CHAPTER 1

Introduction

General

Energy is the basic necessity for the economic development of a country. Many functions necessary to present-day living grind to halt when the supply of energy stops. It is practically impossible to estimate the actual magnitude of the part that energy has played in the building up of present-day civilisation. The availability of huge amount of energy in the modern times has resulted in a shorter working day, higher agricultural and industrial production, a healthier and more balanced diet and better transportation facilities. As a matter of fact, there is a close relationship between the energy used per person and his standard of living. The greater the per capita consumption of energy in a country, the higher is the standard of living of its people.

Energy exists in different forms in nature but the most important form is the *electrical energy*. The modern society is so much dependent upon the use of electrical energy that it has become a part and parcel of our life. In this chapter, we shall focus our attention on the general aspects of electrical energy.

1.1 Importance of Electrical Energy

Energy may be needed as heat, as light, as motive power etc. The present-day advancement in sc and technology has made it possible to convert electrical energy into any desired form. Th given electrical energy a place of pride in the modern world. The survival of industrial underta and our social structures depends primarily upon low cost and uninterrupted supply of elec energy. In fact, the advancement of a country is measured in terms of per capita consumpti electrical energy.

Electrical energy is superior to all other forms of energy due to the following reasons :

(*i*) **Convenient form.** Electrical energy is a very convenient form of energy. It can be converted into other forms of energy. For example, if we want to convert electrical energy into the only thing to be done is to pass electrical current through a wire of high resistance *e.g.*, a h Similarly, electrical energy can be converted into light (*e.g.* electric bulb), mechanical energy electric motors) etc.

(*ii*) **Easy control.** The electrically operated machines have simple and convenient starting, c and operation. For instance, an electric motor can be started or stopped by turning on or off a s Similarly, with simple arrangements, the speed of electric motors can be easily varied over the d range.

(*iii*) **Greater flexibility.** One important reason for preferring electrical energy is the flexi that it offers. It can be easily transported from one place to another with the help of conducto

(*iv*) **Cheapness.** Electrical energy is much cheaper than other forms of energy. Thus it is o economical to use this form of energy for domestic, commercial and industrial purposes.

(*v*) **Cleanliness.** Electrical energy is not associated with smoke, fumes or poisonous Therefore, its use ensures cleanliness and healthy conditions.

(*vi*) **High transmission efficiency.** The consumers of electrical energy are generally sit quite away from the centres of its production. The electrical energy can be transmitted conven and efficiently from the centres of generation to the consumers with the help of overhead cond known as transmission lines.

1.2 Generation of Electrical Energy

The conversion of energy available in different forms in nature into electrical energy is kno **generation of electrical energy**.

Electrical energy is a manufactured commodity like clothing, furniture or tools. Just manufacture of a commodity involves the conversion of raw materials available in nature in desired form, similarly electrical energy is produced from the forms of energy available in n However, electrical energy differs in one important respect. Whereas other commodities m produced at will and consumed as needed, the electrical energy must be produced and transmit the point of use at the instant it is needed. The entire process takes only a fraction of a second. instantaneous production of electrical energy introduces technical and economical considera unique to the electrical power industry.

Energy is available in various forms from different natural sources such as pressure head of water, chemical energy of fuels, nuclear energy of radioactive substances

such as burning of fuel, pressure of water, force of wind etc. For example, chemical energy of a fuel (*e.g.*, coal) can be used to produce steam at high temperature and pressure. The steam is fed to a prime mover which may be a steam engine or a steam turbine. The turbine converts heat energy of steam into mechanical energy which is further converted into electrical energy by the alternator. Similarly, other forms of energy can be converted into electrical energy by employing suitable machinery and equipment.

1.3. Sources of Energy

Since electrical energy is produced from energy available in various forms in nature, it is desirable to look into the various sources of energy. These sources of energy are :

(*i*) The Sun (*ii*) The Wind (*iii*) Water (*iv*) Fuels (*v*) Nuclear energy.

Out of these sources, the energy due to Sun and wind has not been utilised on large scale due to a number of limitations. At present, the other three sources *viz.*, water, fuels and nuclear energy are primarily used for the generation of electrical energy.

(*i*) **The Sun.** The Sun is the primary source of energy. The heat energy radiated by the Sun can be focussed over a small area by means of reflectors. This heat can be used to raise steam and electrical energy can be produced with the help of turbine-alternator combination. However, this method has limited application because :

(*a*) it requires a large area for the generation of even a small amount of electric power

(*b*) it cannot be used in cloudy days or at night

(*c*) it is an uneconomical method.

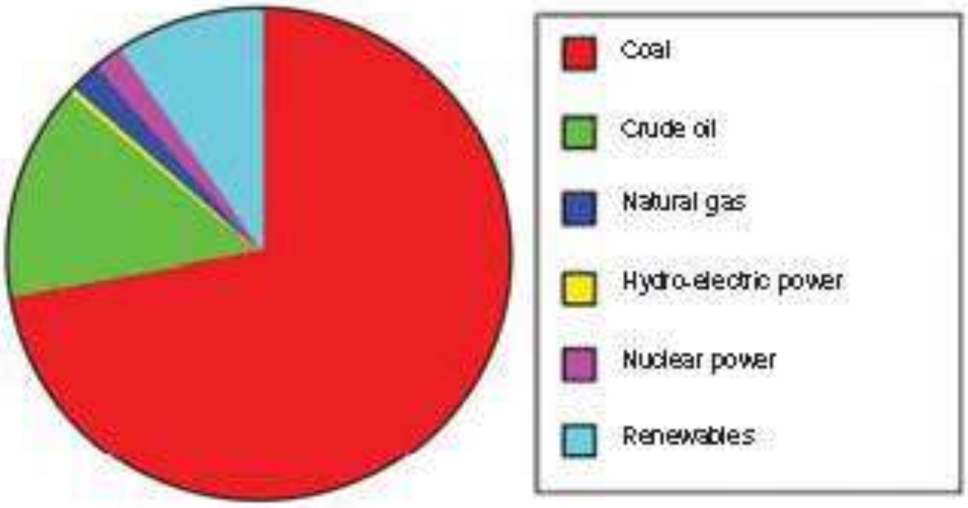
Nevertheless, there are some locations in the world where strong solar radiation is received very regularly and the sources of mineral fuel are scanty or lacking. Such locations offer more interest to the solar plant builders.

(*ii*) **The Wind.** This method can be used where wind flows for a considerable length of time. The wind energy is used to run the wind mill which drives a small generator. In order to obtain the electrical energy from a wind mill continuously, the generator is arranged to charge the batteries. These batteries supply the energy when the wind stops. This method has the advantages that maintenance and generation costs are negligible. However, the drawbacks of this method are (*a*) variable output, (*b*) unreliable because of uncertainty about wind pressure and (*c*) power generated is quite small.

(*iii*) **Water.** When water is stored at a suitable place, it possesses potential energy because of the head created. This water energy can be converted into mechanical energy with the help of water turbines. The water turbine drives the alternator which converts mechanical energy into electrical energy. This method of generation of electrical energy has become very popular because it has low production and maintenance costs.

(*iv*) **Fuels.** The main sources of energy are fuels *viz.*, solid fuel as coal, liquid fuel as oil and gas fuel as natural gas. The heat energy of these fuels is converted into mechanical energy by suitable prime movers such as steam engines, steam turbines, internal combustion engines etc. The prime mover drives the alternator which converts mechanical energy into electrical energy. Although fuels continue to enjoy the place of chief source for the generation of electrical energy, yet their reserves are diminishing day by day. Therefore, the present trend is to harness water power which is more or less a permanent source of power.

(*v*) **Nuclear energy.** Towards the end of Second World War, it was discovered that large amount of heat energy is liberated by the *fission* of uranium and other fissionable materials. It is estimated that heat produced by 1 kg of nuclear fuel is equal to that produced by 4500 tonnes of coal. The heat produced due to nuclear fission can be utilised to raise steam with suitable arrangements. The steam can run the steam turbine which in turn can drive the alternator to produce electrical energy. However, there are some difficulties in the use of nuclear energy. The principal ones are (*a*) high cost of nuclear plant (*b*) problem of disposal of radioactive waste and dearth of trained personnel to handle the plant.



Energy Utilisation

1.4 Comparison of Energy Sources

The chief sources of energy used for the generation of electrical energy are water, fuels and nuclear energy. Below is given their comparison in a tabular form :

S.No.	Particular	Water-power	Fuels	Nuclear energy
1.	*Initial cost*	High	Low	Highest
2.	*Running cost*	Less	High	Least
3.	*Reserves*	Permanent	Exhaustable	Inexhaustible
4.	*Cleanliness*	Cleanest	Dirtiest	Clean
5.	*Simplicity*	Simplest	Complex	Most complex
6.	*Reliability*	Most reliable	Less reliable	More reliable

1.5 Units of Energy

The capacity of an agent to do work is known as its energy. The most important forms of energy are mechanical energy, electrical energy and thermal energy. Different units have been assigned to various forms of energy. However, it must be realised that since mechanical, electrical and thermal energies are interchangeable, it is possible to assign the same unit to them. This point is clarified in Art 1.6.

(*i*) **Mechanical energy.** The unit of mechanical energy is *newton-metre* or *joule* on the M.K.S. or SI system.

The work done on a body is one newton-metre (or joule) if a force of one newton moves it through a distance of one metre *i.e.*,

Mechanical energy in joules = Force in newton × distance in metres

(*ii*) **Electrical energy.** The unit of electrical energy is *watt-sec* or *joule* and is defined as follows:

One watt-second (or joule) energy is transferred between two points if a p.d. of 1 volt exists between them and 1 ampere current passes between them for 1 second *i.e.*, Electrical energy in watt-sec (or joules)

$$= \text{voltage in volts} \times \text{current in amperes} \times \text{time in seconds}$$

Joule or watt-sec is a very small unit of electrical energy for practical purposes. In practice, for the measurement of electrical energy, bigger units *viz.*, watt-hour and kilowatt hour are used.

$$1 \text{ watt-hour} = 1 \text{ watt} \times 1 \text{ hr}$$
$$= 1 \text{ watt} \times 3600 \text{ sec} = 3600 \text{ watt-sec}$$
$$1 \text{ kilowatt hour (kWh)} = 1 \text{ kW} \times 1 \text{ hr} = 1000 \text{ watt} \times 3600 \text{ sec} = 36 \times 10^5 \text{ watt-sec.}$$

***(iii)* Heat.** Heat is a form of energy which produces the sensation of warmth. The unit* of heat is calorie, British thermal unit (B.Th.U.) and centigrade heat units (C.H.U.) on the various systems.

Calorie. It is the amount of heat required to raise the temperature of 1 gm of water through 1°C *i.e.*,

$$1 \text{ calorie} = 1 \text{ gm of water} \times 1°\text{C}$$

Sometimes a bigger unit namely *kilocalorie* is used. A kilocalorie is the amount of heat required to raise the temperature of 1 kg of water through 1°C *i.e.*,

$$1 \text{ kilocalorie} = 1 \text{ kg} \times 1°\text{C} = 1000 \text{ gm} \times 1°\text{C} = 1000 \text{ calories}$$

B.Th.U. It is the amount of heat required to raise the temperature of 1 lb of water through 1°F *i.e.*,

$$1 \text{ B.Th.U.} = 1 \text{ lb} \times 1°\text{F}$$

C.H.U. It is the amount of heat required to raise the temperature of 1 lb of water through 1°C *i.e.*,

$$1 \text{ C.H.U.} = 1 \text{ lb} \times 1°\text{C}$$

1.6 Relationship Among Energy Units

The energy whether possessed by an electrical system or mechanical system or thermal system has the same thing in common *i.e.*, it can do some work. Therefore, mechanical, electrical and thermal energies must have the same unit. This is amply established by the fact that there exists a definite relationship among the units assigned to these energies. It will be seen that these units are related to each other by some constant.

(*i*) Electrical and Mechanical

$$1 \text{ kWh} = 1 \text{ kW} \times 1 \text{ hr}$$
$$= 1000 \text{ watts} \times 3600 \text{ seconds} = 36 \times 10^5 \text{ watt-sec. or Joules}$$
$$\therefore \quad 1 \text{ kWh} = 36 \times 10^5 \text{ Joules}$$

It is clear that electrical energy can be expressed in Joules instead of kWh.

(*ii*) Heat and Mechanical

(*a*) $1 \text{ calorie} = 4{\cdot}18 \text{ Joules}$ (By experiment)

(*b*) $1 \text{ C.H.U.} = 1 \text{ lb} \times 1°\text{C} = 453{\cdot}6 \text{ gm} \times 1°\text{C}$

$$= 453{\cdot}6 \text{ calories} = 453{\cdot}6 \times 4{\cdot}18 \text{ Joules} = 1896 \text{ Joules}$$
$$\therefore \quad 1 \text{C.H.U.} = 1896 \text{ Joules}$$

(*c*) $1 \text{ B.Th.U.} = 1 \text{ lb} \times 1°\text{F} = 453{\cdot}6 \text{ gm} \times 5/9 °\text{C}$

$$= 252 \text{ calories} = 252 \times 4{\cdot}18 \text{ Joules} = 1053 \text{ Joules}$$
$$\therefore \quad 1 \text{ B.Th.U.} = 1053 \text{ Joules}$$

It may be seen that heat energy can be expressed in Joules instead of thermal units *viz.* calorie, B.Th.U. and C.H.U.

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* The SI or MKS unit of thermal energy being used these days is the *joule*—exactly as for mechanical and electrical energies. The thermal units *viz.* calorie, B.Th.U. and C.H.U. are obsolete.

(iii) Electrical and Heat

(a) $1 \text{ kWh} = 1000 \text{ watts} \times 3600 \text{ seconds} = 36 \times 10^5 \text{ Joules}$

$$= \frac{36 \times 10^5}{4 \cdot 18} \text{ calories} = 860 \times 10^3 \text{ calories}$$

$\therefore$ $1 \text{ kWh} = 860 \times 10^3 \text{ calories} \quad \text{or} \quad 860 \text{ kcal}$

(b) $1 \text{ kWh} = 36 \times 10^5 \text{ Joules} = 36 \times 10^5/1896 \text{ C.H.U.} = 1898 \text{ C.H.U.}$

$[\because 1 \text{ C.H.U.} = 1896 \text{ Joules}]$

$\therefore$ $1 \text{ kWh} = 1898 \text{ C.H.U.}$

(c) $1 \text{ kWh} = 36 \times 10^5 \text{ Joules} = \frac{36 \times 10^5}{1053} \text{ B.Th.U.} = 3418 \text{ B.Th.U.}$

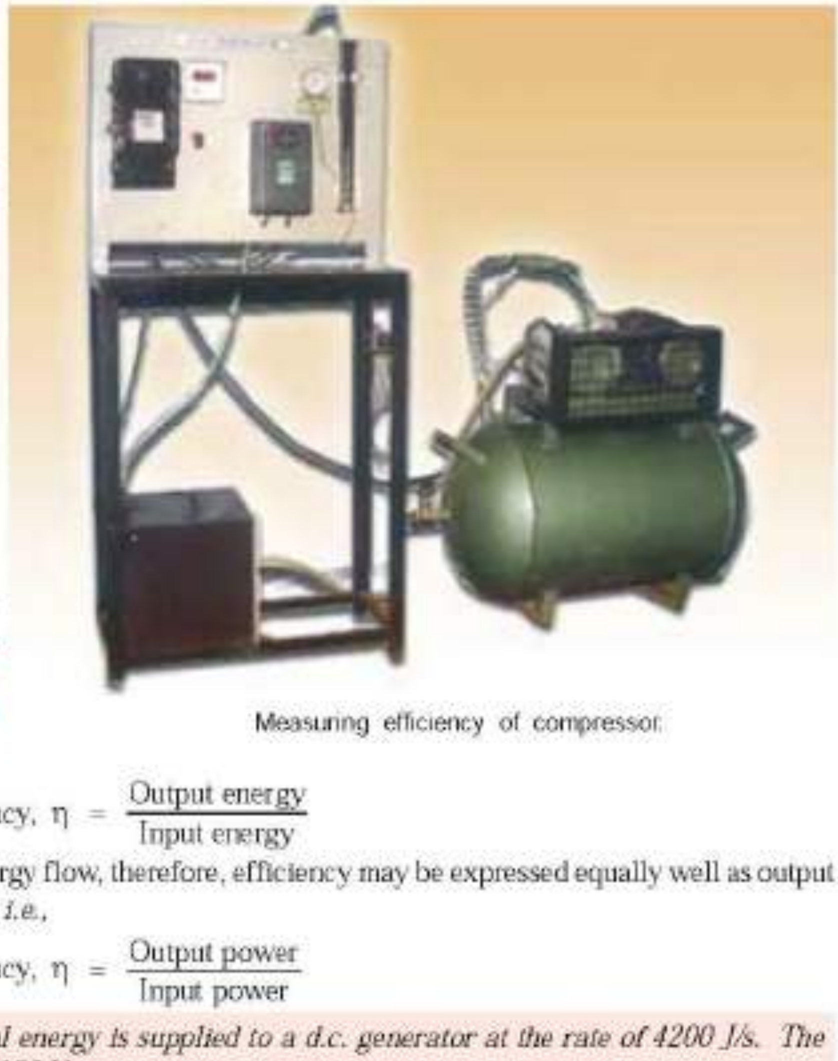
$[\because 1 \text{ B.Th.U.} = 1053 \text{ Joules}]$

$\therefore$ $1 \text{ kWh} = 3418 \text{ B.Th.U.}$

The reader may note that units of electrical energy can be converted into heat and *vice-versa*. This is expected since electrical and thermal energies are interchangeable.

1.7 Efficiency

Energy is available in various forms from different natural sources such as pressure head of water, chemical energy of fuels, nuclear energy of radioactive substances etc. All these forms of energy can be converted into electrical energy by the use of suitable arrangement. In this process of conversion, some energy is *lost* in the sense that it is converted to a form different from electrical energy. Therefore, the output energy is less than the input energy. *The output energy divided by the input energy is called* **energy efficiency** *or simply* **efficiency** *of the system.*



Measuring efficiency of compressor.

$$\text{Efficiency, } \eta = \frac{\text{Output energy}}{\text{Input energy}}$$

As power is the rate of energy flow, therefore, efficiency may be expressed equally well as output power divided by input power *i.e.*,

$$\text{Efficiency, } \eta = \frac{\text{Output power}}{\text{Input power}}$$

Example 1.1. *Mechanical energy is supplied to a d.c. generator at the rate of 4200 J/s. The generator delivers 32·2 A at 120 V.*

(i) *What is the percentage efficiency of the generator ?*

(ii) *How much energy is lost per minute of operation ?*

Solution.

(*i*) Input power, $P_I = 4200 \text{ J/s} = 4200 \text{ W}$

Output power, $P_o = EI = 120 \times 32.2 = 3864 \text{ W}$

∴ Efficiency, $\eta = \frac{P_o}{P_I} \times 100 = \frac{3864}{4200} \times 100 = $ **92 %**

(*ii*) Power lost, $P_L = P_I - P_o = 4200 - 3864 = 336 \text{ W}$

∴ Energy lost per minute (= 60 s) of operation

$= P_L \times t = 336 \times 60 = $ **20160 J**

Note that efficiency *is always less than 1 (or 100 %)*. In other words, every system is less than 100 % efficient.

1.8 Calorific Value of Fuels

The amount of heat produced by the complete combustion of a unit weight of fuel is known as its **calorific value.**

Calorific value indicates the amount of heat available from a fuel. The greater the calorific value of fuel, the larger is its ability to produce heat. In case of solid and liquid fuels, the calorific value is expressed in *cal/gm* or *kcal/kg*. However, in case of gaseous fuels, it is generally stated in *cal/litre* or *kcal/litre*. Below is given a table of various types of fuels and their calorific values along with composition.

S.No.	Particular	Calorific value	Composition
1.	**Solid fuels**		
	(*i*) Lignite	5,000 kcal/kg	C = 67%, H = 5%, O = 20%, ash = 8%
	(*ii*) Bituminous coal	7,600 kcal/kg	C = 83%, H = 5·5%, O = 5%, ash = 6·5%
	(*iii*) Anthracite coal	8,500 kcal/kg	C = 90%, H = 3%, O = 2%, ash = 5%
2.	**Liquid fuels**		
	(*i*) Heavy oil	11,000 kcal/kg	C = 86%, H = 12%, S = 2%
	(*ii*) Diesel oil	11,000 kcal/kg	C = 86·3%, H = 12·8%, S = 0·9%
	(*iii*) Petrol	11,110 kcal/kg	C = 86%, H = 14%
3.	**Gaseous fuels**		
	(*i*) Natural gas	520 kcal/m^3	CH_4 = 84%, C_2H_6 = 10% Other hydrocarbons = 5%
	(*ii*) Coal gas	7,600 kcal/m^3	CH_4 = 35%, H = 45%, CO= 8%, N = 6% CO_2 = 2%, Other hydrocarbons = 4%

1.9 Advantages of Liquid Fuels over Solid Fuels

The following are the advantages of liquid fuels over the solid fuels :

(*i*) The handling of liquid fuels is easier and they require less storage space.

(*ii*) The combustion of liquid fuels is uniform.

(*iii*) The solid fuels have higher percentage of moisture and consequently they burn with great difficulty. However, liquid fuels can be burnt with a fair degree of ease and attain high temperature very quickly compared to solid fuels.

(*iv*) The waste product of solid fuels is a large quantity of ash and its disposal becomes a problem. However, liquid fuels leave no or very little ash after burning.

(*v*) The firing of liquid fuels can be easily controlled. This permits to meet the variation in load demand easily.

1.10 Advantages of Solid Fuels over Liquid Fuels

The following are the advantages of solid fuels over the liquid fuels :

(*i*) In case of liquid fuels, there is a danger of explosion.

(*ii*) Liquids fuels are costlier as compared to solid fuels.

(*iii*) Sometimes liquid fuels give unpleasant odours during burning.

(*iv*) Liquid fuels require special types of burners for burning.

(*v*) Liquid fuels pose problems in cold climates since the oil stored in the tanks is to be heated in order to avoid the stoppage of oil flow.

SELF-TEST

1. Fill in the blanks by inserting appropriate words/figures.

(*i*) The primary source of energy is the ....

(*ii*) The most important form of energy is the ....

(*iii*) 1 kWh = ... kcal

(*iv*) The calorific value of a solid fuel is expressed in ....

(*v*) The three principal sources of energy used for the generation of electrical energy are .... and ....

2. Pick up the correct words/figures from the brackets and fill in the blanks.

(*i*) Electrical energy is .... than other forms of energy. (*cheaper, costlier*)

(*ii*) The electrical, heat and mechanical energies .... be expressed in the same units. (*can, cannot*)

(*iii*) .... continue to enjoy the chief source for the generation of electrical energy. (*fuels, radioactive substances, water*)

(*iv*) The basic unit of energy is .... (*Joule, watt*)

(*v*) An alternator is a machine which converts .... into ....
(*mechanical energy, electrical energy*)

ANSWERS TO SELF-TEST

1. (*i*) Sun, (*ii*) electrical energy, (*iii*) 860, (*iv*) cal/gm or kcal/kg, (*v*) water, fuels and radioactive substances.
2. (*i*) Cheaper, (*ii*) can, (*iii*) fuels, (*iv*) Joule, (*v*) mechanical energy, electrical energy.

CHAPTER REVIEW TOPICS

1. Why is electrical energy preferred over other forms of energy ?
2. Write a short note on the generation of electrical energy.
3. Discuss the different sources of energy available in nature.
4. Compare the chief sources of energy used for the generation of electrical energy.
5. Establish the following relations :
   (*i*) 1 kWh = 36×10^5 Joules (*ii*) 1 kWh = 860 kcal
   (*iii*) 1 B.Th.U. = 1053 Joules (*iv*) 1 C.H.U. = 1896 Joules
6. What do you mean by efficiency of a system ?
7. What are the advantages of liquid fuels over the solid fuels ?
8. What are the advantages of solid fuels over the liquid fuels ?

DISCUSSION QUESTIONS

1. Why do we endeavour to use water power for the generation of electrical energy ?
2. What is the importance of electrical energy ?
3. What are the problems in the use of nuclear energy ?
4. Give one practical example where wind-mill is used.
5. What is the principal source of generation of electrical energy ?

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CHAPTER

2

Generating Stations

Introduction

In this modern world, the dependence on electricity is so much that it has become a part and parcel of our life. The ever increasing use of electric power for domestic, commercial and industrial purposes necessitates to provide bulk electric power economically. This is achieved with the help of suitable power producing units, known as *Power plants or Electric power generating stations*. The design of a power plant should incorporate two important aspects. Firstly, the selection and placing of necessary power-generating equipment should be such so that a maximum of return will result from a minimum of expenditure over the working life of the plant. Secondly, the operation of the plant should be such so as to provide cheap, reliable and continuous service. In this chapter, we shall focus our attention on various types of generating stations with special reference to their advantages and disadvantages.

2.1 Generating Stations

Bulk electric power is produced by special plants known as ***generating stations*** *or* ***power plants.***

A generating station essentially employs a

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prime mover coupled to an alternator for the production of electric power. The prime mover (*e.g.*, steam turbine, water turbine etc.) converts energy from some other form into mechanical energy. The alternator converts mechanical energy of the prime mover into electrical energy. The electrical energy produced by the generating station is transmitted and distributed with the help of conductors to various consumers. It may be emphasised here that apart from prime mover-alternator combination, a modern generating station employs several auxiliary equipment and instruments to ensure cheap, reliable and continuous service.

Depending upon the form of energy converted into electrical energy, the generating stations are classified as under :

(*i*) Steam power stations (*ii*) Hydroelectric power stations

(*iii*) Diesel power stations (*iv*) Nuclear power stations

2.2 Steam Power Station (Thermal Station)

A generating station which converts heat energy of coal combustion into electrical energy is known as a **steam power station.**

A steam power station basically works on the Rankine cycle. Steam is produced in the boiler by utilising the heat of coal combustion. The steam is then expanded in the prime mover (*i.e.*, steam turbine) and is condensed in a condenser to be fed into the boiler again. The steam turbine drives the alternator which converts mechanical energy of the turbine into electrical energy. This type of power station is suitable where coal and water are available in abundance and a large amount of electric power is to be generated.

Advantages

(*i*) The fuel (*i.e.*, coal) used is quite cheap.

(*ii*) Less initial cost as compared to other generating stations.

(*iii*) It can be installed at any place irrespective of the existence of coal. The coal can be transported to the site of the plant by rail or road.

(*iv*) It requires less space as compared to the hydroelectric power station.

(*v*) The cost of generation is lesser than that of the diesel power station.

Disadvantages

(*i*) It pollutes the atmosphere due to the production of large amount of smoke and fumes.

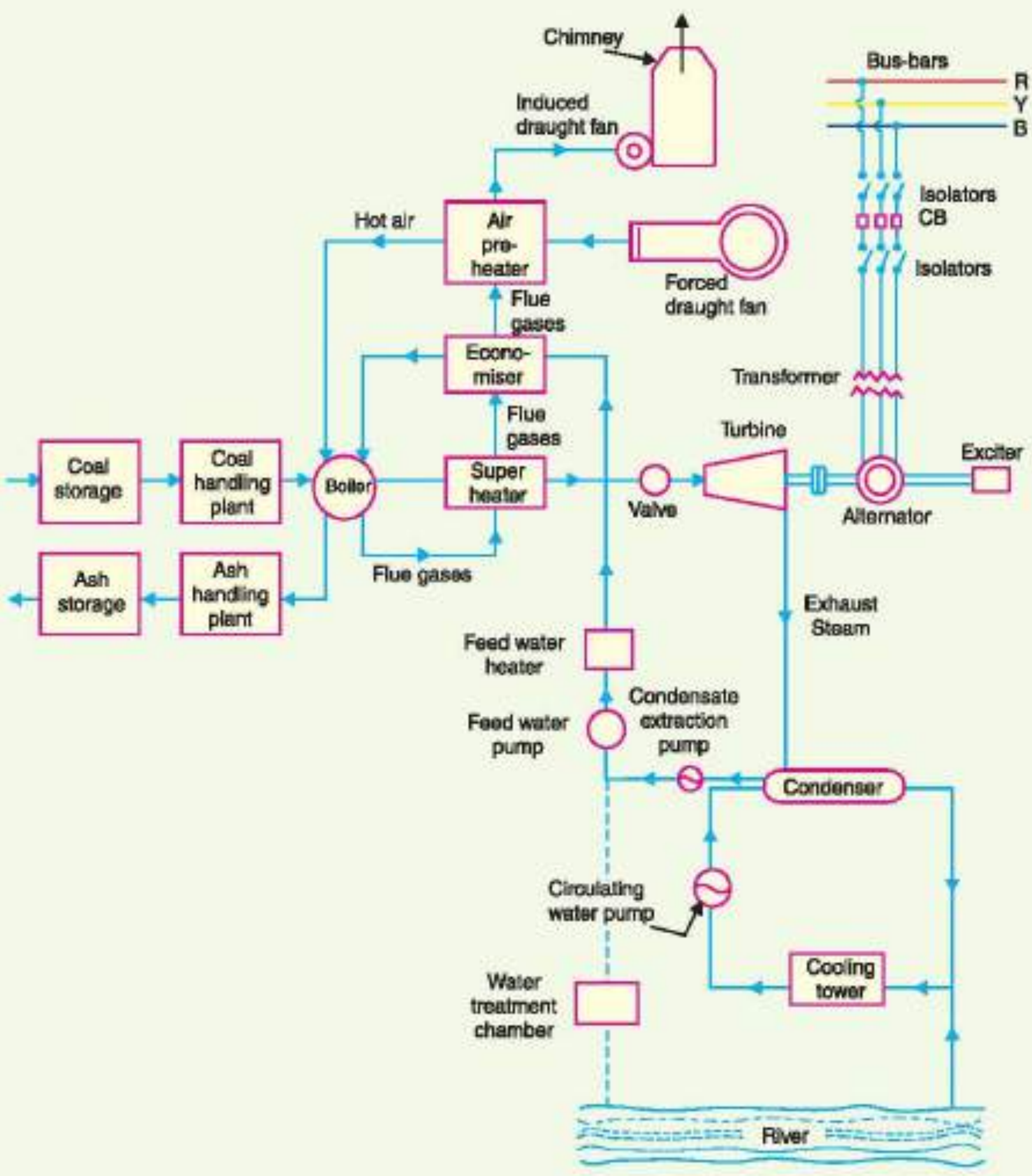
(*ii*) It is costlier in running cost as compared to hydroelectric plant.

2.3 Schematic Arrangement of Steam Power Station

Although steam power station simply involves the conversion of heat of coal combustion into electrical energy, yet it embraces many arrangements for proper working and efficiency. The schematic arrangement of a modern steam power station is shown in Fig. 2.1. The whole arrangement can be divided into the following stages for the sake of simplicity :

1. Coal and ash handling arrangement
2. Steam generating plant
3. Steam turbine
4. Alternator
5. Feed water
6. Cooling arrangement

1. Coal and ash handling plant. The coal is transported to the power station by road or rail and is stored in the coal storage plant. Storage of coal is primarily a matter of protection against coal strikes, failure of transportation system and general coal shortages. From the coal storage plant, coal is delivered to the coal handling plant where it is pulverised (*i.e.*, crushed into small pieces) in order to increase its surface exposure, thus promoting rapid combustion without using large quantity of



Schematic arrangement of Steam Power Station

Fig. 2.1

excess air. The pulverised coal is fed to the boiler by belt conveyors. The coal is burnt in the boiler and the ash produced after the complete combustion of coal is removed to the ash handling plant and then delivered to the ash storage plant for disposal. The removal of the ash from the boiler furnace is necessary for proper burning of coal.

It is worthwhile to give a passing reference to the amount of coal burnt and ash produced in a modern thermal power station. A 100 MW station operating at 50% load factor may burn about 20,000 tons of coal per month and ash produced may be to the tune of 10% to 15% of coal fired *i.e.*, 2,000 to 3,000 tons. In fact, in a thermal station, about 50% to 60% of the total operating cost consists of fuel purchasing and its handling.

2. Steam generating plant. The steam generating plant consists of a boiler for the production of steam and other auxiliary equipment for the utilisation of flue gases.

(*i*) *Boiler.* The heat of combustion of coal in the boiler is utilised to convert water into steam at high temperature and pressure. The flue gases from the boiler make their journey through superheater, economiser, air pre-heater and are finally exhausted to atmosphere through the chimney.

(*ii*) *Superheater.* The steam produced in the boiler is wet and is passed through a superheater where it is dried and superheated (*i.e.*, steam temperature increased above that of boiling point of water) by the flue gases on their way to chimney. Superheating provides two principal benefits. Firstly, the overall efficiency is increased. Secondly, too much condensation in the last stages of turbine (which would cause blade corrosion) is avoided. The superheated steam from the superheater is fed to steam turbine through the main valve.

(*iii*) *Economiser.* An economiser is essentially a feed water heater and derives heat from the flue gases for this purpose. The feed water is fed to the economiser before supplying to the boiler. The economiser extracts a part of heat of flue gases to increase the feed water temperature.

(*iv*) *Air preheater.* An air preheater increases the temperature of the air supplied for coal burning by deriving heat from flue gases. Air is drawn from the atmosphere by a forced draught fan and is passed through air preheater before supplying to the boiler furnace. The air preheater extracts heat from flue gases and increases the temperature of air used for coal combustion. The principal benefits of preheating the air are : increased thermal efficiency and increased steam capacity per square metre of boiler surface.

3. Steam turbine. The dry and superheated steam from the superheater is fed to the steam turbine through main valve. The heat energy of steam when passing over the blades of turbine is converted into mechanical energy. After giving heat energy to the turbine, the steam is exhausted to the *condenser* which condenses the exhausted steam by means of cold water circulation.

4. Alternator. The steam turbine is coupled to an alternator. The alternator converts mechanical energy of turbine into electrical energy. The electrical output from the alternator is delivered to the bus bars through transformer, circuit breakers and isolators.

5. Feed water. The condensate from the condenser is used as feed water to the boiler. Some water may be lost in the cycle which is suitably made up from external source. The feed water on its way to the boiler is heated by water heaters and economiser. This helps in raising the overall efficiency of the plant.

6. Cooling arrangement. In order to improve the efficiency of the plant, the steam exhausted from the turbine is condensed* by means of a condenser. Water is drawn from a natural source of supply such as a river, canal or lake and is circulated through the condenser. The circulating water takes up the heat of the exhausted steam and itself becomes hot. This hot water coming out from the condenser is discharged at a suitable location down the river. In case the availability of water from the source of supply is not assured throughout the year, *cooling towers* are used. During the scarcity of water in the river, hot water from the condenser is passed on to the cooling towers where it is cooled. The cold water from the cooling tower is reused in the condenser.

2.4 Choice of Site for Steam Power Stations

In order to achieve overall economy, the following points should be considered while selecting a site for a steam power station :

(*i*) *Supply of fuel.* The steam power station should be located near the coal mines so that transportation cost of fuel is minimum. However, if such a plant is to be installed at a place

* Efficiency of the plant is increased by reducing turbine exhaust pressure. Low pressure at the exhaust can be achieved by condensing the steam at the turbine exhaust.

where coal is not available, then care should be taken that adequate facilities exist for the transportation of coal.

(*ii*) *Availability of water.* As huge amount of water is required for the condenser, therefore, such a plant should be located at the bank of a river or near a canal to ensure the continuous supply of water.

(*iii*) *Transportation facilities.* A modern steam power station often requires the transportation of material and machinery. Therefore, adequate transportation facilities must exist *i.e.*, the plant should be well connected to other parts of the country by rail, road. etc.

(*iv*) *Cost and type of land.* The steam power station should be located at a place where land is cheap and further extension, if necessary, is possible. Moreover, the bearing capacity of the ground should be adequate so that heavy equipment could be installed.

(*v*) *Nearness to load centres.* In order to reduce the transmission cost, the plant should be located near the centre of the load. This is particularly important if *d.c.* supply system is adopted. However, if *a.c.* supply system is adopted, this factor becomes relatively less important. It is because *a.c.* power can be transmitted at high voltages with consequent reduced transmission cost. Therefore, it is possible to install the plant away from the load centres, provided other conditions are favourable.

(*vi*) *Distance from populated area.* As huge amount of coal is burnt in a steam power station, therefore, smoke and fumes pollute the surrounding area. This necessitates that the plant should be located at a considerable distance from the populated areas.

Conclusion. It is clear that all the above factors cannot be favourable at one place. However, keeping in view the fact that now-a-days the supply system is *a.c.* and more importance is being given to generation than transmission, a site away from the towns may be selected. In particular, a site by river side where sufficient water is available, no pollution of atmosphere occurs and fuel can be transported economically, may perhaps be an ideal choice.

2.5 Efficiency of Steam Power Station

The overall efficiency of a steam power station is quite low (about 29%) due mainly to two reasons. Firstly, a huge amount of heat is lost in the condenser and secondly heat losses occur at various stages of the plant. The heat lost in the condenser cannot be avoided. It is because heat energy cannot be converted into mechanical energy without temperature difference. The greater the temperature difference, the greater is the heat energy converted* into mechanical energy. This necessitates to keep the steam in the condenser at the lowest temperature. But we know that greater the temperature difference, greater is the amount of heat lost. This explains for the low efficiency of such plants.

(*i*) **Thermal efficiency.** *The ratio of heat equivalent of mechanical energy transmitted to the turbine shaft to the heat of combustion of coal is known as* **thermal efficiency** *of steam power station.*

$$\text{Thermal efficiency,} \quad \eta_{thermal} = \frac{\text{Heat equivalent of mech. energy transmitted to turbine shaft}}{\text{Heat of coal combustion}}$$

The thermal efficiency of a modern steam power station is about 30%. It means that if 100 calories of heat is supplied by coal combustion, then mechanical energy equivalent of 30 calories will be available at the turbine shaft and rest is lost. It may be important to note that more than 50% of total heat of combustion is lost in the condenser. The other heat losses occur in flue gases, radiation, ash etc.

(*ii*) **Overall efficiency.** *The ratio of heat equivalent of electrical output to the heat of combustion of coal is known as* **overall efficiency** *of steam power station i.e.*

* Thermodynamic laws.

Overall efficiency, $\eta_{overall} = \frac{\text{Heat equivalent of electrical ouput}}{\text{Heat of combustion of coal}}$

The overall efficiency of a steam power station is about 29%. It may be seen that overall efficiency is less than the thermal efficiency. This is expected since some losses (about 1%) occur in the alternator. The following relation exists among the various efficiencies.

$$\text{Overall efficiency} = \text{Thermal efficiency} \times \text{Electrical efficiency}$$

2.6 Equipment of Steam Power Station

A modern steam power station is highly complex and has numerous equipment and auxiliaries. However, the most important constituents of a steam power station are :

1. Steam generating equipment
2. Condenser
3. Prime mover
4. Water treatment plant
5. Electrical equipment.

1. Steam generating equipment. This is an important part of steam power station. It is concerned with the generation of superheated steam and includes such items as boiler, boiler furnace, superheater, economiser, air pre-heater and other heat reclaiming devices.

(*i*) *Boiler.* A boiler is closed vessel in which water is converted into steam by utilising the heat of coal combustion. Steam boilers are broadly classified into the following two types :

(*a*) Water tube boilers (*b*) Fire tube boilers

In a water tube boiler, water flows through the tubes and the hot gases of combustion flow over these tubes. On the other hand, in a fire tube boiler, the hot products of combustion pass through the tubes surrounded by water. Water tube boilers have a number of advantages over fire tube boilers *viz*., require less space, smaller size of tubes and drum, high working pressure due to small drum, less liable to explosion etc. Therefore, the use of water tube boilers has become universal in large capacity steam power stations.

(*ii*) *Boiler furnace.* A boiler furnace is a chamber in which fuel is burnt to liberate the heat energy. In addition, it provides support and enclosure for the combustion equipment *i.e.*, burners. The boiler furnace walls are made of refractory materials such as fire clay, silica, kaolin etc. These materials have the property to resist change of shape, weight or physical properties at high temperatures. There are following three types of construction of furnace walls :

(*a*) Plain refractory walls

(*b*) Hollow refractory walls with an arrangement for air cooling

(*c*) Water walls.

The plain refractory walls are suitable for small plants where the furnace temperature may not be high. However, in large plants, the furnace temperature is quite high* and consequently, the refractory material may get damaged. In such cases, refractory walls are made hollow and air is circulated through hollow space to keep the temperature of the furnace walls low. The recent development is to use water walls. These consist of plain tubes arranged side by side and on the inner face of the refractory walls. The tubes are connected to the upper and lower headers of the boiler. The boiler water is made to circulate through these tubes. The water walls absorb the radiant heat in the furnace which would otherwise heat up the furnace walls.

(*iii*) *Superheater.* A superheater is a device which superheats the steam *i.e.*, it raises the temperature of steam above boiling point of water. This increases the overall efficiency of the plant. A superheater consists of a group of tubes made of special alloy steels such as chromium-molybdenum. These tubes are heated by the heat of flue gases during their journey from the furnace to the chimney.

* The size of furnace has to be limited due to space, cost and other considerations. This means that furnace of a large plant should develop more kilocalories per square metre of furnace which implies high furnace temperature.

The steam produced in the boiler is led through the superheater where it is superheated by the heat of flue gases. Superheaters are mainly classified into two types according to the system of heat transfer from flue gases to steam *viz.*

(*a*) Radiant superheater (*b*) Convection superheater

The radiant superheater is placed in the furnace between the water walls and receives heat from the burning fuel through radiation process. It has two main disadvantages. Firstly, due to high furnace temperature, it may get overheated and, therefore, requires a careful design. Secondly, the temperature of superheater falls with increase in steam output. Due to these limitations, radiant superheater is not finding favour these days. On the other hand, a convection superheater is placed in the boiler tube bank and receives heat from flue gases entirely through the convection process. It has the advantage that temperature of superheater increases with the increase in steam output. For this reason, this type of superheater is commonly used these days.

(*iv*) *Economiser.* It is a device which heats the feed water on its way to boiler by deriving heat from the flue gases. This results in raising boiler efficiency, saving in fuel and reduced stresses in the boiler due to higher temperature of feed water. An economiser consists of a large number of closely spaced parallel steel tubes connected by headers of drums. The feed water flows through these tubes and the flue gases flow outside. A part of the heat of flue gases is transferred to feed water, thus raising the temperature of the latter.

(*v*) *Air Pre-heater.* Superheaters and economisers generally cannot fully extract the heat from flue gases. Therefore, pre-heaters are employed which recover some of the heat in the escaping gases. The function of an air pre-heater is to extract heat from the flue gases and give it to the air being supplied to furnace for coal combustion. This raises the furnace temperature and increases the thermal efficiency of the plant. Depending upon the method of transfer of heat from flue gases to air, air pre-heaters are divided into the following two classes :

(*a*) Recuperative type (*b*) Regenerative type

The recuperative type air-heater consists of a group of steel tubes. The flue gases are passed through the tubes while the air flows externally to the tubes. Thus heat of flue gases is transferred to air. The regenerative type air pre-heater consists of slowly moving drum made of corrugated metal plates. The flue gases flow continuously on one side of the drum and air on the other side. This action permits the transference of heat of flue gases to the air being supplied to the furnace for coal combustion.

2. Condensers. A condenser is a device which condenses the steam at the exhaust of turbine. It serves two important functions. Firstly, it creates a very low *pressure at the exhaust of turbine, thus permitting expansion of the steam in the prime mover to a very low pressure. This helps in converting heat energy of steam into mechanical energy in the prime mover. Secondly, the condensed steam can be used as feed water to the boiler. There are two types of condensers, namely :

(*i*) Jet condenser (*ii*) Surface condenser

In a jet condenser, cooling water and exhausted steam are mixed together. Therefore, the temperature of cooling water and condensate is the same when leaving the condenser. Advantages of this type of condenser are : low initial cost, less floor area required, less cooling water required and low maintenance charges. However, its disadvantages are : condensate is wasted and high power is required for pumping water.

In a surface condenser, there is no direct contact between cooling water and exhausted steam. It consists of a bank of horizontal tubes enclosed in a cast iron shell. The cooling water flows through the tubes and exhausted steam over the surface of the tubes. The steam gives up its heat to water and is itself condensed. Advantages of this type of condenser are : condensate can be used as feed water, less pumping power required and creation of better vacuum at the turbine exhaust. However, disad-

* By liquidating steam at the exhaust of turbine, a region of emptiness is created. This results in a very low pressure at the exhaust of turbine.

vantages of this type of condenser are : high initial cost, requires large floor area and high maintenance charges.

3. Prime movers. The prime mover converts steam energy into mechanical energy. There are two types of steam prime movers *viz.*, steam engines and steam turbines. A steam turbine has several advantages over a steam engine as a prime mover *viz.*, high efficiency, simple construction, higher speed, less floor area requirement and low maintenance cost. Therefore, all modern steam power stations employ steam turbines as prime movers.

Steam turbines are generally classified into two types according to the action of steam on moving blades *viz.*

(*i*) Impulse turbines (*ii*) Reactions turbines

In an impulse turbine, the steam expands completely in the stationary nozzles (or fixed blades), the pressure over the moving blades remaining constant. In doing so, the steam attains a high velocity and impinges against the moving blades. This results in the impulsive force on the moving blades which sets the rotor rotating. In a reaction turbine, the steam is partially expanded in the stationary nozzles, the remaining expansion takes place during its flow over the moving blades. The result is that the momentum of the steam causes a reaction force on the moving blades which sets the rotor in motion.

4. Water treatment plant. Boilers require clean and soft water for longer life and better efficiency. However, the source of boiler feed water is generally a river or lake which may contain suspended and dissolved impurities, dissolved gases etc. Therefore, it is very important that water is first purified and softened by chemical treatment and then delivered to the boiler.

The water from the source of supply is stored in storage tanks. The suspended impurities are removed through sedimentation, coagulation and filtration. Dissolved gases are removed by aeration and degasification. The water is then *'softened'* by removing temporary and permanent hardness through different chemical processes. The pure and soft water thus available is fed to the boiler for steam generation.

5. Electrical equipment. A modern power station contains numerous electrical equipment. However, the most important items are :

(*i*) *Alternators.* Each alternator is coupled to a steam turbine and converts mechanical energy of the turbine into electrical energy. The alternator may be hydrogen or air cooled. The necessary excitation is provided by means of main and pilot exciters directly coupled to the alternator shaft.

(*ii*) *Transformers.* A generating station has different types of transformers, *viz.*,

(*a*) main step-up transformers which step-up the generation voltage for transmission of power.

(*b*) station transformers which are used for general service (*e.g.*, lighting) in the power station.

(*c*) auxiliary transformers which supply to individual unit-auxiliaries.

(*iii*) *Switchgear.* It houses such equipment which locates the fault on the system and isolate the faulty part from the healthy section. It contains circuit breakers, relays, switches and other control devices.

Example 2.1. *A steam power station has an overall efficiency of 20% and 0·6 kg of coal is burnt per kWh of electrical energy generated. Calculate the calorific value of fuel.*

Solution.

Let x kcal/kg be the calorific value of fuel.

Heat produced by 0·6 kg of coal = 0·6 x kcal

Heat equivalent of 1 kWh = 860 kcal

Now, $\eta_{overall} = \dfrac{\text{Electrical output in heat units}}{\text{Heat of combustion}}$

or $0 \cdot 2 = \dfrac{860}{0 \cdot 6x}$

$\therefore \quad x = \dfrac{860}{0 \cdot 6 \times 0 \cdot 2} =$ **7166·67 kcal/kg**

Example 2.2. *A thermal station has the following data :*

Max. demand	=	*20,000 kW*	;	*Load factor*	=	*40%*
Boiler efficiency	=	*85%*	;	*Turbine efficiency*	=	*90%*
Coal consumption	=	*0·9 kg/kWh*	;	*Cost of 1 ton of coal*	=	*Rs. 300*

Determine (i) thermal efficiency and (ii) coal bill per annum.

Solution.

(*i*) Thermal efficiency = $\eta_{boiler} \times \eta_{turbine} = 0{\cdot}85 \times {\cdot}9 =$ **0·765** or **76·5 %**

(*ii*) Units generated/annum = Max. demand × L.F. × Hours in a year

$= 20{,}000 \times 0{\cdot}4 \times 8760 = 7008 \times 10^4$ kWh

Coal consumption/annum $= \dfrac{(0 \cdot 9)\left(7008 \times 10^4\right)}{1000} = 63{,}072$ tons

$\therefore$ Annual coal bill = Rs 300 × 63072 = **Rs 1,89,21,600**

Example 2.3. *A steam power station spends Rs. 30 lakhs per annum for coal used in the station. The coal has a calorific value of 5000 kcal/kg and costs Rs. 300 per ton. If the station has thermal efficiency of 33% and electrical efficiency of 90%, find the average load on the station.*

Solution.

Overall efficiency, $\eta_{overall} = 0{\cdot}33 \times 0{\cdot}9 = 0{\cdot}297$

Coal used/annum $= 30 \times 10^5/300 = 10^4$ tons $= 10^7$ kg

Heat of combustion = Coal used/annum × Calorific value

$= 10^7 \times 5000 = 5 \times 10^{10}$ kcal

Heat output $= \eta_{overall} \times$ Heat of combustion

$= (0{\cdot}297) \times (5 \times 10^{10}) = 1485 \times 10^7$ kcal

Units generated/annum $= 1485 \times 10^7/860$ kWh

$\therefore$ Average load on station $= \dfrac{\text{Units generated / annum}}{\text{Hours in a year}} = \dfrac{1485 \times 10^7}{860 \times 8760} =$ **1971 kW**

Example 2.4. *The relation between water evaporated (W kg), coal consumption (C kg) and kWh generated per 8-hour shift for a steam generating station is as follows :*

$$W = 13500 + 7{\cdot}5 \text{ kWh} \quad \text{....(i)}$$

$$C = 5000 + 2{\cdot}9 \text{ kWh} \quad \text{....(ii)}$$

(i) To what limiting value does the water evaporating per kg of coal consumed approach as the station output increases ? (ii) How much coal per hour would be required to keep the station running on no load ?

Solution.

(*i*) For an 8-hour shift, weight of water evaporated per kg of coal consumed is

$$\frac{W}{C} = \frac{13500 + 7 \cdot 5 \text{ kWh}}{5000 + 2 \cdot 9 \text{ kWh}}$$

As the station output (*i.e.*, kWh) increases towards infinity, the limiting value of *W*/*C* approaches $7 \cdot 5/2 \cdot 9 = 2 \cdot 6$. Therefore, the weight of water evaporated per kg of coal consumed approaches a limiting value of **2·6 kg** as the kWh output increases.

(*ii*) At no load, the station output is zero *i.e.*, kWh = 0. Therefore, from expression (*ii*), we get, coal consumption at no load

$$= 5000 + 2 \cdot 9 \times 0 = 5000 \text{ kg}$$

$\therefore$ Coal consumption/hour $= 5000/8 =$ **625 kg**

Example 2.5. *A 100 MW steam station uses coal of calorific value 6400 kcal/kg. Thermal efficiency of the station is 30% and electrical efficiency is 92%. Calculate the coal consumption per hour when the station is delivering its full rated output.*

Solution.

Overall efficiency of the power station is

$$\eta_{overall} = \eta_{thermal} \times \eta_{elect} = 0 \cdot 30 \times 0 \cdot 92 = 0 \cdot 276$$

$$\text{Units generated/hour} = (100 \times 10^3) \times 1 = 10^5 \text{ kWh}$$

$$\text{Heat produced/hour, } H = \frac{\text{Electrical output in heat units}}{\eta_{overall}}$$

$$= \frac{10^5 \times 860}{0 \cdot 276} = 311 \cdot 6 \times 10^6 \text{ kcal} \qquad (\because 1 \text{ kWh} = 860 \text{ kcal})$$

$$\therefore \quad \text{Coal consumption/hour} = \frac{H}{\text{Calorific value}} = \frac{311 \cdot 6 \times 10^6}{6400} = \mathbf{48687 \text{ kg}}$$

TUTORIAL PROBLEMS

1. A generating station has an overall efficiency of 15% and 0·75 kg of coal is burnt per kWh by the station. Determine the calorific value of coal in kilocalories per kilogram. **[7644 kcal/kg]**
2. A 75 MW steam power station uses coal of calorific value of 6400 kcal/kg. Thermal efficiency of the station is 30% while electrical efficiency is 80%. Calculate the coal consumption per hour when the station is delivering its full output. **[42 tons]**
3. A 65,000 kW steam power station uses coal of calorific value 15,000 kcal per kg. If the coal consumption per kWh is 0·5 kg and the load factor of the station is 40%, calculate (*i*) the overall efficiency (*ii*) coal consumption per day. **[(*i*) 28·7% (*ii*) 312 tons]**
4. A 60 MW steam power station has a thermal efficiency of 30%. If the coal burnt has a calorific value of 6950 kcal/kg, calculate :
   (*i*) the coal consumption per kWh,
   (*ii*) the coal consumption per day. **[(*i*) 0·413 kg (*ii*) 238 tons]**
5. A 25 MVA turbo-alternator is working on full load at a power factor of 0·8 and efficiency of 97%. Find the quantity of cooling air required per minute at full load, assuming that 90% of the total losses are dissipated by the internally circulating air. The inlet air temperature is 20° C and the temperature rise is 30° C. Given that specific heat of air is 0·24 and that 1 kg of air occupies 0·8 m³. **[890 m³/minute]**
6. A thermal station has an efficiency of 15% and 1·0 kg of coal burnt for every kWh generated. Determine the calorific value of coal. **[5733 kcal/kg]**

2.7 Hydro-electric Power Station

start from here

A generating station which utilises the potential energy of water at a high level for the generation of electrical energy is known as a **hydro-electric power station.**

Hydro-electric power stations are generally located in hilly areas where dams can be built conveniently and large water reservoirs can be obtained. In a hydro-electric power station, water head is created by constructing a dam across a river or lake. From the dam, water is led to a water turbine. The water turbine captures the energy in the falling water and changes the hydraulic energy (*i.e.*, product of head and flow of water) into mechanical energy at the turbine shaft. The turbine drives the alternator which converts mechanical energy into electrical energy. Hydro-electric power stations are becoming very popular because the reserves of fuels (*i.e.*, coal and oil) are depleting day by day. They have the added importance for flood control, storage of water for irrigation and water for drinking purposes.

Advantages

(*i*) It requires no fuel as water is used for the generation of electrical energy.

(*ii*) It is quite neat and clean as no smoke or ash is produced.

(*iii*) It requires very small running charges because water is the source of energy which is available free of cost.

(*iv*) It is comparatively simple in construction and requires less maintenance.

(*v*) It does not require a long starting time like a steam power station. In fact, such plants can be put into service instantly.

(*vi*) It is robust and has a longer life.

(*vii*) Such plants serve many purposes. In addition to the generation of electrical energy, they also help in irrigation and controlling floods.

(*viii*) Although such plants require the attention of highly skilled persons at the time of construction, yet for operation, a few experienced persons may do the job well.

Disadvantages

(*i*) It involves high capital cost due to construction of dam.

(*ii*) There is uncertainty about the availability of huge amount of water due to dependence on weather conditions.

(*iii*) Skilled and experienced hands are required to build the plant.

(*iv*) It requires high cost of transmission lines as the plant is located in hilly areas which are quite away from the consumers.

2.8 Schematic Arrangement of Hydro-electric Power Station

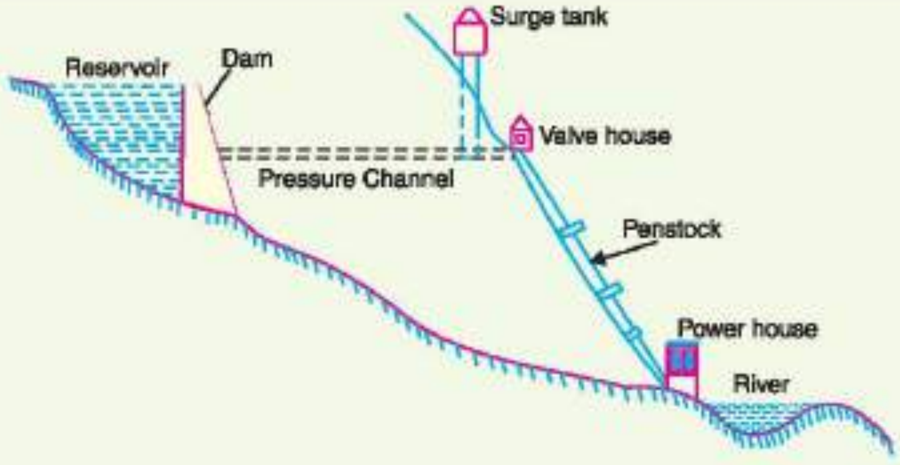
Although a hydro-electric power station simply involves the conversion of hydraulic energy into electrical energy, yet it embraces many arrangements for proper working and efficiency. The schematic arrangement of a modern hydro-electric plant is shown in Fig. 2.2.

The dam is constructed across a river or lake and water from the catchment area collects at the back of the dam to form a reservoir. A pressure tunnel is taken off from the reservoir and water brought to the valve house at the start of the penstock. The valve house contains main sluice valves and automatic isolating valves. The former controls the water flow to the power house and the latter cuts off supply of water when the penstock bursts. From the valve house, water is taken to water turbine through a huge steel pipe known as *penstock*. The water turbine converts hydraulic energy into mechanical energy. The turbine drives the alternator which converts mechanical energy into electrical energy.

A surge tank (open from top) is built just before the valve house and protects the penstock from bursting in case the turbine gates suddenly close* due to electrical load being thrown off. When the

* The governor opens or closes the turbine gates in accordance with the changes in electrical load. If the electrical load increases, the governor opens the turbine gates to allow more water and *vice-versa*.

gates close, there is a sudden stopping of water at the lower end of the penstock and consequently the penstock can burst like a paper log. The surge tank absorbs this pressure swing by increase in its level of water.



Schematic arrangement of a Hydro-electric plant

Fig. 2.2

2.9 Choice of Site for Hydro-electric Power Stations

The following points should be taken into account while selecting the site for a hydro-electric power station :

(i) *Availability of water.* Since the primary requirement of a hydro-electric power station is the availability of huge quantity of water, such plants should be built at a place (*e.g.*, river, canal) where adequate water is available at a good head.

(ii) *Storage of water.* There are wide variations in water supply from a river or canal during the year. This makes it necessary to store water by constructing a dam in order to ensure the generation of power throughout the year. The storage helps in equalising the flow of water so that any excess quantity of water at a certain period of the year can be made available during times of very low flow in the river. This leads to the conclusion that site selected for a hydro-electric plant should provide adequate facilities for erecting a dam and storage of water.

(iii) *Cost and type of land.* The land for the construction of the plant should be available at a reasonable price. Further, the bearing capacity of the ground should be adequate to withstand the weight of heavy equipment to be installed.

(iv) *Transportation facilities.* The site selected for a hydro-electric plant should be accessible by rail and road so that necessary equipment and machinery could be easily transported.

It is clear from the above mentioned factors that ideal choice of site for such a plant is near a river in hilly areas where dam can be conveniently built and large reservoirs can be obtained.

2.10 Constituents of Hydro-electric Plant

The constituents of a hydro-electric plant are (1) hydraulic structures (2) water turbines and (3) electrical equipment. We shall discuss these items in turn.

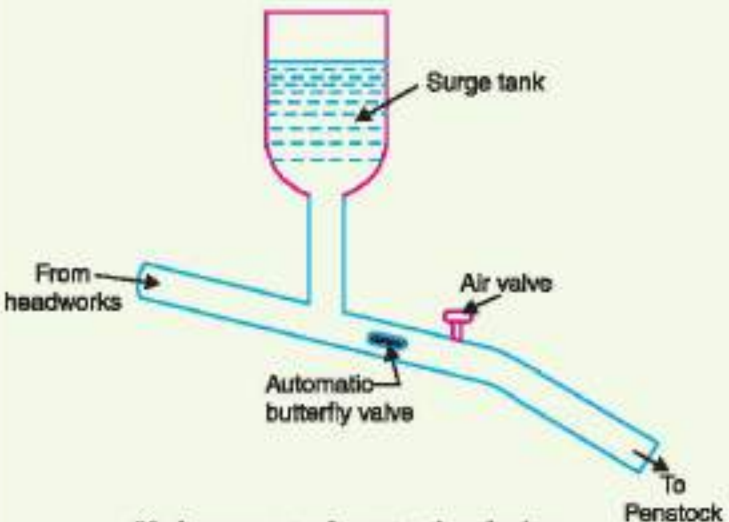
1. **Hydraulic structures.** Hydraulic structures in a hydro-electric power station include dam, spillways, headworks, surge tank, penstock and accessory works.

(i) *Dam.* A dam is a barrier which stores water and creates water head. Dams are built of concrete or stone masonary, earth or rock fill. The type and arrangement depends upon the topography of the site. A masonary dam may be built in a narrow canyon. An earth dam may be best suited for a wide valley. The type of dam also depends upon the foundation conditions, local materials and transportation available, occurrence of earthquakes and other hazards. At most of sites, more than one type of dam may be suitable and the one which is most economical is chosen.

(*ii*) *Spillways.* There are times when the river flow exceeds the storage capacity of the reservoir. Such a situation arises during heavy rainfall in the catchment area. In order to discharge the surplus water from the storage reservoir into the river on the down-stream side of the dam, spillways are used. Spillways are constructed of concrete piers on the top of the dam. Gates are provided between these piers and surplus water is discharged over the crest of the dam by opening these gates.

(*iii*) *Headworks.* The headworks consists of the diversion structures at the head of an intake. They generally include booms and racks for diverting floating debris, sluices for by-passing debris and sediments and valves for controlling the flow of water to the turbine. The flow of water into and through headworks should be as smooth as possible to avoid head loss and cavitation. For this purpose, it is necessary to avoid sharp corners and abrupt contractions or enlargements.

(*iv*) *Surge tank.* Open conduits leading water to the turbine require no* protection. However, when closed conduits are used, protection becomes necessary to limit the abnormal pressure in the conduit. For this reason, closed conduits are always provided with a surge tank. A surge tank is a small reservoir or tank (open at the top) in which water level rises or falls to reduce the pressure swings in the conduit.



Various penstock protective devices

Fig. 2.3

A surge tank is located near the beginning of the conduit. When the turbine is running at a steady load, there are no surges in the flow of water through the conduit *i.e.*, the quantity of water flowing in the conduit is just sufficient to meet the turbine requirements. However, when the load on the turbine decreases, the governor closes the gates of turbine, reducing water supply to the turbine. The excess water at the lower end of the conduit rushes back to the surge tank and increases its water level. Thus the conduit is prevented from bursting. On the other hand, when load on the turbine increases, additional water is drawn from the surge tank to meet the increased load requirement. Hence, a surge tank overcomes the abnormal pressure in the conduit when load on the turbine falls and acts as a reservoir during increase of load on the turbine.

(*v*) *Penstocks.* Penstocks are open or closed conduits which carry water to the turbines. They are generally made of reinforced concrete or steel. Concrete penstocks are suitable for low

* Because in case of open conduits, regulating gates control the inflow at the headworks and the spillway discharges the surplus water.

heads (< 30 m) as greater pressure causes rapid deterioration of concrete. The steel penstocks can be designed for any head; the thickness of the penstock increases with the head or working pressure.

Various devices such as automatic butterfly valve, air valve and surge tank (See Fig. 2.3) are provided for the protection of penstocks. Automatic butterfly valve shuts off water flow through the penstock promptly if it ruptures. Air valve maintains the air pressure inside the penstock equal to outside atmospheric pressure. When water runs out of a penstock faster than it enters, a vacuum is created which may cause the penstock to collapse. Under such situations, air valve opens and admits air in the penstock to maintain inside air pressure equal to the outside air pressure.

2. Water turbines. Water turbines are used to convert the energy of falling water into mechanical energy. The principal types of water turbines are :

(*i*) Impulse turbines (*ii*) Reaction turbines

(*i*) *Impulse turbines.* Such turbines are used for high heads. In an impulse turbine, the entire pressure of water is converted into kinetic energy in a nozzle and the velocity of the jet drives the wheel. The example of this type of turbine is the Pelton wheel (See Fig. 2.4). It consists of a wheel fitted with elliptical buckets along its periphery. The force of water jet striking the buckets on the wheel drives the turbine. The quantity of water jet falling on the turbine is controlled by means of a *needle* or *spear* (not shown in the figure) placed in the tip of the nozzle. The movement of the needle is controlled by the governor. If the load on the turbine decreases, the governor pushes the needle into the nozzle, thereby reducing the quantity of water striking the buckets. Reverse action takes place if the load on the turbine increases.

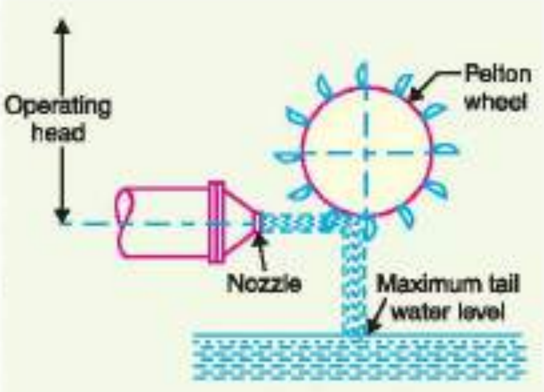


Fig. 2.4 Pelton Wheel

(*ii*) *Reaction turbines.* Reaction turbines are used for low and medium heads. In a reaction turbine, water enters the runner partly with pressure energy and partly with velocity head. The important types of reaction turbines are :

(*a*) Francis turbines (*b*) Kaplan turbines

A Francis turbine is used for low to medium heads. It consists of an outer ring of stationary guide blades fixed to the turbine casing and an inner ring of rotating blades forming the runner. The guide blades control the flow of water to the turbine. Water flows radially inwards and changes to a downward direction while passing through the runner. As the water passes over the "rotating blades" of the runner, both pressure and velocity of water are reduced. This causes a reaction force which drives the turbine.

A Kaplan turbine is used for low heads and large quantities of water. It is similar to Francis turbine except that the runner of Kaplan turbine receives water axially. Water flows radially inwards


Bhakra Dam

through regulating gates all around the sides, changing direction in the runner to axial flow. This causes a reaction force which drives the turbine.

3. Electrical equipment. The electrical equipment of a hydro-electric power station includes alternators, transformers, circuit breakers and other switching and protective devices.

Example 2.6. *A hydro-electric generating station is supplied from a reservoir of capacity 5×10^6 cubic metres at a head of 200 metres. Find the total energy available in kWh if the overall efficiency is 75%.*

Solution.

Weight of water available is

$$W = \text{Volume of water} \times \text{density}$$
$$= (5 \times 10^6) \times (1000) \quad (\because \text{mass of } 1\text{m}^3 \text{ of water is } 1000 \text{ kg})$$
$$= 5 \times 10^9 \text{ kg} = 5 \times 10^9 \times 9.81 \text{ N}$$

$$\text{Electrical energy available} = W \times H \times \eta_{overall} = (5 \times 10^9 \times 9.81) \times (200) \times (0.75) \text{ watt sec}$$
$$= \frac{(5 \times 10^9 \times 9.81) \times (200) \times (0.75)}{3600 \times 1000} \text{ kWh} = \mathbf{2.044 \times 106 \text{ kWh}}$$

Example 2.7. *It has been estimated that a minimum run off of approximately 94 m^3/sec will be available at a hydraulic project with a head of 39 m. Determine (i) firm capacity (ii) yearly gross output. Assume the efficiency of the plant to be 80%.*

Solution.

Weight of water available, $W = 94 \times 1000 = 94000$ kg/sec

Water head, $H = 39$ m

$$\text{Work done/sec} = W \times H = 94000 \times 9.81 \times 39 \text{ watts}$$
$$= 35,963 \times 10^3 \text{ W} = 35,963 \text{ kW}$$

This is gross plant capacity.

(*i*) Firm capacity $= \text{Plant efficiency} \times \text{Gross plant capacity}$
$= 0.80 \times 35,963 = \mathbf{28,770 \text{ kW}}$

(*ii*) Yearly gross output $= \text{Firm capacity} \times \text{Hours in a year}$
$= 28,770 \times 8760 = \mathbf{252 \times 10^6 \text{ kWh}}$

Example 2.8. *Water for a hydro-electric station is obtained from a reservoir with a head of 100 metres. Calculate the electrical energy generated per hour per cubic metre of water if the hydraulic efficiency be 0·86 and electrical efficiency 0·92.*

Solution.

Water head, $H = 100$ m ; discharge, $Q = 1 \text{ m}^3/\text{sec}$; $\eta_{overall} = 0.86 \times 0.92 = 0.79$

Wt. of water available/sec, $W = Q \times 1000 \times 9.81 = 9810\ N$

Power produced $= W \times H \times \eta_{overall} = 9810 \times 100 \times 0.79$ watts
$= 775 \times 10^3 \text{ watts} = 775$ kW

$\therefore$ Energy generated/hour $= 775 \times 1 = \mathbf{775 \text{ kWh}}$

Example 2.9. *Calculate the average power in kW that can be generated in a hydro-electric project from the following data*

Catchment area = 5×10^9 m^2 ; Mean head, H = 30 m

Annual rainfall, F = 1·25 m ; Yield factor, K = 80 %

Overall efficiency, $\eta_{overall}$ = 70 %

If the load factor is 40% , what is the rating of generators installed ?

Solution.

Volume of water which can be utilised per annum

$$= \text{Catchment area} \times \text{Annual rainfall} \times {}^*\text{yield factor}$$

$$= (5 \times 10^9) \times (1{\cdot}25) \times (0{\cdot}8) = 5 \times 10^9 \text{ m}^3$$

Weight of water available per annum is

$$W = 5 \times 10^9 \times 9{\cdot}81 \times 1000 = 49{\cdot}05 \times 10^{12} \text{ N}$$

Electrical energy available per annum

$$= W \times H \times \eta_{overall} = (49{\cdot}05 \times 10^{12}) \times (30) \times (0{\cdot}7) \text{ watt-sec}$$

$$= \frac{(49 \cdot 05 \times 10^{12}) \times (30) \times (0 \cdot 7)}{1000 \times 3600} \text{ kWh} = 2 \cdot 86 \times 10^8 \text{ kWh}$$

$\therefore$ Average power $= 2{\cdot}86 \times 10^8/8760 =$ **32648 kW**

Max. demand $= \dfrac{\text{Average demand}}{\text{Load factor}} = \dfrac{32648}{0 \cdot 4} =$ **81620 kW**

Therefore, the maximum capacity of the generators should be 81620 kW.

Example 2.10. *A hydro-electric power station has a reservoir of area 2·4 square kilometres and capacity 5×10^6 m³. The effective head of water is 100 metres. The penstock, turbine and generation efficiencies are respectively 95%, 90% and 85%.*

(*i*) *Calculate the total electrical energy that can be generated from the power station.*

(*ii*) *If a load of 15,000 kW has been supplied for 3 hours, find the fall in reservoir level.*

Solution.

(*i*) Wt. of water available, W = Volume of reservoir × wt. of 1m³ of water

$$= (5 \times 10^6) \times (1000) \text{ kg} = 5 \times 10^9 \times 9{\cdot}81 \text{ N}$$

Overall efficiency, $\eta_{overall} = 0{\cdot}95 \times 0{\cdot}9 \times 0{\cdot}85 = 0{\cdot}726$

Electrical energy that can be generated

$$= W \times H \times \eta_{overall} = (5 \times 10^9 \times 9{\cdot}81) \times (100) \times (0{\cdot}726) \text{ watt-sec.}$$

$$= \frac{(5 \times 10^9 \times 9 \cdot 81) \times (100) \times (0 \cdot 726)}{1000 \times 3600} \text{ kWh} = \mathbf{9{,}89{,}175 \text{ kWh}}$$

(*ii*) Let x metres be the fall in reservoir level in 3 hours.

$$\text{Average discharge/sec} = \frac{\text{Area of reservoir} \times x}{3 \times 3600} = \frac{2 \cdot 4 \times 10^6 \times x}{3 \times 3600} = 222 \cdot 2\, x \text{ m}^3$$

$$\text{Wt. of water available/sec, } W = 222{\cdot}2x \times 1000 \times 9{\cdot}81 = 21{\cdot}8x \times 10^5 \text{ N}$$

$$\text{Average power produced} = W \times H \times \eta_{overall}$$

$$= (21{\cdot}8x \times 10^5) \times (100) \times (0{\cdot}726) \text{ watts}$$

$$= 15{\cdot}84x \times 10^7 \text{ watts} = 15{\cdot}84x \times 10^4 \text{ kW}$$

But kW produced $= 15{,}000$ (*given*)

$$\therefore \quad 15{\cdot}84x \times 10^4 = 15{,}000$$

or

$$x = \frac{15{,}000}{15 \cdot 84 \times 10^4} = 0 \cdot 0947 \text{ m} = \mathbf{9.47 \text{ cm}}$$

Therefore, the level of reservoir will fall by 9·47 cm.

---

* The total rainfall cannot be utilised as a part of it is lost by evaporation or absorption by ground. Yield factor indicates the percentage of rainfall available for utilisation. Thus 80% yield factor means that only 80% of total rainfall can be utilised.

Alternative method

$$\text{Level of reservoir} = \frac{\text{Vol. of reservoir}}{\text{Area of reservoir}} = \frac{5 \times 10^6}{2 \cdot 4 \times 10^6} = 2 \cdot 083 \text{ m}$$

$$\text{kWh generated in 3 hrs} = 15000 \times 3 = 45{,}000 \text{ kWh}$$

If kWh generated are 9,89,175 kWh, fall in reservoir level = 2·083 m

If kWh generated are 45,000 kWh, fall in reservoir level

$$= \frac{2 \cdot 083}{9{,}89{,}175} \times 45{,}000 = 0 \cdot 0947 \text{ m} = \mathbf{9.47 \text{ cm}}$$

Example 2.11. *A factory is located near a water fall where the usable head for power generation is 25 m. The factory requires continuous power of 400 kW throughout the year. The river flow in a year is (a) 10 m^3/sec for 4 months, (b) 6 m^3/sec for 2 months and (c) 1·5 m^3/sec for 6 months.*

(i) If the site is developed as a run-of-river type of plant, without storage, determine the standby capacity to be provided. Assume that overall efficiency of the plant is 80%.

(ii) If a reservoir is arranged upstream, will any standby unit be necessary ? What will be the excess power available ?

Solution.

(i) **Run of river Plant.** In this type of plant, the whole water of stream is allowed to pass through the turbine for power generation. The plant utilises the water as and when available. Consequently, more power can be generated in a rainy season than in dry season.

(a) *When discharge* $= 10 \text{ m}^3\text{/sec}$

Wt. of water available/sec, $w = 10 \times 1000 \text{ kg} = 10^4 \times 9{\cdot}81 \text{ N}$

Power developed $= w \times H \times \eta_{overall} = (10^4 \times 9{\cdot}81) \times (25) \times (0{\cdot}8)$ watts

$= 1962 \times 10^3$ watts $= 1962$ kW

(b) *When discharge* $= 6 \text{ m}^3\text{/sec}$

Power developed $= 1962 \times 6^*/10 = 1177{\cdot}2$ kW

(c) *When discharge* $= 1{\cdot}5 \text{ m}^3\text{/sec}$

Power developed $= 1962 \times 1{\cdot}5/10 = 294$ kW

It is clear that when discharge is 10 m^3/sec or 6 m^3/sec, power developed by the plant is more than 400 kW required by the factory. However, when the discharge is 1·5 m^3/sec, power developed falls short and consequently standby unit is required during this period.

∴ Capacity of standby unit = 400 − 294 = **106 kW**

(ii) **With reservoir.** When reservoir is arranged upstream, we can store water. This permits regulated supply of water to the turbine so that power output is constant throughout the year.

$$\text{Average discharge} = \frac{(10 \times 4) + (2 \times 6) + (1 \cdot 5 \times 6)}{12} = 5 \cdot 08 \text{ m}^3/\text{sec.}$$

∴ Power developed = 1962 × 5·08/10 = 996·7 kW

Since power developed is more than required by the factory, no standby unit is needed.

∴ Excess power available = 996·7 − 400 = **596·7 kW**

Example 2.12. *A run-of-river hydro-electric plant with pondage has the following data :*

Installed capacity = 10 MW ; Water head, H = 20 m

Overall efficiency, $\eta_{overall}$ = 80% ; Load factor = 40%

* If discharge is 10 m^3/sec, power devloped = 1962 kW

If discharge is 1 m^3/sec, power devloped = 1962/10

If discharge is 6 m^3/sec, power devloped = 1962 × 6/10 (*i*) *Determine the river discharge in m^3/sec required for the plant.*

(*ii*) *If on a particular day, the river flow is 20 m^3/sec, what load factor can the plant supply ?*

Solution.

(*i*) Consider the duration to be of one week.

Units generated/week = Max. demand × L.F. × Hours in a week

$= (10 \times 10^3) \times (0{\cdot}4) \times (24 \times 7)$ kWh

$= 67{\cdot}2 \times 10^4$ kWh ... (*i*)

Let Q m^3/sec be the river discharge required.

Wt. of water available/sec, $w = Q \times 9{\cdot}81 \times 1000 = 9810\, Q$ newton

Average power produced $= w \times H \times \eta_{overall} = (9810\, Q) \times (20) \times (0{\cdot}8)$ W

$= 156960\, Q$ watt $= 156{\cdot}96\, Q$ kW

Units generated/week $= (156{\cdot}96\, Q) \times 168$ kWh $= 26{,}369$ Q kWh ... (*ii*)

Equating exps. (*i*) and (*ii*), we get,

$$26{,}369\, Q = 67{\cdot}2 \times 10^4$$

$$\therefore \quad Q = \frac{67{\cdot}2 \times 10^4}{26{,}369} = \mathbf{25.48\ m^3/sec}$$

(*ii*) If the river discharge on a certain day is 20 m^3/sec, then,

Power developed $= 156{\cdot}96 \times 20 = 3139{\cdot}2$ kW

Units generated on that day $= 3139{\cdot}2 \times 24 = 75{,}341$ kWh

Load factor $= \dfrac{75{,}341}{10^4 \times 24} \times 100 = \mathbf{31.4\%}$

Example 2.13. *The weekly discharge of a typical hydroelectric plant is as under :*

Day	*Sun*	*Mon*	*Tues*	*Wed*	*Thurs*	*Fri*	*Sat*
Discharge (m^3/sec)	*500*	*520*	*850*	*800*	*875*	*900*	*546*

The plant has an effective head of 15 m and an overall efficiency of 85%. If the plant operates on 40% load factor, estimate (i) the average daily discharge (ii) pondage required and (iii) installed capacity of proposed plant.

