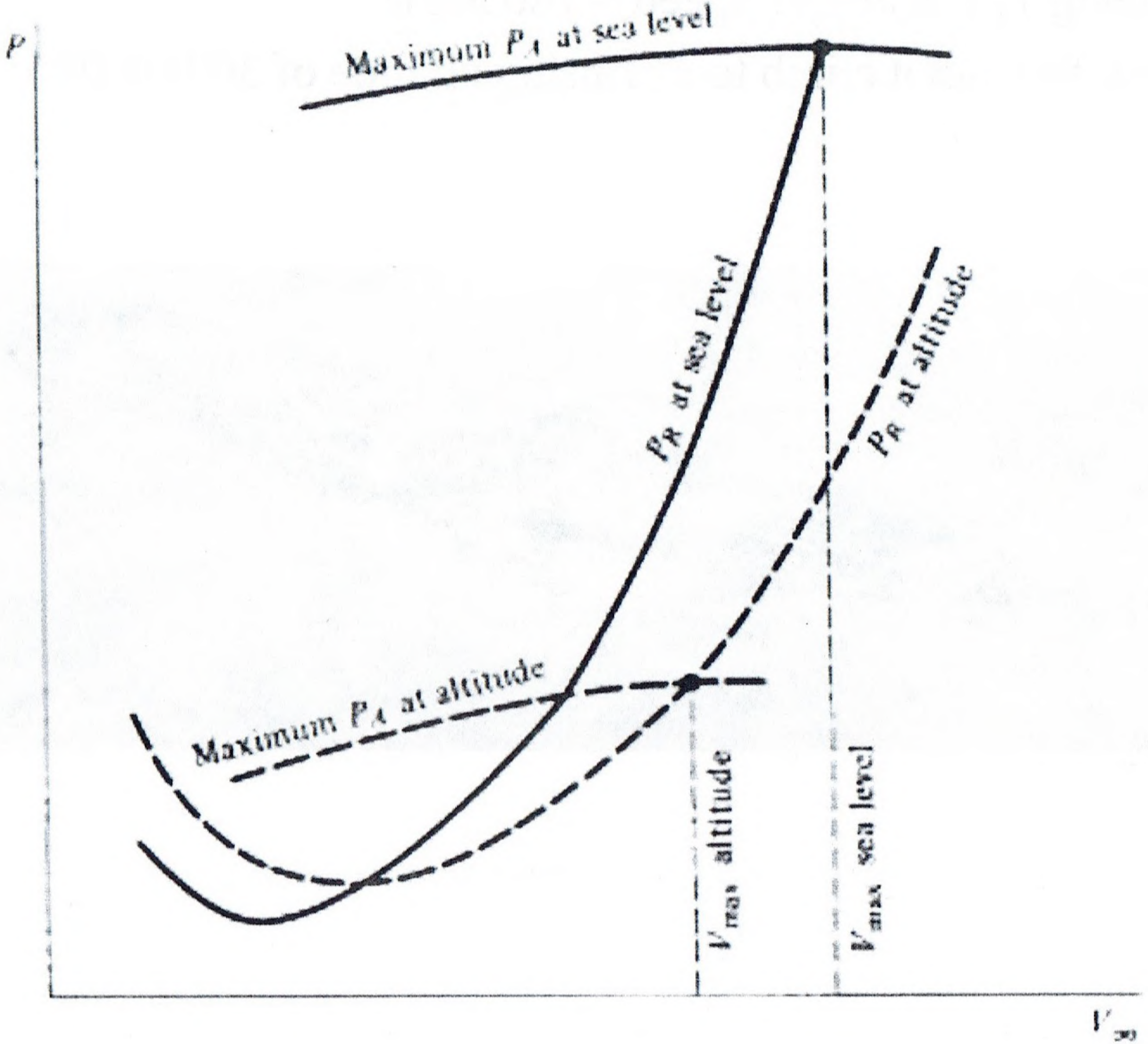
$$V_{ALT} = V_0 \left(\frac{\rho_0}{\rho} \right)^{1/2}$$

$$P_{R,ALT} = P_{R,0} \left(\frac{\rho_0}{\rho} \right)^{1/2}$$



ALTITUDE EFFECTS ON POWER REQUIRED AND AVAILABLE (6.7)

