Steam Nozzles

2.1. Introduction

A nozzle is a duct of smoothly varying cross sectional area in which a steadily flow fluid can be made to accelerate by a pressure drop along the duct. There are many applications in practice which require a nozzle such as steam and gas turbines, jet engine and flow measurement.

2.2. Types of nozzles

There are two types of nozzles.

① Convergent Nozzle.

In this nozzle the cross section area diminishes from the inlet section to the outlet section.

② Convergent – Divergent Nozzle.

In this type the cross-sectional area diminishes from inlet section to another section which is known as throat and beyond this section, the nozzle area increases till the exit area.

2.3. Definitions:

1. Mach number (M) = velocity of fluid / velocity of sound through a fluid

2. Back Pressure: It is a Pressure beyond exit section of nozzle

3. Chocking: When M=1 at the throat of nozzle, then the nozzle is chocking and then there is a maximum mass flow rate for any inlet conditions and all Properties at the throat known as critical Properties

4. Design condition:

A. Convergent nozzle: It is a condition when M=1 at the throat and back Pressure = exit Pressure = critical Pressure.

B. Convergent Divergent nozzle:

It is a condition when M=1 at the throat and the steam is expanding from inlet pressure to the back Pressure (fully expansion) and back Pressure = exit Pressure.

2.4. Phenomena in nozzles operating off the design condi..

It is also of interest to know what happens to the fluid in a nozzle of a given dimensions when the back Pressure differs from the value for which the nozzle is designed.

A: Convergent nozzle:

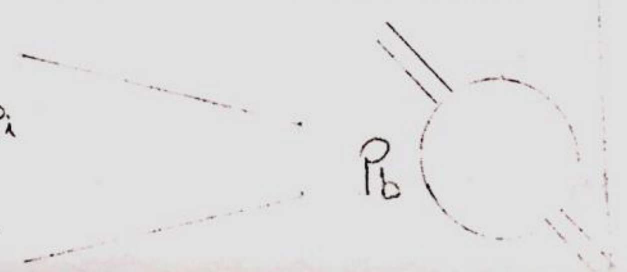
The inlet conditions are maintained constant and the back Pressure (P_b) is gradually reduced from the inlet Pressure (P_i)

1- $P_b = P_i$ there is no pressure drop and no flow ($\dot{m}=0$) ∴ Curve (i)

2. When P_b is reduced then the axial pressure distribution shown in curve (ii) and there is mass flow rate through a nozzle and $M<1$ along nozzle.

3. $P_b = P_{critical}$ the pressure distribution as shown in curve (iii), there is mass flow rate which is more than that in (2) and it is the maximum mass flow for this inlet conditions. $M=1$ at exit of nozzle and <1 for any section through nozzle.

4. $P_b < P_c$. Pressure distribution as shown in (iv), and there is no changing in mass flow rate that achieve in ③, in this condition the fluid undergoes an irreversible expantion out of exit nozzle.

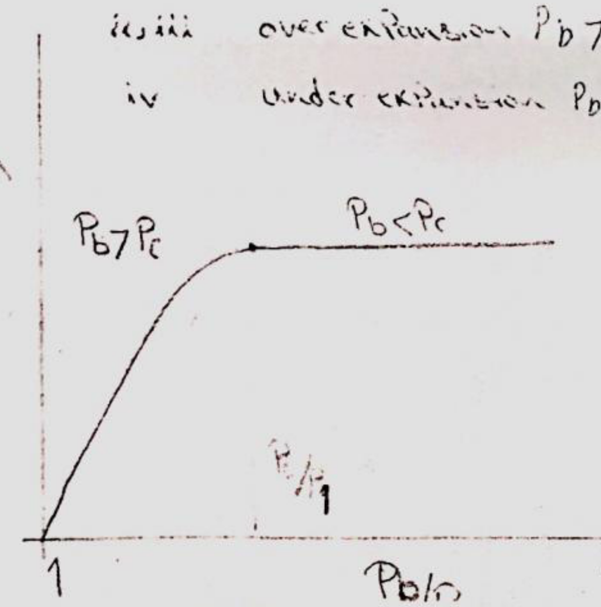


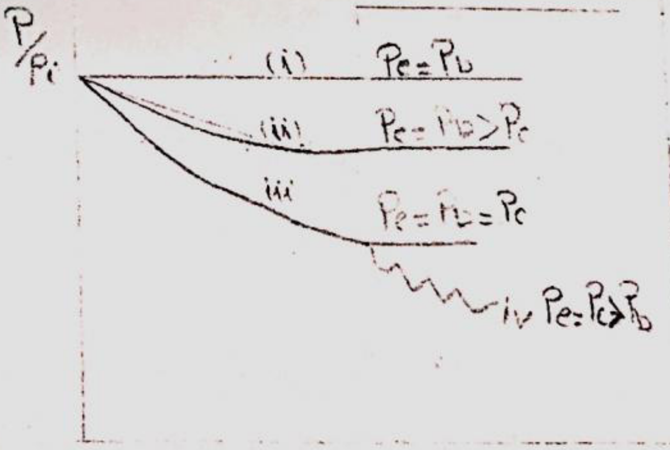
Note, 1_3 $P_{exit} = P_{back}$

4 $P_{exit} = P_{critical} > P_b$

ii, iii over expansion $P_b > P_c$

iv under expansion $P_b < P_c$





D) Convergent - Divergent nozzle:-

(1) $P_b = P_i$ .. $M = 0$ curve (i)

(2) $P_b < P_i$ there is (M) and the convergent part works as nozzle (velocity increases), divergent part works as diffuser (Pressure increases). In this condition $M<1$ along Con-Div duct which likes a venturimeter in this condition curve (ii)

(3) $P_b < P_i$, $P_{throat} = P_{critical}$. in this condition. Convergent part works as nozzle $M<1$ along it and divergent part works as diffuser $M<1$ along it, in this condition $M=1$ at throat and then the nozzle is choking. Curve (iii)

(4) P_b is less than that in (3), the process of acceleration continues beyond convergent part to a point at which shock wave is formed. A shock wave involves a rise of pressure, entropy and deceleration from supersonic ($M>1$) to subsonic ($M<1$), and in (4) no changing occures in m from that in (3) (iv)

(5) P_b is less than that in (4) the shock wave moves towards the exit of the nozzle (v)

(6) P_b = design value the expansion continues from inlet to the exit without any shock wave in or out of the nozzle, this condition is known as design condition. curve (vi)

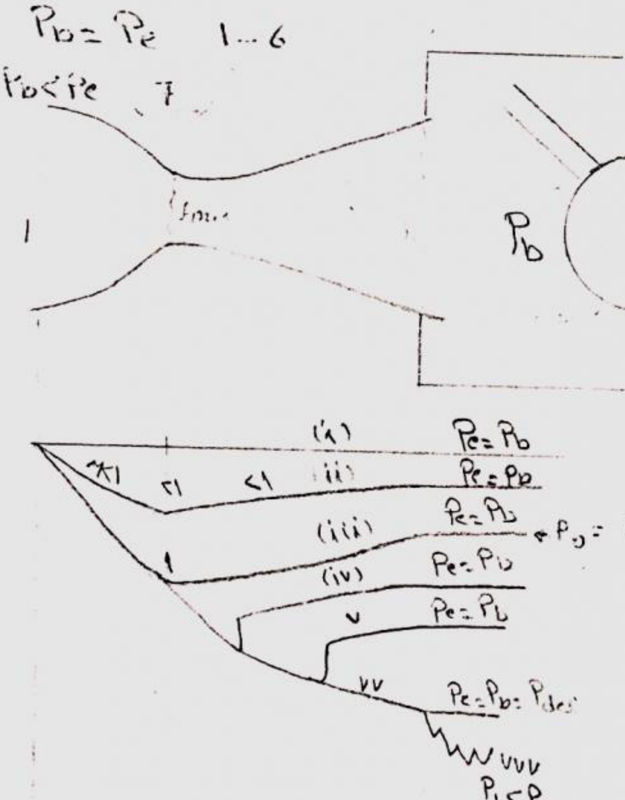
(7) P_b < design value the same expansion occures as (6)

nozzle   curve (vvv)

$P_b > P_{design}$

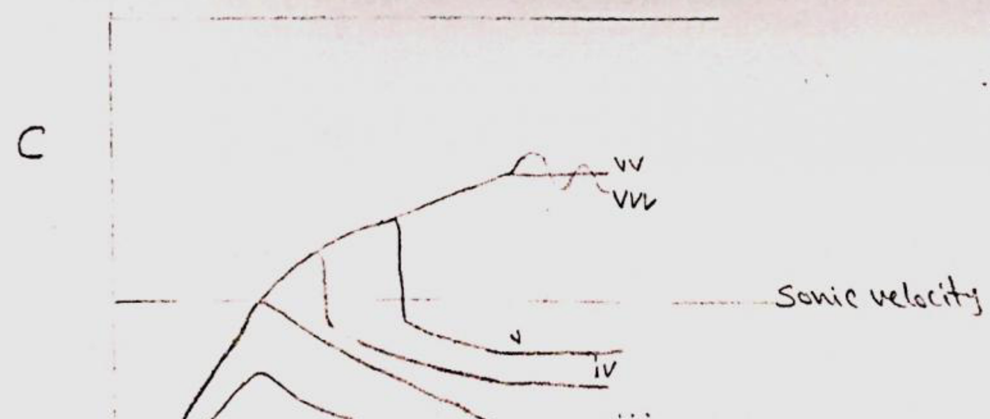
Note :- * from Point 2 - 5 expansion is said to be over-expansion