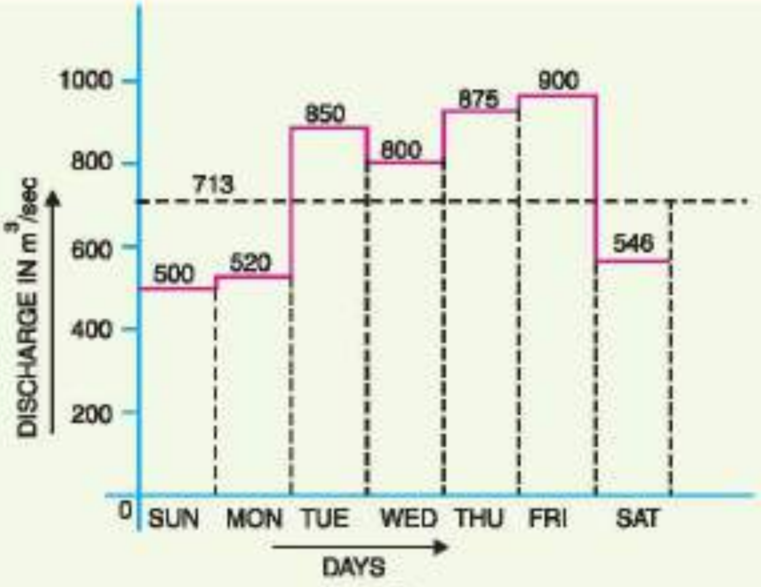


Fig. 2.5

Solution.

Fig. 2.5 shows the plot of weekly discharge. In this graph, discharge is taken along Y-axis and days along X-axis.

(*i*) Average daily discharge

$$= \frac{500 + 520 + 850 + 800 + 875 + 900 + 546}{7}$$

$$= \frac{4991}{7} = \mathbf{713\ m^3/sec}$$

(*ii*) It is clear from graph that on three dyas (*viz.*, Sun, Mon. and Sat.), the discharge is less than the average discharge.

Volume of water actually available on these three days

$$= (500 + 520 + 546) \times 24 \times 3600\ \text{m}^3 = 1566 \times 24 \times 3600\ \text{m}^3$$

Volume of water required on these three days

$$= 3 \times 713 \times 24 \times 3600\ \text{m}^3 = 2139 \times 24 \times 3600\ \text{m}^3$$

Pondage required $= (2139 - 1566) \times 24 \times 3600\ \text{m}^3 = \mathbf{495 \times 10^5\ m^3}$

(*iii*) Wt. of water available/sec, $w = 713 \times 1000 \times 9{\cdot}81$ N

Average power produced $= w \times H \times \eta_{overall} = (713 \times 1000 \times 9{\cdot}81) \times (15) \times (0{\cdot}85)$ watts

$$= 89180 \times 10^3 \text{ watts} = 89180 \text{ kW}$$

Installed capacity of the plant

$$= \frac{\text{Output power}}{\text{Load factor}} = \frac{89180}{0{\times}4} = 223 \times 10^3 \text{ kW} = \mathbf{223\ MW}$$

TUTORIAL PROBLEMS

1. A hydro-electric station has an average available head of 100 metres and reservoir capacity of 50 million cubic metres. Calculate the total energy in kWh that can be generated, assuming hydraulic efficiency of 85% and electrical efficiency of 90%. **[$10{\cdot}423 \times 10^6$ kWh]**
2. Calculate the continuous power that will be available from hydroelectric plant having an available head of 300 meters, catchment area of 150 sq. km, annual rainfall 1·25 m and yield factor of 50%. Assume penstock, turbine and generator efficiencies to be 96%, 86% and 97% respectively. If the load factor is 40% what should be the rating of the generators installed ? **[7065 kW, 17662 kW]**
3. A hydroelectric plant has a reservoir of area 2 sq. kilometres and of capacity 5 million cubic meters. The net head of water at the turbine is 50 m. If the efficiencies of turbine and generator are 85% and 95% respectively, calculate the total energy in kWh that can be generated from this station. If a load of 15000 kW has been supplied for 4 hours, find the fall in reservoir. **[$5{\cdot}5 \times 10^5$ kWh ; 27·8 cm]**
4. It has been estimated that a minimum run-off of approximately 94 m^3/sec will be available at a hydraulic project with a head of 39 m. Determine the firm capacity and yearly gross output. **[3600 kW, $315{\cdot}36 \times 10^6$ kWh]**

   Hint. Wt. of water flowing/sec $= \dfrac{94 \times (100)^3}{1000}$ kg
5. A hydroelectric power station is supplied from a reservoir having an area of 50 km^2 and a head of 50 m. If overall efficiency of the plant is 60%, find the rate at which the water level will fall when the station is generating 30,000 kW. **[7·337 mm/hour]**
6. A hydro-electric plant has a catchment are of 120 square km. The available run off is 50% with annual rainfall of 100 cm. A head of 250 m is available on the average. Efficiency of the power plant is 70%. Find (*i*) average power produced (*ii*) capacity of the power plant. Assume a load factor of 0.6. **[(*i*) 3266 kW (*ii*) 5443 kW]**

2.11 Diesel Power Station

A generating station in which diesel engine is used as the prime mover for the generation of electrical energy is known as **diesel power station.**

In a diesel power station, diesel engine is used as the prime mover. The diesel burns inside the engine and the products of this combustion act as the "working fluid" to produce mechanical energy. The diesel engine drives the alternator which converts mechanical energy into electrical energy. As the generation cost is considerable due to high price of diesel, therefore, such power stations are only used to produce small power.

Although steam power stations and hydro-electric plants are invariably used to generate bulk power at cheaper cost, yet diesel power stations are finding favour at places where demand of power is less, sufficient quantity of coal and water is not available and the transportation facilities are inadequate. These plants are also used as standby sets for continuity of supply to important points such as hospitals, radio stations, cinema houses and telephone exchanges.

Advantages

(*i*) The design and layout of the plant are quite simple.
(*ii*) It occupies less space as the number and size of the auxiliaries is small.
(*iii*) It can be located at any place.
(*iv*) It can be started quickly and can pick up load in a short time.
(*v*) There are no standby losses.
(*vi*) It requires less quantity of water for cooling.
(*vii*) The overall cost is much less than that of steam power station of the same capacity.
(*viii*) The thermal efficiency of the plant is higher than that of a steam power station.
(*ix*) It requires less operating staff.

Disadvantages

(*i*) The plant has high running charges as the fuel (*i.e.*, diesel) used is costly.
(*ii*) The plant does not work satisfactorily under overload conditions for a longer period.
(*iii*) The plant can only generate small power.
(*iv*) The cost of lubrication is generally high.
(*v*) The maintenance charges are generally high.

2.12 Schematic Arrangement of Diesel Power Station

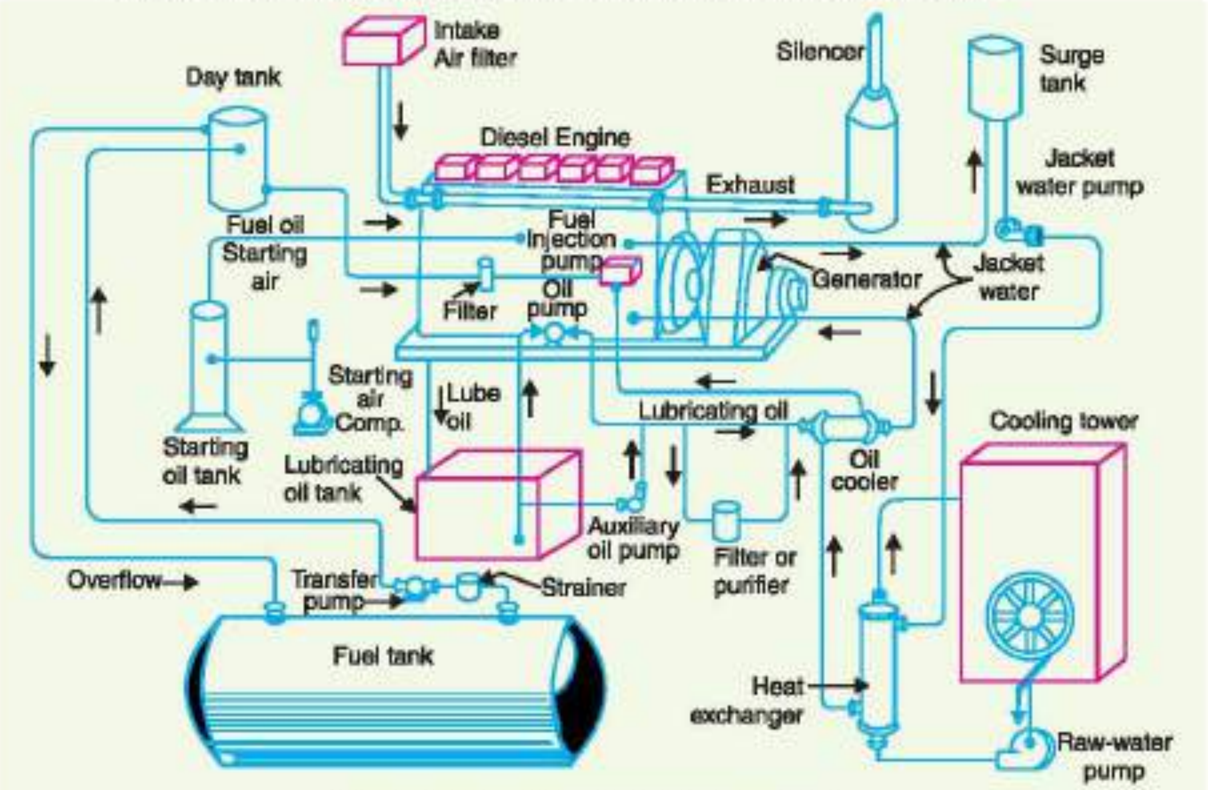
Fig. 2.6 shows the schematic arrangement of a typical diesel power station. Apart from the diesel-generator set, the plant has the following auxiliaries :

(*i*) *Fuel supply system.* It consists of storage tank, strainers, fuel transfer pump and all day fuel tank. The fuel oil is supplied at the plant site by rail or road. This oil is stored in the storage tank. From the storage tank, oil is pumped to smaller all day tank at daily or short intervals. From this tank, fuel oil is passed through strainers to remove suspended impurities. The clean oil is injected into the engine by fuel injection pump.

(*ii*) *Air intake system.* This system supplies necessary air to the engine for fuel combustion. It consists of pipes for the supply of fresh air to the engine manifold. Filters are provided to remove dust particles from air which may act as abrasive in the engine cylinder.

(*iii*) *Exhaust system.* This system leads the engine exhaust gas outside the building and discharges it into atmosphere. A silencer is usually incorporated in the system to reduce the noise level.

(*iv*) *Cooling system.* The heat released by the burning of fuel in the engine cylinder is partially converted into work. The remainder part of the heat passes through the cylinder walls, piston, rings etc. and may cause damage to the system. In order to keep the temperature of the engine parts within the safe operating limits, cooling is provided. The cooling system consists of a water source, pump and cooling towers. The pump circulates water through cylinder and head jacket. The water takes away heat form the engine and itself becomes hot. The hot water is cooled by cooling towers and is recirculated for cooling.



Schematic arrangement of Diesel Power Plant

Fig. 2.6

(*v*) *Lubricating system.* This system minimises the wear of rubbing surfaces of the engine. It comprises of lubricating oil tank, pump, filter and oil cooler. The lubricating oil is drawn from the lubricating oil tank by the pump and is passed through filters to remove impurities. The clean lubricating oil is delivered to the points which require lubrication. The oil coolers incorporated in the system keep the temperature of the oil low.

(*vi*) *Engine starting system.* This is an arrangement to rotate the engine initially, while starting, until firing starts and the unit runs with its own power. Small sets are started manually by handles but for larger units, compressed air is used for starting. In the latter case, air at high pressure is admitted to a few of the cylinders, making them to act as reciprocating air motors to turn over the engine shaft. The fuel is admitted to the remaining cylinders which makes the engine to start under its own power.

Example 2.14. *A diesel power station has fuel consumption of 0·28 kg per kWh, the calorific value of fuel being 10,000 kcal/kg. Determine (i) the overall efficiency, and (ii) efficiency of the engine if alternator efficiency is 95%.*

Solution.

Heat produced by 0·28 kg of oil = $10{,}000 \times 0{\cdot}28 = 2800$ kcal

Heat equivalent of 1 kWh = 860 kcal

(i) Overall efficiency $= \dfrac{\text{Electrical output in heat units}}{\text{Heat of combustion}} = 860/2800 = 0{\cdot}307 =$ **30·7%**

(ii) Engine efficiency $= \dfrac{\text{Overall efficiency}}{\text{Alternator efficiency}} = \dfrac{30{\cdot}7}{0{\cdot}95} =$ **32·3%**

Example 2.15. *A diesel power station has the following data :*

Fuel consumption/day = 1000 kg
Units generated/day = 4000 kWh
Calorific value of fuel = 10,000 kcal/kg
Alternator efficiency = 96%
Engine mech. efficiency = 95%

Estimate (i) specific fuel consumption, (ii) overall efficiency, and (iii) thermal efficiency of engine.

Solution.

(i) Specific fuel consumption = 1000/4000 = **0·25 kg/kWh**

(ii) Heat produced by fuel per day

= Coal consumption/day × calorific value

$= 1000 \times 10{,}000 = 10^7$ kcal

Electrical output in heat units per day

$= 4000 \times 860 = 344 \times 10^4$ kcal

$$\text{Overall efficiency} = \frac{344 \times 10^4}{10^7} \times 100 = \mathbf{34{\cdot}4\%}$$

(iii) Engine efficiency, $\eta_{engine} = \dfrac{\eta_{overall}}{\eta_{alt}} = \dfrac{34{\cdot}4}{0{\cdot}96} = 35{\cdot}83\%$

$$\text{Thermal efficiency, } \eta_{ther} = \frac{\eta_{engine}}{\text{Mech. } \eta \text{ of engine}} = \frac{35{\cdot}83}{0{\cdot}95} = \mathbf{37{\cdot}71\%}$$

Example 2.16. *A diesel engine power plant has one 700 kW and two 500 kW generating units. The fuel consumption is 0·28 kg per kWh and the calorific value of fuel oil is 10200 kcal/kg. Estimate (i) the fuel oil required for a month of 30 days and (ii) overall efficiency. Plant capacity factor = 40%.*

Solution.

(i) Maximum energy that can be produced in a month

= Plant capacity × Hours in a month

$= (700 + 2 \times 500) \times (30 \times 24) = 1700 \times 720$ kWh

$$\text{Plant capacity factor} = \frac{\text{Actual energy produced}}{\text{Max. energy that could have been produced}}$$

or $$0{\cdot}4 = \frac{\text{Actual energy produced}}{1700 \times 720}$$

∴ Actual energy produced in a month

$= 0{\cdot}4 \times 1700 \times 720 = 489600$ kWh

Fuel oil consumption in a month

$= 489600 \times 0{\cdot}28 =$ **137088 kg**

(ii) Output $= 489600$ kWh $= 489600 \times 860$ kcal

Input $= 137088 \times 10200$ kcal

$\therefore$ Overall efficiency $= \frac{\text{Output}}{\text{Input}} = \frac{489600 \times 860}{137088 \times 10200} =$ **0·3 or 30%**

2.13 Nuclear Power Station

A generating station in which nuclear energy is converted into electrical energy is known as a **nuclear power station.**

In nuclear power station, heavy elements such as Uranium (U^{235}) or Thorium (Th^{232}) are subjected to nuclear fission* in a special apparatus known as a *reactor*. The heat energy thus released is utilised in raising steam at high temperature and pressure. The steam runs the steam turbine which converts steam energy into mechanical energy. The turbine drives the alternator which converts mechanical energy into electrical energy.

The most important feature of a nuclear power station is that huge amount of electrical energy can be produced from a relatively small amount of nuclear fuel as compared to other conventional types of power stations. It has been found that complete fission of 1 kg of Uranium (U^{235}) can produce as much energy as can be produced by the burning of 4,500 tons of high grade coal. Although the recovery of principal nuclear fuels (*i.e.*, Uranium and Thorium) is difficult and expensive, yet the total energy content of the estimated world reserves of these fuels are considerably higher than those of conventional fuels, *viz.*, coal, oil and gas. At present, energy crisis is gripping us and, therefore, nuclear energy can be successfully employed for producing low cost electrical energy on a large scale to meet the growing commercial and industrial demands.

Advantages

(*i*) The amount of fuel required is quite small. Therefore, there is a considerable saving in the cost of fuel transportation.

(*ii*) A nuclear power plant requires less space as compared to any other type of the same size.

(*iii*) It has low running charges as a small amount of fuel is used for producing bulk electrical energy.

(*iv*) This type of plant is very economical for producing bulk electric power.

(*v*) It can be located near the load centres because it does not require large quantities of water and need not be near coal mines. Therefore, the cost of primary distribution is reduced.

(*vi*) There are large deposits of nuclear fuels available all over the world. Therefore, such plants can ensure continued supply of electrical energy for thousands of years.

(*vii*) It ensures reliability of operation.

Disadvantages

(*i*) The fuel used is expensive and is difficult to recover.

(*ii*) The capital cost on a nuclear plant is very high as compared to other types of plants.

(*iii*) The erection and commissioning of the plant requires greater technical know-how.

(*iv*) The fission by-products are generally radioactive and may cause a dangerous amount of radioactive pollution.

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* **Fission.** The breaking up of nuclei of heavy atoms into two nearly equal parts with release of huge amount of energy is known as nuclear fission. The release of huge amount of energy during fission is due to *mass defect i.e.* the mass of the final product comes out to be less than the initial product. This mass defect is converted into heat energy according to Einstein's relation, $E = mc^2$.

(v) Maintenance charges are high due to lack of standardisation. Moreover, high salaries of specially trained personnel employed to handle the plant further raise the cost.

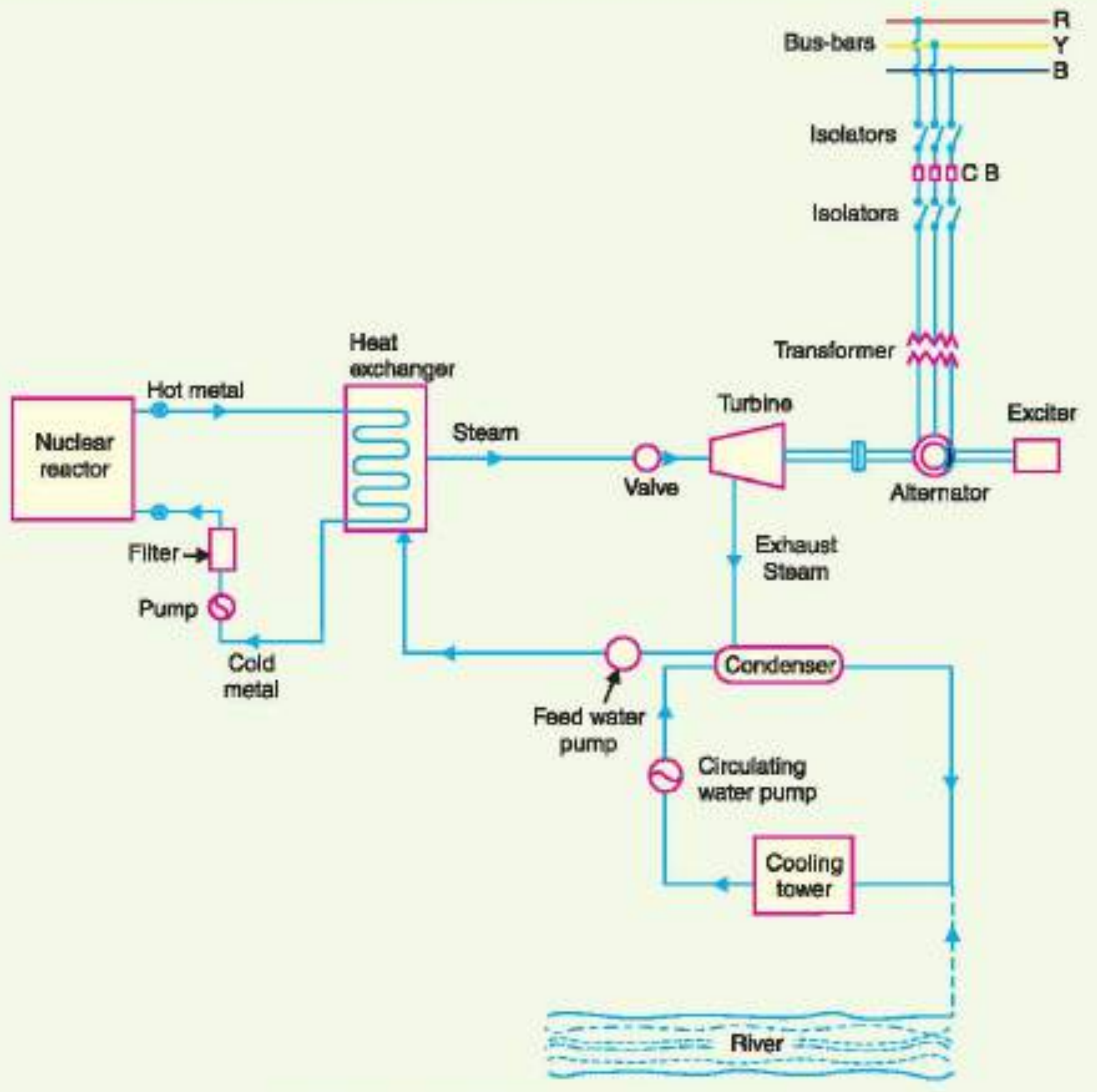
(vi) Nuclear power plants are not well suited for varying loads as the reactor does not respond to the load fluctuations efficiently.

(vii) The disposal of the by-products, which are radioactive, is a big problem. They have either to be disposed off in a deep trench or in a sea away from sea-shore.

2.14 Schematic Arrangement of Nuclear Power Station

The schematic arrangement of a nuclear power station is shown in Fig. 2.7. The whole arrangement can be divided into the following main stages :

(i) Nuclear reactor (ii) Heat exchanger (iii) Steam turbine (iv) Alternator.

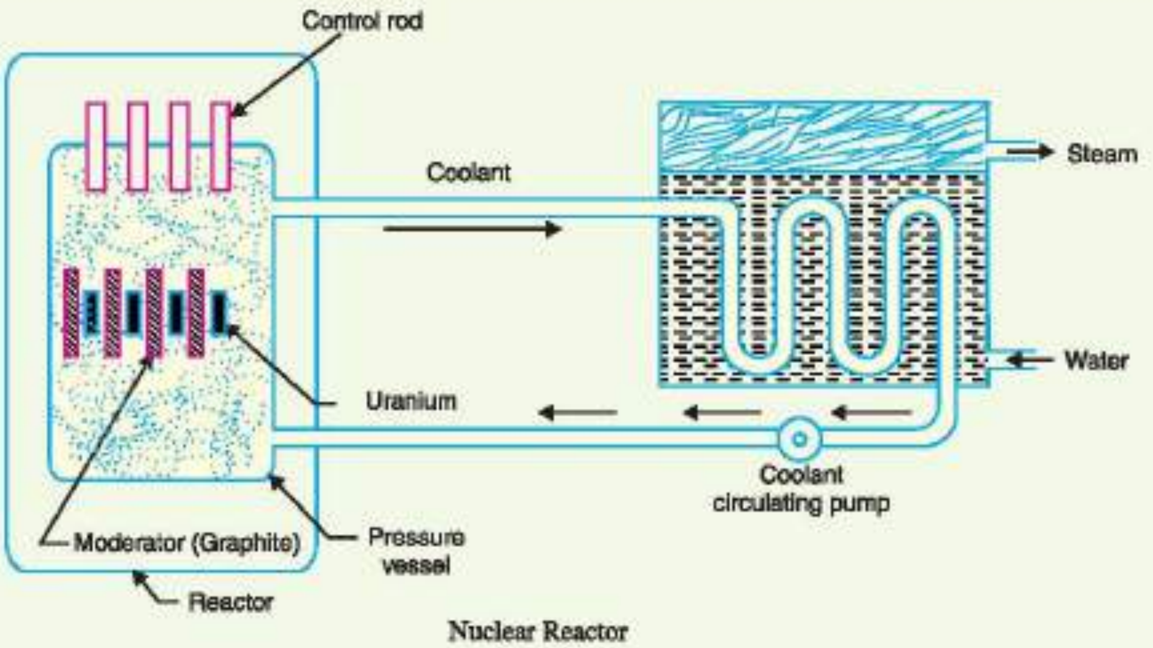


Schematic arrangement of Nuclear Power Station

Fig. 2.7

(*i*) **Nuclear reactor.** It is an apparatus in which nuclear fuel (U^{235}) is subjected to nuclear fission. It controls the *chain reaction** that starts once the fission is done. If the chain reaction is not controlled, the result will be an explosion due to the fast increase in the energy released.

A nuclear reactor is a cylindrical stout pressure vessel and houses fuel rods of Uranium, moderator and control rods (See Fig. 2.8). The fuel rods constitute the fission material and release huge amount of energy when bombarded with slow moving neutrons. The moderator consists of graphite rods which enclose the fuel rods. The moderator slows down the neutrons before they bombard the fuel rods. The control rods are of cadmium and are inserted into the reactor. Cadmium is strong neutron absorber and thus regulates the supply of neutrons for fission. When the control rods are pushed in deep enough, they absorb most of fission neutrons and hence few are available for chain reaction which, therefore, stops. However, as they are being withdrawn, more and more of these fission neutrons cause fission and hence the *intensity* of chain reaction (or heat produced) is increased. Therefore, by pulling out the control rods, power of the nuclear reactor is increased, whereas by pushing them in, it is reduced. In actual practice, the lowering or raising of control rods is accomplished automatically according to the requirement of load. The heat produced in the reactor is removed by the coolant, generally a sodium metal. The coolant carries the heat to the heat exchanger.



Nuclear Reactor
Fig. 2.8

(*ii*) **Heat exchanger.** The coolant gives up heat to the heat exchanger which is utilised in raising the steam. After giving up heat, the coolant is again fed to the reactor.

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* **Chain reaction.** Nuclear fission is done by bombarding Uranium nuclei with slow moving neutrons. This splits the Uranium nuclei with the release of huge amount of energy and emission of neutrons (called fission neutrons). These fission neutrons cause further fission. If this process continues, then in a very short time huge amount of energy will be released which may cause explosion. This is known as *explosive chain reaction*. But in a reactor, controlled chain reaction is allowed. This is done by systematically removing the fission neutrons from the reactor. The greater the number of fission neutrons removed, the lesser is the intensity (*i.e.*, fission rate) of energy released.

(*iii*) **Steam turbine.** The steam produced in the heat exchanger is led to the steam turbine through a valve. After doing a useful work in the turbine, the steam is exhausted to condenser. The condenser condenses the steam which is fed to the heat exchanger through feed water pump.

(*iv*) **Alternator.** The steam turbine drives the alternator which converts mechanical energy into electrical energy. The output from the alternator is delivered to the bus-bars through transformer, circuit breakers and isolators.

2.15 Selection of Site for Nuclear Power Station

The following points should be kept in view while selecting the site for a nuclear power station :

(*i*) *Availability of water.* As sufficient water is required for cooling purposes, therefore, the plant site should be located where ample quantity of water is available, *e.g.*, across a river or by sea-side.

(*ii*) *Disposal of waste.* The waste produced by fission in a nuclear power station is generally radioactive which must be disposed off properly to avoid health hazards. The waste should either be buried in a deep trench or disposed off in sea quite away from the sea shore. Therefore, the site selected for such a plant should have adequate arrangement for the disposal of radioactive waste.

(*iii*) *Distance from populated areas.* The site selected for a nuclear power station should be quite away from the populated areas as there is a danger of presence of radioactivity in the atmosphere near the plant. However, as a precautionary measure, *a dome* is used in the plant which does not allow the radioactivity to spread by wind or underground waterways.

(*iv*) *Transportation facilities.* The site selected for a nuclear power station should have adequate facilities in order to transport the heavy equipment during erection and to facilitate the movement of the workers employed in the plant.

From the above mentioned factors it becomes apparent that ideal choice for a nuclear power station would be near sea or river and away from thickly populated areas.

Nuclear Power Station

Example 2.17. *An atomic power reactor can deliver 300 MW. If due to fission of each atom of* ${}_{92}U^{235}$, *the energy released is 200 MeV, calculate the mass of uranium fissioned per hour.*

Solution.

Energy received from the reactor

$$= 300 \text{ MW} = 3 \times 10^8 \text{ W (or Js}^{-1})$$

$$\text{Energy received/hour} = (3 \times 10^8) \times 3600 = 108 \times 10^{10} \text{ J}$$

$$\text{Energy released/fission} = 200 \text{ MeV} = 200 \times 10^6 \times 1{\cdot}6 \times 10^{-19} \text{ J} = 3{\cdot}2 \times 10^{-11} \text{J}$$

Number of atoms fissioned per hour

$$= \frac{108 \times 10^{10}}{3{\cdot}2 \times 10^{-11}} = 33{\cdot}75 \times 10^{21}$$

Now 1 gram-atom (*i.e.*, 235g) has $6{\cdot}023 \times 10^{23}$ atoms.

∴ Mass of Uranium fissioned per hour

$$= \frac{235}{6{\cdot}023 \times 10^{23}} \times 33{\cdot}75 \times 10^{21} = \mathbf{13{\cdot}17g}$$

Example 2.18. *What is the power ouput of a $_{92}U^{235}$ reactor if it takes 30 days to use up 2 kg of fuel? Given that energy released per fission is 200 MeV and Avogadro's number = $6{\cdot}023 \times 10^{26}$ per kilomole.*

Solution.

$$\text{Number of atoms in 2 kg fuel} = \frac{2}{235} \times 6{\cdot}023 \times 10^{26} = 5{\cdot}12 \times 10^{24}$$

These atoms fission in 30 days. Therefore, the fission rate (*i.e.*, number of fissions per second)

$$= \frac{5{\cdot}12 \times 10^{24}}{30 \times 24 \times 60 \times 60} = 1{\cdot}975 \times 10^{18}$$

$$\text{Energy released per fission} = 200 \text{ MeV} = (200 \times 10^6) \times 1{\cdot}6 \times 10^{-19} = 3{\cdot}2 \times 10^{-11} \text{J}$$

∴ Energy released per second *i.e.*, power output P is

$$P = (3{\cdot}2 \times 10^{-11}) \times (1{\cdot}975 \times 10^{18}) \text{ W}$$

$$= 63{\cdot}2 \times 10^6 \text{ W} = \mathbf{63{\cdot}2 \text{ MW}}$$

2.16 Gas Turbine Power Plant

A generating station which employs gas turbine as the prime mover for the generation of electrical energy is known as a **gas turbine power plant**.

In a gas turbine power plant, air is used as the working fluid. The air is compressed by the compressor and is led to the combustion chamber where heat is added to air, thus raising its temperature. Heat is added to the compressed air either by burning fuel in the chamber or by the use of air heaters. The hot and high pressure air from the combustion chamber is then passed to the gas turbine where it expands and does the mechanical work. The gas turbine drives the alternator which converts mechanical energy into electrical energy.

It may be mentioned here that compressor, gas turbine and the alternator are mounted on the same shaft so that a part of mechanical power of the turbine can be utilised for the operation of the compressor. Gas turbine power plants are being used as standby plants for hydro-electric stations, as a starting plant for driving auxiliaries in power plants etc.

Advantages

(*i*) It is simple in design as compared to steam power station since no boilers and their auxiliaries are required.

(*ii*) It is much smaller in size as compared to steam power station of the same capacity. This is expected since gas turbine power plant does not require boiler, feed water arrangement etc.

(*iii*) The initial and operating costs are much lower than that of equivalent steam power station.

(*iv*) It requires comparatively less water as no condenser is used.

(*v*) The maintenance charges are quite small.

(*vi*) Gas turbines are much simpler in construction and operation than steam turbines.

(*vii*) It can be started quickly form cold conditions.

(*viii*) There are no standby losses. However, in a steam power station, these losses occur because boiler is kept in operation even when the steam turbine is supplying no load.

Disadvantages

(*i*) There is a problem for starting the unit. It is because before starting the turbine, the compressor has to be operated for which power is required from some external source. However, once the unit starts, the external power is not needed as the turbine itself supplies the necessary power to the compressor.

(*ii*) Since a greater part of power developed by the turbine is used in driving the compressor, the net output is low.

(*iii*) The overall efficiency of such plants is low (about 20%) because the exhaust gases from the turbine contain sufficient heat.

(*iv*) The temperature of combustion chamber is quite high (3000°F) so that its life is comparatively reduced.

2.17 Schematic Arrangement of Gas Turbine Power Plant

The schematic arrangement of a gas turbine power plant is shown in Fig. 2.9. The main components of the plant are :

(*i*) Compressor (*ii*) Regenerator

(*iii*) Combustion chamber (*iv*) Gas turbine

(*v*) Alternator (*vi*) Starting motor

(*i*) **Compressor.** The compressor used in the plant is generally of rotatory type. The air at atmospheric pressure is drawn by the compressor *via* the filter which removes the dust from air. The rotatory blades of the compressor push the air between stationary blades to raise its pressure. Thus air at high pressure is available at the output of the compressor.

(*ii*) **Regenerator.** A regenerator is a device which recovers heat from the exhaust gases of the turbine. The exhaust is passed through the regenerator before wasting to atmosphere. A regenerator consists of a nest of tubes contained in a shell. The compressed air from the compressor passes through the tubes on its way to the combustion chamber. In this way, compressed air is heated by the hot exhaust gases.

(*iii*) **Combustion chamber.** The air at high pressure from the compressor is led to the combustion chamber *via* the regenerator. In the combustion chamber, heat* is added to the air by burning oil. The oil is injected through the burner into the chamber at high pressure to ensure atomisation of oil and its thorough mixing with air. The result is that the chamber attains a very high temperature (about 3000°F). The combustion gases are suitably cooled to 1300°F to 1500°F and then delivered to the gas turbine.

(*iv*) **Gas turbine.** The products of combustion consisting of a mixture of gases at high temperature and pressure are passed to the gas turbine. These gases in passing over the turbine blades expand and thus do the mechanical work. The temperature of the exhaust gases from the turbine is about 900°F.

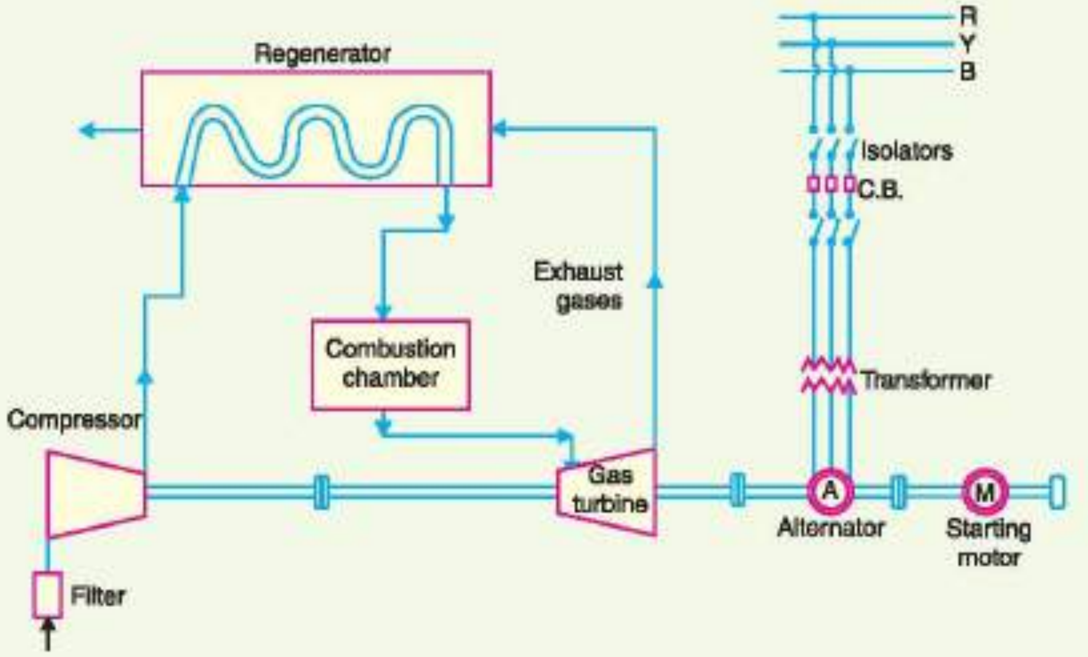
* Only hot pressurised air makes it possible to convert heat into mechanical work. Heating air at atmospheric pressure generally does not make it permissible to convert heat into mehanical work.

2.18 Comparison of the Various Power Plants

The comparison of steam power plant, hydro-electric plant, diesel power plant and nuclear power plant is given below in the tabular form :

S.No.	Item	Steam Power Station	Hydro-electric Power Plant	Diesel Power Plant	Nuclear power Plant
1.	*Site*	Such plants are located at a place where ample supply of water and coal is available, transportation facilities are adequate	Such plants are located where large reservoirs can be obtained by constructing a dam e.g in hilly areas.	Such plants can be located at any place because they require less space and small quantity of water.	These plants are located away from thickly populated areas to avoid radioactive pollution.
2.	*Initial cost*	Initial cost is lower than those of hydroelectric and nuclear power plants.	Initial cost is very high because of dam construction and excavation work.	Initial cost is less as compared to other plants.	Initial cost is highest because of huge investement on building a nuclear reactor.
3.	*Running cost*	Higher than hydroelectric and nuclear plant because of the requirement of huge amount of coal.	Practically nil because no fuel is required.	Highest among all plants because of high price of diesel.	Except the hydroelectric plant, it has the minimum running cost because small amount of fuel can produce relatively large amount of power.
4.	*Limit of source of power*	Coal is the source of power which has limited reserves all over the world.	Water is the source of power which is not dependable becuase of wide variations in the rainfall every year.	Diesel is the source of power which is not available in huge quantities due to limited reserves.	The source of power is the nuclear fuel which is available in sufficient quantity. It is because small amount of fuel can produce huge power.
5.	*Cost of fuel transportation*	Maximum because huge amount of coal is transported to the plant site.	Practically nil.	Higher than hydro and nuclear power plants	Minimum because small quantity of fuel is required.
6.	*Cleanliness and simplicity*	Least clean as atmosphere is polluted due to smoke.	Most simple and clean.	More clean than steam power and nuclear power plants.	Less cleaner than hydroelectric and diesel power plants.

S.No.	Item	Steam Power Station	Hydro-electric Power Plant	Diesel Power Plant	Nuclear power Plant
7.	*Overall efficiency*	Least efficient. Overall efficiency is about 25%.	Most efficient. Overall efficiency is about 85%.	More efficient than steam power station. Efficiency is about 35%.	More efficient than steam power station.
8.	*Starting*	Requires a lot of time for starting.	Can be started instantly.	Can be started quickly.	Can be started easily.
9.	*Space required*	These plants need sufficient space because of boilers and other auxiliaries.	Require very large area because of the reservoir.	Require less space.	These require minimum space as compared to any other plant of equivalent capacity.
10.	*Maintenance cost*	Quite high as skilled operating staff is required.	Quite low.	Less	Very high as highly trained personnel are required to handle the plant.
11.	*Transmission and distribution cost*	Quite low as these are generally located near the load centres.	Quite high as these are located quite away from the load centres.	Least as they are generally located at the centre of gravity of the load.	Quite low as these are located near load centres.
12.	*Standby losses*	Maximum as the boiler remains in operation even when the turbine is not working.	No standby losses.	Less standby losses.	Less.



Schematic arrangement of gas turbine power plant.

Fig. 2.9

(*v*) **Alternator.** The gas turbine is coupled to the alternator. The alternator converts mechanical energy of the turbine into electrical energy. The output from the alternator is given to the bus-bars through transformer, circuit breakers and isolators.

(*vi*) **Starting motor.** Before starting the turbine, compressor has to be started. For this purpose, an electric motor is mounted on the same shaft as that of the turbine. The motor is energised by the batteries. Once the unit starts, a part of mechanical power of the turbine drives the compressor and there is no need of motor now.

SELF-TEST

1. Fill in the blanks by inserting appropriate words/figures :

(*i*) The major heat loss in a steam power station occurs in ....

(*ii*) The thermal efficiency of a steam power station is about ....

(*iii*) Cooling towers are used where ....

(*iv*) The running cost of medium power stations is about .... paise per unit.

(*v*) In a hydro-electric plant, spillways are used ....

(*vi*) The running cost of a hydro-electric plant is about .... paise per unit.

(*vii*) For high head hydro-electric plants, the turbine used is ....

(*viii*) Francis and Kaplan turbines are used for .... heads.

(*ix*) Surge tank is provided for the protection of ....

(*x*) Of all the plants, minimum quantity of fuel is required in .... plant.

2. Pick up the correct words from the brackets and fill in the blanks :

(*i*) The cost of fuel transportation is minimum in .... plant.
(*steam power, hydro-electric, nuclear power*)

(*ii*) The cheapest plant in operation and maintenance is .... plant.
(*diesel power, hydro-electric, steam power*)

(*iii*) Economisers are used to heat .... (*air, feed water, steam*) (*iv*) The running cost of a nuclear power plant is about .... paise per unit. (20, 48, 64)

(*v*) Diesel power plants are used as .... plants. (*base load, standby*)

(*vi*) India's first nuclear power plant was built at .... (*Tarapur, Rana Partap Sagar, Kalpakkam*)

(*vii*) The most simple and clean plant is .... plant (*steam power, hydro-electric, nuclear power*)

(*viii*) The first nuclear power plant in the world was commissioned in .... (*U.S.A., U.S.S.R., England*)

(*ix*) Gas turbine power plant is .... efficient than steam power plant. (*more, less*)

(*x*) Draft tube is used in .... turbines. (*impulse, reaction*)

ANSWERS TO SELF-TEST

1. (*i*) Condenser, about 53% (*ii*) 28% (*iii*) water is not available in sufficient quantity (*iv*) 15 (*v*) to discharge surplus water on the downstream side of dam (*vi*) 5 (*vii*) pelton wheel (*viii*) medium and low (*ix*) penstock (*x*) nuclear power.
2. (*i*) Hydro-electric (*ii*) hydro-electric (*iii*) feed water (*iv*) 20 (*v*) standby (*vi*) Tarapur (*vii*) hydro-electric (*viii*) U.S.S.R. in 1954 (*ix*) more (*x*) reaction.

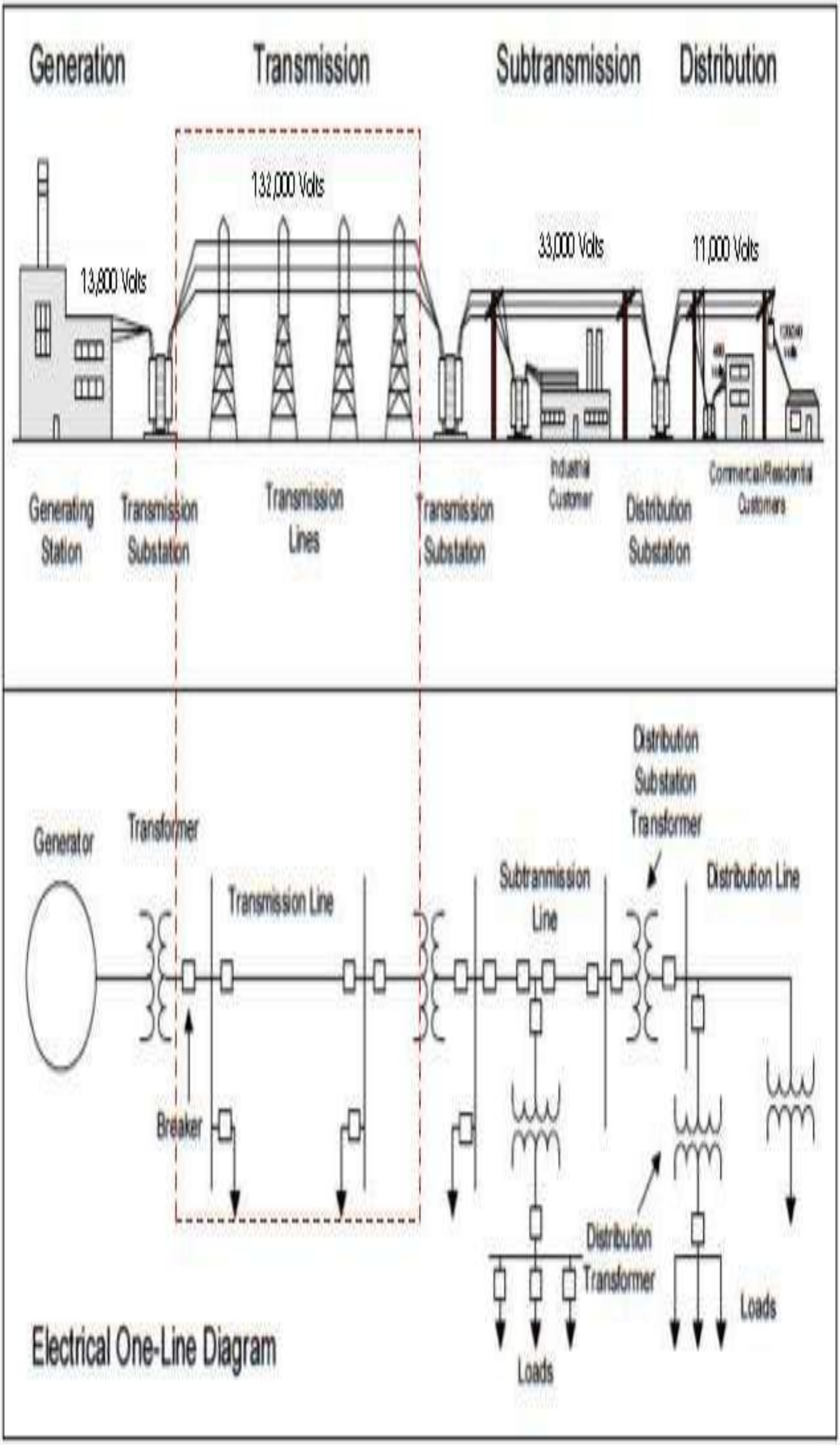
CHAPTER REVIEW TOPICS

1. What is a power generating station ?
2. What is a steam power station ? Discuss its advantages and disadvantages.
3. Draw the schematic diagram of a modern steam power station and explain its operation.
4. Explain the important components of a steam power station.
5. What factors are taken into account while selecting the site for a steam power station ?
6. Discuss the merits and demerits of a hydro-electric plant.
7. Draw a neat schematic diagram of a hydro-electric plant and explain the functions of various components.
8. Explain the essential factors which influence the choice of site for a hydro-electric plant.
9. Explain the functions of the following :
   (*i*) dam (*ii*) spillways (*iii*) surge tank (*iv*) headworks (*v*) draft tube.
10. Draw the flow diagram of a diesel power station and discuss its operation.
11. Discuss the advantages and disadvantages of a diesel power station.
12. Draw the schematic diagram of a nuclear power station and discuss its operation.
13. Explain with a neat sketch the various parts of a nuclear reactor.
14. Discuss the factors for the choice of site for a nuclear power plant.
15. Explain the working of a gas turbine power plant with a schematic diagram.
16. Give the comparison of steam power plant, hydro-electric power plant, diesel power plant and nuclear power plant on the basis of operating cost, initial cost, efficiency, maintenance cost and availability of source of power.

DISCUSSION QUESTIONS

1. Why is the overall efficiency of a steam power station very low ?
2. Why is a condenser used in a steam power station ?
3. Why hydro-electric stations have high transmission and distribution costs ?
4. Why are nuclear power stations becoming very popular ?
5. Why hot gas at high pressure and not hot gas at atmospheric pressure is used in gas turbine power plants?
6. How do the various devices protect the penstock ?
7. Why cannot diesel power stations be employed to generate bulk power ?
8. Why is regenerator used in gas turbine power plant ?

GO


Generation
Transmission
Subtransmission
Distribution
132,000 Volts
13,800 Volts
33,000 Volts
11,000 Volts
Generating Station
Transmission Substation
Transmission Lines
Transmission Substation
Industrial Customer
Distribution Substation
Commercial/Residential Customers
Generator
Transformer
Transmission Line
Breaker
Subtranmission Line
Distribution Substation Transformer
Distribution Line
Distribution Transformer
Loads
Loads
Electrical One-Line Diagram

CONTENTS

CHAPTER

8

Mechanical Design of Overhead Lines

Introduction

Electric power can be transmitted or distributed either by means of underground cables or by overhead lines. The underground* cables are rarely used for power transmission due to two main reasons. Firstly, power is generally transmitted over long distances to load centres. Obviously, the installation costs for underground transmission will be very heavy. Secondly, electric power has to be transmitted at high voltages for economic reasons. It is very difficult to provide proper insulation† to the cables to withstand such higher pressures. Therefore, as a rule, power trasmission over long distances is carried out by using overhead lines. With the growth in power demand and consequent rise in voltage levels, power transmission by overhead lines has assumed considerable importance.

* The undergound system is much more expensive than overhead system. Therefore, it has limited use for distribution in congested areas where safety and good appearances are the main considerations.

† In overhead lines, bare conductors are used and air acts as the insulation. The necessary insulation between the conductors can be provided by adjusting the spacing between them.

CONTENTS

An overhead line is subjected to uncertain weather conditions and other external interferences. This calls for the use of proper mechanical factors of saftey in order to ensure the continuity of operation in the line. In general, the strength of the line should be such so as to provide against the worst *probable* weather conditions. In this chapter, we shall focus our attention on the various aspects of mechanical design of overhead lines.

8.1 Main Components of Overhead Lines

An overhead line may be used to transmit or distribute electric power. The succcessful operation of an overhead line depends to a great extent upon the mechanical design of the line. While constructing an overhead line, it should be ensured that mechanical strength of the line is such so as to provide against the most *probable* weather conditions. In general, the main components of an overhead line are:

(*i*) *Conductors* which carry electric power from the sending end station to the receiving end station.

(*ii*) *Supports* which may be poles or towers and keep the conductors at a suitable level above the ground.

(*iii*) *Insulators* which are attached to supports and insulate the conductors from the ground.

(*iv*) *Cross arms* which provide support to the insulators.

(*v*) *Miscellaneous items* such as phase plates, danger plates, lightning arrestors, anti-climbing wires etc.

The continuity of operation in the overhead line depends upon the judicious choice of above components. Therefore, it is profitable to have detailed discussion on them.

8.2 Conductor MaterialS

The conductor is one of the important items as most of the capital outlay is invested for it. Therefore, proper choice of material and size of the conductor is of considerable importance. The conductor material used for transmission and distribution of electric power should have the following properties :

(*i*) high electrical conductivity.

(*ii*) high tensile strength in order to withstand mechanical stresses.

(*iii*) low cost so that it can be used for long distances.

(*iv*) low specific gravity so that weight per unit volume is small.

All above requirements are not found in a single material. Therefore, while selecting a conductor material for a particular case, a compromise is made between the cost and the required electrical and mechanical properties.

Commonly used conductor materials. The most commonly used conductor materials for overhead lines are *copper, aluminium, steel-cored aluminium, galvanised steel* and *cadmium copper*. The choice of a particular material will depend upon the cost, the required electrical and mechanical properties and the local conditions.

All conductors used for overhead lines are preferably stranded* in order to increase the flexibility. In stranded conductors, there is generally one central wire and round this, successive layers of wires containing 6, 12, 18, 24 ...... wires. Thus, if there are n layers, the total number of individual wires is $3n(n+1)+1$. In the manufacture of stranded conductors, the consecutive layers of wires are twisted or spiralled in opposite directions so that layers are bound together.

1. Copper. Copper is an ideal material for overhead lines owing to its high electrical conductivity and greater tensile strength. It is always used in the hard drawn form as stranded conductor.

* Solid wires are only used when area of X-section is small. If solid wires are used for larger X-section and longer spans, continuous vibrations and swinging would produce mechanical fatigue and they would fracture at the points of support.

Although hard drawing decreases the electrical conductivity slightly yet it increases the tensile strength considerably.

Copper has high current density *i.e.*, the current carrying capacity of copper per unit of X-sectional area is quite large. This leads to two advantages. Firstly, smaller X-sectional area of conductor is required and secondly, the area offered by the conductor to wind loads is reduced. Moreover, this metal is quite homogeneous, durable and has high scrap value.

There is hardly any doubt that copper is an ideal material for transmission and distribution of electric power. However, due to its higher cost and non-availability, it is rarely used for these purposes. Now-a-days the trend is to use aluminium in place of copper.

2. Aluminium. Aluminium is cheap and light as compared to copper but it has much smaller conductivity and tensile strength. The relative comparison of the two materials is briefed below :

(*i*) The conductivity of aluminium is 60% that of copper. The smaller conductivity of aluminium means that for any particular transmission efficiency, the X-sectional area of conductor must be larger in aluminium than in copper. For the same resistance, the diameter of aluminium conductor is about 1·26 times the diameter of copper conductor.

The increased X-section of aluminium exposes a greater surface to wind pressure and, therefore, supporting towers must be designed for greater transverse strength. This often requires the use of higher towers with consequence of greater sag.

(*ii*) The specific gravity of aluminium (2·71 gm/cc) is lower than that of copper (8·9 gm/cc). Therefore, an aluminium conductor has almost one-half the weight of equivalent copper conductor. For this reason, the supporting structures for aluminium need not be made so strong as that of copper conductor.

(*iii*) Aluminium conductor being light, is liable to greater swings and hence larger cross-arms are required.

(*iv*) Due to lower tensile strength and higher co-efficient of linear expansion of aluminium, the sag is greater in aluminium conductors.

Considering the combined properties of cost, conductivity, tensile strength, weight etc., aluminium has an edge over copper. Therefore, it is being widely used as a conductor material. It is particularly profitable to use aluminium for heavy-current transmission where the conductor size is large and its cost forms a major proportion of the total cost of complete installation.

3. Steel cored aluminium. Due to low tensile strength, aluminium conductors produce greater sag. This prohibits their use for larger spans and makes them unsuitable for long distance transmission. In order to increase the tensile strength, the aluminium conductor is reinforced with a core of galvanised steel wires. The *composite conductor thus obtained is known as *steel cored aluminium* and is abbreviated as A.C.S.R. (aluminium conductor steel reinforced).

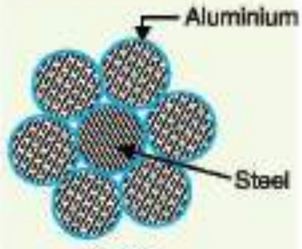


Fig. 8.1

Steel-cored aluminium conductor consists of central core of †galvanised steel wires surrounded by a number of aluminium strands. Usually, diameter of both steel and aluminium wires is the same. The X-section of the two metals are generally in the ratio of 1 : 6 but can be modified to 1 : 4 in order to get more tensile strength for the conductor. Fig. 8.1 shows steel cored aluminium conductor having one steel wire surrounded by six wires of aluminium. The result of this composite conductor is that steel core takes greater percentage of

* The reader may think that reinforcement with steel increases the weight but actually the weight of composite conductor is 25% less as compared with equivalent copper conductor.

† The galvanised steel is used in order to prevent rusting and electrolytic corrosion.

mechanical strength while aluminium strands carry the bulk of current. The steel cored aluminium conductors have the following advantages :

(*i*) The reinforcement with steel increases the tensile strength but at the same time keeps the composite conductor light. Therefore, steel cored aluminium conductors will produce smaller sag and hence longer spans can be used.

(*ii*) Due to smaller sag with steel cored aluminium conductors, towers of smaller heights can be used.

4. **Galvanised steel.** Steel has very high tensile strength. Therefore, galvanised steel conductors can be used for extremely long spans or for short line sections exposed to abnormally high stresses due to climatic conditions. They have been found very suitable in rural areas where cheapness is the main consideration. Due to poor conductivity and high resistance of steel, such conductors are not suitable for transmitting large power over a long distance. However, they can be used to advantage for transmitting a small power over a small distance where the size of the copper conductor desirable from economic considerations would be too small and thus unsuitable for use because of poor mechanical strength.

5. **Cadmium copper.** The conductor material now being employed in certain cases is copper alloyed with cadmium. An addition of 1% or 2% cadmium to copper increases the tensile strength by about 50% and the conductivity is only reduced by 15% below that of pure copper. Therefore, cadmium copper conductor can be useful for exceptionally long spans. However, due to high cost of cadmium, such conductors will be economical only for lines of small X-section *i.e.*, where the cost of conductor material is comparatively small compared with the cost of supports.

8.3 Line Supports

The supporting structures for overhead line conductors are various types of poles and towers called *line supports*. In general, the line supports should have the following properties :

(*i*) High mechanical strength to withstand the weight of conductors and wind loads etc.

(*ii*) Light in weight without the loss of mechanical strength.

(*iii*) Cheap in cost and economical to maintain.

(*iv*) Longer life.

(*v*) Easy accessibility of conductors for maintenance.

The line supports used for transmission and distribution of electric power are of various types including *wooden poles, steel poles, R.C.C. poles* and *lattice steel towers*. The choice of supporting structure for a particular case depends upon the line span, X-sectional area, line voltage, cost and local conditions.

1. **Wooden poles.** These are made of seasoned wood (sal or chir) and are suitable for lines of moderate X-sectional area and of relatively shorter spans, say upto 50 metres. Such supports are cheap, easily available, provide insulating properties and, therefore, are widely used for distirbution purposes in rural areas as an economical proposition. The wooden poles generally tend to rot below the ground level, causing foundation failure. In order to prevent this, the portion of the pole below the ground level is impregnated with preservative compounds like *creosote oil*. Double pole structures of the '*A*' or '*H*' type are often used (See Fig. 8.2) to obtain a higher transverse strength than could be economically provided by means of single poles.

The main objections to wooden supports are : (*i*) tendency to rot below the ground level (*ii*) comparatively smaller life (20-25 years) (*iii*) cannot be used for voltages higher than 20 kV (*iv*) less mechanical strength and (*v*) require periodical inspection.

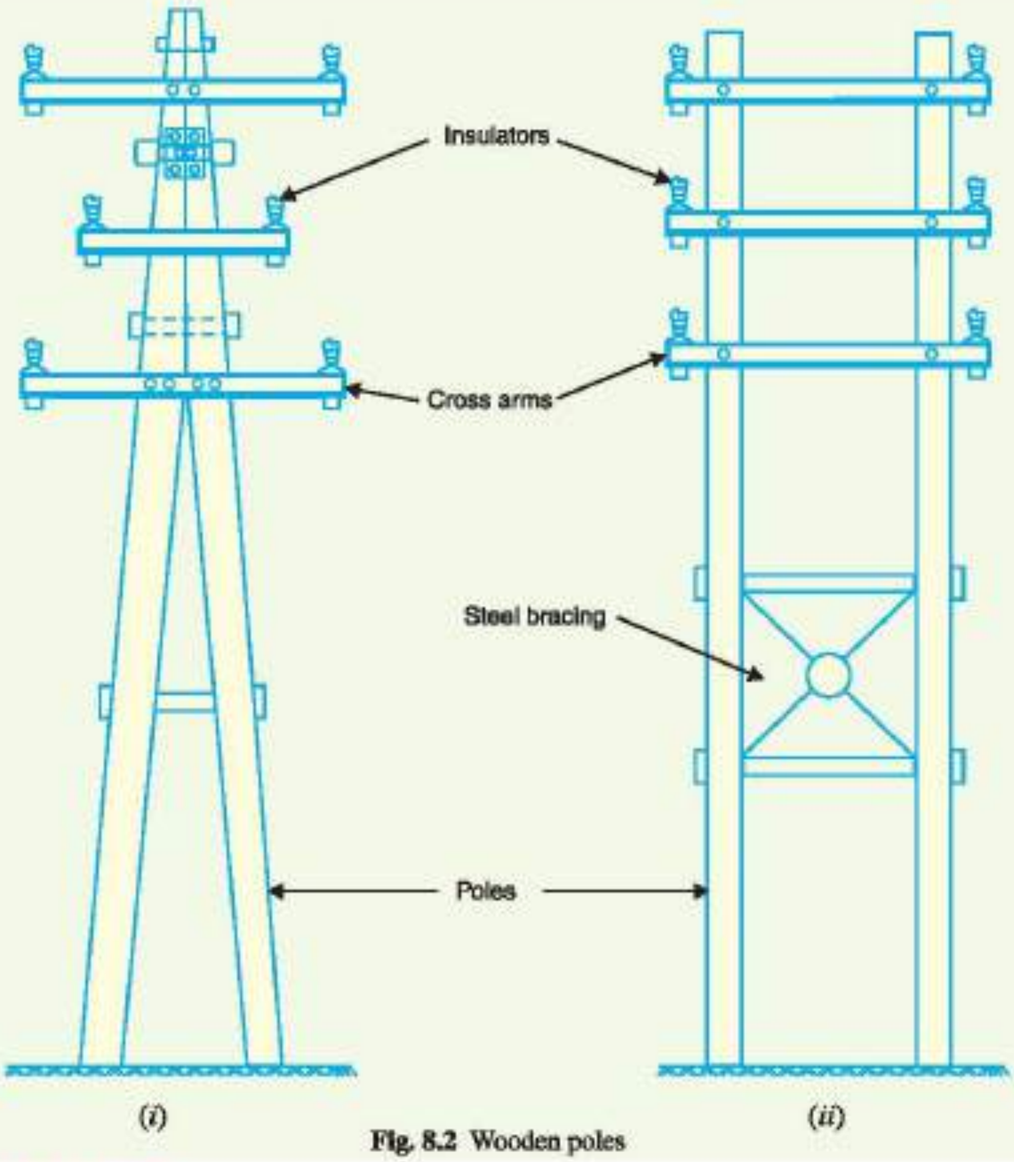


Fig. 8.2 Wooden poles

2. Steel poles. The steel poles are often used as a substitute for wooden poles. They possess greater mechanical strength, longer life and permit longer spans to be used. Such poles are generally used for distribution purposes in the cities. This type of supports need to be galvanised or painted in order to prolong its life. The steel poles are of three types *viz.*, (*i*) rail poles (*ii*) tubular poles and (*iii*) rolled steel joints.

3. RCC poles. The reinforced concrete poles have become very popular as line supports in recent years. They have greater mechanical strength, longer life and permit longer spans than steel poles. Moreover, they give good outlook, require little maintenance and have good insulating properties. Fig. 8.3 shows R.C.C. poles for single and double circuit. The holes in the poles facilitate the climbing of poles and at the same time reduce the weight of line supports.

The main difficulty with the use of these poles is the high cost of transport owing to their heavy weight. Therefore, such poles are often manufactured at the site in order to avoid heavy cost of transportation.

4. Steel towers. In practice, wooden, steel and reinforced concrete poles are used for distribution purposes at low voltages, say upto 11 kV. However, for long distance transmission at higher voltage, steel towers are invariably employed. Steel towers have greater mechanical strength, longer

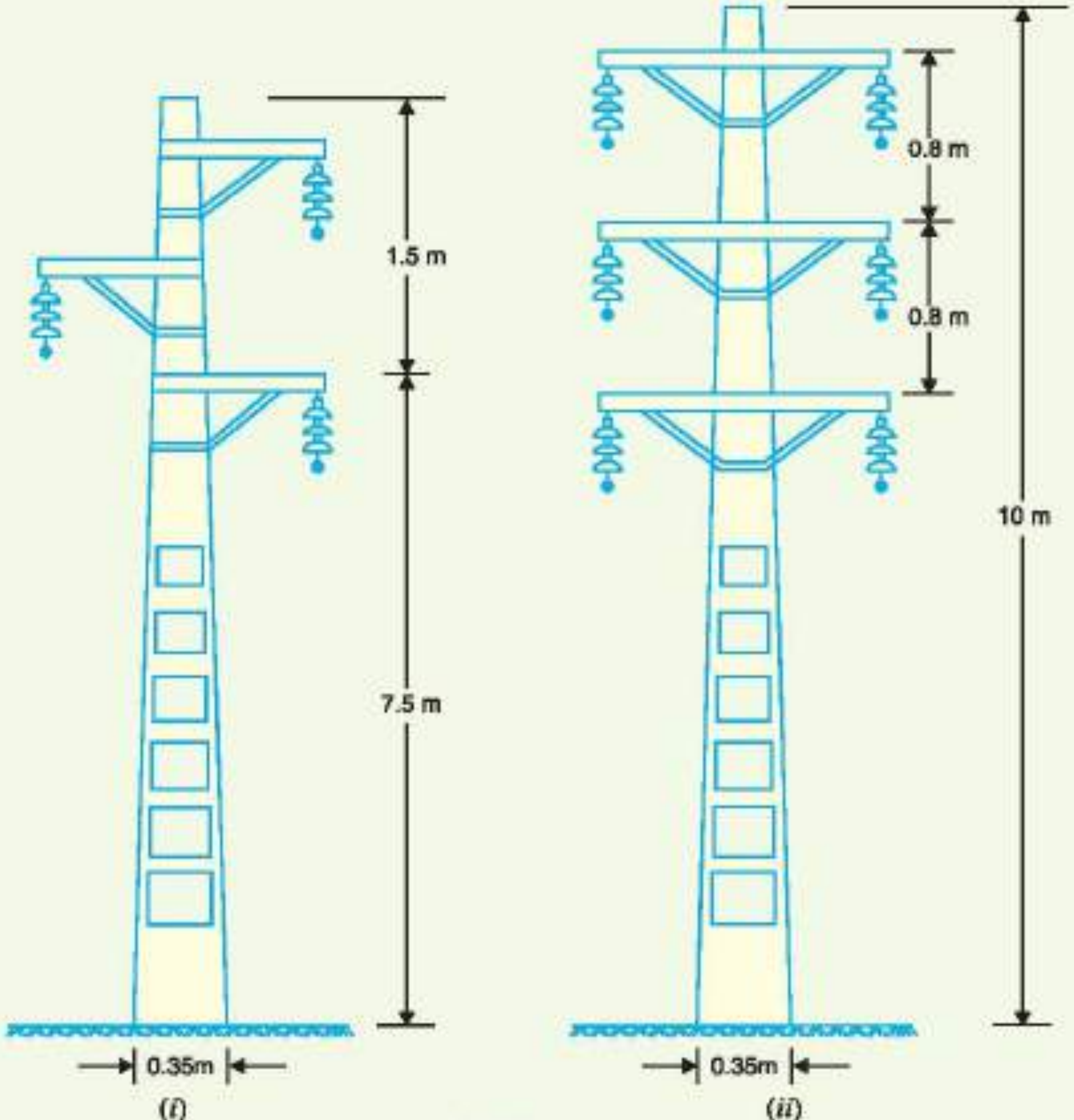


Fig. 8.3

life, can withstand most severe climatic conditions and permit the use of longer spans. The risk of interrupted serivce due to broken or punctured insulation is considerably reduced owing to longer spans. Tower footings are usually grounded by driving rods into the earth. This minimises the lightning troubles as each tower acts as a lightning conductor.

Fig. 8.4 (*i*) shows a single circuit tower. However, at a moderate additional cost, double circuit tower can be provided as shown in Fig. 8.4 (*ii*). The double circuit has the advantage that it ensures continuity of supply. It case there is breakdown of one circuit, the continuity of supply can be maintained by the other circuit.

8.4 Insulators

The overhead line conductors should be supported on the poles or towers in such a way that currents from conductors do not flow to earth through supports *i.e.*, line conductors must be properly insulated from supports. This is achieved by securing line conductors to supports with the help of *insulators*. The insulators provide necessary insulation between line conductors and supports and thus prevent any leakage current from conductors to earth. In general, the insulators should have the following desirable properties :

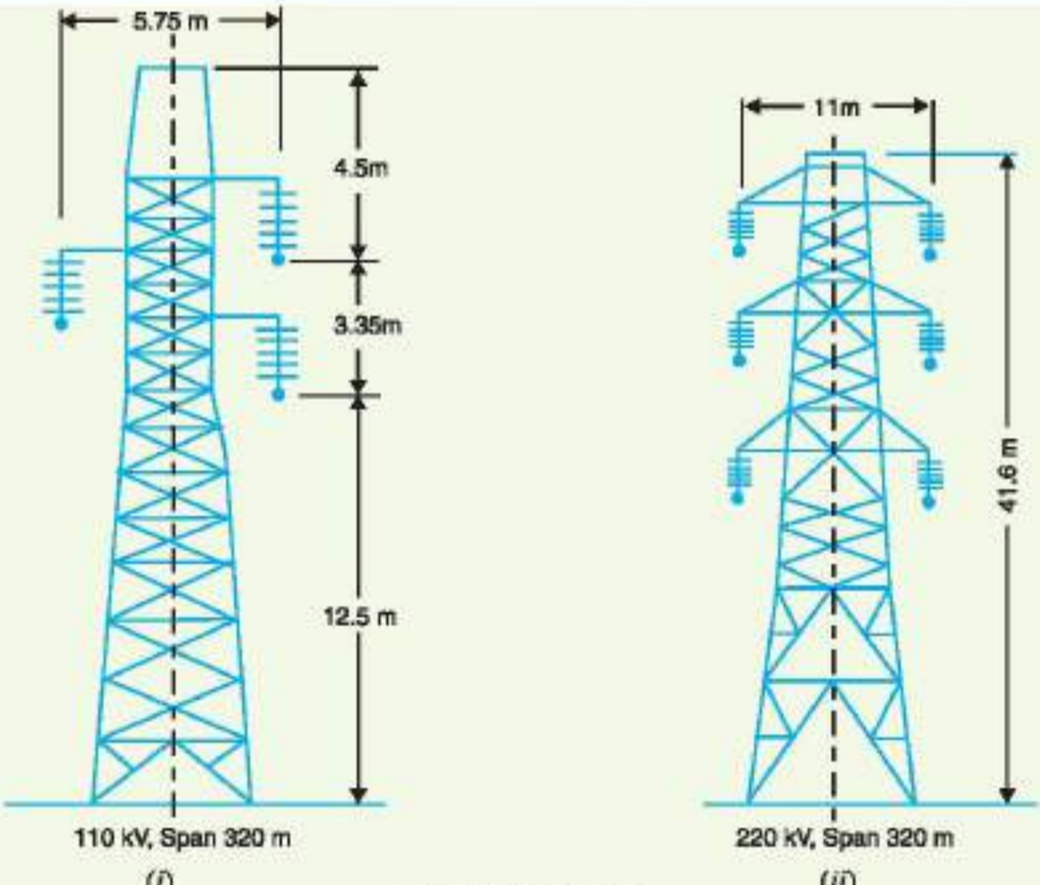


Fig. 8.4 Steel Towers

(i) High mechanical strength in order to withstand conductor load, wind load etc.

(ii) High electrical resistance of insulator material in order to avoid leakage currents to earth.

(iii) High relative permittivity of insulator material in order that dielectric strength is high.

(iv) The insulator material should be non-porous, free from impurities and cracks otherwise the permittivity will be lowered.

(v) High ratio of puncture strength to flashover.

The most commonly used material for insulators of overhead line is *porcelain* but glass, steatite and special composition materials are also used to a limited extent. Porcelain is produced by firing at a high temperature a mixture of kaolin, feldspar and quartz. It is stronger mechanically than glass, gives less trouble from leakage and is less effected by changes of temperature.

8.5 Types of Insulators

The successful operation of an overhead line depends to a considerable extent upon the proper selection of insulators. There are several types of insulators but the most commonly used are pin type, suspension type, strain insulator and shackle insulator.

1. Pin type insulators. The part section of a pin type insulator is shown in Fig. 8.5 (i). As the name suggests, the pin type insulator is secured to the cross-arm on the

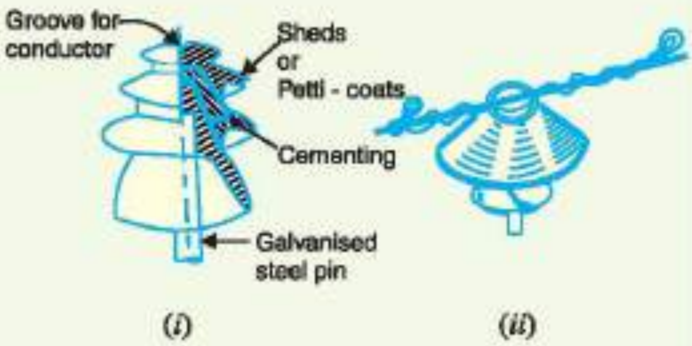


Fig. 8.5. Pin-type insulator

pole. There is a groove on the upper end of the insulator for housing the conductor. The conductor passes through this groove and is bound by the annealed wire of the same material as the conductor [See Fig. 8.5 (*ii*)].

Pin type insulators are used for transmission and distribution of electric power at voltages upto 33 kV. Beyond operating voltage of 33 kV, the pin type insulators become too bulky and hence uneconomical.

Causes of insulator failure. Insulators are required to withstand both mechanical and electrical stresses. The latter type is pirmarily due to line voltage and may cause the breakdown of the insulator. The electrical breakdown of the insulator can occur either by *flash-over* or *puncture*. In flash-over, an arc occurs between the line conductor and insulator pin (*i.e.*, earth) and the discharge jumps across the *air gaps, following shortest distance. Fig. 8.6 shows the arcing distance (*i.e.* $a + b + c$) for the insulator. In case of flash-over, the insulator will continue to act in its proper capacity unless extreme heat produced by the arc destroys the insulator.

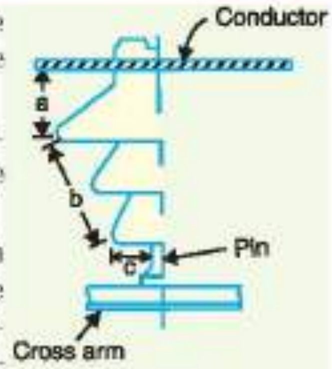


Fig. 8.6

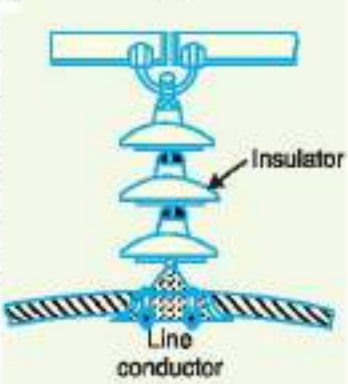


Fig. 8.7

In case of puncture, the discharge occurs from conductor to pin through the body of the insulator. When such breakdown is involved, the insulator is permanently destroyed due to excessive heat. In practice, sufficient thickness of porcelain is provided in the insulator to avoid puncture by the line voltage. The ratio of puncture strength to flash-over voltage is known as safety factor *i.e.*,

$$\text{Safety factor of insulator} = \frac{\text{Puncture strength}}{\text{Flash - over voltage}}$$


Pin type insulator


Suspension insulator

It is desirable that the value of safety factor is high so that flash-over takes place before the insulator gets punctured. For pin type insulators, the value of safety factor is about 10.

2 Suspension type insulators. The cost of pin type insulator increases rapidly as the working voltage is increased. Therefore, this type of insulator is not economical beyond 33 kV. For high voltages (>33 kV), it is a usual practice to use suspension type insulators shown in Fig. 8.7. They

* The insulator is generally dry and its surfaces have proper insulating properties. Therefore, arc can only occur through air gap between conductor and insulator pin.

consist of a number of porcelain discs connected in series by metal links in the form of a string. The conductor is suspended at the bottom end of this string while the other end of the string is secured to the cross-arm of the tower. Each unit or disc is designed for low voltage, say 11 kV. The number of discs in series would obviously depend upon the working voltage. For instance, if the working voltage is 66 kV, then six discs in series will be provided on the string.

Advantages

(*i*) Suspension type insulators are cheaper than pin type insulators for voltages beyond 33 kV.

(*ii*) Each unit or disc of suspension type insulator is designed for low voltage,usually 11 kV. Depending upon the working voltage, the desired number of discs can be connected in series.

(*iii*) If any one disc is damaged, the whole string does not become useless because the damaged disc can be replaced by the sound one.

(*iv*) The suspension arrangement provides greater flexibility to the line. The connection at the cross arm is such that insulator string is free to swing in any direction and can take up the position where mechanical stresses are minimum.

(*v*) In case of increased demand on the transmission line, it is found more satisfactory to supply the greater demand by raising the line voltage than to provide another set of conductors. The additional insulation required for the raised voltage can be easily obtained in the suspension arrangement by adding the desired number of discs.

(*vi*) The suspension type insulators are generally used with steel towers. As the conductors run below the earthed cross-arm of the tower, therefore, this arrangement provides partial protection from lightning.

3. Strain insulators. When there is a dead end of the line or there is corner or sharp curve, the line is subjected to greater tension. In order to relieve the line of excessive tension, strain insulators are used. For low voltage lines (< 11 kV), shackle insulators are used as strain insulators. However, for high voltage transmission lines, strain insulator consists of an assembly of suspension insulators as shown in Fig. 8.8. The discs of strain insulators are used in the vertical plane. When the tension in lines is exceedingly high, as at long river spans, two or more strings are used in parallel.

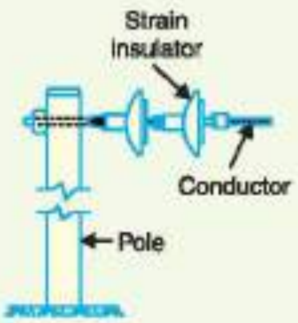


Fig. 8.8. Strain insulator.

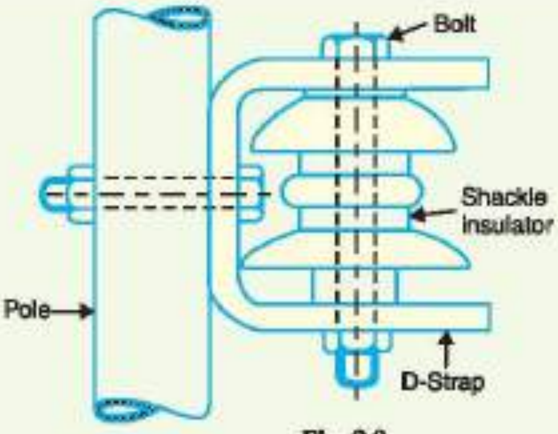


Fig. 8.9

4. Shackle insulators. In early days, the shackle insulators were used as strain insulators. But now a days, they are frequently used for low voltage distribution lines. Such insulators can be used either in a horizontal position or in a vertical position. They can be directly fixed to the pole with a bolt or to the cross arm. Fig. 8.9 shows a shackle insulator fixed to the pole. The conductor in the groove is fixed with a soft binding wire.

8.6 Potential Distribution over Suspension Insulator String

A string of suspension insulators consists of a number of porcelain discs connected in series through metallic links. Fig. 8.10 (*i*) shows 3-disc string of suspension insulators. The porcelain portion of each disc is inbetween two metal links. Therefore, each disc forms a capacitor C as shown in Fig. 8.10 (*ii*). This is known as *mutual capacitance* or *self-capacitance*. If there were mutual capacitance alone, then charging current would have been the same through all the discs and consequently voltage across each unit would have been the same *i.e.*, $V/3$ as shown in Fig. 8.10 (*ii*). However, in actual practice, capacitance also exists between metal fitting of each disc and tower or earth. This is known as *shunt capacitance* C_1. Due to shunt capacitance, charging current is not the same through all the discs of the string [See Fig. 8.10 (*iii*)]. Therefore, voltage across each disc will be different. Obviously, the disc nearest to the line conductor will have the maximum* voltage. Thus referring to Fig. 8.10 (*iii*), V_3 will be much more than V_2 or V_1.

