Subject : Mechanics II — موضوع : ميكانيك (II)

Weekly Hours : Theoretical:2 UNITS:4 — الساعات الأسبوعية : نظري : 2 الوحدات :4

Tutorial: 1 — مناقشة : 1

Experimental : — عملي :

week	Contents	المحتويات	الأسبوع
1.	Rectilinear motion	الحركة على خطوط مستقيمة	.1
2.	Curvilinear motion / x-y coordinates	الحركة على مسارات منحنية (احداثيات متعامدة)	.2
3.	Normal / tangential coordinates	الإحداثيات لا عمودية المماسة	.3
4.	Polar coordinates	الاحداثيات القطبية	.4
5.	Force / mass / acceleration for particles	القوة ، الكتلة ، التعجيل للجسيمات	.5
6.	Rectilinear motion / Kinetics	الحركة على خطوط مستقيمة	.6
7.	Curvilinear motion / Kinetics	الحركة على مسارات منحنية	.7
8.	Work and energy for particles	الشغل والطاقة للجسيمات	.8
9.	Impales and momentum for particles	الدفع والزخم للجسيمات	.9
10.	=	=	.10
11.	Conservation of momentum	حفظ الزخم	.11
12.	Rigid body rotation	الدوران للاجسام الصلبة	.12
13.	=	=	.13
14.	=	=	.14
15.	=	=	.15
16.	Absolute motion	الحركة المطلقة	.16
17.	Relative velocity	السرعة النسبية	.17
18.	=	=	.18
19.	=	=	.19
20.	Instantaneous center of velocity	المركز الحظي للسرعة	.20
21.	Relative acceleration	التعجيل النسبي	.21
22.	=	=	.22
23.	Mass moment of inertia	عزم القصور الذاتي للكتلة	.23
24.	Force / mass / acceleration for Rigid body	القوة ، الكتلة ، التعجيل للأجسام الصلدة	.24
25.	Rotation Kinetics	الدوران	.25
26.	General plane motion	الحركة المستوية العامة	.26
27.	=	=	.27
28.	Work and energy for Rigid body	الشغل والطاقة للاجسام الصلدة	.28
29.	=	=	.29
30.	Impulse and momentum for rigid bodies	الدفع والزخم للاجسام الصلدة	.30

Dynamic

Rectilinear Motion of particles:

Position ,velocity and acceleration.

A particle moving along a straight line is said to be in rectilinear motion .at any instant (t) ,the particle will occupy a certain position the straight line.

The position x, with the

Appropriate sign ,

Completely defines the position

Of the particle, it is called the

Position coordinate of the particle.

The motion of the particle may be given in the form of an equation in (x) and (t) such as :

$X=6t^2 - t^3$

P : position occupied by the particle at time (t) and coordinate (x)

P`: position occupied by particle at time (t+Δt) and coordinate (x+Δx).

$\frac{\Delta x}{\Delta t}$Average velocity =

m/sec or ft/sec$\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$Instantaneous velocity □=

------------------ (1)$\frac{dx}{dt}$ □=

The magnitude of (□) is known as the speed .

P=the particle has vel. (□) at time (t)

p`=the particle has vel.(□+Δ□) at time (t+Δt)

------- --------- $\frac{\Delta v}{\Delta t}$average acc.=

Instantaneous acc. $\lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$=

------------- (2)$\frac{dv}{dt}$a=

substituting for □ from eq. (1)

$\frac{d^2}{dt^2}$ a=

$\frac{dx}{v}$from eq. (1) dt =

substitute in eq.(2)

$\frac{dv}{dx}$ a=□ ------------

EX.

A particle moving in a straight line, and assume that its position is defined by the equation $x=6t^2 - t^3$ (where x in m and t is in seconds). Determine the position ,velocity and acceleration at sec.=3 .

At t=3 sec.

□= $6(3)^2 - (3)^3$

= 54 – 27 =27 m

= $12t – 3t^2 \frac{dx}{dt}$□=

=$12 (3) – 3(3)^2$

□= 9 m/sec.

= 12 – 6t = 12 – 6(3) $\frac{dv}{dt}$a=

a = - 6m/sec.

the negative sign of acc. is called deceleration or retardation.