* Point is under expansion $P_b < P_{design}$
* $\dot{m}$ from 3 — 7 is the same = Maximum for this inlet conditions
* $P_b = P_e$ 1 ... 6
* $P_b < P_e$ 7



$P_D = P_1 \left[0.546 + 0.454 \cdot \sqrt{\frac{f_1 - f_{min}}{f_1}}\right]$

$\sqrt{1 - \left(\frac{f_{min}}{f_1}\right)^2}$



2.5. Relationship between A, c and Pressure in nozzle

From any two steady flow energy equation between any two planes an infinitesimal distance apart

$$dQ = dh + d\left(\frac{c^2}{2}\right) + dw \quad \text{------} ①$$

for nozzle $dQ = dw = 0$

$$dh + d\left(\frac{c^2}{2}\right) = 0 \quad \text{------} ②$$

From second law of thermodynamics for reversible flow

$$dQ = Tds \quad \text{------} ③$$

$h = u + pv$

$$dh = de + Pdv + vdP$$

$$de = dh - pdv - vdp$$

first law

$dQ = dh - pdv - vdp + pdv$

$$dQ = de + Pdv \quad \text{------} ④$$

$$Tds = dh - vdP \quad \text{------} ⑤$$

for isentropic flow

$$dh = vdP \quad \text{------} ⑥$$

from Continuity equation

$$m = Ac/v = \text{Constant}$$

$$\log A + \log c - \log v = c$$

by differentiation

$$\frac{dA}{A} + \frac{dc}{c} - \frac{dv}{v} = 0 \quad \text{------} ⑦$$

from eq. 2 & 6

$vdp + d\left(\frac{c^2}{2}\right) = 0$

$$vdP + d\left(\frac{c^2}{2}\right) = 0 \quad \text{------} ⑧$$

$$vdP + 2c\frac{dc}{2} = 0 \qquad dc = -\frac{v\,dP}{c} \quad \text{—} ⑧$$

$$\frac{1}{2}(C_2^2 - C_1^2) = -\int_1^2 v\,dP \quad \text{-----} (9)$$

from eq. 7, 8

$$\frac{dA}{A} = \frac{v}{C^2}dP + \frac{dv}{v}$$

$$= v\,dP\left(\frac{1}{C^2} + \frac{1}{v^2}\frac{dv}{dP}\right) \quad \text{-----} (10)$$

Velocity of sound (a) is given by the following relationship

$$a^2 = -v^2\frac{dP}{dv} \quad \text{-----} (11)$$

from 10, 11

$$\frac{dA}{A} = v\,dP\left[\frac{1}{C^2} - \frac{1}{a^2}\right] \quad \text{-----} (12)$$

The following conclusion can be drawn from eq 12.

(1) Accelerated flow (nozzle)

dP must be negative so that

(i) when $C < a$

$\therefore$ dA is negative (nozzle is convergent)

(ii) When $C > a$

$\therefore$ dA is positive (nozzle is divergent)

② Decelerated flow (Diffuser)

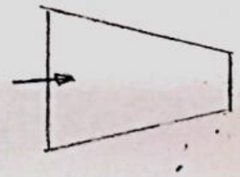
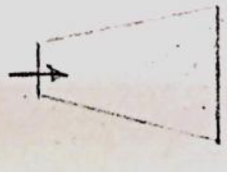
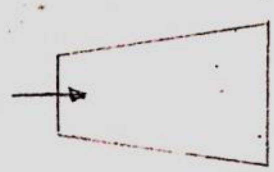
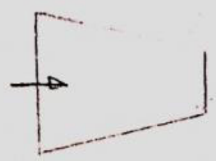
dP must be positive so that

① when $C < a$
dA is positive diffuse is divergent

ii) when $C > a$
dA is negative diffuser is convergent

③ Constant velocity flow

dP is Zero
$\therefore C \gtrless a \quad dA = 0$

type of flow	Nozzle P decrease	diffuser P↑
subsonic flow		
supersonic		

2.6 Flow through the nozzle and its basic equations:

(1) Velocity.

From energy equation between any two sections through the nozzle

$Q - W = h_2 - h_1 + \frac{C_2^2}{2} - \frac{C_1^2}{2}$

$Q = W = 0$

$\therefore h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}$

$\therefore C_2 = \sqrt{2(h_1 - h_2) + C_1^2}$ ---- (13)

Equation (13) gives velocity of fluid through nozzle at any section and it is true for reversible and irreversible flow

If inlet velocity C_1 is negligible then C_2.

$C_2 = \sqrt{2(h_1 - h_2)}$ ----- (14)

and for perfect gas

$C_2 = \sqrt{2\,C_P\,(T_1 - T_2)}$ ---- (15)

(2) Critical pressure.

It is a pressure at the throat of nozzle when nozzle is chocking.

The isentropic flow of steam through the nozzle may be approximated represented by an equation of the form.

$Pv^n = C$ $\qquad$ $n = 1.135$ for steam initially dry
$\qquad\qquad\qquad$ $n = 1.3$ for steam initially superheated

From eq^n (9) $\quad$ from energy equation and second law of thermodynamics we can prove for reversible flow that

$$\frac{1}{2}(C_2^2 - C_1^2) = -\int_1^2 v\,dP$$

if $C_1 = 0$ and $Pv^n = C$ $\quad \therefore v = \frac{\text{Constant}}{P^{\frac{1}{n}}}$

then

$$\frac{1}{2}(C_2^2) = -\int_1^2 \frac{\text{Constant}}{P^{\frac{1}{n}}}\,dP$$

$$= -\left[\text{Constant} \times \frac{P^{\frac{n-1}{n}}}{\frac{n-1}{n}}\right]_1^2$$

$$= -\left[P^{\frac{1}{n}} v \times \frac{n}{n-1} \times P^{\frac{n-1}{n}}\right]_1^2$$

$$\frac{1}{2} C_2^2 = \frac{n}{1-n}(P_2 v_2 - P_1 v_1) \qquad \frac{v_2}{v_1} = \left(\frac{P_1}{P_2}\right)^{\frac{1}{n}}$$

$$C_2 = \sqrt{\frac{2n}{n-1} P_1 v_1 \left(1 - \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}\right)} \quad \text{----} (16)$$

Equation (16) gives approximated velocity of steam at any section through the nozzle when flow is reversible only.

From Continuity equation

$$m = \frac{AC}{v} = \frac{A_2 C_2}{v_2}$$

$$P_2 v_2^n = P_1 v_1^n$$

Mass flow per unit area

$$\frac{m}{A_2} = \frac{C_2}{v_1 (P_1/P_2)^{\frac{1}{n}}}$$

Sub. C_2 from eq 16

$$\therefore \frac{m}{A_2} = \sqrt{\frac{2n}{n-1} P_1 v_1 \left(1 - \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}\right)} \Big/ \left(\frac{P_1}{P_2}\right)^{\frac{1}{n}} v_1$$

$$\frac{m}{A_2} = \sqrt{\frac{2n}{1-n} \frac{P_1}{v_1} \left\{ \left(\frac{P_2}{P_1}\right)^{\frac{n+1}{n}} - \left(\frac{P_2}{P_1}\right)^{\frac{2}{n}} \right\}} \quad \text{------ (17)}$$

The mass flow per unit area is maximum at the throat and from eq (17) for any inlet condition m/A changes with P_2 only; then $\left(\frac{m}{A}\right)_{max}$ achieved by equating
and 2 refer to throat section

$$\frac{d\,(m/A_2)}{d P_2}_{P_1} = 0$$

$$\frac{d\,(m/A_2)}{d P_2}_{P_1} = \frac{\frac{2n}{1-n} \frac{P_1}{v_1} \left[\frac{n+1}{n} \left(\frac{P_2}{P_1}\right)^{\frac{1}{n}} - \frac{2}{n} \left(\frac{P_2}{P_1}\right)^{\frac{2-n}{n}} \right]}{2\sqrt{\frac{2n}{1-n} P_1 v_1 \left(\left(\frac{P_2}{P_1}\right)^{\frac{n+1}{n}} - \left(\frac{P_2}{P_1}\right)^{\frac{2}{n}} \right)}} = 0$$

$$\therefore \left[\frac{n+1}{n} \left(\frac{P_2}{P_1}\right)^{\frac{1}{n}} - \frac{2}{n} \left(\frac{P_2}{P_1}\right)^{\frac{2-n}{n}} \right] = 0$$

$$\frac{2}{n+1} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{n}} * \left(\frac{P_2}{P_1}\right)^{\frac{n-2}{n}}$$

$$\therefore \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = \frac{2}{n+1}$$