Propeller-Driven Airplane



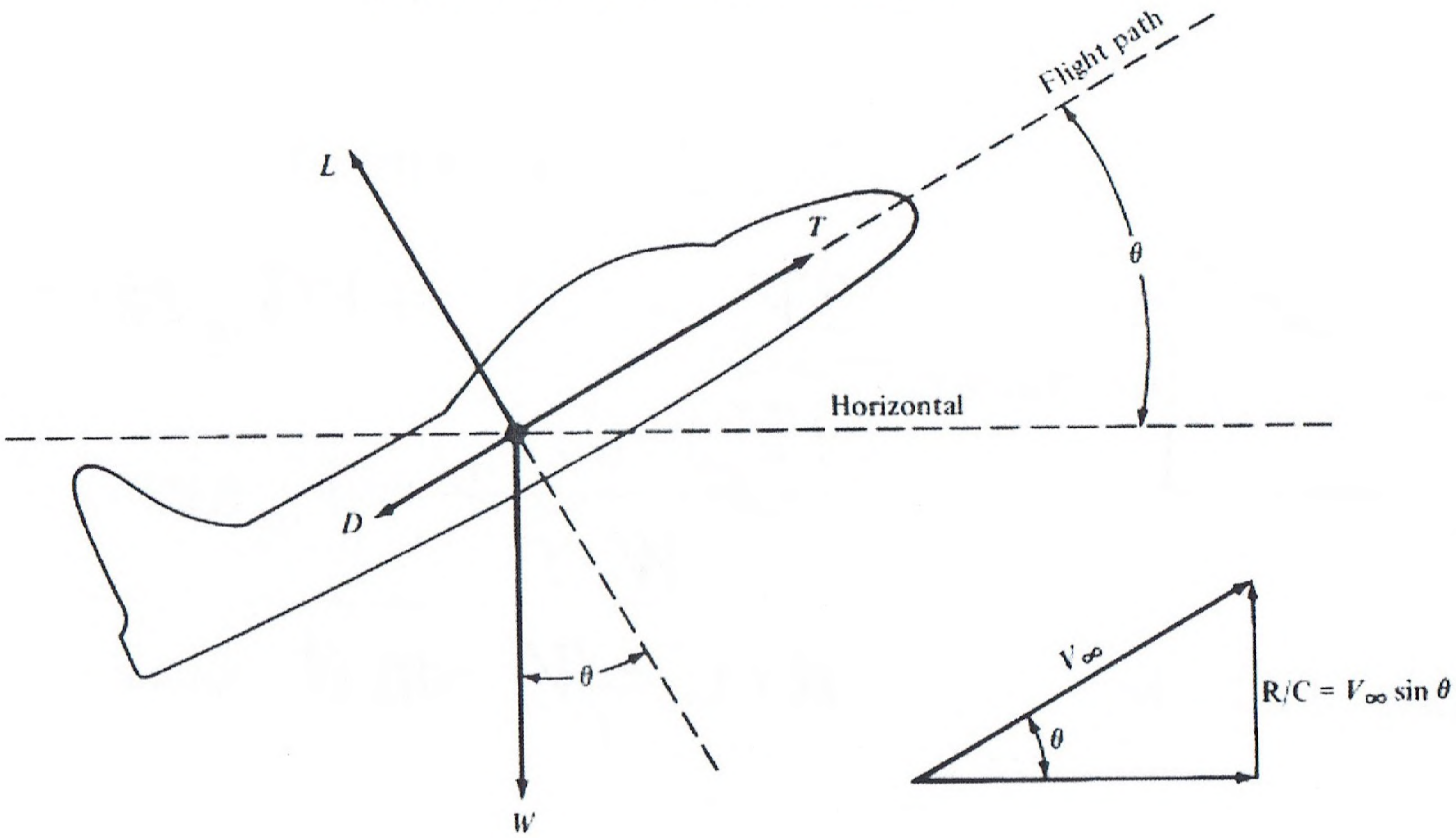
$$V_{max,ALT} < V_{max,sea\text{-}level}$$

CHAPTER 6: RATE OF CLIMB

Boeing 777: Lift-Off Speed ~ 180 MPH

How fast can it climb to a cruising altitude of 30,000 ft?

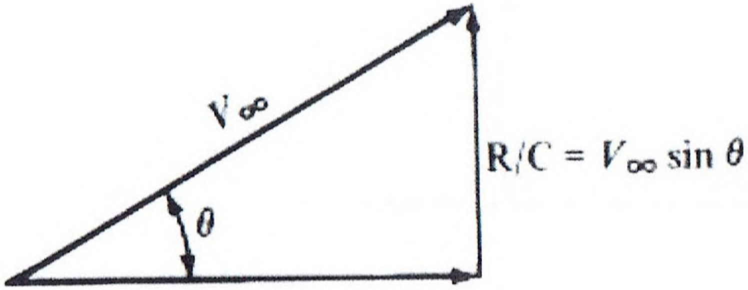
RATE OF CLIMB



Governing Equations: $$T = D + W \sin\theta$$

$$L = W\cos\theta$$

RATE OF CLIMB



$$T = D + W \sin\theta$$

$$TV_\infty = DV_\infty + WV_\infty \sin\theta$$

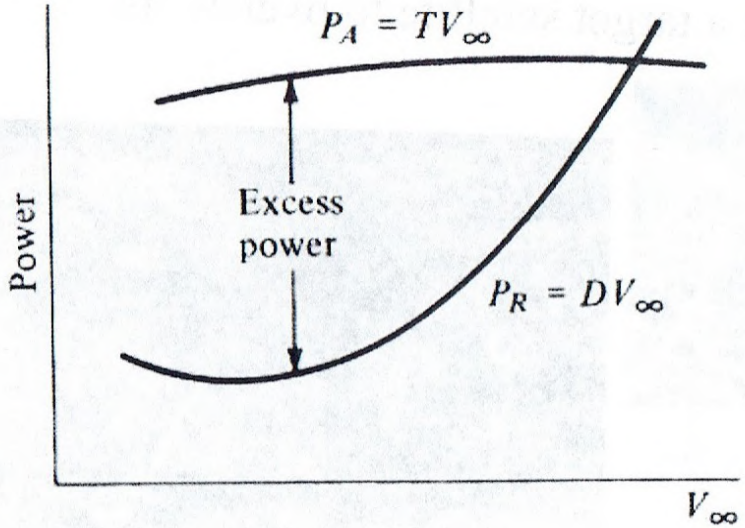
$$\frac{TV_\infty - DV_\infty}{W} = V_\infty \sin\theta$$