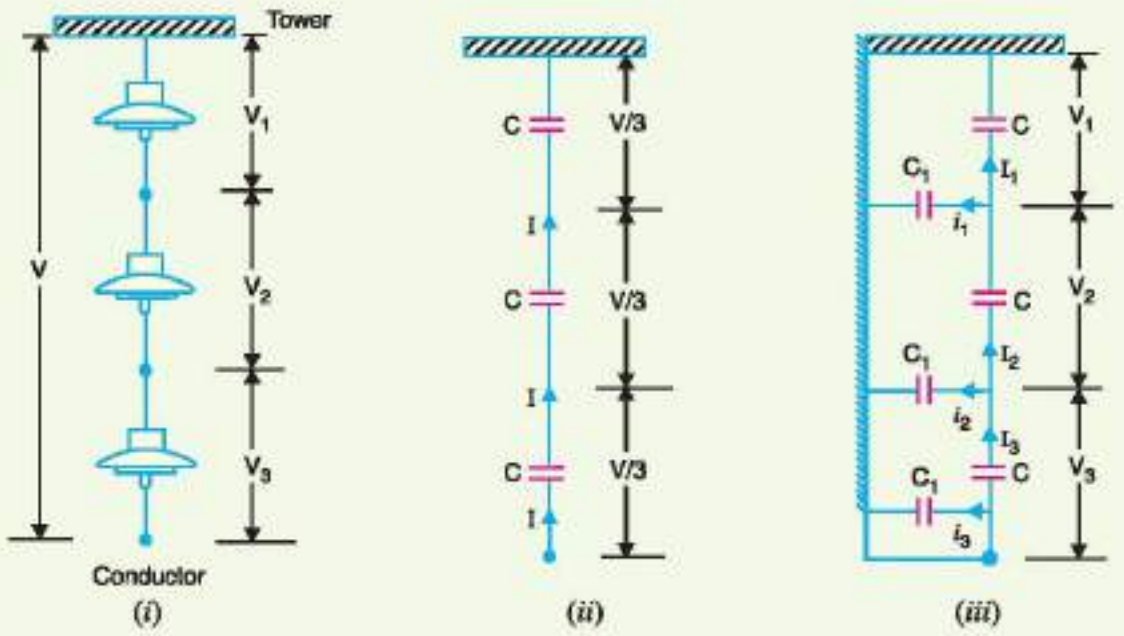


Fig. 8.10

The following points may be noted regarding the potential distribution over a string of suspension insulators :

(*i*) The voltage impressed on a string of suspension insulators does not distribute itself uniformly across the individual discs due to the presence of shunt capacitance.

(*ii*) The disc nearest to the conductor has maximum voltage across it. As we move towards the cross-arm, the voltage across each disc goes on decreasing.

(*iii*) The unit nearest to the conductor is under maximum electrical stress and is likely to be punctured. Therefore, means must be provided to equalise the potential across each unit. This is fully discussed in Art. 8.8.

(*iv*) If the voltage impressed across the string were d.c., then voltage across each unit would be the same. It is because insulator capacitances are ineffective for d.c.

8.7 String Efficiency

As stated above, the voltage applied across the string of suspension insulators is not uniformly distributed across various units or discs. The disc nearest to the conductor has much higher potential than the other discs. This unequal potential distribution is undesirable and is usually expressed in

* Because charging current through the string has the maximum value at the disc nearest to the conductor.

terms of string efficiency.

The ratio of voltage across the whole string to the product of number of discs and the voltage across the disc nearest to the conductor is known as **string efficiency** *i.e.,*

$$\text{String efficiency} = \frac{\text{Voltage across the string}}{n \times \text{Voltage across disc nearest to conductor}}$$

where n = number of discs in the string.

String efficiency is an important consideration since it decides the potential distribution along the string. The greater the string efficiency, the more uniform is the voltage distribution. Thus 100% string efficiency is an ideal case for which the volatge across each disc will be exactly the same. Although it is impossible to achieve 100% string efficiency, yet efforts should be made to improve it as close to this value as possible.

Mathematical expression. Fig. 8.11 shows the equivalent circuit for a 3-disc string. Let us suppose that self capacitance of each disc is C. Let us further assume that shunt capacitance C_1 is some fraction K of self-capacitance *i.e.*, $C_1 = KC$. Starting from the cross-arm or tower, the voltage across each unit is V_1, V_2 and V_3 respectively as shown.

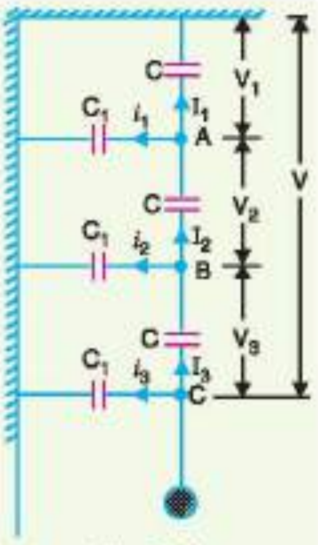


Fig. 8.11

Applying Kirchhoff's current law to node A, we get,

$$I_2 = I_1 + i_1$$

or $$V_2 \omega C^* = V_1 \omega C + V_1 \omega C_1$$

or $$V_2 \omega C = V_1 \omega C + V_1 \omega K C$$

$\therefore$ $$V_2 = V_1 (1 + K) \quad \text{...(i)}$$

Applying Kirchhoff's current law to node B, we get,

$$I_3 = I_2 + i_2$$

or $$V_3 \omega C = V_2 \omega C + (V_1 + V_2) \omega C_1 †$$

or $$V_3 \omega C = V_2 \omega C + (V_1 + V_2) \omega K C$$

or $$V_3 = V_2 + (V_1 + V_2)K$$
$$= KV_1 + V_2 (1 + K)$$
$$= KV_1 + V_1 (1 + K)^2 \quad [\because V_2 = V_1 (1 + K)]$$
$$= V_1 [K + (1 + K)^2]$$

$\therefore$ $$V_3 = V_1[1 + 3K + K^2] \quad \text{...(ii)}$$

Voltage between conductor and earth (*i.e.*, tower) is

$$V = V_1 + V_2 + V_3$$
$$= V_1 + V_1(1 + K) + V_1 (1 + 3K + K^2)$$
$$= V_1 (3 + 4K + K^2)$$

$\therefore$ $$V = V_1(1 + K)(3 + K) \quad \text{...(iii)}$$

From expressions (*i*), (*ii*) and (*iii*), we get,

$$\frac{V_1}{1} = \frac{V_2}{1 + K} = \frac{V_3}{1 + 3K + K^2} = \frac{V}{(1 + K)(3 + K)} \quad \text{...(iv)}$$

$\therefore$ Voltage across top unit, $$V_1 = \frac{V}{(1 + K)(3 + K)}$$

* Note that current through capacitor = $\dfrac{\text{Voltage}}{\text{Capacitive reactance}}$

† Voltage across second shunt capacitance C_1 from top = $V_1 + V_2$. It is because one point of it is connected to B and the other point to the tower.

Voltage across second unit from top, $V_2 = V_1 (1 + K)$

Voltage across third unit from top, $V_3 = V_1 (1 + 3K + K^2)$

$$\text{\%age String efficiency} = \frac{\text{Voltage across string}}{n \times \text{Voltage across disc nearest to conductor}} \times 100$$

$$= \frac{V}{3 \times V_3} \times 100$$

The following points may be noted from the above mathematical analysis :

(*i*) If $K = 0{\cdot}2$ (Say), then from exp. (*iv*), we get, $V_2 = 1{\cdot}2\ V_1$ and $V_3 = 1{\cdot}64\ V_1$. This clearly shows that disc nearest to the conductor has maximum voltage across it; the voltage across other discs decreasing progressively as the cross-arm in approached.

(*ii*) The greater the value of $K (= C_1/C)$, the more non-uniform is the potential across the discs and lesser is the string efficiency.

(*iii*) The inequality in voltage distribution increases with the increase of number of discs in the string. Therefore, shorter string has more efficiency than the larger one.

8.8 Methods of Improving String Efficiency

It has been seen above that potential distribution in a string of suspension insulators is not uniform. The maximum voltage appears across the insulator nearest to the line conductor and decreases progressively as the cross-arm is approached. If the insulation of the highest stressed insulator (*i.e.* nearest to conductor) breaks down or flash over takes place, the breakdown of other units will take place in succession. This necessitates to equalise the potential across the various units of the string *i.e.* to improve the string efficiency. The various methods for this purpose are :

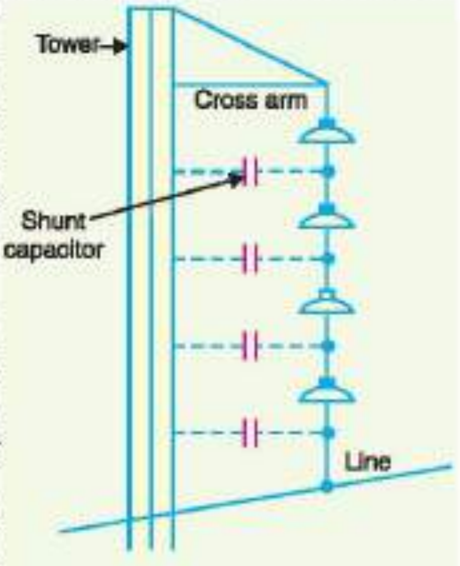


Fig. 8.12

(*i*) *By using longer cross-arms.* The value of string efficiency depends upon the value of K *i.e.*, ratio of shunt capacitance to mutual capacitance. The lesser the value of K, the greater is the string efficiency and more uniform is the voltage distribution. The value of K can be decreased by reducing the shunt capacitance. In order to reduce shunt capacitance, the distance of conductor from tower must be increased *i.e.*, longer cross-arms should be used. However, limitations of cost and strength of tower do not allow the use of very long cross-arms. In practice, $K = 0{\cdot}1$ is the limit that can be achieved by this method.

(*ii*) *By grading the insulators.* In this method, insulators of different dimensions are so chosen that each has a different capacitance. The insulators are capacitance graded *i.e.* they are assembled in the string in such a way that the top unit has the minimum capacitance, increasing progressively as the bottom unit (*i.e.*, nearest to conductor) is reached. Since voltage is inversely proportional to capacitance, this method tends to equalise the potential distribution across the units in the string. This method has the disadvantage that a large number of different-sized insulators are required. However, good results can be obtained by using standard insulators for most of the string and larger units for that near to the line conductor.

(*iii*) *By using a guard ring.* The potential across each unit in a string can be equalised by using a guard ring which is a metal ring electrically connected to the conductor and surrounding the bottom insulator as shown in the Fig. 8.13. The guard ring introduces capacitance between metal fittings and the line conductor. The guard ring is contoured in such a way that shunt capacitance currents i_1, i_2 etc. are equal to metal fitting line capacitance currents i'_1, i'_2 etc. The result is that same charging current I flows through each unit of string. Consequently, there will be uniform potential distribution across the units.

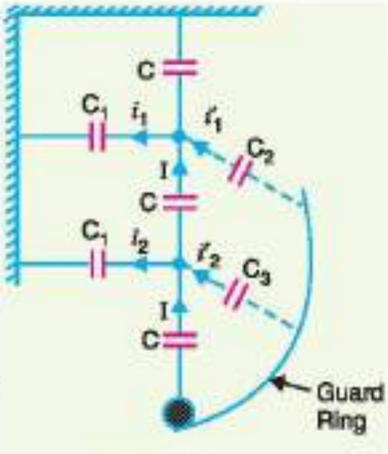


Fig. 8.13

8.9 Important Points

While solving problems relating to string efficiency, the following points must be kept in mind:

(*i*) The maximum voltage appears across the disc nearest to the conductror (*i.e.*, line conductor).

(*ii*) The voltage across the string is equal to phase voltage *i.e.*,

Voltage across string = Voltage between line and earth = Phase Voltage

(*iii*) Line Voltage = $\sqrt{3}$ × Voltage across string

Example 8.1. *In a 33 kV overhead line, there are three units in the string of insulators. If the capacitance between each insulator pin and earth is 11% of self-capacitance of each insulator, find (i) the distribution of voltage over 3 insulators and (ii) string efficiency.*

Solution. Fig. 8.14. shows the equivalent circuit of string insulators. Let V_1, V_2 and V_3 be the voltage across top, middle and bottom unit respectively. If C is the self-capacitance of each unit, then KC will be the shunt capacitance.

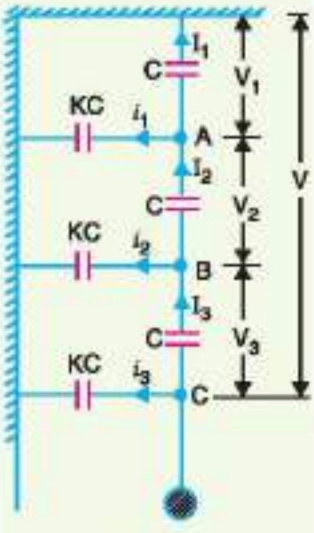


Fig. 8.14

$$K = \frac{\text{Shunt Capacitance}}{\text{Self - capacitance}} = 0.11$$

$$\text{Voltage across string, } V = 33/\sqrt{3} = 19.05 \text{ kV}$$

At Junction A

$$I_2 = I_1 + i_1$$

or $$V_2 \omega C = V_1 \omega C + V_1 K \omega C$$

or $$V_2 = V_1 (1 + K) = V_1 (1 + 0.11)$$

or $$V_2 = 1.11\ V_1 \quad ...(i)$$

At Junction B

$$I_3 = I_2 + i_2$$

or $$V_3 \omega C = V_2 \omega C + (V_1 + V_2)\ K \omega C$$

or $$V_3 = V_2 + (V_1 + V_2)\ K$$

$$= 1.11 V_1 + (V_1 + 1.11\ V_1)\ 0.11$$

∴ $$V_3 = 1.342\ V_1$$

(*i*) Voltage across the whole string is

$$V = V_1 + V_2 + V_3 = V_1 + 1.11\ V_1 + 1.342\ V_1 = 3.452\ V_1$$

or $$19.05 = 3.452\ V_1$$

∴ Voltage across top unit, V_1 = 19.05/3.452 = **5.52 kV**

Voltage across middle unit, V_2 = 1.11 V_1 = 1.11 × 5.52 = **6.13 kV**

Voltage across bottom unit, V_3 = 1.342 V_1 = 1.342 × 5.52 = **7.4 kV**

(*ii*) $$\text{String efficiency} = \frac{\text{Voltage across string}}{\text{No. of insulators} \times V_3} \times 100 = \frac{19.05}{3 \times 7.4} \times 100 = \mathbf{85.8\%}$$

Example 8.2. *A 3-phase transmission line is being supported by three disc insulators. The potentials across top unit (i.e., near to the tower) and middle unit are 8 kV and 11 kV respectively. Calculate (i) the ratio of capacitance between pin and earth to the self-capacitance of each unit (ii)the line voltage and (iii) string efficiency.*

Solution. The equivalent circuit of string insulators is the same as shown in Fig. 8.14. It is given that $V_1 = 8$ kV and $V_2 = 11$ kV.

(*i*) Let K be the ratio of capacitance between pin and earth to self capacitance. If C farad is the self capacitance of each unit, then capacitance between pin and earth = KC.

Applying Kirchoff's current law to Junction A,

$$I_2 = I_1 + i_1$$

or $$V_2 \omega C = V_1 \omega C + V_1 K \omega C$$

or $$V_2 = V_1 (1 + K)$$

$\therefore$ $$K = \frac{V_2 - V_1}{V_1} = \frac{11-8}{8} = \mathbf{0{\cdot}375}$$

(*ii*) Applying Kirchoff's current law to Junction B,

$$I_3 = I_2 + i_2$$

or $$V_3 \omega C = V_2 \omega C + (V_1 + V_2) K \omega C$$

or $$V_3 = V_2 + (V_1 + V_2) K = 11 + (8 + 11) \times 0{\cdot}375 = 18{\cdot}12 \text{ kV}$$

Voltage between line and earth $= V_1 + V_2 + V_3 = 8 + 11 + 18{\cdot}12 = 37{\cdot}12$ kV

$\therefore$ Line Voltage $= \sqrt{3} \times 37{\cdot}12 = \mathbf{64{\cdot}28\ kV}$

(*iii*) String efficiency $$= \frac{\text{Voltage across string}}{\text{No. of insulators} \times V_3} \times 100 = \frac{37{\cdot}12}{3 \times 18{\cdot}12} \times 100 = \mathbf{68{\cdot}28\%}$$

Example 8.3. *Each line of a 3-phase system is suspended by a string of 3 similar insulators. If the voltage across the line unit is 17·5 kV, calculate the line to neutral voltage. Assume that the shunt capacitance betwen each insulator and earth is 1/8th of the capacitance of the insulator itself. Also find the string efficiency.*

Solution. Fig. 8·15 shows the equivalent circuit of string insulators. If C is the self capacitance of each unit, then KC will be the shunt capacitance where $K = 1/8 = 0{\cdot}125$.

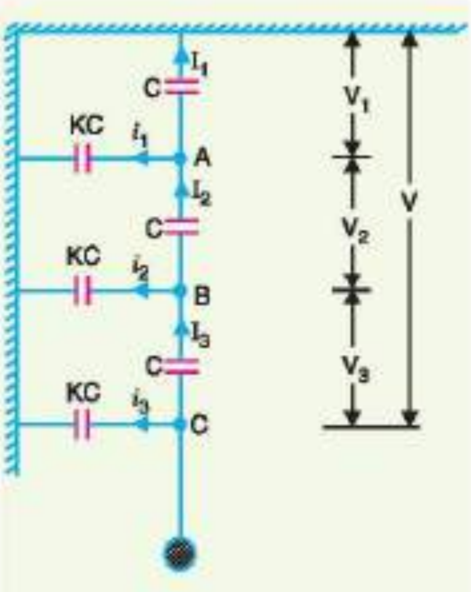


Fig. 8.15

Voltage across line unit, $V_3 = 17{\cdot}5$ kV

At Junction A

$$I_2 = I_1 + i_1$$

$$V_2 \omega C = V_1 \omega C + V_1 K \omega C$$

or $$V_2 = V_1 (1 + K) = V_1 (1 + 0.125)$$

$\therefore$ $$V_2 = 1{\cdot}125\ V_1$$

At Junction B

$$I_3 = I_2 + i_2$$

or $$V_3 \omega C = V_2 \omega C + (V_1 + V_2) K \omega C$$

or $$V_3 = V_2 + (V_1 + V_2) K$$

$$= 1{\cdot}125\ V_1 + (V_1 + 1{\cdot}125\ V_1) \times 0.125$$

$\therefore$ $$V_3 = 1{\cdot}39\ V_1$$

Voltage across top unit, $V_1 = V_3/1{\cdot}39 = 17{\cdot}5/1{\cdot}39$

$= 12{\cdot}59$ kV

Voltage across middle unit, $V_2 = 1{\cdot}125\ V_1 = 1{\cdot}125 \times 12{\cdot}59 = 14{\cdot}16$ kV

$\therefore$ Voltage between line and earth (*i.e.*, line to neutral)

$$= V_1 + V_2 + V_3 = 12{\cdot}59 + 14{\cdot}16 + 17{\cdot}5 = \mathbf{44.25\ kV}$$

$$\text{String efficiency} = \frac{44\cdot25}{3\times17\cdot5}\times100 = \mathbf{84{\cdot}28\%}$$

Example 8.4. *The three bus-bar conductors in an outdoor substation are supported by units of post type insulators. Each unit consists of a stack of 3 pin type insulators fixed one on the top of the other. The voltage across the lowest insulator is 13·1 kV and that across the next unit is 11 kV. Find the bus-bar voltage of the station.*

Solution. The equivalent circuit of insulators is the same as shown in Fig. 8.15. It is given that $V_3 = 13{\cdot}1$ kV and $V_2 = 11$ kV. Let K be the ratio of shunt capacitance to self capacitance of each unit. Applying Kirchhoff's current law to Junctions A and B, we can easily derive the following equations (See example 8.3) :

$$V_2 = V_1(1+K)$$

or $$V_1 = \frac{V_2}{1+K} \qquad ...(i)$$

and $$V_3 = V_2 + (V_1 + V_2)K \qquad ...(ii)$$

Putting the value of $V_1 = V_2/(1+K)$ in eq. (*ii*), we get,

$$V_3 = V_2 + \left[\frac{V_2}{1+K} + V_2\right]K$$

or $$V_3(1+K) = V_2(1+K) + [V_2 + V_2(1+K)]\,K$$

$$= V_2[(1+K) + K + (K+K^2)]$$

$$= V_2(1+3K+K^2)$$

$\therefore$ $$13{\cdot}1(1+K) = 11[1+3K+K^2]$$

or $$11K^2 + 19{\cdot}9\,K - 2{\cdot}1 = 0$$

Solving this equation, we get, $K = 0.1$.

$\therefore$ $$V_1 = \frac{V_2}{1+K} = \frac{11}{1+0\cdot1} = 10\text{ kV}$$

Voltage between line and earth = $V_1 + V_2 + V_3 = 10 + 11 + 13{\cdot}1 = 34{\cdot}1$ kV

$\therefore$ Voltage between bus-bars (*i.e.*, line voltage)

$$= 34{\cdot}1\times\sqrt{3} = \mathbf{59\ kV}$$

Example 8.5. *An insulator string consists of three units, each having a safe working voltage of 15 kV. The ratio of self-capacitance to shunt capacitance of each unit is 8 : 1. Find the maximum safe working voltage of the string. Also find the string efficiency.*

Solution. The equivalent circuit of string insulators is the same as shown in Fig. 8.15. The maximum voltage will appear across the lowest unit in the string.

$\therefore$ $V_3 = 15$ kV ; $K = 1/8 = 0{\cdot}125$

Applying Kirchhoff's current law to junction A, we get,

$$V_2 = V_1(1+K)$$

or $$V_1 = V_2/(1+K) = V_2/(1+0{\cdot}125) = 0{\cdot}89\ V_2 \qquad ...(i)$$

Applying Kirchhoff's current law to Junction B, we get,

$$V_3 = V_2 + (V_1 + V_2)K = V_2 + (0.89\ V_2 + V_2)\times 0.125$$

$\therefore \quad V_3 = 1.236\ V_2$ ...(ii)

$\therefore$ Voltage across middle unit, $V_2 = V_3/1.236 = 15/1{\cdot}236 = 12{\cdot}13$ kV

Voltage across top unit, $V_1 = 0{\cdot}89\ V_2 = 0{\cdot}89 \times 12{\cdot}13 = 10{\cdot}79$ kV

Voltage across the String $= V_1 + V_2 + V_3 = 10{\cdot}79 + 12{\cdot}13 + 15 =$ **37·92 kV**

String efficiency $= \dfrac{37 \cdot 92}{3 \times 15} \times 100 =$ **84·26 %**

Example 8.6. *A string of 4 insulators has a self-capacitance equal to 10 times the pin to earth capacitance. Find (i) the voltage distribution across various units expressed as a percentage of total voltage across the string and (ii) string efficiency.*

Solution. When the number of insulators in a string exceeds 3, the nodal equation method becomes laborious. Under such circumstances, there is a simple method to solve the problem. In this method*, shunt capacitance (C_1) and self capacitance (C) of each insulator are represented by their equivalent reactances. As it is only the ratio of capacitances which determines the voltage distribution, therefore, the problem can be simplified by assigning unity value to X_C *i.e.*, assuming $X_C = 1\ \Omega$. If ratio of $C/C_1 = 10$, then we have $X_C = 1\ \Omega$ and $X_{C1} = 10\ \Omega$.

(*i*) Suppose $X_C = 1\ \Omega$. As the ratio of self-capacitance to shunt capacitance (*i.e.*, C/C_1) is 10, therefore, $X_{C1} = 10\ \Omega$ as shown in Fig. 8·16 (*i*). Suppose that potential V across the string is such that 1 A current flows in the top insulator. Now the potential across each insulator can be easily determined. Thus :

Voltage across top unit, $V_1 = 1\ \Omega \times 1\ \text{A} = 1$ volt

Voltage across **2nd unit, $V_2 = 1\ \Omega \times 1{\cdot}1\ \text{A} = 1{\cdot}1$ volts

Voltage across †3rd unit, $V_3 = 1\ \Omega \times 1{\cdot}31\ \text{A} = 1{\cdot}31$ volts

Voltage across 4th unit, $V_4 = 1\ \Omega \times 1{\cdot}65\ \text{A} = 1{\cdot}65$ volts

Voltage obtained across the string, $V = 1 + 1{\cdot}1 + 1{\cdot}31 + 1{\cdot}65 = 5{\cdot}06$ volts

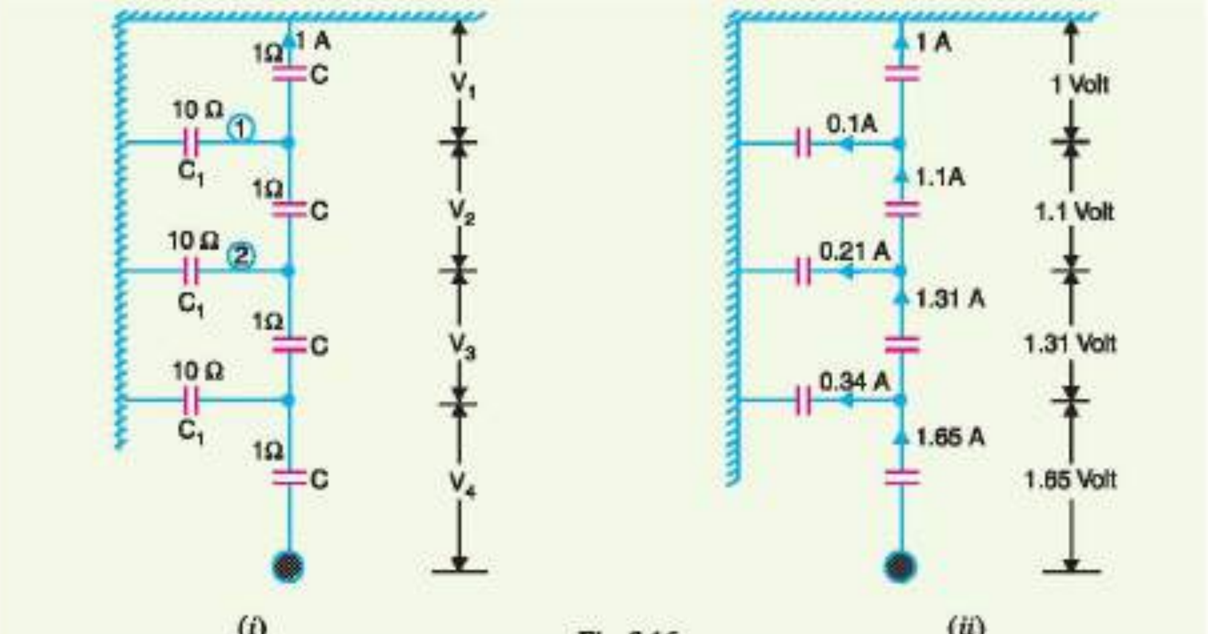


Fig. 8.16

* This method is equally applicable for a string having 3 or less than 3 insulators.

** Current through first shunt capacitance [marked 1, see Fig. 8·16] is $V_1/10 = 1/10 = 0{\cdot}1$ A. Therefore, the current through second unit from top is $= 1 + 0{\cdot}1 = 1{\cdot}1$ A and voltage across it is $= 1\Omega \times 1{\cdot}1\ \text{A} = 1{\cdot}1$ volts.

† Current through second shunt capacitance [marked 2 in Fig. 8·16] is $(V_1 + V_2)/10 = (1 + 1{\cdot}1)/10 = 0{\cdot}21$ A. Therefore, current thro' 3rd unit from top $= 1{\cdot}1 + 0{\cdot}21 = 1{\cdot}31$ A and voltage across it is $1\ \Omega \times 1{\cdot}31\ \text{A} = 1{\cdot}31$ volts.

The voltage across each unit expressed as a percentage of V (*i.e.*, 5·06 volts) becomes :

Top unit $= (1/5{\cdot}06) \times 100 =$ **19·76%**

Second from top $= (1{\cdot}1/5{\cdot}06) \times 100 =$ **21·74%**

Third from top $= (1{\cdot}31/5{\cdot}06) \times 100 =$ **25·9%**

Fourth from top $= (1{\cdot}65/5{\cdot}06) \times 100 =$ **32·6%**

(*ii*) String efficiency $= \dfrac{V}{4 \times V_4} \times 100 = \dfrac{5{\cdot}06}{4 \times 1{\cdot}65} \times 100 =$ **76·6%**

Example 8.7. *A string of 5 insulators is connected across a 100 kV line. If the capacitance of each disc to earth is 0·1 of the capacitance of the insulator, calculate (i) the distribution of voltage on the insulator discs and (ii) the string efficiency.*

Solution. Suppose $X_C = 1\,\Omega$. As the ratio of self capacitance to shunt capacitance is 10, therefore, $X_{C1} = 10\,\Omega$ as shown in Fig. 8·17 (*i*). Suppose that potential V across the string is such that 1A current flows in the top insulator. Then potential across each insulator will be as shown in Fig. 8·17 (*ii*).

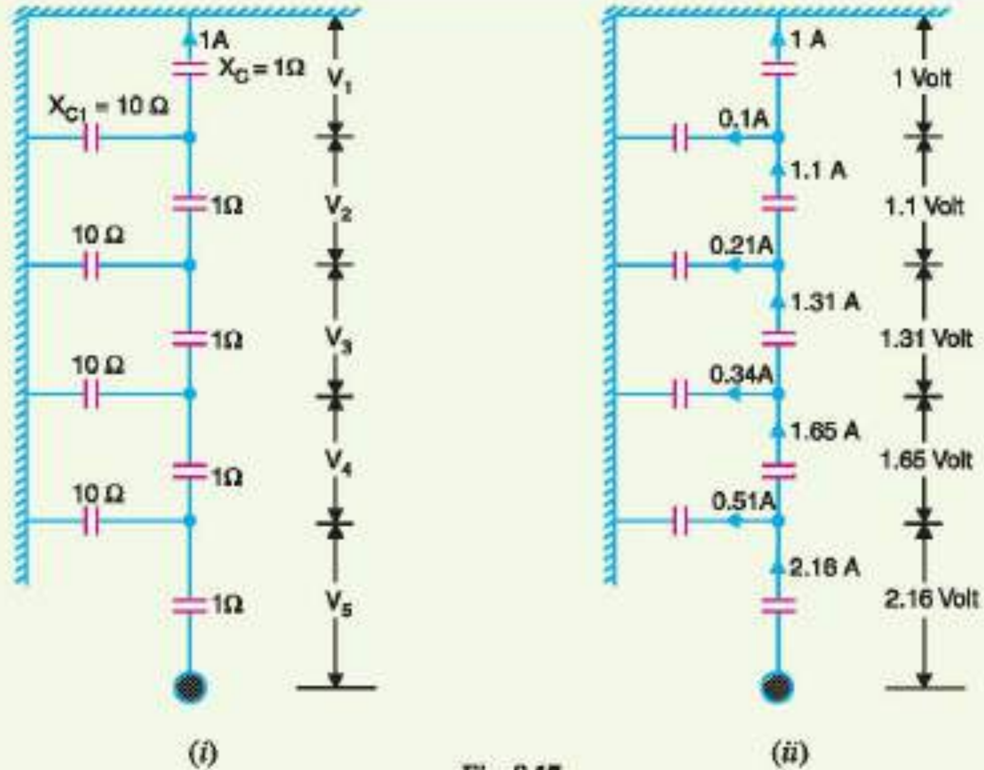


Fig. 8.17

The value obtained for $V = 1 + 1{\cdot}1 + 1{\cdot}31 + 1{\cdot}65 + 2{\cdot}16 = 7{\cdot}22$ volts and starting from top, the percentage of V (*i.e.*, 7·22 volts) across various units are :

*13·8 %, 15·2 %, 18·2 %, 22·8 % and 30%

Voltage across string $= 100/\sqrt{3} = 57{\cdot}7$ kV

(*i*) Voltage across top insulator, $V_1 = 0{\cdot}138 \times 57{\cdot}7 =$ **7·96 kV**

Voltage across 2nd from top, $V_2 = 0{\cdot}152 \times 57{\cdot}7 =$ **8·77 kV**

---

* % age of V (*i.e.*, 7·22 volts) across top unit $= \dfrac{1}{7{\cdot}22} \times 100 = 13{\cdot}8\%$

% age of V across 2nd from top $= \dfrac{1{\cdot}1}{7{\cdot}22} \times 100 = 15{\cdot}2\%$

Voltage across 3rd from top, $V_3 = 0{\cdot}182 \times 57{\cdot}7 =$ **10·5 kV**

Voltage across 4th from top, $V_4 = 0{\cdot}228 \times 57{\cdot}7 =$ **13·16 kV**

Voltage across 5th from top, $V_5 = 0{\cdot}3 \times 57{\cdot}7 =$ **17·3 kV**

(*ii*) String efficiency $= \dfrac{57 \cdot 7}{5 \times 17 \cdot 3} \times 100 =$ **66·7%**

Example 8.8. *Each conductor of a 3-phase high-voltage transmission line is suspended by a string of 4 suspension type disc insulators. If the potential difference across the second unit from top is 13·2 kV and across the third from top is 18 kV, determine the voltage between conductors.*

Solution. Suppose $X_C = 1\ \Omega$. If K is the ratio of shunt-capacitance to self-capacitance, then $X_{C1} = 1/K$ ohms as shown in Fig. 8.18 (*i*). Suppose voltage across string is such that current in top insulator disc is 1 A. Then voltage across each insulator can be easily determined [see Fig. 8.18 (*ii*)]. Thus the voltage across first shunt capacitance from top is 1 volt and its reactance is $1/K$ ohms. Therefore, current through it is K ampere. Hence current through second insulator from top is $(1 + K)$ amperes and voltage across it is $(1 + K) \times 1 = (1 + K)$ volts.

Referring to Fig. 8.18 (*ii*), we have,

$$V_2/V_1 = (1 + K)/1$$

or $$V_2 = V_1 (1 + K) \quad \text{...(}i\text{)}$$

Also $$V_3/V_1 = (1 + 3K + K^2)/1$$

$\therefore$ $$V_3 = V_1 (1 + 3K + K^2) \quad \text{...(}ii\text{)}$$

Dividing (*ii*) by (*i*), we get,

$$\frac{V_3}{V_2} = \frac{1 + 3K + K^2}{1 + K}$$

It is given that $V_3 = 18$ kV and $V_2 = 13{\cdot}2$ kV

$\therefore$ $$\frac{18}{13 \cdot 2} = \frac{1 + 3K + K^2}{1 + K}$$

or $$13{\cdot}2\,K^2 + 21{\cdot}6\,K - 4{\cdot}8 = 0$$

Solving this equation, we get, $K = 0{\cdot}2$.

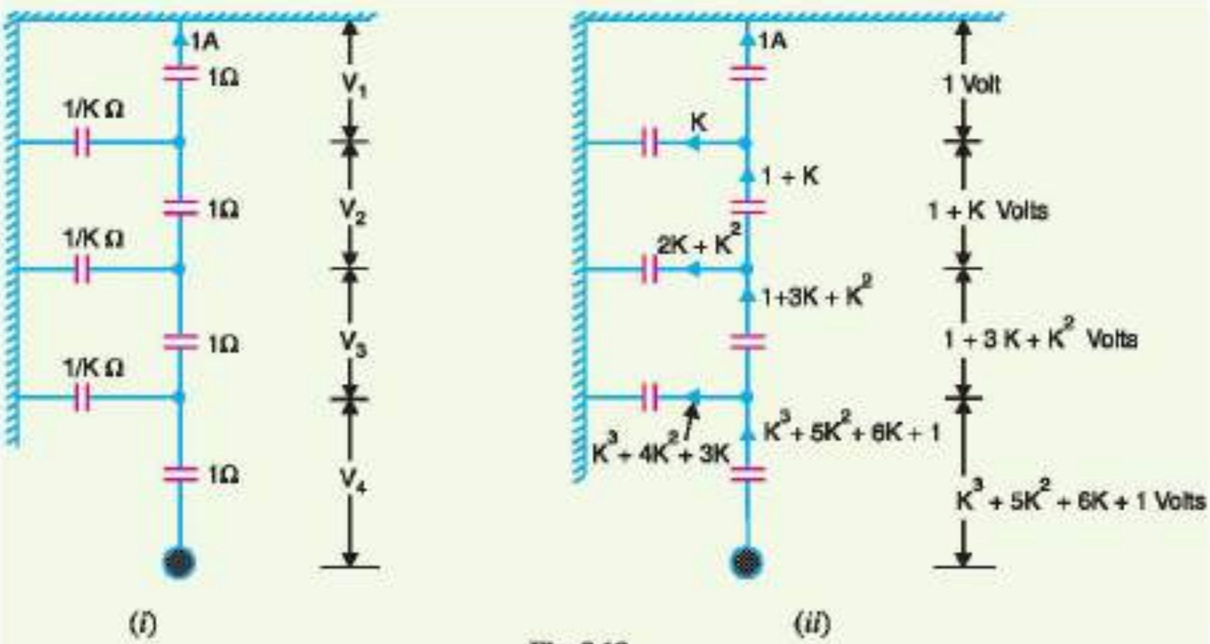


Fig. 8.18

$\therefore$
$$V_1 = V_2/(1+K) = 13\cdot2/1\cdot2 = 11 \text{ kV}$$
$$V_4 = V_1(1+K^3+5K^2+6K) = 11(1+0\cdot008+0\cdot2+1\cdot2) = 26\cdot49 \text{kV}$$

Voltage between line and earth (*i.e.*, phase voltage)
$$= V_1 + V_2 + V_3 + V_4$$
$$= 11 + 13\cdot2 + 18 + 26\cdot49 = 68\cdot69 \text{ kV}$$

Voltage between conductors (*i.e.*, line voltage)
$$= 68\cdot69 \times \sqrt{3} = \mathbf{119 \text{ kV}}$$

Example 8.9. *A string of four insulators has a self-capacitance equal to 5 times pin to earth capacitance. Find (i) the voltage distribution across various units as a percentage of total voltage across the string and (ii) string efficiency.*

Solution. The ratio of self-capacitance (C) to pin-earth capacitance (C_1) is $C/C_1 = 5$. Suppose $X_C = 1\ \Omega$. Then $X_{C1} = 5\ \Omega$. Suppose the voltage V across string is such that current in the top insulator is 1 A as shown in Fig. 8.19 (*i*). The potential across various insulators will be as shown in Fig. 8.19 (*ii*).

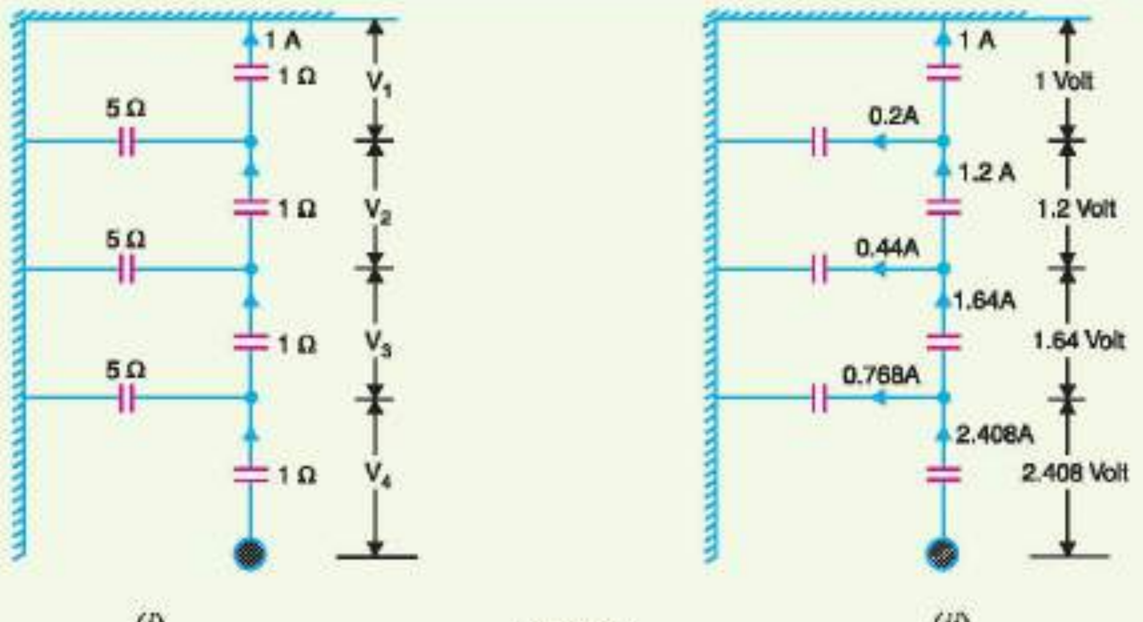


Fig. 8.19

The voltage obtained across the string is given by ;
$$V = 1 + 1\cdot2 + 1\cdot64 + 2\cdot408 = 6\cdot248 \text{ volts}$$

(*i*) The voltage across each unit expressed as a percentage of V (*i.e.*, 6·248 volts) is given by :

Top Unit $= (1/6\cdot248) \times 100 = \mathbf{16\%}$

Second from top $= (1\cdot2/6\cdot248) \times 100 = \mathbf{19\cdot2\%}$

Third from top $= (1\cdot64/6\cdot248) \times 100 = \mathbf{26\cdot3\%}$

Fourth from top $= (2\cdot408/6\cdot248) \times 100 = \mathbf{38\cdot5\%}$

(*ii*) String efficiency $= \dfrac{6\cdot248}{4 \times 2\cdot408} \times 100 = \mathbf{64\cdot86\%}$

Example 8.10. *The self capacitance of each unit in a string of three suspension insulators is C. The shunting capacitance of the connecting metal work of each insulator to earth is 0·15 C while for line it is 0·1 C. Calculate (i) the voltage across each insulator as a percentage of the line voltage to earth and (ii) string efficiency.*

Solution. In an actual string of insulators, three capacitances exist *viz.*, self-capacitance of each insulator, shunt capacitance and capacitance of each unit to line as shown in Fig. 8.20 (*i*). However, capacitance of each unit to line is very small and is usually neglected. Fig. 8.20 (*ii*) shows the equivalent circuit of string insulators.

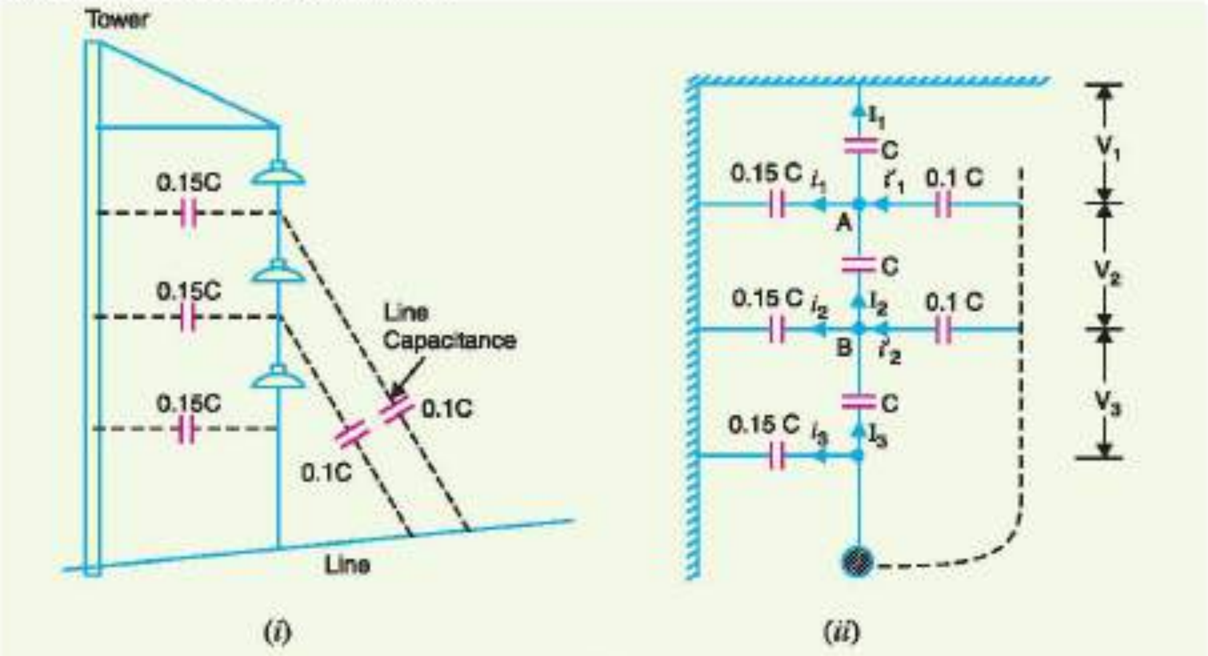


Fig. 8.20

At Junction A

$$I_2 + i'_1 = I_1 + i_1$$

or $$V_2 \omega C + (V_2 + V_3)\, 0{\cdot}1 \omega C = V_1 \omega C + 0{\cdot}15\, C V_1 \omega$$

or $$0{\cdot}1\, V_3 = 1{\cdot}15\, V_1 - 1{\cdot}1\, V_2$$

or $$V_3 = 11{\cdot}5\, V_1 - 11\, V_2 \qquad \ldots(i)$$

At Junction B

$$I_3 + i'_2 = I_2 + i_2$$

or $$V_3 \omega C + V_3 \times 0{\cdot}1\, C \times \omega = V_2 \omega C + (V_1 + V_2)\, \omega \times 0{\cdot}15\, C$$

or $$1{\cdot}1\, V_3 = 1{\cdot}15\, V_2 + 0{\cdot}15\, V_1 \qquad \ldots(ii)$$

Putting the value of V_3 from exp (*i*). into exp. (*ii*), we get,

$$1{\cdot}1\,(11{\cdot}5\, V_1 - 11\, V_2) = 1{\cdot}15\, V_2 + 0{\cdot}15\, V_1$$

or $$13{\cdot}25\, V_2 = 12{\cdot}5\, V_1$$

or $$V_2 = \frac{12 \cdot 5}{13 \cdot 25} V_1 \qquad \ldots(iii)$$

Putting the value of V_2 from exp. (*iii*) into exp. (*i*), we get,

$$V_3 = 11{\cdot}5\, V_1 - 11\left(\frac{12 \cdot 5\, V_1}{13 \cdot 25}\right) = \left(\frac{14 \cdot 8}{13 \cdot 25}\right) V_1$$

Now voltage between conductor and earth is

$$V = V_1 + V_2 + V_3 = V_1\left(1 + \frac{12 \cdot 5}{13 \cdot 25} + \frac{14 \cdot 8}{13 \cdot 25}\right) = \left(\frac{40 \cdot 55\, V_1}{13 \cdot 25}\right) \text{volts}$$

$\therefore$

$$V_1 = 13{\cdot}25\, V/40{\cdot}55 = 0{\cdot}326\, V \text{ volts}$$

$$V_2 = 12{\cdot}5 \times 0{\cdot}326\, V/13{\cdot}25 = 0{\cdot}307\, V \text{ volts}$$

$$V_3 = 14{\cdot}8 \times 0{\cdot}326\, V/13{\cdot}25 = 0{\cdot}364\, V \text{ volts}$$

(*i*) The voltage across each unit expressed as a percentage of V becomes:

Top unit $= V_1 \times 100/V = 0.326 \times 100 =$ **32.6%**

Second from top $= V_2 \times 100/V = 0.307 \times 100 =$ **30.7%**

Third from top $= V_3 \times 100/V = 0.364 \times 100 =$ **36.4%**

(*ii*) String efficiency $= \dfrac{V}{3 \times 0.364\, V} \times 100 =$ **91.5 %**

Example 8.11. *Each line of a 3-phase system is suspended by a string of 3 indentical insulators of self-capacitance C farad. The shunt capacitance of connecting metal work of each insulator is 0·2 C to earth and 0·1 C to line. Calculate the string efficiency of the system if a guard ring increases the capacitance to the line of metal work of the lowest insulator to 0·3 C.*

Solution. The capacitance between each unit and line is artificially increased by using a guard ring as shown in Fig. 8.21. This arrangement tends to equalise the potential across various units and hence leads to improved string efficiency. It is given that with the use of guard ring, capacitance of the insulator link-pin to the line of the lowest unit is increased from 0·1 C to 0·3 C.

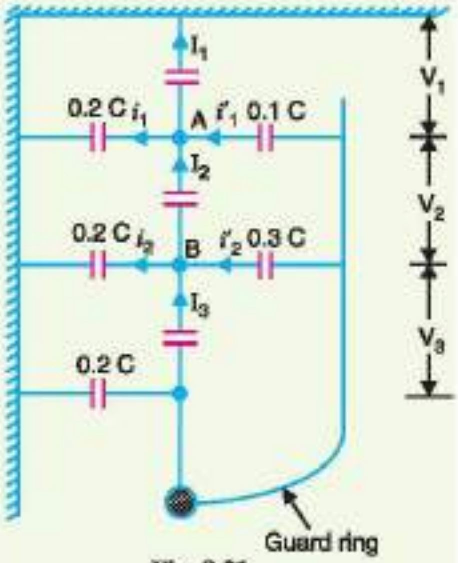


Fig. 8.21

At Junction A

$$I_2 + i'_1 = I_1 + i_1$$

or $\quad V_2 \omega C + (V_2 + V_3)\, \omega \times 0.1\, C = V_1 \omega C + V_1 \times 0.2\, C \omega$

$$V_3 = 12\, V_1 - 11\, V_2 \qquad \ldots(i)$$

At Junction B

$$I_3 + i'_2 = I_2 + i_2$$

or $\quad V_3 \omega C + V_3 \times 0.3\, C \times \omega = V_2 \omega C + (V_1 + V_2)\, \omega \times 0.2\, C$

or
$$1.3\, V_3 = 1.2\, V_2 + 0.2\, V_1 \qquad \ldots(ii)$$

Substituting the value of V_3 from exp. (*i*) into exp. (*ii*), we get,

$$1.3\,(12\, V_1 - 11\, V_2) = 1.2\, V_2 + 0.2\, V_1$$

or
$$15.5\, V_2 = 15.4\, V_1$$

$\therefore$
$$V_2 = 15.4\, V_1/15.5 = 0.993\, V_1 \qquad \ldots(iii)$$

Substituting the value of V_2 from exp. (*iii*) into exp. (*i*), we get,

$$V_3 = 12\, V_1 - 11 \times 0.993\, V_1 = 1.077\, V_1$$

Voltage between conductor and earth (*i.e.* phase voltage)

$$= V_1 + V_2 + V_3 = V_1 + 0.993\, V_1 + 1.077\, V_1 = 3.07\, V_1$$

String efficiency $= \dfrac{3.07\, V_1}{3 \times 1.077\, V_1} \times 100 =$ **95%**

Example 8.12. *It is required to grade a string having seven suspension insulators. If the pin to earth capacitance are all equal to C, determine the line to pin capacitance that would give the same voltage across each insulator of the string.*

Solution. Let $C_1, C_2 \ldots C_6$ respectively be the required line to pin capacitances of the units as shown in Fig. 8.22. As the voltage across each insulator has to be the same, therefore,

$$I_1 = I_2 = I_3 = I_4 = I_5 = I_6 = I_7$$

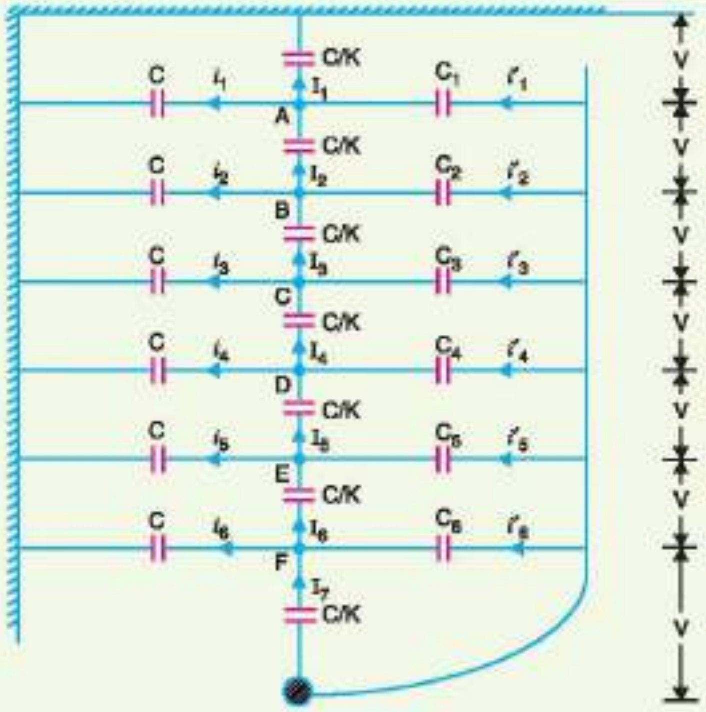


Fig. 8.22

At Junction A

$$i_1' + I_2 = i_1 + I_1$$

or $\quad i'_1 = i_1 \qquad (\because I_1 = I_2)$

or $\quad \omega C_1 (6\ V) = \omega CV \qquad (\because$ Voltage across $C_1 = 6\ V)$

$\therefore \quad C_1 = C/6 = \mathbf{0{\cdot}167}\ C$

At Junction B

$$i_2' = i_2$$

or $\quad \omega C_2 (5\ V) = \omega C (2\ V)$

$\therefore \quad C_2 = \dfrac{2C}{5} = \mathbf{0{\cdot}4}\ C$

At Junction C

$$i_3' = i_3$$

or $\quad \omega C_3 (4\ V) = \omega C (3\ V)$

$\therefore \quad C_3 = 3C/4 = \mathbf{0{\cdot}75}\ C$

At Junction D

$$i_4' = i_4$$

or $\quad \omega C_4 (3\ V) = \omega C (4\ V)$

$\therefore \quad C_4 = 4C/3 = \mathbf{1{\cdot}33}\ C$

At Junction E

$$i_5' = i_5$$

or $\quad \omega C_5 (2\ V) = \omega C (5\ V)$

$\therefore \quad C_5 = 5C/2 = \mathbf{2{\cdot}5}\ C$

At Junction F

$$i_6' = i_6$$

or $\quad \omega C_6\ V = \omega C (6\ V)$

$\therefore \quad C_6 = \mathbf{6\ C}$

TUTORIAL PROBLEMS

1. In a 3-phase overhead system, each line is suspended by a string of 3 insulators. The voltage across the top unit (*i.e.* near the tower) and middle unit are 10 kV and 11 kV respectively. Calculate (*i*) the ratio of shunt capacitance to self capacitance of each insulator, (*ii*) the string efficiency and (*iii*) line voltage.
[(*i*) 0·1 (*ii*) 86·76% (*iii*) 59 kV]

2. Each line of a 3-phase system is suspended by a string of 3 similar insulators. If the voltage across the line unit is 17·5 kV, calculate the line to neutral voltage and string efficiency. Assume that shunt capacitance between each insulator and earthed metal work of tower to be 1/10th of the capacitance of the insulator. **[52 kV, 86·67%]**

3. The three bus-bar conductors in an outdoor sub-station are supplied by units of post insulators. Each unit consists of a stack of 3-pin insulators fixed one on the top of the other. The voltage across the lowest insulator is 8·45 kV and that across the next is 7·25 kV. Find the bus-bar voltage of the station. **[38·8 kV]**

4. A string of suspension insulators consists of three units. The capacitance between each link pin and earth is one-sixth of the self-capacitance of each unit. If the maximum voltage per unit is not to exceed 35 kV, determine the maximum voltage that the string can withstand. Also calculate the string efficiency. **[84·7 kV; 80·67%]**

5. A string of 4 insulators has self-capacitance equal to 4 times the pin-to-earth capacitance. Calculate (*i*) the voltage distribution across various units as a percentage of total voltage across the string and (*ii*) string efficiency. **[(*i*) 14·5%, 18·1%, 26·2% and 40·9% (*ii*) 61·2 %]**

6. A string of four suspension insulators is connected across a 285 kV line. The self-capacitance of each unit is equal to 5 times pin to earth capacitance. Calculate :
   (*i*) the potential difference across each unit, (*ii*) the string efficiency.
   [(*i*) 27·65 kV, 33·04 kV, 43·85 kV, 60 kV (*ii*) 68·5%]

7. Each of three insulators forming a string has self-capacitance of "*C*" farad. The shunt capacitance of each cap of insulator is 0·25 C to earth and 0·15 C to line. Calculate the voltage distribution across each insulator as a pecentage of line voltage to earth and the string efficiency. **[31·7%, 29·4%, 38·9%; 85·7%]**

8. Each of the three insulators forming a string has a self capacitance of *C* farad. The shunt capacitance of each insulator is 0·2 C to earth and 0·1 C to line. A guard-ring increases the capacitance of line of the metal work of the lowest insulator to 0·3 C. Calculate the string efficiency of the arrangement :
   (*i*) with the guard ring, (*ii*) without guard ring. **[(*i*) 95% (*ii*) 86·13%]**

9. A three-phase overhead transmission line is being supported by three-disc suspension insulators; the potentials across the first and second insulator from the top are 8 kV and 11 kV respectively. Calcualte (*i*) the line voltage (*ii*) the ratio of capacitance between pin and earth to self capacitance of each unit (*iii*) the string efficiency. **[(*i*) 64·28 V (*ii*) 0·375 (*iii*) 68·28%]**

10. A 3-phase overhead transmission line is supported on 4-disc suspension insulators. The voltage across the second and third discs are 13·2 kV and 18 kV respectively. Calculate the line voltage and mention the nearest standard voltage. **[118·75 kV; 120 kV]**

8.10 Corona

When an alternating potential difference is applied across two conductors whose spacing is large as compared to their diameters, there is no apparent change in the condition of atmospheric air surrounding the wires if the applied voltage is low. However, when the applied voltage exceeds a certain value, called *critical disruptive voltage*, the conductors are surrounded by a faint violet glow called corona.

The phenomenon of corona is accompanied by a hissing sound, production of ozone, power loss and radio interference. The higher the voltage is raised, the larger and higher the luminous envelope becomes, and greater are the sound, the power loss and the radio noise. If the applied voltage is increased to breakdown value, a flash-over will occur between the conductors due to the breakdown of air insulation.

The phenomenon of violet glow, hissing noise and production of ozone gas in an overhead transmission line is known as **corona.**

If the conductors are polished and smooth, the corona glow will be uniform throughout the length of the conductors, otherwise the rough points will appear brighter. With d.c. voltage, there is difference in the appearance of the two wires. The positive wire has uniform glow about it, while the negative conductor has spotty glow.

Theory of corona formation. Some ionisation is always present in air due to cosmic rays, ultraviolet radiations and radioactivity. Therefore, under normal conditions, the air around the conductors contains some ionised particles (*i.e.*, free electrons and +ve ions) and neutral molecules. When p.d. is applied between the conductors, potential gradient is set up in the air which will have maximum value at the conductor surfaces. Under the influence of potential gradient, the existing free electrons acquire greater velocities. The greater the applied voltage, the greater the potential gradient and more is the velocity of free electrons.

When the potential gradient at the conductor surface reaches about 30 kV per cm (max. value), the velocity acquired by the free electrons is sufficient to strike a neutral molecule with enough force to dislodge one or more electrons from it. This produces another ion and one or more free electrons, which is turn are accelerated until they collide with other neutral molecules, thus producing other ions. Thus, the process of ionisation is cummulative. The result of this ionisation is that either corona is formed or spark takes place between the conductors.

8.11 Factors Affecting Corona

The phenomenon of corona is affected by the physical state of the atmosphere as well as by the conditions of the line. The following are the factors upon which corona depends :

(*i*) *Atmosphere.* As corona is formed due to ionsiation of air surrounding the conductors, therefore, it is affected by the physical state of atmosphere. In the stormy weather, the number of ions is more than normal and as such corona occurs at much less voltage as compared with fair weather.

(*ii*) *Conductor size.* The corona effect depends upon the shape and conditions of the conductors. The rough and irregular surface will give rise to more corona because unevenness of the surface decreases the value of breakdown voltage. Thus a stranded conductor has irregular surface and hence gives rise to more corona that a solid conductor.

(*iii*) *Spacing between conductors.* If the spacing between the conductors is made very large as compared to their diameters, there may not be any corona effect. It is because larger distance between conductors reduces the electro-static stresses at the conductor surface, thus avoiding corona formation.

(*iv*) *Line voltage.* The line voltage greatly affects corona. If it is low, there is no change in the condition of air surrounding the conductors and hence no corona is formed. However, if the line voltage has such a value that electrostatic stresses developed at the conductor surface make the air around the conductor conducting, then corona is formed.

8.12 Important Terms

The phenomenon of corona plays an important role in the design of an overhead transmission line. Therefore, it is profitable to consider the following terms much used in the analysis of corona effects:

(*i*) **Critical disruptive voltage.** *It is the minimum phase-neutral voltage at which corona occurs.*

Consider two conductors of radii *r* cm and spaced *d* cm apart. If *V* is the phase-neutral potential, then potential gradient at the conductor surface is given by:

$$g = \frac{V}{r \log_e \frac{d}{r}} \text{ volts / cm}$$

In order that corona is formed, the value of *g* must be made equal to the breakdown strength of air. The breakdown strength of air at 76 cm pressure and temperature of 25°C is 30 kV/cm (*max*) or

21·2 kV/cm (*r.m.s.*) and is denoted by g_o. If V_c is the phase-neutral potential required under these conditions, then,

$$g_o = \frac{V_c}{r \log_e \frac{d}{r}}$$

where g_o = breakdown strength of air at 76 cm of mercury and 25°C
= 30 kV/cm (*max*) or 21·2 kV/cm (*r.m.s.*)

∴ Critical disruptive voltage, $V_c = g_o\, r \log_e \frac{d}{r}$

The above expression for disruptive voltage is under standard conditions *i.e.*, at 76 cm of Hg and 25°C. However, if these conditions vary, the air density also changes, thus altering the value of g_o. The value of g_o is directly proportional to air density. Thus the breakdown strength of air at a barometric pressure of b cm of mercury and temperature of t°C becomes $\delta\, g_o$ where

$$\delta = \text{air density factor} = \frac{3\cdot92b}{273+t}$$

Under standard conditions, the value of $\delta = 1$.

∴ Critical disruptive voltage, $V_c = g_o\, \delta\, r \log_e \frac{d}{r}$

Correction must also be made for the surface condition of the conductor. This is accounted for by multiplying the above expression by irregularity factor m_o.

∴ Critical disruptive voltage, $V_c = m_o g_o\, \delta\, r \log_e \frac{d}{r}$ kV/phase

where m_o = 1 for polished conductors
= 0·98 to 0·92 for dirty conductors
= 0·87 to 0·8 for stranded conductors

(*ii*) Visual critical voltage. *It is the minimum phase-neutral voltage at which corona glow appears all along the line conductors.*

It has been seen that in case of parallel conductors, the corona glow does not begin at the disruptive voltage V_c but at a higher voltage V_v, called *visual critical voltage*. The phase-neutral effective value of visual critical voltage is given by the following empirical formula :

$$V_v = m_v g_o\, \delta\, r \left(1 + \frac{0\cdot3}{\sqrt{\delta\, r}}\right) \log_e \frac{d}{r} \text{ kV/phase}$$

where m_v is another irregularity factor having a value of 1·0 for polished conductors and 0·72 to 0·82 for rough conductors.

(*iii*) Power loss due to corona. Formation of corona is always accompanied by energy loss which is dissipated in the form of light, heat, sound and chemical action. When disruptive voltage is exceeded, the power loss due to corona is given by :

$$P = 242\cdot2 \left(\frac{f+25}{\delta}\right)\sqrt{\frac{r}{d}}\,(V - V_c)^2 \times 10^{-5} \text{ kW/km/phase}$$

where f = supply frequency in Hz
V = phase-neutral voltage (*r.m.s.*)
V_c = disruptive voltage (*r.m.s.*) per phase

8.13 Advantages and Disadvantages of Corona

Corona has many advantages and disadvantages. In the correct design of a high voltage overhead line, a balance should be struck between the advantages and disadvantages.

Advantages

(*i*) Due to corona formation, the air surrounding the conductor becomes conducting and hence virtual diameter of the conductor is increased. The increased diameter reduces the electrostatic stresses between the conductors.

(*ii*) Corona reduces the effects of transients produced by surges.

Disadvantages

(*i*) Corona is accompanied by a loss of energy. This affects the transmission efficiency of the line.

(*ii*) Ozone is produced by corona and may cause corrosion of the conductor due to chemical action.

(*iii*) The current drawn by the line due to corona is non-sinusoidal and hence non-sinusoidal voltage drop occurs in the line. This may cause inductive interference with neighbouring communication lines.

8.14 Methods of Reducing Corona Effect

It has been seen that intense corona effects are observed at a working voltage of 33 kV or above. Therefore, careful design should be made to avoid corona on the sub-stations or bus-bars rated for 33 kV and higher voltages otherwise highly ionised air may cause flash-over in the insulators or between the phases, causing considerable damage to the equipment. The corona effects can be reduced by the following methods :

(*i*) *By increasing conductor size.* By increasing conductor size, the voltage at which corona occurs is raised and hence corona effects are considerably reduced. This is one of the reasons that *ACSR* conductors which have a larger cross-sectional area are used in transmission lines.

(*ii*) *By increasing conductor spacing.* By increasing the spacing between conductors, the voltage at which corona occurs is raised and hence corona effects can be eliminated. However, spacing cannot be increased too much otherwise the cost of supporting structure (*e.g.*, bigger cross arms and supports) may increase to a considerable extent.

Example 8.13. *A 3-phase line has conductors 2 cm in diameter spaced equilaterally 1 m apart. If the dielectric strength of air is 30 kV (max) per cm, find the disruptive critical voltage for the line. Take air density factor $\delta = 0{\cdot}952$ and irregularity factor $m_o = 0{\cdot}9$.*

Solution.

Conductor radius, $r = 2/2 = 1$ cm

Conductor spacing, $d = 1$ m $= 100$ cm

Dielectric strength of air, $g_o = 30$ kV/cm (*max.*) $= 21{\cdot}2$ kV (*r.m.s.*) per cm

Disruptive critical voltage, $V_c = m_o\, g_o\, \delta\, r \log_e (d/r)$ kV*/phase (*r.m.s.* value)

$= 0{\cdot}9 \times 21{\cdot}2 \times 0{\cdot}952 \times 1 \times \log_e 100/1 = 83{\cdot}64$ kV/phase

$\therefore$ Line voltage (*r.m.s.*) $= \sqrt{3} \times 83{\cdot}64 =$ **144·8 kV**

Example 8.14. *A 132 kV line with 1·956 cm dia. conductors is built so that corona takes place if the line voltage exceeds 210 kV (r.m.s.). If the value of potential gradient at which ionisation occurs can be taken as 30 kV per cm, find the spacing between the conductors.*

* As g_o is taken in kV/cm, therefore, V_c will be in kV.

Solution.

Assume the line is 3-phase.

Conductor radius, $r = 1{\cdot}956/2 = 0{\cdot}978$ cm

Dielectric strength of air, $g_o = 30/\sqrt{2} = 21{\cdot}2$ kV (*r.m.s.*) per cm

Disruptive voltage/phase, $V_c = 210/\sqrt{3} = 121{\cdot}25$ kV

Assume smooth conductors (*i.e.*, irregularity factor $m_o = 1$) and standard pressure and temperature for which air density factor $\delta = 1$. Let d cm be the spacing between the conductors.

$\therefore$ Disruptive voltage (*r.m.s.*) per phase is

$$V_c = m_o\, g_o\, \delta\, r \log_e (d/r) \text{ kV}$$
$$= 1 \times 21{\cdot}2 \times 1 \times 0{\cdot}978 \times \log_e (d/r)$$

or $$121{\cdot}25 = 20{\cdot}733 \log_e (d/r)$$

or $$\log_e \frac{d}{r} = \frac{121{\cdot}25}{20{\cdot}733} = 5{\cdot}848$$

or $$2{\cdot}3 \log_{10} d/r = 5{\cdot}848$$

or $$\log_{10} d/r = 5{\cdot}848/2{\cdot}3 = 2{\cdot}5426$$

or $$d/r = \text{Antilog } 2{\cdot}5426$$

or $$d/r = 348{\cdot}8$$

$\therefore$ Conductor spacing, $d = 348{\cdot}8 \times r = 348{\cdot}8 \times 0{\cdot}978 =$ **341 cm**

Example 8.15. *A 3-phase, 220 kV, 50 Hz transmission line consists of 1·5 cm radius conductor spaced 2 metres apart in equilateral triangular formation. If the temperature is 40°C and atmospheric pressure is 76 cm, calculate the corona loss per km of the line. Take* $m_o = 0{\cdot}85$.

Solution.

As seen from Art. 8.12, the corona loss is given by :

$$P = \frac{242{\cdot}2}{\delta}(f + 25)\sqrt{\frac{r}{d}}\,(V - V_c)^2 \times 10^{-5} \text{ kW/km/phase}$$

Now, $$\delta = \frac{3{\cdot}92\, b}{273 + t} = \frac{3{\cdot}92 \times 76}{273 + 40} = 0{\cdot}952$$

Assuming $$g_o = 21{\cdot}2 \text{ kV/cm } (r.m.s.)$$

$\therefore$ Critical disruptive voltage per phase is

$$V_c = m_o\, g_o\, \delta\, r \log_e d/r \text{ kV}$$
$$= 0{\cdot}85 \times 21{\cdot}2 \times 0{\cdot}952 \times 1{\cdot}5 \times \log_e 200/1{\cdot}5 = 125{\cdot}9 \text{ kV}$$

Supply voltage per phase, $V = 220/\sqrt{3} = 127$ kV

Substituting the above values, we have corona loss as:

$$P = \frac{242{\cdot}2}{0{\cdot}952}(50 + 25) \times \sqrt{\frac{1{\cdot}5}{200}} \times (127 - 125{\cdot}9)^2 \times 10^{-5} \text{ kW/phase/km}$$
$$= \frac{242{\cdot}2}{0{\cdot}952} \times 75 \times 0{\cdot}0866 \times 1{\cdot}21 \times 10^{-5} \text{ kW/km/phase}$$
$$= 0{\cdot}01999 \text{ kW/km/phase}$$

$\therefore$ Total corona loss per km for three phases

$$= 3 \times 0{\cdot}01999 \text{ kW} = \mathbf{0{\cdot}05998 \text{ kW}}$$

Example 8.16. *A certain 3-phase equilateral transmission line has a total corona loss of 53 kW at 106 kV and a loss of 98 kW at 110·9 kV. What is the disruptive critical voltage? What is the corona loss at 113 kV?*

Solution.

The power loss due to corona for 3 phases is given by :

$$P = 3 \times \frac{242 \cdot 2\,(f+25)}{\delta} \sqrt{\frac{r}{d}}\,(V - V_c)^2 \times 10^{-5} \text{ kW/km}$$

As f, δ, r and d are the same for the two cases,

$$\therefore \quad P \propto (V - V_c)^2$$

For first case, $P = 53$ kW and $V = 106/\sqrt{3} = 61{\cdot}2$ kV

For second case, $P = 98$ kW and $V = 110{\cdot}9/\sqrt{3} = 64$ kV

$$\therefore \quad 53 \propto (61{\cdot}2 - V_c)^2 \qquad ...(i)$$

$$\text{and} \quad 98 \propto (64 - V_c)^2 \qquad ...(ii)$$

Dividing $[(ii)/(i)]$, we get,

$$\frac{98}{53} = \frac{(64 - V_c)^2}{(61 \cdot 2 - V_c)^2}$$

or $\quad V_c =$ **54 kV**

Let W kilowatt be the power loss at 113 kV.

$$\therefore \quad W \propto \left(\frac{113}{\sqrt{3}} - V_c\right)^2$$

$$\propto (65{\cdot}2 - 54)^2 \qquad ...(iii)$$

Dividing $[(iii)/(i)]$, we get,

$$\frac{W}{53} = \frac{(65 \cdot 2 - 54)^2}{(61 \cdot 2 - 54)^2}$$

$\therefore \quad W = (11{\cdot}2/7{\cdot}2)^2 \times 53 =$ **128 kW**

TUTORIAL PROBLEMS

1. Estimate the corona loss for a three-phase, 110 kV, 50 Hz, 150 km long transmission line consisting of three conductors each of 10 mm diameter and spaced 2·5 m apart in an equilateral triangle formation. The temperature of air is 30°C and the atmospheric pressure is 750 mm of mercury. Take irregularity factor as 0·85. Ionisation of air may be assumed to take place at a maximum voltage gradient of 30 kV/cm. **[316·8 kW]**
2. Taking the dielectric strength of air to be 30 kV/cm, calculate the disruptive critical voltage for a 3-phase line with conductors of 1 cm radius and spaced symmetrically 4 m apart. **[220 kV line voltage]**
3. A 3-phase, 220 kV, 50 Hz transmission line consists of 1·2 cm radius conductors spaced 2 m at the corners of an equilateral triangle. Calculate the corona loss per km of the line. The condition of the wire is smoothly weathered and the weather is fair with temperature of 20°C and barometric pressure of 72·2 cm of Hg. **[2·148 kW]**

8.15 Sag in Overhead Lines

While erecting an overhead line, it is very important that conductors are under safe tension. If the conductors are too much stretched between supports in a bid to save conductor material, the stress in the conductor may reach unsafe value and in certain cases the conductor may break due to excessive tension. In order to permit safe tension in the conductors, they are not fully stretched but are allowed to have a dip or sag.

The difference in level between points of supports and the lowest point on the conductor is called **sag.**

Fig. 8.23. (*i*) shows a conductor suspended between two equilevel supports *A* and *B*. The conductor is not fully stretched but is allowed to have a dip. The lowest point on the conductor is *O* and the sag is *S*. The following points may be noted :

Fig. 8.23

(*i*) When the conductor is suspended between two supports at the same level, it takes the shape of catenary. However, if the sag is very small compared with the span, then sag-span curve is like a parabola.

(*ii*) The tension at any point on the conductor acts tangentially. Thus tension T_O at the lowest point *O* acts horizontally as shown in Fig. 8.23. (*ii*).

(*iii*) The horizontal component of tension is constant throughout the length of the wire.

(*iv*) The tension at supports is approximately equal to the horizontal tension acting at any point on the wire. Thus if *T* is the tension at the support *B*, then $T = T_O$.

Conductor sag and tension. This is an important consideration in the mechanical design of overhead lines. The conductor sag should be kept to a minimum in order to reduce the conductor material required and to avoid extra pole height for sufficient clearance above ground level. It is also desirable that tension in the conductor should be low to avoid the mechanical failure of conductor and to permit the use of less strong supports. However, low conductor tension and minimum sag are not possible. It is because low sag means a tight wire and high tension, whereas a low tension means a loose wire and increased sag. Therefore, in actual practice, a compromise in made between the two.

8.16 Calculation of Sag

In an overhead line, the sag should be so adjusted that tension in the conductors is within safe limits. The tension is governed by conductor weight, effects of wind, ice loading and temperature variations. It is a standard practice to keep conductor tension less than 50% of its ultimate tensile strength *i.e.*, minimum factor of safety in respect of conductor tension should be 2. We shall now calculate sag and tension of a conductor when (*i*) supports are at equal levels and (*ii*) supports are at unequal levels.

(*i*) **When supports are at equal levels.** Consider a conductor between two equilevel supports *A* and *B* with *O* as the lowest point as shown in Fig. 8.24. It can be proved that lowest point will be at the mid-span.

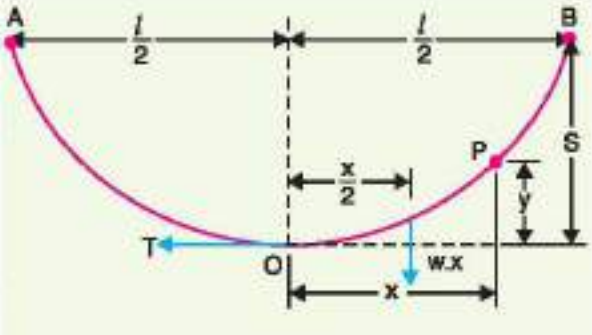


Fig. 8.24

Let

l = Length of span

w = Weight per unit length of conductor

T = Tension in the conductor.

Consider a point *P* on the conductor. Taking the lowest point *O* as the origin, let the co-ordinates of point *P* be *x* and *y*. Assuming that the curvature is so small that curved length is equal to its horizontal projection (*i.e.*, $OP = x$), the two forces acting on the portion *OP* of the conductor are :

(*a*) The weight wx of conductor acting at a distance $x/2$ from *O*.

(*b*) The tension *T* acting at *O*.

Equating the moments of above two forces about point O, we get,

$$Ty = wx \times \frac{x}{2}$$

or

$$y = \frac{wx^2}{2T}$$

The maximum dip (sag) is represented by the value of y at either of the supports A and B.

At support A, $x = l/2$ and $y = S$

$$\therefore \quad \text{Sag,} \quad S = \frac{w(l/2)^2}{2T} = \frac{wl^2}{8T}$$

(*ii*) **When supports are at unequal levels.** In hilly areas, we generally come across conductors suspended between supports at unequal levels. Fig. 8.25 shows a conductor suspended between two supports A and B which are at different levels. The lowest point on the conductor is O.

Let

l = Span length

h = Difference in levels between two supports

x_1 = Distance of support at lower level (*i.e.*, A) from O

x_2 = Distance of support at higher level (*i.e.* B) from O

T = Tension in the conductor

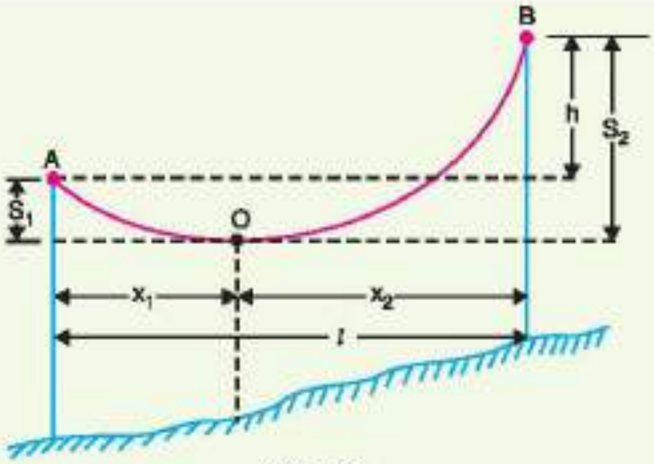


Fig. 8.25

If w is the weight per unit length of the conductor, then,

$$\text{Sag } S_1 = \frac{wx_1^{2*}}{2T}$$

and

$$\text{Sag } S_2 = \frac{wx_2^2}{2T}$$

Also $\quad x_1 + x_2 = l \quad$ ...(*i*)

$$* \; y = \frac{wx^2}{2T}$$

At support A, $x = x_1$ and $y = S_1$.

$$\therefore \quad S_1 = \frac{wx_1^2}{2T}$$

Now $$S_2 - S_1 = \frac{w}{2T}[x_2^2 - x_1^2] = \frac{w}{2T}(x_2 + x_1)(x_2 - x_1)$$

∴ $$S_2 - S_1 = \frac{w\,l}{2T}(x_2 - x_1) \qquad [\because x_1 + x_2 = l]$$

But $$S_2 - S_1 = h$$

∴ $$h = \frac{w\,l}{2T}(x_2 - x_1)$$

or $$x_2 - x_1 = \frac{2\,Th}{w\,l} \qquad ...(ii)$$

Solving exps. (*i*) and (*ii*), we get,

$$x_1 = \frac{l}{2} - \frac{T\,h}{w\,l}$$

$$x_2 = \frac{l}{2} + \frac{T\,h}{w\,l}$$

Having found x_1 and x_2, values of S_1 and S_2 can be easily calculated.