2 refres to throat section $P_2 = P_{throat}$ and because mass flow is max. then nozzle is chocking and $P_{throat} = P_{critical}$

$$\therefore \left(\frac{P_c}{P_1}\right)^{\frac{n-1}{n}} = \frac{2}{n+1} \quad ---- \quad (18)$$

eq (18) is true for Isentropic flow and when initial velocity $\simeq 0$

and for perfect gas only

$$\left(\frac{P_c}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \frac{2}{\gamma+1} \quad ---- \quad (19)$$

$$\left(\frac{T_c}{T_1}\right) = \left(\frac{P_c}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_c}{T_1} = \frac{2}{\gamma+1} \quad ---- \quad (20)$$

eq. (19 & 20) are true for Perfect gas when flow is isentropic and $C_1 = 0$

When there is a friction critical pressure ratio becomes

$$\left(\frac{P_c}{P_1}\right)^{\frac{m-1}{m}} = \frac{2}{m+1} \quad --- \quad (21)$$

$$m = \frac{n}{\eta_n + n(1-\eta_n)} \qquad \eta_n = \text{Nozzle effic.}$$

eq 21 for actual flow & $C_1 = 0$

When nozzle is checking $C_t = a$ (sound velocity)

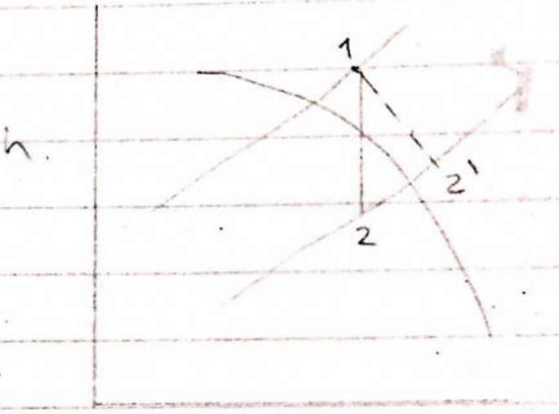
$$C_t = \sqrt{n P_t v_t} \quad ----- \quad (22)$$

eq (22) gives approximate value of critical velocity

2-7 Nozzle efficiency

η_n is the ratio of the actual enthalpy drop to the isentropic enthalpy drop between the same pressures

$$\eta_n = \frac{h_1 - h_{2'}}{h_1 - h_2}$$



$$\eta_n = \frac{c_{2'}^2 - c_1^2}{c_2^2 - c_1^2}$$

if $c_1 \simeq 0$

$$\eta_n = c_{2'}^2 / c_2^2$$

But $c_{2'}/c_2$ called velocity coefficient (C_v)

$\therefore \eta_n = (\text{velocity coefficient})^2$

From eq 16

$$c_2 = \sqrt{\frac{2n}{n-1} P_1 v_1 \left(1 - \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}\right)}$$

$$c_c = \sqrt{\frac{2n}{n-1} \cdot \frac{P_c}{\left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}} \cdot v_1 \left(1 - \left(\frac{2}{n+1}\right)\right)}$$

$$P_1 v_1^n = P_c v_c^n \qquad v_1 = \left(\frac{P_c}{P_1}\right)^{\frac{1}{n}} v_c = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}\cdot\frac{1}{n}} \cdot v_c =$$

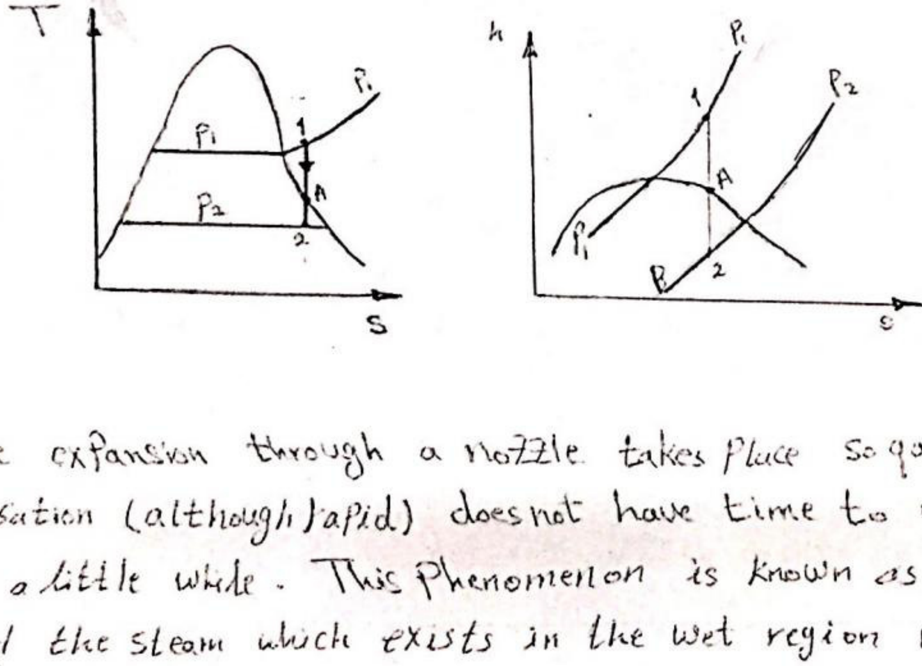
$$c_c = \sqrt{\frac{2n}{n-1} \cdot \frac{P_c}{\left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}} \cdot v_c \left(\frac{2}{n+1}\right)^{\frac{1}{n-1}} \cdot \left(1 - \frac{2}{n+1}\right)}$$

$$\sqrt{\frac{2n}{n-1} \cdot P_c v_c \cdot \left(\frac{2}{n+1}\right)^{\frac{-n}{n-1}} \cdot \left(\frac{2}{n+1}\right)^{\frac{1}{n-1}} \cdot \frac{n-1}{n+1}}$$

Supersaturation

The approach to the problem of steam flow through the nozzle will depend upon whether the steam can be considered as being in equilibrium or supersaturated.

* When a superheated vapour expands isentropically and slowly (ideal case), condensation within the vapour begins to form when the saturated vapour line is reached. As the expansion continues below this line into the wet regions then condensation proceeds gradually and the dryness fraction of the steam becomes less. This type of expansion called equilibrium and is illustrated on T-s and h-s diagrams.

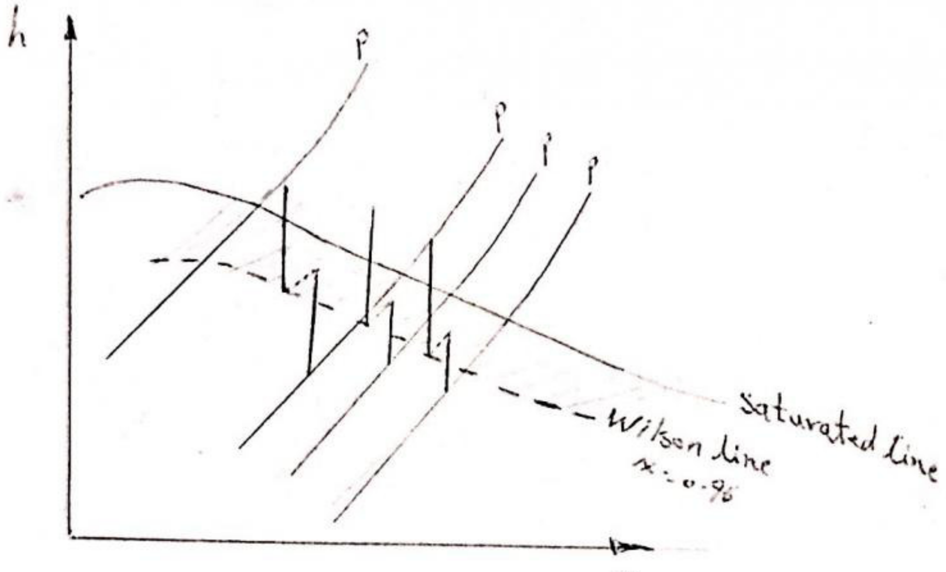


A. Point represents level at which condensation within vapour just begins.

* The expansion through a nozzle takes place so quickly that condensation (although rapid) does not have time to occur, and delay for a little while. This phenomenon is known as supersaturation. and the steam which exists in the wet region without containing any liquid is called supersaturated steam. Supersaturation states are non equilibrium (or metastable) states.

and steam in this region expands as superheat steam.

* During the expansion process, the steam reaches a temperature lower than that normally required for the condensation process to begin. Once the temperature drops a sufficient amount below the saturation temperature corresponding to the local pressure, groups of steam moisture droplets of sufficient size are formed, and condensation occurs rapidly.
The locus of points where condensation will take place regardless of the initial temperature and pressure at nozzle entrance is called Wilson line which lies between $0.95 \leq x \leq 0.96$ curves.



* Steam between saturated line and Wilson line is called supersaturated steam and expansion is called supersaturation (metastable) expansion.

* From above figure we can say that the point at which condensation occurs may be within the nozzle or after the steam leaves the nozzle.