Rate of Climb: $R/C = V_\infty \sin\theta$ Vertical velocity

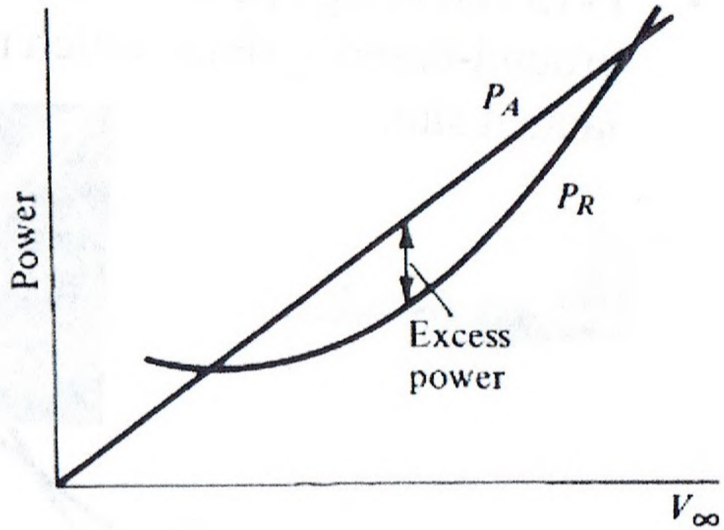
TV$_\infty$ is power available
DV$_\infty$ is level-flight power required (for small θ neglect W)
TV$_\infty$**- DV**$_\infty$ **is excess power**