Effect of wind and ice loading. The above formulae for sag are true only in still air and at normal temperature when the conductor is acted by its weight only. However, in actual practice, a conductor may have ice coating and simultaneously subjected to wind pressure. The weight of ice acts vertically downwards *i.e.*, in the same direction as the weight of conductor. The force due to the wind is assumed to act horizontally *i.e.*, at right angle to the projected surface of the conductor. Hence, the total force on the conductor is the vector sum of horizontal and vertical forces as shown in Fig. 8.26 (*iii*).

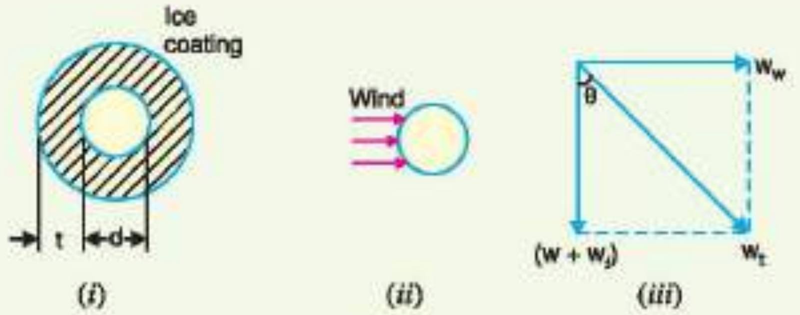


Fig. 8.26

Total weight of conductor per unit length is

$$w_t = \sqrt{(w + w_i)^2 + (w_w)^2}$$

where w = weight of conductor per unit length
= conductor material density × volume per unit length

w_i = weight of ice per unit length
= density of ice × volume of ice per unit length
= density of ice × $\frac{\pi}{4}$ $[(d + 2t)^2 - d^2] \times 1$
= density of ice × $\pi\, t\,(d + t)$*

w_w = wind force per unit length
= wind pressure per unit area × projected area per unit length
= wind pressure × $[(d + 2t) \times 1]$

* Volume of ice per unit length $= \frac{\pi}{4}[(d + t)^2 - d^2] \times 1 = \frac{\pi}{4}[4dt + 4t^2] = \pi t\,(d + t)$

When the conductor has wind and ice loading also, the following points may be noted :

(*i*) The conductor sets itself in a plane at an angle θ to the vertical where

$$\tan\theta = \frac{w_w}{w + w_i}$$

(*ii*) The sag in the conductor is given by :

$$S = \frac{w_t\, l^2}{2T}$$

Hence *S* represents the slant sag in a direction making an angle θ to the vertical. *If no specific mention is made in the problem, then slant slag is calculated by using the above formula.*

(*iii*) The vertical sag = $S\cos\theta$

Example 8.17. *A 132 kV transmission line has the following data :*

Wt. of conductor = 680 kg/km ; Length of span = 260 m

Ultimate strength = 3100 kg ; Safety factor = 2

Calculate the height above ground at which the conductor should be supported. Ground clearance required is 10 metres.

Solution.

Wt. of conductor/metre run, $w = 680/1000 = 0{\cdot}68$ kg

Working tension, $T = \dfrac{\text{Ultimate strength}}{\text{Safety factor}} = \dfrac{3100}{2} = 1550$ kg

Span length, $l = 260$ m

$$\therefore \quad \text{Sag} = \frac{w\,l^2}{8T} = \frac{0{\cdot}68 \times (260)^2}{8 \times 1550} = 3{\cdot}7 \text{ m}$$

∴ Conductor should be supported at a height of 10 + 3·7 = **13·7 m**

Example 8.18. *A transmission line has a span of 150 m between level supports. The conductor has a cross-sectional area of 2 cm². The tension in the conductor is 2000 kg. If the specific gravity of the conductor material is 9·9 gm/cm³ and wind pressure is 1·5 kg/m length, calculate the sag. What is the vertical sag?*

Solution.

Span length, $l = 150$ m; Working tension, $T = 2000$ kg

Wind force/m length of conductor, $w_w = 1{\cdot}5$ kg

Wt. of conductor/m length, w = Sp. Gravity × Volume of 1 m conductor

= 9·9 × 2 × 100 = 1980 gm = 1·98 kg

Total weight of 1 m length of conductor is

$$w_t = \sqrt{w^2 + w_w^2} = \sqrt{(1{\cdot}98)^2 + (1{\cdot}5)^2} = 2{\cdot}48 \text{ kg}$$

$$\therefore \quad \text{Sag, } S = \frac{w_t\, l^2}{8T} = \frac{2{\cdot}48 \times (150)^2}{8 \times 2000} = \mathbf{3{\cdot}48\ m}$$

This is the value of slant sag in a direction making an angle θ with the vertical. Referring to Fig. 8.27, the value of θ is given by :

$$\tan\theta = w_w/w = 1{\cdot}5/1{\cdot}98 = 0{\cdot}76$$

$$\therefore \quad \theta = \tan^{-1} 0{\cdot}76 = 37{\cdot}23°$$

$$\therefore \quad \text{Vertical sag} = S\cos\theta$$

$$= 3{\cdot}48 \times \cos 37{\cdot}23° = \mathbf{2{\cdot}77\ m}$$

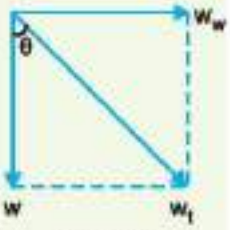


Fig. 8.27

Example 8.19. *A transmission line has a span of 200 metres between level supports. The conductor has a cross-sectional area of 1·29 cm², weighs 1170 kg/km and has a breaking stress of 4218 kg/cm². Calculate the sag for a safety factor of 5, allowing a wind pressure of 122 kg per square metre of projected area. What is the vertical sag?*

Solution.

Span length, $l = 200$ m

Wt. of conductor/m length, $w = 1170/1000 = 1{\cdot}17$ kg

Working tension, $^{*}T = 4218 \times 1{\cdot}29/5 = 1088$ kg

Diameter of conductor, $d = \sqrt{\dfrac{4 \times \text{area}}{\pi}} = \sqrt{\dfrac{4 \times 1{\cdot}29}{\pi}} = 1{\cdot}28$ cm

Wind force/m length, w_w = Pressure × projected area in m²

$= (122) \times (1{\cdot}28 \times 10^{-2} \times 1) = 1{\cdot}56$ kg

Total weight of conductor per metre length is

$$w_t = \sqrt{w^2 + w_w^2} = \sqrt{(1{\cdot}17)^2 + (1{\cdot}56)^2} = 1{\cdot}95 \text{ kg}$$

∴ Slant sag, $S = \dfrac{w_t\, l^2}{8T} = \dfrac{1{\cdot}95 \times (200)^2}{8 \times 1088} =$ **8·96 m**

The slant sag makes an angle θ with the vertical where value of θ is given by :

$$\theta = \tan^{-1}(w_w/w) = \tan^{-1}(1{\cdot}56/1{\cdot}17) = 53{\cdot}13°$$

∴ Vertical sag $= S\cos\theta = 8{\cdot}96 \times \cos 53{\cdot}13° =$ **5·37 m**

Example 8.20. *A transmission line has a span of 275 m between level supports. The conductor has an effective diameter of 1·96 cm and weighs 0·865 kg/m. Its ultimate strength is 8060 kg. If the conductor has ice coating of radial thickness 1·27 cm and is subjected to a wind pressure of 3·9 gm/cm² of projected area, calculate sag for a safety factor of 2. Weight of 1 c.c. of ice is 0·91 gm.*

Solution.

Span length, $l = 275$ m ; Wt. of conductor/m length, $w = 0{\cdot}865$ kg

Conductor diameter, $d = 1{\cdot}96$ cm ; Ice coating thickness, $t = 1{\cdot}27$ cm

Working tension, $T = 8060/2 = 4030$ kg

Volume of ice per metre (*i.e.*, 100 cm) length of conductor

$= \pi\, t\,(d + t) \times 100 \text{ cm}^3$

$= \pi \times 1{\cdot}27 \times (1{\cdot}96 + 1{\cdot}27) \times 100 = 1288 \text{ cm}^3$

Weight of ice per metre length of conductor is

$$w_i = 0{\cdot}91 \times 1288 = 1172 \text{ gm} = 1{\cdot}172 \text{ kg}$$

Wind force/m length of conductor is

w_w = [Pressure] × [(d + 2t) × 100]

$= [3{\cdot}9] \times (1{\cdot}96 + 2 \times 1{\cdot}27) \times 100$ gm $= 1755$ gm $= 1{\cdot}755$ kg

Total weight of conductor per metre length of conductor is

$$w_t = \sqrt{(w + w_i)^2 + (w_w)^2}$$

$$= \sqrt{(0{\cdot}865 + 1{\cdot}172)^2 + (1{\cdot}755)^2} = 2{\cdot}688 \text{ kg}$$

---

* Working stress $= \dfrac{\text{Ultimate Strength}}{\text{Safety factor}} = \dfrac{4218}{5}$

∴ Working Tension, T = Working stress × conductor area = 4218 × 1·29/5

$$\therefore \quad \text{Sag} = \frac{w_t l^2}{8T} = \frac{2 \cdot 688 \times (275)^2}{8 \times 4030} = \mathbf{6 \cdot 3 \ m}$$

Example 8.21. *A transmission line has a span of 214 metres between level supports. The conductors have a cross-sectional area of 3·225 cm². Calculate the factor of safety under the following conditions :*

Vertical sag = 2·35 m ; *Wind pressure = 1·5 kg/m run*
Breaking stress = 2540 kg/cm² ; *Wt. of conductor = 1·125 kg/m run*

Solution.

Here, $l = 214$ m ; $w = 1{\cdot}125$ kg ; $w_w = 1{\cdot}5$ kg

Total weight of one metre length of conductor is

$$w_t = \sqrt{w^2 + w_w^2} = \sqrt{(1 \cdot 125)^2 + (1 \cdot 5)^2} = 1 \cdot 875 \text{ kg}$$

If f is the factor of safety, then,

$$\text{Working tension, } T = \frac{\text{Breaking stress} \times \text{conductor area}}{\text{safety factor}} = 2540 \times 3 \cdot 225/f = 8191/f \text{ kg}$$

$$\text{Slant Sag, } S = \frac{\text{Vertical sag}}{{}^*\cos\theta} = \frac{2 \cdot 35 \times 1 \cdot 875}{1 \cdot 125} = 3 \cdot 92 \text{ m}$$

Now $$S = \frac{w_t l^2}{8T}$$

or $$T = \frac{w_t l^2}{8S}$$

$$\therefore \quad \frac{8191}{f} = \frac{1 \cdot 875 \times (214)^2}{8 \times 3 \cdot 92}$$

or Safety factor, $$f = \frac{8191 \times 8 \times 3 \cdot 92}{1 \cdot 875 \times (214)^2} = \mathbf{3}$$

Example 8.22. *An overhead line has a span of 150 m between level supports. The conductor has a cross-sectional area of 2 cm². The ultimate strength is 5000 kg/cm² and safety factor is 5. The specific gravity of the material is 8·9 gm/cc. The wind pressure is 1·5 kg/m. Calculate the height of the conductor above the ground level at which it should be supported if a minimum clearance of 7 m is to be left between the ground and the conductor.*

Solution.

Span length, $l = 150$ m ; Wind force/m run, $w_w = 1{\cdot}5$ kg

Wt. of conductor/m run, w = conductor area × 100 cm × sp. gravity
= 2 × 100 × 8·9 = 1780 gm = 1·78 kg

Working tension, $T = 5000 \times 2/5 = 2000$ kg

Total weight of one metre length of conductor is

$$w_t = \sqrt{w^2 + w_w^2} = \sqrt{(1 \cdot 78)^2 + (1 \cdot 5)^2} = 2 \cdot 33 \text{ kg}$$

$$\text{Slant sag, } S = \frac{w_t l^2}{8T} = \frac{2 \cdot 33 \times (150)^2}{8 \times 2000} = 3 \cdot 28 \text{ m}$$

$$\text{Vertical sag} = S\cos\theta = 3 \cdot 28 \times w/w_t = 3 \cdot 28 \times 1 \cdot 78/2 \cdot 33 = 2 \cdot 5 \text{ m}$$

Conductor should be supported at a height of 7 + 2·5 = **9·5 m**

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* The slant sag makes an angle θ with the vertical.

$$\therefore \quad \cos\theta = w/w_t = 1 \cdot 125/1 \cdot 875$$

Example 8.23. *The towers of height 30 m and 90 m respectively support a transmission line conductor at water crossing. The horizontal distance betwen the towers is 500 m. If the tension in the conductor is 1600 kg, find the minimum clearance of the conductor and water and clearance mid-way between the supports. Weight of conductor is 1·5 kg/m. Bases of the towers can be considered to be at water level.*

Solution. Fig. 8.28 shows the conductor suspended between two supports *A* and *B* at different levels with *O* as the lowest point on the conductor.

Here, $l = 500$ m ; $w = 1·5$ kg ; $T = 1600$ kg.

Difference in levels between supports, $h = 90 - 30 = 60$ m. Let the lowest point *O* of the conductor be at a distance x_1 from the support at lower level (*i.e.*, support *A*) and at a distance x_2 from the support at higher level (*i.e.*, support *B*).

Obviously, $x_1 + x_2 = 500$ m ...(*i*)

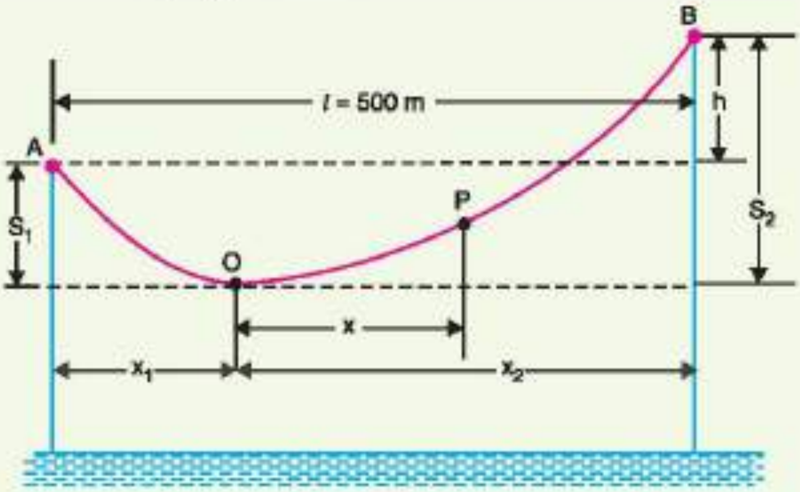

Fig. 8.28

Now Sag $S_1 = \frac{w x_1^2}{2T}$ and Sag $S_2 = \frac{w x_2^2}{2T}$

$$\therefore \quad h = S_2 - S_1 = \frac{w x_2^2}{2T} - \frac{w x_1^2}{2T}$$

or $$60 = \frac{w}{2T}(x_2 + x_1)(x_2 - x_1)$$

$$\therefore \quad x_2 - x_1 = \frac{60 \times 2 \times 1600}{1·5 \times 500} = 256 \text{ m} \quad ...(ii)$$

Solving exps. (*i*) and (*ii*), we get, $x_1 = 122$ m; $x_2 = 378$ m

Now, $$S_1 = \frac{w x_1^2}{2T} = \frac{1·5 \times (122)^2}{2 \times 1600} = 7 \text{ m}$$

Clearance of the lowest point *O* from water level

$= 30 - 7 =$ **23 m**

Let the mid-point *P* be at a distance *x* from the lowest point *O*.

Clearly, $x = 250 - x_1 = 250 - 122 = 128$ m

Sag at mid-point *P*, $$S_{mid} = \frac{w x^2}{2T} = \frac{1·5 \times (128)^2}{2 \times 1600} = 7·68 \text{ m}$$

Clearance of mid-point P from water level

$$= 23 + 7{\cdot}68 = \mathbf{30{\cdot}68\ m}$$

Example 8.24. *An overhead transmission line conductor having a parabolic configuration weighs 1·925 kg per metre of length. The area of X-section of the conductor is 2·2 cm² and the ultimate strength is 8000 kg/cm². The supports are 600 m apart having 15 m difference of levels. Calculate the sag from the taller of the two supports which must be allowed so that the factor of safety shall be 5. Assume that ice load is 1 kg per metre run and there is no wind pressure.*

Solution. Fig. 8.29. shows the conductor suspended between two supports at A and B at different levels with O as the lowest point on the conductor.

Here,
$$l = 600\text{ m}\ ;\ w_i = 1\text{ kg}\ ;\ h = 15\text{m}$$
$$w = 1{\cdot}925\text{ kg}\ ;\ T = 8000 \times 2{\cdot}2/5 = 3520\text{ kg}$$

Total weight of 1 m length of conductor is

$$w_t = w + w_i = 1{\cdot}925 + 1 = 2{\cdot}925\text{ kg}$$

Let the lowest point O of the conductor be at a distance x_1 from the support at lower level (*i.e.*, A) and at a distance x_2 from the support at higher level (*i.e.*, B).

Clearly, $$x_1 + x_2 = 600\text{ m} \quad ...(i)$$

Now, $$h = S_2 - S_1 = \frac{w_t\, x_2^2}{2T} - \frac{w_t\, x_1^2}{2T}$$

or $$15 = \frac{w_t}{2T}(x_2 + x_1)(x_2 - x_1)$$

$\therefore$ $$x_2 - x_1 = \frac{2 \times 15 \times 3520}{2{\cdot}925 \times 600} = 60\text{ m} \quad ...(ii)$$

Solving exps. (*i*) and (*ii*), we have, $x_1 = 270$ m and $x_2 = 330$ m

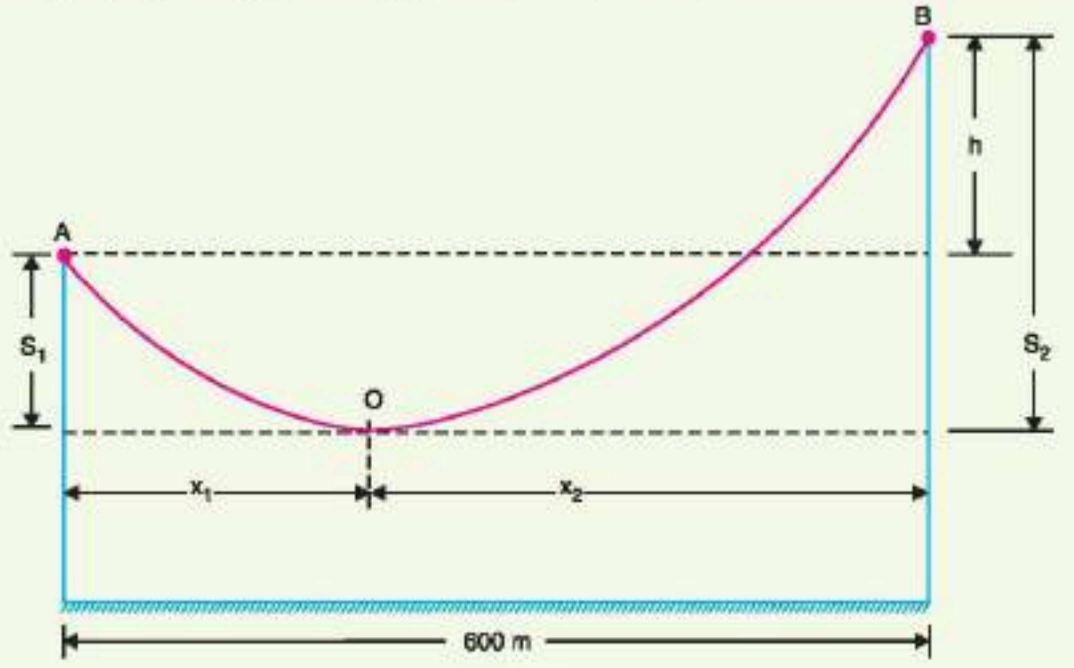

Fig. 8.29

Sag from the taller of the two towers is

$$S_2 = \frac{w_t\, x_2^2}{2T} = \frac{2{\cdot}925 \times (330)^2}{2 \times 3520} = \mathbf{45{\cdot}24\ m}$$

Example 8.25. *An overhead transmission line at a river crossing is supported from two towers at heights of 40 m and 90 m above water level, the horizontal distance between the towers being 400 m. If the maximum allowable tension is 2000 kg, find the clearance between the conductor and water at a point mid-way between the towers. Weight of conductor is 1 kg/m.*

Solution. Fig. 8.30 shows the whole arrangement.

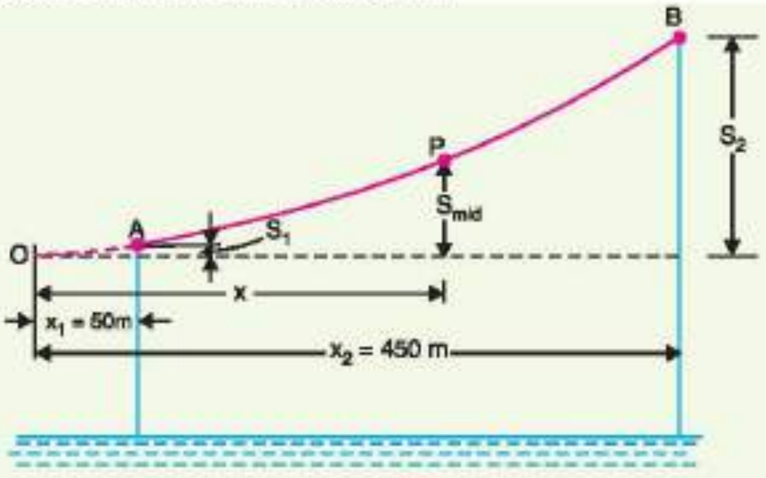

Fig. 8.30

Here, $h = 90 - 40 = 50$ m; $l = 400$ m

$T = 2000$ kg; $w = 1$ kg/m

Obviously, $x_1 + x_2 = 400$ m ...(*i*)

Now $h = S_2 - S_1 = \frac{wx_2^2}{2T} - \frac{wx_1^2}{2T}$

or $50 = \frac{w}{2T}(x_2 + x_1)(x_2 - x_1)$

$\therefore$ $x_2 - x_1 = \frac{50 \times 2 \times 2000}{400} = 500$ m ...(*ii*)

Solving exps. (*i*) and (*ii*), we get, $x_2 = 450$ m and $x_1 = -50$ m

Now x_2 is the distance of higher support B from the lowest point O on the conductor, whereas x_1 is that of lower support A. As the span is 400 m, therefore, point A lies on the same side of O as B (see Fig. 8.30).

Horizontal distance of mid-point P from lowest point O is

x = Distance of A from O + 400/2 = 50 + 200 = 250 m

$\therefore$ Sag at point P, $S_{mid} = \frac{wx^2}{2T} = \frac{1 \times (250)^2}{2 \times 2000} = 15.6$ m

Now Sag $S_2 = \frac{wx_2^2}{2T} = \frac{1 \times (450)^2}{2 \times 2000} = 50.6$ m

Height of point B above mid-point P

$= S_2 - S_{mid} = 50.6 - 15.6 = 35$ m

$\therefore$ Clearance of mid-point P above water level

$= 90 - 35 =$ **55 m**

Example 8.26. *A transmission line over a hillside where the gradient is 1 : 20, is supported by two 22 m high towers with a distance of 300 m between them. The lowest conductor is fixed 2 m below the top of each tower. Find the clearance of the conductor from the ground. Given that conductor weighs 1 kg/m and the allowable tension is 1500 kg.*

Solution. The conductors are supported between towers *AD* and *BE* over a hillside having gradient of 1 : 20 as shown in Fig. 8.31. The lowest point on the conductor is *O* and $\sin \theta = 1/20$.

Effective height of each tower (*AD* or *BE*)

$$= 22 - 2 = 20 \text{ m}$$

Vertical distance between towers is

$$h = EC = DE \sin \theta = 300 \times 1/20 = 15 \text{ m}$$

Horizontal distance between two towers is

$$DC = \sqrt{DE^2 - EC^2} = \sqrt{(300)^2 - (15)^2} \simeq 300 \text{ m}$$

or $$x_1 + x_2 = 300 \text{ m} \quad \text{...(i)}$$

Now $$h = \frac{w x_2^2}{2T} - \frac{w x_1^2}{2T} = \frac{w}{2T}(x_2 + x_1)(x_2 - x_1)$$

or $$x_2 - x_1 = \frac{2Th}{w(x_2 + x_1)} = \frac{2 \times 1500 \times 15}{1 \times 300} = 150 \text{ m} \quad \text{...(ii)}$$

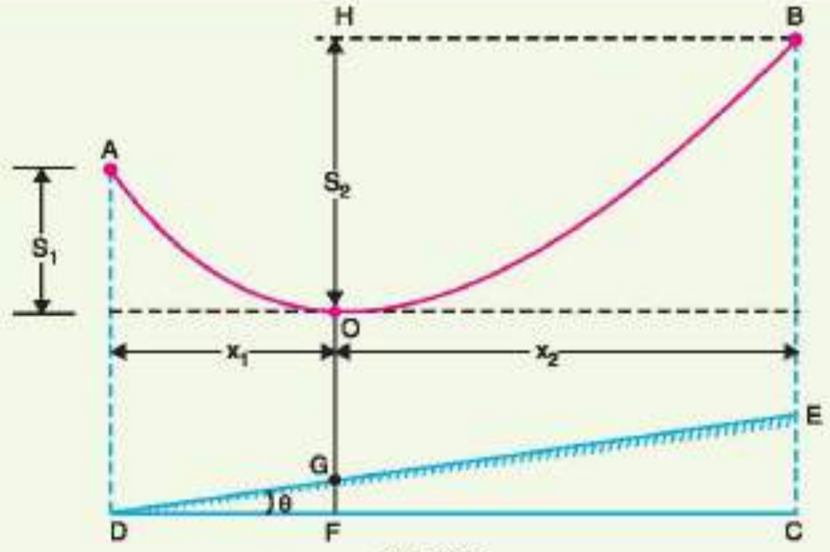

Fig. 8.31

Solving exps. (*i*) and (*ii*), we have, $x_1 = 75$ m and $x_2 = 225$ m

$$\text{Sag } S_2 = \frac{w x_2^2}{2T} = \frac{1 \times (225)^2}{2 \times 1500} = 16{\cdot}87 \text{ m}$$

Now $$BC = BE + EC = 20 + 15 = 35 \text{ m}$$

Clearance of the lowest point *O* from the ground is

$$OG = HF - S_2 - GF$$
$$= BC - S_2 - GF \quad (\because BC = HF)$$
$$[\text{Now } GF = x_1 \tan \theta = 75 \times 0{\cdot}05 = 3{\cdot}75 \text{ m}]$$
$$= 35 - 16{\cdot}87 - 3{\cdot}75 = \mathbf{14{\cdot}38 \text{ m}}$$

Example 8.27. *A transmission tower on a level ground gives a minimum clearance of 8 metres for its lowest conductor with a sag of 10 m for a span of 300 m. If the same tower is to be used over a slope of 1 in 15, find the minimum ground clearance obtained for the same span, same conductor and same weather conditions.*

Solution. On level ground

$$\text{Sag, } S = \frac{w l^2}{8T}$$

$$\therefore \quad \frac{w}{T} = \frac{8S}{l^2} = \frac{8 \times 10}{(300)^2} = \frac{8}{9 \times 10^3}$$

$$\text{Height of tower} = \text{Sag} + \text{Clearance} = 10 + 8 = 18 \text{ m}$$

On sloping ground. The conductors are supported between towers *AD* and *BE* over a sloping ground having a gradient 1 in 15 as shown in Fig. 8.32. The height of each tower (*AD* or *BE*) is 18 m.

Vertical distance between the two towers is

$$h = EC = {}^{*}DE \sin\theta = 300 \times 1/15 = 20 \text{ m}$$

Now $$x_1 + x_2 = 300 \text{ m} \quad \ldots(i)$$

Also $$h = \frac{w x_2^2}{2T} - \frac{w x_1^2}{2T} = \frac{w}{2T}(x_2 + x_1)(x_2 - x_1)$$

$$\therefore \quad x_2 - x_1 = \frac{2Th}{w(x_2 + x_1)} = \frac{2 \times 9 \times 10^3 \times 20}{8 \times 300} = 150 \text{ m} \quad \ldots(ii)$$

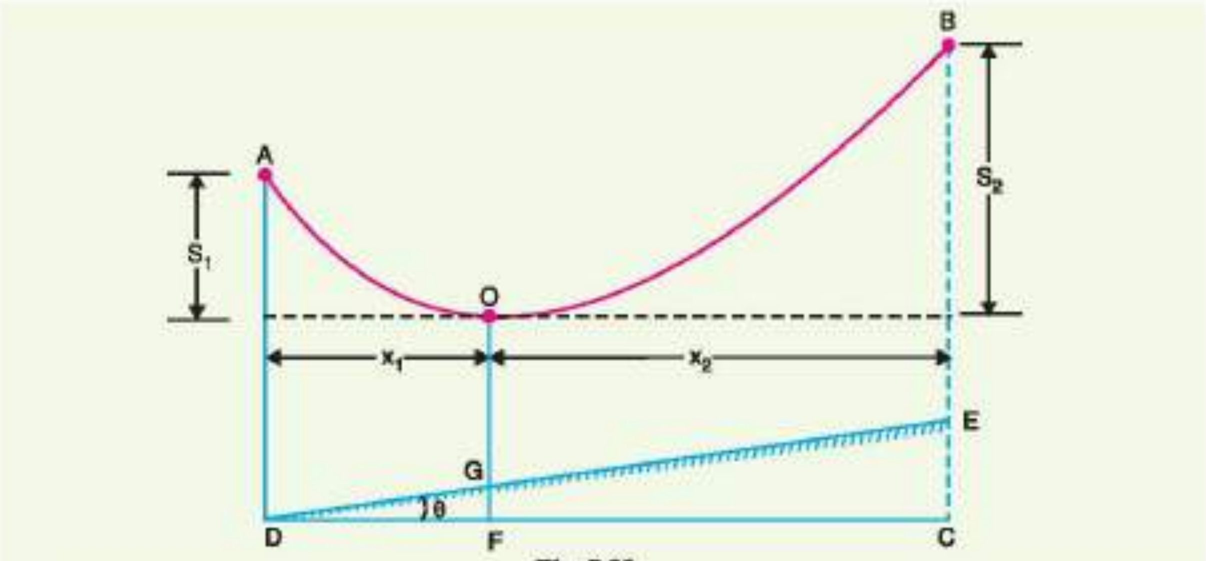

Fig. 8.32

Solving exps. (*i*) and (*ii*), we have, $x_1 = 75$ m and $x_2 = 225$ m

Now $$S_1 = \frac{w x_1^2}{2T} = \frac{8 \times (75)^2}{2 \times 9 \times 10^3} = 2{\cdot}5 \text{ m}$$

$$S_2 = \frac{w x_2^2}{2T} = \frac{8 \times (225)^2}{2 \times 9 \times 10^3} = 22{\cdot}5 \text{ m}$$

Clearance of point *O* from the ground is

$$OG = BC - S_2 - GF = 38 - 22{\cdot}5 - 5 = 10{\cdot}5 \text{ m}$$

$$[\because GF = x_1 \tan\theta = 75 \times 1/15 = 5 \text{m}]$$

Since *O* is the origin, the equation of slope of ground is given by :

$$y = mx + A$$

Here $$m = 1/15 \text{ and } A = OG = -10{\cdot}5 \text{ m}$$

$$\therefore \quad y = \frac{x}{15} - 10{\cdot}5$$

$\therefore$ Clearance *C* from the ground at any point *x* is

* $DE \simeq DC = 300$ m

$$C = \text{Equation of conductor curve} - y = \left(\frac{w\,x^2}{2T}\right) - \left(\frac{x}{15} - 10\cdot 5\right)$$

$$= \frac{8x^2}{2\times 9\times 10^3} - \left(\frac{x}{15} - 10\cdot 5\right)$$

$$\therefore \quad C = \frac{x^2}{2250} - \frac{x}{15} + 10\cdot 5$$

Clearance will be minimum when $dC/dx = 0$ *i.e.*,

$$\frac{d}{dx}\left[\frac{x^2}{2250} - \frac{x}{15} + 10\cdot 5\right] = 0$$

or $$\frac{2x}{2250} - \frac{1}{15} = 0$$

or $$x = \frac{1}{15}\times\frac{2250}{2} = \mathbf{75\ m}$$

i.e., minimum clearance will be at a point 75 m from *O*.

$$\text{Minimum clearance} = \frac{x^2}{2250} - \frac{x}{15} + 10\cdot 5 = (75)^2/2250 - 75/15 + 10\cdot 5$$

$$= 2\cdot 5 - 5 + 10\cdot 5 = \mathbf{8\ m}$$

TUTORIAL PROBLEMS

1. A transmission line conductor is supported from two towers at heights of 70 m above water level. The horizontal distance between the towers is 300 m. If the tension in the conductors is 1500 kg, find the clearance at a point mid-way between the towers. The size of the conductor is 0·9 cm^2 and density of conductor material is 8·9 gm/cm^3. **[64 m]**
2. An overhead line has a span of 260 m, the weight of the line conductor is 0·68 kg per metre run. Calculate the maximum sag in the line. The maximum allowable tension in the line is 1550 kg. **[3·7 m]**
3. A transmission line has a span of 150 m between level supports. The cross-sectional area of the conductor is 1·25 cm^2 and weighs 100 kg per 100 m. The breaking stress is 4220 kg/cm^2. Calculate the factor of safety if the sag of the line is 3·5 m. Assume a maximum wind pressure of 100 kg per sq. metre. **[4]**
4. A transmission line has a span of 150 m between the level supports. The conductor has a cross-sectional area of 2 cm^2. The ultimate strength is 5000 kg/cm^2. The specific gravity of the material is 8·9 gm/cm^3. If the wind pressure is 1·5 kg/m length of conductor, calculate the sag at the centre of the conductor if factor of safety is 5. **[3·28 m]**
5. A transmission line has a span of 250 m between supports, the supports being at the same level. The conductor has a cross-sectional area of 1·29 cm^2. The ultimate strength is 4220 kg/cm^2 and factor of safety is 2. The wind pressure is 40 kg/cm^2. Calculate the height of the conductor above ground level at which it should be supported if a minimum clearance of 7m is to be kept between the ground and the conductor. **[10·24 m]**
6. A transmission line has a span of 150 m between level supports. The conductor has a cross-sectional area of 2 cm^2. The ultimate strength is 5000 kg/cm^2. The specific gravity of the material is 8·9 gm/cm^3. If the wind pressure is 1·5 kg/m length of the conductor, calculate the sag if factor of safety is 5. **[3·5 m]**
7. Two towers of height 40 m and 30 m respectively support a transmission line conductor at water crossing. The horizontal distance between the towers is 300 m. If the tension in the conductor is 1590 kg, find the clearance of the conductor at a point mid-way between the supports. Weight of conductor is 0·8 kg/m. Bases of the towers can be considered to be at the water level. **[59 m]**
8. An overhead transmission line at a river crossing is supported from two towers at heights of 50 m and 100 m above the water level. The horizontal distance between the towers is 400 m. If the maximum allowable tension is 1800 kg and the conductor weighs 1 kg/m, find the clearance between the conductor and water at a point mid-way between the supports. **[63·8 m]**

8.17 Some Mechanical Principles

Mechanical factors of safety to be used in transmission line design should depend to some extent on the importance of continuity of operation in the line under consideration. In general, the strength of the line should be such as to provide against the worst *probable* weather conditions. We now discuss some important points in the mechanical design of overhead transmission lines.

(*i*) **Tower height :** Tower height depends upon the length of span. With long spans, relatively few towers are required but they must be tall and correspondingly costly. It is not usually possible to determine the tower height and span length on the basis of direct construction costs because the lightning hazards increase greatly as the height of the conductors above ground is increased. This is one reason that horizontal spacing is favoured inspite of the wider right of way required.

(*ii*) **Conductor clearance to ground :** The conductor clearance to ground at the time of greatest sag should not be less than some specified distance (usually between 6 and 12 m), depending on the voltage, on the nature of the country and on the local laws. The greatest sag may occur on the hottest day of summer on account of the expansion of the wire or it may occur in winter owing to the formation of a heavy coating of ice on the wires. Special provisions must be made for melting ice from the power lines.

(*iii*) **Sag and tension :** When laying overhead transmission lines, it is necessary to allow a reasonable factor of safety in respect of the tension to which the conductor is subjected. The tension is governed by the effects of wind, ice loading and temperature variations. The relationship between tension and sag is dependent on the loading conditions and temperature variations. For example, the tension increases when the temperature decreases and there is a corresponding decrease in the sag. Icing-up of the line and wind loading will cause stretching of the conductor by an amount dependent on the line tension.

In planning the sag, tension and clearance to ground of a given span, a maximum stress is selected. It is then aimed to have this stress developed at the worst probable weather conditions (*i.e.* minimum expected temperature, maximum ice loading and maximum wind). Wind loading increases the sag in the direction of resultant loading but decreases the vertical component. Therefore, in clearance calculations, the effect of wind should not be included unless horizontal clearance is important.

(*iv*) **Stringing charts :** For use in the field work of stringing the conductors, temperature-sag and temperature-tension charts are plotted for the given conductor and loading conditions. Such curves are called stringing charts (see Fig. 8.33). These charts are very helpful while stringing overhead lines.

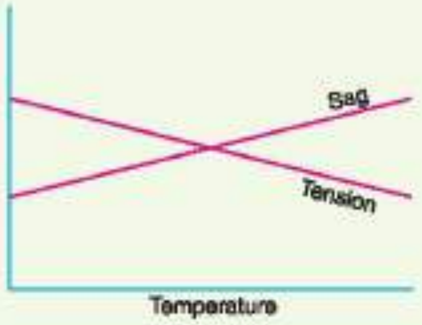

Fig. 8.33

(*v*) **Conductor spacing :** Spacing of conductors should be such so as to provide safety against flash-over when the wires are swinging in the wind. The proper spacing is a function of span length, voltage and weather conditions. The use of horizontal spacing eliminates the danger caused by unequal ice loading. Small wires or wires of light material are subjected to more swinging by the wind than heavy conductors. Therefore, light wires should be given greater spacings.

(*vi*) **Conductor vibration :** Wind exerts pressure on the exposed surface of the conductor. If the wind velocity is small, the swinging of conductors is harmless provided the clearance is sufficiently large so that conductors do not approach within the sparking distance of each other. A completely different type of vibration, called *dancing*, is caused by the action of fairly strong wind on a wire covered with ice, when the ice coating happens to take a form which makes a good air-foil section. Then the whole span may sail up like a kite until it reaches the limit of its slack, stops with a jerk and falls or sails back. The harmful effects of these vibrations occur at the clamps or supports where the conductor suffers fatigue and breaks eventually. In order to protect the conductors, dampers are used.

SELF-TEST

1. Fill in the blanks by inserting appropriate words/figures.

(*i*) Cross-arms are used on poles or towers to provide .... to the insulators.
(*ii*) The most commonly used material for insulators of overhead lines is ....
(*iii*) The potential across the various discs of suspension string is different because of .... capacitance.
(*iv*) In a string of suspension insulators, the maximum voltage appears across the unit ...... to the conductor.
(*v*) If the string efficiency is 100%, it means that ....
(*vi*) If shunt capacitance is reduced, then string efficiency is ....
(*vii*) If the spacing between the conductors is increased, then corona effect is ....
(*viii*) If sag in an overhead line increases, tension in the line ....
(*ix*) By using a guard ring, string efficiency is ....
(*x*) Shunt capacitance in suspension insulators can be decreased by increasing the distance of .... from ....

2. Pick up the correct words/figures from the brackets and fill in the blanks.

(*i*) The insulator is so designed that it should fail only by .... (*flash-over, puncture*)
(*ii*) Suspension type insulators are used for voltages beyond .... (*33 kV, 400 V, 11 kV*)
(*iii*) In a string of suspension insulators, if the unit nearest to the conductor breaks down, then other units will .... (*also breakdown, remain intact*)
(*iv*) A shorter string has .... string efficiency than a larger one. (*less, more*)
(*v*) Corona effect is .... pronounced in stormy weather as compared to fair weather. (*more, less*)
(*vi*) If the conductor size is increased, the corona effect is .... (*increased, decreased*)
(*vii*) The longer the cross arm, the .... the string efficiency. (*greater, lesser*)
(*viii*) The discs of the strain insulators are used in .... plane. (*vertical, horizontal*)
(*ix*) Sag is provided in overhead lines so that .... (*Safe tension is not exceeded, repair can be done*)
(*x*) When an insulator breaks down by puncture, it is .... damaged. (*permanently, only partially*)

ANSWERS

1. (*i*) support (*ii*) porcelain (*iii*) shunt (*iv*) nearest (*v*) potential across each disc is the same (*vi*) increased (*vii*) reduced (*viii*) decreases (*ix*) increased (*x*) conductor, tower.
2. (*i*) flash-over (*ii*) 33 kV (*iii*) also breakdown (*iv*) more (*v*) more (*vi*) decreased (*vii*) greater (*viii*) vertical (*ix*) safe tension is not exceeded (*x*) permanently.

CHAPTER REVIEW TOPICS

1. Name the important components of an overhead transmission line.
2. Discuss the various conductor materials used for overhead lines. What are their relative advantages and disadvantages ?
3. Discuss the various types of line supports.
4. Why are insulators used with overhead lines ? Discuss the desirable properties of insulators.

5. Discuss the advantages and disadvantages of (*i*) pin-type insulators (*ii*) suspension type insulators.
6. Explain how the electrical breakdown can occur in an insulator.
7. What is a strain insulator and where is it used ? Give a sketch to show its location.
8. Give reasons for unequal potential distribution over a string of suspension insulators.
9. Define and explain string efficiency. Can its value be equal to 100% ?
10. Show that in a string of suspension insulators, the disc nearest to the conductor has the highest voltage across it.
11. Explain various methods of improving string efficiency.
12. What is corona? What are the factors which affect corona ?
13. Discuss the advantages and disadvantages of corona.
14. Explain the following terms with reference to corona :
    (*i*) Critical disruptive voltage
    (*ii*) Visual critical voltage
    (*iii*) Power loss due to corona
15. Describe the various methods for reducing corona effect in an overhead transmission line.
16. What is a sag in overhead lines ? Discuss the disadvantages of providing too small or too large sag on a line.
17. Deduce an approximate expression for sag in overhead lines when
    (*i*) supports are at equal levels
    (*ii*) supports are at unequal levels.

DISCUSSION QUESTIONS

1. What is the need for stranding the conductors ?
2. Is sag a necessity or an evil ? Discuss.
3. String efficiency for a d.c. system is 100% ? Discuss.
4. Can string efficiency in an a.c. system be 100% ?
5. Why are suspension insulators preferred for high voltage power transmission ?
6. Give reasons for the following :
    (*i*) A.C.S.R. conductors are preferred for transmission and distribution lines.
    (*ii*) Conductors are not fully stretched between supports.

CONTENTS

CHAPTER

10

Performance of Transmission Lines

Introduction

The important considerations in the design and operation of a transmission line are the determination of voltage drop, line losses and efficiency of transmission. These values are greatly influenced by the line constants *R*, *L* and *C* of the transmission line. For instance, the voltage drop in the line depends upon the values of above three line constants. Similarly, the resistance of transmission line conductors is the most important cause of power loss in the line and determines the transmission efficiency. In this chapter, we shall develop formulas by which we can calculate voltage regulation, line losses and efficiency of transmission lines. These formulas are important for two principal reasons. Firstly, they provide an opportunity to understand the effects of the parameters of the line on bus voltages and the flow of power. Secondly, they help in developing an overall understanding of what is occuring on electric power system.

CONTENTS

10.1 Classification of Overhead Transmission Lines

A transmission line has *three constants R, L and C distributed uniformly along the whole length of the line. The resistance and inductance form the series impedance. The capacitance existing between conductors for 1-phase line or from a conductor to neutral for a 3-phase line forms a shunt path throughout the length of the line. Therefore, capacitance effects introduce complications in transmission line calculations. Depending upon the manner in which capacitance is taken into account, the overhead transmission lines are classified as :

(i) *Short transmission lines.* When the length of an overhead transmission line is upto about 50 km and the line voltage is comparatively low (< 20 kV), it is usually considered as a short transmission line. Due to smaller length and lower voltage, the capacitance effects are small and hence can be neglected. Therefore, while studying the performance of a short transmisison line, only resistance and inductance of the line are taken into account.

(ii) *Medium transmission lines.* When the length of an overhead transmission line is about 50-150 km and the line voltage is moderatly high (>20 kV < 100 kV), it is considered as a medium transmission line. Due to sufficient length and voltage of the line, the capacitance effects are taken into account. For purposes of calculations, the distributed capacitance of the line is divided and lumped in the form of condensers shunted across the line at one or more points.

(iii) *Long transmission lines.* When the length of an overhead transmission line is more than 150 km and line voltage is very high (> 100 kV), it is considered as a long transmission line. For the treatment of such a line, the line constants are considered uniformly distributed over the whole length of the line and rigorous methods are employed for solution.

It may be emphasised here that exact solution of any tranmission line must consider the fact that the constants of the line are not lumped but are distributed unfiormly throughout the length of the line. However, reasonable accuracy can be obtained by considering these constants as lumped for short and medium transmission lines.

10.2 Important Terms

While studying the performance of a transmission line, it is desirable to determine its voltage regulation and transmission efficiency. We shall explain these two terms in turn.

(i) **Voltage regulation.** When a transmission line is carrying current, there is a voltage drop in the line due to resistance and inductance of the line. The result is that receiving end voltage (V_R) of the line is generally less than the sending end voltage (V_S). This voltage drop ($V_S - V_R$) in the line is expressed as a percentage of receiving end voltage V_R and is called voltage regulation.

*The difference in voltage at the receiving end of a transmission line **between conditions of no load and full load is called* **voltage regulation** and is expressed as a percentage of the receiving end voltage.

---

* There is also a fourth constant *i.e.*, shunt conductance. It represents the conductance between conductors or between conductor and ground and accounts for the leakage current at the insulators. It is very small in case of overhead lines and may be assumed zero.

** At no load, there is no drop in the line so that at no load, $V_R = V_S$. However, at full load, there is a voltage drop in the line so that receiving end voltage is V_R.

∴ Difference in voltage at receiving end between no load and full load

$$= V_S - V_R$$

Mathematically,

$$\%\text{ age Voltage regulation} = \frac{V_S - V_R}{V_R} \times 100$$

Obviously, it is desirable that the voltage regulation of a transmission line should be low *i.e.*, the increase in load current should make very little difference in the receiving end voltage.

(*ii*) **Transmission efficiency.** The power obtained at the receiving end of a transmission line is generally less than the sending end power due to losses in the line resistance.

The ratio of receiving end power to the sending end power of a transmission line is known as the **transmission efficiency** *of the line i.e.*

$$\%\text{ age Transmission efficiency, } \eta_T = \frac{\text{Receiving end power}}{\text{Sending end power}} \times 100$$

$$= \frac{V_R I_R \cos\phi_R}{V_S I_S \cos\phi_S} \times 100$$

where V_R, I_R and $\cos\phi_R$ are the receiving end voltage, current and power factor while V_S, I_S and $\cos\phi_S$ are the corresponding values at the sending end.

10.3 Performance of Single Phase Short Transmission Lines

As stated earlier, the effects of line capacitance are neglected for a short transmission line. Therefore, while studying the performance of such a line, only resistance and inductance of the line are taken into account. The equivalent circuit of a single phase short transmission line is shown in Fig. 10.1 (*i*). Here, the total line resistance and inductance are shown as concentrated or lumped instead of being distributed. The circuit is a simple a.c. series circuit.

Let

I = load current

R = loop resistance *i.e.*, resistance of both conductors

X_L = loop reactance

V_R = receiving end voltage

$\cos\phi_R$ = receiving end power factor (lagging)

V_S = sending end voltage

$\cos\phi_S$ = sending end power factor

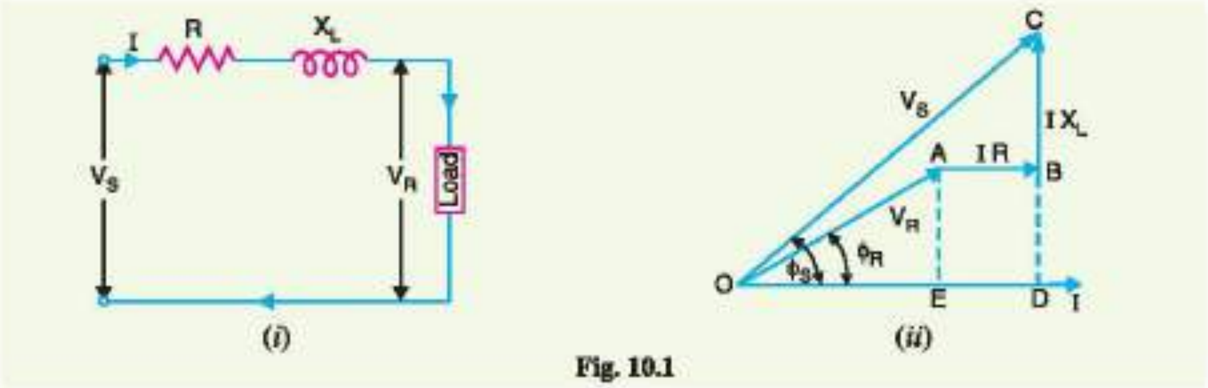

Fig. 10.1

The *phasor diagram of the line for lagging load power factor is shown in Fig. 10.1 (*ii*). From the right angled traingle *ODC*, we get,

* **Phasor diagram.** Current I is taken as the reference phasor. OA represents the receiving end voltage V_R leading I by ϕ_R. AB represents the drop IR in phase with I. BC represents the inductive drop IX_L and leads I by 90°. OC represents the sending end voltage V_S and leads I by ϕ_S.

$$(OC)^2 = (OD)^2 + (DC)^2$$

or

$$V_S^2 = (OE + ED)^2 + (DB + BC)^2$$

$$= (V_R \cos \phi_R + IR)^2 + (V_R \sin \phi_R + IX_L)^2$$

$\therefore$

$$V_S = \sqrt{(V_R \cos \phi_R + IR)^2 + (V_R \sin \phi_R + IX_L)^2}$$

(*i*) %age Voltage regulation $= \dfrac{V_S - V_R}{V_R} \times 100$

(*ii*) Sending end *p.f.*, $\cos \phi_S = \dfrac{OD}{OC} = \dfrac{V_R \cos \phi_R + IR}{V_S}$

(*iii*) Power delivered $= V_R I_R \cos \phi_R$

Line losses $= I^2 R$

Power sent out $= V_R I_R \cos \phi_R + I^2 R$

%age Transmission efficiency $= \dfrac{\text{Power delivered}}{\text{Power sent out}} \times 100$

$$= \frac{V_R I_R \cos \phi_R}{V_R I_R \cos \phi_R + I^2 R} \times 100$$

An approximate expression for the sending end voltage V_S can be obtained as follows. Draw perpendicular from *B* and *C* on *OA* produced as shown in Fig. 10.2. Then *OC* is *nearly* equal to *OF* *i.e.*,

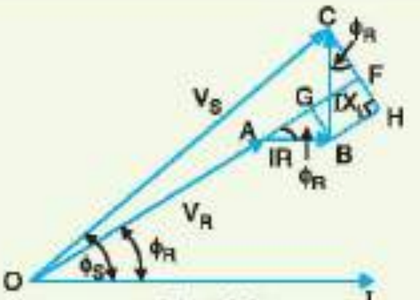

Fig. 10.2

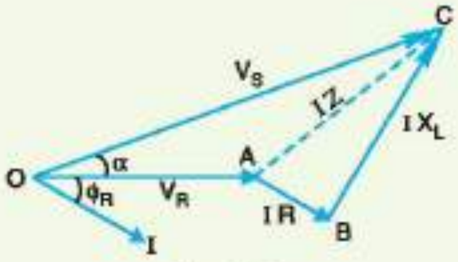

Fig. 10.3

$$OC = OF = OA + AF = OA + AG + GF$$

$$= OA + AG + BH$$

$\therefore$

$$V_S = V_R + IR \cos \phi_R + IX_L \sin \phi_R$$

Solution in complex notation. It is often convenient and profitable to make the line calculations in complex notation.

Taking $\vec{V_R}$ as the reference phasor, draw the phasor diagram as shown in Fig 10.3. It is clear that $\vec{V_S}$ is the phasor sum of $\vec{V_R}$ and $\vec{I}\,\vec{Z}$.

$$^*\vec{V_R} = V_R + j0$$

$$\vec{I} = \vec{I} \angle -\phi_R = I(\cos \phi_R - j \sin \phi_R)$$

$$\vec{Z} = R + jX_L$$

$\therefore$

$$\vec{V_S} = \vec{V_R} + \vec{I}\,\vec{Z}$$

$$= (V_R + j0) + I(\cos \phi_R - j \sin \phi_R)(R + jX_L)$$

* Phasors are shown by arrows and their magnitudes without arrow. Thus $\vec{V_R}$ is the receiving end voltage phasor, whereas V_R is its magnitude.

$$= (V_R + IR\cos\phi_R + IX_L\sin\phi_R) + j\,(IX_L\cos\phi_R - IR\sin\phi_R)$$

$$\therefore \quad V_S = \sqrt{(V_R + IR\cos\phi_R + IX_L\sin\phi_R)^2 + (IX_L\cos\phi_R - IR\sin\phi_R)^2}$$

The second term under the root is quite small and can be neglected with reasonable accuracy. Therefore, approximate expression for V_S becomes :

$$V_S = V_R + IR\cos\phi_R + IX_L\sin\phi_R$$

The following poins may be noted :

(*i*) The approximate formula for $V_S (= V_R + IR\cos\phi_R + IX_L\sin\phi_R)$ gives fairly correct results for lagging power factors. However, appreciable error is caused for leading power factors. Therefore, approximate expression for V_S should be used for lagging p.f. only.

(*ii*) The solution in complex notation is in more presentable form.

10.4 Three-Phase Short Transmission Lines

For reasons associated with economy, transmission of electric power is done by 3-phase system. This system may be regarded as consisting of three single phase units, each wire transmitting one-third of the total power. As a matter of convenience, we generally analyse 3-phase system by considering

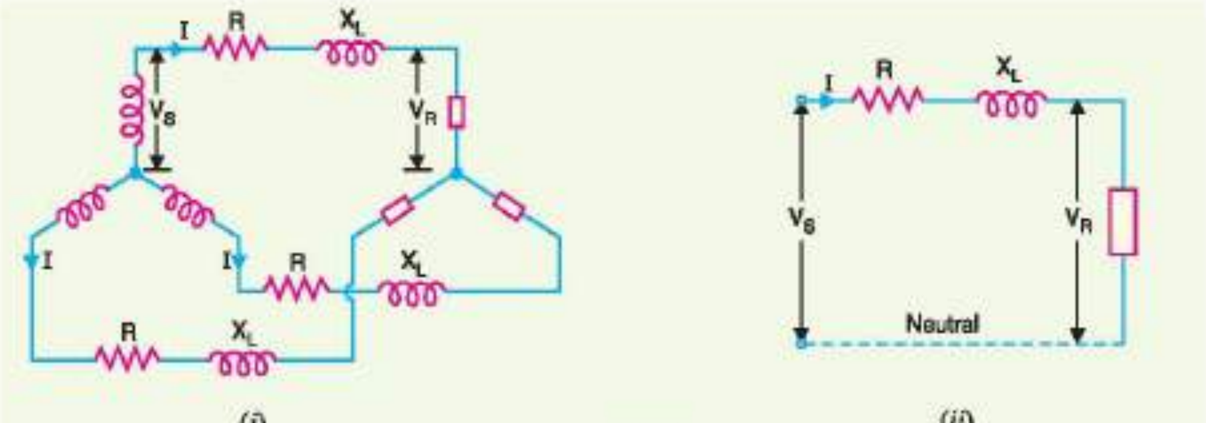

Fig. 10.4

*one phase only. Therefore, expression for regulation, efficiency etc. derived for a single phase line can also be applied to a 3-phase system. Since only one phase is considered, phase values of 3-phase system should be taken. Thus, V_S and V_R are the phase voltages, whereas R and X_L are the resistance and inductive reactance per phase respectively.

Fig. 10.4 (*i*) shows a *Y*-connected generator supplying a balanced *Y*-connected load through a transmission line. Each conductor has a resistance of $R\,\Omega$ and inductive reactance of $X_L\,\Omega$. Fig. 10.4 (*ii*) shows one phase separately. The calculations can now be made in the same way as for a single phase line.

10.5 Effect of Load p.f. on Regulation and Efficiency

The regulation and efficiency of a transmission line depend to a considerable extent upon the power factor of the load.

1. Effect on regulation. The expression for voltage regulation of a short transmission line is given by :

$$\%\text{age Voltage regulation} = \frac{IR\cos\phi_R + IX_L\sin\phi_R}{V_R} \times 100 \qquad (\textit{for lagging p.f.})$$

* As similar conditions prevail in the three phases.

$$\%\text{age Voltage regulation} = \frac{I R \cos \phi_R - I X_L \sin \phi_R}{V_R} \times 100 \qquad \textit{(for leading p.f.)}$$

The following conclusions can be drawn from the above expressions :

(*i*) When the load p.f. is lagging or unity or such leading that $I R \cos \phi_R > I X_L \sin \phi_R$, then voltage regulation is positive *i.e.*, receiving end voltage V_R will be less than the sending end voltage V_S.

(*ii*) For a given V_R and I, the voltage regulation of the line increases with the decrease in p.f. for lagging loads.

(*iii*) When the load p.f. is leading to this extent that $I X_L \sin \phi_R > I R \cos \phi_R$, then voltage regulation is negative *i.e.* the receiving end voltage V_R is more than the sending end voltage V_S.

(*iv*) For a given V_R and I, the voltage regulation of the line decreases with the decrease in p.f. for leading loads.

2. Effect on transmission efficiency. The power delivered to the load depends upon the power factor.

$$P = V_R{}^* I \cos \phi_R \quad \text{(For 1-phase line)}$$

$$\therefore \quad I = \frac{P}{V_R \cos \phi_R}$$

$$P = 3\, V_R I \cos \phi_R \quad \text{(For 3-phase line)}$$

$$\therefore \quad I = \frac{P}{3 V_R \cos \phi_R}$$

It is clear that in each case, for a given amount of power to be transmitted (P) and receiving end voltage (V_R), the load current I is inversely proportional to the load p.f. $\cos \phi_R$. Consequently, with the decrease in load p.f., the load current and hence the line losses are increased. This leads to the conclusion that transmission efficiency of a line decreases with the decrease in load p.f. and *vice-versa*.

Power Factor Meter


Power Factor Regulator

Example 10.1. *A single phase overhead transmission line delivers 1100 kW at 33 kV at 0·8 p.f. lagging. The total resistance and inductive reactance of the line are 10 Ω and 15 Ω respectively. Determine : (i) sending end voltage (ii) sending end power factor and (iii) transmission efficiency.*

Solution.

Load power factor, $\cos \phi_R = 0{\cdot}8$ lagging

Total line impedance, $\vec{Z} = R + jX_L = 10 + j\,15$

---

* $I_R = I_S = I$

Receiving end voltage, $V_R = 33$ kV = 33,000 V

$$\therefore \quad \text{Line current, } I = \frac{kW \times 10^3}{V_R \cos\phi_R} = \frac{1100 \times 10^3}{33{,}000 \times 0 \cdot 8} = 41 \cdot 67 \text{ A}$$

$$\text{As } \cos\phi_R = 0 \cdot 8 \quad \therefore \quad \sin\phi_R = 0 \cdot 6$$

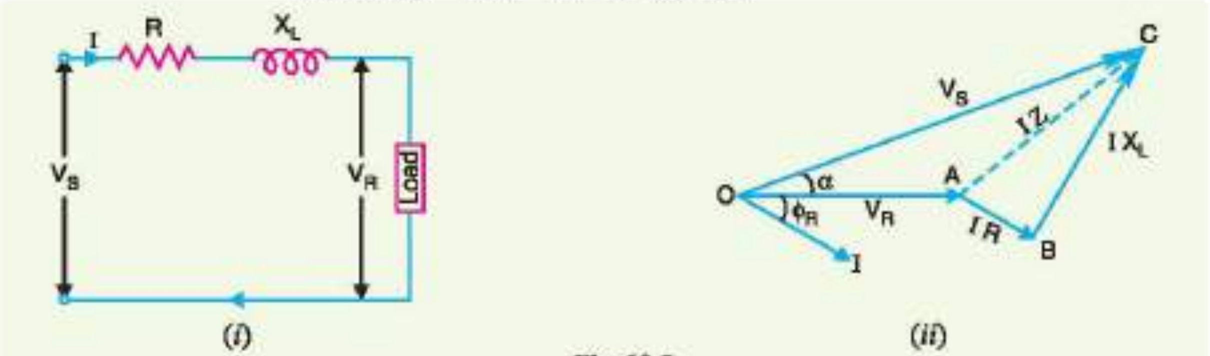

Fig. 10.5

The equivalent circuit and phasor diagram of the line are shown in Figs. 10.5 (*i*) and 10.5 (*ii*) respectively. Taking receiving end voltage $\vec{V_R}$ as the reference phasor,

$$\vec{V_R} = V_R + j0 = 33000 \text{ V}$$

$$\vec{I} = I(\cos\phi_R - j\sin\phi_R)$$

$$= 41 \cdot 67\,(0 \cdot 8 - j0 \cdot 6) = 33 \cdot 33 - j25$$

(*i*) Sending end voltage, $\vec{V_S} = \vec{V_R} + \vec{I}\,\vec{Z}$

$$= 33{,}000 + (33 \cdot 33 - j25 \cdot 0)\,(10 + j15)$$

$$= 33{,}000 + 333 \cdot 3 - j250 + j500 + 375$$

$$= 33{,}708 \cdot 3 + j250$$

$$\therefore \quad \text{Magnitude of } V_S = \sqrt{(33{,}708 \cdot 3)^2 + (250)^2} = \textbf{33,709 V}$$

(*ii*) Angle between $\vec{V_S}$ and $\vec{V_R}$ is

$$\alpha = \tan^{-1}\frac{250}{33{,}708 \cdot 3} = \tan^{-1} 0 \cdot 0074 = 0 \cdot 42^\circ$$

∴ Sending end power factor angle is

$$\phi_S = \phi_R + \alpha = 36 \cdot 87^\circ + 0 \cdot 42^\circ = 37 \cdot 29^\circ$$

∴ Sending end p.f., $\cos\phi_S = \cos 37 \cdot 29^\circ =$ **0·7956 lagging**

(*iii*) Line losses $= I^2R = (41 \cdot 67)^2 \times 10 = 17{,}364 \text{ W} = 17 \cdot 364$ kW

Output delivered = 1100 kW

Power sent = 1100 + 17·364 = 1117·364 kW

$$\therefore \quad \text{Transmission efficiency} = \frac{\text{Power delivered}}{\text{Power sent}} \times 100 = \frac{1100}{1117 \cdot 364} \times 100 = \textbf{98·44\%}$$

Note. V_S and ϕ_S can also be calculated as follows :

$$V_S = V_R + IR\cos\phi_R + IX_L\sin\phi_R \text{ (approximately)}$$

$$= 33{,}000 + 41 \cdot 67 \times 10 \times 0 \cdot 8 + 41 \cdot 67 \times 15 \times 0 \cdot 6$$

$$= 33{,}000 + 333 \cdot 36 + 375 \cdot 03$$

= 33708·39 V which is approximately the same as above

$$\cos\phi_S = \frac{V_R\cos\phi_R + IR}{V_S} = \frac{33{,}000 \times 0 \cdot 8 + 41 \cdot 67 \times 10}{33{,}708 \cdot 39} = \frac{26{,}816 \cdot 7}{33{,}708 \cdot 39}$$

$$= 0 \cdot 7958$$

As stated earlier, this method gives fairly correct results for lagging p.f. The reader will find that this method is used in the solution of some numericals.

Example 10.2. *What is the maximum length in km for a 1-phase transmission line having copper conductor of 0·775 cm² cross-section over which 200 kW at unity power factor and at 3300V are to be delivered ? The efficiecny of transmission is 90%. Take specific resistance as 1.725 μ Ω cm.*

Solution.

$$\text{Receiving end power} = 200 \text{ kW} = 2{,}00{,}000 \text{ W}$$

$$\text{Transmission efficiency} = 0{\cdot}9$$

$$\therefore \quad \text{Sending end power} = \frac{2{,}00{,}000}{0{\cdot}9} = 2{,}22{,}222 \text{ W}$$

$$\therefore \quad \text{Line losses} = 2{,}22{,}222 - 2{,}00{,}000 = 22{,}222 \text{ W}$$

$$\text{Line current, } I = \frac{200 \times 10^3}{3{,}300 \times 1} = 60{\cdot}6 \text{ A}$$

Let $R\,\Omega$ be the resistance of one conductor.

$$\text{Line losses} = 2\, I^2 R$$

or

$$22{,}222 = 2\,(60{\cdot}6)^2 \times R$$

$$\therefore \quad R = \frac{22{,}222}{2 \times (60 \cdot 6)^2} = 3{\cdot}025\ \Omega$$

Now,

$$R = \rho\, l/a$$

$$\therefore \quad l = \frac{Ra}{\rho} = \frac{3{\cdot}025 \times 0{\cdot}775}{1{\cdot}725 \times 10^{-6}} = 1{\cdot}36{\times}10^6 \text{ cm} = \mathbf{13{\cdot}6\ km}$$

Example 10.3. *An overhead 3-phase transmission line delivers 5000 kW at 22 kV at 0·8 p.f. lagging. The resistance and reactance of each conductor is 4 Ω and 6 Ω respectively. Determine : (i) sending end voltage (ii) percentage regulation (iii) transmission efficiency.*

Solution.

$$\text{Load power factor, } \cos\phi_R = 0{\cdot}8 \text{ lagging}$$

$$\text{Receiving end voltage/phase,* } V_R = 22{,}000/\sqrt{3} = 12{,}700 \text{ V}$$

$$\text{Impedance/phase, } \quad \vec{Z} = 4 + j\,6$$

$$\text{Line current, } \quad I = \frac{5000 \times 10^3}{3 \times 12700 \times 0{\cdot}8} = 164 \text{ A}$$

$$\text{As } \cos\phi_R = 0{\cdot}8 \quad \therefore \sin\phi_R = 0{\cdot}6$$

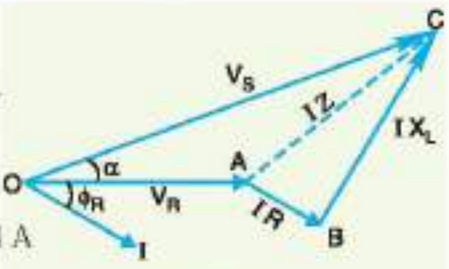

Fig. 10.6

Taking $\vec{V_R}$ as the reference phasor (see Fig. 10.6),

$$\vec{V_R} = V_R + j\,0 = 12700 \text{ V}$$

$$\vec{I} = I(\cos\phi_R - j\sin\phi_R) = 164\,(0{\cdot}8 - j\,0{\cdot}6) = 131{\cdot}2 - j\,98{\cdot}4$$

(*i*) Sending end voltage per phase is

$$\begin{aligned}\vec{V_S} &= \vec{V_R} + \vec{I}\,\vec{Z} = 12700 + (131{\cdot}2 - j\,98{\cdot}4)\,(4 + j\,6)\\ &= 12700 + 524{\cdot}8 + j\,787{\cdot}2 - j\,393{\cdot}6 + 590{\cdot}4\\ &= 13815{\cdot}2 + j\,393{\cdot}6\end{aligned}$$

$$\text{Magnitude of } V_S = \sqrt{(13815 \cdot 2)^2 + (393 \cdot 6)^2} = 13820{\cdot}8 \text{ V}$$

* If not mentioned in the problem, star-connection is understood.

$$\text{Line value of } V_S = \sqrt{3} \times 13820{\cdot}8 = 23938 \text{ V} = \mathbf{23{\cdot}938 \text{ kV}}$$

(*ii*) $$\% \text{age Regulation} = \frac{V_S - V_R}{V_R} \times 100 = \frac{13820 \cdot 8 - 12700}{12700} \times 100 = \mathbf{8{\cdot}825\%}$$

(*iii*) $$\text{Line losses} = 3I^2R = 3 \times (164)^2 \times 4 = 3{,}22{,}752 \text{ W} = 322{\cdot}752 \text{ kW}$$

$$\therefore \quad \text{Transmission efficiency} = \frac{5000}{5000 + 322 \cdot 752} \times 100 = \mathbf{93{\cdot}94\%}$$

Example 10.4. *Estimate the distance over which a load of 15000 kW at a p.f. 0·8 lagging can be delivered by a 3-phase transmission line having conductors each of resistance 1 Ω per kilometre. The voltage at the receiving end is to be 132 kV and the loss in the transmission is to be 5%.*

Solution.

$$\text{Line current,} \quad I = \frac{\text{Power delivered}}{\sqrt{3} \times \text{line voltage} \times \text{power factor}} = \frac{15000 \times 10^3}{\sqrt{3} \times 132 \times 10^3 \times 0 \cdot 8} = 82 \text{ A}$$

$$\text{Line losses} = 5\% \text{ of power delivered} = 0{\cdot}05 \times 15000 = 750 \text{ kW}$$

Let R Ω be the resistance of one conductor.

$$\text{Line losses} = 3I^2R$$

or $$750 \times 10^3 = 3 \times (82)^2 \times R$$

$$\therefore \quad R = \frac{750 \times 10^3}{3 \times (82)^2} = 37{\cdot}18 \ \Omega$$

Resistance of each conductor per km is 1 Ω (*given*).

$\therefore$ Length of line = **37·18 km**

Example 10.5. *A 3-phase line delivers 3600 kW at a p.f. 0·8 lagging to a load. If the sending end voltage is 33 kV, determine (i) the receiving end voltage (ii) line current (iii) transmission efficiency. The resistance and reactance of each conductor are 5·31 Ω and 5·54 Ω respectively.*

Solution.

Resistance of each conductor, $R = 5{\cdot}31 \ \Omega$

Reactance of each conductor, $X_L = 5{\cdot}54 \ \Omega$

Load power factor, $\cos \phi_R = 0{\cdot}8$ (*lagging*)

Sending end voltage/phase, $V_S = 33{,}000/\sqrt{3} = 19{,}052 \text{ V}$

Let V_R be the phase voltage at the receiving end.

$$\text{Line current,} \quad I = \frac{\text{Power delivered / phase}}{V_R \times \cos \phi_R} = \frac{1200 \times 10^3}{V_R \times 0 \cdot 8}$$

$$= \frac{150 \times 10^5}{V_R} \quad \text{...(i)}$$

(*i*) Using approximate expression for V_S, we get,

$$V_S = V_R + IR \cos \phi_R + IX_L \sin \phi_R$$

or $$19{,}052 = V_R + \frac{15 \times 10^5}{V_R} \times 5{\cdot}31 \times 0{\cdot}8 + \frac{15 \times 10^5}{V_R} \times 5{\cdot}54 \times 0{\cdot}6$$

or $$V_R^2 - 19{,}052 \ V_R + 1{,}13{,}58{,}000 = 0$$

Solving this equation, we get, $V_R = 18{,}435$ V

$\therefore$ Line voltage at the receiving end $= \sqrt{3} \times 18{,}435 = 31{,}930 \text{ V} = \mathbf{31{\cdot}93 \text{ kV}}$

(*ii*) Line current, $I = \frac{15 \times 10^5}{V_R} = \frac{15 \times 10^5}{18{,}435} = \mathbf{81 \cdot 36\ A}$

(*iii*) Line losses, $= 3 I^2 R = 3 \times (81 \cdot 36)^2 \times 5 \cdot 31 = 1{,}05{,}447\ W = 105 \cdot 447\ kW$

∴ Transmission efficiency $= \frac{3600}{3600 + 105 \cdot 447} \times 100 = \mathbf{97 \cdot 15\%}$

Example 10.6. *A short 3-ϕ transmission line with an impedance of (6 + j 8) Ω per phase has sending and receiving end voltages of 120 kV and 110 kV respectively for some receiving end load at a p.f. of 0·9 lagging. Determine (i) power output and (ii) sending end power factor.*

Solution.

Resistance of each conductor, $R = 6\ \Omega$

Reactance of each conductor, $X_L = 8\ \Omega$

Load power factor, $\cos \phi_R = 0 \cdot 9$ lagging

Receiving end voltage/phase, $V_R = 110 \times 10^3/\sqrt{3} = 63508\ V$

Sending end voltage/phase, $V_S = 120 \times 10^3/\sqrt{3} = 69282\ V$

Let I be the load current. Using approximate expression for V_S, we get,

$$V_S = V_R + IR \cos \phi_R + I X_L \sin \phi_R$$

or $$69282 = 63508 + I \times 6 \times 0 \cdot 9 + I \times 8 \times 0 \cdot 435$$

or $$8 \cdot 88\ I = 5774$$

or $$I = 5774/8 \cdot 88 = 650 \cdot 2\ A$$

(*i*) $$\text{Power output} = \frac{3\ V_R\ I \cos \phi_R}{1000}\ kW = \frac{3 \times 63508 \times 650 \cdot 2 \times 0 \cdot 9}{1000} = \mathbf{1{,}11{,}490\ kW}$$

(*ii*) $$\text{Sending end p.f.}, \cos \phi_S = \frac{V_R \cos \phi_R + IR}{V_S} = \frac{63508 \times 0 \cdot 9 + 650 \cdot 2 \times 6}{69282} = \mathbf{0 \cdot 88\ lag}$$

Example 10.7. *An 11 kV, 3-phase transmission line has a resistance of 1·5 Ω and reactance of 4 Ω per phase. Calculate the percentage regulation and efficiency of the line when a total load of 5000 kVA at 0.8 lagging power factor is supplied at 11 kV at the distant end.*

Solution.

Resistance of each conductor, $R = 1 \cdot 5\ \Omega$

Reactance of each conductor, $X_L = 4\ \Omega$

Receiving end voltage/phase, $V_R = \frac{11 \times 10^3}{\sqrt{3}} = 6351\ V$

Load power factor, $\cos \phi_R = 0 \cdot 8$ lagging

Load current, $$I = \frac{\text{Power delivered in kVA} \times 1000}{3 \times V_R} = \frac{5000 \times 1000}{3 \times 6351} = 262 \cdot 43 A$$

Using the approximate expression for V_S (sending end voltage per phase), we get,

$$V_S = V_R + IR \cos \phi_R + I X_L \sin \phi_R$$
$$= 6351 + 262 \cdot 43 \times 1 \cdot 5 \times 0 \cdot 8 + 262 \cdot 43 \times 4 \times 0 \cdot 6 = 7295 \cdot 8\ V$$

$$\%\ \text{regulation} = \frac{V_S - V_R}{V_R} \times 100 = \frac{7295 \cdot 8 - 6351}{6351} \times 100 = \mathbf{14 \cdot 88\%}$$

$$\text{Line losses} = 3 I^2 R = 3 \times (262 \cdot 43)^2 \times 1 \cdot 5 = 310 \times 10^3\ W = 310\ kW$$

$$\text{Output power} = 5000 \times 0{\cdot}8 = 4000 \text{ kW}$$

$$\text{Input power} = \text{Ouput power + line losses} = 4000 + 310 = 4310 \text{ kW}$$

$$\text{Transmission efficiency} = \frac{\text{Output power}}{\text{Input power}} \times 100 = \frac{4000}{4310} \times 100 = \mathbf{92{\cdot}8\%}$$

Example 10.8. *A 3-phase, 50 Hz, 16 km long overhead line supplies 1000 kW at 11kV, 0·8 p.f. lagging. The line resistance is 0·03 Ω per phase per km and line inductance is 0·7 mH per phase per km. Calculate the sending end voltage, voltage regulation and efficiency of transmission.*

Solution.

Resistance of each conductor, $R = 0{\cdot}03 \times 16 = 0{\cdot}48\ \Omega$

Reactance of each conductor, $X_L = 2\pi f L \times 16 = 2\pi \times 50 \times 0{\cdot}7 \times 10^{-3} \times 16 = 3{\cdot}52\ \Omega$

Receiving end voltage/phase, $V_R = \dfrac{11 \times 10^3}{\sqrt{3}} = 6351 \text{ V}$

Load power factor, $\cos \phi_R = 0{\cdot}8$ lagging

$$\text{Line current, } I = \frac{1000 \times 10^3}{3 \times V_R \times \cos\phi} = \frac{1000 \times 10^3}{3 \times 6351 \times 0{\cdot}8} = 65{\cdot}6 \text{A}$$

$$\text{Sending end voltage/phase, } V_S = V_R + I R \cos\phi_R + I X_L \sin\phi_R$$

$$= 6351 + 65{\cdot}6 \times 0{\cdot}48 \times 0{\cdot}8 + 65{\cdot}6 \times 3{\cdot}52 \times 0{\cdot}6 = 6515 \text{ V}$$

$$\therefore \quad \text{\%age Voltage regulation} = \frac{V_S - V_R}{V_R} \times 100 = \frac{6515 - 6351}{6351} \times 100 = \mathbf{2{\cdot}58\%}$$

$$\text{Line losses} = 3 I^2 R = 3 \times (65{\cdot}6)^2 \times 0{\cdot}48 = 6{\cdot}2 \times 10^3 \text{ W} = 6{\cdot}2 \text{ kW}$$

$$\text{Input power} = \text{Output power + Line losses} = 1000 + 6{\cdot}2 = 1006{\cdot}2 \text{ kW}$$

$$\therefore \quad \text{Transmission efficiency} = \frac{\text{Output power}}{\text{Input power}} \times 100 = \frac{1000}{1006{\cdot}2} \times 100 = \mathbf{99{\cdot}38\%}$$

Example 10.9. *A 3-phase load of 2000 kVA, 0·8 p.f. is supplied at 6·6 kV, 50 Hz by means of a 33 kV transmission line 20 km long and 33/6·6 kV step-down transfomer. The resistance and reactance of each conductor are 0·4 Ω and 0·5 Ω per km respectively. The resistance and reactance of transformer primary are 7.5 Ω and 13.2 Ω, while those of secondary are 0.35 Ω and 0.65 Ω respectively. Find the voltage necessary at the sending end of transmission line when 6.6 kV is maintained at the receiving end. Determine also the sending end power factor and transmission efficiency.*

Solution. Fig. 10.7 shows the single diagram of the transmission system. Here, the voltage drop will be due to the impedance of transmission line and also due to the impedance of transformer.

Resistance of each conductor $= 20 \times 0{\cdot}4 = 8\ \Omega$

Reactance of each conductor $= 20 \times 0{\cdot}5 = 10\ \Omega$

Let us transfer the impedance of transformer secondary to high tension side *i.e.*, 33 kV side.