$\therefore A_2 = \dot{m} * C_R / v_R = 0,1 * 5$

$$A_2 : \frac{m v_R}{C_R} = \frac{0,1 * 0,576 * 10^6}{568} = 101,5\ mm^2$$

$A_{2_{supersaturated}} < A_{2_{eql.}}$ for the same mass flow rate

To find the degree of supercooling

since for supersaturated steam

$$\frac{T_1}{T_R} = \left(\frac{P_1}{P_2}\right)^{\frac{n-1}{n}}$$

$$T_R = 389\ K = 116^\circ C$$

from steam tables at P_2 $t_s = 133,5^\circ C$

$\therefore$ degree of super cooling $= 133,5 - 116 = 17,5\ K$

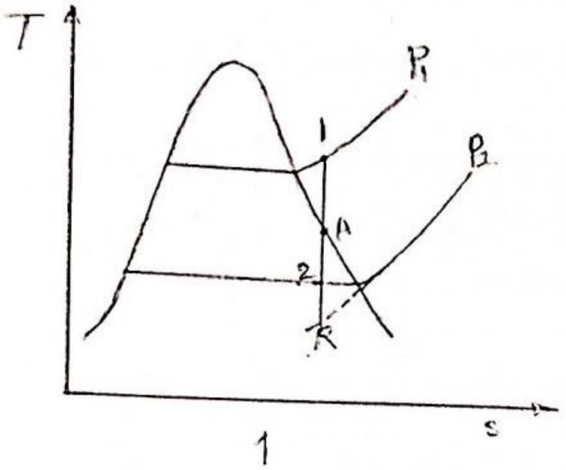
To fin the degree of supersaturation

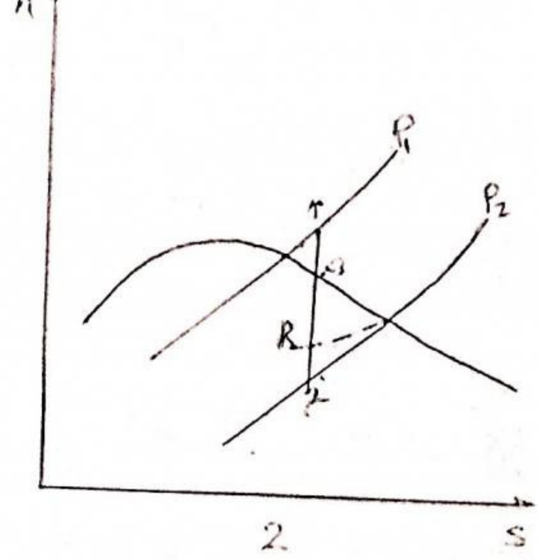
$$\text{degree of supersaturation} = \frac{P_2}{P_s \text{ at } t_R}$$

$$P_s \text{ at } t_R = 1,7 + \left(\frac{116 - 115,2}{116,9 - 115,2}\right) \cdot 0,1 = 1,75\ \text{bar}$$

$$\text{degree of super saturation} = \frac{3}{1,75} = 1,713$$

Properties of super-saturated steam:





* . In this Case Condensation not occurs within the nozzle.
* $1 \longrightarrow 2$ equilibrium expansion.
  $1 \longrightarrow R$ Metastable expansion.

From figures (1) ~~and~~

$T_R < T_2$ Steam Sometimes Called Super-Cooled steam because at any Pressures between A & 2 the temperature of steam is always less than the saturation temperature Corresponding to that Pressure.

Degree of undercooling (supercooling) = $T_2 - T_R$

T_2 : Saturation temperature corresponding to P_2

$T_R = T_1 \left(P_2/P_1\right)^{\frac{n-1}{n}}$ $\quad n = 1.3$

Degree of supersaturation = $\dfrac{\text{Actual Pressure } P_2}{\text{Saturation Pressure at } T_R}$

From figure (2)

$$h_R > h_2$$

$$\therefore \; h_1 - h_R < h_1 - h_2$$

$$C_R = \sqrt{2\Delta h} < C_2 = \quad \text{(the difference is small)}$$

$$C_2 = 44.72\sqrt{(h_1 - h_2)} \quad (h_1 - h_2) \text{ from mollier chart}$$

but $C_R = 44.72\sqrt{(h_1 - h_R)}$ mollier chart cannot be used to find h_R.

from $Pv^n = c$

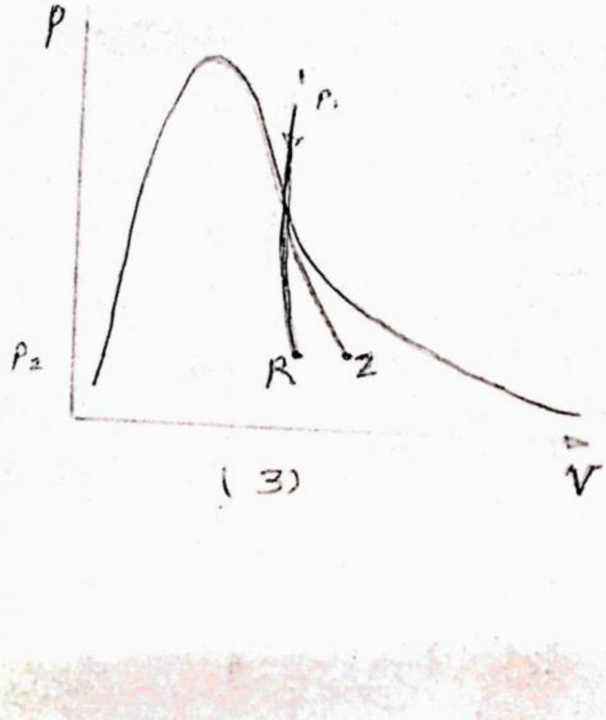
$$C_R = \sqrt{\frac{2n}{n-1}\left(P_1 v_1 - P_2 v_R\right)}$$

$\dot{m} = A_2 C_2 / v_2$ for equilibrium

$\dot{m} = A_2 C_R / v_R$ for super saturated

$C_R < C_2$ (are very nearly equal)

but from figure(3) $v_R < v_2$



(3)

$$\therefore \; \dot{m}_s > \dot{m}_e$$

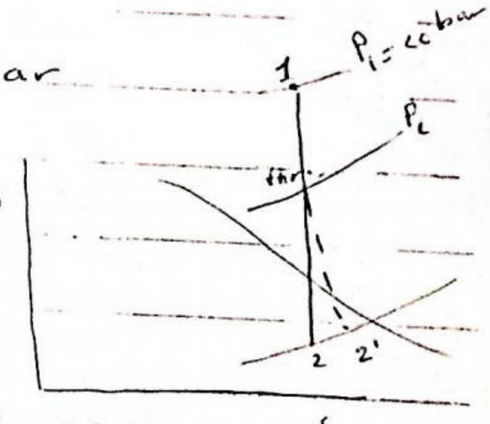
Ex. Steam enters a converging - diverging nozzle at 20 bar and 400°c with a negligible velocity and mass flow rate of 2.5 kg/s, and it exits at a pressure of 3 bar. The flow is isentropic between the nozzle entrance and throat, and the overall efficiency is 93%. Determine the throat and exit areas and the Mach number at the throat.

Solution.

$$P_t = P_c = P_1 \cdot \left(\frac{2}{n+1}\right)^{n/n-1} = 10.9 \text{ bar}$$

$h_1 = 3247.6$ kJ/kg (at P_1 & t_1)

$S_1 = S_t = S_{2s} = 7.1271$ kJ/kg.K



at throat

$P_t = 10.9$ bar , $h_t = 3076$ kJ/kg , $v_t = 0.242$ m³/kg

$$C_{throat} = \sqrt{2 \cdot 10^3 \cdot (3247.6 - 3076)} = \boxed{585.7} \text{ m/s}$$

$$\text{throat area} = m \cdot v_t / C_{th.} = \frac{0.242 \cdot 2.5}{585.7} = \boxed{10.33 \cdot 10^{-4}} \text{ m}^2$$

at exit,

for isentropic,

$h_2 = 2781.9$ kJ/kg

$$\eta_N = \frac{h_1 - h_2'}{h_1 - h_2} \qquad 0.93 = \frac{3247.6 - h_2'}{3247.6 - 2781.9} \qquad h_2' = 2814.5 \frac{kJ}{kg}$$

at (2') $v_2' = 0.6764$ m³/kg , $s_2' = 7.2009$ kJ/kg.K

$$C_2' = \sqrt{2 \cdot 10^3 \cdot (h_1 - h_2')} = \sqrt{2 \cdot 10^3 (3247.6 - 2814.5)} = \boxed{930.7 \text{ m/s}}$$

$$A_2 = \frac{m\, v_2'}{C_2'} = \frac{2.5 \cdot 0.6764}{930.7} = \boxed{18.17 \cdot 10^{-4}} \text{ m}^2$$

$$M_{throat} = \frac{C}{C_s} = 585.7 / \sqrt{n P_c v_c} = 585.7 / \sqrt{1.3 \cdot 10.9 \cdot 0.242 \cdot 10^5} =$$

$=$

Ex.

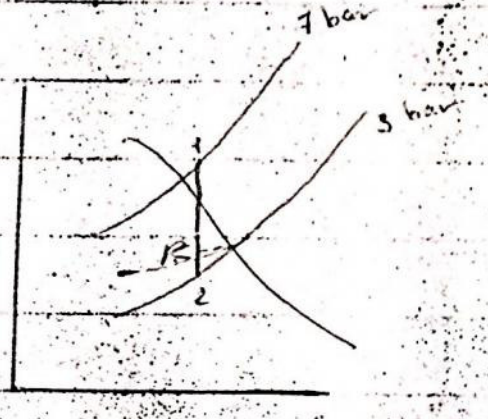
A convergent-divergent nozzle ~~recieve~~ receives steam at 7 bar and 200°c and expands it isentropically into a space at 3 bar. Neglecting the inlet velocity, calculate the exit velocity, area required for a mass flow rate of 0,1 Kg/s.

a) when the flow is in equilibrium throughout,

b) when the flow is supersaturated

Calculate also for part (b) the degree of supercooling and the degree of supersaturation.