RATE OF CLIMB

Propeller Drive Engine



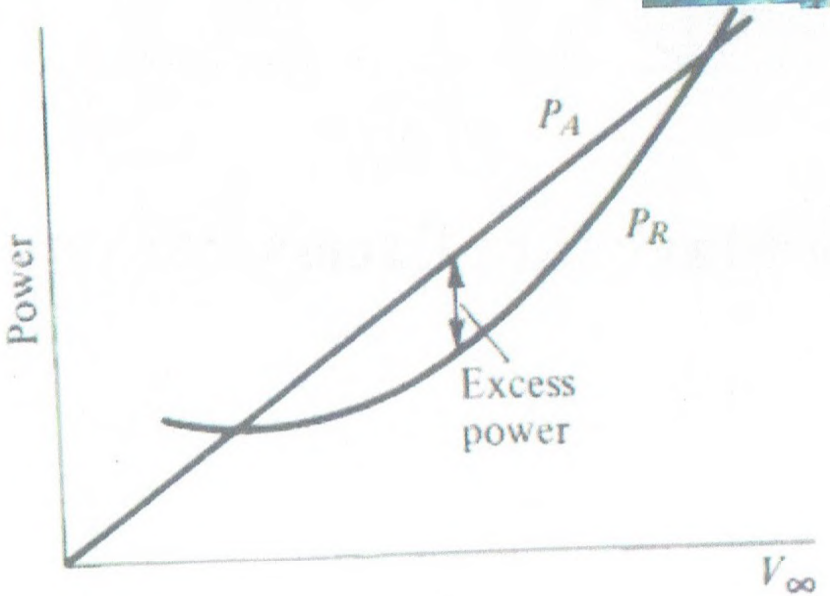
Jet Engine



Maximum R/C Occurs when Maximum Excess Power

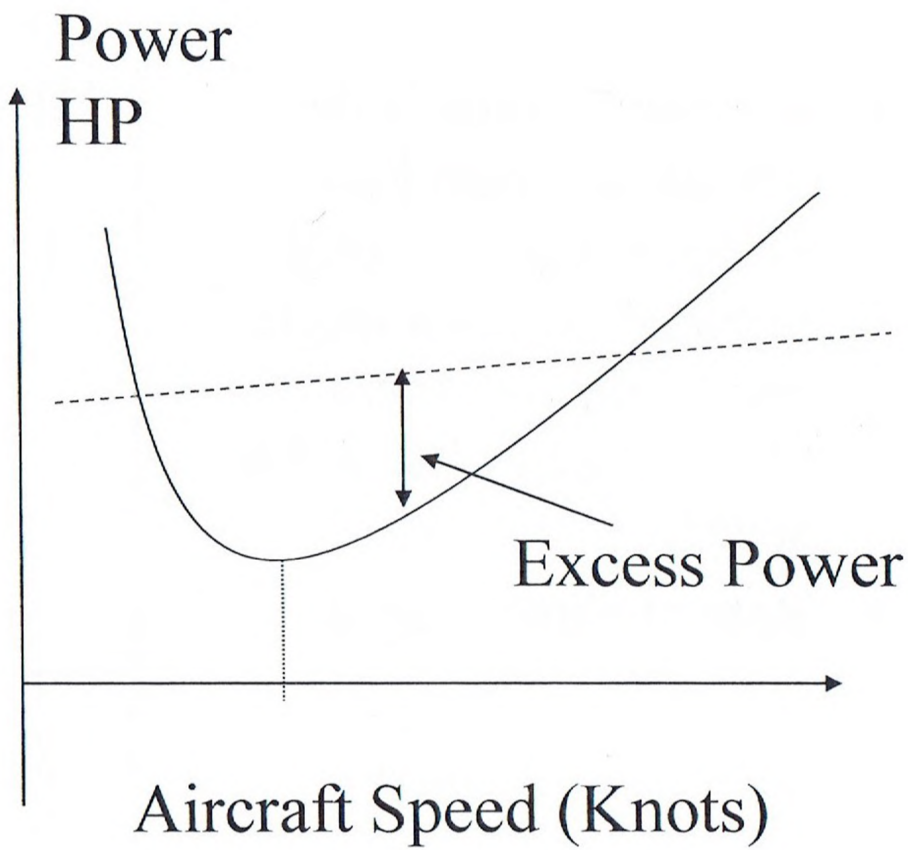
EXAMPLE: F-15 K

- Weapon launched from an F-15 fighter by a small two stage rocket, carries a heat-seeking Miniature Homing Vehicle (MHV) which destroys target by direct impact at high speed (kinetic energy weapon)
- F-15 can bring ALMV under the ground track of its target, as opposed to a ground-based system, which must wait for a target satellite to overfly its launch site.



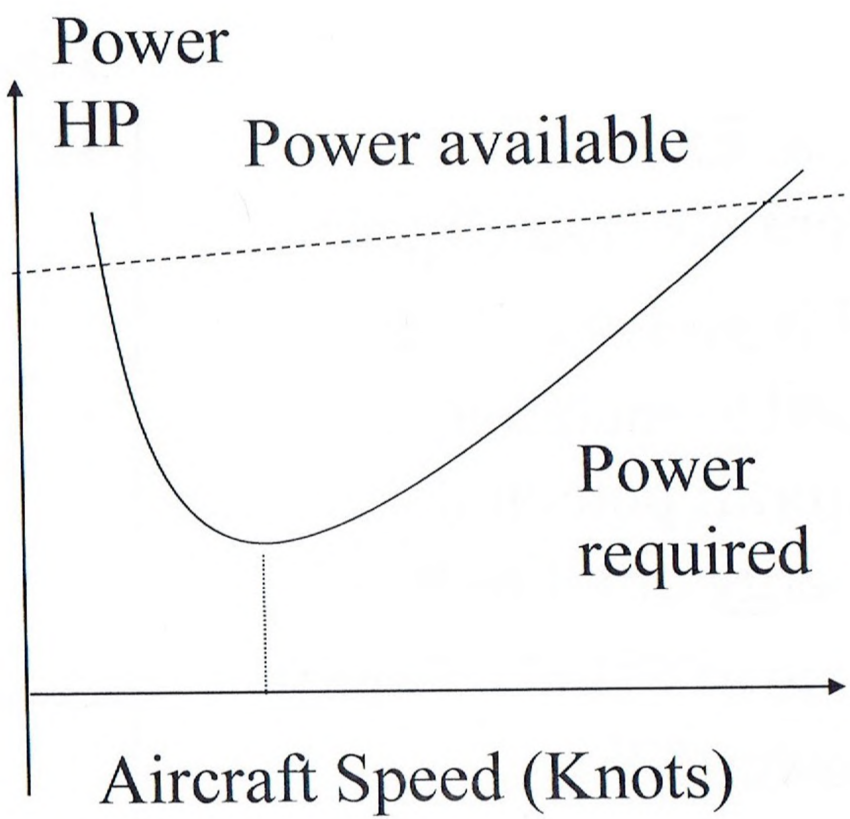
Maximum Rate of Climb

- Find Excess Power from previous figure.
- This power can be used to increase aircraft potential energy or altitude
- Rate of Climb=Excess Power/GW