<u>Determination of the motion of a particle</u>:

$\frac{dv}{dt}$a=

□ ⊟ adt = f(t)dt (integrating both members)

$$\int dv = \int \quad f(t)dt$$

$$\int_{v}^{v} dv = \int_{0}^{t} f(t)dt$$

$\int_0^t f(t)dt$□-□=

2) a= f(x)

$\frac{vdv}{dx}$ .a=

.vdv= adx= f(x)dx

$$\int_{v}^{v} vdv = \int_{x}^{x} f(x)dx$$

$\int_x^x f(x)dx$½□²-½□² =

3) a= f(v)

$\frac{dv}{dt}$ .a=

$\frac{vdv}{dx}$.a=

$\frac{dv}{dt}$(□)=

$\frac{vdv}{dx}$□(□)=

$\frac{dv}{f(v)}$□ ⊟

$\frac{vdv}{f(v)}$□ ⊟

UNIFORM RECTILINEAR MOTION

The vel. is therefore constant and eq. (1) becomes

= const.$\frac{dx}{dt}$□=

□ =□□ □ □

$$\int_{x}^{x} dx = v \int_{0}^{t} dt$$

□-□ =□ □

□=□-□ □ ..(5)

This equation may be used only if the vel. of the particle is known to constant.

Uniformly Accelerated Rectilinear Motion:

The acc. is therefore constant and eq.(2) becomes

=const.$\frac{dv}{dt}$a =

□ =□a□ □

$$\int_{v}^{v} dv = a \int_{0}^{t} dt$$

□- =a□

□= +a□--------- ..(6)

Substitute for □ in eq.(1)

a□$v +$ = $\frac{dx}{dt}$

□ □=(□ + a□)□ □

$$\int_x^x dx = \int_0^t (v + at)dt$$

□ – □ =□ +½ □ ²□

□ =□ + □ +½ □ ²□ ..(7)

we may also use eq.(4) and write

= const.$\frac{vdv}{dx}$a =

□ □ =□ □ □

$$\int_v^v vdv = a\int_x^x dx$$

½ (□² - □²)=a(□ –□)

□² =□²+2a(□ – □) …. (8)

The three equations derived provide useful relations among position coordinate, vel. and time in case of a uniformly accelerated motion.

<u>EX.</u>

An automobile travels 240m in 30 sec. while being accelerated at a constant rate of 0.2 m/sec². Find

a) its initial vel. b) its final vel. c) the distance traveled during the first 10 sec.

a) □ =□ + □ +½ □ ²□

240= 0 + (30) +½ (0.2)(30)²

240= 30 + 90

□= 5m/sec.^{2}

b) when t = 30 sec.

□= +a□

□=5+0.2(30)

□=11m/sec.

c) □ =□ + □ +½ □ ^{2}□

□ = 0+ 5(10) + ½ (0.2)(10)^2 □ = 60m

Curvilinear Motion of Particles:

When a particle moves along a curve other than a straight line , then it is in curvilinear motion

Average velocity $= \frac{\Delta r}{\Delta t}$

Velocity = v = $\lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t}$

$$\therefore V = \frac{dr}{dt}$$

$$V = \lim_{\Delta t \to 0} \frac{P\acute{P}}{\Delta t} = \lim_{\Delta t \to 0} = \frac{\Delta s}{\Delta t}$$

$$V = \frac{ds}{dt}$$

$$\vec{\acute{V}} = \vec{V} + \overrightarrow{\Delta V}$$

Average acceleration = $\frac{\Delta V}{\Delta t}$

Acceleration = a = $\lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t}$

a = $\frac{dV}{dt}$

Rectangular Components of Velocity and acceleration:

$$\vec{r} = x\vec{i} + y\vec{i} + z\vec{k}$$

$$\vec{V} = \frac{d\vec{r}}{dt} = \dot{x}\vec{i} + \dot{y}\vec{i} + \dot{z}\vec{i}$$

$$\vec{a} = \frac{d\vec{V}}{dt} = \ddot{x}\vec{i} + \ddot{y}\vec{i} + \ddot{z}\vec{i}$$

$$\therefore V_x = \dot{x} \ , \ V_y = \dot{y} \ , \ V_z = \dot{z} \quad \rightarrow \quad V = \sqrt{V_x^2 + V_y^2 + V_z^2}$$

$$a_x = \ddot{x} \ , \ a_y = \ddot{y} \ , \ a_z = \ddot{z} \quad \rightarrow \quad a = \sqrt{V_x^2 + V_y^2 + V_z^2}$$