Solution



(a)

$C_2 = \sqrt{2(h_1 - h_2)}$

$h_1 = 2846$ KJ/Kg from superheat tables at 7 bar & 200°C

$h_2 = 2682$ KJ/Kg from h-s chart at s=c

$C_2 = \sqrt{2 * 10^3 * (2846 - 2682)} = 573$ m/s

$X_2 = 0,98$ then $v_2 = X_2 v_g$

$v_2 = 0,98 * 0,6057 = 0,594 \ m^3/Kg$

$A_2 = \frac{m v_2}{C_2} = 10^6 \, 0,1 * 0,594 / 573 = 103,7 \ mm^2$

b) $C_R = \sqrt{\frac{2 \cdot n}{n-1} (P_1 v_1 - P_2 v_R)}$

$P_1 v_1^n = P_2 v_R^n$ $\quad v_1 = 0,3001$ from steam tables at 7 bar & 200°c

$v_R = \left(\frac{P_1}{P_2}\right)^{\frac{1}{1,3}} v_1 = 0,576 \ m^3/Kg$

$C_R = \sqrt{10^5 \times \frac{2 * 1,3}{0,3} (7 * 0,3001 - 3 * 0,576)} = 568$ m/s

$C_R < C_2$ $\quad$ 0,8%

$v_R < v_2$ $\quad$ 3%

"Steam turbines"

3-1. General Principle

A steam turbine is a machine which converts part of heat energy in the steam in to mechanical work.
Steam turbine essentially consists of the following two parts :-

1- The nozzle :- In which the steam expands from high pressure and a state of comparative rest to a lower pressure and state of comparatively rapid motion.
2. The moving blades :- In which the momentum of the stream of steam particles change.

The nozzles are attached to the stationary part of the turbine ; which is usually termed the stator (casing) , whereas the moving blades are attached to the rotating element of the machine (rotor).

3.2. Types of steam turbines :-

Steam turbines are classified according to the method of expansion in to

1- Impulse turbine
2- Impulse - reaction turbine (reaction).

1- Impulse turbine :-

The steam is caused to fall in pressure in nozzles , due to this fall in pressure a certain amount of

Pressure and thereby acquires a high velocity. This high velocity steam is next directed on to the blades and hence due to the change of direction there is a change in momentum, then a force is applied to the blades which causes the turbine wheel and the shaft to rotate.

Simple impulse turbine is show diagramutically in fig ①. The top portion of the fig. shows a longitudinal section through the upper half of turbine, the middle portion shows a development of the nozzles and blades while the lower part of the diagram shows approximatly how the absolute pressure and the absolute velocity of steam vary from point to point during the passage of the steam through the turbine.

The simple turbine described above is called De laval turbine and it is also knowns as single-stage impulse turbine.

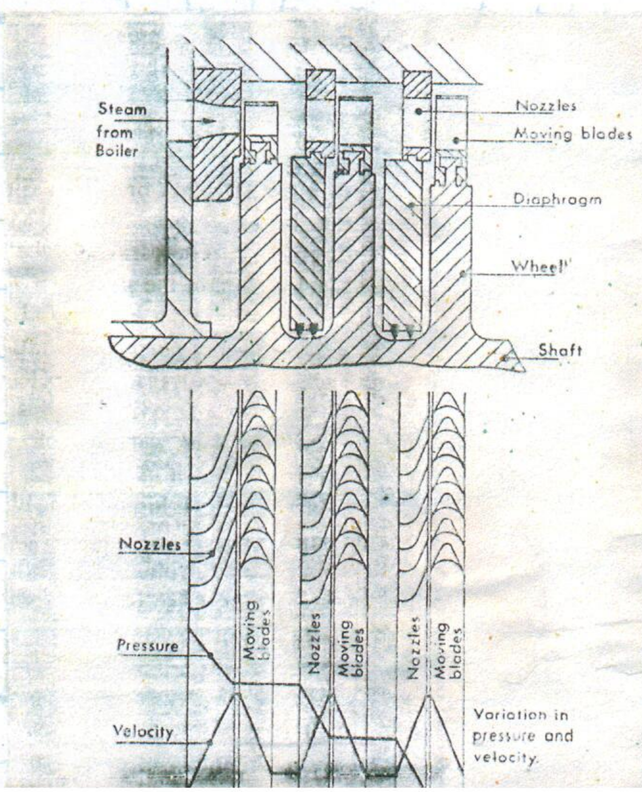
This turbine is not in common use due to the following disadvantages

① Since all pressure drop occures in one set of nozzles and all K.E. absorbes in only one ring of moving blades turbine rotation is to high about 30000 rpm.

② steam velocity at exit is quite high which means that there is a considerable loss of K.E.

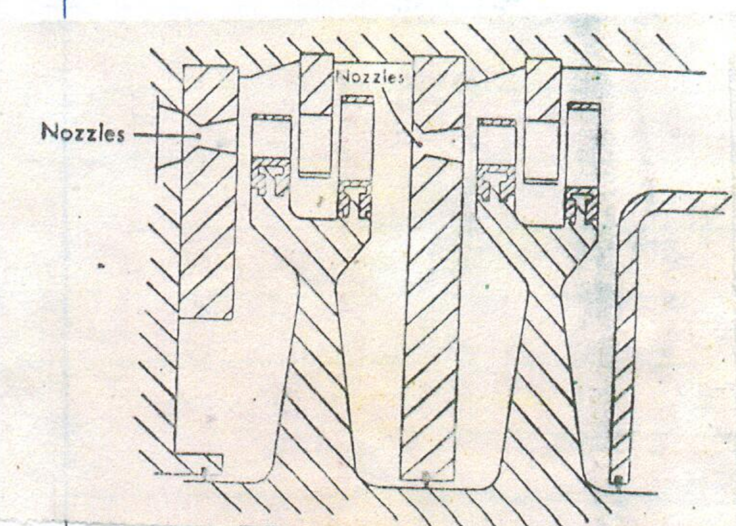
1. Pressure - Compound impulse turbine.

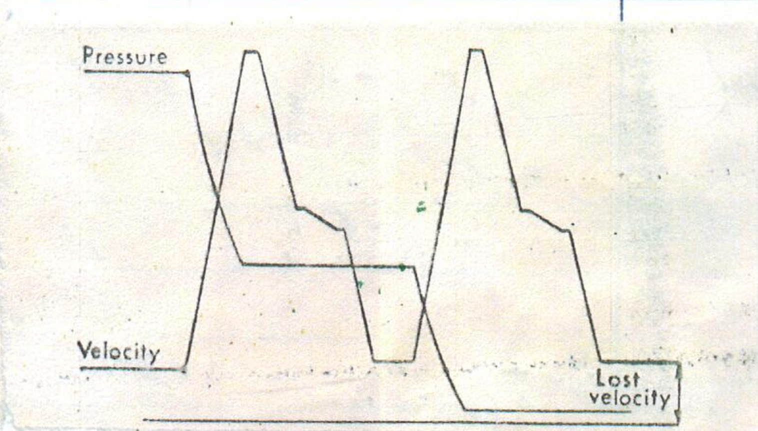
In this turbine the compounding is done by arranging the expansion of steam in anumber of steps, each step is a simple impulse turbine consists of one set of nozzle and one row of blades and is known as stage of the turbine. The exhaust steam from each row of moving blades enters the succeeding set of nozzles. The expansion of steam takes place only in the nozzles. The moving blades in rotor change the direction of the entering steam and cause force to be applied and torque to be developed. Since the drop of pressure per stage is reduced (compare with simple impulse turbine) the steam velocity leaving the nozzle is reduced. Thus the shaft speed is reduced. The leaving (carry over) loss in the pressure compounded turbine is also reduced. This type of turbine is also called Rateau turbine. This turbine has disadvantage of long number of stages, hence it is most expensive.

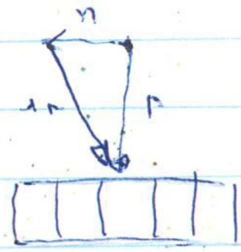


3. Pressure - velocity Compounding:

This type of turbine is a Combination of Pressure and velocity Compounding. The total pressure drop of steam from boiler to Condenser pressure is divided into a number of stages as is done in pressure compounded method.
Each stage may be having a number of rows of fixed and moving blades. Thus the velocity obtained in each stage is utilized in a number of moving blades as is done in the velocity Compounded method. Hence each stage of this turbine will consists of a set of nozzles followed by a set of moving blades. Steam is partially expanded in a row of nozzles where its velocity is increased, the steam then enters a few rows of velocity compounding. From this stage the steam then enters a second row of nozzles where its velocity is again increased. This is followed by another few rows of velocity compounding and so on. This type of turbine is simple in construction but its efficiency is low compare with Pressure Compounded.







V_1: absolute velocity of steam at outlet from nozzle.

V_2: 〃 〃 〃 〃 〃 〃 〃 blades.

V_{r_1}: Steam velocity relative to blades at inlet

V_{r_2}: 〃 〃 〃 〃 〃 outlet

U: mean peripheral velocity of blades.