Equivalent resistance of transformer referred to 33 kV side

$$= \text{Primary resistance} + 0{\cdot}35\ (33/6{\cdot}6)^2$$

$$= 7{\cdot}5 + 8{\cdot}75 = 16{\cdot}25\ \Omega$$

Equivalent reactance of transformer referred to 33 kV side

$$= \text{Primary reactance} + 0{\cdot}65\ (33/6{\cdot}6)^2$$

$$= 13{\cdot}2 + 16{\cdot}25 = 29{\cdot}45\ \Omega$$

Total resistance of line and transformer is

$$R = 8 + 16{\cdot}25 = 24{\cdot}25\ \Omega$$

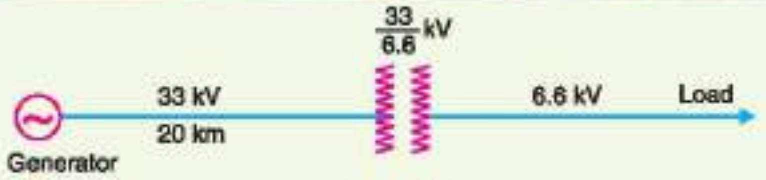

Fig. 10.7

Total reactance of line and transformer is

$$X_L = 10 + 29{\cdot}45 = 39{\cdot}45\ \Omega$$

Receiving end voltage per phase is

$$V_R = 33{,}000/\sqrt{3} = 19052\ \text{V}$$

Line current, $$I = \frac{2000 \times 10^3}{\sqrt{3} \times 33000} = 35\ \text{A}$$

Using the approximate expression for sending end voltage V_S per phase,

$$V_S = V_R + IR\cos\phi_R + IX_L\sin\phi_R$$
$$= 19052 + 35 \times 24{\cdot}25 \times 0{\cdot}8 + 35 \times 39{\cdot}45 \times 0{\cdot}6$$
$$= 19052 + 679 + 828 = 20559\ \text{V} = 20{\cdot}559\ \text{kV}$$

Sending end line voltage $= \sqrt{3} \times 20{\cdot}559$ kV $=$ **35·6 kV**

Sending end p.f., $\cos\phi_S = \dfrac{V_R\cos\phi_R + IR}{V_S} = \dfrac{19052 \times 0{\cdot}8 + 35 \times 24{\cdot}25}{20559} =$ **0·7826 lag**

Line losses $= \dfrac{3I^2R}{1000}\ \text{kW} = \dfrac{3 \times (35)^2 \times 24{\cdot}25}{1000} = 89{\cdot}12\ \text{kW}$

Output power $= 2000\ \text{kVA} \times 0{\cdot}8 = 1600\ \text{kW}$

∴ Transmission efficiency $= \dfrac{1600}{1600 + 89{\cdot}12} \times 100 =$ **94·72%**

TUTORIAL PROBLEMS

1. A single phase overhead transmission line delivers 4000 kW at 11 kV at 0·8 p.f. lagging. If resistance and reactance per conductor are 0·15 Ω and 0·02 Ω respectively, calculate :
   (*i*) percentage regulation (*ii*) sending end power factor
   (*iii*) line losses **[(*i*) 19·83% (*ii*) 0·77 lag (*iii*) 620 kW]**
2. A single phase 11 kV line with a length of 15 km is to transmit 500 kVA. The inductive reactance of the line is 0·5 Ω/km and the resistance is 0·3 Ω/km. Calculate the efficiency and regulation of the line for 0·8 lagging power factor. **[97·74%, 3·34%]**
3. A load of 1000 kW at 0·8 p.f. lagging is received at the end of a 3-phase line 20 km long. The resistance and reactance of each conductor are 0·25 Ω and 0·28 Ω per km. If the receiving end line voltage is maintained at 11 kV, calculate :
   (*i*) sending end voltage (line-to-line) (*ii*) percentage regulation
   (*iii*) transmission efficiency **[(*i*) 11·84 kV (*ii*) 7·61% (*iii*) 94·32%]**
4. Estimate the distance over which a load of 15000 kW at 0·85 p.f. can be delivered by a 3-phase transmission line having conductors of steel-cored aluminium each of resistance 0·905 Ω/phase per kilometre. The voltage at the receiving end is to be 132 kV and the loss in transmission is to be 7·5% of the load. **[69·55 km]**
5. A 3-phase line 3 km long delivers 3000 kW at a p.f. 0·8 lagging to a load. The resistance and reactance per km of each conductor are 0·4 Ω and 0·3 Ω respectively. If the voltage at the supply end is maintained at 11 kV, calculate :

(i) receiving end voltage (line-to-line) (ii) line current
(iii) transmission efficiency. **[(i) 10.46 kV (ii) 207 A (iii) 95%]**

6. A short 3-φ transmission line with an impedance of $(5 + j\,20)\ \Omega$ per phase has sending end and receiving end voltages of 46·85 kV and 33 kV respectively for some receiving end load at a p.f.of 0·8 lagging. Determine :
   (i) power output
   (ii) sending end power factor **[(i) 22·86 kW (ii) 0·657 lag]**
7. A substation receives 6000 kVA at 6 kV, 0.8 p.f. lagging on low voltage side of a transformer from a generating station through a 3-phase cable system having resistance of 7 Ω and reactance of 2 Ω per phase. Identical 6600/33000 V transformers are installed at each end, 6600 V side being delta connected and 33000 V side star connected. The resistance and reactance of each transformer are 1 Ω and 9 Ω respectively, referred to *h.v.* side. Calculate the voltage at the generating station bus bars **[6778 V]**
8. A short 3-phase transmission line connected to a 33kV, 50 Hz generating station at the sending end is required to supply a load of 10 MW at 0·8 lagging power factor at 30 kV at the receiving end. If the minimum transmission efficiency is to be limited to 96%, estimate the per phase value of resistance and inductance of the line. **[2·4 Ω; 0·028 H]**
9. A single phase transmission line is delivering 500 kVA load at 2 kV. Its resistance is 0·2 Ω and inductive reactance is 0·4 Ω. Determine the voltage regulation if the load power factor is (i) 0·707 lagging (ii) 0·707 leading. **[(i) 5·3% (ii) −1·65%]**

V_S = sending end voltage per phase

The *phasor diagram for the circuit is shown in Fig 10.9. Taking the receiving end voltage $\vec{V_R}$ as the reference phasor, we have, $\vec{V_R} = V_R + j\,0$

Load current, $\vec{I_R} = I_R\,(\cos\phi_R - j\sin\phi_R)$

Capacitive current, $\vec{I_C} = j\,\vec{V_R}\,\omega\,C = j\,2\,\pi\,f\,C\,\vec{V_R}$

The sending end current $\vec{I_S}$ is the phasor sum of load current $\vec{I_R}$ and capacitive current $\vec{I_C}$ *i.e.*,

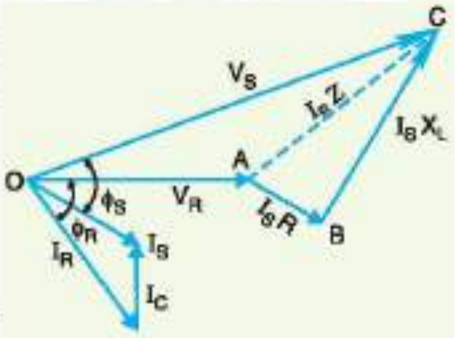

Fig. 10.9

$$\begin{aligned}\vec{I_S} &= \vec{I_R} + \vec{I_C}\\ &= I_R\,(\cos\phi_R - j\sin\phi_R) + j\,2\,\pi\,f\,C\,V_R\\ &= I_R\cos\phi_R + j\,(-I_R\sin\phi_R + 2\,\pi\,f\,C\,V_R)\end{aligned}$$

Voltage drop/phase $= \vec{I_S}\,\vec{Z} = \vec{I_S}\,(R + jX_L)$

Sending end voltage, $\vec{V_S} = \vec{V_R} + \vec{I_S}\,\vec{Z} = \vec{V_R} + \vec{I_S}\,(R + j\,X_L)$

Thus, the magnitude of sending end voltage V_S can be calculated.

$$\%\text{ Voltage regulation} = \frac{V_S - V_R}{V_R} \times 100$$

$$\%\text{ Voltage transmission efficiency} = \frac{\text{Power delivered / phase}}{\text{Power delivered / phase + losses / phase}} \times 100$$

$$= \frac{V_R\,I_R\cos\phi_R}{V_R\,I_R\cos\phi_R + I_S^2\,R} \times 100$$

Limitations. Although end condenser method for the solution of medium lines is simple to work out calculations, yet it has the following drawbacks :

(*i*) There is a considerable error (about 10%) in calculations because the distributed capacitance has been assumed to be lumped or concentrated.

(*ii*) This method overestimates the effects of line capacitance.

Example 10.10. *A (medium) single phase transmission line 100 km long has the following constants :*

Resistance/km = 0·25 Ω ; *Reactance/km = 0·8 Ω*

Susceptance/km = 14 × 10^{-6} siemen ; *Receiving end line voltage = 66,000 V*

Assuming that the total capacitance of the line is localised at the receiving end alone, determine (i) the sending end current (ii) the sending end voltage (iii) regulation and (iv) supply power factor. The line is delivering 15,000 kW at 0.8 power factor lagging. Draw the phasor diagram to illustrate your calculations.

Solution. Figs. 10.10 (*i*) and (*ii*) show the circuit diagram and phasor diagram of the line respectively.

---

* Note the construction of phasor diagram. The load current $\vec{I_R}$ lags behind $\vec{V_R}$ by ϕ_R. The capacitive current $\vec{I_C}$ leads $\vec{V_R}$ by 90° as shown. The phasor sum of $\vec{I_C}$ and $\vec{I_R}$ is the sending end current $\vec{I_S}$. The drop in the line resistance is $\vec{I_S}R$ (*AB*) in phase with $\vec{I_S}$ whereas inductive drop $\vec{I_S}X_L$ (*BC*) leads I_S by 90°. Therefore, *OC* represents the sending end voltage $\vec{V_S}$. The angle ϕ_S between the sending end voltage $\vec{V_S}$ and sending end current $\vec{I_S}$ determines the sending end power factor cos ϕ_S.

Total resistance, $R = 0.25 \times 100 = 25\ \Omega$

Total reactance, $X_L = 0.8 \times 100 = 80\ \Omega$

Total susceptance, $Y = 14 \times 10^{-6} \times 100 = 14 \times 10^{-4}\ S$

Receiving end voltage, $V_R = 66{,}000$ V

∴ Load current, $I_R = \dfrac{15{,}000 \times 10^3}{66{,}000 \times 0.8} = 284$ A

$\cos\phi_R = 0.8\,; \qquad \sin\phi_R = 0.6$

Taking receiving end voltage as the reference phasor [see Fig. 10.10 (*ii*)], we have,

$$\vec{V_R} = V_R + j0 = 66{,}000\,V$$

Load current, $\vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R) = 284\,(0.8 - j\,0.6) = 227 - j\,170$

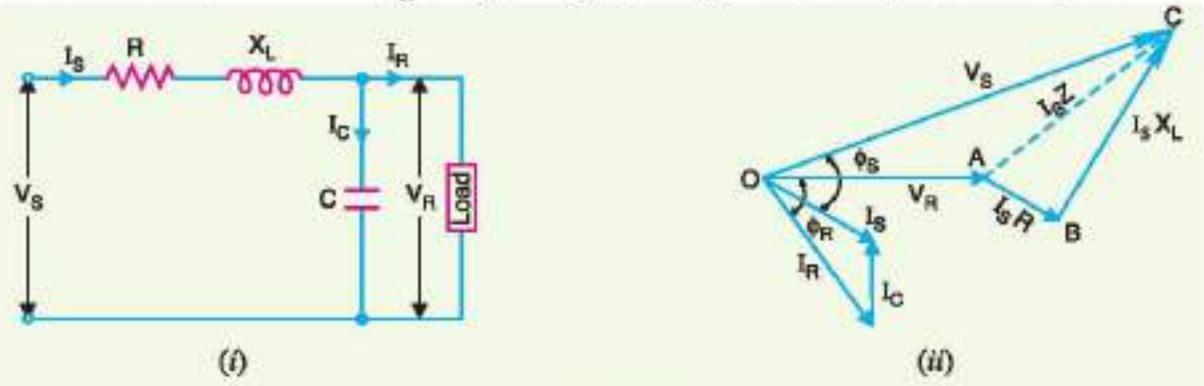

Fig. 10.10

Capacitive current, $\vec{I_C} = jY \times V_R = j\,14 \times 10^{-4} \times 66000 = j\,92$

(*i*) Sending end current, $\vec{I_S} = \vec{I_R} + \vec{I_C} = (227 - j\,170) + j\,92$

$= 227 - j\,78$ ...(*i*)

Magnitude of $I_S = \sqrt{(227)^2 + (78)^2} =$ **240 A**

(*ii*) Voltage drop $= \vec{I_S}\vec{Z} = \vec{I_S}(R + jX_L) = (227 - j\,78)\,(25 + j\,80)$

$= 5{,}675 + j\,18{,}160 - j\,1950 + 6240$

$= 11{,}915 + j\,16{,}210$

Sending end voltage, $\vec{V_S} = \vec{V_R} + \vec{I_S}\vec{Z} = 66{,}000 + 11{,}915 + j\,16{,}210$

$= 77{,}915 + j\,16{,}210$ ...(*ii*)

Magnitude of $V_S = \sqrt{(77915)^2 + (16210)^2} =$ **79583V**

(*iii*) % Voltage regulation $= \dfrac{V_S - V_R}{V_R} \times 100 = \dfrac{79{,}583 - 66{,}000}{66{,}000} \times 100 =$ **20·58%**

(*iv*) Referring to exp. (*i*), phase angle between $\vec{V_R}$ and $\vec{I_R}$ is :

$$\theta_1 = \tan^{-1} - 78/227 = \tan^{-1}(-0.3436) = -18.96°$$

Referring to exp. (*ii*), phase angle between $\vec{V_R}$ and $\vec{V_S}$ is :

$$\theta_2 = \tan^{-1}\frac{16210}{77915} = \tan^{-1}(0.2036) = 11.50°$$

∴ Supply power factor angle, $\phi_S = 18.96° + 11.50° = 30.46°$

∴ Supply p.f. $= \cos\phi_S = \cos 30.46° =$ **0·86 lag**

10.8 Nominal T Method

In this method, the whole line capacitance is assumed to be concentrated at the middle point of the line and half the line resistance and reactance are lumped on its either side as shown in Fig. 10.11. Therefore, in this arrangement, full charging current flows over half the line. In Fig. 10.11, one phase of 3-phase transmission line is shown as it is advantageous to work in phase instead of line-to-line values.

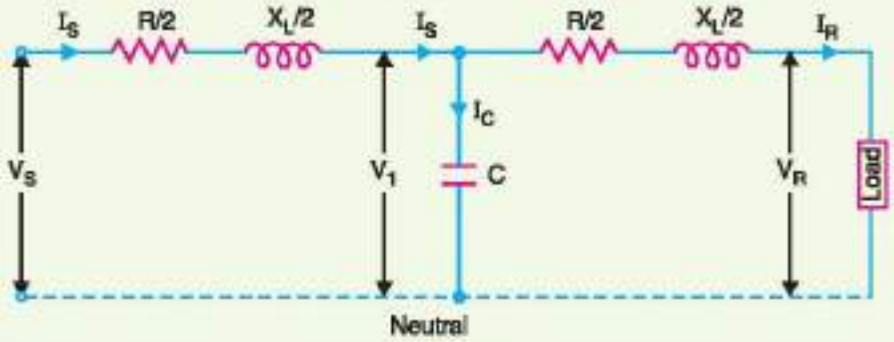

Fig. 10.11

Let I_R = load current per phase ; R = resistance per phase
X_L = inductive reactance per phase ; C = capacitance per phase
$\cos \phi_R$ = receiving end power factor (*lagging*) ; V_S = sending end voltage/phase
V_1 = voltage across capacitor C

The *phasor diagram for the circuit is shown in Fig. 10.12. Taking the receiving end voltage $\vec{V_R}$ as the reference phasor, we have,

Receiving end voltage, $\vec{V_R} = V_R + j0$

Load current, $\vec{I_R} = I_R (\cos \phi_R - j \sin \phi_R)$

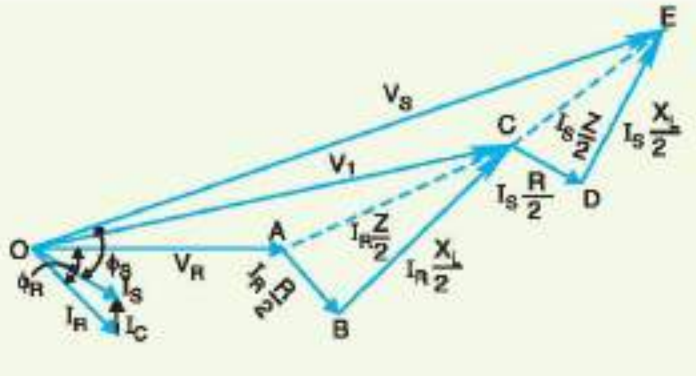

Fig. 10.12

* Note the construction of phasor diagram. $\vec{V_R}$ is taken as the reference phasor represented by OA. The load current $\vec{I_R}$ lags behind $\vec{V_R}$ by ϕ_R. The drop $AB = I_R R/2$ is in phase with $\vec{I_R}$ and $BC = I_R X_L/2$ leads $\vec{I_R}$ by 90°. The phasor OC represents the voltage $\vec{V_1}$ across condenser C. The capacitor current $\vec{I_C}$ leads $\vec{V_1}$ by 90° as shown. The phasor sum of $\vec{I_R}$ and $\vec{I_C}$ gives $\vec{I_S}$. Now $CD = I_S R/2$ is in phase with $\vec{I_S}$ while $DE = I_S X_L/2$ leads $\vec{I_S}$ by 90°. Then, OE represents the sending end voltage $\vec{V_S}$.

Voltage across C, $\vec{V_1} = \vec{V_R} + \vec{I_R}\vec{Z}/2$

$$= V_R + I_R(\cos\phi_R - j\sin\phi_R)\left(\frac{R}{2} + j\frac{X_L}{2}\right)$$

Capacitive current, $\vec{I_C} = j\omega C\vec{V_1} = j2\pi fC\vec{V_1}$

Sending end current, $\vec{I_S} = \vec{I_R} + \vec{I_C}$

Sending end voltage, $\vec{V_S} = \vec{V_1} + \vec{I_S}\frac{\vec{Z}}{2} = \vec{V_1} + \vec{I_S}\left(\frac{R}{2} + j\frac{X_L}{2}\right)$

Example 10.11. *A 3-phase, 50-Hz overhead transmission line 100 km long has the following constants :*

Resistance/km/phase $= 0.1\ \Omega$

Inductive reactance/km/phase $= 0.2\ \Omega$

Capacitive susceptance/km/phase $= 0.04 \times 10^{-4}$ *siemen*

Determine (i) the sending end current (ii) sending end voltage (iii) sending end power factor and (iv) transmission efficiency when supplying a balanced load of 10,000 kW at 66 kV, p.f. 0.8 lagging. Use nominal T method.

Solution. Figs. 10.13 (*i*) and 10.13 (*ii*) show the circuit diagram and phasor diagram of the line respectively.

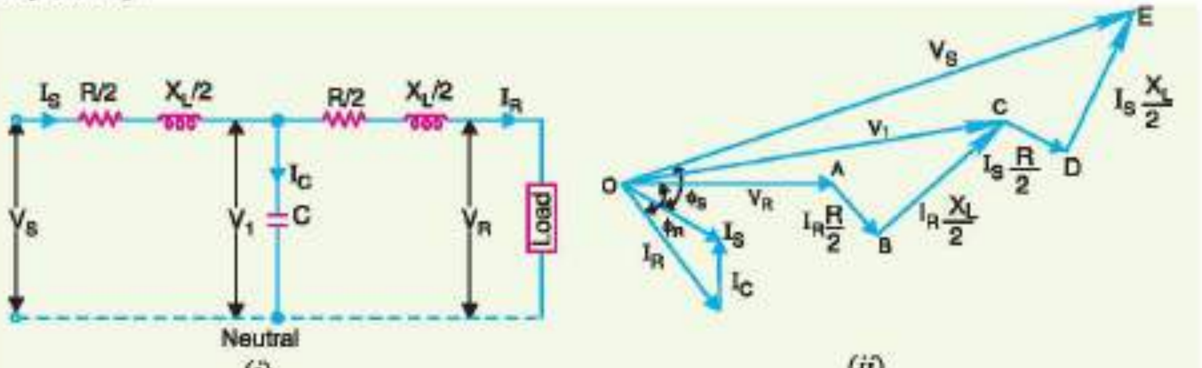

Fig. 10.13

Total resistance/phase, $R = 0.1 \times 100 = 10\ \Omega$

Total reactance/phase, $X_L = 0.2 \times 100 = 20\ \Omega$

Capacitive susceptance, $Y = 0.04 \times 10^{-4} \times 100 = 4 \times 10^{-4}$ S

Receiving end voltage/phase, $V_R = 66{,}000/\sqrt{3} = 38105$ V

Load current, $I_R = \dfrac{10{,}000 \times 10^3}{\sqrt{3} \times 66 \times 10^3 \times 0.8} = 109$ A

$\cos\phi_R = 0.8$; $\sin\phi_R = 0.6$

Impedance per phase, $\vec{Z} = R + jX_L = 10 + j20$

(*i*) Taking receiving end voltage as the reference phasor [see Fig. 10.13 (*ii*)], we have,

Receiving end voltage, $\vec{V_R} = V_R + j0 = 38{,}105$ V

Load current, $\vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R) = 109\ (0.8 - j0.6) = 87.2 - j65.4$

Voltage across C, $\vec{V_1} = \vec{V_R} + \vec{I_R}\vec{Z}/2 = 38,105 + (87.2 - j65.4)(5 + j10)$

$$= 38{,}105 + 436 + j872 - j327 + 654 = 39{,}195 + j545$$

Charging current, $\vec{I_C} = jY\vec{V_1} = j\,4\times10^{-4}(39{,}195 + j\,545) = -0{\cdot}218 + j15{\cdot}6$

Sending end current, $\vec{I_S} = \vec{I_R} + \vec{I_C} = (87{\cdot}2 - j\,65{\cdot}4) + (-0{\cdot}218 + j\,15{\cdot}6)$

$= 87{\cdot}0 - j\,49{\cdot}8 = 100\,\angle -29°47'$ A

∴ Sending end current $=$ **100 A**

(*ii*) Sending end voltage, $\vec{V_S} = \vec{V_1} + \vec{I_S}\,\vec{Z}/2 = (39{,}195 + j\,545) + (87{\cdot}0 - j\,49{\cdot}8)\,(5 + j\,10)$

$= 39{,}195 + j\,545 + 434{\cdot}9 + j\,870 - j\,249 + 498$

$= 40128 + j\,1170 = 40145\,\angle 1°40'$ V

∴ Line value of sending end voltage

$= 40145 \times \sqrt{3} = 69\,533$ V $=$ **69·533 kV**

(*iii*) Referring to phasor diagram in Fig. 10.14,

θ_1 = angle between $\vec{V_R}$ and $\vec{V_S} = 1°40'$

θ_2 = angle between $\vec{V_R}$ and $\vec{I_S} = 29°\,47'$

∴ ϕ_S = angle between $\vec{V_S}$ and $\vec{I_S}$

$= \theta_1 + \theta_2 = 1°40' + 29°47' = 31°27'$

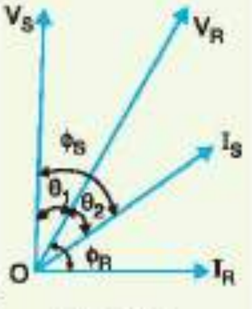

Fig. 10.14

∴ Sending end power factor, $\cos\phi_S = \cos 31°27' =$ **0·853 lag**

(*iv*) Sending end power $= 3\,V_S I_S \cos\phi_S = 3 \times 40{,}145 \times 100 \times 0{\cdot}853$

$= 10273105$ W $= 10273{\cdot}105$ kW

Power delivered $= 10{,}000$ kW

∴ Transmission efficiency $= \dfrac{10{,}000}{10273{\cdot}105} \times 100 =$ **97·34%**

Example 10.12. *A 3-phase, 50 Hz transmission line 100 km long delivers 20 MW at 0·9 p.f. lagging and at 110 kV. The resistance and reactance of the line per phase per km are 0·2 Ω and 0·4 Ω respectively, while capacitance admittance is 2·5 × 10⁻⁶ siemen/km/phase. Calculate : (i) the current and voltage at the sending end (ii) efficiency of transmission. Use nominal T method.*

Solution. Figs. 10.15 (*i*) and 10.15 (*ii*) show the circuit diagram and phasor diagram respectively.

Total resistance/phase, $R = 0{\cdot}2 \times 100 = 20\ \Omega$

Total reactance/phase, $X_L = 0{\cdot}4 \times 100 = 40\ \Omega$

Total capacitance admittance/phase, $Y = 2{\cdot}5 \times 10^{-6} \times 100 = 2{\cdot}5 \times 10^{-4}$ S

Phase impedance, $\vec{Z} = 20 + j40$

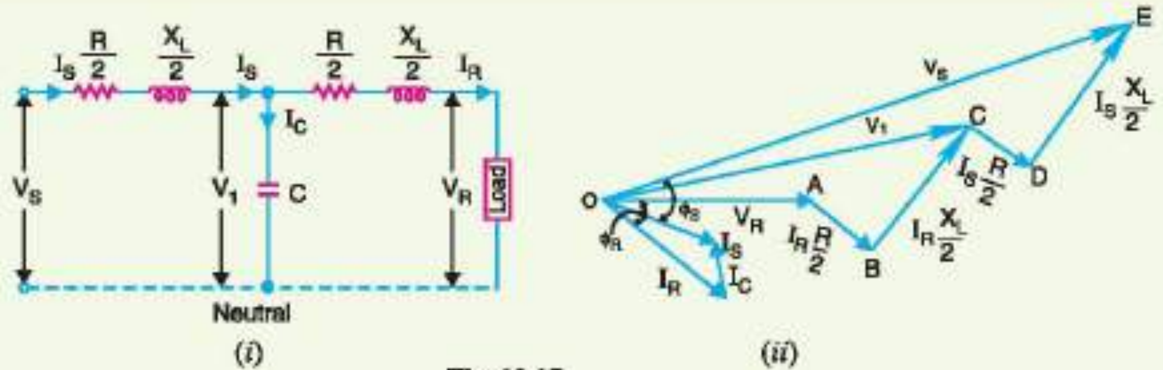

Fig. 10.15

Receiving end voltage/phase, $V_R = 110 \times 10^3/\sqrt{3} = 63508$ V

$$\text{Load current, } I_R = \frac{20 \times 10^6}{\sqrt{3} \times 110 \times 10^3 \times 0{\cdot}9} = 116.6 \text{ A}$$

$$\cos\phi_R = 0{\cdot}9 \text{ ; } \sin\phi_R = 0{\cdot}435$$

(*i*) Taking receiving end voltage as the reference phasor [see phasor diagram 10.15 (*ii*)], we have,

$$\vec{V_R} = V_R + j0 = 63508 \text{ V}$$

Load current, $\vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R) = 116{\cdot}6\,(0{\cdot}9 - j\,0{\cdot}435) = 105 - j50{\cdot}7$

Voltage across C, $\vec{V_1} = \vec{V_R} + \vec{I_R}\,\vec{Z}/2 = 63508 + (105 - j\,50{\cdot}7)\,(10 + j\,20)$

$$= 63508 + (2064 + j1593) = 65572 + j1593$$

Charging current, $\vec{I_C} = j\,Y\vec{V_1} = j\,2{\cdot}5 \times 10^{-4}\,(65572 + j1593) = -0{\cdot}4 + j\,16{\cdot}4$

Sending end current, $\vec{I_S} = \vec{I_R} + \vec{I_C} = (105 - j\,50{\cdot}7) + (-0{\cdot}4 + j16{\cdot}4)$

$$= (104{\cdot}6 - j\,34{\cdot}3) = 110\,\angle -18^\circ 9' \text{ A}$$

∴ Sending end current $=$ **110 A**

Sending end voltage, $\vec{V_S} = \vec{V_1} + \vec{I_S}\,\vec{Z}/2$

$$= (65572 + j1593) + (104{\cdot}6 - j34{\cdot}3)\,(10 + j20)$$

$$= 67304 + j\,3342$$

∴ Magnitude of $V_S = \sqrt{(67304)^2 + (3342)^2} = 67387\ V$

∴ Line value of sending end voltage

$$= 67387 \times \sqrt{3} = 116717 \text{ V} = \mathbf{116{\cdot}717\ kV}$$

(*ii*) Total line losses for the three phases

$$= 3\,I_S^2\,R/2 + 3I_R^2\,R/2$$

$$= 3 \times (110)^2 \times 10 + 3 \times (116{\cdot}6)^2 \times 10$$

$$= 0{\cdot}770 \times 10^6 \text{ W} = 0{\cdot}770 \text{ MW}$$

∴ Transmission efficiency $= \dfrac{20}{20 + 0{\cdot}770} \times 100 = \mathbf{96{\cdot}29\%}$

10.9 Nominal π Method

In this method, capacitance of each conductor (*i.e.*, line to neutral) is divided into two halves; one half being lumped at the sending end and the other half at the receiving end as shown in Fig. 10.16. It is obvious that capacitance at the sending end has no effect on the line drop. However, its charging current must be added to line current in order to obtain the total sending end current.

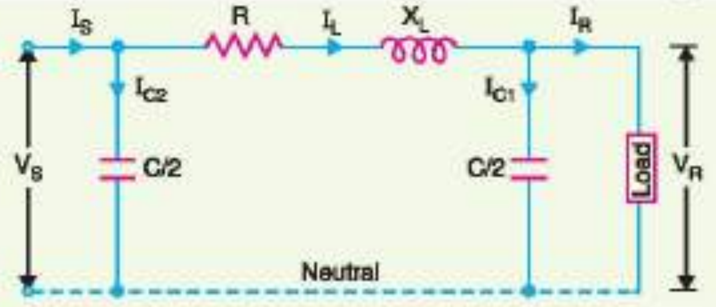

Fig. 10.16

Let I_R = load current per phase
R = resistance per phase
X_L = inductive reactance per phase
C = capacitance per phase
$\cos\phi_R$ = receiving end power factor (*lagging*)
V_S = sending end voltage per phase

The *phasor diagram for the circuit is shown in Fig. 10.17. Taking the receiving end voltage as the reference phasor, we have,

$$\vec{V_R} = V_R + j0$$

Load current,
$$\vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R)$$

Charging current at load end is

$$\vec{I_{C1}} = j\omega(C/2)\vec{V_R} = j\pi fC\vec{V_R}$$

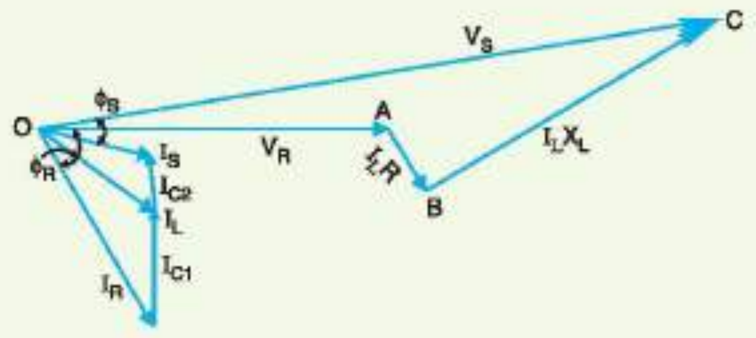

Fig. 10.17

Line current,
$$\vec{I_L} = \vec{I_R} + \vec{I_{C1}}$$

Sending end voltage,
$$\vec{V_S} = \vec{V_R} + \vec{I_L}\vec{Z} = \vec{V_R} + \vec{I_L}(R + jX_L)$$

Charging current at the sending end is

$$\vec{I_{C2}} = j\omega(C/2)\vec{V_S} = j\pi fC\vec{V_S}$$

∴ Sending end current,
$$\vec{I_S} = \vec{I_L} + \vec{I_{C2}}$$

Example 10.13 *A 3-phase, 50Hz, 150 km line has a resistance, inductive reactance and capacitive shunt admittance of 0·1 Ω, 0·5 Ω and 3 × 10^{-6} S per km per phase. If the line delivers 50 MW at 110 kV and 0·8 p.f. lagging, determine the sending end voltage and current. Assume a nominal π circuit for the line.*

---

* Note the construction of phasor diagram. $\vec{V_R}$ is taken as the reference phasor represented by OA. The current $\vec{I_R}$ lags behind $\vec{V_R}$ by ϕ_R. The charging current $\vec{I_{C1}}$ leads $\vec{V_R}$ by 90°. The line current $\vec{I_L}$ is the phasor sum of $\vec{I_R}$ and $\vec{I_{C1}}$. The drop $AB = I_LR$ is in phase with $\vec{I_L}$ whereas drop $BC = I_LX_L$ leads $\vec{I_L}$ by 90°. Then OC represents the sending end voltage $\vec{V_S}$. The charging current $\vec{I_{C2}}$ leads $\vec{V_S}$ by 90°. Therefore, sending end current $\vec{I_S}$ is the phasor sum of the $\vec{I_{C2}}$ and $\vec{I_L}$. The angle ϕ_S between sending end voltage V_S and sending end current I_S determines the sending end p.f. $\cos\phi_S$.

Solution. Fig. 10.18 shows the circuit diagram for the line.

Total resistance/phase, $R = 0{\cdot}1 \times 150 = 15\ \Omega$

Total reactance/phase, $X_L = 0{\cdot}5 \times 150 = 75\ \Omega$

Capacitive admittance/phase, $Y = 3 \times 10^{-6} \times 150 = 45 \times 10^{-5}$ S

Receiving end voltage/phase, $V_R = 110 \times 10^3/\sqrt{3} = 63{,}508$ V

Load current, $$I_R = \frac{50 \times 10^6}{\sqrt{3} \times 110 \times 10^3 \times 0{\cdot}8} = 328 \text{ A}$$

$$\cos\phi_R = 0{\cdot}8;\ \sin\phi_R = 0{\cdot}6$$

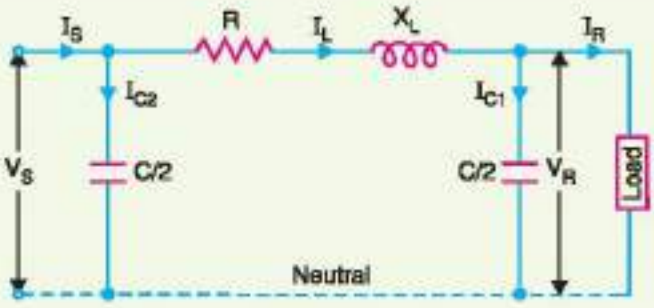

Fig. 10.18

Taking receiving end voltage as the reference phasor, we have,

$$\vec{V_R} = V_R + j\,0 = 63{,}508 \text{ V}$$

Load current, $$\vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R) = 328\,(0{\cdot}8 - j0{\cdot}6) = 262{\cdot}4 - j196{\cdot}8$$

Charging current at the load end is

$$\vec{I_{C1}} = \vec{V_R}\, j\frac{Y}{2} = 63{,}508 \times j\frac{45 \times 10^{-5}}{2} = j\,14{\cdot}3$$

Line current, $$\vec{I_L} = \vec{I_R} + \vec{I_{C1}} = (262{\cdot}4 - j\,196{\cdot}8) + j\,14{\cdot}3 = 262{\cdot}4 - j\,182{\cdot}5$$

Sending end voltage, $$\vec{V_S} = \vec{V_R} + \vec{I_L}\,\vec{Z} = \vec{V_R} + \vec{I_L}\,(R + j\,X_L)$$
$$= 63{,}508 + (262{\cdot}4 - j\,182{\cdot}5)\,(15 + j\,75)$$
$$= 63{,}508 + 3936 + j\,19{,}680 - j\,2737{\cdot}5 + 13{,}687$$
$$= 81{,}131 + j\,16{,}942{\cdot}5 = 82{,}881\ \angle 11°47'\text{ V}$$

∴ Line to line sending end voltage = 82,881 × √3 = 1,43,550 V = **143·55 kV**

Charging current at the sending end is

$$I_{C2} = j\vec{V_S}\,Y/2 = (81{,}131 + j\,16{,}942{\cdot}5)\, j\frac{45 \times 10^{-5}}{2}$$
$$= -3{\cdot}81 + j\,18{\cdot}25$$

Sending end current, $$\vec{I_S} = \vec{I_L} + \vec{I_{C_2}} = (262{\cdot}4 - j\,182{\cdot}5) + (-3{\cdot}81 + j\,18{\cdot}25)$$
$$= 258{\cdot}6 - j\,164{\cdot}25 = 306{\cdot}4\ \angle -32{\cdot}4°\text{ A}$$

∴ Sending end current = **306·4 A**

Example 10·14. *A 100-km long, 3-phase, 50-Hz transmission line has following line constants:*

Resistance/phase/km = 0·1 Ω

Reactance/phase/km = 0·5 Ω

Susceptance/phase/km = 10 × 10⁻⁶ S

If the line supplies load of 20 MW at 0·9 p.f. lagging at 66 kV at the receiving end, calculate by nominal π method :

(*i*) *sending end power factor* (*ii*) *regulation*
(*iii*) *transmission efficiency*

Solution. Fig. 10·19 shows the circuit diagram for the line.

Total resistance/phase, $R = 0{\cdot}1 \times 100 = 10\ \Omega$

Total reactance/phase, $X_L = 0{\cdot}5 \times 100 = 50\ \Omega$

Susceptance/phase, $Y = 10 \times 10^{-6} \times 100 = 10 \times 10^{-4}$ S

Receiving end voltage/phase, $V_R = 66 \times 10^3/\sqrt{3} = 38105$ V

Load current,
$$I_R = \frac{20 \times 10^6}{\sqrt{3} \times 66 \times 10^3 \times 0{\cdot}9} = 195 \text{ A}$$

$$\cos\phi_R = 0{\cdot}9 \quad ; \quad \sin\phi_R = 0{\cdot}435$$

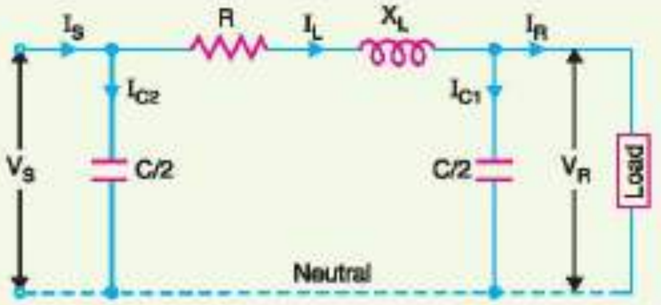

Fig. 10.19

Taking receiving end voltage as the reference phasor, we have,

$$\vec{V_R} = V_R + j0 = 38105 \text{ V}$$

Load current, $\vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R) = 195\,(0{\cdot}9 - j0{\cdot}435) = 176 - j85$

Charging current at the receiving end is

$$\vec{I_{C1}} = \vec{V_R}\, j\frac{Y}{2} = 38105 \times j\frac{10 \times 10^{-4}}{2} = j19$$

Line current, $\vec{I_L} = \vec{I_R} + \vec{I_{C1}} = (176 - j85) + j19 = 176 - j66$

Sending end voltage,
$$\vec{V_S} = \vec{V_R} + \vec{I_L}\vec{Z} = \vec{V_R} + \vec{I_L}(R + jX_L)$$
$$= 38{,}105 + (176 - j66)(10 + j50)$$
$$= 38{,}105 + (5060 + j8140)$$
$$= 43{,}165 + j8140 = 43{,}925 \angle 10{\cdot}65^\circ \text{ V}$$

Sending end line to line voltage $= 43{,}925 \times \sqrt{3} = 76 \times 10^3 \text{ V} = 76$ kV

Charging current at the sending end is

$$\vec{I_{C2}} = \vec{V_S}\, jY/2 = (43165 + j8140)\, j\frac{10 \times 10^{-4}}{2}$$
$$= -4{\cdot}0 + j21{\cdot}6$$

∴ Sending end current,
$$\vec{I_S} = \vec{I_L} + \vec{I_{C2}} = (176 - j66) + (-4{\cdot}0 + j21{\cdot}6)$$
$$= 172 - j44{\cdot}4 = 177{\cdot}6 \angle -14{\cdot}5^\circ \text{ A}$$

(*i*) Referring to phasor diagram in Fig. 10.20,

$$\theta_1 = \text{angle between } \vec{V_R} \text{ and } \vec{V_S} = 10{\cdot}65°$$

$$\theta_2 = \text{angle between } \vec{V_R} \text{ and } \vec{I_S} = -14{\cdot}5°$$

$$\therefore \quad \phi_S = \text{angle between } \vec{V_S} \text{ and } \vec{I_S} = \theta_2 + \theta_1$$

$$= 14{\cdot}5° + 10{\cdot}65° = 25{\cdot}15°$$

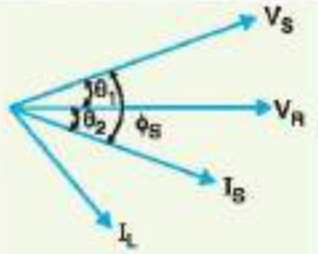

Fig. 10.20

$\therefore$ Sending end p.f., $\cos\phi_S = \cos 25{\cdot}15° =$ **0·905 lag**

(*ii*) % Voltage regulation $= \dfrac{V_S - V_R}{V_R} \times 100 = \dfrac{43925 - 38105}{38105} \times 100 =$ **15·27 %**

(*iii*) Sending end power $= 3\, V_S\, I_S \cos\phi_S = 3 \times 43925 \times 177{\cdot}6 \times 0{\cdot}905$

$= 21{\cdot}18 \times 10^6$ W $= 21{\cdot}18$ MW

Transmission efficiency $= (20/21{\cdot}18) \times 100 =$ **94 %**

TUTORIAL PROBLEMS

1. A (medium) single phase transmission line 100 km long has the following constants :

   Resistance/km/phase = 0·15Ω
   Inductive reactance/km/phase = 0·377 Ω
   Capacitive reactance/km/phase = 31·87 Ω
   Receiving end line voltage = 132 kV

   Assuming that the total capacitance of the line is localised at the receiving end alone, determine :

   (*i*) sending end current (*ii*) line value of sending end voltage
   (*iii*) regulation (*iv*) sending end power factor

   The line is delivering 72 MW at 0·8 p.f. lagging.

   [(*i*) 377·3 A (*ii*) 155·7 kV (*iii*) 17·9% (*iv*) 0·774 lag]

2. A 3-phase, 50 Hz overhead transmission line has the following constants :

   Resistance/phase = 9·6 Ω
   Inductance/phase = 0·097 mH
   Capacitance/phase = 0·765 μF

   If the line is supplying a balanced load of 24,000 kVA 0·8 p.f. lagging at 66 kV, calculate :

   (*i*) sending end current (*ii*) line value of sending end voltage
   (*iii*) sending end power factor (*iv*) percentage regulation
   (*v*) transmission efficiency

   [(*i*) 204 A (*ii*) 75 kV (*iii*) 0·814 lag (*iv*) 13·63 % (*v*) 93·7%]

3. A 3-phase, 50 Hz, overhead transmission line delivers 10 MW at 0·8 p.f. lagging and at 66 kV. The resistance and inductive reactance of the line per phase are 10 Ω and 20 Ω respectively while capacitance admittance is 4×10^{-4} siemen. Calculate :

   (*i*) the sending end current (*ii*) sending end voltage (line-to-line)
   (*iii*) sending end power factor (*iv*) transmission efficiency

   Use nominal *T* method.

   [(*i*) 100 A (*ii*) 69·8 kV (*iii*) 0·852 (*iv*) 97·5%]

4. A 3-phase, 50 Hz, 100 km transmission line has the following constants :

   Resistance/phase/km = 0·1 Ω
   Reactance/phase/km = 0·5 Ω
   Susceptance/phase/km = 10^{-5} siemen

   If the line supplies a load of 20 MW at 0·9 p.f. lagging at 66 kV at the receiving end, calculate by using nominal π method :

   (*i*) sending end current (*ii*) line value of sending end voltage

(*iii*) sending end power factor (*iv*) regulation

[(*i*) 177·6 A (*ii*) 76kV (*iii*) 0·905 lag (*iv*) 15·15%]

5. A 3-phase overhead transmission line has the following constants :

$$\text{Resistance/phase} = 10\ \Omega$$
$$\text{Inductive reactance/phase} = 35\ \Omega$$
$$\text{Capacitive admittance/phase} = 3 \times 10^{-4}\ \text{siemen}$$

If the line supplied a balanced load of 40,000 kVA at 110 kV and 0·8 p.f. lagging, calculate :

(*i*) sending end power factor (*ii*) percentage regulation

(*iii*) transmission efficiency **[(*i*) 0·798 lag (*ii*) 10% (*iii*) 96·38%]**

6. A 3-phase, 50 Hz overhead transmission line, 100 km long, 110 kV between the lines at the receiving end has the following constants :

$$\text{Resistance per km per phase} = 0{\cdot}153\ \Omega$$
$$\text{Inductance per km per phase} = 1{\cdot}21\ \text{mH}$$
$$\text{Capacitance per km per phase} = 0{\cdot}00958\ \mu\text{F}$$

The line supplies a load of 20,000 kW at 0·9 power factor lagging. Calculate using nominal π representation, the sending end voltage, current, power factor, regulation and the efficiency of the line. Neglect leakage. **[115·645 kV (line voltage) ; 109 ∠ – 16·68° A ; 0·923 lag ; 5·13 %; 97·21 %]**

10.10 Long Transmission Lines

It is well known that line constants of the transmission line are uniformly distributed over the entire length of the line. However, reasonable accuracy can be obtained in line calculations for short and medium lines by considering these constants as lumped. If such an assumption of lumped constants is applied to long transmission lines (having length excess of about 150 km), it is found that serious errors are introduced in the performance calculations. Therefore, in order to obtain fair degree of accuracy in the performance calculations of long lines, the line constants are considered as uniformly distributed throughout the length of the line. Rigorous mathematical treatment is required for the solution of such lines.

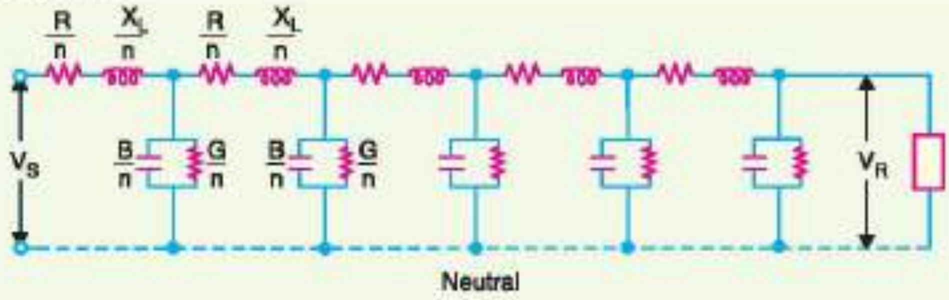

Fig. 10.21

Fig. 10.21 shows the equivalent circuit of a 3-phase long transmission line on a phase-neutral basis. The whole line length is divided into n sections, each section having line constants $\frac{1}{n}$ th of those for the whole line. The following points may by noted :

(*i*) The line constants are uniformly distributed over the entire length of line as is actually the case.

(*ii*) The resistance and inductive reactance are the series elements.

(*iii*) The leakage susceptance (B) and leakage conductance (G) are shunt elements. The leakage susceptance is due to the fact that capacitance exists between line and neutral. The leakage conductance takes into account the energy losses occurring through leakage over the insulators or due to corona effect between conductors. Admittance = $\sqrt{G^2 + B^2}$.

10.6 Medium Transmission Lines

In short transmission line calculations, the effects of the line capacitance are neglected because such lines have smaller lengths and transmit power at relatively low voltages (< 20 kV). However, as the length and voltage of the line increase, the capacitance gradually becomes of greater importance. Since medium transmission lines have sufficient length (50-150 km) and usually operate at voltages greater than 20 kV, the effects of capacitance cannot be neglected. Therefore, in order to obtain reasonable accuracy in medium transmission line calculations, the line capacitance must be taken into consideration.

The capacitance is uniformly distributed over the entire length of the line. However, in order to make the calculations simple, the line capacitance is assumed to be lumped or concentrated in the form of capacitors shunted across the line at one or more points. Such a treatment of localising the line capacitance gives reasonably accurate results. The most commonly used methods (known as *localised capacitance methods*) for the solution of medium transmissions lines are :

(*i*) End condenser method (*ii*) Nominal *T* method (*iii*) Nominal π method.

Although the above methods are used for obtaining the performance calculations of medium lines, they can also be used for short lines if their line capacitance is given in a particular problem.

10.7 End Condenser Method

In this method, the capacitance of the line is lumped or concentrated at the receiving or load end as shown in Fig. 10.8. This method of localising the line capacitance at the load end overestimates the effects of capacitance. In Fig. 10.8, one phase of the 3-phase transmission line is shown as it is more convenient to work in phase instead of line-to-line values.

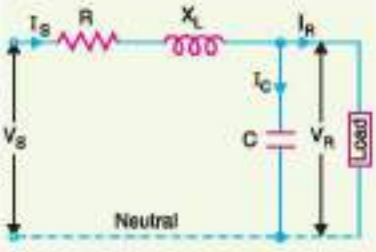

Fig. 10.8

Let I_R = load current per phase

R = resistance per phase

X_L = inductive reactance per phase

C = capacitance per phase

$\cos \phi_R$ = receiving end power factor (*lagging*)

V_S = sending end voltage per phase

The *phasor diagram for the circuit is shown in Fig 10.9. Taking the receiving end voltage $\vec{V_R}$ as the reference phasor, we have, $\vec{V_R} = V_R + j\,0$

Load current, $\vec{I_R} = I_R\,(\cos\phi_R - j\sin\phi_R)$

Capacitive current, $\vec{I_C} = j\,\vec{V_R}\;\omega\,C = j\,2\,\pi\,f\,C\,\vec{V_R}$

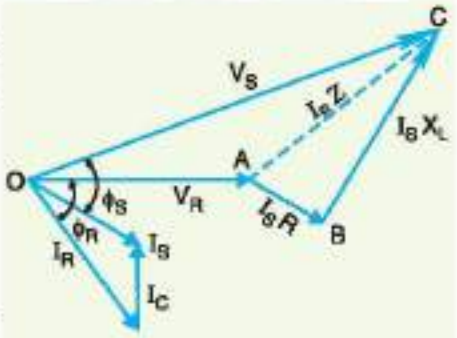

Fig. 10.9

The sending end current $\vec{I_S}$ is the phasor sum of load current $\vec{I_R}$ and capacitive current $\vec{I_C}$ *i.e.*,

$$\vec{I_S} = \vec{I_R} + \vec{I_C}$$
$$= I_R\,(\cos\phi_R - j\sin\phi_R) + j\,2\,\pi\,f\,C\,V_R$$
$$= I_R\cos\phi_R + j\,(-I_R\sin\phi_R + 2\,\pi\,f\,C\,V_R)$$

Voltage drop/phase $= \vec{I_S}\,\vec{Z} = \vec{I_S}\,(R + j\,X_L)$

Sending end voltage, $\vec{V_S} = \vec{V_R} + \vec{I_S}\,\vec{Z} = \vec{V_R} + \vec{I_S}\,(R + j\,X_L)$

Thus, the magnitude of sending end voltage V_S can be calculated.

$$\%\ \text{Voltage regulation} = \frac{V_S - V_R}{V_R} \times 100$$

$$\%\ \text{Voltage transmission efficiency} = \frac{\text{Power delivered / phase}}{\text{Power delivered / phase + losses / phase}} \times 100$$
$$= \frac{V_R\,I_R\cos\phi_R}{V_R\,I_R\cos\phi_R + I_S^2\,R} \times 100$$

Limitations. Although end condenser method for the solution of medium lines is simple to work out calculations, yet it has the following drawbacks :

(*i*) There is a considerable error (about 10%) in calculations because the distributed capacitance has been assumed to be lumped or concentrated.

(*ii*) This method overestimates the effects of line capacitance.

Example 10.10. *A (medium) single phase transmission line 100 km long has the following constants :*

Resistance/km = $0{\cdot}25\,\Omega$; *Reactance/km* = $0{\cdot}8\,\Omega$

Susceptance/km = 14×10^{-6} *siemen* ; *Receiving end line voltage* = *66,000 V*

Assuming that the total capacitance of the line is localised at the receiving end alone, determine (i) the sending end current (ii) the sending end voltage (iii) regulation and (iv) supply power factor. The line is delivering 15,000 kW at 0.8 power factor lagging. Draw the phasor diagram to illustrate your calculations.

Solution. Figs. 10.10 (*i*) and (*ii*) show the circuit diagram and phasor diagram of the line respectively.

* Note the construction of phasor diagram. The load current $\vec{I_R}$ lags behind $\vec{V_R}$ by ϕ_R. The capacitive current $\vec{I_C}$ leads $\vec{V_R}$ by 90° as shown. The phasor sum of $\vec{I_C}$ and $\vec{I_R}$ is the sending end current $\vec{I_S}$. The drop in the line resistance is $\vec{I_S}R$ (*AB*) in phase with $\vec{I_S}$ whereas inductive drop $\vec{I_S}X_L$ (*BC*) leads I_S by 90°. Therefore, *OC* represents the sending end voltage $\vec{V_S}$. The angle ϕ_S between the sending end voltage $\vec{V_S}$ and sending end current $\vec{I_S}$ determines the sending end power factor $\cos\phi_S$.

Total resistance, $R = 0{\cdot}25 \times 100 = 25\ \Omega$

Total reactance, $X_L = 0{\cdot}8 \times 100 = 80\ \Omega$

Total susceptance, $Y = 14 \times 10^{-6} \times 100 = 14 \times 10^{-4}\ S$

Receiving end voltage, $V_R = 66{,}000$ V

∴ Load current, $I_R = \dfrac{15{,}000 \times 10^3}{66{,}000 \times 0{\cdot}8} = 284$ A

$\cos \phi_R = 0{\cdot}8\ ; \quad \sin \phi_R = 0{\cdot}6$

Taking receiving end voltage as the reference phasor [see Fig. 10.10 (*ii*)], we have,

$$\vec{V_R} = V_R + j0 = 66{,}000\ V$$

Load current, $\vec{I_R} = I_R(\cos \phi_R - j \sin \phi_R) = 284\ (0{\cdot}8 - j\,0{\cdot}6) = 227 - j\,170$

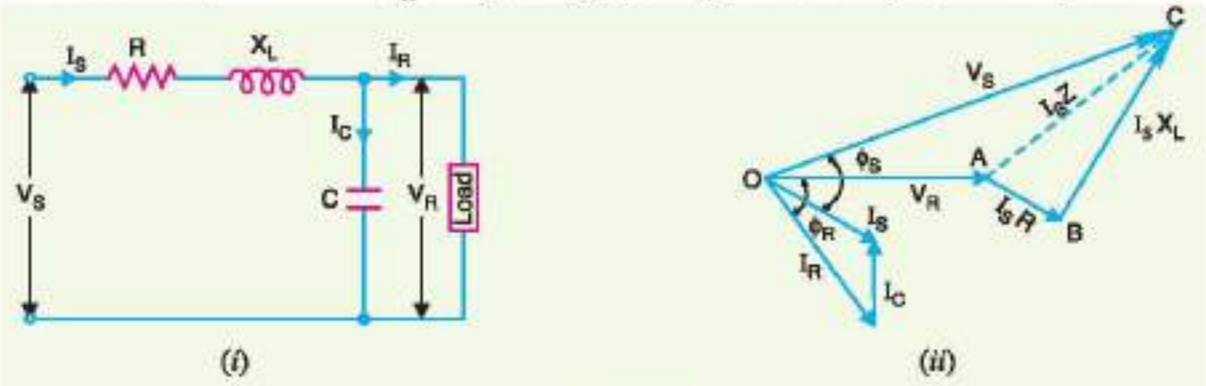

Fig. 10.10

Capacitive current, $\vec{I_C} = jY \times V_R = j\,14 \times 10^{-4} \times 66000 = j\,92$

(*i*) Sending end current, $\vec{I_S} = \vec{I_R} + \vec{I_C} = (227 - j\,170) + j\,92$

$= 227 - j\,78$ ... (*i*)

Magnitude of $I_S = \sqrt{(227)^2 + (78)^2} =$ **240 A**

(*ii*) Voltage drop $= \vec{I_S}\vec{Z} = \vec{I_S}(R + j\,X_L) = (227 - j\,78)\ (25 + j\,80)$

$= 5{,}675 + j\,18{,}160 - j\,1950 + 6240$

$= 11{,}915 + j\,16{,}210$

Sending end voltage, $\vec{V_S} = \vec{V_R} + \vec{I_S}\vec{Z} = 66{,}000 + 11{,}915 + j\,16{,}210$

$= 77{,}915 + j\,16{,}210$ ...(*ii*)

Magnitude of $V_S = \sqrt{(77915)^2 + (16210)^2} =$ **79583V**

(*iii*) % Voltage regulation $= \dfrac{V_S - V_R}{V_R} \times 100 = \dfrac{79{,}583 - 66{,}000}{66{,}000} \times 100 =$ **20·58%**

(*iv*) Referring to exp. (*i*), phase angle between $\vec{V_R}$ and $\vec{I_R}$ is :

$$\theta_1 = \tan^{-1} - 78/227 = \tan^{-1}(-0{\cdot}3436) = -18{\cdot}96°$$

Referring to exp. (*ii*), phase angle between $\vec{V_R}$ and $\vec{V_S}$ is :

$$\theta_2 = \tan^{-1}\frac{16210}{77915} = \tan^{-1}(0{\cdot}2036) = 11{\cdot}50°$$

∴ Supply power factor angle, $\phi_S = 18{\cdot}96° + 11{\cdot}50° = 30{\cdot}46°$

∴ Supply p.f. $= \cos \phi_S = \cos 30{\cdot}46° =$ **0·86 lag**

10.8 Nominal T Method

In this method, the whole line capacitance is assumed to be concentrated at the middle point of the line and half the line resistance and reactance are lumped on its either side as shown in Fig. 10.11. Therefore, in this arrangement, full charging current flows over half the line. In Fig. 10.11, one phase of 3-phase transmission line is shown as it is advantageous to work in phase instead of line-to-line values.

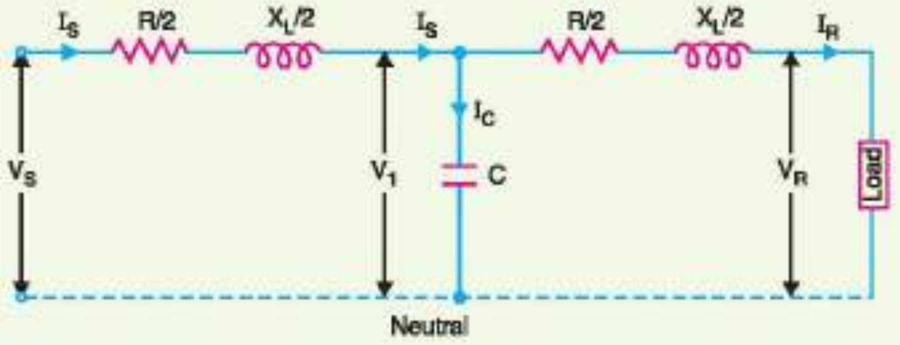

Fig. 10.11

Let I_R = load current per phase ; R = resistance per phase
X_L = inductive reactance per phase ; C = capacitance per phase
$\cos\phi_R$ = receiving end power factor (*lagging*) ; V_S = sending end voltage/phase
V_1 = voltage across capacitor C

The *phasor diagram for the circuit is shown in Fig. 10.12. Taking the receiving end voltage $\vec{V_R}$ as the reference phasor, we have,

Receiving end voltage, $\vec{V_R} = V_R + j0$

Load current, $\vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R)$

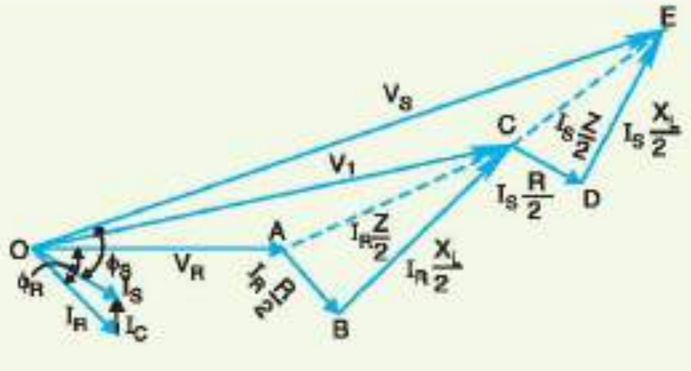

Fig. 10.12

* Note the construction of phasor diagram. $\vec{V_R}$ is taken as the reference phasor represented by OA. The load current $\vec{I_R}$ lags behind $\vec{V_R}$ by ϕ_R. The drop $AB = I_R R/2$ is in phase with $\vec{I_R}$ and $BC = I_R X_L/2$ leads $\vec{I_R}$ by 90°. The phasor OC represents the voltage $\vec{V_1}$ across condenser C. The capacitor current $\vec{I_C}$ leads $\vec{V_1}$ by 90° as shown. The phasor sum of $\vec{I_R}$ and $\vec{I_C}$ gives $\vec{I_S}$. Now $CD = I_S R/2$ is in phase with $\vec{I_S}$ while $DE = I_S X_L/2$ leads $\vec{I_S}$ by 90°. Then, OE represents the sending end voltage $\vec{V_S}$.

Voltage across C, $\vec{V_1} = \vec{V_R} + \vec{I_R}\vec{Z}/2$

$$= V_R + I_R(\cos\phi_R - j\sin\phi_R)\left(\frac{R}{2} + j\frac{X_L}{2}\right)$$

Capacitive current, $\vec{I_C} = j\omega C\vec{V_1} = j2\pi f C\vec{V_1}$

Sending end current, $\vec{I_S} = \vec{I_R} + \vec{I_C}$

Sending end voltage, $\vec{V_S} = \vec{V_1} + \vec{I_S}\frac{\vec{Z}}{2} = \vec{V_1} + \vec{I_S}\left(\frac{R}{2} + j\frac{X_L}{2}\right)$

Example 10.11. *A 3-phase, 50-Hz overhead transmission line 100 km long has the following constants :*

Resistance/km/phase $= 0.1\ \Omega$

Inductive reactance/km/phase $= 0.2\ \Omega$

Capacitive susceptance/km/phase $= 0.04 \times 10^{-4}$ *siemen*

Determine (i) the sending end current (ii) sending end voltage (iii) sending end power factor and (iv) transmission efficiency when supplying a balanced load of 10,000 kW at 66 kV, p.f. 0.8 lagging. Use nominal T method.

Solution. Figs. 10.13 (*i*) and 10.13 (*ii*) show the circuit diagram and phasor diagram of the line respectively.

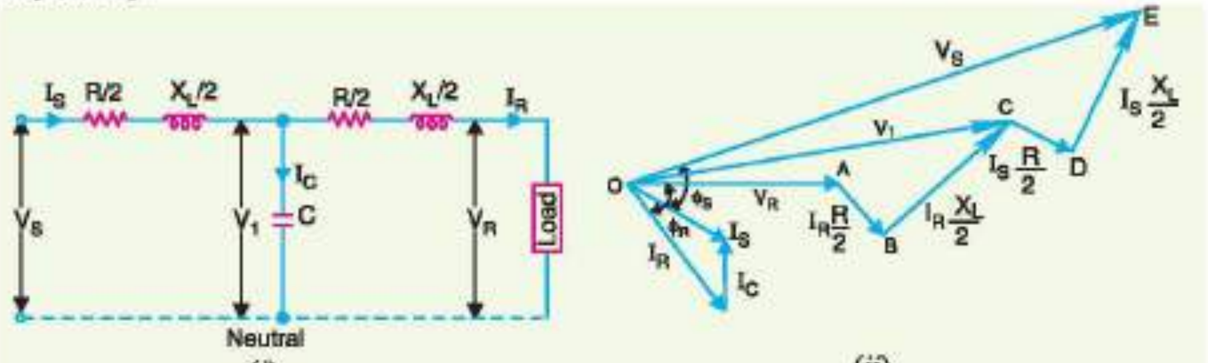

Fig. 10.13

Total resistance/phase, $R = 0.1 \times 100 = 10\ \Omega$

Total reactance/phase, $X_L = 0.2 \times 100 = 20\ \Omega$

Capacitive susceptance, $Y = 0.04 \times 10^{-4} \times 100 = 4 \times 10^{-4}$ S

Receiving end voltage/phase, $V_R = 66{,}000/\sqrt{3} = 38105\ V$

Load current, $I_R = \dfrac{10{,}000 \times 10^3}{\sqrt{3} \times 66 \times 10^3 \times 0.8} = 109$ A

$\cos\phi_R = 0.8$; $\sin\phi_R = 0.6$

Impedance per phase, $\vec{Z} = R + jX_L = 10 + j20$

(*i*) Taking receiving end voltage as the reference phasor [see Fig. 10.13 (*ii*)], we have,

Receiving end voltage, $\vec{V_R} = V_R + j0 = 38{,}105\ V$

Load current, $\vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R) = 109\ (0.8 - j0.6) = 87.2 - j65.4$

Voltage across C, $\vec{V_1} = \vec{V_R} + \vec{I_R}\vec{Z}/2 = 38,105 + (87.2 - j65.4)(5 + j10)$

$$= 38{,}105 + 436 + j872 - j327 + 654 = 39{,}195 + j545$$

Charging current, $\vec{I_C} = jY\vec{V_1} = j\,4\times10^{-4}(39{,}195 + j\,545) = -0{\cdot}218 + j15{\cdot}6$

Sending end current, $\vec{I_S} = \vec{I_R} + \vec{I_C} = (87{\cdot}2 - j\,65{\cdot}4) + (-0{\cdot}218 + j\,15{\cdot}6)$

$= 87{\cdot}0 - j\,49{\cdot}8 = 100\angle -29°47'\text{ A}$

∴ Sending end current $=$ **100 A**

(*ii*) Sending end voltage, $\vec{V_S} = \vec{V_1} + \vec{I_S}\,\vec{Z}/2 = (39{,}195 + j\,545) + (87{\cdot}0 - j\,49{\cdot}8)\,(5 + j\,10)$

$= 39{,}195 + j\,545 + 434{\cdot}9 + j\,870 - j\,249 + 498$

$= 40128 + j\,1170 = 40145\angle 1°40'\text{ V}$

∴ Line value of sending end voltage

$= 40145 \times \sqrt{3} = 69\,533\text{ V} =$ **69·533 kV**

(*iii*) Referring to phasor diagram in Fig. 10.14,

θ_1 = angle between $\vec{V_R}$ and $\vec{V_S} = 1°40'$

θ_2 = angle between $\vec{V_R}$ and $\vec{I_S} = 29°\,47'$

∴ ϕ_S = angle between $\vec{V_S}$ and $\vec{I_S}$

$= \theta_1 + \theta_2 = 1°40' + 29°47' = 31°27'$

∴ Sending end power factor, $\cos\phi_S = \cos 31°27' =$ **0·853 lag**

(*iv*) Sending end power $= 3\,V_S I_S \cos\phi_S = 3 \times 40{,}145 \times 100 \times 0{\cdot}853$

$= 10273105\text{ W} = 10273{\cdot}105\text{ kW}$

Power delivered $= 10{,}000$ kW

∴ Transmission efficiency $= \dfrac{10{,}000}{10273{\cdot}105} \times 100 =$ **97·34%**

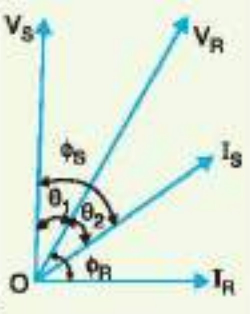

Fig. 10.14

Example 10.12. *A 3-phase, 50 Hz transmission line 100 km long delivers 20 MW at 0·9 p.f. lagging and at 110 kV. The resistance and reactance of the line per phase per km are 0·2 Ω and 0·4 Ω respectively, while capacitance admittance is 2·5 × 10⁻⁶ siemen/km/phase. Calculate : (i) the current and voltage at the sending end (ii) efficiency of transmission. Use nominal T method.*

Solution. Figs. 10.15 (*i*) and 10.15 (*ii*) show the circuit diagram and phasor diagram respectively.

Total resistance/phase, $R = 0{\cdot}2 \times 100 = 20\ \Omega$

Total reactance/phase, $X_L = 0{\cdot}4 \times 100 = 40\ \Omega$

Total capacitance admittance/phase, $Y = 2{\cdot}5 \times 10^{-6} \times 100 = 2{\cdot}5 \times 10^{-4}$ S

Phase impedance, $\vec{Z} = 20 + j40$

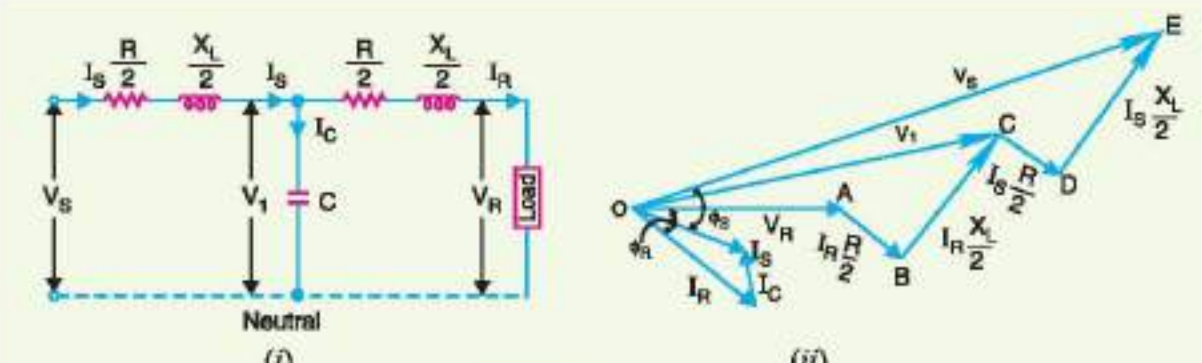

Fig. 10.15

Receiving end voltage/phase, $V_R = 110 \times 10^3/\sqrt{3} = 63508$ V

Load current, $I_R = \dfrac{20 \times 10^6}{\sqrt{3} \times 110 \times 10^3 \times 0{\cdot}9} = 116.6$ A

$\cos \phi_R = 0{\cdot}9$; $\sin \phi_R = 0{\cdot}435$

(*i*) Taking receiving end voltage as the reference phasor [see phasor diagram 10.15 (*ii*)], we have,

$$\vec{V_R} = V_R + j0 = 63508 \text{ V}$$

Load current, $\vec{I_R} = I_R(\cos \phi_R - j \sin \phi_R) = 116{\cdot}6\,(0{\cdot}9 - j0{\cdot}435) = 105 - j50{\cdot}7$

Voltage across C, $\vec{V_1} = \vec{V_R} + \vec{I_R}\,\vec{Z}/2 = 63508 + (105 - j50{\cdot}7)(10 + j20)$

$= 63508 + (2064 + j1593) = 65572 + j1593$

Charging current, $\vec{I_C} = j\,Y\vec{V_1} = j\,2{\cdot}5 \times 10^{-4}\,(65572 + j1593) = -0{\cdot}4 + j\,16{\cdot}4$

Sending end current, $\vec{I_S} = \vec{I_R} + \vec{I_C} = (105 - j\,50{\cdot}7) + (-0{\cdot}4 + j16{\cdot}4)$

$= (104{\cdot}6 - j\,34{\cdot}3) = 110 \angle -18°9'$ A

∴ Sending end current $=$ **110 A**

Sending end voltage, $\vec{V_S} = \vec{V_1} + \vec{I_S}\,\vec{Z}/2$

$= (65572 + j1593) + (104{\cdot}6 - j34{\cdot}3)(10 + j20)$

$= 67304 + j\,3342$

∴ Magnitude of $V_S = \sqrt{(67304)^2 + (3342)^2} = 67387$ V

∴ Line value of sending end voltage

$= 67387 \times \sqrt{3} = 116717$ V $=$ **116·717 kV**

(*ii*) Total line losses for the three phases

$= 3\,I_S^2\,R/2 + 3I_R^2\,R/2$

$= 3 \times (110)^2 \times 10 + 3 \times (116{\cdot}6)^2 \times 10$

$= 0{\cdot}770 \times 10^6$ W $= 0{\cdot}770$ MW

∴ Transmission efficiency $= \dfrac{20}{20 + 0{\cdot}770} \times 100 =$ **96·29%**

10.9 Nominal π Method

In this method, capacitance of each conductor (*i.e.*, line to neutral) is divided into two halves; one half being lumped at the sending end and the other half at the receiving end as shown in Fig. 10.16. It is obvious that capacitance at the sending end has no effect on the line drop. However, its charging current must be added to line current in order to obtain the total sending end current.

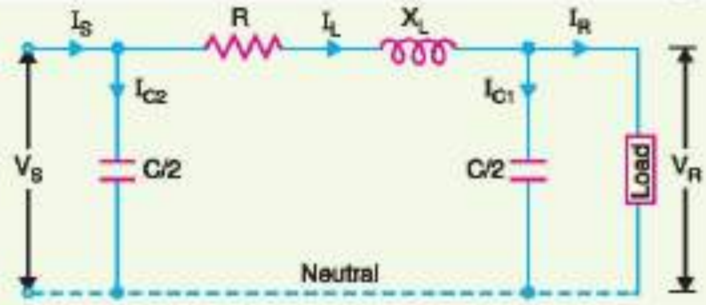

Fig. 10.16

Let

I_R = load current per phase
R = resistance per phase
X_L = inductive reactance per phase
C = capacitance per phase
$\cos\phi_R$ = receiving end power factor (*lagging*)
V_S = sending end voltage per phase

The *phasor diagram for the circuit is shown in Fig. 10.17. Taking the receiving end voltage as the reference phasor, we have,

$$\vec{V_R} = V_R + j0$$

Load current, $\vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R)$

Charging current at load end is

$$\vec{I_{C1}} = j\omega(C/2)\vec{V_R} = j\pi fC\vec{V_R}$$

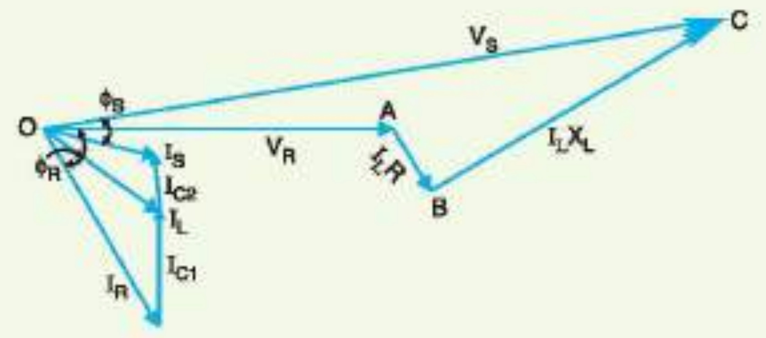

Fig. 10.17

Line current, $\vec{I_L} = \vec{I_R} + \vec{I_{C1}}$

Sending end voltage, $\vec{V_S} = \vec{V_R} + \vec{I_L}\vec{Z} = \vec{V_R} + \vec{I_L}(R + jX_L)$

Charging current at the sending end is

$$\vec{I_{C2}} = j\omega(C/2)\vec{V_S} = j\pi fC\vec{V_S}$$

$\therefore$ Sending end current, $\vec{I_S} = \vec{I_L} + \vec{I_{C2}}$

Example 10.13 *A 3-phase, 50Hz, 150 km line has a resistance, inductive reactance and capacitive shunt admittance of 0·1 Ω, 0·5 Ω and 3 × 10^{-6} S per km per phase. If the line delivers 50 MW at 110 kV and 0·8 p.f. lagging, determine the sending end voltage and current. Assume a nominal π circuit for the line.*

---

* Note the construction of phasor diagram. $\vec{V_R}$ is taken as the reference phasor represented by *OA*. The current $\vec{I_R}$ lags behind $\vec{V_R}$ by ϕ_R. The charging current $\vec{I_{C1}}$ leads $\vec{V_R}$ by 90°. The line current $\vec{I_L}$ is the phasor sum of $\vec{I_R}$ and $\vec{I_{C1}}$. The drop $AB = I_LR$ is in phase with $\vec{I_L}$ whereas drop $BC = I_LX_L$ leads $\vec{I_L}$ by 90°. Then *OC* represents the sending end voltage $\vec{V_S}$. The charging current $\vec{I_{C2}}$ leads $\vec{V_S}$ by 90°. Therefore, sending end current $\vec{I_S}$ is the phasor sum of the $\vec{I_{C2}}$ and $\vec{I_L}$. The angle ϕ_S between sending end voltage V_S and sending end current I_S determines the sending end p.f. $\cos\phi_S$.

Solution. Fig. 10.18 shows the circuit diagram for the line.