In case of a projectile (Motion of a projectile)

$a_x = \ddot{x} = 0$ ($V_x = constant$)

$a_y = \ddot{y} = -g$ ($V_y = variable$)

$V_x = \dot{x} = V_x)_o$, $V_y = \dot{y} = V_y)_o - gt$,

$V_z = \dot{z} = V_z)_o$

$x = x_o + V_x)_o t$, $y = y_o + v_y)_o t - \frac{1}{2} g t$,

$z = z_o + V_z)_o t$

<u>EX:</u> A projectile is fired with an initial velocity of 800 m/sec at a target (B) located (2000m) above gun (A) and at a horizontal distance of (12000m). Determine the value of the firing angle α .

<u>Solution:</u>

<u>H. Motion is uniform</u>

$V_x)_o = 800 \ cos\alpha$

$x = V_x)_o t = (800 \cos \alpha) t$

12000 = (800 cos α)t

$\therefore t = \dfrac{15}{cos\alpha} \quad ..... \ (1)$

<u>V. Motion</u>

$V_y)_o = 800 \ sin\alpha$, a = - 9.81 m/sec^2

y = $V_y)_o t + \frac{1}{2}\, a\, t^2$

=(800 sinα)t – 4.9 t^2 . . . . .(2)

2000 = 800 sinα $\frac{15}{\cos\alpha} - \ 4.9\left(\frac{15}{\cos\alpha}\right)^2$

Since $\frac{1}{cos^2} = sec^2\alpha\ (1 + tan^2\alpha)$

2000 = 800(15) tanα – 4.9 $(15)^2$ $(1 + tan^2\alpha)$

1103.6 $\tan^2\alpha$ – 12000 tanα + 3103.6 = 0

Solve for tan α

Tangential and normal Components:

It is convenient to resolve the acceleration into components tangent and normal to the path of the particle.

$\Delta\vec{i}_t = \ \vec{i}_{t'} - \vec{i}_t$

$\vec{i}_t\ ,\vec{i}_{t'}\ : unit\ vectors$

$\Delta\theta = angle \quad formed\ \vec{i}_t\ and\ \vec{i}_{t'}$

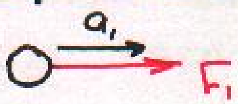
$$\Delta i_t = 2\sin\frac{\Delta\theta}{2}$$

$$\lim_{\Delta t\to 0}\frac{\Delta i_t}{\Delta\theta} = \ \lim_{\Delta t\to 0}\frac{2\sin\frac{\Delta\theta}{2}}{\Delta\theta} = \ \lim_{\Delta t\to 0}\frac{\sin\frac{\Delta\theta}{2}}{\frac{\Delta\theta}{2}} = 1$$

Kinetics of Particles:

Newton's 2nd law: **If the resultant force acting on a particle is not zero, the particle will have an acceleration proportional to the magnitude of the resultant and in the direction of this resultant force.**

***If the particle subjected to a force F1 of const. magnitude and direction, the particle will have a const. magnitude of acc. a1**

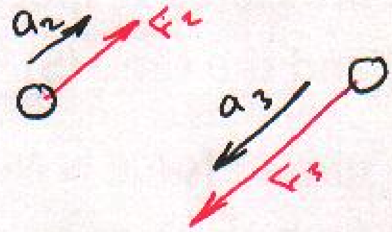


***If the particle is acted upon by F2 and F3 at different magnitude or direction, we**

Find each time that the

Particle moves in direction of the

Force acting on it.

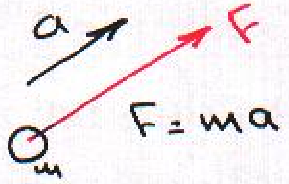


The magnitude of a1,a2,a3 etc.. of the acc. are proportional to the magnitudes F1,F2,F3 etc.. of the corresponding forces

F1/a1 = F2/a2 = F3/a3 = const.