V_f: axial velocity

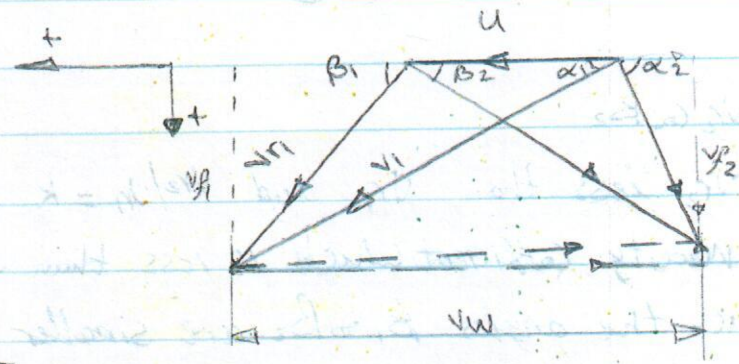
V_w: tangential component.

α_1: Jet angle (nozzle outlet angles)

α_2: absolute direction of steam leaving blades.

β_1: inlet angle of blades.

β_2: outlet 〃 〃 〃



$F =$ $\dot{m}$ * change in momentum

$F =$ $\dot{m}$ * change of velocity in the tangential direction

$F = \dot{m}(-V_{w_2} - V_{w_1})$

$F = -\dot{m}(V_{w_2} + V_{w_1})$

This force is from blade to the jet and force from jet to blade [which is impulsive force on the blade in the direction of motion] is

$$F = \dot{m}(V_{w_2} + V_{w_1}) \quad \text{------} \quad (1)$$

From diagram

$$V_w = V_1 \cos\alpha_1 + V_2 \cos\alpha_2 = V_{r_1} \cos\beta_1 + V_{r_2} \cos\beta_2$$

$$2uV_w = 2u(V_1\cos\alpha_1 - u)(1 + KG)$$

let $u/V_1 = \rho$ this is known as the blade speed ratio

then $u = V_1\rho$

and

$$2uV_w = 2V_1\rho(V_1\cos\alpha_1 - \rho V_1)(1 + KG)$$

$$= 2V_1^2(1 + KG)(\rho\cos\alpha_1 - \rho^2)$$

$$\therefore \eta_b = 2(1 + KG)(\rho\cos\alpha_1 - \rho^2) \quad \text{------} \; (5)$$

If $K \cdot G$ and α_1 are constants then η_b depends on ρ only and for max. blade efficiency

$$\frac{d\eta_b}{d\rho} = 2(1 + KG)(\cos\alpha_1 - 2\rho) = 0$$

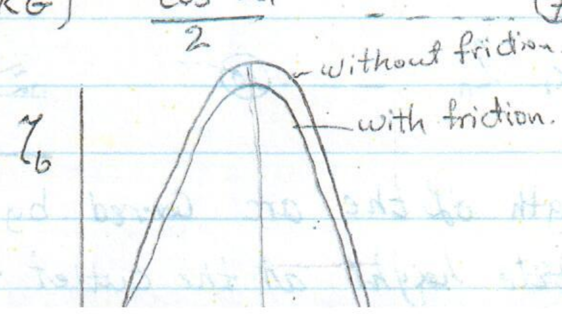
$$\therefore \rho_{opti} = \frac{\cos\alpha_1}{2} \quad \text{----} \; (6)$$

This speed ratio gives max. blade efficiency.

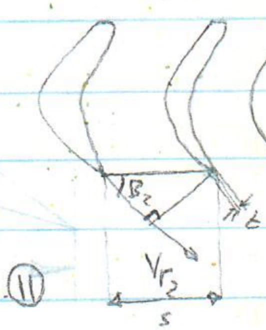
from eq (6) $u = V_1\dfrac{\cos\alpha_1}{2}$ sub. in eq (5)

$$\eta_{b(Max)} = 2(1 + KG)\left(\frac{\cos^2\alpha_1}{2} - \frac{\cos^2\alpha_1}{4}\right)$$

$$\eta_{bMax} = (1 + KG)\frac{\cos^2\alpha_1}{2} \quad \text{------} \; (7)$$



The mass flow of steam, m
Passes through the blade channels
of moving blades



$$m\sigma = V_{r_2}\,(s\sin\beta_2 - t)\,l * \frac{\lambda}{s} \quad \text{--- (11)}$$

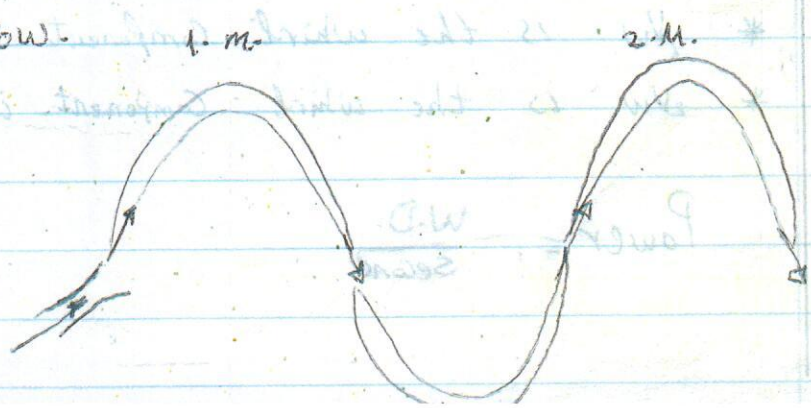
Where

S: is Pitch at exit
t: blade thickness
l: = hight at exit

All above calculations are used for a single stage impulse turbine and because pressure compounded turbine is a multi single stage then these calculations are also used.

B. Velocity Compounded

① Velocity diagram.
The method employed in drawing the diagram is merely an extension of the method used for a single row wheel. The
* figure below shows the velocity diagram for two row wheel which Containes two set of moving blades and one set of fixed (guide) blades which are to direct the steam for entry in to the next moving row.



Power from the first row $= \dot{m} u \; {}_1V_w$

" " " second " $= \dot{m} u \; {}_2V_w$

Total Power from this stage $= \dot{m} u \left({}_1V_w + {}_2V_w \right)$ —— (12)

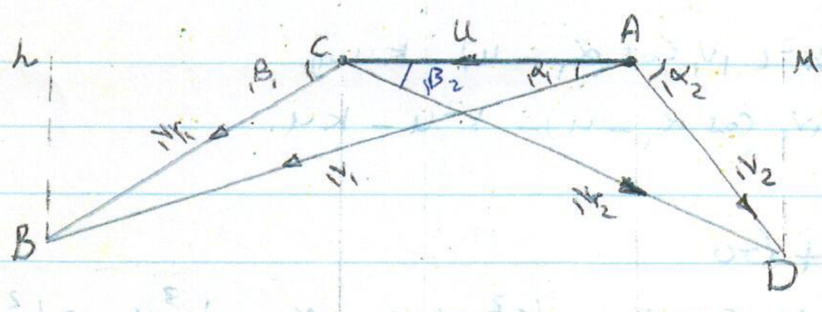
Total axial thrust $= \dot{m}\left[\left({}_1V_{f_1} - {}_1V_{f_2} \right) + \left({}_2V_{f_1} - {}_2V_{f_2} \right) \right]$ —— (13)

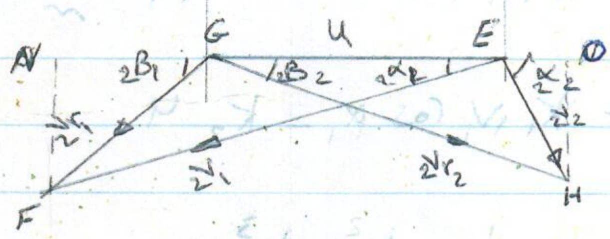
② Influnce of blade speed ratio on blade efficiency.

As in the case of the single row impulse wheel there is a certain ratio of blade velocity to steam velocity which results in the greatest blade efficiency. This ratio will be now determined for two-row wheel, under the following assumptions:-

① That the blades in each row are equiangular.

② That the velocity cofficient (K) is the same for each row of blades (moving and fixed)





Blade efficiency = $\dfrac{2u({}_1V_w + {}_2V_w)}{{}_1V_1^2}$

$\therefore \eta_b = 2u(K_1\, {}_1V_1 \cos {}_1\alpha_1 - K_2 u) / {}_1V_1^2$

$2u\, {}_1V_1 (K_1 \cos \alpha_1 - K_2 \rho) / {}_1V_1^2$

let $\rho = u / {}_1V_1 \quad \therefore u = \rho\, {}_1V_1$

يعوض بدلا u بـ ρV_1

$\eta_b = 2(k_1 \rho \cos \alpha_1 - k_2 \rho^2)$ ...... (14)

k_1 ? k_2 ?

for max. efficiency $\dfrac{d\eta_b}{d\rho} = 0$

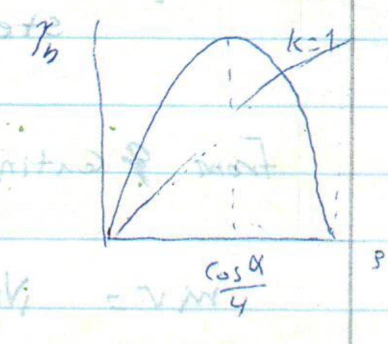
$\therefore 2k_1 \cos {}_1\alpha_1 - 4K_2 \rho = 0$