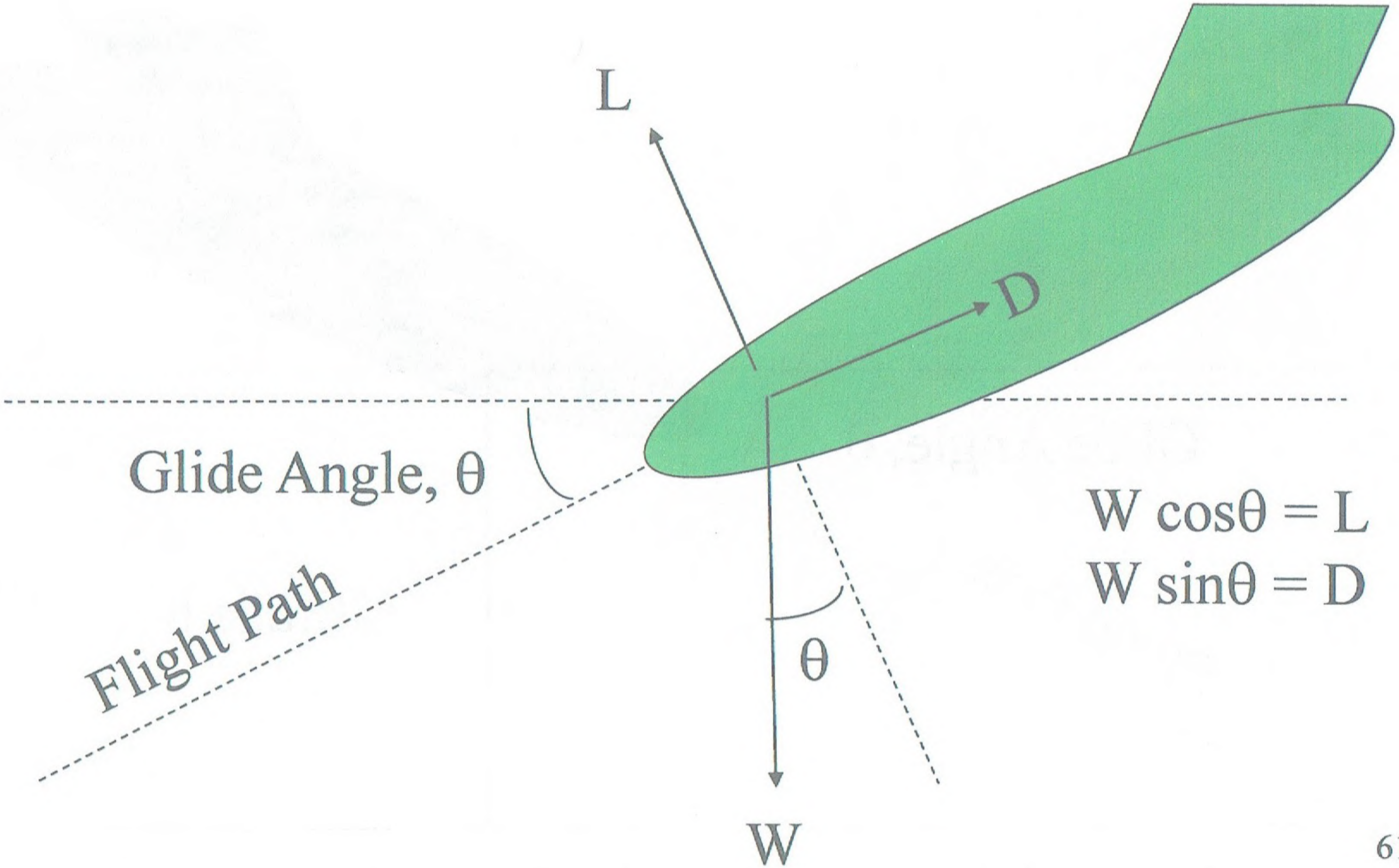


Absolute Ceiling

- Absolute ceiling is the altitude at which Power available equals power required only at a single speed, and no excess power is available at this speed.
- Rate of climb is zero.