Total resistance/phase, $R = 0{\cdot}1 \times 150 = 15\ \Omega$

Total reactance/phase, $X_L = 0{\cdot}5 \times 150 = 75\ \Omega$

Capacitive admittance/phase, $Y = 3 \times 10^{-6} \times 150 = 45 \times 10^{-5}$ S

Receiving end voltage/phase, $V_R = 110 \times 10^3/\sqrt{3} = 63{,}508$ V

Load current, $$I_R = \frac{50 \times 10^6}{\sqrt{3} \times 110 \times 10^3 \times 0{\cdot}8} = 328 \text{ A}$$

$$\cos \phi_R = 0{\cdot}8\ ;\ \sin \phi_R = 0{\cdot}6$$

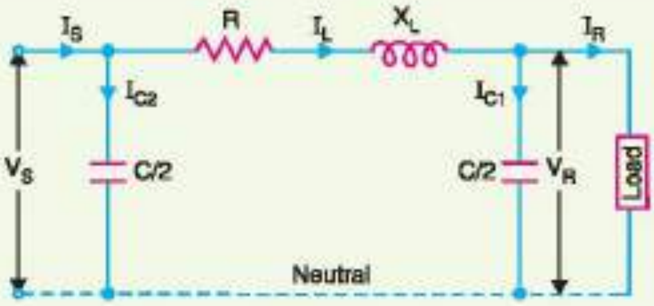

Fig. 10.18

Taking receiving end voltage as the reference phasor, we have,

$$\vec{V_R} = V_R + j\,0 = 63{,}508 \text{ V}$$

Load current, $$\vec{I_R} = I_R\,(\cos \phi_R - j \sin \phi_R) = 328\ (0{\cdot}8 - j0{\cdot}6) = 262{\cdot}4 - j196{\cdot}8$$

Charging current at the load end is

$$\vec{I_{C1}} = \vec{V_R}\ j\frac{Y}{2} = 63{,}508 \times\ j\frac{45 \times 10^{-5}}{2} = j\,14{\cdot}3$$

Line current, $$\vec{I_L} = \vec{I_R} + \vec{I_{C1}} = (262{\cdot}4 - j\,196{\cdot}8) + j\,14{\cdot}3 = 262{\cdot}4 - j\,182{\cdot}5$$

Sending end voltage, $$\vec{V_S} = \vec{V_R} + \vec{I_L}\,\vec{Z} = \vec{V_R} + \vec{I_L}\,(R + j\,X_L)$$
$$= 63{,}\ 508 + (262{\cdot}4 - j\,182{\cdot}5)\ (15 + j\,75)$$
$$= 63{,}508 + 3936 + j\,19{,}680 - j\,2737{\cdot}5 + 13{,}687$$
$$= 81{,}131 + j\,16{,}942{\cdot}5 = 82{,}881\ \angle 11°47'\ \text{V}$$

∴ Line to line sending end voltage = $82{,}881 \times \sqrt{3}$ = 1,43,550 V = **143·55 kV**

Charging current at the sending end is

$$I_{C2} = j\vec{V_S}\ Y/2 = (81{,}131 + j\,16{,}942{\cdot}5)\ j\frac{45 \times 10^{-5}}{2}$$
$$= -3{\cdot}81 + j\,18{\cdot}25$$

Sending end current, $$\vec{I_S} = \vec{I_L} + \vec{I_{C_2}} = (262{\cdot}4 - j\,182{\cdot}5) + (-3{\cdot}81 + j\,18{\cdot}25)$$
$$= 258{\cdot}6 - j\,164{\cdot}25 = 306{\cdot}4\ \angle -32{\cdot}4° \text{ A}$$

∴ Sending end current = **306·4 A**

Example 10·14. *A 100-km long, 3-phase, 50-Hz transmission line has following line constants:*

Resistance/phase/km = 0·1 Ω

Reactance/phase/km = 0·5 Ω

Susceptance/phase/km = 10×10^{-6} S

If the line supplies load of 20 MW at 0·9 p.f. lagging at 66 kV at the receiving end, calculate by nominal π method :

(*i*) *sending end power factor* (*ii*) *regulation*

(*iii*) *transmission efficiency*

Solution. Fig. 10·19 shows the circuit diagram for the line.

Total resistance/phase, $R = 0{\cdot}1 \times 100 = 10\ \Omega$

Total reactance/phase, $X_L = 0{\cdot}5 \times 100 = 50\ \Omega$

Susceptance/phase, $Y = 10 \times 10^{-6} \times 100 = 10 \times 10^{-4}$ S

Receiving end voltage/phase, $V_R = 66 \times 10^3/\sqrt{3} = 38105$ V

Load current, $$I_R = \frac{20 \times 10^6}{\sqrt{3} \times 66 \times 10^3 \times 0{\cdot}9} = 195 \text{ A}$$

$$\cos \phi_R = 0{\cdot}9 \quad ; \quad \sin \phi_R = 0{\cdot}435$$

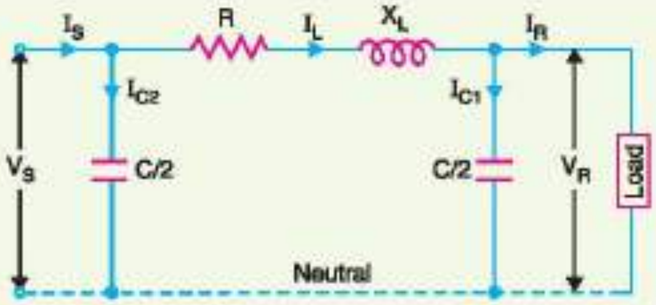

Fig. 10.19

Taking receiving end voltage as the reference phasor, we have,

$$\vec{V_R} = V_R + j0 = 38105 \text{ V}$$

Load current, $$\vec{I_R} = I_R(\cos \phi_R - j \sin \phi_R) = 195\,(0{\cdot}9 - j0{\cdot}435) = 176 - j85$$

Charging current at the receiving end is

$$\vec{I_{C1}} = \vec{V_R}\, j\frac{Y}{2} = 38105 \times j\frac{10 \times 10^{-4}}{2} = j19$$

Line current, $$\vec{I_L} = \vec{I_R} + \vec{I_{C1}} = (176 - j85) + j19 = 176 - j66$$

Sending end voltage, $$\vec{V_S} = \vec{V_R} + \vec{I_L}\,\vec{Z} = \vec{V_R} + \vec{I_L}\,(R + j X_L)$$

$$= 38{,}105 + (176 - j66)\,(10 + j50)$$

$$= 38{,}105 + (5060 + j8140)$$

$$= 43{,}165 + j8140 = 43{,}925 \angle 10{\cdot}65^\circ \text{ V}$$

Sending end line to line voltage $= 43{,}925 \times \sqrt{3} = 76 \times 10^3 \text{ V} = 76$ kV

Charging current at the sending end is

$$\vec{I_{C2}} = \vec{V_S}\, jY/2 = (43165 + j8140)\, j\frac{10 \times 10^{-4}}{2}$$

$$= -4{\cdot}0 + j21{\cdot}6$$

∴ Sending end current, $$\vec{I_S} = \vec{I_L} + \vec{I_{C2}} = (176 - j66) + (-4{\cdot}0 + j21{\cdot}6)$$

$$= 172 - j44{\cdot}4 = 177{\cdot}6 \angle -14{\cdot}5^\circ \text{ A}$$

(i) Referring to phasor diagram in Fig. 10·20,

θ_1 = angle between $\vec{V_R}$ and $\vec{V_S}$ = 10·65°

θ_2 = angle between $\vec{V_R}$ and $\vec{I_S}$ = − 14·5°

∴ ϕ_S = angle between $\vec{V_S}$ and $\vec{I_S}$ = $\theta_2 + \theta_1$

= 14·5° + 10·65° = 25·15°

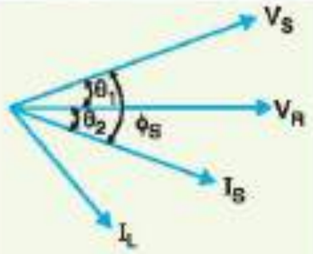

Fig. 10.20

∴ Sending end p.f., $\cos\phi_S = \cos 25{\cdot}15° =$ **0·905 lag**

(ii) % Voltage regulation $= \dfrac{V_S - V_R}{V_R} \times 100 = \dfrac{43925 - 38105}{38105} \times 100 =$ **15·27 %**

(iii) Sending end power $= 3\, V_S\, I_S \cos\phi_S = 3 \times 43925 \times 177{\cdot}6 \times 0{\cdot}905$

$= 21{\cdot}18 \times 10^6$ W = 21·18 MW

Transmission efficiency = (20/21·18) × 100 = **94 %**

TUTORIAL PROBLEMS

1. A (medium) single phase transmission line 100 km long has the following constants :

Resistance/km/phase	= 0·15Ω
Inductive reactance/km/phase	= 0·377 Ω
Capacitive reactance/km/phase	= 31·87 Ω
Receiving end line voltage	= 132 kV

Assuming that the total capacitance of the line is localised at the receiving end alone, determine :

(i) sending end current (ii) line value of sending end voltage

(iii) regulation (iv) sending end power factor

The line is delivering 72 MW at 0·8 p.f. lagging.

[(i) 377·3 A (ii) 155·7 kV (iii) 17·9% (iv) 0·774 lag]

2. A 3-phase, 50 Hz overhead transmission line has the following constants :

Resistance/phase = 9·6 Ω

Inductance/phase = 0·097 mH

Capacitance/phase = 0·765 μF

If the line is supplying a balanced load of 24,000 kVA 0·8 p.f. lagging at 66 kV, calculate :

(i) sending end current (ii) line value of sending end voltage

(iii) sending end power factor (iv) percentage regulation

(v) transmission efficiency **[(i) 204 A (ii) 75 kV (iii) 0·814 lag (iv) 13·63 % (v) 93·7%]**

3. A 3-phase, 50 Hz, overhead transmission line delivers 10 MW at 0·8 p.f. lagging and at 66 kV. The resistance and inductive reactance of the line per phase are 10 Ω and 20 Ω respectively while capacitance admittance is 4×10^{-4} siemen. Calculate :

(i) the sending end current (ii) sending end voltage (line-to-line)

(iii) sending end power factor (iv) transmission efficiency

Use nominal *T* method. **[(i) 100 A (ii) 69·8 kV (iii) 0·852 (iv) 97·5%]**

4. A 3-phase, 50 Hz, 100 km transmission line has the following constants :

Resistance/phase/km = 0·1 Ω

Reactance/phase/km = 0·5 Ω

Susceptance/phase/km = 10^{-5} siemen

If the line supplies a load of 20 MW at 0·9 p.f. lagging at 66 kV at the receiving end, calculate by using nominal π method :

(i) sending end current (ii) line value of sending end voltage

(*iii*) sending end power factor (*iv*) regulation

[(*i*) 177·6 A (*ii*) 76kV (*iii*) 0·905 lag (*iv*) 15·15%]

5. A 3-phase overhead transmission line has the following constants :

Resistance/phase = 10 Ω

Inductive reactance/phase = 35 Ω

Capacitive admittance/phase = 3×10^{-4} siemen

If the line supplied a balanced load of 40,000 kVA at 110 kV and 0·8 p.f. lagging, calculate :

(*i*) sending end power factor (*ii*) percentage regulation

(*iii*) transmission efficiency [(*i*) 0·798 lag (*ii*) 10% (*iii*) 96·38%]

6. A 3-phase, 50 Hz overhead transmission line, 100 km long, 110 kV between the lines at the receiving end has the following constants :

Resistance per km per phase = 0·153 Ω

Inductance per km per phase = 1·21 mH

Capacitance per km per phase = 0·00958 μF

The line supplies a load of 20,000 kW at 0·9 power factor lagging. Calculate using nominal π representation, the sending end voltage, current, power factor, regulation and the efficiency of the line. Neglect leakage. [115·645 kV (line voltage) : 109 $\angle -16·68°$ A : 0·923 lag : 5·13 %; 97·21 %]

(*iii*) sending end power factor (*iv*) regulation

[(*i*) 177·6 A (*ii*) 76kV (*iii*) 0·905 lag (*iv*) 15·15%]

5. A 3-phase overhead transmission line has the following constants :

Resistance/phase = 10 Ω

Inductive reactance/phase = 35 Ω

Capacitive admittance/phase = 3×10^{-4} siemen

If the line supplied a balanced load of 40,000 kVA at 110 kV and 0·8 p.f. lagging, calculate :

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(*iii*) transmission efficiency [(*i*) 0·798 lag (*ii*) 10% (*iii*) 96·38%]

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Capacitance per km per phase = 0·00958 μF

The line supplies a load of 20,000 kW at 0·9 power factor lagging. Calculate using nominal π representation, the sending end voltage, current, power factor, regulation and the efficiency of the line. Neglect leakage. [115·645 kV (line voltage) : 109 ∠ – 16·68° A : 0·923 lag : 5·13 %; 97·21 %]

10.10 Long Transmission Lines

It is well known that line constants of the transmission line are uniformly distributed over the entire length of the line. However, reasonable accuracy can be obtained in line calculations for short and medium lines by considering these constants as lumped. If such an assumption of lumped constants is applied to long transmission lines (having length excess of about 150 km), it is found that serious errors are introduced in the performance calculations. Therefore, in order to obtain fair degree of accuracy in the performance calculations of long lines, the line constants are considered as uniformly distributed throughout the length of the line. Rigorous mathematical treatment is required for the solution of such lines.

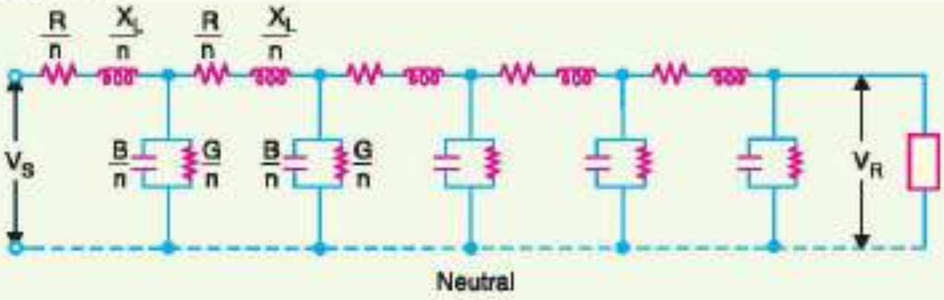


Fig. 10.21

Fig. 10.21 shows the equivalent circuit of a 3-phase long transmission line on a phase-neutral basis. The whole line length is divided into *n* sections, each section having line constants $\frac{1}{n}$ *th* of those for the whole line. The following points may by noted :

(*i*) The line constants are uniformly distributed over the entire length of line as is actually the case.

(*ii*) The resistance and inductive reactance are the series elements.

(*iii*) The leakage susceptance (*B*) and leakage conductance (*G*) are shunt elements. The leakage susceptance is due to the fact that capacitance exists between line and neutral. The leakage conductance takes into account the energy losses occurring through leakage over the insulators or due to corona effect between conductors. Admittance = $\sqrt{G^2 + B^2}$.

(*iv*) The leakage current through shunt admittance is maximum at the sending end of the line and decreases continuously as the receiving end of the circuit is approached at which point its value is zero.

10.11 Analysis of Long Transmission Line (*Rigorous method*)

Fig. 10.22 shows one phase and neutral connection of a 3-phase line with impedance and shunt admittance of the line uniformly distributed.

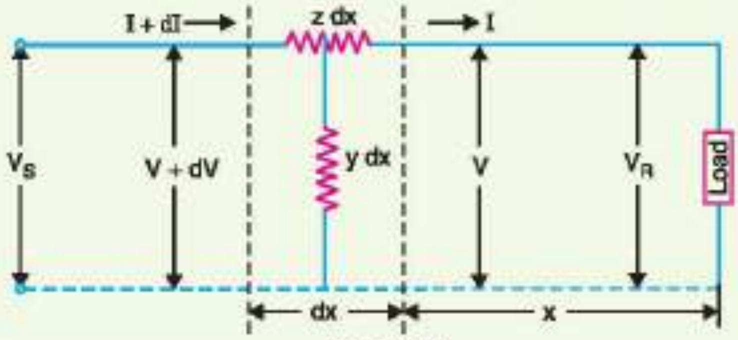


Fig. 10.22

Consider a small element in the line of length dx situated at a distance x from the receiving end.

Let

z = series impedance of the line per unit length

y = shunt admittance of the line per unit length

V = voltage at the end of element towards receiving end

$V + dV$ = voltage at the end of element towards sending end

$I + dI$ = current entering the element dx

I = current leaving the element dx

Then for the small element dx,

$z\,dx$ = series impedance

$y\,dx$ = shunt admittance

Obviously, $dV = I\,z\,dx$

or
$$\frac{dV}{dx} = I\,z \qquad \text{...}(i)$$

Now, the current entering the element is $I + dI$ whereas the current leaving the element is I. The difference in the currents flows through shunt admittance of the element *i.e.*,

dI = Current through shunt admittance of element = $V\,y\,dx$

or
$$\frac{dI}{dx} = V\,y \qquad \text{...}(ii)$$

Differentiating eq. (*i*) w.r.t. x, we get,

$$\frac{d^2V}{dx^2} = z\frac{dI}{dx} = z\,(V\,y) \qquad \left[\because \frac{dI}{dx} = V\,y \text{ from exp. } (ii)\right]$$

or
$$\frac{d^2V}{dx^2} = y\,z\,V \qquad \text{...}(iii)$$

The solution of this differential equation is

$$V = k_1 \cosh\left(x\sqrt{y\,z}\right) + k_2 \sinh\left(x\sqrt{y\,z}\right) \qquad \text{...}(iv)$$

Differentiating exp. (*iv*) w.r.t. *x*, we have,

$$\frac{dV}{dx} = k_1\sqrt{yz}\sinh(x\sqrt{yz}) + k_2\sqrt{yz}\cosh(x\sqrt{yz})$$

But $$\frac{dV}{dx} = Iz \qquad \text{[from exp. (}i\text{)]}$$

$$\therefore \quad Iz = k_1\sqrt{yz}\sinh(x\sqrt{yz}) + k_2\sqrt{zy}\cosh(x\sqrt{yz})$$

or $$I = \sqrt{\frac{y}{z}}\left[k_1\sinh(x\sqrt{yz}) + k_2\cosh\left(x\sqrt{yz}\right)\right] \qquad ...(v)$$

Equations (*iv*) and (*v*) give the expressions for V and I in the form of unknown constants k_1 and k_2. The values of k_1 and k_2 can be found by applying end conditions as under :

$$\text{At } x = 0, \quad V = V_R \text{ and } I = I_R$$

Putting these values in eq. (*iv*), we have,

$$V_R = k_1\cosh 0 + k_2\sinh 0 = k_1 + 0$$

$$\therefore \quad V_R = k_1$$

Similarly, putting $x = 0$, $V = V_R$ and $I = I_R$ in eq. (*v*), we have,

$$I_R = \sqrt{\frac{y}{z}}\left[k_1\sinh 0 + k_2\cosh 0\right] = \sqrt{\frac{y}{z}}\left[0 + k_2\right]$$

$$\therefore \quad k_2 = \sqrt{\frac{z}{y}}\, I_R$$

Substituting the values of k_1 and k_2 in eqs. (*iv*) and (*v*), we get,

$$V = V_R\cosh(x\sqrt{yz}) + \sqrt{\frac{z}{y}}\, I_R\sinh(x\sqrt{yz})$$

and $$I = \sqrt{\frac{y}{z}}\,V_R\sinh(x\sqrt{yz}) + I_R\cosh(x\sqrt{yz})$$

The sending end voltage (V_S) and sending end current (I_S) are obtained by putting $x = l$ in the above equations *i.e.*,

$$V_S = V_R\cosh(l\sqrt{yz}) + \sqrt{\frac{z}{y}}\,I_R\sinh(l\sqrt{yz})$$

$$I_S = \sqrt{\frac{y}{z}}\,V_R\sinh(l\sqrt{yz}) + I_R\cosh(l\sqrt{yz})$$

Now, $$l\sqrt{yz} = \sqrt{ly\cdot lz} = \sqrt{YZ}$$

and $$\sqrt{\frac{y}{z}} = \sqrt{\frac{yl}{zl}} = \sqrt{\frac{Y}{Z}}$$

where Y = total shunt admittance of the line

Z = total series impedance of the line

Therefore, expressions for V_S and I_S become :

$$\mathbf{V_S = V_R\cosh\sqrt{YZ} + I_R\sqrt{\frac{Z}{Y}}\sinh\sqrt{YZ}}$$

$$\mathbf{I_S = V_R\sqrt{\frac{Y}{Z}}\sinh\sqrt{YZ} + I_R\cosh\sqrt{YZ}}$$

It is helpful to expand hyperbolic sine and cosine in terms of their power series.

$$\cosh\sqrt{YZ} = \left(1 + \frac{ZY}{2} + \frac{Z^2Y^2}{24} + \ldots\ldots\right)$$

$$\sinh\sqrt{YZ} = \left(\sqrt{YZ} + \frac{(YZ)^{3/2}}{6} + \ldots\ldots\right)$$

Example 10.15. *A 3-φ transmission line 200 km long has the following constants :*

Resistance/phase/km = *0·16 Ω*

Reactance/phase/km = *0·25 Ω*

Shunt admittance/phase/km = *1·5 × 10⁻⁶ S*

Calculate by rigorous method the sending end voltage and current when the line is delivering a load of 20 MW at 0·8 p.f. lagging. The receiving end voltage is kept constant at 110 kV.

Solution :

Total resistance/phase, $R = 0{\cdot}16 \times 200 = 32\ \Omega$

Total reactance/phase, $X_L = 0{\cdot}25 \times 200 = 50\ \Omega$

Total shunt admittance/phase, $Y = j\,1{\cdot}5 \times 10^{-6} \times 200 = 0{\cdot}0003 \angle 90°$

Series Impedance/phase, $Z = R + j X_L = 32 + j\,50 = 59{\cdot}4 \angle 58°$

The sending end voltage V_S per phase is given by :

$$V_S = V_R \cosh\sqrt{YZ} + I_R\sqrt{\frac{Z}{Y}}\sinh\sqrt{ZY} \quad \ldots(i)$$

Now

$$\sqrt{ZY} = \sqrt{59{\cdot}4 \angle 58° \times 0{\cdot}0003 \angle 90°} = 0{\cdot}133 \angle 74°$$

$$ZY = 0{\cdot}0178 \angle 148°$$

$$Z^2Y^2 = 0{\cdot}00032 \angle 296°$$

$$\sqrt{\frac{Z}{Y}} = \sqrt{\frac{59{\cdot}4 \angle 58°}{0{\cdot}0003 \angle 90°}} = 445 \angle -16°$$

$$\sqrt{\frac{Y}{Z}} = \sqrt{\frac{0{\cdot}0003 \angle 90°}{59{\cdot}4 \angle 58°}} = 0{\cdot}00224 \angle 16°$$

∴

$$\cosh\sqrt{YZ} = 1 + \frac{ZY}{2} + \frac{Z^2Y^2}{24} \text{ approximately}$$

$$= 1 + \frac{0{\cdot}0178}{2}\angle 148° + \frac{0{\cdot}00032}{24}\angle 296°$$

$$= 1 + 0{\cdot}0089 \angle 148° + 0{\cdot}0000133 \angle 296°$$

$$= 1 + 0{\cdot}0089\,(-0{\cdot}848 + j\,0{\cdot}529) + 0{\cdot}0000133\,(0{\cdot}438 - j\,0{\cdot}9)$$

$$= 0{\cdot}992 + j\,0{\cdot}00469 = 0{\cdot}992 \angle 0{\cdot}26°$$

$$\sinh\sqrt{YZ} = \sqrt{YZ} + \frac{(YZ)^{3/2}}{6} \text{ approximately}$$

$$= 0{\cdot}133 \angle 74° + \frac{0{\cdot}0024 \angle 222°}{6}$$

$$= 0{\cdot}133 \angle 74° + 0{\cdot}0004 \angle 222°$$

$$= 0{\cdot}133\,(0{\cdot}275 + j\,0{\cdot}961) + 0{\cdot}0004\,(-0{\cdot}743 - j\,0{\cdot}67)$$

$$= 0{\cdot}0362 + j\,0{\cdot}1275 = 0{\cdot}1325 \angle 74°6'$$

Receiving end voltage per phase is

$$V_R = 110 \times 10^3/\sqrt{3} = 63508 \text{ V}$$

Receiving end current,

$$I_R = \frac{20 \times 10^6}{\sqrt{3} \times 110 \times 10^3 \times 0{\cdot}8} = 131\ A$$

Putting the various values in exp (*i*), we get,

$$V_S = 63508 \times 0.992 \angle 0.26° + 131 \times 445 \angle -16°0' \times 0.1325 \angle 74°6'$$
$$= 63000 \angle 0.26° + 7724 \angle 58°6'$$
$$= 63000\ (0.999 + j\,0.0045) + 7724\ (0.5284 + j\,0.8489)$$
$$= 67018 + j\,6840 = 67366 \angle 5°50'\ \text{V}$$

Sending end line-to-line voltage $= 67366 \times \sqrt{3} = 116.67 \times 10^3$ V = **116·67 kV**

The sending end current I_S is given by :

$$I_S = V_R \sqrt{\frac{Y}{Z}} \sinh \sqrt{YZ} + I_R \cosh \sqrt{YZ}$$

Putting the various values, we get,

$$I_S = 63508 \times 0.00224 \angle 16° \times 0.1325 \angle 74°6' + 131 \times 0.992 \angle 0.26°$$
$$= 18.85 \angle 90°6' + 130 \angle 0.26°$$
$$= 18.85\ (-0.0017 + j\,0.999) + 130\ (0.999 + j\,0.0045)$$
$$= 129.83 + j\,19.42 = 131.1 \angle 8°\ \text{A}$$

∴ Sending end current = **131·1 A**

TUTORIAL PROBLEMS

1. A 3-phase overhead transmission line has a total series impedance per phase of 200 $\angle 80°$ ohms and a total shunt admittance of 0·0013$\angle 90°$ siemen per phase. The line delivers a load of 80 MW at 0·8 p.f. lagging and 220 kV between the lines. Determine the sending end line voltage and current by rigorous method. **[263·574 kV ; 187·5 A]**
2. A 3-phase transmission line, 160 km long, has the following constants :

   Resistance/phase/km = 0·2 Ω

   Reactance/phase/km = 0·3127 Ω

   Shunt admittance/phase/km = 1.875×10^{-6} S

   Determine the sending end voltage and current by rigorous method when the line is delivering a load of 25 MVA at 0·8 p.f. lagging. The receiving end voltage is kept constant at 110 kV. **[116·67 kV ; 131·1 A]**

10.12 Generalised Circuit Constants of a Transmission Line

In any four terminal *network, the input voltage and input current can be expressed in terms of output voltage and output current. Incidentally, a transmission line is a 4-terminal network ; two input terminals where power enters the network and two output terminals where power leaves the network.

Therefore, the input voltage $(\vec{V}_S)$ and input current $(\vec{I}_S)$ of a 3-phase transmission line can be expressed as :

$$\vec{V}_S = \vec{A}\,\vec{V}_R + \vec{B}\,\vec{I}_R$$
$$\vec{I}_S = \vec{C}\,\vec{V}_R + \vec{D}\,\vec{I}_R$$

where $\vec{V}_S$ = sending end voltage per phase

$\vec{I}_S$ = sending end current

$\vec{V}_R$ = receiving end voltage per phase

$\vec{I}_R$ = receiving end current

* The network should be *passive* (containing no source of e.m.f.), *linear* (impedances independent of current flowing) and *bilateral* (impedances independent of direction of current flowing). This condition is fully met in transmission lines.

and $\vec{A}, \vec{B}, \vec{C}$ and $\vec{D}$ (generally complex numbers) are the constants known as *generalised circuit constants* of the transmission line. The values of these constants depned upon the particular method adopted for solving a transmission line. Once the values of these constants are known, performance calculations of the line can be easily worked out. The following points may be kept in mind :

(*i*) The constants $\vec{A}, \vec{B}, \vec{C}$ and $\vec{D}$ are generally complex numbers.

(*ii*) The constants $\vec{A}$ and $\vec{D}$ are dimensionless whereas the dimensions of $\vec{B}$ and $\vec{C}$ are ohms and siemen respectively.

(*iii*) For a given transmisson line,

$$\vec{A} = \vec{D}$$

(*iv*) For a given transmission line,

$$\vec{A}\vec{D} - \vec{B}\vec{C} = 1$$

We shall establish the correctness of above characteristics of generalised circuit constants in the following discussion.

10.13 Determination of Generalised Constants for Transmission Lines

As stated previously, the sending end voltage ($\vec{V_S}$) and sending end current ($\vec{I_S}$) of a transmission line can be expressed as :

$$\vec{V_S} = \vec{A}\,\vec{V_R} + \vec{B}\,\vec{I_R} \quad \text{...(i)}$$

$$\vec{I_S} = \vec{C}\,\vec{V_R} + \vec{D}\,\vec{I_R} \quad \text{...(ii)}$$

We shall now determine the values of these constants for different types of transmission lines.

(*i*) **Short lines.** In short transmission lines, the effect of line capacitance is neglected. Therefore, the line is considered to have series impedance. Fig. 10.23 shows the circuit of a 3-phase transmission line on a single phase basis.

Here, $$\vec{I_S} = \vec{I_R} \quad \text{...(iii)}$$

and $$\vec{V_S} = \vec{V_R} + \vec{I_R}\,\vec{Z} \quad \text{...(iv)}$$

Comparing these with eqs. (*i*) and (*ii*), we have,

$$\vec{A} = 1;\quad \vec{B} = \vec{Z},\quad \vec{C} = 0 \quad \text{and} \quad \vec{D} = 1$$

Incidentally : $\vec{A} = \vec{D}$

and $$\vec{A}\vec{D} - \vec{B}\vec{C} = 1 \times 1 - \vec{Z} \times 0 = 1$$

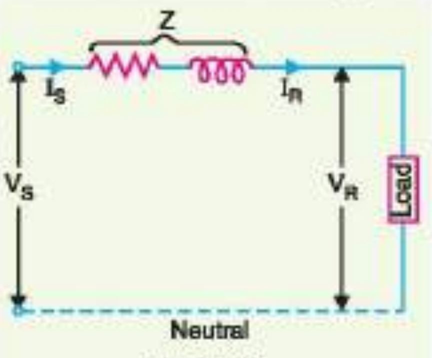


Fig. 10.23

(*ii*) **Medium lines – Nominal T method.** In this method, the whole line to neutral capacitance is assumed to be concentrated at the middle point of the line and half the line resistance and reactance are lumped on either side as shown in Fig. 10.24.

Here, $$\vec{V_S} = \vec{V_1} + \vec{I_S}\,\vec{Z}/2 \quad \text{...(v)}$$

and $$\vec{V_1} = \vec{V_R} + \vec{I_R}\,\vec{Z}/2$$

Now, $$\vec{I_C} = \vec{I_S} - \vec{I_R}$$

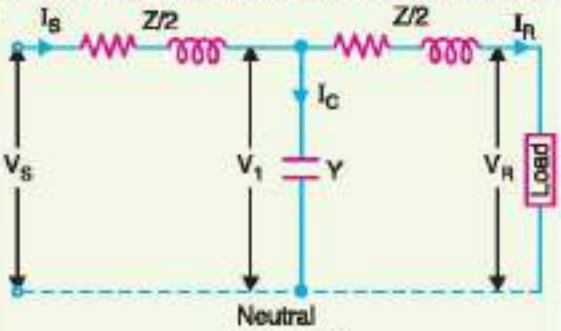


Fig. 10.24

$$= \vec{V_1}\vec{Y} \text{ where } Y = \text{shunt admittance}$$

$$= \vec{Y}\left(\vec{V_R} + \frac{\vec{I_R}\vec{Z}}{2}\right)$$

$$\therefore \quad \vec{I_S} = \vec{I_R} + \vec{Y}\vec{V_R} + \vec{Y}\frac{\vec{I_R}\vec{Z}}{2}$$

$$= \vec{Y}\vec{V_R} + \vec{I_R}\left(1 + \frac{\vec{Y}\vec{Z}}{2}\right) \quad ...(vi)$$

Substituting the value of V_1 in eq. (*v*), we get,

$$\vec{V_S} = \vec{V_R} + \frac{\vec{I_R}\vec{Z}}{2} + \frac{\vec{I_S}\vec{Z}}{2}$$

Substituing the value of I_S, we get,

$$\vec{V_S} = \left(1 + \frac{\vec{Y}\vec{Z}}{2}\right)\vec{V_R} + \left(\vec{Z} + \frac{\vec{Y}\vec{Z}^2}{4}\right)\vec{I_R} \quad ...(vii)$$

Comparing eqs. (*vii*) and (*vi*) with those of (*i*) and (*ii*), we have,

$$\vec{A} = \vec{D} = 1 + \frac{\vec{Y}\vec{Z}}{2}; \quad \vec{B} = \vec{Z}\left(1 + \frac{\vec{Y}\vec{Z}}{4}\right); \vec{C} = \vec{Y}$$

Incidentally :

$$\vec{A}\vec{D} - \vec{B}\vec{C} = \left(1 + \frac{YZ}{2}\right)^2 - Z\left(1 + \frac{YZ}{4}\right)Y$$

$$= 1 + \frac{Y^2Z^2}{4} + YZ - ZY - \frac{Z^2Y^2}{4} = 1$$

(*iii*) Medium lines—Nominal π method. In this method, line-to-neutral capacitance is divided into two halves ; one half being concentrated at the load end and the other half at the sending end as shown in Fig. 10.25.

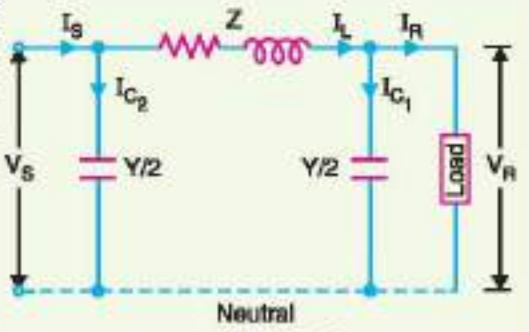


Fig. 10.25

Here, $\vec{Z} = R + jX_L$ = series impedenace/phase

$\vec{Y} = j\omega C$ = shunt admittance

$$\vec{I_S} = \vec{I_L} + \vec{I_{C2}}$$

or

$$\vec{I_S} = \vec{I_L} + \vec{V_S}\vec{Y}/2 \quad ...(viii)$$

Also

$$\vec{I_L} = \vec{I_R} + \vec{I_{C1}} = \vec{I_R} + \vec{V_R}\vec{Y}/2 \quad ...(ix)$$

Now

$$\vec{V_S} = \vec{V_R} + \vec{I_L}\vec{Z} = \vec{V_R} + \left(\vec{I_R} + \vec{V_R}\vec{Y}/2\right)\vec{Z} \text{ (Putting the value of } \vec{I_L})$$

$$\therefore \quad \vec{V_S} = \vec{V_R}\left(1 + \frac{\vec{Y}\vec{Z}}{2}\right) + \vec{I_R}\vec{Z} \quad ...(x)$$

Also

$$\vec{I_S} = \vec{I_L} + \vec{V_S}\vec{Y}/2 = \left(\vec{I_R} + \vec{V_R}\vec{Y}/2\right) + \vec{V_S}\vec{Y}/2$$

(Putting the value of $\vec{I_L}$)

Putting the value of $\vec{V_S}$ from eq. (*x*), we get,

$$\vec{I_S} = \vec{I_R} + \vec{V_R}\frac{\vec{Y}}{2} + \frac{\vec{Y}}{2}\left[\vec{V_R}\left(1 + \frac{\vec{Y}\vec{Z}}{2}\right) + \vec{I_R}\vec{Z}\right]$$

$$= \vec{I}_R + \vec{V}_R\frac{\vec{Y}}{2} + \frac{\vec{V}_R\vec{Y}}{2} + \frac{\vec{V}_R\vec{Y}^2\vec{Z}}{4} + \frac{\vec{Y}\vec{I}_R\vec{Z}}{2}$$

$$= \vec{I}_R\left(1+\frac{\vec{Y}\vec{Z}}{2}\right) + \vec{V}_R\vec{Y}\left(1+\frac{\vec{Y}\vec{Z}}{4}\right) \quad ...(xi)$$

Comparing equations (*x*) and (*xi*) with those of (*i*) and (*ii*), we get,

$$\vec{A} = \vec{D} = \left(1+\frac{\vec{Y}\vec{Z}}{2}\right); \ \vec{B} = \vec{Z}; \ \vec{C} = \vec{Y}\left(1+\frac{\vec{Y}\vec{Z}}{4}\right)$$

Also

$$\vec{A}\vec{D} - \vec{B}\vec{C} = \left(1+\frac{YZ}{2}\right)^2 - ZY\left(1+\frac{YZ}{4}\right)$$

$$= 1 + \frac{Y^2Z^2}{4} + YZ - ZY - \frac{Z^2Y^2}{4} = 1$$

(*iv*) **Long lines—Rigorous method.** By rigorous method, the sending end voltage and current of a long transmission line are given by :

$$V_S = V_R \cosh\sqrt{YZ} + I_R\sqrt{\frac{Z}{Y}}\sinh\sqrt{YZ}$$

$$I_S = V_R\sqrt{\frac{Y}{Z}}\sinh\sqrt{YZ} + I_R\cosh\sqrt{YZ}$$

Comparing these equations with those of (*i*) and (*ii*), we get,

$$\vec{A} = \vec{D} = \cosh\sqrt{YZ}; \ \vec{B} = \sqrt{\frac{Z}{Y}}\sinh\sqrt{YZ}; \quad \vec{C} = \sqrt{\frac{Y}{Z}}\sinh\sqrt{YZ}$$

Incidentally

$$\vec{A}\vec{D} - \vec{B}\vec{C} = \cosh\sqrt{YZ}\times\cosh\sqrt{YZ} - \sqrt{\frac{Z}{Y}}\sinh\sqrt{YZ}\times\sqrt{\frac{Y}{Z}}\sinh\sqrt{YZ}$$

$$= \cosh^2\sqrt{YZ} - \sinh^2\sqrt{YZ} = 1$$

Example 10.16. *A balanced 3-phase load of 30 MW is supplied at 132 kV, 50 Hz and 0·85 p.f. lagging by means of a transmission line. The series impedance of a single conductor is (20 + j52) ohms and the total phase-neutral admittance is 315 × 10⁻⁶ siemen. Using nominal T method, determine: (i) the A, B, C and D constants of the line (ii) sending end voltage (iii) regulation of the line.*

Solution. Fig. 10.26 shows the representation of 3-phase line on the single phase basis.

Series line impedance/phase, $\vec{Z} = (20 + j\,52)\ \Omega$

Shunt admittance/phase, $\vec{Y} = j\,315 \times 10^{-6}$ S

(*i*) **Generalised constants of line.** For nominal *T* method, various constants have the values as under :

$$\vec{A} = \vec{D} = 1 + \vec{Z}\,\vec{Y}/2 = 1 + \frac{20 + j\,52}{2} \times j\,315 \times 10^{-6}$$

$$= 0{\cdot}992 + j\,0{\cdot}00315 = \mathbf{0{\cdot}992\ \angle 0{\cdot}18°}$$

$$\vec{B} = \vec{Z}\left(1+\frac{\vec{Z}\vec{Y}}{4}\right) = (20 + j\,52)\left[1 + \frac{(20 + j\,52)\,j\,315\times10^{-6}}{4}\right]$$

$$= 19{\cdot}84 + j\,51{\cdot}82 = \mathbf{55{\cdot}5\ \angle 69°}$$

$$\vec{C} = \vec{Y} = \mathbf{0{\cdot}000315\ \angle 90°}$$

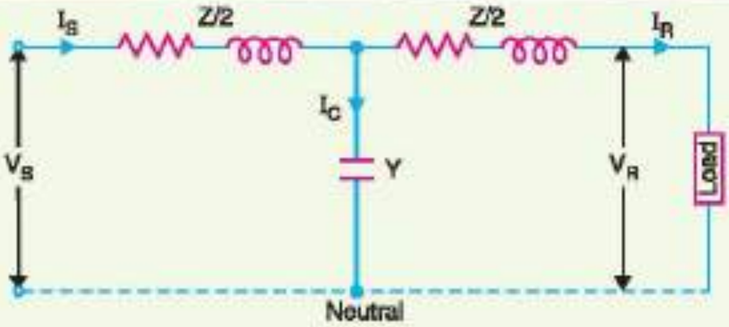


Fig. 10.26

(*ii*) **Sending end voltage.**

Receiving end voltage/phase, $V_R = 132 \times 10^3/\sqrt{3} = 76210$ V

Receiving end current, $I_R = \dfrac{30 \times 10^6}{\sqrt{3} \times 132 \times 10^3 \times 0{\cdot}85} = 154$ A

$\cos\phi_R = 0{\cdot}85;\quad \sin\phi_R = 0{\cdot}53$

Taking receiving end voltage as the reference phasor, we have,

$$\vec{V_R} = V_R + j0 = 76210 \text{ V}$$

$$\vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R) = 154\,(0{\cdot}85 - j\,0{\cdot}53) = 131 - j\,81{\cdot}62$$

Sending end voltage per phase is

$$\vec{V_S} = \vec{A}\,\vec{V_R} + \vec{B}\,\vec{I_R}$$
$$= (0{\cdot}992 + j\,0{\cdot}0032)\ 76210 + (19{\cdot}84 + j\,51{\cdot}82)\ (131 - j\,81{\cdot}62)$$
$$= 82{,}428 + j\,5413$$

∴ Magnitude of sending end voltage is

$$V_S = \sqrt{(82{,}428)^2 + (5413)^2} = 82{\cdot}6 \times 10^3 \text{V} = 82{\cdot}6 \text{ kV}$$

∴ Sending end line-to-line voltage

$$= 82{\cdot}6 \times \sqrt{3} = \mathbf{143\ kV}$$

(*iii*) **Regulation.** Regulation is defined as the change in voltage at the receiving end when full-load is thrown off.

Now, $\vec{V_S} = \vec{A}\,\vec{V_R} + \vec{B}\,\vec{I_R}$

At no load, $\vec{I_R} = 0$

∴ $\vec{V_S} = \vec{A}\,\vec{V_{R0}}$

where $\vec{V_{R0}}$ = voltage at receiving end at no load

or $\vec{V_{R0}} = \vec{V_S}/\vec{A}$

or $V_{R0} = V_S/A$ (in magnitude)

∴ % Regulation $= \dfrac{(V_S/A - V_R)}{V_R} \times 100 = \dfrac{(82{\cdot}6/0{\cdot}992) - 76{\cdot}21}{76{\cdot}21} \times 100 = \mathbf{9{\cdot}25\%}$

Example 10.17. *A 132 kV, 50 Hz, 3-phase transmission line delivers a load of 50 MW at 0·8 p.f. lagging at the receiving end. The generalised constants of the transmission line are :*

$$A = D = 0{\cdot}95\ \angle 1{\cdot}4°;\ B = 96\ \angle 78°;\ C = 0{\cdot}0015\ \angle 90°$$

Find the regulation of the line and charging current. Use Nominal-T method.

Solution.

Receiving end voltage/phase, $V_R = 132 \times 10^3/\sqrt{3} = 76210$ V

Receiving end current, $I_R = \dfrac{50 \times 10^6}{\sqrt{3} \times 132 \times 10^3 \times 0.8} = 273$ A

$\cos \phi_R = 0.8; \quad \sin \phi_R = 0.6$

Taking receiving end voltage as the reference phasor, we have,

$$\vec{V_R} = V_R + j0 = 76210 \angle 0°$$

$$\vec{I_R} = I_R \angle -\phi_R = 273 \angle -36.9°$$

Sending end voltage per phase is

$$\begin{aligned}
\vec{V_S} &= \vec{A}\,\vec{V_R} + \vec{B}\,\vec{I_R}\\
&= 0.95 \angle 1.4° \times 76210 \angle 0° + 96 \angle 78° \times 273 \angle -36.9°\\
&= 72400 \angle 1.4° + 26208 \angle 41.1°\\
&= 72400 (\cos 1.4° + j \sin 1.4°) + 26208 (\cos 41.1° + j \sin 41.1°)\\
&= 72400 (0.9997 + j\,0.0244) + 26208 (0.7536 + j\,0.6574)\\
&= (72378 + j\,1767) + (19750 + j\,17229)\\
&= 92128 + j\,18996 = 94066 \angle 11.65° \text{ V}
\end{aligned}$$

Sending end current,

$$\begin{aligned}
\vec{I_S} &= \vec{C}\,\vec{V_R} + \vec{D}\,\vec{I_R}\\
&= 0.0015 \angle 90° \times 76210 \angle 0° + 0.95 \angle 1.4° \times 273 \angle -36.9°\\
&= 114 \angle 90° + 260 \angle -35.5°\\
&= 114 (\cos 90° + j \sin 90°) + 260 (\cos 35.5° - j \sin 35.5°)\\
&= 114 (0 + j) + 260 (0.814 - j\,0.58)\\
&= j\,114 + 211 - j\,150 = 211 - j\,36
\end{aligned}$$

Charging current,

$$\begin{aligned}
\vec{I_C} &= \vec{I_S} - \vec{I_R} = (211 - j36) - 273 \angle -36.9°\\
&= (211 - j\,36) - (218 - j\,164) = -7 + j\,128 = \mathbf{128.2 \angle 93.1° \text{ A}}
\end{aligned}$$

$$\% \text{ Regulation} = \frac{(V_S/A) - V_R}{V_R} \times 100 = \frac{94066/0.95 - 76210}{76210} \times 100 = \mathbf{30\%}$$

Example 10.18. *Find the following for a single circuit transmission line delivering a load of 50 MVA at 110 kV and p.f. 0·8 lagging :*

(i) sending end voltage (ii) sending end current (iii) sending end power (iv) efficiency of transmission. Given $A = D = 0.98 \angle 3°$; $B = 110 \angle 75°$ ohm ; $C = 0.0005 \angle 80°$ siemen.

Solution.

Receiving end voltage/phase,

$$V_R = \frac{110}{\sqrt{3}} = 63.5 \text{ kV}$$

Receiving end current, $I_R = \dfrac{50 \times 10^6}{\sqrt{3} \times 110 \times 10^3} = 262.4$ A

Taking receiving end voltage as the reference phasor, we have,

$$\vec{V_R} = (63500 + j0)$$

$$\vec{I_R} = 262\cdot4 \angle -\cos^{-1} 0\cdot8 = 262\cdot4\,(0\cdot8 - j\,0\cdot6) = (210 - j\,157\cdot5) \text{ A}$$

(*i*) Now sending-end voltage per phase is

$$\vec{V_S} = \vec{A}\,\vec{V_R} + \vec{B}\,\vec{I_R}$$

Here $\vec{A}\,\vec{V_R} = 0\cdot98 \angle 3° \times 63500 \angle 0° = 62230 \angle 3° = (62145 + j\,3260) \text{ V}$

and $\vec{B}\,\vec{I_R} = 110 \angle 75° \times 262\cdot4 \angle -36\cdot86°$

$$= 28865 \angle 38\cdot14° = (22702 + j\,17826) \text{ V}$$

$\therefore$ $\vec{V_S} = (62145 + j\,3260) + (22702 + j\,17826)$

$$= 84847 + j\,21086 = 87427 \angle 14° \text{ V}$$

$\therefore$ Magnitude of sending-end voltage/phase = **87427 V**

(*ii*) Sending-end current is given by :

$$\vec{I_S} = \vec{C}\,\vec{V_R} + \vec{D}\,\vec{I_R}$$

Here $\vec{C}\,\vec{V_R} = 0\cdot0005 \angle 80° \times 63500 \angle 0° = 31\cdot75 \angle 80° = (5\cdot5 + j\,31\cdot3) \text{ A}$

and $\vec{D}\,\vec{I_R} = 0\cdot98 \angle 3° \times 262\cdot4 \angle -36\cdot86°$

$$= 257\cdot15 \angle -33\cdot8° = (213\cdot5 - j\,143\cdot3) \text{ A}$$

$\therefore$ $\vec{I_S} = (5\cdot5 + j\,31\cdot3) + (213\cdot5 - j\,143\cdot3)$

$$= 219 - j\,112 = 246 \angle -27° \text{ A}$$

$\therefore$ Magnitude of sending-end current = **246 A**

(*iii*) Sending-end power $= 3\, V_S\, I_S \cos\phi_S$

Here $V_S = 87427$ V ; $I_S = 246$ A ; $\cos\phi_S = \cos(-27° - 14°)$

$\therefore$ Sending-end power $= 3 \times 87427 \times 246 \times \cos(-27° - 14°)$

$= 48\cdot6 \times 10^6$ W = **48·6 MW**

(*iv*) Receiving end power $= 50 \times 0\cdot8 = 40$ MW

Transmission efficiency, $\eta = \dfrac{40}{48\cdot6} \times 100 =$ **82·3%**

TUTORIAL PROBLEMS

1. A 150 km, 3-ϕ, 110 kV, 50 Hz transmission line transmits a load of 40,000 kW at 0·8 p.f. lagging at the receiving end. Resistance/km/phase = 0·15Ω ; reactance/km/phase = 0·6 Ω; susceptance/km/phase = 10^{-5} S. Determine (*i*) the *A, B, C* and *D* constants of the line (*ii*) regulation of the line.
   [(*i*) A = D = 0·968 ∠1° ; B = 92·8 ∠7·5°Ω ; C = 0·00145 ∠90·5° S (*ii*) 33·5%]
2. A balanced load of 30 MW is supplied at 132 kV, 50 Hz and 0·85 p.f. lagging by means of a transmission line. The series impedance of a single conductor is (20 + j52) ohms and the total phases-neutral admittance is 315 microsiemens. Shunt leakage may be neglected. Using the nominal *T* approximation, calculate the line voltage at the sending end of the line. If the load is removed and the sending end voltage remains constant, find the percentage rise in voltage at the receiving end.
   [143 kV; 9%]
3. Calculate *A, B, C* and *D* constants of a 3-phase, 50 Hz transmission line 160 km long having the following distributed parameters :
   $R = 0\cdot15$ Ω/km ; $L = 1\cdot20 \times 10^{-3}$ H/km ; $C = 8 \times 10^{-9}$ F/km ; $G = 0$
   [A = D = 0·988 ∠0·3° ; B = 64·2 ∠ 68·3° Ω ; C = 0·4 × 10^{-3} ∠90·2° S]

SELF-TEST

1. **Fill in the blanks by inserting appropriate words/figures.**
   (*i*) In short transmission lines, the effects of .... are neglected.
   (*ii*) .... of transmission lines, is the most important cause of power loss in the line.
   (*iii*) In the analysis of 3-phase transmission line, only .... is considered.
   (*iv*) For a given V_R and I, the regulation of the line .... with the decrease in p.f. for lagging loads.
   (*v*) If the p.f. of the load decreases, the line losses ....
   (*vi*) In medium transmission lines, effects of .... are taken into account.
   (*vii*) The rigorous solution of transmission lines takes into account the .... nature of line constants.
   (*viii*) In any transmission line, $AD - BC =$ ....
   (*ix*) In a transmission line, generalised constants .... and .... are equal.
   (*x*) The dimensions of constants B and C are respectively .... and ....
2. **Pick up the correct words/figures from the brackets and fill in the blanks.**
   (*i*) The line constants of a transmission line are .... [*uniformly distributed, lumped*]
   (*ii*) The length of a short transmission line is upto about .... [*50 km, 120 km, 200 km*]
   (*iii*) The capacitance of a transmission line is a .... element. [*series, shunt*]
   (*iv*) It is desirable that voltage regulation of a transmission line should be .... [*low, high*]
   (*v*) When the regulation is positive, then receiving and voltage (V_R) is .... than sending and voltage (V_S). [*more, less*]
   (*vi*) The shunt admittance of a transmission line is 3 microsiemens. Its complex notation will be .... siemen. [$3 \times 10^{-6} \angle 90°$, $3 \times 10^{-6} \angle 0°$]
   (*vii*) The exact solution of any transmission line must consider the fact that line constants are .... [*uniformly distributed, lumped*]
   (*viii*) The generalised constants A and D of the transmission line have .... [*no dimensions, dimensions of ohm*]
   (*ix*) $30 \angle 10° \times 60 \angle 20° =$ .... [$2 \angle 2°$, $1800 \angle 30°$, $1800 \angle 2°$]
   (*x*) $\sqrt{9 \angle 90° \times 4 \angle 10°} =$ .... [$6 \angle 50°$, $6 \angle 80°$, $6 \angle 10°$]

ANSWERS TO SELF-TEST

1. (*i*) capacitance (*ii*) resistance (*iii*) one phase (*iv*) increases (*v*) increase (*vi*) capacitance (*vii*) distributed (*viii*) 1 (*ix*) A and D (*x*) ohm, siemen
2. (*i*) uniformly distributed (*ii*) 50 km (*iii*) shunt (*iv*) low (*v*) less (*vi*) $3 \times 10^{-6} \angle 90°$ (*vii*) uniformly distributed (*viii*) no dimensions (*ix*) $1800 \angle 30°$ (*x*) $6 \angle 50°$

CHAPTER REVIEW TOPICS

1. What is the purpose of an overhead transmission line ? How are these lines classified ?
2. Discuss the terms voltage regulation and transmission efficiency as applied to transmission line.
3. Deduce an expression for voltage regulation of a short transmission line, giving the vector diagram.
4. What is the effect of load power factor on regulation and efficiency of a transmission line ?
5. What do you understand by medium transmission lines ? How capacitance effects are taken into account in such lines ?
6. Show how regulation and transmission efficiency are determined for medium lines using
   (*i*) end condensor method
   (*ii*) nominal T method
   (*iii*) nominal π method

   Illustrate your answer with suitable vector diagrams.

7. What do you understand by long transmission lines ? How capacitance effects are taken into account in such lines ?
8. Using rigorous method, derive expressions for sending end voltage and current for a long transmission line.
9. What do you understand by generalised circuit constants of a transmission line ? What is their importance?
10. Evaluate the generalised circuit constants for
    (*i*) short transmission line
    (*ii*) medium line — nominal *T* method
    (*iii*) medium line — nominal π method

DISCUSSION QUESTIONS

1. What is the justification in neglecting line capacitance in short transmission lines ?
2. What are the drawbacks of localised capacitance methods ?
3. A long transmission line is open circuited at the receiving end. Will there be any current in the line at the sending end ? Explain your answer.
4. Why is leakage conductance negligible in overhead lines ? What about underground system ?
5. Why do we analyse a 3-phase transmission line on single phase basis ?

CONTENTS

CHAPTER

12

Distribution Systems – General

Introduction

The electrical energy produced at the generating station is conveyed to the consumers through a network of transmission and distribution systems. It is often difficult to draw a line between the transmission and distribution systems of a large power system. It is impossible to distinguish the two merely by their voltage because what was considered as a high voltage a few years ago is now considered as a low voltage. In general, distribution system is that part of power system which distributes power to the consumers for utilisation.

The transmission and distribution systems are similar to man's circulatory system. The transmission system may be compared with arteries in the human body and distribution system with capillaries. They serve the same purpose of supplying the ultimate consumer in the city with the life-giving blood of civilisation–electricity. In this chapter, we shall confine our attention to the general introduction to distribution system.

12.1 Distribution System

That part of power system which distributes electric power for local use is known as **distribution system.**

CONTENTS

In general, the distribution system is the electrical system between the sub-station fed by the transmission system and the consumers meters. It generally consists of *feeders, distributors* and the *service mains*. Fig. 12.1 shows the single line diagram of a typical low tension distribution system.

(*i*) *Feeders.* A feeder is a conductor which connects the sub-station (or localised generating station) to the area where power is to be distributed. Generally, no tappings are taken from the feeder so that current in it remains the same throughout. The main consideration in the design of a feeder is the current carrying capacity.

(*ii*) *Distributor.* A distributor is a conductor from which tappings are taken for supply to the consumers. In Fig. 12.1, *AB, BC, CD* and *DA* are the distributors. The current through a distributor is not constant because tappings are taken at various places along its length. While designing a distributor, voltage drop along its length is the main consideration since the statutory limit of voltage variations is ± 6% of rated value at the consumers' terminals.

(*iii*) *Service mains.* A service mains is generally a small cable which connects the distributor to the consumers' terminals.

12.2 Classification of Distribution Systems

A distribution system may be classified according to ;

(*i*) *Nature of current.* According to nature of current, distribution system may be classified as (*a*) d.c. distribution system (*b*) a.c. distribution system. Now-a-days, a.c. system is universally adopted for distribution of electric power as it is simpler and more economical than direct current method.

(*ii*) *Type of construction.* According to type of construction, distribution system may be classified as (*a*) overhead system (*b*) underground system. The overhead system is generally employed for distribution as it is 5 to 10 times cheaper than the equivalent underground system. In general, the underground system is used at places where overhead construction is impracticable or prohibited by the local laws.

(*iii*) *Scheme of connection.* According to scheme of connection, the distribution system may be classified as (*a*) radial system (*b*) ring main system (*c*) inter-connected system. Each scheme has its own advantages and disadvantages and those are discussed in Art.12.7.

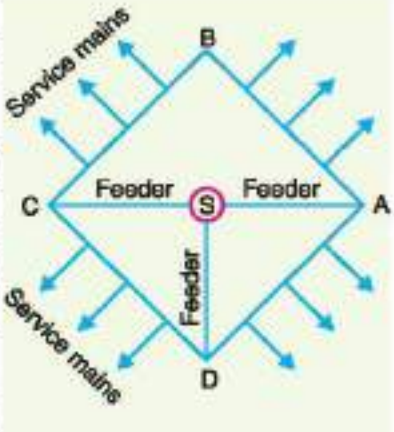


Fig. 12.1

12.3 A.C. Distribution

Now-a-days electrical energy is generated, transmitted and distributed in the form of alternating current. One important reason for the widespread use of alternating current in preference to direct current is the fact that alternating voltage can be conveniently changed in magnitude by means of a transformer. Transformer has made it possible to transmit a.c. power at high voltage and utilise it at a safe potential. High transmission and distribution voltages have greatly reduced the current in the conductors and the resulting line losses.

There is no definite line between transmission and distribution according to voltage or bulk capacity. However, in general, the a.c. distribution system is the electrical system between the step-down substation fed by the transmission system and the consumers' meters. The a.c. distribution system is classified into (*i*) primary distribution system and (*ii*) secondary distribution system.

(*i*) **Primary distribution system.** It is that part of a.c. distribution system which operates at voltages somewhat higher than general utilisation and handles large blocks of electrical energy than the average low-voltage consumer uses. The voltage used for primary distribution depends upon the amount of power to be conveyed and the distance of the substation required to be fed. The most commonly used primary distribution voltages are 11 kV, 6·6 kV and 3·3 kV. Due to economic considerations, primary distribution is carried out by 3-phase, 3-wire system.

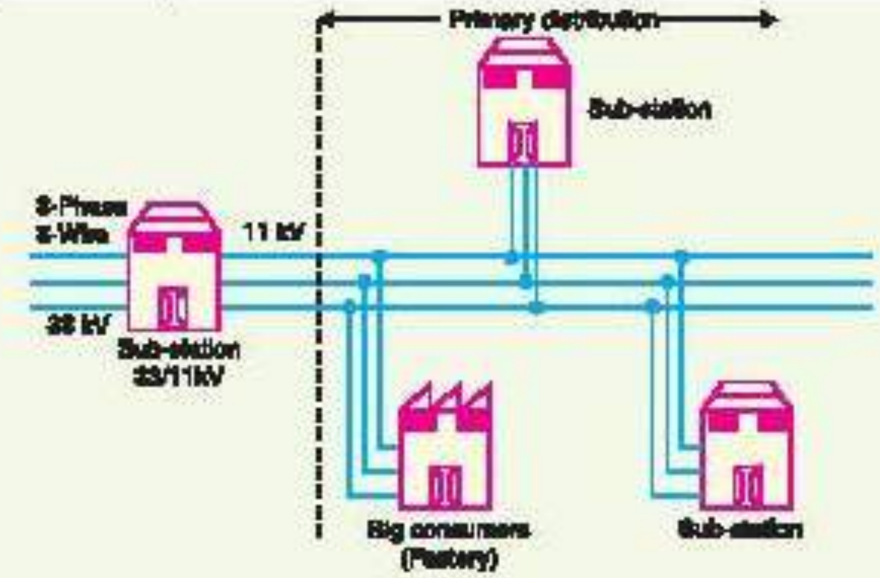


Fig. 12.2

Fig. 12.2 shows a typical primary distribution system. Electric power from the generating station is transmitted at high voltage to the substation located in or near the city. At this substation, voltage is stepped down to 11 kV with the help of step-down transformer. Power is supplied to various substations for distribution or to big consumers at this voltage. This forms the high voltage distribution or primary distribution.

(ii) Secondary distribution system. It is that part of a.c. distribution system which includes the range of voltages at which the ultimate consumer utilises the electrical energy delivered to him. The secondary distribution employs 400/230 V, 3-phase, 4-wire system.

Fig. 12.3 shows a typical secondary distribution system. The primary distribution circuit delivers power to various substations, called distribution substations. The substations are situated near the consumers' localities and contain step-down transformers. At each distribution substation, the voltage is stepped down to 400 V and power is delivered by 3-phase, 4-wire a.c. system. The voltage between any two phases is 400 V and between any phase and neutral is 230 V. The single phase domestic loads are connected between any one phase and the neutral, whereas 3-phase 400 V motor loads are connected across 3-phase lines directly.

Power transformer

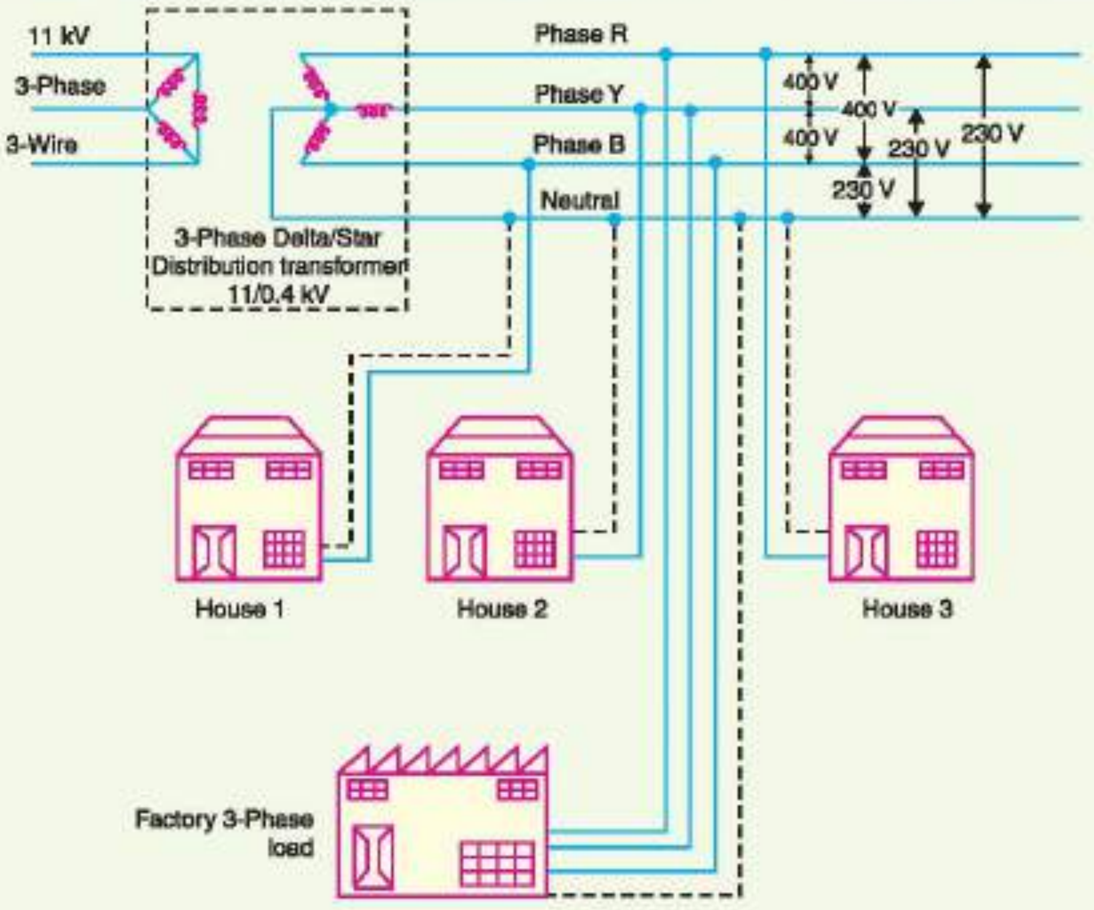


Fig. 12.3

12.4 D.C. Distribution

It is a common knowledge that electric power is almost exclusively generated, transmitted and distributed as a.c. However, for certain applications, d.c. supply is absolutely necessary. For instance, d.c. supply is required for the operation of variable speed machinery (*i.e.*, d.c. motors), for electrochemical work and for congested areas where storage battery reserves are necessary. For this purpose, a.c. power is converted into d.c. power at the substation by using converting machinery *e.g.*, mercury arc rectifiers, rotary converters and motor-generator sets. The d.c. supply from the substation may be obtained in the form of (*i*) 2-wire or (*ii*) 3-wire for distribution.

(*i*) *2-wire d.c. system.* As the name implies, this system of distribution consists of two wires. One is the outgoing or positive wire and the other is the return or negative wire. The loads such as lamps, motors etc. are connected in parallel between the two wires as shown in Fig. 12.4. This system is never used for transmission purposes due to low efficiency but may be employed for distribution of d.c. power.

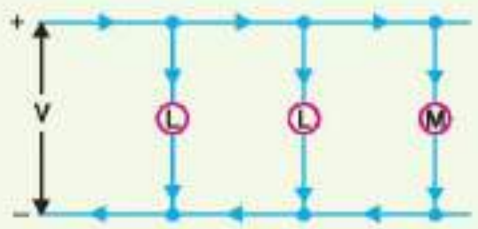


Fig. 12.4

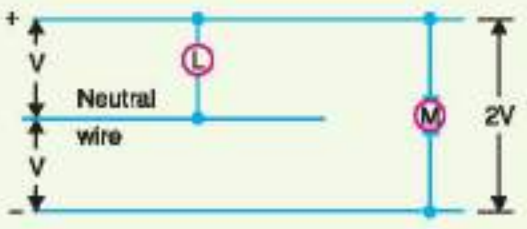


Fig. 12.5

(*ii*) *3-wire d.c. system.* It consists of two outers and a middle or neutral wire which is earthed at the substation. The voltage between the outers is twice the voltage between either outer and neutral wire as shown in Fig. 12.5. The principal advantage of this system is that it makes available two voltages at the consumer terminals *viz.*, V between any outer and the neutral and 2V between the outers. Loads requiring high voltage (*e.g.*, motors) are connected across the outers, whereas lamps and heating circuits requiring less voltage are connected between either outer and the neutral. The methods of obtaining 3-wire system are discussed in the following article.

12.5 Methods of Obtaining 3-wire D.C. System

There are several methods of obtaining 3-wire d.c. system. However, the most important ones are:

(*i*) *Two generator method.* In this method, two shunt wound d.c. generators G_1 and G_2 are connected in series and the neutral is obtained from the common point between generators as shown in Fig. 12.6 (*i*). Each generator supplies the load on its own side. Thus generator G_1 supplies a load current of I_1, whereas generator G_2 supplies a load current of I_2. The difference of load currents on the two sides, known as out of balance current $(I_1 - I_2)$ flows through the neutral wire. The principal disadvantage of this method is that two separate generators are required.

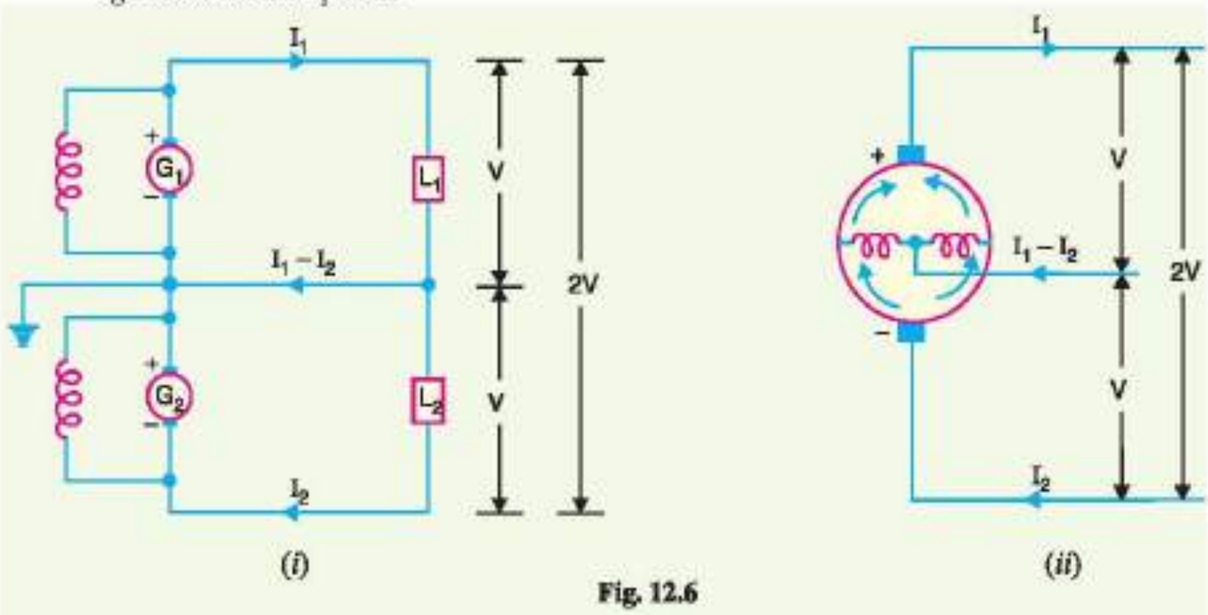


Fig. 12.6

(*ii*) *3-wire d.c. generator.* The above method is costly on account of the necessity of two generators. For this reason, 3-wire d.c. generator was developed as shown in Fig. 12.6 (*ii*). It consists of a standard 2-wire machine with one or two coils of high reactance and low resistance, connected permanently to diametrically opposite points of the armature winding. The neutral wire is obtained from the common point as shown.

(*iii*) *Balancer set.* The 3-wire system can be obtained from 2-wire d.c. system by the use of balancer set as shown in Fig. 12.7. G is the main 2-wire d.c. gen-

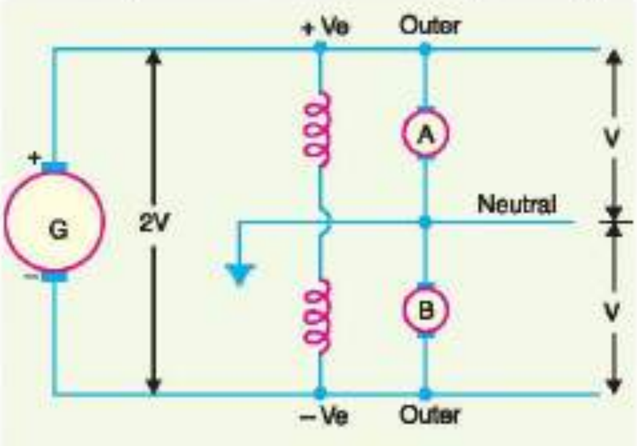


Fig. 12.7

erator and supplies power to the whole system. The balancer set consists of two identical d.c shunt machines *A* and *B* coupled mechanically with their armatures and field windings joined in series across the outers. The junction of their armatures is earthed and neutral wire is taken out from here. The balancer set has the additional advantage that it maintains the potential difference on two sides of neutral equal to each other. This method is discussed in detail in the next chapter.

12.6 Overhead Versus Underground System

The distribution system can be overhead or underground. Overhead lines are generally mounted on wooden, concrete or steel poles which are arranged to carry distribution transformers in addition to the conductors. The underground system uses conduits, cables and manholes under the surface of streets and sidewalks. The choice between overhead and underground system depends upon a number of widely differing factors. Therefore, it is desirable to make a comparison between the two.

(*i*) *Public safety.* The underground system is more safe than overhead system because all distribution wiring is placed underground and there are little chances of any hazard.

(*ii*) *Initial cost.* The underground system is more expensive due to the high cost of trenching, conduits, cables, manholes and other special equipment. The initial cost of an underground system may be five to ten times than that of an overhead system.

(*iii*) *Flexibility.* The overhead system is much more flexible than the underground system. In the latter case, manholes, duct lines etc., are permanently placed once installed and the load expansion can only be met by laying new lines. However, on an overhead system, poles, wires, transformers etc., can be easily shifted to meet the changes in load conditions.

(*iv*) *Faults.* The chances of faults in underground system are very rare as the cables are laid underground and are generally provided with better insulation.

(*v*) *Appearance.* The general appearance of an underground system is better as all the distribution lines are invisible. This factor is exerting considerable public pressure on electric supply companies to switch over to underground system.

(*vi*) *Fault location and repairs.* In general, there are little chances of faults in an underground system. However, if a fault does occur, it is difficult to locate and repair on this system. On an overhead system, the conductors are visible and easily accessible so that fault locations and repairs can be easily made.

(*vii*) *Current carrying capacity and voltage drop.* An overhead distribution conductor has a considerably higher current carrying capacity than an underground cable conductor of the same material and cross-section. On the other hand, underground cable conductor has much lower inductive reactance than that of an overhead conductor because of closer spacing of conductors.

(*viii*) *Useful life.* The useful life of underground system is much longer than that of an overhead system. An overhead system may have a useful life of 25 years, whereas an underground system may have a useful life of more than 50 years.

(*ix*) *Maintenance cost.* The maintenance cost of underground system is very low as compared with that of overhead system because of less chances of faults and service interruptions from wind, ice, lightning as well as from traffic hazards.

(*x*) *Interference with communication circuits.* An overhead system causes electromagnetic interference with the telephone lines. The power line currents are superimposed on speech currents, resulting in the potential of the communication channel being raised to an undesirable level. However, there is no such interference with the underground system.

It is clear from the above comparison that each system has its own advantages and disadvantages. However, comparative economics (*i.e.*, annual cost of operation) is the most powerful factor influencing the choice between underground and overhead system. The greater capital cost of underground system prohibits its use for distribution. But sometimes non-economic factors (*e.g.*, general appearance, public safety etc.) exert considerable influence on choosing underground system. In general, overhead system is adopted for distribution and the use of underground system is made only where overhead construction is impracticable or prohibited by local laws.

12.7 Connection Schemes of Distribution System

All distribution of electrical energy is done by constant voltage system. In practice, the following distribution circuits are generally used :

(*i*) **Radial System.** In this system, separate feeders radiate from a single substation and feed the distributors at one end only. Fig. 12.8 (*i*) shows a single line diagram of a radial system for d.c. distribution where a feeder *OC* supplies a distributor *AB* at point *A*. Obviously, the distributor is fed at one end only *i.e.*, point *A* is this case. Fig. 12.8 (*ii*) shows a single line diagram of radial system for a.c. distribution. The radial system is employed only when power is generated at low voltage and the substation is located at the centre of the load.

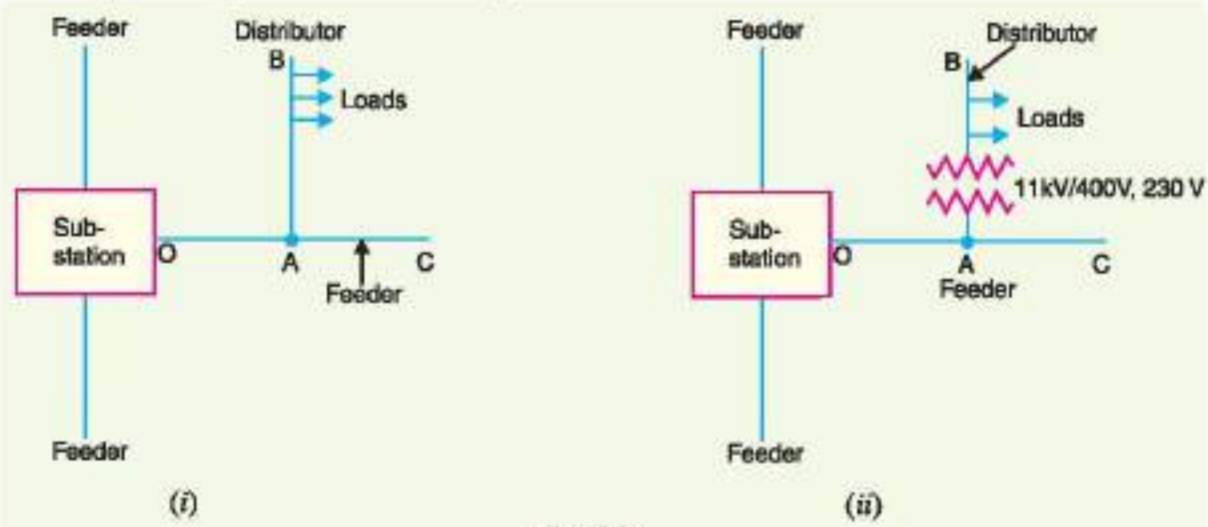


Fig. 12.8

This is the simplest distribution circuit and has the lowest initial cost. However, it suffers from the following drawbacks :

(*a*) The end of the distributor nearest to the feeding point will be heavily loaded.

(*b*) The consumers are dependent on a single feeder and single distributor. Therefore, any fault on the feeder or distributor cuts off supply to the consumers who are on the side of the fault away from the substation.

(*c*) The consumers at the distant end of the distributor would be subjected to serious voltage fluctuations when the load on the distributor changes.

Due to these limitations, this system is used for short distances only.

(*ii*) **Ring main system.** In this system, the primaries of distribution transformers form a loop. The loop circuit starts from the substation bus-bars, makes a loop through the area to be served, and returns to the substation. Fig. 12.9 shows the single line diagram of ring main system for a.c. distribution where substation supplies to the closed feeder LMNOPQRS. The distributors are tapped from different points *M*, *O* and *Q* of the feeder through distribution transformers. The ring main system has the following advantages :

(a) There are less voltage fluctuations at consumer's terminals.

(b) The system is very reliable as each distributor is fed *via* *two feeders. In the event of fault on any section of the feeder, the continuity of supply is maintained. For example, suppose that fault occurs at any point *F* of section SLM of the feeder. Then section SLM of the feeder can be isolated for repairs and at the same time continuity of supply is maintained to all the consumers *via* the feeder *SRQPONM*.

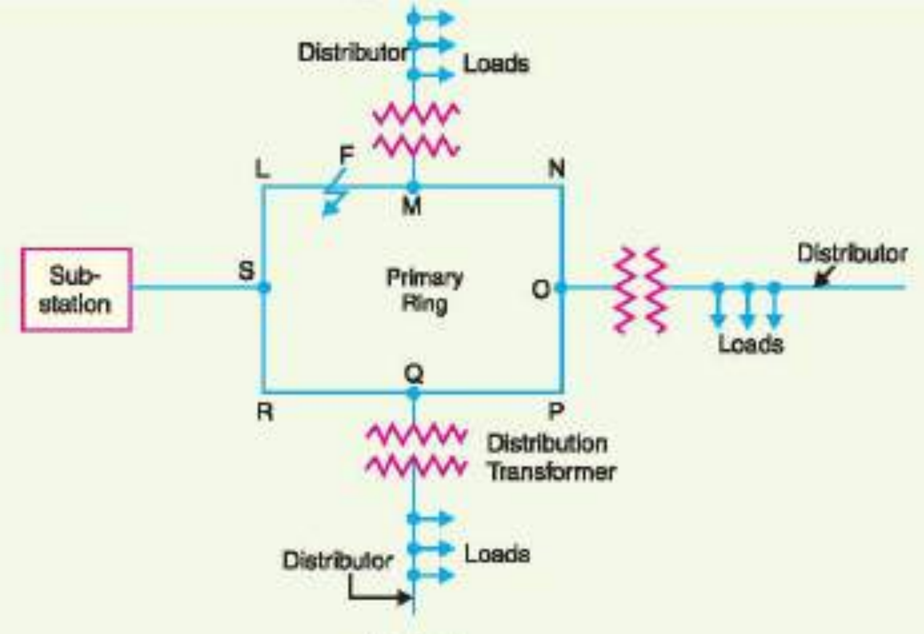


Fig. 12.9

(iii) **Interconnected system.** When the feeder ring is energised by two or more than two generating stations or substations, it is called inter-connected system. Fig. 12.10 shows the single line diagram of interconnected system where the closed feeder ring *ABCD* is supplied by two substations S_1 and S_2 at points *D* and *C* respectively. Distributors are connected to

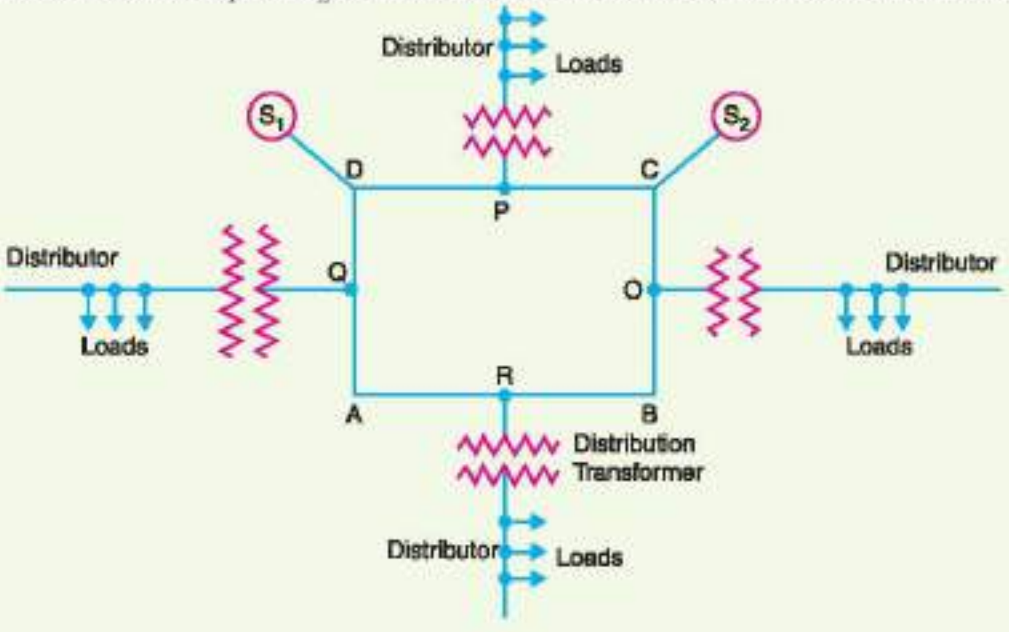


Fig. 12.10

* Thus the distributor from point *M* is supplied by the feeders SLM and SRQPONM.

points *O*, *P*, *Q* and *R* of the feeder ring through distribution transformers. The interconnected system has the following advantages :

(*a*) It increases the service reliability.