The const. value obtained is called the mass and is denoted by m.

When a particle of mass (m) is acted upon by a force F1 the force F and the acc. (a) of the particle must therefore satisfy the relation



$F = ma$ (Newton's 2nd law)

When a particle is subjected simultaneously to several forces. $\sum F = ma$

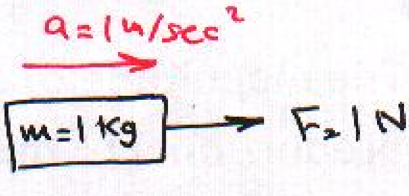
System of Units:

1.International system of units (SI units):

The base units are the units of length, mass and time and are called respectively, the meter (m), the kilogram (kg) and the time (S).

The unit of force is Newton (N) and is defind as the force which gives an acc. of $1m/sec^2$ to a mass of 1kg

$1N = (1kg)(1m/sec^2) = 1kg.m/sec^2$



The weight of a body W is a force expressed in Newton

$W = mg$

$g = 9.81\ m/sec^2$

The weight of a body of mass of 1kg

$W = (1kg)(9.81\ m/sec^2) = 9.81\ N$

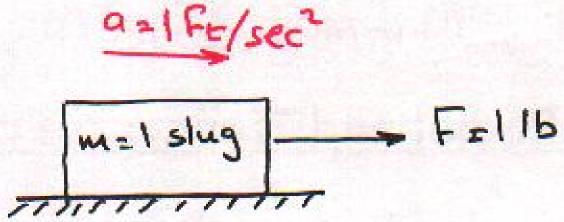
2. U.S. customary units system:

The base units are the units of length, force and time. These units are respectively, the foot (ft), the pound (lb) and the second (sec).

The unit of mass is called slug. $1slug. = 1lb/(1ft/sec^2) = 1\ lb.sec^2/ft$.

$W = mg$

$g = 32.2\ ft/sec^2$



Equations of Motion:

1.rectangular components:

Resolving each force (F) and the acc.(a) into rectangular components, we write

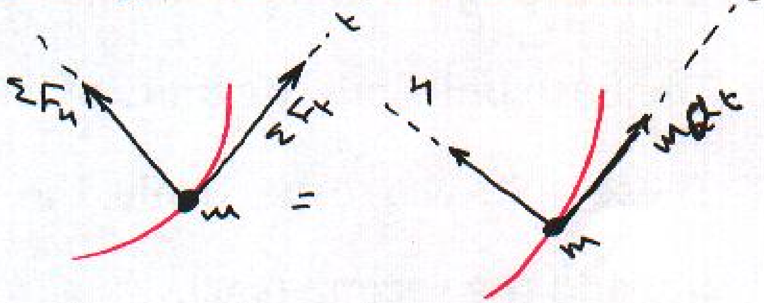
$$\sum(F_x i + F_y j + F_z k) = m(a_x i + a_y j + a_z k)$$

$$\sum F_x = ma_x \ , \ \sum F_y = ma_y \ , \sum F_z = ma_z$$

$$\sum F_x = m_x^{\cdot\cdot} \ , \ \sum F_y = m_y^{\cdot\cdot} \ , \sum F_z = m_z^{\cdot\cdot}$$

2.Tangential and Normal components:

Resolving the forces and acc. of the particle into components along the tangent to the path and the normal.

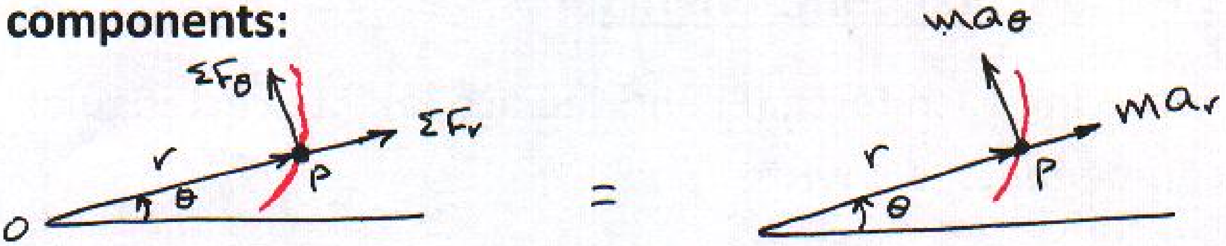


$\sum F_t = ma_t$, $\sum F_n = ma_n$

$\sum F_t = m\ _{dv/dt}$, $\sum F_n = m\ _{v^2/p}$

3.Radial and Transverse components:

Particle (P) of polar coordinates (r,θ) which moves in plane under the action of several forces. Resolving the forces and acc. of the particle into radial and transverse components:

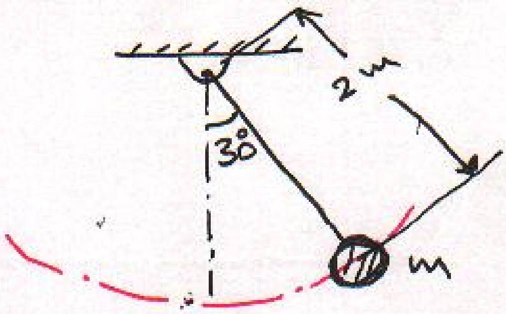


$\sum F_r = ma_r$ $\sum F_\theta = ma_\theta$

$\sum F_r = m(\ddot{r} - r\dot{\theta}^2)$ $\sum F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$

Ex: The bob of 2-m pendulum describe an arc of circle in a vertical plane. If the tension in the cord is 2.5 times the weight of the bob for the position shown, find the velocity and acc. of the bob in that position.

Solution:

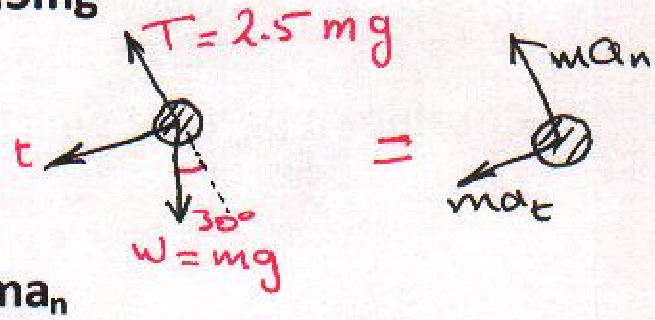


w = mg

the tension T = 2.5mg

apply newton 's 2nd law:

$\sum F_t = ma_t$: mgsin30 = ma_t T = 2.5mg



a_t = gSin30

a_t = 9.81(0.5) = 4.9 m/sec^2

$\sum F_n = ma_n$: 2.5mg – mgCos30 = ma_n

a_n = 1.634 = 16.03m/sec^2

a_n = 16.03m/sec^2

since $a_n = N^2/e \rightarrow N^2 = a_n e$ = (2)(16.03)

v^2 = 32.06

v = 5.66m/sec (up or down)

a = $\sqrt{(at)^2 + (an)^2}$ = $\sqrt{(4.9)^2 + (16.03)^2}$ =

$\sqrt{24.01 + 256.96}$

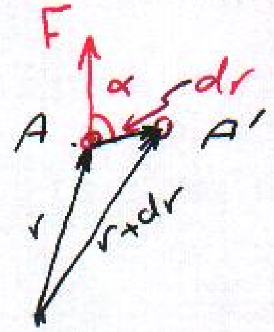
a = $\sqrt{280.97}$

a = 16.762m/sec^2

Work and Energy of Particles:

Work of a Force:

Consider a particle which moves from point (A) to point (A`)



$\vec{r}$: position vector corresponding to (A)

$\vec{r}$+ $\vec{dr}$: position vector corresponding to (A`)

$\vec{dr}$: small vector joining A and A`

F : Force acting on the particle

.:. The work of the force (F) corresponding to the displacement (dr) is defined as the quantity

$du^{\rightarrow} = F^{\rightarrow}.dr^{\rightarrow}$

The scalar quantity

$Du = F \, ds \, Cos \propto$

The work du may be expressed in terms of the rectangular components of the force and displacement.