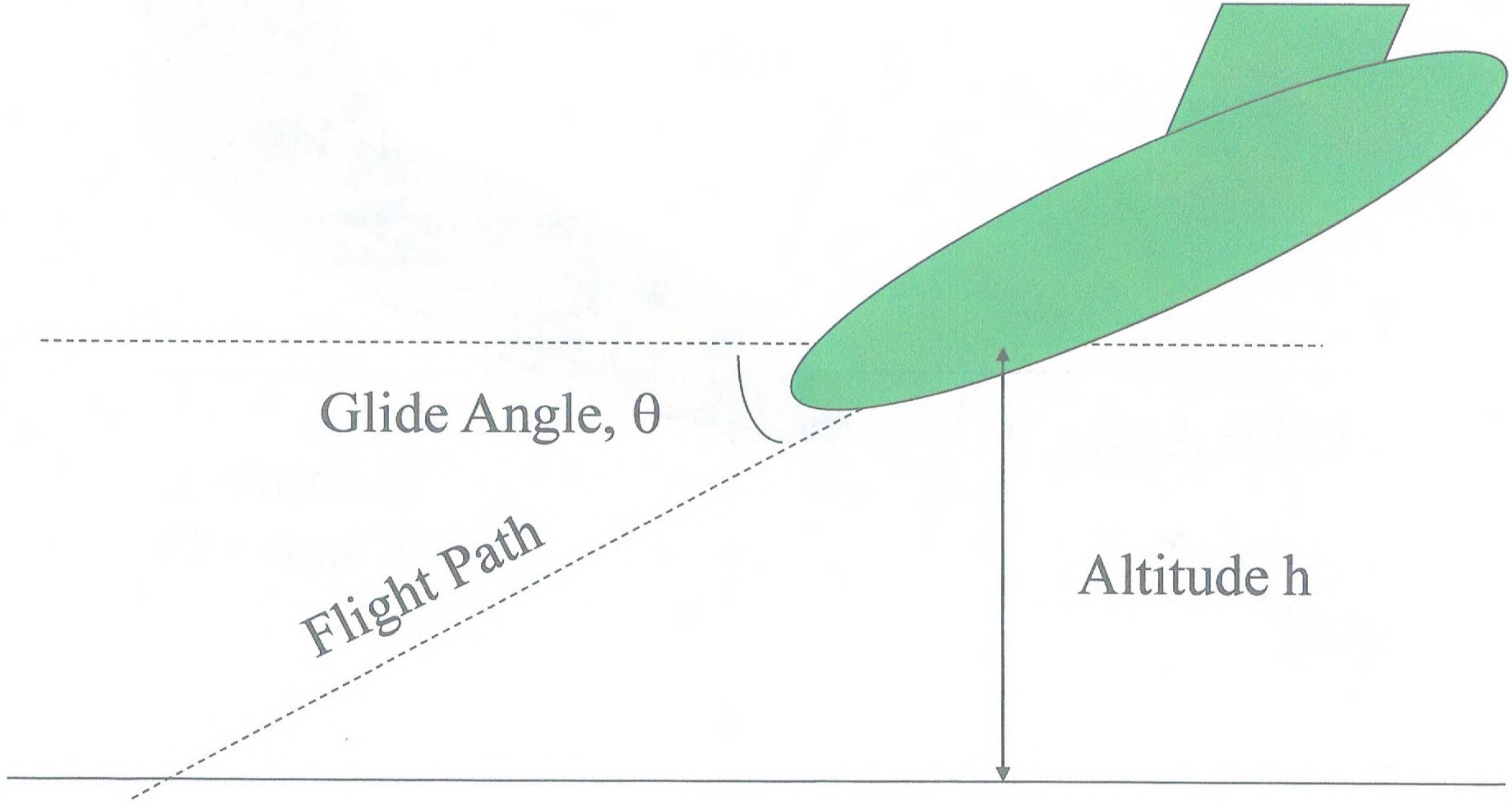
CHAPTER 7: Equilibrium Gliding Flight



$W \cos\theta = L$

$W \sin\theta = D$

Gliding Distance



Gliding Distance = $h/\tan\theta = h * L/D$

Gliding Flight

- $D=W \sin\theta$ where θ is the equilibrium glide angle.
- $L= W \cos\theta$
- $\tan\theta = D/L$
- Glide distance $= h/ \tan\theta = h (L/D)$.

CHAPTER 7: RANGE AND ENDURANCE

RANGE: How far can we fly?

ENDURANCE: How long can we stay aloft?

propeller-driven Airplane and jet-engine Airplane?

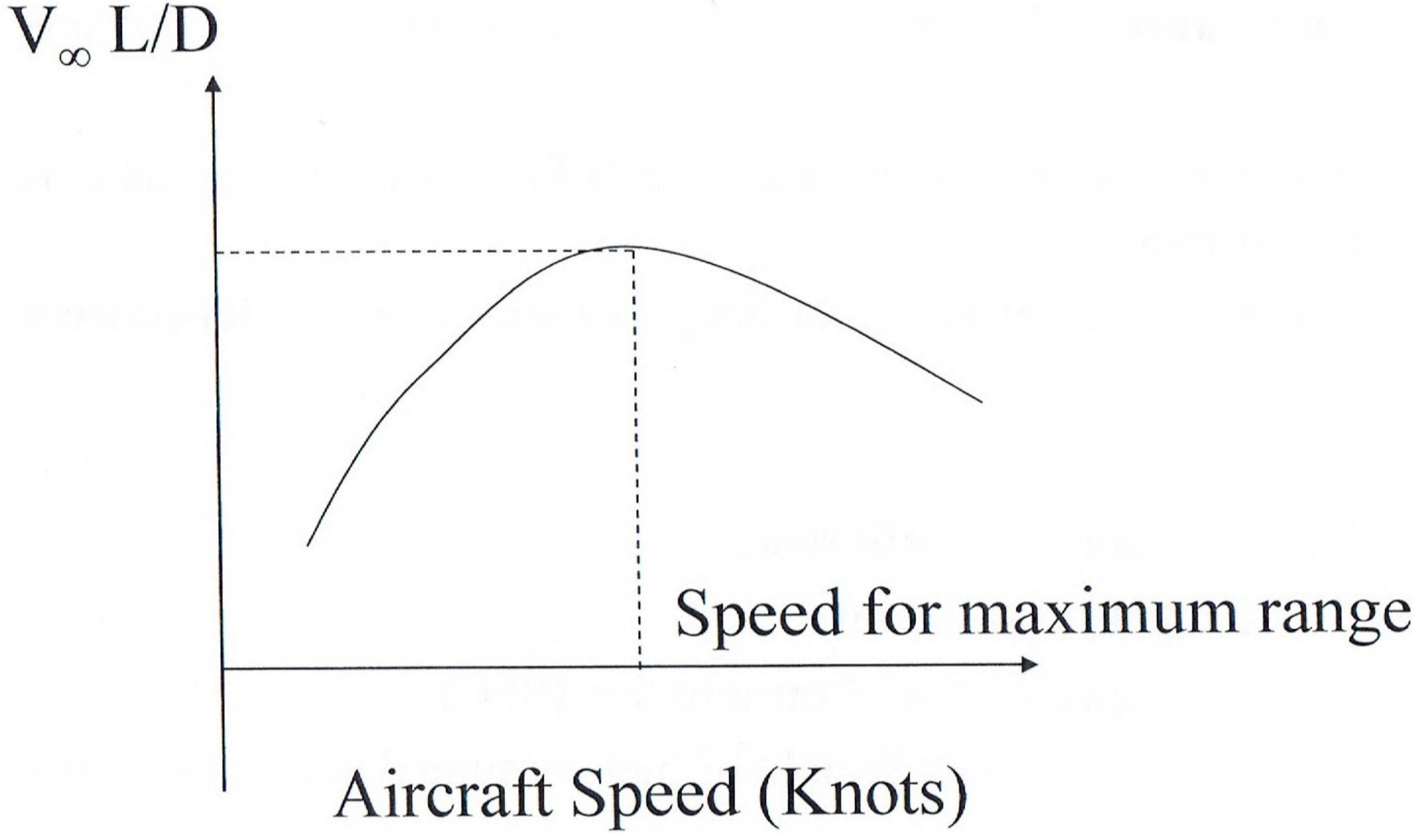
RANGE AND ENDURANCE

- **Range:** Total distance (measured with respect to the ground) traversed by airplane on a single tank of fuel
- **Endurance:** Total time that airplane stays in air on a single tank of fuel