(*b*) Any area fed from one generating station during peak load hours can be fed from the other generating station. This reduces reserve power capacity and increases efficiency of the system.

12.8 Requirements of a Distribution System

A considerable amount of effort is necessary to maintain an electric power supply within the requirements of various types of consumers. Some of the requirements of a good distribution system are : proper voltage, availability of power on demand and reliability.

(*i*) **Proper voltage.** One important requirement of a distribution system is that voltage variations at consumer's terminals should be as low as possible. The changes in voltage are generally caused due to the variation of load on the system. Low voltage causes loss of revenue, inefficient lighting and possible burning out of motors. High voltage causes lamps to burn out permanently and may cause failure of other appliances. Therefore, a good distribution system should ensure that the voltage variations at consumers terminals are within permissible limits. The statutory limit of voltage variations is ± 6% of the rated value at the consumer's terminals. Thus, if the declared voltage is 230 V, then the highest voltage of the consumer should not exceed 244 V while the lowest voltage of the consumer should not be less than 216 V.

(*ii*) **Availability of power on demand.** Power must be available to the consumers in any amount that they may require from time to time. For example, motors may be started or shut down, lights may be turned on or off, without advance warning to the electric supply company. As electrical energy cannot be stored, therefore, the distribution system must be capable of supplying load demands of the consumers. This necessitates that operating staff must continuously study load patterns to predict in advance those major load changes that follow the known schedules.

(*iii*) **Reliability.** Modern industry is almost dependent on electric power for its operation. Homes and office buildings are lighted, heated, cooled and ventilated by electric power. This calls for reliable service. Unfortunately, electric power, like everything else that is man-made, can never be absolutely reliable. However, the reliability can be improved to a considerable extent by (*a*) interconnected system (*b*) reliable automatic control system (*c*) providing additional reserve facilities.

12.9 Design Considerations in Distribution System

Good voltage regulation of a distribution network is probably the most important factor responsible for delivering good service to the consumers. For this purpose, design of feeders and distributors requires careful consideration.

(*i*) *Feeders.* A feeder is designed from the point of view of its current carrying capacity while the voltage drop consideration is relatively unimportant. It is because voltage drop in a feeder can be compensated by means of voltage regulating equipment at the substation.

(*ii*) *Distributors.* A distributor is designed from the point of view of the voltage drop in it. It is because a distributor supplies power to the consumers and there is a statutory limit of voltage variations at the consumer's terminals (± 6% of rated value). The size and length of the distributor should be such that voltage at the consumer's terminals is within the permissible limits.

SELF - TEST

1. **Fill in the blanks by inserting appropriate words/figures.**
   - (*i*) The underground system has .... initial cost than the overhead system.
   - (*ii*) A ring main system of distribution is .... reliable than the radial system.
   - (*iii*) The distribution transformer links the primary and .... distribution systems.
   - (*iv*) The most common system for secondary distribution is .... 3-phase, ... wire system.
   - (*v*) The statutory limit for voltage variations at the consumer's terminals is .... % of rated value.
   - (*vi*) The service mains connect the .... and the ....
   - (*vii*) The overhead system is .... flexible than underground system.
2. **Fill in the blanks by picking up correct words/figures from brackets.**
   - (*i*) The main consideration in the design of a feeder is the .... (*current carrying capacity, voltage drop*)
   - (*ii*) A 3-wire d.c. distribution makes available .... voltages. (*one, two, three*)
   - (*iii*) Now-a-days .... system is used for distribution. (*a, c, d.c.*)
   - (*iv*) The interconnected system .... the reserve capacity of the systems. (*increases, decreases*)
   - (*v*) The major part of investment on secondary distribution is made on .... (*Distribution transformers, conductors, pole fittings*)
   - (*vi*) The chances of faults in underground system are .... as compared to overhead system. (*less, more*)

ANSWERS TO SELF-TEST

1. (*i*) more (*ii*) more (*iii*) secondary (*iv*) 400/230 V, 4 (*v*) = 6 (*vi*) distributor, consumer terminals (*vii*) more
2. (*i*) current carrying capacity (*ii*) two (*iii*) a.c. (*iv*) increases (*v*) distribution transformers (*vi*) less

CHAPTER REVIEW TOPICS

1. What do you understand by distribution system ?
2. Draw a single line diagram showing a typical distribution system.
3. Define and explain the terms : feeder, distributor and service mains.
4. Discuss the relative merits and demerits of underground and overhead systems.
5. Explain the following systems of distribution :
   - (*i*) Radial system
   - (*ii*) Ring main system
   - (*iii*) Interconnected system
6. Discuss briefly the design considerations in distribution system.
7. With a neat diagram, explain the complete a.c. system for distribution of electrical energy.
8. Write short notes on the following :
   - (*i*) Distribution transformers
   - (*ii*) 3-wire d.c. distribution
   - (*iii*) Primary distribution

DISCUSSION QUESTIONS

1. Can transmission and distribution systems be distinguished merely by their voltages ? Explain your answer.
2. It is suggested that since distribution transformer links the primary and utilisation voltage, secondary system is not essential. Is it a feasible proposition ?
3. What are the situations where the cost of underground system becomes comparable to overhead system ?
4. What are the effects of high primary voltage on the distribution system ?

CHAPTER

3

Variable Load on Power Stations

Introduction

The function of a power station is to deliver power to a large number of consumers. However, the power demands of different consumers vary in accordance with their activities. The result of this variation in demand is that load on a power station is never constant, rather it varies from time to time. Most of the complexities of modern power plant operation arise from the inherent variability of the load demanded by the users. Unfortunately, electrical power cannot be stored and, therefore, the power station must produce power as and when demanded to meet the requirements of the consumers. On one hand, the power engineer would like that the alternators in the power station should run at their rated capacity for maximum efficiency and on the other hand, the demands of the consumers have wide variations. This makes the design of a power station highly complex. In this chapter, we shall focus our attention on the problems of variable load on power stations.

3.1 Structure of Electric Power System

The function of an electric power system is to connect the power station to the consumers' loads (*iv*) The power demanded by the consumers is supplied by the power station through the transmission and distribution networks. As the consumers' load demand changes, the power supply by the power station changes accordingly.

3.2 Variable Load on Power Station

The load on a power station varies from time to time due to uncertain demands of the consumers and is known as **variable load on the station.**

Transmission line

A power station is designed to meet the load requirements of the consumers. An ideal load on the station, from stand point of equipment needed and operating routine, would be one of constant magnitude and steady duration. However, such a steady load on the station is never realised in actual practice. The consumers require their small or large block of power in accordance with the demands of their activities. Thus the load demand of one consumer at any time may be different from that of the other consumer. The result is that load on the power station varies from time to time.

Effects of variable load. The variable load on a power station introduces many perplexities in its operation. Some of the important effects of variable load on a power station are :

(*i*) **Need of additional equipment.** The variable load on a power station necessitates to have additional equipment. By way of illustration, consider a steam power station. Air, coal and water are the raw materials for this plant. In order to produce variable power, the supply of these materials will be required to be varied correspondingly. For instance, if the power demand on the plant increases, it must be followed by the increased flow of coal, air and water to the boiler in order to meet the increased demand. Therefore, additional equipment has to be installed to accomplish this job. As a matter of fact, in a modern power plant, there is much equipment devoted entirely to adjust the rates of supply of raw materials in accordance with the power demand made on the plant.

(*ii*) **Increase in production cost.** The variable load on the plant increases the cost of the production of electrical energy. An alternator operates at maximum efficiency near its rated capacity. If a single alternator is used, it will have poor efficiency during periods of light loads on the plant. Therefore, in actual practice, a number of alternators of different capacities are installed so that most of the alternators can be operated at nearly full load capacity. However, the use of a number of generating units increases the initial cost per kW of the plant capacity as well as floor area required. This leads to the increase in production cost of energy.

3.3 Load Curves

The curve showing the variation of load on the power station with respect to (w.r.t) time is known as a **load curve.**

The load on a power station is never constant; it varies from time to time. These load variations during the whole day (*i.e.*, 24 hours) are recorded half-hourly or hourly and are plotted against time on the graph. The curve thus obtained is known as *daily load curve* as it shows the variations of load *w.r.t.* time during the day. Fig. 3.2. shows a typical daily load curve of a power station. It is clear that load on the power station is varying, being maximum at 6 P.M. in this case. It may be seen that load curve indicates at a glance the general character of the load that is being imposed on the plant. Such a clear representation cannot be obtained from tabulated figures.

The *monthly load curve* can be obtained from the daily load curves of that month. For this purpose, average* values of power over a month at different times of the day are calculated and then plotted on the graph. The monthly load curve is generally used to fix the rates of energy. The *yearly load curve* is obtained by considering the monthly load curves of that particular year. The yearly load curve is generally used to determine the annual load factor.

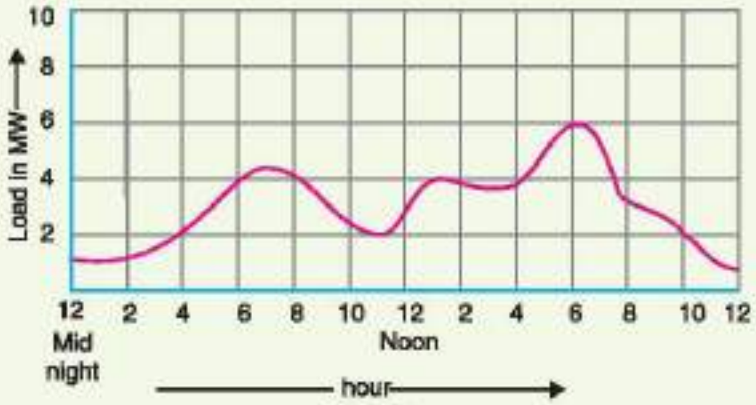


Fig. 3.2

Importance. The daily load curves have attained a great importance in generation as they supply the following information readily :

(*i*) The daily load curve shows the variations of load on the power station during different hours of the day.

(*ii*) The area under the daily load curve gives the number of units generated in the day.
Units generated/day = Area (in kWh) under daily load curve.

(*iii*) The highest point on the daily load curve represents the maximum demand on the station on that day.

(*iv*) The area under the daily load curve divided by the total number of hours gives the average load on the station in the day.

$$\text{Average load} = \frac{\text{Area (in kWh) under daily load curve}}{\text{24 hours}}$$

(*v*) The ratio of the area under the load curve to the total area of rectangle in which it is contained gives the load factor.

$$\text{Load factor} = \frac{\text{Average load}}{\text{Max. demand}} = \frac{\text{Average load} \times 24}{\text{Max. demand} \times 24}$$

$$= \frac{\text{Area (in kWh) under daily load curve}}{\text{Total area of rectangle in which the load curve is contained}}$$

* For instance, if we consider the load on power station at mid-night during the various days of the month, it may vary slightly. Then the average will give the load at mid-night on the monthly curve.

(vi) The load curve helps in selecting* the size and number of generating units.

(vii) The load curve helps in preparing the operation schedule** of the station.

3.4 Important Terms and Factors

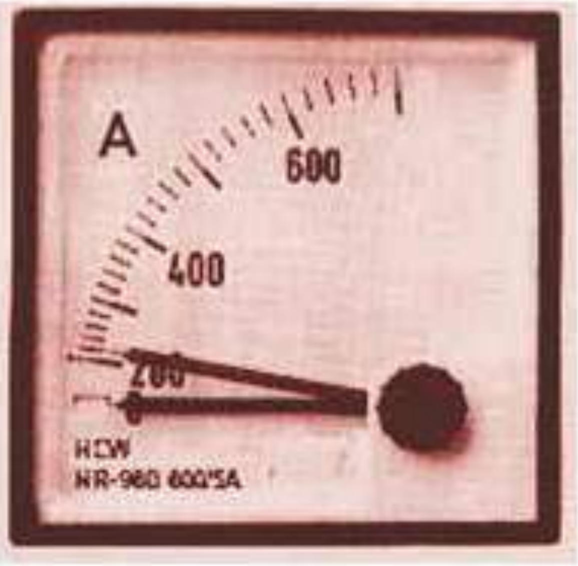
The variable load problem has introduced the following terms and factors in power plant engineering:

(i) Connected load. *It is the sum of continuous ratings of all the equipments connected to supply system.*

A power station supplies load to thousands of consumers. Each consumer has certain equipment installed in his premises. The sum of the continuous ratings of all the equipments in the consumer's premises is the "connected load" of the consumer. For instance, if a consumer has connections of five 100-watt lamps and a power point of 500 watts, then connected load of the consumer is 5 × 100 + 500 = 1000 watts. The sum of the connected loads of all the consumers is the connected load to the power station.

(ii) Maximum demand : *It is the greatest demand of load on the power station during a given period.*

The load on the power station varies from time to time. The maximum of all the demands that have occurred during a given period (*say* a day) is the maximum demand. Thus referring back to the load curve of Fig. 3.2, the maximum demand on the power station during the day is 6 MW and it occurs at 6 P.M. Maximum demand is generally less than the connected load because all the consumers do not switch on their connected load to the system at a time. The knowledge of maximum demand is very important as it helps in determining the installed capacity of the station. The station must be capable of meeting the maximum demand.



Maximum demand meter

(iii) Demand factor. *It is the ratio of maximum demand on the power station to its connected load i.e.,*

$$\text{Demand factor} = \frac{\text{Maximum demand}}{\text{Connected load}}$$

The value of demand factor is usually less than 1. It is expected because maximum demand on the power station is generally less than the connected load. If the maximum demand on the power station is 80 MW and the connected load is 100 MW, then demand factor = 80/100 = 0·8. The knowledge of demand factor is vital in determining the capacity of the plant equipment.


Energy meter

(iv) Average load. *The average of loads occurring on the power station in a given period (day or month or year) is known as* average load *or* average demand.

* It will be shown in Art. 3.9 that number and size of the generating units are selected to fit the load curve. This helps in operating the generating units at or near the point of maximum efficiency.

** It is the sequence and time for which the various generating units (*i.e.*, alternators) in the plant will be put in operation.

$$\text{Daily average load} = \frac{\text{No. of units (kWh) generated in a day}}{\text{24 hours}}$$

$$\text{Monthly average load} = \frac{\text{No. of units (kWh) generated in a month}}{\text{Number of hours in a month}}$$

$$\text{Yearly average load} = \frac{\text{No. of units (kWh) generated in a year}}{\text{8760 hours}}$$

(*v*) **Load factor.** *The ratio of average load to the maximum demand during a given period is known as* **load factor** *i.e.*,

$$\text{Load factor} = \frac{\text{Average load}}{\text{Max. demand}}$$

If the plant is in operation for T hours,

$$\text{Load factor} = \frac{\text{Average load} \times \text{T}}{\text{Max. demand} \times \text{T}}$$

$$= \frac{\text{Units generated in T hours}}{\text{Max. demand} \times \text{T hours}}$$

The load factor may be daily load factor, monthly load factor or annual load factor if the time period considered is a day or month or year. Load factor is always less than 1 because average load is smaller than the maximum demand. The load factor plays key role in determining the overall cost per unit generated. Higher the load factor of the power station, lesser* will be the cost per unit generated.

(*vi*) **Diversity factor.** *The ratio of the sum of individual maximum demands to the maximum demand on power station is known as* **diversity factor** *i.e.*,

$$\text{Diversity factor} = \frac{\text{Sum of individual max. demands}}{\text{Max. demand on power station}}$$

A power station supplies load to various types of consumers whose maximum demands generally do not occur at the same time. Therefore, the maximum demand on the power station is always less than the sum of individual maximum demands of the consumers. Obviously, diversity† factor will always be greater than 1. The greater the diversity factor, the lesser‡ is the cost of generation of power.

(*vii*) **Plant capacity factor.** *It is the ratio of actual energy produced to the maximum possible energy that could have been produced during a given period i.e.*,

$$\text{Plant capacity factor} = \frac{\text{Actual energy produced}}{\text{Max. energy that could have been produced}}$$

$$= \frac{\text{Average demand} \times \text{T}^{**}}{\text{Plant capacity} \times \text{T}}$$

$$= \frac{\text{Average demand}}{\text{Plant capacity}}$$

* It is because higher load factor factor means lesser maximum demand. The station capacity is so selected that it must meet the maximum demand. Now, lower maximum demand means lower capacity of the plant which, therefore, reduces the cost of the plant.

† There is diversification in the individual maximum demands *i.e.*, the maximum demand of some consumers may occur at one time while that of others at some other time. Hence, the name diversity factor

‡ Greater diversity factor means lesser maximum demand. This in turn means that lesser plant capcity is required. Thus, the capital investment on the plant is reduced.

** Suppose the period is *T* hours.

Thus if the considered period is one year,

$$\text{Annual plant capacity factor} = \frac{\text{Annual kWh output}}{\text{Plant capacity} \times 8760}$$

The plant capacity factor is an indication of the reserve capacity of the plant. A power station is so designed that it has some reserve capacity for meeting the increased load demand in future. Therefore, the installed capacity of the plant is always somewhat greater than the maximum demand on the plant.

$$\text{Reserve capacity} = \text{Plant capacity} - \text{Max. demand}$$

It is interesting to note that difference between load factor and plant capacity factor is an indication of reserve capacity. If the maximum demand on the plant is equal to the plant capacity, then load factor and plant capacity factor will have the same value. In such a case, the plant will have no reserve capacity.

(*viii*) **Plant use factor.** *It is ratio of kWh generated to the product of plant capacity and the number of hours for which the plant was in operation i.e.*

$$\text{Plant use factor} = \frac{\text{Station output in kWh}}{\text{Plant capacity} \times \text{Hours of use}}$$

Suppose a plant having installed capacity of 20 MW produces annual output of $7{\cdot}35 \times 10^6$ kWh and remains in operation for 2190 hours in a year. Then,

$$\text{Plant use factor} = \frac{7{\cdot}35 \times 10^6}{(20 \times 10^3) \times 2190} = 0{\cdot}167 = 16{\cdot}7\%$$

3.5 Units Generated per Annum

It is often required to find the kWh generated per annum from maximum demand and load factor. The procedure is as follows :

$$\text{Load factor} = \frac{\text{Average load}}{\text{Max. demand}}$$

$$\therefore \quad \text{Average load} = \text{Max. demand} \times \text{L.F.}$$

$$\text{Units generated/annum} = \text{Average load (in kW)} \times \text{Hours in a year}$$

$$= \text{Max. demand (in kW)} \times \text{L.F.} \times 8760$$

3.6 Load Duration Curve

When the load elements of a load curve are arranged in the order of descending magnitudes, the curve thus obtained is called a **load duration curve.**

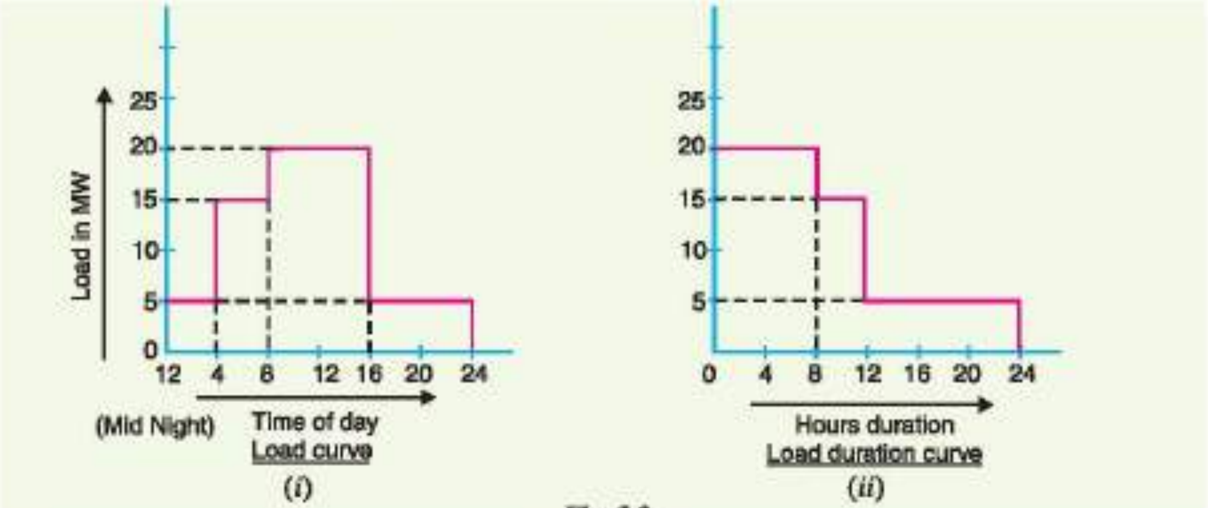


Fig. 3.3

The load duration curve is obtained from the same data as the load curve but the ordinates are arranged in the order of descending magnitudes. In other words, the maximum load is represented to the left and decreasing loads are represented to the right in the descending order. Hence the area under the load duration curve and the area under the load curve are equal. Fig. 3.3 (*i*) shows the daily load curve. The daily load duration curve can be readily obtained from it. It is clear from daily load curve [See Fig. 3.3. (*i*)], that load elements in order of descending magnitude are : 20 MW for 8 hours; 15 MW for 4 hours and 5 MW for 12 hours. Plotting these loads in order of descending magnitude, we get the daily load duration curve as shown in Fig. 3.3 (*ii*).

The following points may be noted about load duration curve :

(*i*) The load duration curve gives the data in a more presentable form. In other words, it readily shows the number of hours during which the given load has prevailed.

(*ii*) The area under the load duration curve is equal to that of the corresponding load curve. Obviously, area under daily load duration curve (in kWh) will give the units generated on that day.

(*iii*) The load duration curve can be extended to include any period of time. By laying out the abscissa from 0 hour to 8760 hours, the variation and distribution of demand for an entire year can be summarised in one curve. The curve thus obtained is called the *annual load duration curve*.

3.7 Types of Loads

A device which taps electrical energy from the electric power system is called a load on the system. The load may be resistive (*e.g.*, electric lamp), inductive (*e.g.*, induction motor), capacitive or some combination of them. The various types of loads on the power system are :

(*i*) **Domestic load.** Domestic load consists of lights, fans, refrigerators, heaters, television, small motors for pumping water etc. Most of the residential load occurs only for some hours during the day (*i.e.*, 24 hours) *e.g.*, lighting load occurs during night time and domestic appliance load occurs for only a few hours. For this reason, the load factor is low (10% to 12%).

(*ii*) **Commercial load.** Commercial load consists of lighting for shops, fans and electric appliances used in restaurants etc. This class of load occurs for more hours during the day as compared to the domestic load. The commercial load has seasonal variations due to the extensive use of air-conditioners and space heaters.

(*iii*) **Industrial load.** Industrial load consists of load demand by industries. The magnitude of industrial load depends upon the type of industry. Thus small scale industry requires load upto 25 kW, medium scale industry between 25kW and 100 kW and large-scale industry requires load above 500 kW. Industrial loads are generally not weather dependent.

(*iv*) **Municipal load.** Municipal load consists of street lighting, power required for water supply and drainage purposes. Street lighting load is practically constant throughout the hours of the night. For water supply, water is pumped to overhead tanks by pumps driven by electric motors. Pumping is carried out during the off-peak period, usually occurring during the night. This helps to improve the load factor of the power system.

(*v*) **Irrigation load.** This type of load is the electric power needed for pumps driven by motors to supply water to fields. Generally this type of load is supplied for 12 hours during night.

(*vi*) **Traction load.** This type of load includes tram cars, trolley buses, railways etc. This class of load has wide variation. During the morning hour, it reaches peak value because people have to go to their work place. After morning hours, the load starts decreasing and again rises during evening since the people start coming to their homes.

3.8 Typical Demand and Diversity Factors

The demand factor and diversity factor depend on the type of load and its magnitude.

TYPICAL DEMAND FACTORS

Type of consumer		**Demand factor**
Residence lighting	$\frac{1}{4}$ kW	1·00
	$\frac{1}{2}$ kW	0·60
	Over 1 kW	0·50
Commercial lighting	Restaurants	0·70
	Theatres	0·60
	Hotels	0·50
	Schools	0·55
	Small industry	0·60
	Store	0·70
General power service	0–10 H.P.	0·75
	10–20 H.P.	0·65
	20–100 H.P.	0·55
	Over 100 H.P.	0·50

TYPICAL DIVERSITY FACTORS

	Residential lighting	*Commercial lighting*	*General power supply*
Between consumers	3 – 4	1·5	1·5
Between transformers	1·3	1·3	1·3
Between feeders	1·2	1·2	1·2
Between substations	1·1	1·1	1·1

Illustration. Load and demand factors are always less than 1 while diversity factors are more than unity. High load and diversity factors are the desirable qualities of the power system. Indeed, these factors are used to predict the load. Fig. 3.4 shows a small part of electric power system where a distribution transformer is supplying power to the consumers. For simplicity, only three consumers *a*, *b*, and *c* are shown in the figure. The maximum demand of consumer *a* is the product of its connected load and the appropriate demand factor. Same is the case for consumers *b* and *c*. The maximum demand on the transformer is the sum of *a*, *b* and *c*'s maximum demands divided by the diversity factors between the consumers. Similarly, the maximum demand on the feeder is the sum of maximum demands on the distribution transformers connected to it divided by the diversity factor between transformers. Likewise diversification between feeders is recognised when obtaining substation maximum demands and substation diversification when predicting maximum load on the power station. Note that diversity factor is the sum of the individual maximum demands of the subdivisions of a system taken as they may occur during the daily cycle divided by the maximum simultaneous demand of the system. The "system" may be a group of consumers served by a certain transformer, a group of transformers served by a feeder etc. Since individual variations have diminishing effect as one goes

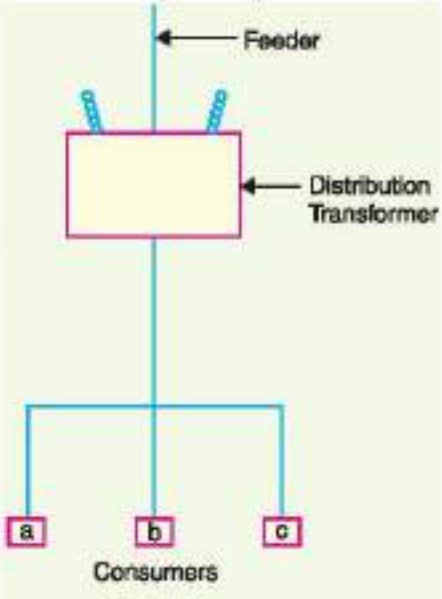


Fig. 3.4

farther from the ultimate consumer in making measurements, one should expect decreasing numerical values of diversity factor as the power plant end of the system is approached. This is clear from the above table showing diversity factors between different elements of the power system.

Example 3.1. *The maximum demand on a power station is 100 MW. If the annual load factor is 40% , calculate the total energy generated in a year.*

Solution.

$$\text{Energy generated/year} = \text{Max. demand} \times \text{L.F.} \times \text{Hours in a year}$$
$$= (100 \times 10^3) \times (0.4) \times (24 \times 365) \text{ kWh}$$
$$= \mathbf{3504 \times 10^5 \text{ kWh}}$$

Example 3.2. *A generating station has a connected load of 43MW and a maximum demand of 20 MW; the units generated being 61·5 × 10⁶ per annum. Calculate (i) the demand factor and (ii) load factor.*

Solution.

(*i*) $$\text{Demand factor} = \frac{\text{Max. demand}}{\text{Connected load}} = \frac{20}{43} = \mathbf{0.465}$$

(*ii*) $$\text{Average demand} = \frac{\text{Units generated / annum}}{\text{Hours in a year}} = \frac{61 \cdot 5 \times 10^6}{8760} = 7020 \text{ kW}$$

∴ $$\text{Load factor} = \frac{\text{Average demand}}{\text{Max. demand}} = \frac{7020}{20 \times 10^3} = \mathbf{0.351} \text{ or } \mathbf{35.1\%}$$

Example 3.3. *A 100 MW power station delivers 100 MW for 2 hours, 50 MW for 6 hours and is shut down for the rest of each day. It is also shut down for maintenance for 45 days each year. Calculate its annual load factor.*

Solution.

Energy supplied for each working day

$$= (100 \times 2) + (50 \times 6) = 500 \text{ MWh}$$

$$\text{Station operates for} = 365 - 45 = 320 \text{ days in a year}$$

∴ $$\text{Energy supplied/year} = 500 \times 320 = 160{,}000 \text{ MWh}$$

$$\text{Annual load factor} = \frac{\text{MWh supplied per annum}}{\text{Max. demand in MW} \times \text{Working hours}} \times 100$$
$$= \frac{160{,}000}{(100) \times (320 \times 24)} \times 100 = \mathbf{20.8\%}$$

Example 3.4. *A generating station has a maximum demand of 25MW, a load factor of 60%, a plant capacity factor of 50% and a plant use factor of 72%. Find (i) the reserve capacity of the plant (ii) the daily energy produced and (iii) maximum energy that could be produced daily if the plant while running as per schedule, were fully loaded.*

Solution.

(*i*) $$\text{Load factor} = \frac{\text{Average demand}}{\text{Maximum demand}}$$

or $$0.60 = \frac{\text{Average demand}}{25}$$

∴ $$\text{Average demand} = 25 \times 0.60 = 15 \text{ MW}$$

$$\text{Plant capacity factor} = \frac{\text{Average demand}}{\text{Plant capacity}}$$

∴ $$\text{Plant capacity} = \frac{\text{Average demand}}{\text{Plant capacity factor}} = \frac{15}{0 \cdot 5} = 30 \text{ MW}$$

$\therefore$ Reserve capacity of plant = Plant capacity − maximum demand

= 30 − 25 = **5 MW**

(*ii*) Daily energy produced = Average demand × 24

= 15 × 24 = **360 MWh**

(*iii*) Maximum energy that could be produced

$$= \frac{\text{Actual energy produced in a day}}{\text{Plant use factor}}$$

$$= \frac{360}{0 \cdot 72} = \textbf{500 MWh/day}$$

Example 3.5. *A diesel station supplies the following loads to various consumers :*

Industrial consumer = 1500 kW ; Commercial establishment = 750 kW

Domestic power = 100 kW; Domestic light = 450 kW

If the maximum demand on the station is 2500 kW and the number of kWh generated per year is 45×10^5*, determine (i) the diversity factor and (ii) annual load factor.*

Solution.

(*i*) Diversity factor $= \dfrac{1500 + 750 + 100 + 450}{2500} = \mathbf{1{\cdot}12}$

(*ii*) Average demand $= \dfrac{\text{kWh generated / annum}}{\text{Hours in a year}} = 45 \times 10^5/8760 = 513{\cdot}7$ kW

$\therefore$ Load factor $= \dfrac{\text{Average load}}{\text{Max. demand}} = \dfrac{513 \cdot 7}{2500} = 0{\cdot}205 = \mathbf{20{\cdot}5\%}$

Example 3.6. *A power station has a maximum demand of 15000 kW. The annual load factor is 50% and plant capacity factor is 40%. Determine the reserve capacity of the plant.*

Solution.

Energy generated/annum = Max. demand × L.F. × Hours in a year

= (15000) × (0·5) × (8760) kWh

= $65{\cdot}7 \times 10^6$ kWh

Plant capacity factor $= \dfrac{\text{Units generated / annum}}{\text{Plant capacity} \times \text{Hours in a year}}$

$\therefore$ Plant capacity $= \dfrac{65 \cdot 7 \times 10^6}{0 \cdot 4 \times 8760} = 18{,}750$ kW

Reserve capacity = Plant capacity − Max. demand

= 18,750 − 15000 = **3750 kW**

Example 3.7. *A power supply is having the following loads :*

Type of load	*Max. demand (kW)*	*Diversity of group*	*Demand factor*
Domestic	*1500*	*1·2*	*0·8*
Commercial	*2000*	*1·1*	*0·9*
Industrial	*10,000*	*1·25*	*1*

If the overall system diversity factor is 1·35, determine (i) the maximum demand and (ii) connected load of each type.

Solution.

(*i*) The sum of maximum demands of three types of loads is = 1500 + 2000 + 10,000 = 13,500 kW. As the system diversity factor is 1·35,

$\therefore$ Max. demand on supply system = 13,500/1·35 = **10,000 kW** (*ii*) Each type of load has its own diversity factor among its consumers.

Sum of max. demands of different domestic consumers

= Max. domestic demand × diversity factor

= 1500 × 1·2 = 1800 kW

∴ Connected domestic load = 1800/0·8 = **2250 kW**

Connected commercial load = 2000 × 1·1/0·9 = **2444 kW**

Connected industrial load = 10,000 × 1·25/1= **12,500 kW**

Example 3.8. *At the end of a power distribution system, a certain feeder supplies three distribution transformers, each one supplying a group of customers whose connected loads are as under:*

Transformer	*Load*	*Demand factor*	*Diversity of groups*
Transformer No. 1	*10 kW*	*0·65*	*1·5*
Transformer No. 2	*12 kW*	*0·6*	*3·5*
Transformer No. 3	*15 kW*	*0·7*	*1·5*

If the diversity factor among the transformers is 1·3, find the maximum load on the feeder.

Solution. Fig. 3.5 shows a feeder supplying three distribution transformers.

Sum of max. demands of customers on Transformer 1

= connected load × demand factor = 10 × 0·65 = 6·5 kW

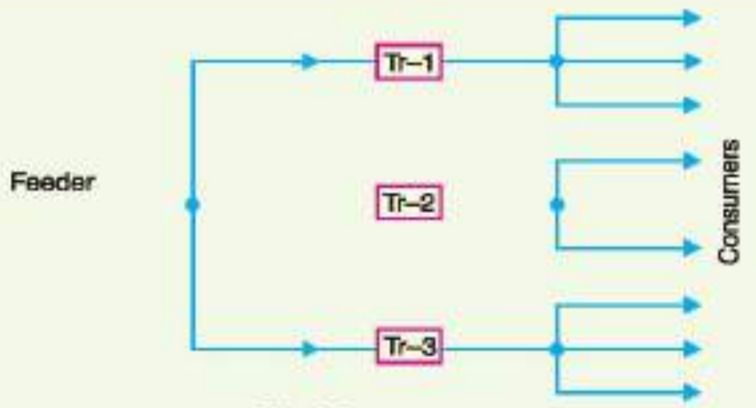


Fig. 3.5

As the diversity factor among consumers connected to transformer No. 1 is 1·5,

∴ Maximum demand on Transformer 1 = 6·5/1·5 = 4·33 kW

Maximum demand on Transformer 2 = 12 × 0·6/3·5 = 2·057 kW

Maximum demand on Transformer 3 = 15 × 0·7/1·5 = 7 kW

As the diversity factor among transformers is 1·3,

$$\therefore \quad \text{Maximum demand on feeder} = \frac{4 \cdot 33 + 2 \cdot 057 + 7}{1 \cdot 3} = \mathbf{10 \cdot 3 \ kW}$$

Example 3.9. *It has been desired to install a diesel power station to supply power in a suburban area having the following particulars :*

(*i*) *1000 houses with average connected load of 1·5 kW in each house. The demand factor and diversity factor being 0·4 and 2·5 respectively.*

(*ii*) *10 factories having overall maximum demand of 90 kW.*

(*iii*) *7 tubewells of 7 kW each and operating together in the morning.*

The diversity factor among above three types of consumers is 1·2. What should be the minimum capacity of power station ?

Solution.

Sum of max. demands of houses $= (1{\cdot}5 \times 0{\cdot}4) \times 1000 = 600$ kW

Max. demand for domestic load $= 600/2{\cdot}5 = 240$ kW

Max. demand for factories $= 90$ kW

Max. demand for tubewells $= 7^* \times 7 = 49$ kW

The sum of maximum demands of three types of loads is $= 240 + 90 + 49 = 379$ kW. As the diversity factor among the three types of loads is 1·2,

∴ Max. demand on station $= 379/1{\cdot}2 = 316$ kW

∴ Minimum capacity of station requried = **316 kW**

Example 3.10. *A generating station has the following daily load cycle :*

Time (Hours)	*0—6*	*6—10*	*10—12*	*12—16*	*16—20*	*20—24*
Load (MW)	*40*	*50*	*60*	*50*	*70*	*40*

Draw the load curve and find (i) maximum demand (ii) units generated per day (iii) average load and (iv) load factor.

Solution. Daily curve is drawn by taking the load along *Y*-axis and time along *X*-axis. For the given load cycle, the load curve is shown in Fig. 3.6.

(*i*) It is clear from the load curve that maximum demand on the power station is 70 MW and occurs during the period 16—20 hours.

∴ Maximum demand = **70 MW**

Fig. 3.6

(*ii*) Units generated/day = Area (in kWh) under the load curve

$= 10^3\,[40 \times 6 + 50 \times 4 + 60 \times 2 + 50 \times 4 + 70 \times 4 + 40 \times 4]$

$= 10^3\,[240 + 200 + 120 + 200 + 280 + 160]$ kWh

$= \mathbf{12 \times 10^5}$ **kWh**

(*iii*) Average load $= \dfrac{\text{Units generated / day}}{24 \text{ hours}} = \dfrac{12 \times 10^5}{24} =$ **50,000 kW**

(*iv*) Load factor $= \dfrac{\text{Average load}}{\text{Max. demand}} = \dfrac{50{,}000}{70 \times 10^3} = 0{\cdot}714 =$ **71·4%**

* Since the tubewells operate together, the diversity factor is 1.

Example 3.11. *A power station has to meet the following demand :*

Group A : 200 kW between 8 A.M. and 6 P.M.

Group B : 100 kW between 6 A.M. and 10 A.M.

Group C : 50 kW between 6 A.M. and 10 A.M.

Group D : 100 kW between 10 A.M. and 6 P.M. and then between 6 P.M. and 6 A.M.

Plot the daily load curve and determine (i) diversity factor (ii) units generated per day (iii) load factor.

Solution. The given load cycle can be tabulated as under :

Time (Hours)	0—6	6—8	8—10	10—18	18—24
Group A	—	—	200 kW	200 kW	—
Group B	—	100 kW	100 kW	—	—
Group C	—	50 kW	50 kW	—	—
Group D	100 kW	—	—	100 kW	100 kW
Total load on power station	100 kW	150 kW	350 kW	300 kW	100 kW

From this table, it is clear that total load on power station is 100 kW for 0—6 hours, 150 kW for 6—8 hours, 350 kW for 8—10 hours, 300 kW for 10—18 hours and 100 kW for 18—24 hours. Plotting the load on power station versus time, we get the daily load curve as shown in Fig. 3.7. It is clear from the curve that maximum demand on the station is 350 kW and occurs from 8 A.M. to 10 A. M. *i.e.*,

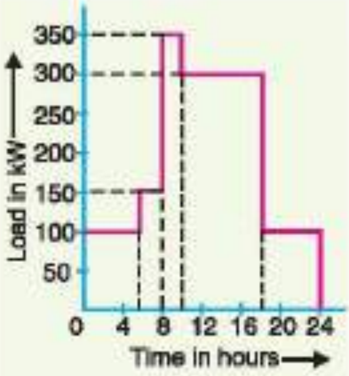


Fig. 3.7

$$\text{Maximum demand} = 350 \text{ kW}$$

Sum of individual maximum demands of groups

$$= 200 + 100 + 50 + 100$$

$$= 450 \text{ kW}$$

(*i*) $$\text{Diversity factor} = \frac{\text{Sum of individual max. demands}}{\text{Max. demand on station}} = 450/350 = \mathbf{1{\cdot}286}$$

(*ii*) $$\text{Units generated/day} = \text{Area (in kWh) under load curve}$$

$$= 100 \times 6 + 150 \times 2 + 350 \times 2 + 300 \times 8 + 100 \times 6$$

$$= \mathbf{4600 \text{ kWh}}$$

(*iii*) $$\text{Average load} = 4600/24 = 191{\cdot}7 \text{ kW}$$

$$\therefore \quad \text{Load factor} = \frac{191{\cdot}7}{350} \times 100 = \mathbf{54{\cdot}8\%}$$

Example 3.12. *The daily demands of three consumers are given below :*

Time	*Consumer 1*	*Consumer 2*	*Consumer 3*
12 midnight to 8 A.M.	*No load*	*200 W*	*No load*
8 A.M. to 2 P.M.	*600 W*	*No load*	*200 W*
2 P.M. to 4 P.M.	*200 W*	*1000 W*	*1200 W*
4 P.M. to 10 P.M.	*800 W*	*No load*	*No load*
10 P.M. to midnight	*No load*	*200 W*	*200 W*

Plot the load curve and find (i) maximum demand of individual consumer (ii) load factor of individual consumer (iii) diversity factor and (iv) load factor of the station.

Solution. Fig. 3.8 shows the load curve.

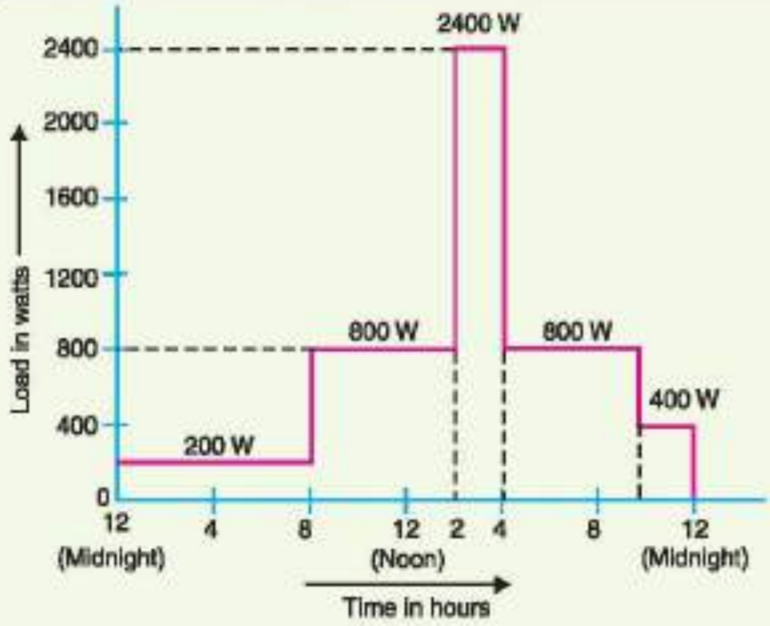


Fig. 3.8

(*i*) Max. demand of consumer 1 = **800 W**

Max. demand of consumer 2 = **1000 W**

Max. demand of consumer 3 = **1200 W**

(*ii*)
$$\text{L.F. of consumer 1} = \frac{\text{Energy consumed / day}}{\text{Max. demand} \times \text{Hours in a day}} \times 100$$

$$= \frac{600 \times 6 + 200 \times 2 + 800 \times 6}{800 \times 24} \times 100 = \mathbf{45{\cdot}8\%}$$

$$\text{L.F. of consumer 2} = \frac{200 \times 8 + 1000 \times 2 + 200 \times 2}{1000 \times 24} \times 100 = \mathbf{16{\cdot}7\%}$$

$$\text{L.F. of consumer 3} = \frac{200 \times 6 + 1200 \times 2 + 200 \times 2}{1200 \times 24} \times 100 = \mathbf{13{\cdot}8\%}$$

(*iii*) The simultaneous maximum demand on the station is 200 + 1000 + 1200 = 2400 W and occurs from 2 P.M. to 4 P.M.

$$\therefore \quad \text{Diversity factor} = \frac{800 + 1000 + 1200}{2400} = \mathbf{1{\cdot}25}$$

(*iv*)
$$\text{Station load factor} = \frac{\text{Total energy consumed / day}}{\text{Simultaneous max. demand} \times 24} \times 100$$

$$= \frac{8800 + 4000 + 4000}{2400 \times 24} \times 100 = \mathbf{29{\cdot}1\%}$$

Example 3.13. *A daily load curve which exhibited a 15-minute peak of 3000 kW is drawn to scale of 1 cm = 2 hours and 1 cm = 1000 kW. The total area under the load curve is measured by planimeter and is found to be 12 cm². Calculate the load factor based on 15-min. peak.*

Solution.

1 cm^2 of load curve represents 1000 × 2 = 2000 kWh

$$\text{Average demand} = \frac{2000 \times \text{Area of load curve}}{\text{Hours in a day}} = 2000 \times \frac{12}{24} = 1000 \text{ kW}$$

$\therefore$ Load factor $= \frac{1000}{3000} \times 100 = \mathbf{33 \cdot 3\%}$

Example 3.14. *A power station has a daily load cycle as under :*

260 MW for 6 hours ; 200 MW for 8 hours : 160 MW for 4 hours, 100 MW for 6 hours.

If the power station is equipped with 4 sets of 75 MW each, calculate (i) daily load factor (ii) plant capacity factor and (iii) daily requirement if the calorific value of oil used were 10,000 kcal/kg and the average heat rate of station were 2860 kcal/kWh.

Solution. Max. demand on the station is 260×10^3 kW.

$$\text{Units supplied/day} = 10^3 [260 \times 6 + 200 \times 8 + 160 \times 4 + 100 \times 6] = 4400 \times 10^3 \text{ kWh}$$

(*i*) $$\text{Daily load factor} = \frac{4400 \times 10^3}{260 \times 10^3 \times 24} \times 100 = \mathbf{70 \cdot 5\%}$$

(*ii*) $$\text{Average demand/day} = 4400 \times 10^3/24 = 1{,}83{,}333 \text{ kW}$$

$$\text{Station capacity} = (75 \times 10^3) \times 4 = 300 \times 10^3 \text{ kW}$$

$$\therefore \text{Plant capacity factor} = \frac{1{,}83{,}333}{300 \times 10^3} \times 100 = \mathbf{61 \cdot 1\ \%}$$

(*iii*) $$\text{Heat required/day} = \text{Plant heat rate} \times \text{units per day} = (2860) \times (4400 \times 10^3) \text{ kcal}$$

$$\text{Fuel required/day} = \frac{2860 \times 4400 \times 10^3}{10000} = 1258 \cdot 4 \times 10^3 \text{ kg} = \mathbf{1258 \cdot 4 \text{ tons}}$$

Example 3.15. *A power station has the following daily load cycle :*

Time in Hours	*6—8*	*8—12*	*12—16*	*16—20*	*20—24*	*24—6*
Load in MW	*20*	*40*	*60*	*20*	*50*	*20*

Plot the load curve and load duratoin curve. Also calculate the energy generated per day.

Solution. Fig. 3.9 (*i*) shows the daily load curve, whereas Fig. 3.9 (*ii*) shows the daily load duraton curve. It can be readily seen that area under the two load curves is the same. Note that load duration curve is drawn by arranging the loads in the order of descending magnitudes.

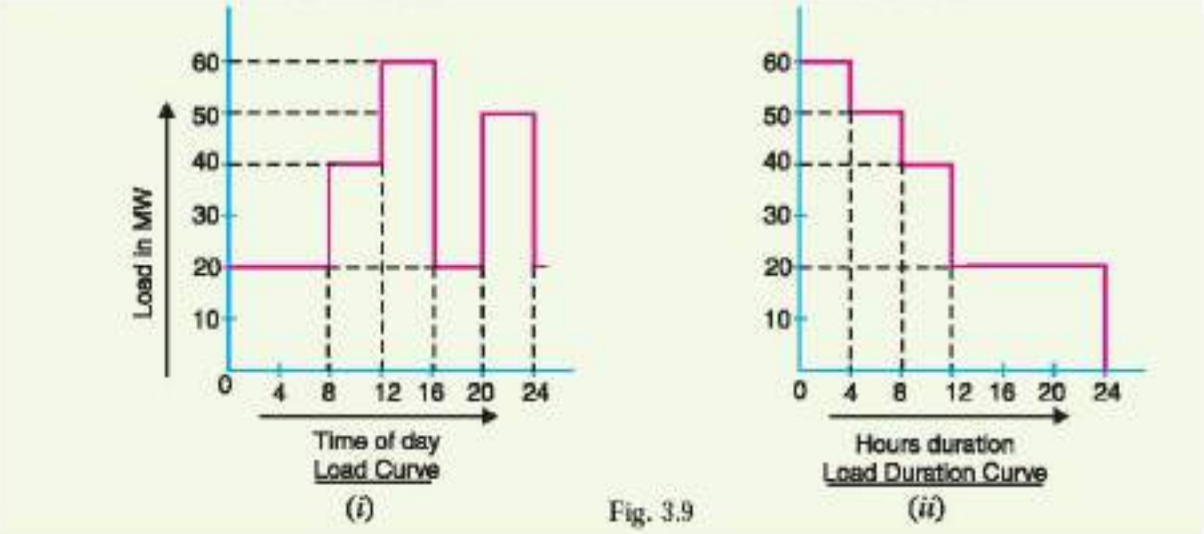


Fig. 3.9

$$\text{Units generated/day} = \text{Area (in kWh) under daily load curve}$$
$$= 10^3 [20 \times 8 + 40 \times 4 + 60 \times 4 + 20 \times 4 + 50 \times 4]$$
$$= \mathbf{840 \times 10^3 \text{ kWh}}$$

Alternatively :

$$\text{Units generated/day} = \text{Area (in kWh) under daily load duration curve}$$
$$= 10^3 [60 \times 4 + 50 \times 4 + 40 \times 4 + 20 \times 12]$$
$$= \mathbf{840 \times 10^3\ kWh}$$

which is the same as above.

Example 3.16. *The annual load duration curve of a certain power station can be considered as a straight line from 20 MW to 4 MW. To meet this load, three turbine-generator units, two rated at 10 MW each and one rated at 5 MW are installed. Determine (i) installed capacity (ii) plant factor (iii) units generated per annum (iv) load factor and (v) utilisation factor.*

Solution. Fig. 3.10 shows the annual load duration curve of the power station.

(*i*) Installed capacity = 10 + 10 + 5 = **25 MW**

(*ii*) Referring to the load duration curve,

$$\text{Average demand} = \frac{1}{2}[20 + 4] = 12 \text{ MW}$$

$$\therefore \quad \text{Plant factor} = \frac{\text{Average demand}}{\text{Plant capacity}} = \frac{12}{25} = 0{\cdot}48 = \mathbf{48\%}$$

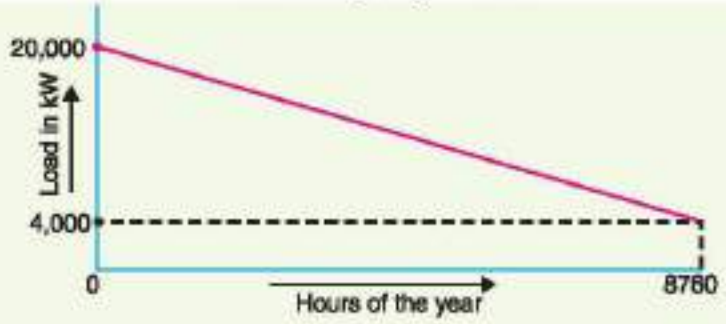


Fig. 3.10

(*iii*) Units generated/annum = Area (in kWh) under load duration curve

$$= \frac{1}{2}[4000 + 20{,}000] \times 8760 \text{ kWh} = \mathbf{105{\cdot}12 \times 10^6\ kWh}$$

(*iv*)
$$\text{Load factor} = \frac{12{,}000}{20{,}000} \times 100 = \mathbf{60\%}$$

(*v*)
$$\text{Utilisation factor} = \frac{\text{Max.demand}}{\text{Plant capacity}} = \frac{20{,}000}{25000} = 0{\cdot}8 = \mathbf{80\%}.$$

Example 3.17. *At the end of a power distribution system, a certain feeder supplies three distribution transformers, each one supplying a group of customers whose connected load are listed as follows :*

Transformer 1	Transformer 2	Transformer 3
General power service and lighting	*Residence lighting*	*Store lighting and power*
a : 10 H.P., 5 kW	*e : 5 kW*	*j : 10 kW, 5 H.P.*
b : 7·5 H.P., 4 kW	*f : 4 kW*	*k : 8 kW, 25 H.P.*
c : 15 H.P.	*g : 8 kW*	*l : 4 kW*
d : 5 H.P., 2 kW	*h : 15 kW*	
	i : 20 kW	

Use the factors given in Art. 3.8 and predict the maximum demand on the feeder. The H.P. load is motor load and assume an efficiency of 72%.

Solution. The individual maximum demands of the group of consumers connected to transformer 1 are obtained with factors from the table on page 49.

$$a: \quad \left(10 \times \frac{0.746}{0.72}\right) \times 0.65 + 5 \times 0.60^{*} = 9.74 \text{ kW}$$

$$b: \quad \left(7.5 \times \frac{0.746}{0.72}\right) \times 0.75 + 4 \times 0.60 = 8.23 \text{ kW}$$

$$c: \quad \left(15 \times \frac{0.746}{0.72}\right) \times 0.65 = 10.10 \text{ kW}$$

$$d: \quad \left(5 \times \frac{0.746}{0.72}\right) \times 0.75 + 2 \times 0.60 = 5.09 \text{ kW}$$

$$\text{Total} = 33.16 \text{ kW}$$

The diversity factor between consumers of this type of service is 1·5 (From the table of article 3.8).

$$\therefore \quad \text{Maximum demand on transformer 1} = \frac{33.16}{1.5} = 22.10 \text{ kW}$$

In a similar manner, the other transformer loads are determined to be

	Total	*Simultaneous*
Transformer 2	26 kW	7·43 kW
Transformer 3	29·13 kW	19·40 kW

The diversity factor between transformers is 1·3.

$$\therefore \quad \text{Maximum load on feeder} = \frac{22.10 + 7.43 + 19.40}{1.3} = \frac{48.93}{1.3} = \mathbf{37.64 \text{ kW}}$$

TUTORIAL PROBLEMS

1. A generating station has a connected load of 40 MW and a maximum demand of 20 MW ; the units generated being 60×10^6. Calculate (*i*) the demand factor (*ii*) the load factor. **[(*i*) 0·5 (*ii*) 34·25%]**
2. A 100 MW powers stations delivers 100 MW for 2 hours, 50 MW for 8 hours and is shut down for the rest of each day. It is also shut down for maintenance for 60 days each year. Calculate its annual load factor. **[21%]**
3. A power station is to supply four regions of loads whose peak values are 10,000 kW, 5000 kW, 8000 kW and 7000 kW. The diversity factor of the load at the station is 1.5 and the average annual load factor is 60%. Calculate the maximum demand on the station and annual energy supplied from the station. **[20,000 kW ; 105·12 × 10^6 kWh]**
4. A generating station supplies the following loads : 15000 kW, 12000 kW, 8500 kW, 6000 kW and 450 kW. The station has a maximum demand of 22000 kW. The annual load factor of the station is 48%. Calculate (*i*) the number of units supplied annually (*ii*) the diversity factor and (*iii*) the demand factor. **[(*i*) 925 × 10^5 kWh (*ii*) 52·4% (*iii*) 1·9]**
5. A generating station has a maximum demand of 20 MW, a load factor of 60%, a plant capacity factor of 48% and a plant use factor of 80% . Find :
   (*i*) the daily energy produced
   (*ii*) the reserve capacity of the plant

---

* Since demand factor for a particular load magnitude in not given in the table, it is reasonable to assume the average value *i.e.*

$$\text{Demand Factor} = \frac{0.7 + 0.5}{2} = \frac{1.2}{2} = 0.6$$

(*iii*) the maximum energy that could be produced daily if the plant was running all the time

(*iv*) the maximum energy that could be produced daily if the plant was running fully loaded and operating as per schedule. [(*i*) 288×10^3 kWh (*ii*) 0 (*iii*) 480×10^3 kWh (*iv*) 600×10^3 kWh]

6. A generating station has the following daily load cycle :

Time (hours)	0—6	6—10	10—12	12—16	16—20	20—24
Load (MW)	20	25	30	25	35	20

Draw the load curve and find

(*i*) maximum demand,

(*ii*) units generated per day,

(*iii*) average load,

(*iv*) load factor. [(*i*) 35 MW (*ii*) 560×10^3 kWh (*iii*) 23333 kW (*iv*) 66·67%]

7. A power station has to meet the following load demand :

Load *A*	50 kW	between	10 A.M. and 6 P.M.
Load *B*	30 kW	between	6 P.M. and 10 P.M.
Load *C*	20 kW	between	4 P.M. and 10 A.M.

Plot the daily load curve and determine (*i*) diversity factor (*ii*) units generated per day (*iii*) load factor.
[(*i*) 1·43 (*ii*) 880 kWh (*iii*) 52·38%]

8. A substation supplies power by four feeders to its consumers. Feeder no. 1 supplies six consumers whose individual daily maximum demands are 70 kW, 90 kW, 20 kW, 50 kW, 10 kW and 20 kW while the maximum demand on the feeder is 200 kW. Feeder no. 2 supplies four consumers whose daily maximum demands are 60 kW, 40 kW, 70 kW and 30 kW, while the maximum demand on the feeder is 160 kW. Feeder nos. 3 and 4 have a daily maximum demand of 150 kW and 200 kW respectively while the maximum demand on the station is 600 kW.

Determine the diversity factors for feeder no. 1, feeder no. 2 and for the four feeders. [1·3, 1·25, 1·183]

9. A central station is supplying energy to a community through two substations. Each substation feeds four feeders. The maximum daily recorded demands are :

POWER STATION....... 12,000 KW

Substation *A* ...... 6000 kW	Sub-station *B* .... 9000 kW
Feeder 1 .... 1700 kW	Feeder 1 .... 2820 kW
Feeder 2 .... 1800 kW	Feeder 2 .... 1500 kW
Feeder 3 .... 2800 kW	Feeder 3 .... 4000 kW
Feeder 4 .... 600 kW	Feeder 4 .... 2900 kW

Calculate the diversity factor between (*i*) substations (*ii*) feeders on substation A and (*iii*) feeders on substation B. [(*i*) 1·25 (*ii*) 1·15 (*iii*) 1·24]

10. The yearly load duration curve of a certain power station can be approximated as a straight line ; the maximum and minimum loads being 80 MW and 40 MW respectively. To meet this load, three turbine-generator units, two rated at 20 MW each and one at 10 MW are installed. Determine (*i*) installed capacity (*ii*) plant factor (*iii*) kWh output per year (*iv*) load factor.

[(*i*) 50MW (*ii*) 48% (*iii*) 210×10^6 (*iv*) 60%]

3.9 Load Curves and Selection of Generating Units

The load on a power station is seldom constant; it varies from time to time. Obviously, a single generating unit (*i.e.*, alternator) will not be an economical proposition to meet this varying load. It is because a single unit will have very poor* efficiency during the periods of light loads on the power station. Therefore, in actual practice, a number of generating units of different sizes are installed in a power station. The selection of the number and sizes of the units is decided from the annual load curve of the station. *The number and size of the units are selected in such a way that they correctly*

* The efficiency of a machine (alternator in this case) is maximum at nearly 75% of its rated capacity.

fit the station load curve. Once this underlying principle is adhered to, it becomes possible to operate the generating units at or near the point of maximum efficiency.

Illustration. The principle of selection of number and sizes of generating units with the help of load curve is illustrated in Fig. 3.11. In Fig. 3.11 (*i*), the annual load curve of the station is shown. It is clear form the curve that load on the station has wide variations ; the minimum load being somewhat near 50 kW and maximum load reaching the value of 500 kW. It hardly needs any mention that use of a single unit to meet this varying load will be highly uneconomical.

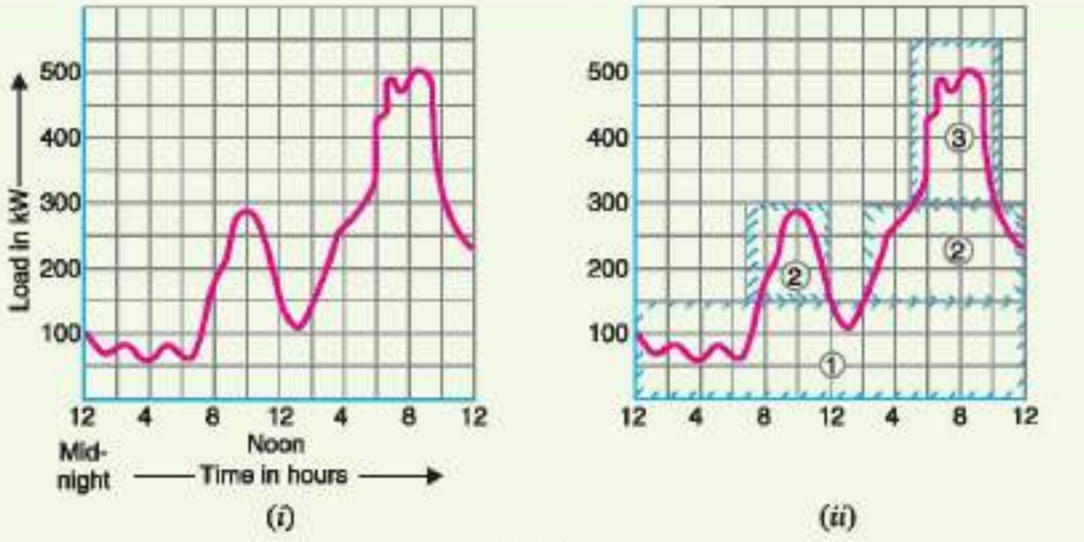


Fig. 3.11

As discussed earlier, the total plant capacity is divided into several generating units of different sizes to fit the load curve. This is illustrated in Fig. 3.11(*ii*) where the plant capacity is divided into three* units numbered as 1, 2 and 3. The cyan colour outline shows the units capacity being used. The three units employed have different capacities and are used according to the demand on the station. In this case, the operating schedule can be as under :

Time	*Units in operation*
From 12 midnight to 7 A.M.	Only unit no.1 is put in operation.
From 7 A.M. to 12.00 noon	Unit no. 2 is also started so that both units 1 and 2 are in operation.
From 12.00 noon to 2 P.M.	Unit no. 2 is stopped and only unit 1 operates.
From 2 P.M. to 5 P.M.	Unit no. 2 is again started. Now units 1 and 2 are in operation.
From 5 P.M. to 10.30 P.M.	Units 1, 2 and 3 are put in operation.
From 10. 30 P.M. to 12.00 midnight	Units 1 and 2 are put in operation.

Thus by selecting the proper number and sizes of units, the generating units can be made to operate near maximum efficiency. This results in the overall reduction in the cost of production of electrical energy.

3.10 Important Points in the Selection of Units

While making the selection of number and sizes of the generating units, the following points should be kept in view :

(*i*) The number and sizes of the units should be so selected that they approximately fit the annual load curve of the station.

* It may be seen that the generating units can fit the load curve more closely if more units of smaller sizes are employed. However, using greater number of units increases the investment cost per kW of the capacity.

(ii) The units should be *preferably* of different capacities to meet the load requirements. Although use of identical units (*i.e.*, having same capacity) ensures saving* in cost, they often do not meet the load requirement.

(iii) The capacity of the plant should be made 15% to 20% more than the maximum demand to meet the future load requirements.

(iv) There should be a spare generating unit so that repairs and overhauling of the working units can be carried out.

(v) The tendency to select a large number of units of smaller capacity in order to fit the load curve very accurately should be avoided. It is because the investment cost per kW of capacity increases as the size of the units decreases.

Example 3.18. *A proposed station has the following daily load cycle :*

Time in hours	*6—8*	*8—11*	*11—16*	*16—19*	*19—22*	*22—24*	*24—6*
Load in MW	*20*	*40*	*50*	*35*	*70*	*40*	*20*

Draw the load curve and select suitable generator units from the 10,000, 20,000, 25,000, 30,000 kVA. Prepare the operation schedule for the machines selected and determine the load factor from the curve

Solution. The load curve of the power station can be drawn to some suitable scale as shown in Fig. 3.12.

$$\begin{aligned}\text{Units generated per day} &= \text{Area (in kWh) under the load curve}\\ &= 10^3\,[20\times 8+40\times 3+50\times 5+35\times 3+70\times 3+40\times 2]\\ &= 10^3\,[160+120+250+105+210+80]\text{ kWh}\\ &= 925\times 10^3\text{ kWh}\end{aligned}$$

$$\text{Average load} = \frac{925\times 10^3}{24} = 38541{\cdot}7\text{ kW}$$

$$\text{Load factor} = \frac{38541{\cdot}7}{70\times 10^3}\times 100 = \mathbf{55{\cdot}06\%}$$

Selection of number and sizes of units : Assuming power factor of the machines to be 0·8, the output of the generating units available will be 8, 16, 20 and 24 MW. There can be several possibilities. However, while selecting the size and number of units, it has to be borne in mind that (i) one set of highest capacity should be kept as standby unit (ii) the units should meet the maximum demand (70 MW in this case) on the station (iii) there should be overall economy.

Keeping in view the above facts, *4 sets of 24 MW each may be chosen.* Three sets will meet the maximum demand of 70 MW and one unit will serve as a standby unit.

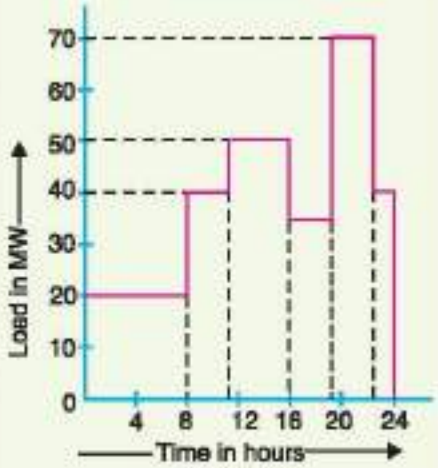


Fig. 3.12

Operational schedule. Referring to the load curve shown in Fig. 3.12, the operational schedule will be as under :

(i) Set No. 1 will run for 24 hours.

(ii) Set No. 2 will run from 8.00 hours to midnight.

(iii) Set No. 3 will run from 11.00 hours to 16 hours and again from 19 hours to 22 hours.

Example 3.19. *A generating station is to supply four regions of load whose peak loads are 10 MW, 5 MW, 8 MW and 7 MW. The diversity factor at the station is 1·5 and the average annual load factor is 60%. Calculate :*

* Due to duplication of sizes and dimensions of pipes, foundations etc.

(i) *the maximum demand on the station.*

(ii) *annual energy supplied by the station.*

(iii) *Suggest the installed capacity and the number of units.*

Solution.

(i) Max. demand on station $= \dfrac{\text{Sum of max. demands of the regions}}{\text{Diversity factor}}$

$= (10 + 5 + 8 + 7)/1{\cdot}5 = \mathbf{20\ MW}$

(ii) Units generated/annum $=$ Max. demand × L.F. × Hours in a year

$= (20 \times 10^3) \times (0{\cdot}6) \times (8760)$ kWh

$= \mathbf{105{\cdot}12 \times 10^6\ kWh}$

(iii) The installed capacity of the station should be 15% to 20% more than the maximum demand in order to meet the future growth of load. Taking installed capacity to be 20% more than the maximum demand,

Installed capacity $= 1{\cdot}2 \times$ Max. demand $= 1{\cdot}2 \times 20 = \mathbf{24\ MW}$

Suitable unit sizes are 4, each of 6 MW capacity.

3.11 Base Load and Peak Load on Power Station

The changing load on the power station makes its load curve of variable nature. Fig. 3.13. shows the typical load curve of a power station. It is clear that load on the power station varies from time to time. However, a close look at the load curve reveals that load on the power station can be considered in two parts, namely;

(i) Base load

(ii) Peak load

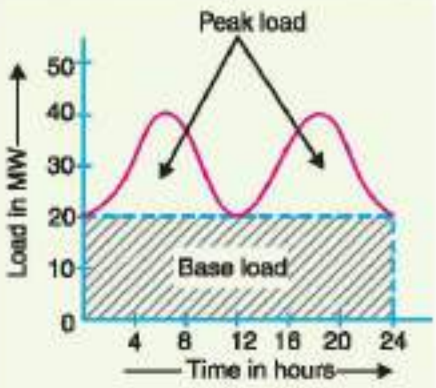


Fig. 3.13

(i) **Base load.** *The unvarying load which occurs almost the whole day on the station is known as* **base load.**

Referring to the load curve of Fig. 3.13, it is clear that 20 MW of load has to be supplied by the station at all times of day and night *i.e.* throughout 24 hours. Therefore, 20 MW is the base load of the station. As base load on the station is almost of constant nature, therefore, it can be suitably supplied (as discussed in the next Article) without facing the problems of variable load.

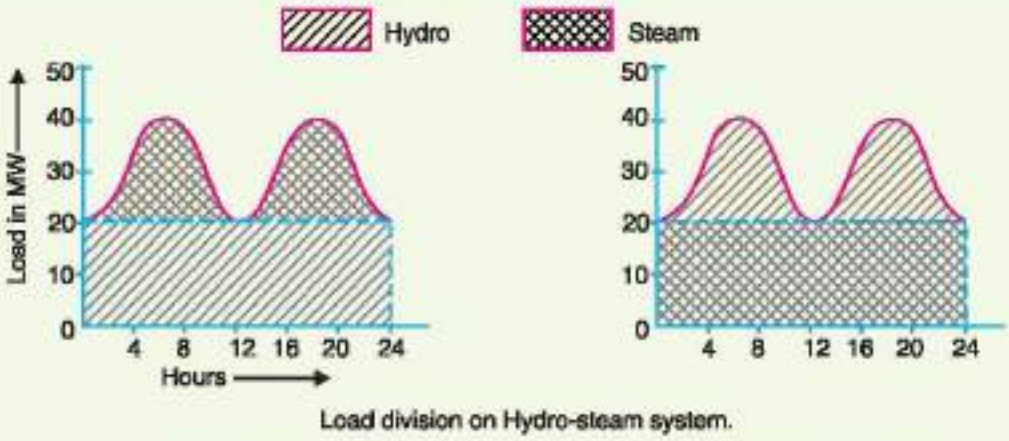
(ii) **Peak load.** *The various peak demands of load over and above the base load of the station is known as* **peak load.**

Referring to the load curve of Fig. 3.13, it is clear that there are peak demands of load excluding base load. These peak demands of the station generally form a small part of the total load and may occur throughout the day.

3.12 Method of Meeting the Load

The total load on a power station consists of two parts *viz.*, base load and peak load. In order to achieve overall economy, *the best method to meet load is to interconnect two different power stations.* The more efficient plant is used to supply the base load and is known as *base load power station.* The less efficient plant is used to supply the peak loads and is known as *peak load power station.* There is no hard and fast rule for selection of base load and peak load stations as it would depend upon the particular situation. For example, both hydro-electric and steam power stations are quite efficient and can be used as base load as well as peak load station to meet a particular load requirement.

Illustration. The interconnection of steam and hydro plants is a beautiful illustration to meet the load. When water is available in sufficient quantity as in summer and rainy season, the hydro-electric plant is used to carry the base load and the steam plant supplies the peak load as shown in Fig 3.14 (*i*).



(*i*) High Flow day (*ii*) Low Flow day

Fig. 3.14

However, when the water is not available in sufficient quantity as in winter, the steam plant carries the base load, whereas the hydro-electric plant carries the peak load as shown in Fig. 3.14 (*ii*).

3.13 Interconnected Grid System

The connection of several generating stations in parallel is known as **interconnected grid system.**

The various problems facing the power engineers are considerably reduced by interconnecting different power stations in parallel. Although interconnection of station involves extra cost, yet considering the benefits derived from such an arrangement, it is gaining much favour these days. Some of the advantages of interconnected system are listed below :

(*i*) *Exchange of peak loads* : An important advantage of interconnected system is that the peak load of the power station can be exchanged. If the load curve of a power station shows a peak demand that is greater than the rated capacity of the plant, then the excess load can be shared by other stations interconnected with it.

(*ii*) *Use of older plants* : The interconnected system makes it possible to use the older and less efficient plants to carry peak loads of short durations. Although such plants may be inadequate when used alone, yet they have sufficient capacity to carry short peaks of loads when interconnected with other modern plants. Therefore, interconnected system gives a direct key to the use of obsolete plants.

(*iii*) *Ensures economical operation* : The interconnected system makes the operation of concerned power stations quite economical. It is because sharing of load among the stations is arranged in such a way that more efficient stations work continuously throughouts the year at a high load factor and the less efficient plants work for peak load hours only.

(*iv*) *Increases diversity factor* : The load curves of different interconnected stations are generally different. The result is that the maximum demand on the system is much reduced as compared to the sum of individual maximum demands on different stations. In other words, the diversity factor of the system is improved, thereby increasing the effective capacity of the system.

(*v*) *Reduces plant reserve capacity* : Every power station is required to have a standby unit for emergencies. However, when several power stations are connected in parallel, the reserve capacity of the system is much reduced. This increases the efficiency of the system.

(*vi*) *Increases reliability of supply* : The interconnected system increases the reliability of supply. If a major breakdown occurs in one station, continuity of supply can be maintained by other healthy stations.

Example 3.20. *A base load station having a capacity of 18 MW and a standby station having a capacity of 20 MW share a common load. Find the annual load factors and plant capacity factors of two power stations from the following data :*

Annual standby station output	$= 7{\cdot}35 \times 10^6$ *kWh*
Annual base load station output	$= 101{\cdot}35 \times 10^6$ *kWh*
Peak load on standby station	*= 12 MW*
Hours of use by standby station/year	*= 2190 hours*

Solution.

Installed capacity of standby unit

$$= 20 \text{ MW} = 20 \times 10^3 \text{ kW}$$

Installed capacity of base load plant

$$= 18 \text{ MW} = 18 \times 10^3 \text{ kW}$$

Standby station

$$\text{Annual load factor} = \frac{\text{kWh generated / annum}}{\text{Max. demand} \times \text{Annual working hours}} \times 100$$

$$= \frac{7{\cdot}35 \times 10^6}{(12 \times 10^3) \times 2190} \times 100 = \mathbf{28\%}$$

$$\text{Annual plant capacity factor} = \frac{\text{kWh output / annum}}{\text{Installed capacity} \times \text{Hours in a year}} \times 100$$

$$= \frac{7{\cdot}35 \times 10^6}{(20 \times 10^3) \times 8760} \times 100 = \mathbf{4{\cdot}2\%}$$

Base load station. It is reasonable to assume that the maximum demand on the base load station is equal to the installed capacity (*i.e.*, 18 MW). It operates throughout the year *i.e.*, for 8760 hours.

$$\therefore \quad \text{Annual load factor} = \frac{101{\cdot}35 \times 10^6}{(18 \times 10^3) \times 8760} = \mathbf{64{\cdot}2\%}$$

As the base load station has no reserves above peak load and it is in continuous operation, therefore, its capacity factor is also **64.2%**.

Example 3.21. *The load duration curve for a typical heavy load being served by a combined hydro-steam system may be approximated by a straight line; maximum and minimum loads being 60,000 kW and 20,000 kW respectively. The hydro power available at the time of minimum regulated flow is just sufficient to take a peak load of 50,000 kWh per day. It is observed that it will be economical to pump water from tail race to the reservoir by utilising the steam power plant during the off-peak periods and thus running the station at 100% load factor. Determine the maximum capacity of each type of plant. Assume the efficiency of steam conversion to be 60%.*

Solution. *OCBA* represents the load duration curve for the combined system as shown in Fig. 3.15. The total maximum demand (*i.e.*, 60,000 kW) is represented by *OC*, whereas the minimum demand (*i.e.*, 20,000 kW) is represented by *OD*.

Let

OE = Capacity of steam plant

EC = Capacity of hydro plant

Area CHI = The energy available from hydro plant in the low flow period.

$$\text{Area } FGB = \text{The off-peak* period energy available from steam plant}$$

Obviously, the energy of hydro plant represented by area *HEFI* and available from reservoir has been supplied by steam power plant represented by area *FGB*. As steam electric conversion is 60%,

$$\therefore \quad \text{Area } HEFI = 0{\cdot}6 \times \text{Area } FGB \quad \text{... (i)}$$

But

$$\text{Area } HEFI = \text{Area } CFE - \text{Area } CHI$$

$$= \frac{1}{2}xy - 50{,}000\dagger$$

Now

$$\text{Area } FGB = \frac{1}{2} \times FG \times GB = \frac{1}{2}(24 - x)(40{,}000 - y)$$

Putting the various values in exp. (*i*), we get,

$$\frac{1}{2}xy - 50{,}000 = 0{\cdot}6\left[\frac{1}{2}(24 - x)(40{,}000 - y)\right]$$

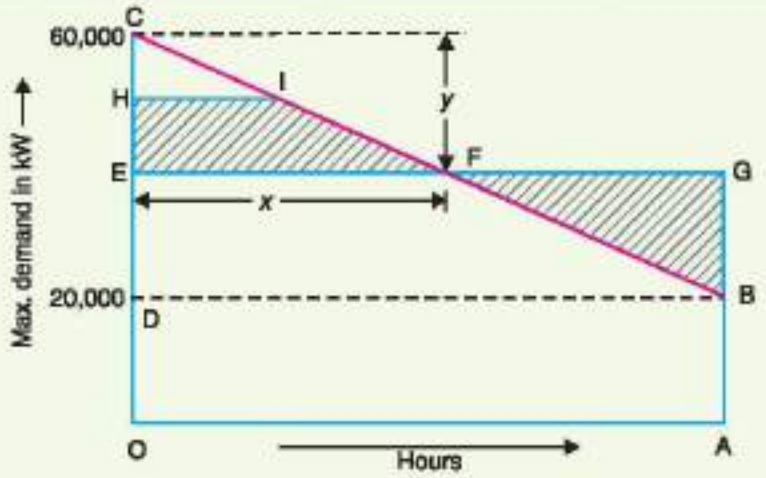


Fig. 3.15

or $0{\cdot}2\,xy + 12000\,x + 7{\cdot}2\,y - 3{,}38{,}000 = 0$ ... (*ii*)

Also from similar triangles *CEF* and *CDB*, we get,

$$\frac{y}{40{,}000} = \frac{x}{24}$$

$$\therefore \quad y = \frac{40{,}000\,x}{24} \quad \text{... (iii)}$$

Putting $y = 40{,}000\,x/24$ from exp. (*iii*) into exp. (*ii*), we get,

$$333\,x^2 + 24000\,x - 3{,}38{,}000 = 0$$

or

$$x^2 + 72x - 1015 = 0$$

$$\therefore \quad x = \frac{-72 \pm \sqrt{5184 + 4060}}{2} = \frac{-72 \pm 96}{24} = 12$$

$\therefore$ Capacity of the hydro plant is

$$y\,(= EC) = \frac{40{,}000 \times 12}{24} = \mathbf{20{,}000\ kW}$$

$$\text{Capacity of steam plant} = 60{,}000 - 20{,}000 = \mathbf{40{,}000\ kW}$$

Example 3.22. *The annual load duration curve for a typical heavy load being served by a steam station, a run-of-river station and a reservoir hydro-electric station is as shown in Fig. 3.16. The ratio of number of units supplied by these stations is as follows :*

* It is clear from load duration curve that the capacity of steam plant represented by area *FGB* is not being utilised efficiently. This steam energy can be used to pump water in tail race back to the reservoir.