$\rho_{opt.} = \dfrac{k_1 \cos \alpha_1}{2k_2}$ ..... (15)

When there is no friction on blades $k = 1$

$k_1 = 4 \;,\; k_2 = 8$

$\rho_{opt} = \dfrac{\cos {}_1\alpha_1}{4}$ 16



For general optimum ρ is given by

$\rho = \dfrac{\cos \alpha_1}{2n}$ .... (17)

n : number of moving rows

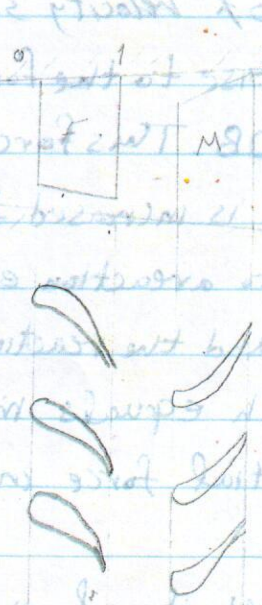
Sub. (16) in (14)

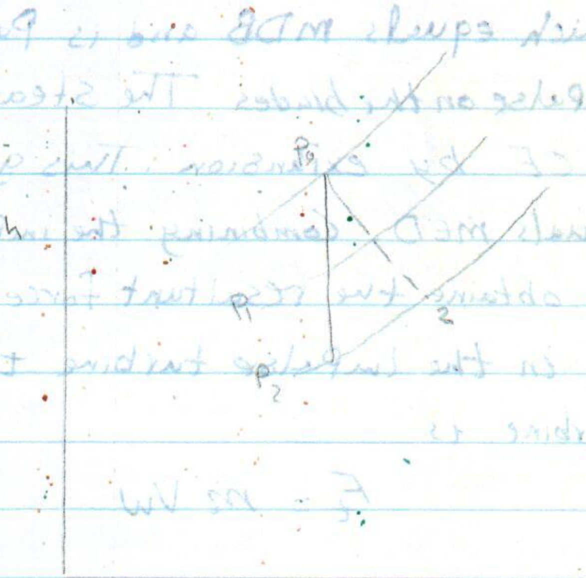
$\eta_{max} = \dfrac{k_1^2}{k_2} \dfrac{\cos^2 \alpha_1}{2}$ ... (18) when $k = 1$

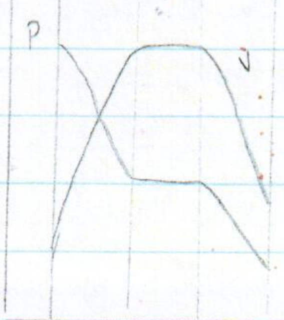
$\therefore \eta = \cos^2 \alpha_1$ —— (19)

Flow of steam through impulse_ reaction turbine (reaction tur.)

The reaction turbine applies the principle of both the pure impulse turbine and pure reaction turbine. Each stage of the reaction turbine consists of a fixed row of blades over the whole of the circumferential annulas, and an equal number of blades on a wheel. The fixed blade channels are of nozzle shape and there is a comparatively small drop in pressure accompained by an increase in velocity. The steam then passes over the moving blades and as in the pure impulse turbine a force is exerted on the blades by the steam. There is a further drop in pressure as the steam passes through the moving blades and there is an increase in the steam velocity relative to the blades and adds to the propelling force which is applied to the turbine rotor.







② Degree of reaction:-

The degree of reaction (R) is defined as the ratio of heat drop in the moving blades (Δh_b) to the sum of the heat drops in the fixed and moving blades ($\Delta h_n + \Delta h_b$)

$$R = \frac{\Delta h_b}{\Delta h_n + \Delta b} \quad \text{---} (24)$$

Δh_b = heat drop in moving blades Δh_n: heat drop in fixed.

In each row of fixed blades except the first, the steam enters with a kinetic energy equal to

$$V_2^2/2$$

Owing to losses at inlet, the available energy will be reduced to $\phi V_2^2/2$

where ϕ is a carry over coefficient.

The gain of k.E in the fixed blades $(V_1^2 - \phi V_2^2)/2$

and if η_n is the efficiency of the blades when considered as nozzles (expansion efficiency) then the isentropic heat drop (Δhis) in each row of fixed blades will be

$$\Delta his_n = \frac{V_1^2 - \phi V_2^2}{2\eta_n} \quad \text{---} (25)$$

For moving blades the gain of k.E is

$$(Vr_2^2 - k^2 Vr_1^2)/2$$

Then the isentropic heat drop (Δhis) in each row of moving blades will be

$$\Delta his_b = \frac{Vr_2^2 - \phi Vr_1^2}{2\eta_n} \quad \text{where } \phi = k^2 \quad \text{---} (26)$$

④ Blade efficiency :-

$$\eta_b = \text{Work / energy input}$$

$$\text{Energy input} = \frac{V_1^2}{2} + \frac{V_{r_2}^2 - V_{r_1}^2}{2} \qquad (\phi = 1)$$

and for Parsons turbine

$$V_1 = V_{r2}$$

$$\therefore \text{Energy input} = V_1^2 - \frac{V_{r_1}^2}{2}$$

$$\text{But } V_{r_1}^2 = V_1^2 + u^2 - 2u V_1 \cos\alpha_1$$

$\therefore$ Energy input

$$= V_1^2 - \frac{(V_1^2 + u^2 - 2uV_1\cos\alpha_1)}{2}$$

$$= \frac{V_1^2 - u^2 + 2uV_1\cos\alpha_1}{2}$$

Rate of doing work per kg/s $= u\,V_w$

$$V_w = V_1\cos\alpha_1 + V_2\cos\alpha_2$$

$$= V_1\cos\alpha_1 + V_{r2}\cos\beta_2 - u$$

$$= 2V_1\cos\alpha_1 - u$$

$$\therefore \text{Rate of doing work per kg/s} = u(2V_1\cos\alpha_1 - u) \text{ ——— } (28)$$

$$\therefore \eta_b = \frac{2u(2V_1\cos\alpha_1 - u)}{V_1^2 - u^2 + 2uV_1\cos\alpha_1}$$

$$\therefore \eta_b = \frac{2\rho(2\cos\alpha_1 - \rho)}{1 - \rho^2 + 2\rho\cos\alpha_1} \text{ ——— } (29)$$

For max. blade efficiency then $d\eta_b / d\rho = 0$

$$\therefore \rho_{opt} = \cos\alpha_1 \text{ ----- } (30)$$

Sub (30) in (28)

⑤ stage efficiency

$$\eta_s = \frac{W.D}{(\Delta h_{is})_s} \quad \text{......} \quad (33)$$

$$\Delta h_{is})_s = \Delta h_n)_{is} + \Delta h_b)_{is}$$

$$\Delta h_n)_{is} = \frac{v_1^2 - \phi v_2^2}{2\eta_n}, \quad \Delta h_b)_{is} = \frac{v_{r2}^2 - \phi v_{r1}^2}{2\eta_n}$$

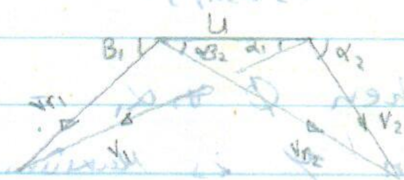
$$\Delta h_{is})_s = \frac{v_1^2 - \phi v_2^2 + v_{r2}^2 - \phi v_{r1}^2}{2\eta_n}$$

For Parsons turbin $\quad v_{r2} = v_1, \; v_{r1} = v_2$

(34)

$$\therefore \Delta h_{is})_s = \frac{v_1^2 - \phi v_{r1}^2 + v_1^2 - \phi v_{r1}^2}{2\eta_n}$$

$$\Delta h_{is})_s = \frac{v_1^2 - \phi v_{r1}^2}{\eta_n}$$



From diagram

$$v_{r1}^2 = v_1^2 + u^2 - 2uv_1\cos\alpha_1$$

$$\rho = u/v_1$$

$$v_{r1}^2 = v_1^2 + \rho^2 v_1^2 - 2\rho v_1^2 \cos\alpha_1$$

$$= v_1^2(1 + \rho^2 - 2\rho\cos\alpha_1)$$

$$\therefore \Delta h_{is})_s = v_1^2[1 - \phi(1 + \rho^2 - 2\rho\cos\alpha_1)]/\eta_n$$

$$W.D. = u\,V_W = u[v_1\cos\alpha_1 + v_2\cos\alpha_2]$$

$$= u[v_1\cos\alpha_1 + v_{r2}\cos\beta_2 - u]$$

$$= u[v_1\cos\alpha_1 + v_1\cos\alpha_1 - u]$$

$$= 2uv_1\cos\alpha_1 - u^2$$

$$= 2\rho v_1^2\cos\alpha_1 - \rho^2 v_1^2$$

$$= v_1^2(2\rho\cos\alpha_1 - \rho^2)$$

⑥ Blade Height

If l is the radial height of blade

s is the Circumferential Pitch of the blade

D is the mean diameter of the blade ring

From Continuity equation at outlet of fixed blade

$$\dot{m}v = A * V$$

$$\dot{m}v = A * \text{velocity}$$

The outlet area of fixed blades $= s.l.\sin\alpha$

The number of blades Per ring $= \dfrac{\pi D}{s}$

$$\therefore \dot{m}v = \frac{\pi D}{s} * s.l.\sin\alpha_1 . v_1$$

$$\therefore \dot{m}v = \pi D . l . \sin\alpha_1 v_1 = \pi D l V_{f_1} \quad \text{—— } 36$$

And at outlet of moving blade

$$\dot{m}v = \pi D l . \sin\beta_2 V_{r_2} = \pi D l V_{f_2} \quad \text{—— } 37$$

Example :- P. 369

A stage of a turbine with Parsons blading delivers dry saturated steam at 2.7 bar from the fixed blades at 90 m/s. The mean blade height is 40 mm, and the moving blade exit angle is 20°. The axial velocity of the steam is 3/4 of the blade velocity at the mean radius. Steam is supplied to the stage at the rate of 9000 kg/h. The effect of the blade tip thickness on the annulus area can be neglected. Calculate :- 1. The wheel speed in rpm b) The diagram Power c) The diagram efficiency d) The enthalpy drop

losses in steam turbines

Losses in a turbine may be divided in two groups. There will be external losses due to bearing friction and the Power required to drive auxiliaries. The second and major group, the internal losses which are divided into:

1. Friction losses - Which are least for reaction turbine compared with impulse turbine. This is because the average velocity through the nozzle and rotor blading is less for the reaction stage than impulse stage and because laminar layer continues over a greater part of the blades surface.