1. Parameters to maximize **range** are different from those that maximize **endurance**
2. Parameters are different for **propeller-powered** and **jet-powered** aircraft

- Fuel Consumption Definitions
  - **Propeller-Powered:**
    - Specific Fuel Consumption (SFC)
    - Definition: Weight of fuel consumed per unit **power** per unit time
  - **Jet-Powered:**
    - Thrust Specific Fuel Consumption (TSFC)
    - Definition: Weight of fuel consumed per unit **thrust** per unit time

Cruise Speed for Maximum Range



From your level flight performance data plot V_∞ L/D vs. V_∞
As will be seen later, the speed at which V_∞ L/D is maximum gives maximum range.

Calculation of Range

We have selected a cruise V_∞.

Over a small period of time dt, the vehicle will travel a distance equal to V_∞ dt

The aircraft weight will decrease by dW as fuel is burned.

If we know the engine we use, we know the fuel burn rate per pound of thrust T. This ratio is called thrust-specific fuel consumption (Symbol used: *sfc* or just *c*).

dt = Change in the aircraft weight dW/(fuel burn rate)
= dW / (Thrust times c)
= dW/(Tc)

Distance Traveled during dt=V_∞dW/(Tc) =V_∞ [W/T](1/c) dW/W

Calculation of Range (Contd…)

- From previous slide:
  - Distance Traveled during dt=V_∞[W/T](1/c) dW/W
- Since T=D and W=L, W/T = L/D
- The aircraft is usually flown at a fixed L/D.
- The L/D is kept as high as possible during cruise.
  - Distance Traveled during dt= V_∞[L/D](1/c) dW/W

Calculation of Range (Contd...)

- From previous slide:
  - Distance Traveled during dt= $V_\infty[L/D](1/c)\ dW/W$
- Integrate between start of cruise phase, and end of cruise phase. The aircraft weight changes from W_i to W_f.
- Integral of $dx/x = \log(x)$ where natural log is used.
- Range = $V_\infty[L/D](1/c)\log(W_i/W_f)$

Breguet Range Equation

$$Range = \frac{1}{c} \cdot \frac{V_{\infty} L}{D} \cdot \log_e \left(\frac{W_{initial}}{W_{final}} \right)$$

Structures & Weights Group/
Designer Responsibility
to keep W_{final} small.

Propulsion Group/
Designer Responsibility
to choose an engine
with a low specific
fuel consumption c

Aerodynamics Group/
Designer Responsibility
to maximize this factor.

<u>PROPELLER-DRIVEN:</u> RANGE AND ENDURANCE

- SFC: Weight of fuel consumed per unit power per unit time

$$SFC = \frac{\text{lb of fuel}}{(\text{HP})(\text{hour})}$$

- **ENDURANCE:** To stay in air for longest amount of time, use minimum number of pounds of fuel per hour

$$\frac{\text{lb of fuel}}{(\text{hour})} \propto SFC(\text{HP})$$

- Minimum lb of fuel per hour obtained with minimum HP
- **Maximum endurance for a propeller-driven airplane occurs when airplane is flying at minimum power required**
- **Maximum endurance for a propeller-driven airplane occurs when airplane is flying at a velocity such that $C_L^{3/2}/C_D$ is a maximized**

PROPELLER-DRIVEN: RANGE AND ENDURANCE

- SFC: Weight of fuel consumed per unit power per unit time

$$\text{SFC} = \frac{\text{lb of fuel}}{(\text{HP})(\text{hour})}$$

- **RANGE:** To cover longest distance use minimum pounds of fuel per mile

$$\frac{\text{lb of fuel}}{(\text{mile})} \propto \frac{\text{SFC}(\text{HP})}{V_\infty}$$

- Minimum lb of fuel per hour obtained with minimum HP/V_∞
- **Maximum range for a propeller-driven airplane occurs when airplane is flying at a velocity such that C_L/C_D is a maximum**

PROPELLER-DRIVEN: RANGE BREGUET FORMULA

$$R = \frac{\eta}{SFC}\frac{C_L}{C_D}\ln\left(\frac{W_{initial}}{W_{final}}\right)$$

To maximize range:

- Largest propeller efficiency, η
- Lowest possible SFC
- Highest ratio of $W_{initial}$ to W_{final}, which is obtained with the largest fuel weight
- Fly at maximum L/D

PROPELLER-DRIVEN: RANGE BREGUET FORMULA

$$R = \boxed{\frac{\eta}{SFC}} \boxed{\frac{C_L}{C_D}} \boxed{\ln\left(\frac{W_{initial}}{W_{final}}\right)}$$

Propulsion

Aerodynamics

Structures and Materials

PROPELLER-DRIVEN: ENDURACE BREGUET FORMULA

$$E = \frac{\eta}{SFC}\frac{C_L^{3/2}}{C_D}(2\rho_\infty S)^{1/2}\left(W_{final}^{-1/2} - W_{initial}^{-1/2}\right)$$

- To maximize endurance:
  - Largest propeller efficiency, η
  - Lowest possible SFC
  - Largest fuel weight
  - Fly at maximum $C_L^{3/2}/C_D$
  - Flight at sea level