† Because during minimum regulated flow, hydro energy supplied is 50,000 kWh.

Steam : Run-of-river : Reservoir :: 7 : 4 : 1

The run-of-river station is capable of generating power continuously and works as a base load station. The reservoir station works as a peak load station. Determine (i) the maximum demand of each station and (ii) load factor of each station.

Solution. *ODCA* is the annual load duration curve for the system as shown in Fig. 3.16. The energy supplied by the reservoir plant is represented by area *DFG*; steam station by area *FGCBE* and run-of-river by area *OEBA*. The maximum and minimum loads on the system are 320 MW and 160 MW respectively.

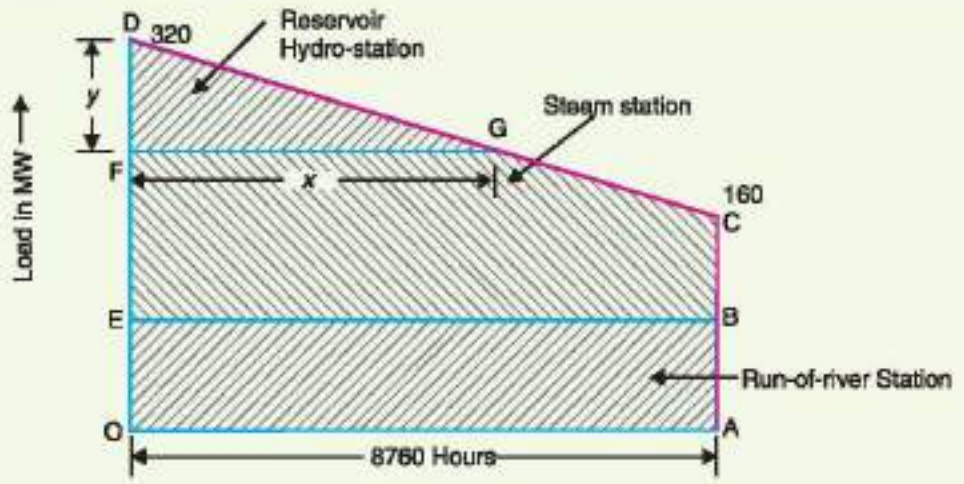


Fig. 3.16

$$\text{Units generated/annum} = \text{Area (in kWh) under annual load duration curve}$$

$$= 10^3\left[\frac{1}{2}(320+160)\times 8760\right]\text{kWh} = 2102\cdot 4\times 10^6\text{ kWh}$$

As the steam plant, run-of-river plant and hydro plant generate units in the ratio of 7 : 4 : 1, therefore, units generated by each plant are given by :

$$\text{Steam plant} = 2102{\cdot}4\times 10^6\times 7/12 = 1226{\cdot}4\times 10^6\text{ kWh}$$

$$\text{Run-of-river plant} = 2102{\cdot}4\times 10^6\times 4/12 = 700{\cdot}8\times 10^6\text{ kWh}$$

$$\text{Reservoir plant} = 2102{\cdot}4\times 10^6\times 1/12 = 175{\cdot}2\times 10^6\text{ kWh}$$

(*i*) Maximum demand on run-of-river plant

$$= \frac{\text{Area } OEBA}{OA} = \frac{700\cdot 8\times 10^6}{8760} = \mathbf{80{,}000\ kW}$$

Suppose the maximum demand of reservoir plant is *y* MW and it operates for *x* hours (See Fig. 3.16).

Then,
$$\frac{y}{160} = \frac{x}{8760} \text{ or } x = \frac{8760\,y}{160}$$

Units generated per annum by reservoir plant

$$= \text{Area (in kWh) } DFG$$

$$= 10^3\left(\frac{1}{2}xy\right) = 10^3\left(\frac{1}{2}\times\frac{8760\,y}{160}\,y\right)$$

$$= \frac{y^2}{32}\times 8{,}76{,}000$$

But the units generated by reservoir plant are $175{\cdot}2\times 10^6$ kWh.

$$\therefore \quad \frac{y^2}{32}\times 8{,}76{,}000 = 175{\cdot}2\times 10^6$$

$$y^2 = 6400 \text{ or } y = \sqrt{6400} = 80 \text{ MW}$$

∴ Maximum demand on reservoir station is

$$FD = \textbf{80 MW}$$

Maximum demand on steam station is

$$EF = 320 - 80 - 80 = \textbf{160 MW}$$

(*ii*) L.F. of run of river plant = **100* %**

$$\text{L.F. of reservoir plant} = \frac{\text{Units generated / annum}}{\text{Maximum demand} \times 8760} \times 100$$

$$= \frac{175 \cdot 2 \times 10^6}{(80 \times 10^3) \times 8760} \times 100 = \textbf{25\%}$$

$$\text{L.F. of steam plant} = \frac{1226 \cdot 4 \times 10^6}{(160 \times 10^3) \times 8760} \times 100 = \textbf{87.5\%}$$

SELF - TEST

1. Fill in the blanks by inserting appropriate words/figures :

(*i*) The area under the daily load curve gives ....

(*ii*) The connected load is generally .... than the maximum demand.

(*iii*) The value of demand factor is .... than 1.

(*iv*) The higher the load factor of a power station, the .... is the cost per unit generated.

(*v*) The value of diversity factor is .... than 1.

(*vi*) The lesser the diversity factor, the .... is the cost of generation of power.

(*vii*) A generating unit operates with maximum efficiency at about .... % of its rated capacity.

(*viii*) According to Indian Electricity Supply Act (1948), the capacity of the spare set should be ....

(*ix*) In an annual load curve, ... is taken along Y-axis and .... along X-axis.

(*x*) Base load occurs on the power station for .... hours in a day.

2. Pick up the correct words/figures from the brackets and fill in the blanks :

(*i*) Area under the daily load curve divided by 24 gives ....
(*average load, maximum demand, units generated*)

(*ii*) The knowledge of diversity factor helps in determining ....
(*average load, units generated, plant capacity*)

(*iii*) More efficient plants are used as .... (*base load stations, peak load stations*)

(*iv*) A diesel power plant is generally used as a .... (*base load station, peak load station*)

(*v*) In a hydro-steam system, steam power station carries the base load during ....
(*high flow day, low flow day*)

(*vi*) In an interconnected grid system, the diversity factor of the whole system ....
(*increases, decreases, remains constant*)

(*vii*) Installed capacity of a power station is .... then the maximum demand. (*less, more*)

(*viii*) Annual load factor is determined from .... load curve. (*daily, monthly, annual*)

ANSWERS TO SELF-TEST

1. (*i*) units generated in the day (*ii*) more (*iii*) less (*iv*) lesser (*v*) more (*vi*) greater (*vii*) 75% (*viii*) highest of all sets (*ix*) load, hours (*x*) 24.
2. (*i*) average load (*ii*) plant capacity (*iii*) base load stations (*iv*) peak load station (*v*) low flow day (*vi*) increases (*vii*) more (*viii*) annual.

* Since it operates continuously at rated capacity (*i.e.* it is a base load station).

CHAPTER REVIEW TOPICS

1. Why is the load on a power station variable ? What are the effects of variable load on the operation of the power station ?
2. What do you understand by the load curve ? What informations are conveyed by a load curve ?
3. Define and explain the importance of the following terms in generation :
   (*i*) connected load (*ii*) maximum demand (*iii*) demand factor (*iv*) average load.
4. Explain the terms load factor and divesity factor. How do these factors influence the cost of generation?
5. Explain how load curves help in the selection of size and number of generating units.
6. Discuss the important points to be taken into consideration while selecting the size and number of units.
7. What do you understand by (*i*) base load and (*ii*) peak load of a power station ?
8. Discuss the method of meeting the peak load of an electrified area.
9. Discuss the advantages of interconnected grid system.
10. Write short notes on the following :
    (*i*) load curves,
    (*ii*) load division on hydro-steam system,
    (*iii*) load factor,
    (*iv*) plant capacity factor.

DISCUSSION QUESTIONS

1. Why are load curves drawn ?
2. How will you improve the diversity factor of a power station ?
3. What is the importance of load factor ?
4. What is the importance of diversity factor ?
5. The values of demand factor and load factor are always less than 1. Why ?

CHAPTER

14

A.C. Distribution

Introduction

In the beginning of electrical age, electricity was generated, transmitted and distributed as direct current. The principal disadvantage of d.c. system was that voltage level could not readily be changed, except by the use of rotating machinery, which in most cases was too expensive. With the development of transformer by George Westinghouse, a.c. system has become so predominant as to make d.c. system practically extinct in most parts of the world. The present day large power system has been possible only due to the adoption of a.c. system.

Now-a-days, electrical energy is generated, transmitted and distributed in the form of alternating current as an economical proposition. The electrical energy produced at the power station is transmitted at very high voltages by 3-phase, 3-wire system to step-down sub-stations for distribution. The distribution system consists of two parts *viz.* primary distribution and secondary distribution. The primary distribution circuit is 3-phase, 3-wire and operates at voltages (3·3 or 6·6 or 11kV) somewhat higher than general utilisation levels. It delivers power to the secondary distribution circuit through distribution transformers situated near consumers' localities. Each distribution transformer steps down the voltage to 400 V and power is distributed to ultimate consumers' by 400/230 V, 3-phase, 4-wire system. In this chapter, we shall focus our attention on the various aspects of a.c. distribution.

14.1 A.C. Distribution Calculations

A.C. distribution calculations differ from those of d.c. distribution in the following respects :

(*i*) In case of d.c. system, the voltage drop is due to resistance alone. However, in a.c. system, the voltage drops are due to the combined effects of resistance, inductance and capacitance.

(*ii*) In a d.c. system, additions and subtractions of currents or voltages are done arithmetically but in case of a.c. system, these operations are done vectorially.

(*iii*) In an a.c. system, power factor (p.f.) has to be taken into account. Loads tapped off form the distributor are generally at different power factors. There are two ways of referring power factor *viz*

(*a*) It may be referred to supply or receiving end voltage which is regarded as the reference vector.

(*b*) It may be referred to the voltage at the load point itself.

There are several ways of solving a.c. distribution problems. However, symbolic notation method has been found to be most convenient for this purpose. In this method, voltages, currents and impedances are expressed in complex notation and the calculations are made exactly as in d.c. distribution.

14.2 Methods of Solving A.C. Distribution Problems

In a.c. distribution calculations, power factors of various load currents have to be considered since currents in different sections of the distributor will be the vector sum of load currents and not the arithmetic sum. The power factors of load currents may be given (*i*) *w.r.t.* receiving or sending end voltage or (*ii*) *w.r.t.* to load voltage itself. Each case shall be discussed separately.

(*i*) **Power factors referred to receiving end voltage.** Consider an a.c. distributor AB with concentrated loads of I_1 and I_2 tapped off at points C and B as shown in Fig. 14.1. Taking the receiving end voltage V_B as the reference vector, let lagging power factors at C and B be $\cos\phi_1$ and $\cos\phi_2$ *w.r.t.* V_B. Let R_1, X_1 and R_2, X_2 be the resistance and reactance of sections AC and CB of the distributor.

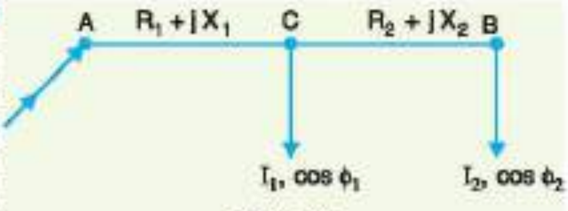


Fig. 14.1

Impedance of section AC, $\overrightarrow{Z_{AC}} = R_1 + j X_1$

Impedance of section CB, $\overrightarrow{Z_{CB}} = R_2 + j X_2$

Load current at point C, $\overrightarrow{I_1} = I_1 (\cos\phi_1 - j\sin\phi_1)$

Load current at point B, $\overrightarrow{I_2} = I_2 (\cos\phi_2 - j\sin\phi_2)$

Current in section CB, $\overrightarrow{I_{CB}} = \overrightarrow{I_2} = I_2 (\cos\phi_2 - j\sin\phi_2)$

Current in section AC,
$$\overrightarrow{I_{AC}} = \overrightarrow{I_1} + \overrightarrow{I_2}$$
$$= I_1 (\cos\phi_1 - j\sin\phi_1) + I_2 (\cos\phi_2 - j\sin\phi_2)$$

Voltage drop in section CB, $\overrightarrow{V_{CB}} = \overrightarrow{I_{CB}}\,\overrightarrow{Z_{CB}} = I_2 (\cos\phi_2 - j\sin\phi_2)(R_2 + j X_2)$

Voltage drop in section AC, $\overrightarrow{V_{AC}} = \overrightarrow{I_{AC}}\,\overrightarrow{Z_{AC}} = \left(\overrightarrow{I_1} + \overrightarrow{I_2}\right) Z_{AC}$

$$= [I_1(\cos\phi_1 - j\sin\phi_1) + I_2(\cos\phi_2 - j\sin\phi_2)]\,[R_1 + jX_1]$$

Sending end voltage, $\quad \vec{V_A} = \vec{V_B} + \vec{V_{CB}} + \vec{V_{AC}}$

Sending end current, $\quad \vec{I_A} = \vec{I_1} + \vec{I_2}$

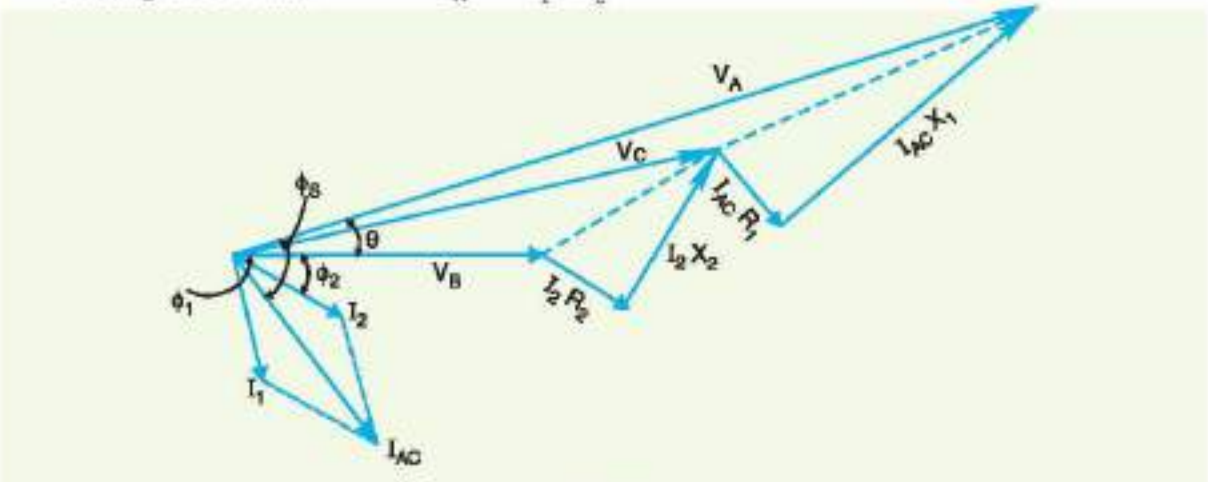


Fig. 14.2

The vector diagram of the a.c. distributor under these conditions is shown in Fig. 14.2. Here, the receiving end voltage V_B is taken as the reference vector. As power factors of loads are given *w.r.t.* V_B, therefore, I_1 and I_2 lag behind V_B by ϕ_1 and ϕ_2 respectively.

(*ii*) Power factors referred to respective load voltages. Suppose the power factors of loads in the previous Fig. 14.1 are referred to their respective load voltages. Then ϕ_1 is the phase angle between V_C and I_1 and ϕ_2 is the phase angle between V_B and I_2. The vector diagram under these conditions is shown in Fig. 14.3.

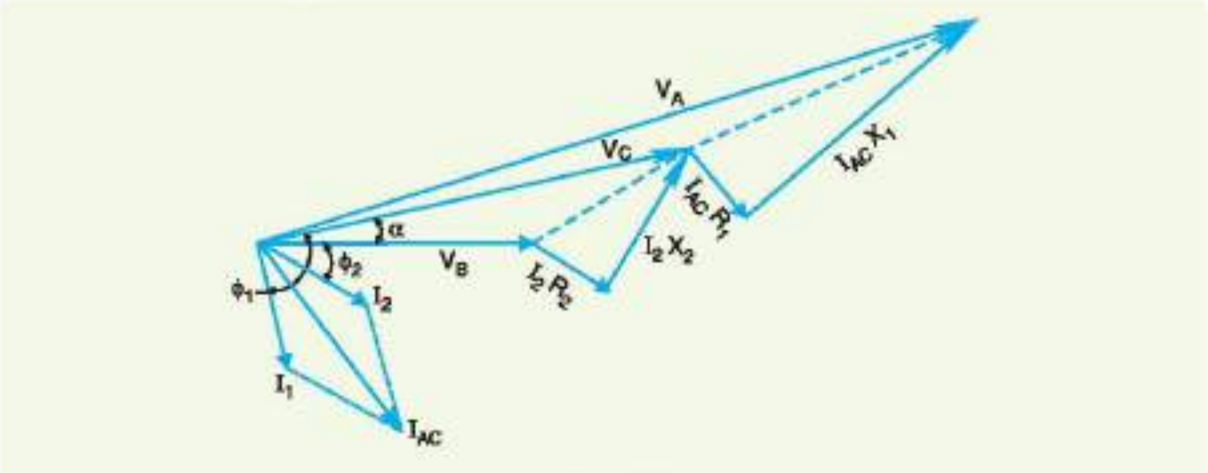


Fig. 14.3

Voltage drop in section $CB = \vec{I_2}\,\vec{Z_{CB}} = I_2(\cos\phi_2 - j\sin\phi_2)(R_2 + jX_2)$

Voltage at point $C = \vec{V_B}$ + Drop in section $CB = V_C \angle \alpha$ (say)

Now $\quad \vec{I_1} = I_1 \angle -\phi_1 \quad$ *w.r.t.* voltage V_C

$\therefore \quad \vec{I_1} = I_1 \angle -(\phi_1 - \alpha) \quad$ *w.r.t.* voltage V_B

i.e. $\quad \vec{I_1} = I_1[\cos(\phi_1 - \alpha) - j\sin(\phi_1 - \alpha)]$

Now $\quad \vec{I_{AC}} = \vec{I_1} + \vec{I_2}$

$$= I_1 [\cos(\phi_1 - \alpha) - j\sin(\phi_1 - \alpha)] + I_2 (\cos\phi_2 - j\sin\phi_2)$$

Voltage drop in section $AC = \overrightarrow{I_{AC}}\,\overrightarrow{Z_{AC}}$

$\therefore$ Voltage at point $A = V_B$ + Drop in CB + Drop in AC

Example 14.1. *A single phase a.c. distributor AB 300 metres long is fed from end A and is loaded as under :*

(*i*) *100 A at 0·707 p.f. lagging 200 m from point A*

(*ii*) *200 A at 0·8 p.f. lagging 300 m from point A*

The load resistance and reactance of the distributor is 0·2 Ω and 0·1 Ω per kilometre. Calculate the total voltage drop in the distributor. The load power factors refer to the voltage at the far end.

Solution. Fig. 14.4 shows the single line diagram of the distributor.

Impedance of distributor/km $= (0{\cdot}2 + j\,0{\cdot}1)\ \Omega$

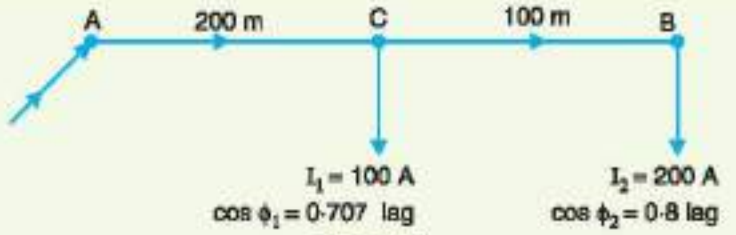


Fig. 14.4

Impedance of section AC, $\overrightarrow{Z_{AC}} = (0{\cdot}2 + j\,0{\cdot}1) \times 200/1000 = (0{\cdot}04 + j\,0{\cdot}02)\ \Omega$

Impedance of section CB, $\overrightarrow{Z_{CB}} = (0{\cdot}2 + j\,0{\cdot}1) \times 100/1000 = (0{\cdot}02 + j\,0{\cdot}01)\ \Omega$

Taking voltage at the far end B as the reference vector, we have,

Load current at point B, $\overrightarrow{I_2} = I_2(\cos\phi_2 - j\sin\phi_2) = 200\,(0{\cdot}8 - j\,0{\cdot}6)$
$= (160 - j\,120)$ A

Load current at point C, $\overrightarrow{I_1} = I_1(\cos\phi_1 - j\sin\phi_1) = 100\,(0{\cdot}707 - j\,0{\cdot}707)$
$= (70{\cdot}7 - j\,70{\cdot}7)$ A

Current in section CB, $\overrightarrow{I_{CB}} = \overrightarrow{I_2} = (160 - j\,120)$ A

Current in section AC, $\overrightarrow{I_{AC}} = \overrightarrow{I_1} + \overrightarrow{I_2} = (70{\cdot}7 - j\,70{\cdot}7) + (160 - j\,120)$
$= (230{\cdot}7 - j\,190{\cdot}7)$ A

Voltage drop in section CB, $\overrightarrow{V_{CB}} = \overrightarrow{I_{CB}}\,\overrightarrow{Z_{CB}} = (160 - j\,120)\,(0{\cdot}02 + j\,0{\cdot}01)$
$= (4{\cdot}4 - j\,0{\cdot}8)$ volts

Voltage drop in section AC, $\overrightarrow{V_{AC}} = \overrightarrow{I_{AC}}\,\overrightarrow{Z_{AC}} = (230{\cdot}7 - j\,190{\cdot}7)\,(0{\cdot}04 + j\,0{\cdot}02)$
$= (13{\cdot}04 - j\,3{\cdot}01)$ volts

Voltage drop in the distributor $= \overrightarrow{V_{AC}} + \overrightarrow{V_{CB}} = (13{\cdot}04 - j\,3{\cdot}01) + (4{\cdot}4 - j\,0{\cdot}8)$
$= (17{\cdot}44 - j\,3{\cdot}81)$ volts

Magnitude of drop $= \sqrt{(17{\cdot}44)^2 + (3{\cdot}81)^2} =$ **17·85 V**

Example 14.2. *A single phase distributor 2 kilometres long supplies a load of 120 A at 0·8 p.f. lagging at its far end and a load of 80 A at 0·9 p.f. lagging at its mid-point. Both power factors are* *referred to the voltage at the far end. The resistance and reactance per km (go and return) are 0·05 Ω and 0·1 Ω respectively. If the voltage at the far end is maintained at 230 V, calculate :*

(*i*) *voltage at the sending end*

(*ii*) *phase angle between voltages at the two ends.*

Solution. Fig. 14.5 shows the distributor *AB* with *C* as the mid-point

Impedance of distributor/km $= (0{\cdot}05 + j0{\cdot}1)\ \Omega$

Impedance of section *AC*, $\vec{Z}_{AC} = (0{\cdot}05 + j0{\cdot}1) \times 1000/1000 = (0{\cdot}05 + j0{\cdot}1)\ \Omega$

Impedance of section *CB*, $\vec{Z}_{CB} = (0{\cdot}05 + j0{\cdot}1) \times 1000/1000 = (0{\cdot}05 + j0{\cdot}1)\ \Omega$

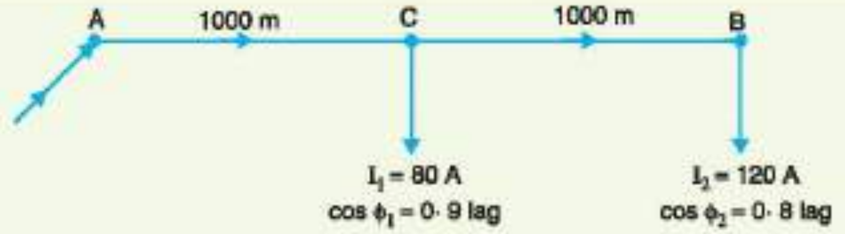


Fig. 14.5

Let the voltage V_B at point *B* be taken as the reference vector.

Then, $\vec{V_B} = 230 + j0$

(*i*) Load current at point *B*, $\vec{I_2} = 120\ (0{\cdot}8 - j0{\cdot}6) = 96 - j72$

Load current at point *C*, $\vec{I_1} = 80\ (0{\cdot}9 - j0{\cdot}436) = 72 - j34{\cdot}88$

Current in section *CB*, $\overrightarrow{I_{CB}} = \vec{I_2} = 96 - j72$

Current in section *AC*, $\overrightarrow{I_{AC}} = \vec{I_1} + \vec{I_2} = (72 - j34{\cdot}88) + (96 - j72)$

$= 168 - j106{\cdot}88$

Drop in section *CB*, $\overrightarrow{V_{CB}} = \overrightarrow{I_{CB}}\ \overrightarrow{Z_{CB}} = (96 - j72)\ (0{\cdot}05 + j0{\cdot}1)$

$= 12 + j6$

Drop in section *AC*, $\overrightarrow{V_{AC}} = \overrightarrow{I_{AC}}\ \overrightarrow{Z_{AC}} = (168 - j106{\cdot}88)\ (0{\cdot}05 + j0{\cdot}1)$

$= 19{\cdot}08 + j11{\cdot}45$

∴ Sending end voltage, $\vec{V_A} = \vec{V_B} + \overrightarrow{V_{CB}} + \overrightarrow{V_{AC}}$

$= (230 + j0) + (12 + j6) + (19.08 + j11.45)$

$= 261.08 + j17.45$

Its magnitude is $= \sqrt{(261{\cdot}08)^2 + (17{\cdot}45)^2} =$ **261·67 V**

(*ii*) The phase difference θ between V_A and V_B is given by :

$$\tan\theta = \frac{17{\cdot}45}{261{\cdot}08} = 0{\cdot}0668$$

∴ $\theta = \tan^{-1} 0{\cdot}0668 =$ **3·82°**

Example 14.3. *A single phase distributor one km long has resistance and reactance per conductor of 0·1 Ω and 0·15 Ω respectively. At the far end, the voltage V_B = 200 V and the current is 100 A at a p.f. of 0·8 lagging. At the mid-point M of the distributor, a current of 100 A is tapped at a p.f.*

of 0·6 lagging with reference to the voltage V_M at the mid-point. Calculate :

(i) voltage at mid-point

(ii) sending end voltage V_A

(iii) phase angle between V_A and V_B

Solution. Fig. 14.6 shows the single line diagram of the distributor *AB* with *M* as the mid-point.

Total impedance of distributor $= 2(0{\cdot}1 + j0{\cdot}15) = (0{\cdot}2 + j0{\cdot}3)\ \Omega$

Impedance of section *AM*, $\overrightarrow{Z_{AM}} = (0{\cdot}1 + j0{\cdot}15)\ \Omega$

Impedance of section *MB*, $\overrightarrow{Z_{MB}} = (0{\cdot}1 + j0{\cdot}15)\ \Omega$

Let the voltage V_B at point *B* be taken as the reference vector.

Then, $\overrightarrow{V_B} = 200 + j0$

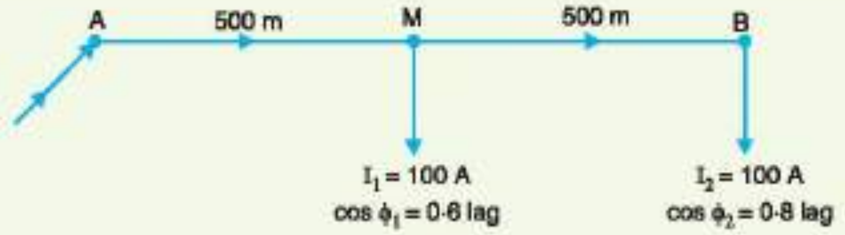


Fig. 14.6

(*i*) Load current at point *B*, $\overrightarrow{I_2} = 100\ (0{\cdot}8 - j0{\cdot}6) = 80 - j60$

Current in section *MB*, $\overrightarrow{I_{MB}} = \overrightarrow{I_2} = 80 - j60$

Drop in section *MB*, $\overrightarrow{V_{MB}} = \overrightarrow{I_{MB}}\ \overrightarrow{Z_{MB}}$

$= (80 - j60)\ (0{\cdot}1 + j0{\cdot}15) = 17 + j6$

∴ Voltage at point *M*, $\overrightarrow{V_M} = \overrightarrow{V_B} + \overrightarrow{V_{MB}} = (200 + j0) + (17 + j6)$

$= 217 + j6$

Its magnitude is $= \sqrt{(217)^2 + (6)^2} =$ **217·1 V**

Phase angle between V_M and V_B, $\alpha = \tan^{-1} 6/217 = \tan^{-1} 0{\cdot}0276 = 1{\cdot}58°$

(*ii*) The load current I_1 has a lagging p.f. of 0·6 *w.r.t.* V_M. It lags behind V_M by an angle $\phi_1 = \cos^{-1} 0{\cdot}6 = 53{\cdot}13°$

∴ Phase angle between I_1 and V_B, $\phi'_1 = \phi_1 - \alpha = 53{\cdot}13° - 1{\cdot}58 = 51{\cdot}55°$

Load current at *M*, $\overrightarrow{I_1} = I_1\ (\cos\phi_1' - j\sin\phi_1') = 100\ (\cos 51{\cdot}55° - j\sin 51{\cdot}55°)$

$= 62{\cdot}2 - j78{\cdot}3$

Current in section *AM*, $\overrightarrow{I_{AM}} = \overrightarrow{I_1} + \overrightarrow{I_2} = (62{\cdot}2 - j78{\cdot}3) + (80 - j60)$

$= 142{\cdot}2 - j138{\cdot}3$

Drop in section *AM*, $\overrightarrow{V_{AM}} = \overrightarrow{I_{AM}}\ \overrightarrow{Z_{AM}} = (142{\cdot}2 - j138{\cdot}3)\ (0{\cdot}1 + j0{\cdot}15)$

$= 34{\cdot}96 + j7{\cdot}5$

Sending end voltage, $\overrightarrow{V_A} = \overrightarrow{V_M} + \overrightarrow{V_{AM}} = (217 + j6) + (34{\cdot}96 + j7{\cdot}5)$

$$= 251 \cdot 96 + j13 \cdot 5$$

Its magnitude is $= \sqrt{(251 \cdot 96)^2 + (13 \cdot 5)^2} =$ **252·32 V**

(*iii*) The phase difference θ between V_A and V_B is given by :

$$\tan \theta = 13 \cdot 5/251 \cdot 96 = 0 \cdot 05358$$

$$\therefore \quad \theta = \tan^{-1} 0 \cdot 05358 = \mathbf{3 \cdot 07°}$$

Hence supply voltage is 252·32 V and leads V_B by 3·07°.

Example 14.4. *A single phase ring distributor ABC is fed at A. The loads at B and C are 20 A at 0.8 p.f. lagging and 15 A at 0.6 p.f. lagging respectively ; both expressed with reference to the voltage at A. The total impedance of the three sections AB, BC and CA are (1 + j 1), (1+ j2) and (1 + j3) ohms respectively. Find the total current fed at A and the current in each section. Use Thevenin's theorem to obtain the results.*

Solution. Fig. 14.7 (*i*) shows the ring distributor *ABC*. Thevenin's theorem will be used to solve this problem. First, let us find the current in *BC*. For this purpose, imagine that section *BC* is removed as shown in Fig. 14.7 (*ii*).

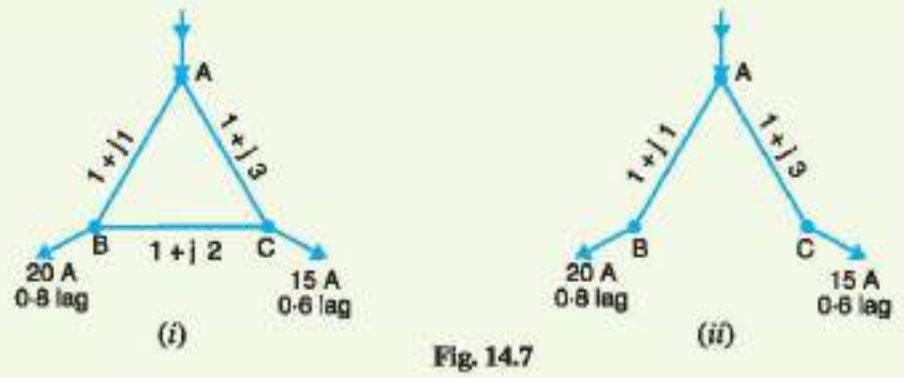


Fig. 14.7

Referring to Fig.14.7 (*ii*), we have,

Current in section *AB* $= 20\,(0 \cdot 8 - j\,0 \cdot 6) = 16 - j\,12$

Current in section *AC* $= 15\,(0 \cdot 6 - j\,0 \cdot 8) = 9 - j\,12$

Voltage drop in section *AB* $= (16 - j\,12)\,(1 + j1) = 28 + j\,4$

Voltage drop in section *AC* $= (9 - j\,12)\,(1 + j\,3) = 45 + j\,15$

Obviously, point *B* is at higher potential than point *C*. The p.d. between *B* and *C* is Thevenin's equivalent circuit e.m.f. E_0 *i.e.*

Thevenin's equivalent circuit e.m.f., E_0 = p.d. between *B* and *C*

$$= (45 + j\,15) - (28 + j\,4) = 17 + j\,11$$

Thevenin's equivalent impedance Z_0 can be found by looking into the network from points *B* and *C*.

Obviously, $Z_0 = (1 + j1) + (1 + j\,3) = 2 + j4$

$$\therefore \quad \text{Current in } BC = \frac{E_0}{Z_0 + \text{Impedance of } BC}$$

$$= \frac{17 + j11}{(2 + j4) + (1 + j2)} = \frac{17 + j11}{3 + j6}$$

$$= 2 \cdot 6 - j\,1 \cdot 53 = \mathbf{3\angle{-30 \cdot 48°}\ A}$$

$$\text{Current in } AB = (16 - j\,12) + (2 \cdot 6 - j\,1 \cdot 53)$$

$$= 18{\cdot}6 - j\,13{\cdot}53 = \mathbf{23\angle{-36{\cdot}03}^\circ\ A}$$

$$\text{Current in } AC = (9 - j\,12) - (2{\cdot}6 - j\,1{\cdot}53)$$
$$= 6{\cdot}4 - j\,10{\cdot}47 = \mathbf{12{\cdot}27\angle{-58{\cdot}56}^\circ\ A}$$

$$\text{Current fed at } A = (16 - j\,12) + (9 - j\,12)$$
$$= 25 - j\,24 = \mathbf{34{\cdot}65\angle{-43{\cdot}83}^\circ\ A}$$

Example 14.5. *A 3-phase, 400V distributor AB is loaded as shown in Fig.14.8. The 3-phase load at point C takes 5A per phase at a p.f. of 0·8 lagging. At point B, a 3-phase, 400 V induction motor is connected which has an output of 10 H.P. with an efficiency of 90% and p.f. 0·85 lagging.*

If voltage at point B is to be maintained at 400 V, what should be the voltage at point A ? The resistance and reactance of the line are 1Ω and 0·5Ω per phase per kilometre respectively.

Solution. It is convenient to consider one phase only. Fig.14.8 shows the single line diagram of the distributor. Impedance of the distributor per phase per kilometre = $(1 + j0{\cdot}5)\ \Omega$

Impedance of section AC, $\quad \vec{Z_{AC}} = (1 + j\,0{\cdot}5) \times 600/1000 = (0{\cdot}6 + j\,0{\cdot}3)\ \Omega$

Impedance of section CB, $\quad \vec{Z_{CB}} = (1 + j\,0{\cdot}5) \times 400/1000 = (0{\cdot}4 + j\,0{\cdot}2)\ \Omega$

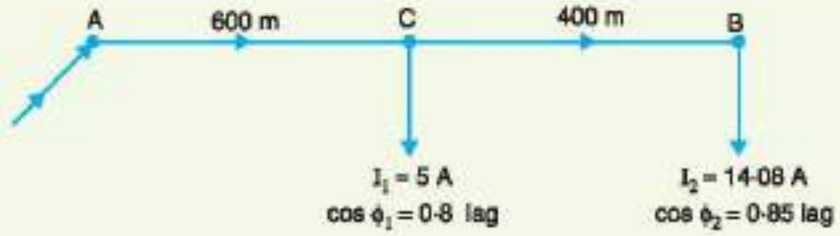


Fig. 14.8

Phase voltage at point B, $\quad V_B = 400/\sqrt{3} = 231\ \text{V}$

Let the voltage V_B at point B be taken as the reference vector.

Then, $\quad \vec{V_B} = 231 + j0$

Line current at B
$$= \frac{\text{H.P.} \times 746}{\sqrt{3} \times \text{line voltage} \times \text{p.f.} \times \text{efficiency}}$$
$$= \frac{10 \times 746}{\sqrt{3} \times 400 \times 0{\cdot}85 \times 0{\cdot}9} = 14{\cdot}08\ \text{A}$$

∴ *Current/phase at B, $\quad I_2 = 14{\cdot}08\ \text{A}$

Load current at B, $\quad \vec{I_2} = 14{\cdot}08\,(0{\cdot}85 - j0{\cdot}527) = 12 - j\,7{\cdot}4$

Load current at C, $\quad \vec{I_1} = 5\,(0{\cdot}8 - j0{\cdot}6) = 4 - j3$

Current in section AC, $\quad \vec{I_{AC}} = \vec{I_1} + \vec{I_2} = (4 - j\,3) + (12 - j\,7{\cdot}4)$
$$= 16 - j\,10{\cdot}4$$

Current in section CB, $\quad \vec{I_{CB}} = \vec{I_2} = 12 - j\,7{\cdot}4$

Voltage drop in CB, $\quad \vec{V_{CB}} = \vec{I_{CB}}\vec{Z_{CB}} = (12 - j\,7{\cdot}4)\,(0{\cdot}4 + j\,0{\cdot}2)$
$$= 6{\cdot}28 - j\,0{\cdot}56$$

Voltage drop in AC, $\quad \vec{V_{AC}} = \vec{I_{AC}}\vec{Z_{AC}} = (16 - j\,10{\cdot}4)\,(0{\cdot}6 + j\,0{\cdot}3)$
$$= 12{\cdot}72 - j\,1{\cdot}44$$

* In a 3-phase system, if the type of connection is not mentioned, then star connection is understood.

Voltage at A per phase, $\vec{V}_A = \vec{V}_B + \vec{V}_{CB} + \vec{V}_{AC}$

$$= (231 + j0) + (6{\cdot}28 - j0{\cdot}56) + (12{\cdot}72 - j1{\cdot}44)$$
$$= 250 - j2$$

Magnitude of V_A/phase $= \sqrt{(250)^2 + (2)^2} = 250$ V

∴ Line voltage at A $= \sqrt{3} \times 250 =$ **433 V**

Example 14.6. *A 3-phase ring main ABCD fed at A at 11 kV supplies balanced loads of 50 A at 0.8 p.f. lagging at B, 120 A at unity p.f. at C and 70 A at 0·866 lagging at D, the load currents being referred to the supply voltage at A. The impedances of the various sections are :*

Section AB $= (1 + j\,0{\cdot}6)\ \Omega$; *Section BC* $= (1{\cdot}2 + j\,0{\cdot}9)\ \Omega$

Section CD $= (0{\cdot}8 + j\,0{\cdot}5)\ \Omega$; *Section DA* $= (3 + j\,2)\ \Omega$

Calculate the currents in various sections and station bus-bar voltages at B, C and D.

Solution. Fig.14.9 shows one phase of the ring main. The problem will be solved by Kirchhoff's laws. Let current in section AB be $(x + j\,y)$.

∴ Current in section BC, $\overrightarrow{I_{BC}} = (x + j\,y) - 50\,(0{\cdot}8 - j0{\cdot}6) = (x - 40) + j\,(y + 30)$

Current in section CD, $\overrightarrow{I_{CD}} = [(x - 40) + j\,(y + 30)] - [120 + j\,0]$

$$= (x - 160) + j\,(y + 30)$$

Current in section DA, $\overrightarrow{I_{DA}} = [(x - 160) + j\,(y + 30)] - [70\,(0{\cdot}866 - j0{\cdot}5)]$

$$= (x - 220{\cdot}6) + j\,(y + 65)$$

Drop in section AB $= \overrightarrow{I_{AB}}\,\overrightarrow{Z_{AB}} = (x + jy)\,(1 + j0{\cdot}6)$

$$= (x - 0{\cdot}6y) + j\,(0{\cdot}6x + y)$$

Drop in section BC $= \overrightarrow{I_{BC}}\,\overrightarrow{Z_{BC}}$

$$= [(x - 40) + j\,(y + 30)]\,[(1{\cdot}2 + j0{\cdot}9)]$$
$$= (1{\cdot}2x - 0{\cdot}9\,y - 75) + j\,(0{\cdot}9x + 1{\cdot}2\,y)$$

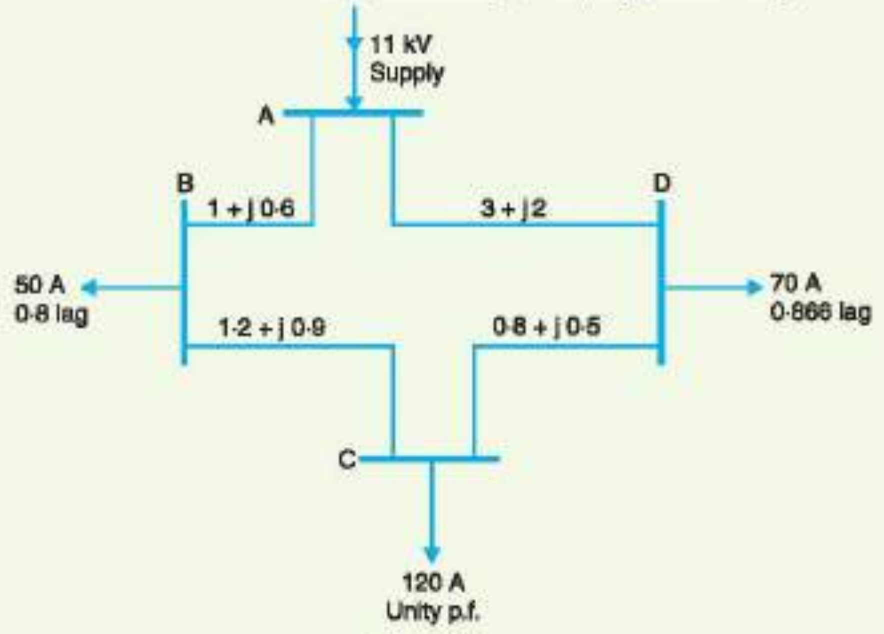


Fig. 14.9

$$\text{Drop in section } CD = \overrightarrow{I_{CD}}\ \overrightarrow{Z_{CD}}$$
$$= [(x-160)+j(y+30)]\,[(0{\cdot}8+j0{\cdot}5)]$$
$$= (0{\cdot}8x-0{\cdot}5y-143)+j(0{\cdot}5x+0{\cdot}8y-56)$$

$$\text{Drop in section } DA = \overrightarrow{I_{DA}}\ \overrightarrow{Z_{DA}}$$
$$= [(x-220{\cdot}6)+j(y+65)]\,[(3+j2)]$$
$$= (3x-2y-791{\cdot}8)+j(2x+3y-246{\cdot}2)$$

Applying Kirchhoff's voltage law to mesh *ABCDA*, we have,

Drop in *AB* + Drop in *BC* + Drop in *CD* + Drop in *DA* = 0

or
$$[(x-0{\cdot}6y)+j(0{\cdot}6x+y)]+[(1{\cdot}2x-0{\cdot}9y-75)+j(0{\cdot}9x+1{\cdot}2y)]$$
$$+[(0{\cdot}8x-0{\cdot}5y-143)+j(0{\cdot}5x+0{\cdot}8y-56)]$$
$$+[(3x-2y-791{\cdot}8)+j(2x+3y-246{\cdot}2)]=0$$

or
$$(6x-4y-1009{\cdot}8)+j(4x+6y-302{\cdot}2)=0$$

As the real (or active) and imaginary (or reactive) parts have to be separately zero,

$\therefore$ $6x-4y-1009{\cdot}8 = 0$

and $4x+6y-302{\cdot}2 = 0$

Solving for x and y, we have,

$$x = 139{\cdot}7 \text{ A} \ ; \ y = -42{\cdot}8 \text{ A}$$

Current in section *AB* $= \mathbf{(139{\cdot}7 - j42{\cdot}8)\ A}$

Current in section *BC* $= (x-40)+j(y+30)$

$= (139{\cdot}7-40)+j(-42{\cdot}8+30) = \mathbf{(99{\cdot}7 - j12{\cdot}8)\ A}$

Current in section *CD* $= (x-160)+j(y+30)$

$= (139{\cdot}7-160)+j(-42{\cdot}8+30)$

$= \mathbf{(-20{\cdot}3 - j12{\cdot}8)\ A}$

Current in section *DA* $= (x-220{\cdot}6)+j(y+65)$

$= (139{\cdot}7-220{\cdot}6)+j(-42{\cdot}8+65)$

$= \mathbf{(-80{\cdot}9 + j22{\cdot}2)\ A}$

Voltage at supply end *A*, $V_A = 11000/\sqrt{3} = 6351$ V/phase

$\therefore$ Voltage at station *B*, $\overrightarrow{V_B} = \overrightarrow{V_A} - \overrightarrow{I_{AB}}\ \overrightarrow{Z_{AB}}$

$= (6351+j0)-(139{\cdot}7-j42{\cdot}8)\,(1+j0{\cdot}6)$

$= \mathbf{(6185{\cdot}62 - j41{\cdot}02)}$ **volts/phase**

Voltage at station *C*, $\overrightarrow{V_C} = \overrightarrow{V_B} - \overrightarrow{I_{BC}}\ \overrightarrow{Z_{BC}}$

$= (6185{\cdot}62-j41{\cdot}02)-(99{\cdot}7-j12{\cdot}8)\,(1{\cdot}2+j0{\cdot}9)$

$= \mathbf{(6054{\cdot}46 - j115{\cdot}39)}$ **volts/phase**

Voltage at station *D*, $\overrightarrow{V_D} = \overrightarrow{V}_C - \overrightarrow{I_{CD}}\ \overrightarrow{Z_{CD}}$

$= (6054{\cdot}46-j115{\cdot}39)-(-20{\cdot}3-j12{\cdot}8)\times(0{\cdot}8+j0{\cdot}5)$

$= \mathbf{(6064{\cdot}3 - j95)}$ **volts/phase**

TUTORIAL PROBLEMS

1. A single phase distributor AB has a total impedance of $(0{\cdot}1 + j0{\cdot}2)$ ohm. At the far end B, a current of 80 A at 0·8 p.f. lagging and at mid-point C a current of 100 A at 0·6 p.f. lagging are tapped. If the voltage of the far end is maintained at 200 V, determine :
   (*i*) Supply end voltage V_A
   (*ii*) Phase angle between V_A and V_B
   The load power factors are w.r.t. the voltage at the far end. **[(*i*) 227·22 V (*ii*) 2°31′]**
2. A single-phase a.c. distributor AB is fed from end A and has a total impedance of $(0{\cdot}2 + j03)$ ohm. At the far end, the voltage $V_B = 240$ V and the current is 100 A at a p.f. of 0·8 lagging. At the mid-point M, a current of 100 A is tapped at a p.f. of 0·6 lagging with reference to the voltage V_M at the mid-point. Calculate the supply voltage V_A and phase angle between V_A and V_B. **[292 V, 2·6°]**
3. A single phase ring distributor ABC is fed at A. The loads at B and C are 40 A at 0·8 p.f. lagging and 60 A at 0·6 p.f. lagging respectively. Both power factors expressed are referred to the voltage at point A. The total impedance of sections AB, BC and CA are $2 + j1$, $2 + j3$ and $1 + j2$ ohms respectively. Determine the current in each section.
   [Current in $AB = (39{\cdot}54 - j\,25{\cdot}05)$ amp ; $BC = (7{\cdot}54 - j\,1{\cdot}05)$ amp ; $CA = (28{\cdot}46 - j\,46{\cdot}95)$ amp.]
4. A 3-phase ring distributor $ABCD$ fed at A at 11 kV supplies balanced loads of 40 A at 0·8 p.f. lagging at B, 50 A at 0·707 p.f. lagging at C and 30 A at 0·8 p.f. lagging at D, the load currents being referred to the supply voltage at A.
   The impedances per phase of the various sections are :
   Section $AB = (1 + j\,2)\ \Omega$; Section $BC = (2 + j\,3)\ \Omega$
   Section $CD = (1 + j\,1)\ \Omega$; Section $DA = (3 + j\,4)\ \Omega$
   Calculate the currents in various sections and station bus-bar voltages at B, C and D.
   [Current in $AB = (53{\cdot}8 - j\,46)$ amp ; $BC = (21{\cdot}8 - j\,22)$ amp.
   $CD = (-13{\cdot}55 + j\,13{\cdot}35)$ amp ; $DA = (-40{\cdot}55 - j\,26{\cdot}45)$ amp.
   $V_B = (6212{\cdot}5 - j\,61{\cdot}6)$ volts/phase ; $V_C = (6103 - j\,83)$ volts/phase
   $V_D = (6129{\cdot}8 - j\,82{\cdot}8)$ volts/phase]

Phase Sequence Indicator

14.3 3-Phase Unbalanced Loads

The 3-phase loads that have the same impedance and power factor in each phase are called balanced loads. The problems on balanced loads can be solved by considering one phase only ; the conditions in the other two phases being similar. However, we may come across a situation when loads are unbalanced *i.e.* each load phase has different impedance and/or power factor. In that case, current and power in each phase will be different. In practice, we may come across the following unbalanced loads :

(i) Four-wire star-connected unbalanced load

(ii) Unbalanced Δ-connected load

(iii) Unbalanced 3-wire, *Y*-connected load

The 3-phase, 4-wire system is widely used for distribution of electric power in commercial and industrial buildings. The single phase load is connected between any line and neutral wire while a 3-phase load is connected across the three lines. The 3-phase, 4-wire system invariably carries *unbalanced loads. In this chapter, we shall only discuss this type of unbalanced load.

14.4 Four-Wire Star-Connected Unbalanced Loads

We can obtain this type of load in two ways. First, we may connect a 3-phase, 4-wire unbalanced load to a 3-phase, 4-wire supply as shown in Fig. 14.10. Note that star point *N* of the supply is connected to the load star point *N'*. Secondly, we may connect single phase loads between any line and the neutral wire as shown in Fig.14.11. This will also result in a 3-phase, 4-wire **unbalanced load because it is rarely possible that single phase loads on all the three phases have the same magnitude and power factor. Since the load is unbalanced, the line currents will be different in magnitude and displaced from one another by unequal angles. The current in the neutral wire will be the phasor sum of the three line currents *i.e.*

Current in neutral wire, $I_N = I_R + I_Y + I_B$ ...*phasor sum*

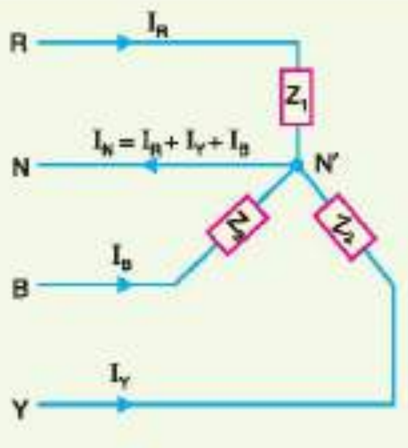


Fig. 14.10

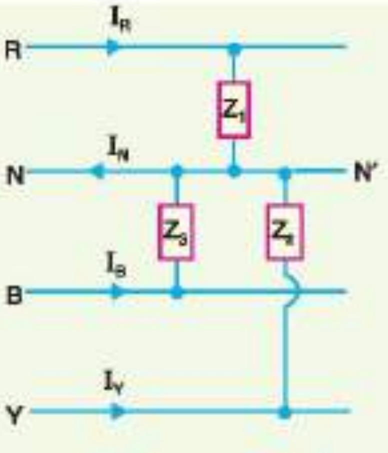


Fig. 14.11

The following points may be noted carefully :

(i) Since the neutral wire has negligible resistance, supply neutral *N* and load neutral *N'* will be at the same potential. It means that voltage across each impedance is equal to the phase voltage of the supply. However, current in each phase (or line) will be different due to unequal impedances.

(ii) The amount of current flowing in the neutral wire will depend upon the magnitudes of line currents and their phasor relations. In most circuits encountered in practice, the neutral current is equal to or smaller than one of the line currents. The exceptions are those circuits having severe unbalance.

---

* No doubt 3-phase loads (*e.g.* 3-phase motors) connected to this supply are balanced but when we add single phase loads (*e.g.* lights, fans etc.), the balance is lost. It is because it is rarely possible that single phase loads on all the three phases have the same magnitude and power factor.

** In actual practice, we never have an unbalanced 3-phase, 4-wire load. Most of the 3-phase loads (*e.g.* 3-phase motors) are 3-phase, 3-wire and are balanced loads. In fact, these are the single phase loads on the 3-phase, 4-wire supply which constitute unbalanced, 4-wire *Y*-connected load.

Example 14.7. *Non-reactive loads of 10 kW, 8 kW and 5 kW are connected between the neutral and the red, yellow and blue phases respectively of a 3-phase, 4-wire system. The line voltage is 400V. Calculate (i) the current in each line and (ii) the current in the neutral wire.*

Solution. This is a case of unbalanced load so that the line currents (and hence the phase currents) in the three lines will be different. The current in the *neutral wire will be equal to the phasor sum of three line currents as shown in Fig. 14.12.

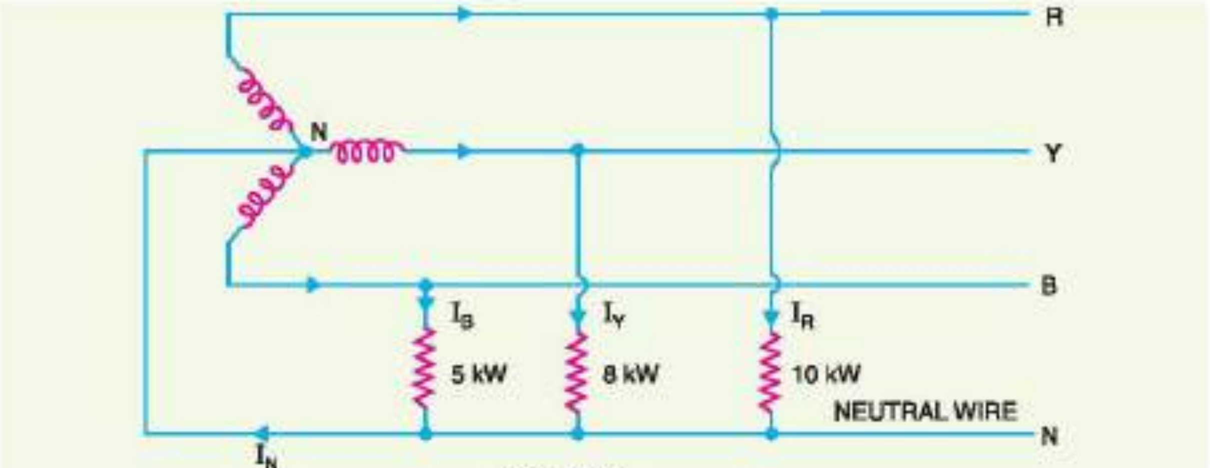


Fig. 14.12

(*i*)

$$\text{Phase voltage} = 400/\sqrt{3} = 231 \text{ V}$$
$$I_R = 10 \times 10^3/231 = \mathbf{43{\cdot}3\ A}$$
$$I_Y = 8 \times 10^3/231 = \mathbf{34{\cdot}6\ A}$$
$$I_B = 5 \times 10^3/231 = \mathbf{21{\cdot}65\ A}$$

(*ii*) The three lines currents are represented by the respective phasors in Fig. 14.13. Note that the three line currents are of different magnitude but displaced 120° from one another. The current in the neutral wire will be the phasor sum of the three line currents.

Resolving the three currents along *x*-axis and *y*-axis, we have,

Resultant horizontal component $= I_Y \cos 30° - I_B \cos 30°$

$= 34{\cdot}6 \times 0{\cdot}866 - 21{\cdot}65 \times 0{\cdot}866 = 11{\cdot}22 \text{ A}$

Resultant vertical component $= I_R - I_Y \cos 60° - I_B \cos 60°$

$= 43{\cdot}3 - 34{\cdot}6 \times 0{\cdot}5 - 21{\cdot}65 \times 0{\cdot}5 = 15{\cdot}2 \text{ A}$

As shown in Fig. 14.14, current in neutral wire is

$$I_N = \sqrt{(11{\cdot}22)^2 + (15{\cdot}2)^2} = \mathbf{18{\cdot}9\ A}$$

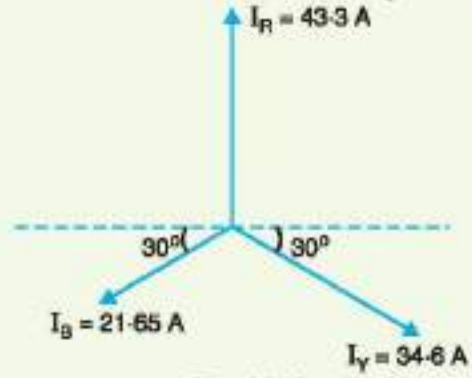


Fig. 14.13

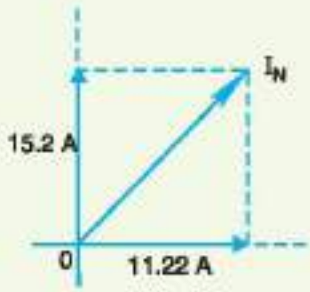


Fig. 14.14

* Had the load been balanced (*i.e.* each phase having identical load), the current in the neutral wire would have been zero.

Example 14.8. *A 3-phase, 4-wire system supplies power at 400 V and lighting at 230 V. If the lamps is use require 70, 84 and 33 amperes in each of the three lines, what should be the current in the neutral wire ? If a 3-phase motor is now started, taking 200 A from the lines at a p.f. of 0·2 lagging, what should be the total current in each line and the neutral wire ? Find also the total power supplied to the lamps and the motor.*

Solution. Fig. 14.15 shows the lamp load and motor load on 400 V/230 V, 3-phase, 4-wire sypply.

Lamp load alone. If there is lamp load alone, the line currents in phases *R*, *Y* and *B* are 70 *A*, 84 *A* and 33 *A* respectively. These currents will be 120° apart (assuming phase sequence *RYB*) as shown in Fig.14.16.

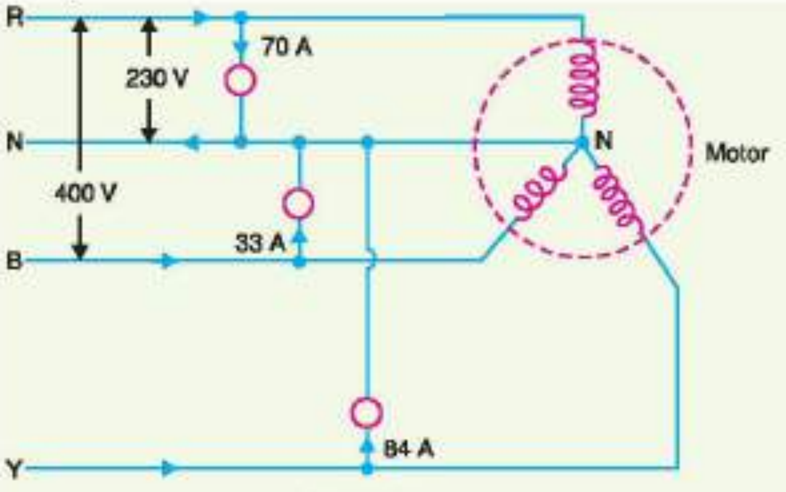


Fig. 14.15

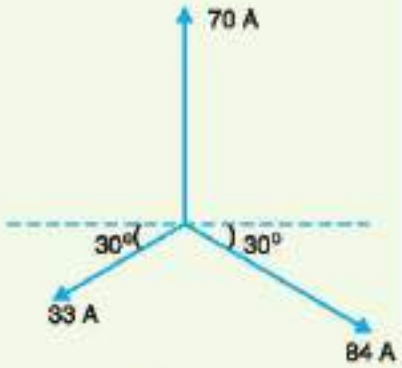


Fig. 14.16

Resultant *H*-component $= 84 \cos 30° - 33 \cos 30° = 44{\cdot}17$ A

Resultant V-component $= 70 - 33 \cos 60° - 84 \cos 60° = 11{\cdot}5$ A

$\therefore$ Neutral current, $I_N = \sqrt{(44{\cdot}17)^2 + (11{\cdot}5)^2} = \mathbf{45{\cdot}64\ A}$

Both lamp load and motor load

When motor load is also connected along with lighting load, there will be no change in current in the neutral wire. It is because the motor load is balanced and hence no current will flow in the neutral wire due to this load.

$\therefore$ Neutral current, $I_N = \mathbf{45{\cdot}64\ A}$ *...same as before*

The current in each line is the phasor sum of the line currents due to lamp load and motor load.

Active component of motor current $= 200 \times \cos\phi_m = 200 \times 0{\cdot}2 = 40$ A

Reactive component of motor current $= 200 \times \sin\phi_m = 200 \times 0{\cdot}98 = 196$ A

$$\therefore \quad I_R = \sqrt{(\text{sum of active comp.})^2 + (\text{reactive comp.})^2}$$

$$= \sqrt{(40+70)^2 + (196)^2} = \mathbf{224{\cdot}8\ A}$$

$$I_Y = \sqrt{(40+84)^2 + (196)^2} = \mathbf{232\ A}$$

$$I_B = \sqrt{(40+33)^2 + (196)^2} = \mathbf{209{\cdot}15\ A}$$

Power supplied

Power supplied to lamps $= 230\ (70 + 84 + 33) \times 1 = \mathbf{43010\ W}$ $(\because \cos\phi_L = 1)$

Power supplied to motor $= \sqrt{3}\ V_L I_L \cos\phi_m$

$= \sqrt{3} \times 400 \times 200 \times 0{\cdot}2 =$ **27712 W**

Example 14.9. *The three line leads of a 400/230 V, 3-phase, 4-wire supply are designated as R, Y and B respectively. The fourth wire or neutral wire is designated as N. The phase sequence is RYB. Compute the currents in the four wires when the following loads are connected to this supply :*

From R to N : 20 kW, unity power factor

From Y to N : 28·75 kVA, 0·866 lag

From B to N : 28·75 kVA, 0·866 lead

If the load from B to N is removed, what will be the value of currents in the four wires ?

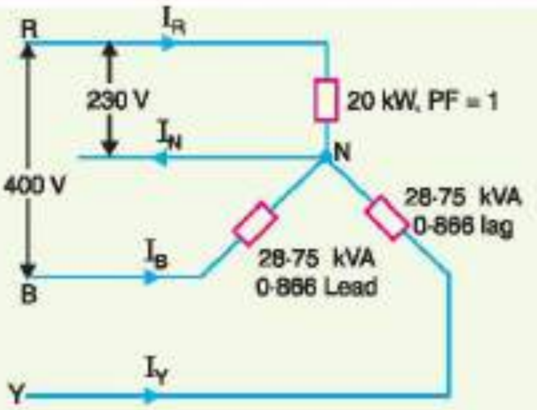


Fig. 14.17

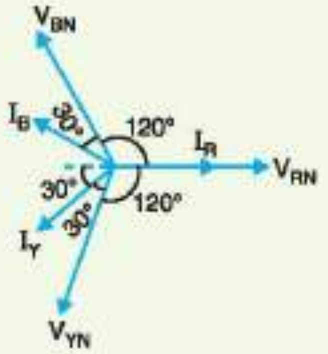


Fig. 14.18

Solution. Fig. 14.17 shows the circuit diagram whereas Fig.14.18 shows its phasor diagram. The current I_R is in phase with V_{RN}, current I_Y lags behind its phase voltage V_{YN} by $\cos^{-1} 0{\cdot}866 = 30°$ and the current I_B leads its phase voltage V_{BN} by $\cos^{-1} 0{\cdot}866 = 30°$.

$I_R = 20 \times 10^3/230 =$ **89·96 A**

$I_Y = 28{\cdot}75 \times 10^3/230 =$ **125 A**

$I_B = 28{\cdot}75 \times 10^3/230 =$ **125 A**

The current in the neutral wire will be equal to the phasor sum of the three line currents I_R, I_Y and I_B. Referring to the phasor diagram in Fig.14.18 and resolving these currents along x-axis and y-axis, we have,

Resultant X-component $= 86{\cdot}96 - 125\cos 30° - 125 \cos 30°$

$= 86{\cdot}96 - 108{\cdot}25 - 108{\cdot}25 = -129{\cdot}54$ A

Resultant Y-component $= 0 + 125 \sin 30° - 125 \sin 30° = 0$

∴ Neutral current, $I_N = \sqrt{(-129{\cdot}54)^2 + (0)^2} =$ **129·54A**

When load from B to N removed. When the load from B to N is removed, the various line currents are :

$I_R =$ **86·96A** in phase with V_{RN} ; $I_Y =$ **125A** lagging V_{YN} by 30° ; $I_B =$ **0 A**

The current in the neutral wire is equal to the phasor sum of these three line currents. Resolving the currents along x-axis and y-axis, we have,

Resultant X-component $= 86{\cdot}96 - 125 \cos 30° = 86{\cdot}96 - 108{\cdot}25 = -21{\cdot}29$ A

Resultant Y-component $= 0 - 125 \sin 30° = 0 - 125 \times 0{\cdot}5 = -62{\cdot}5$ A

∴ Neutral current, $I_N = \sqrt{(-21{\cdot}29)^2 + (-62{\cdot}5)^2} =$ **66·03 A**

Example 14.10. *A 3-phase, 4-wire distributor supplies a balanced voltage of 400/230 V to a load consisting of 30 A at p.f. 0·866 lagging for R-phase, 30 A at p.f. 0·866 leading for Y phase and 30 A at unity p.f. for B phase. The resistance of each line conductor is 0·2 Ω. The area of X-section of neutral is half of any line conductor. Calculate the supply end voltage for R phase. The phase sequence is RYB.*

Solution. The circuit diagram is shown in Fig. 14.19. Since neutral is half the cross-section, its resistance is 0·4 Ω. Considering the load end and taking $\vec{V_R}$ as the reference vector, the phase voltages can be written as :

$$\vec{V_R} = 230 \angle 0° \text{ volts} \; ; \; \vec{V_Y} = 230 \angle -120° \text{ volts} \; ; \; \vec{V_B} = 230 \angle 120° \text{ volts}$$

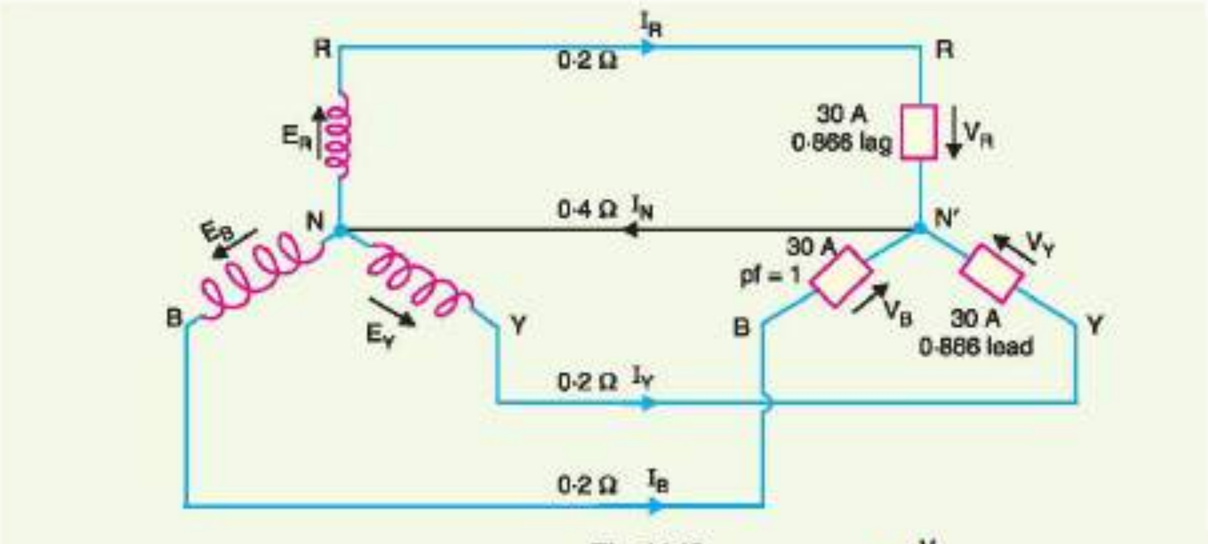


Fig. 14.19

The vector diagram of the circuit is shown in Fig. 14.20. The line current I_R lags behind V_R by an angle $\cos^{-1} 0·866 = 30°$. The current I_Y leads V_Y by 30° and the current I_B is in phase with V_B. Referring to the vector diagram of Fig.14.20, the line currents can be expressed as :

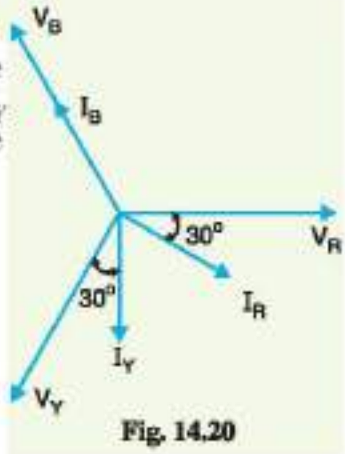


Fig. 14.20

$$\vec{I_R} = 30 \angle -30° \text{ amperes}$$

$$\vec{I_Y} = 30 \angle -90° \text{ amperes}$$

$$\vec{I_B} = 30 \angle 120° \text{ amperes}$$

Current in neutral wire,

$$\vec{I_N} = \vec{I_R} + \vec{I_Y} + \vec{I_B}$$

$$= 30 \angle -30° + 30 \angle -90° + 30 \angle 120°$$

$$= 30\,(0·866 - j\,0·5) - 30\,(j) + 30\,(-0·5 + j\,0·866)$$

$$= 10·98 - j\,19·02$$

Let the supply voltage of phase R to neutral be $\vec{E_R}$. Then,

$$\vec{E_R} = \vec{V_R} + \text{Drop in } R \text{ phase} + \text{Drop in neutral}$$

$$= (230 + j\,0) + 0·2 \times 30 \angle -30° + (10·98 - j\,19·02) \times 0·4$$

$$= 230 + 6\,(0·866 - j\,0·5) + 0·4\,(10·98 - j\,19·02)$$

$$= 239·588 - j\,10·608$$

$$= \mathbf{239·8 \angle -2·54° \text{ volts}}$$

Example 14.11. *In a 3-phase, 4-wire, 400/230 V system, a lamp of 100 watts is connected to one phase and neutral and a lamp of 150 watts is connected to the second phase and neutral. If the neutral wire is disconnected accidentally, what will be the voltage across each lamp?*

Solution. Fig. 14.21 (*i*) shows the lamp connections. The lamp L_1 of 100 watts is connected between phase R and neutral whereas lamp L_2 of 150 watts is connected between phase Y and the neutral.

Resistance of lamp L_1, $$R_1 = \frac{(230)^2}{100} = 529\ \Omega$$

Resistance of lamp L_2, $$R_2 = \frac{(230)^2}{150} = 352{\cdot}67\ \Omega$$

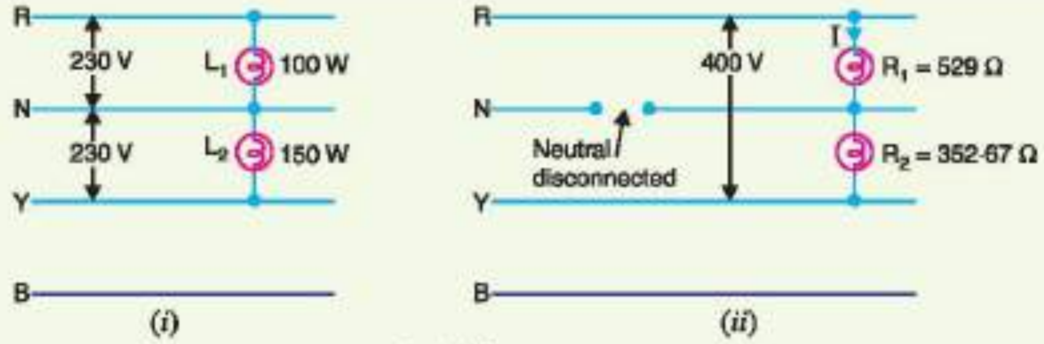


Fig. 14.21

When the neutral wire is disconnected as shown in Fig. 14.21 (*ii*), the two lamps are connected in series and the p.d. across the combination becomes equal to the line voltage E_L (= 400 V).

$$\text{Current through lamps, } I = \frac{E_L}{R_1 + R_2} = \frac{400}{529 + 352{\cdot}67} = 0{\cdot}454 \text{ A}$$

$$\text{Voltage across lamp } L_1 = I R_1 = 0{\cdot}454 \times 529 = \mathbf{240\ V}$$

$$\text{Voltage across lamp } L_2 = I R_2 = 0{\cdot}454 \times 352{\cdot}67 = \mathbf{160\ V}$$

Comments. The voltage across 100-watt lamp is increased to 240 V whereas that across 150-watt is decreased to 160 V. Therefore, 100-watt lamp becomes brighter and 150-watt lamp becomes dim. It may be noted here that if 100-watt lamp happens to be rated at 230 V, it may burn out due to 240 V coming across it.

TUTORIAL PROBLEMS

1. Non-reactive loads of 10 kW, 6kW and 4 kW are connected between the neutral and red, yellow and blue phases respectively of a 3-phase, 4-wire 400/230V supply. Find the current in each line and in the neutral wire. **[I_R = 43·3 A; I_Y = 26 A; I_B = 17·3 A; I_N = 22·9 A]**
2. A factory has the following loads with a power factor of 0·9 lagging in each case. Red phase 40 A, yellow phase 50 A and blue phase 60 A. If the supply is 400V, 3-phase, 4-wire, calculate the current in the neutral wire and the total power. **[17·3A, 31·2 kW]**
3. In a 3-phase, 4-wire system, two phases have currents of 10A and 6A at lagging power factors of 0·8 and 0·6 respectively, while the third phase is open-circuited. Calculate the current in the neutral wire. **[7A]**
4. A 3-phase, 4-wire system supplies a lighting load of 40A, 30A and 20A respectively in the three phases. If the line voltage is 400 V, determine the current in the neutral wire. **[17·32A]**

14.5. Ground Detectors

Ground detectors are the devices that are used to detect the ground fault for ungrounded a.c. systems.

When a ground fault occurs on such a system, immediate steps should be taken to clear it. If this is not done and a second ground fault happens, a short circuit occurs.

Fig.14.22 shows how lamps are connected to an un-grounded 3-phase system for the detection of ground fault. If ground fault occurs on any wire, the lamp connected to that wire will be dim and the lamps connected to healthy (un-grounded) wire will become brighter.

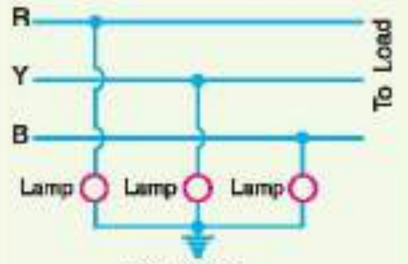


Fig. 14.22

SELF - TEST

1. **Fill in the blanks by inserting appropriate words/figures.**
   (*i*) The most common system for secondary distribution is 400/..... V, 3-phase, ... wire system.
   (*ii*) In a 3-phase, 4-wire a.c. system, if the loads are balanced, then current in the neutral wire is ....
   (*iii*) Distribution transformer links the .... and .... systems.
   (*iv*) The 3-phase, 3-wire a.c. system of distribution is used for .... loads.
   (*v*) For combined power and lighting load, ... system is used.
2. **Pick up the correct words/figures from brackets and fill in the blanks.**
   (*i*) 3-phase, 4-wire a.c. system of distribution is used for .... load. (*balanced, unbalanced*)
   (*ii*) In a balanced 3-phase, 4-wire a.c. system, the phase sequence is *RYB*. If the voltage of *R* phase = $230 \angle 0°$ volts, then for *B* phase it will be .... (*$230 \angle -120°$ volts, $230 \angle 120°$ volts*)
   (*iii*) In a.c. system, additions and subtractions of currents are done .... (*vectorially, arithmetically*)
   (*iv*) The area of X-section of neutral is generally .... that of any line conductor. (*the same, half*)
   (*v*) For purely domestic loads, ... a.c. system is employed for distribution. (*single phase 2-wire, 3-phase 3-wire*)

ANSWERS TO SELF-TEST

1. (*i*) 230, 4 (*ii*) zero (*iii*) primary, secondary (*iv*) balanced (*v*) 3-phase 4-wire.
2. (*i*) unbalanced (*ii*) $230 \angle 120°$ (*iii*) vectorially (*iv*) half (*v*) single phase 2-wire.

CHAPTER REVIEW TOPICS

1. How does a.c. distribution differ from d.c. distribution ?
2. What is the importance of load power factors in a.c. distribution ?
3. Describe briefly how will you solve a.c. distribution problems ?
4. Write short notes on the following :
   (*i*) Difference between d.c. and a.c. distribution
   (*ii*) Systems of a.c. distribution

DISCUSSION QUESTIONS

1. What are the undesirable effects of too much voltage variation on a distribution circuit ?
2. What are the effects of diversity factor on the maximum load of a distribution transformer ?
3. Where does the greatest current density occur in a distribution feeder ?
4. What is the controlling factor in determining the size of a distributor ?
5. In which situation is secondary distribution eliminated ?

GO TO