Friction losses are:

a. Nozzle losses

losses of energy in nozzle $= \frac{V_{th}^2 - V_a^2}{2}$

V_{th} is theoretical velocity

V_a is actual velocity $= K_n V_{th}$

K_n is the velocity coefficient in nozzle

$$\eta_n = \frac{\Delta h_a}{\Delta h_{th}} = \frac{V_a^2}{V_{th}^2} = K_n^2$$

$$\therefore \text{loss of energy in nozzle} = \frac{V_{th}^2(1 - K_n^2)}{2} = \frac{V_{th}^2(1 - \eta_n)}{2} \quad \text{--- 36}$$

B. Blade friction losses

$$\text{loss of energy in moving blades Per Stage} = \frac{V_{r1}^2 - V_{r2}^2}{2} \quad \text{--- 37}$$

$$= \frac{V_{r1}^2(1 - K^2)}{2}$$

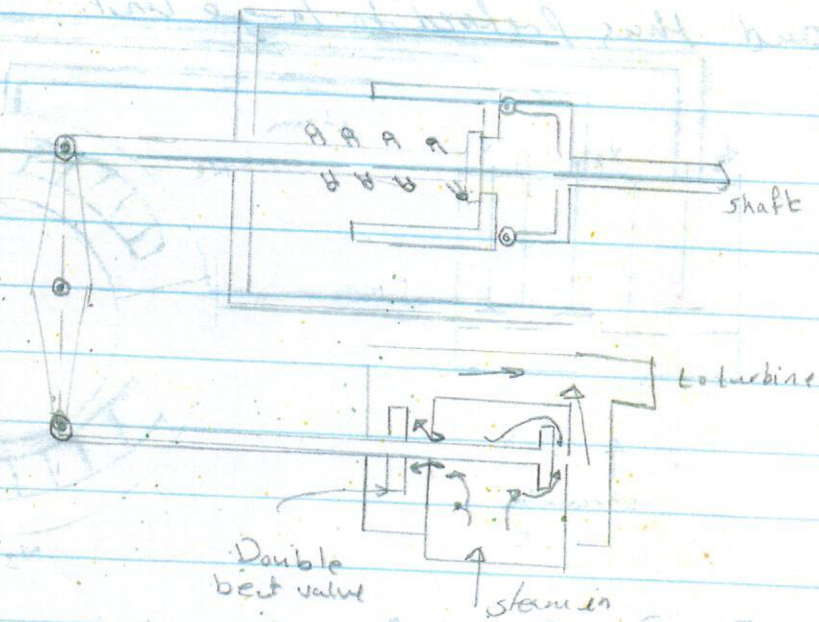
c. Dis friction

which occures due to relative motion between the disc and steam particles

Steam turbine governing:-

The function of a governing is to regulate the supply of steam to the turbine in such a way as to keep its speed fairly constant from no-load to full load. Following methods are commonly used for governing of steam turbines.

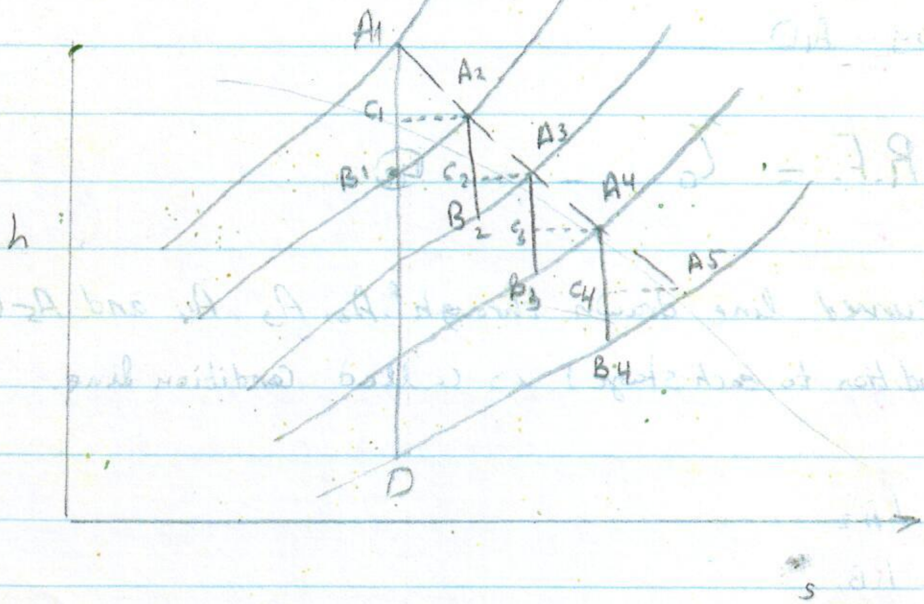
1- Throttle governing:-
In this method flow of steam entering the turbine is varied with the help of a double beat valve which itself is controlled by a centrifugal governor as shown in fig.



When load on the turbine decreases, its speed tends to increase the centrifugal force ~~on~~ and hence it moves outwards and then the throttling tends to restor the original speed.
Throttle govnor is mechanically simple but thermodynamically inefficient hence it's used for small units only.

Stage efficiency, overall efficiency and Reheat factor:-

Figure shows the expansion of steam through a number of turbine stages, A_1B_1 represent the isentropic expansion in the first stage. The actual state of steam with frictional reheating, is shown by the Point A_2. There fore the actual heat drop is equivalant to A_1C_1. Similarly the isentropic and actual heat drops in the succeeding stages are shown in the figure by A_2B_2, A_3B_3 and A_2C_2 and A_3C_3 and so on. The drop) represent the isentropic heat drop (direct isentropic heat drop)



* The sum of the isentropic drops in all the turbine stages $(A_1B_1 + A_2B_2 + A_3B_3 + A_4B_4)$ is called "the cumlative enthalpy drop".

The cumlative enthalpy drop is always found to be greater than the direct isentropic heat drops. (A_1D) as the constant pressure lines diverage from left to right on h-s chart.

* $$\eta_s = \frac{\text{Actual enthalpy drop}}{\text{isentropic enthalpy drop}} = \frac{A_1C_1}{A_1B_1}, \quad \eta_s = \frac{A_2C_2}{A_2B_2} \text{ and so on.}$$

Ex. Steam at 15 bar and 350°C is expanded through a 50% reaction turbine to a pressure of 0.14 bar. The stage efficiency is 75% for each stage, and the reheat factor is 1.04. The expansion is to be carried out in twenty stages and the diagram Power is required to be 12000 kW. Calculate the flow of steam required, assuming that the stages all develop equal work. At one stage the pressure is 1 bar and the steam is dry saturated. The exit angle of the blades is 20°, and the blade speed ratio is 0.7. If the blade height is 1/12 of the blade mean diameter, calculate the value of the mean blade diameter and the rotor speed.

Solution:-

$$\eta_o = \eta_s * R.F. = 0.75 \cdot 1.04 = 0.78$$

From h,s chart $h_1 = 3148$ kJ/kg at $P = 15$ & $t = 350°C$

$h_2 = 2293$ kJ/kg at $s_1 = s_2$ & $P_2 = 0.14$

direct isentropic overall enthalpy drop $= 3148 - 2293 = 855$ kJ/kg

actual overall enthalpy drop $= \eta_o \cdot \Delta h_{is} = 0.78 \cdot 855 = 667$ kJ/kg

enthalpy drop per stage $= 667/20 = 33.35$ kJ/kg

Total Power $= \dot{m}_s \Delta h_{ac} =$

$$\dot{m}_s = 12000/667 = \boxed{17.99 \text{ kg/s}} = 64770 \text{ kg/h}$$

Work done per kg of steam $= u V_w = u(V_1 \cos\alpha_1 + V_2 \cos\alpha_2)$

$$= u(2V_1 \cos\alpha_1 - u)$$

$$\frac{u}{V_1} = 0.7 \quad V_1 \doteqdot 1.43u$$

$$\therefore 33.35 = \frac{u}{10^3}(2 \cdot 1.43u \cos 20 - u)$$

$$u = 141.4 \text{ m/s}$$