<u>JET-POWERED:</u> RANGE AND ENDURANCE

- TSFC: Weight of fuel consumed per thrust per unit time

$$\text{TSFC} = \frac{\text{lb of fuel}}{(\text{lb of thrust})(\text{hour})}$$

- **ENDURANCE:** To stay in air for longest amount of time, use minimum number of pounds of fuel per hour

$$\frac{\text{lb of fuel}}{(\text{hour})} \propto \text{TSFC}(\text{Thrust})$$

- Minimum lb of fuel per hour obtained with minimum thrust
- **Maximum endurance for a jet-powered airplane occurs when airplane is flying at minimum thrust required**
- **Maximum endurance for a jet-powered airplane occurs when airplane is flying at a velocity such that C_L/C_D is a maximum**

JET-POWERED: RANGE AND ENDURANCE

- TSFC: Weight of fuel consumed per unit power per unit time

$$\text{TSFC} = \frac{\text{lb of fuel}}{(\text{lb of thrust})(\text{hour})}$$

- **RANGE:** To cover longest distance use minimum pounds of fuel per mile

$$\frac{\text{lb of fuel}}{(\text{mile})} \propto \frac{\text{SFC}(\text{Thrust})}{V_\infty}$$

- Minimum lb of fuel per hour obtained with minimum Thrust/V_∞

$$\frac{T_R}{V_\infty} = \frac{1}{2}\rho_\infty S \sqrt{\frac{2W}{\rho_\infty S C_L}}\, C_D \propto \frac{1}{C_L^{1/2} \big/ C_D}$$

- **Maximum range for a jet-powered airplane occurs when airplane is flying at a velocity such that $C_L^{1/2}/C_D$ is a maximum**

JET-POWERED: RANGE BREGUET FORMULA

$$R = 2\sqrt{\frac{2}{\rho_\infty S}}\frac{1}{TSFC}\frac{C_L^{1/2}}{C_D}\left(W_{initial}^{1/2} - W_{final}^{1/2}\right)$$

- To maximize range:
  - Minimum TSFC
  - Maximum fuel weight
  - Flight at maximum $C_L^{1/2}/C_D$
  - Fly at high altitudes

<u>JET-POWERED:</u> ENDURACE BREGUET FORMULA

$$E = \frac{1}{TSFC} \frac{C_L}{C_D} \ln\left(\frac{W_{initial}}{W_{final}}\right)$$

- To maximize endurance:
  - Minimum TSFC
  - Maximum fuel weight
  - Flight at maximum L/D

SUMMARY: ENDURANCE AND RANGE

- Maximum Endurance
  - Propeller-Driven
    - Maximum endurance for a propeller-driven airplane occurs when airplane is flying at minimum power required
    - Maximum endurance for a propeller-driven airplane occurs when airplane is flying at a velocity such that $C_L^{3/2}/C_D$ is a maximized
  - Jet Engine-Driven
    - Maximum endurance for a jet-powered airplane occurs when airplane is flying at minimum thrust required
    - Maximum endurance for a jet-powered airplane occurs when airplane is flying at a velocity such that C_L/C_D is a maximum

- Maximum Range
  - Propeller-Driven
    - Maximum range for a propeller-driven airplane occurs when airplane is flying at a velocity such that C_L/C_D is a maximum
  - Jet Engine-Driven
    - Maximum range for a jet-powered airplane occurs when airplane is flying at a velocity such that $C_L^{1/2}/C_D$ is a maximum

Breguet Eqns - Summary

- Here is a summary of the Breguet equations:

- For piston-propellers

$$R = \frac{\eta}{c}\frac{C_L}{C_D}\ln\left(\frac{W_o}{W_1}\right)$$

$$E = \frac{\eta C_L^{3/2}}{c C_D}\sqrt{2\rho_\infty S}\left(\frac{1}{W_1^{1/2}} - \frac{1}{W_0^{1/2}}\right)$$

- For turbojets:

$$R = \frac{2}{c_t}\frac{C_L^{1/2}}{C_D}\sqrt{\frac{2}{\rho_\infty S}}\left(W_0^{1/2} - W_1^{1/2}\right)$$

$$E = \frac{1}{c_t}\frac{C_L}{C_D}\ln\left(\frac{W_o}{W_1}\right)$$

C_L and C_D relations - Summary

- For $(C_L^{3/2}/C_D)_{max}$, $C_{D,i}=3C_{D,0}$

$$C_L = \sqrt{3\pi e ARC_{D,0}}$$

$$\frac{C_L^{3/2}}{C_D} = \frac{\left(3\pi e ARC_{D,0}\right)^{3/4}}{4C_{D,0}}$$

- For $(C_L/C_D)_{max}$, $C_{D,i}=C_{D,0}$

$$C_L = \sqrt{\pi e ARC_{D,0}}$$

$$\frac{C_L}{C_D} = \frac{1}{2}\sqrt{\frac{\pi e AR}{C_{D,0}}}$$

- For $(C_L^{1/2}/C_D)_{max}$, $C_{D,i}=1/3\ C_{D,0}$

$$C_L = \sqrt{\tfrac{1}{3}\pi e ARC_{D,0}}$$

$$\frac{C_L^{1/2}}{C_D} = \frac{\left(\tfrac{1}{3}\pi e ARC_{D,0}\right)^{1/4}}{\tfrac{4}{3}C_{D,0}}$$