$du = F_x dx + F_y dy + F_z dz$

scalar quantity work has magnitude and sign, but no direction.

-The work is positive if   is acute

-The work is negative if   is obtuse

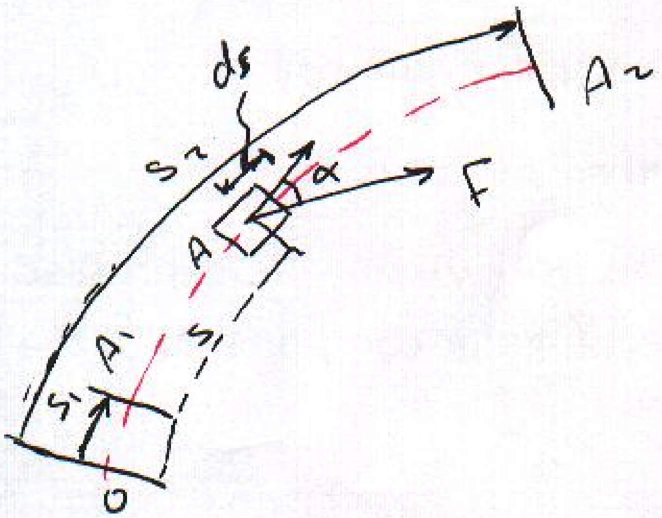
Three particular cases are of special interest

(i) F has same direction as dr the work reduce to Fds

(ii) If F has a direction opposite to that of dr the work is = _Fds

(iii) If F is perpendicular to dr th work du is Zero

The work of F during a Finite displacement of the particle from A_1 to A_2 is obtained by Integration the equation

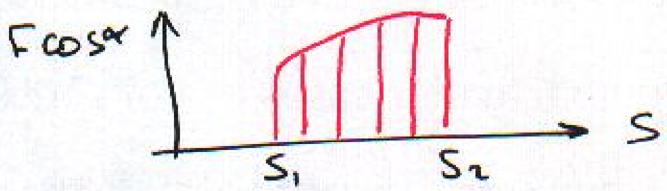


$$U_{1-2} = \int_{A_1}^{A_2} F.dr$$

$$U_{1-2} = \int_{S_1}^{S_2} (F\cos\alpha)ds$$

$$= \int_{S_1}^{S_2} F_t ds$$ (since Fcos $= F_t =$ tangential comp. of the force)

The work U_{1-} is represented by the area under the curve obtained by plotting $F_t = F\cos\alpha$ against S

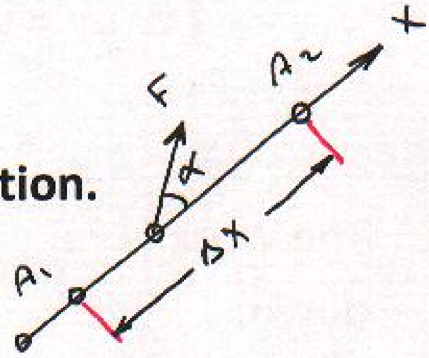


When the Force F is defined by its rectangular components

$$U_{1-2} = \int_{A_1}^{A_2} (F_x dx + F_y dy + F_z dz)$$

Work of a constant Force in Rectilinear motion:

When a particle moving in a straight line is acted upon by a force F of constant magnitude and of const. direction.



$U_{1\text{-}2} = (F\cos\propto)\,\Delta x$

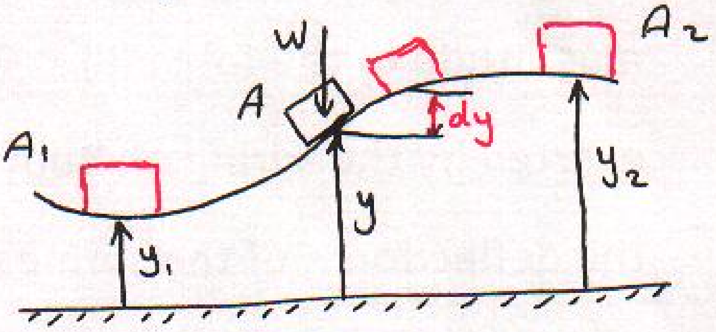
where

$\propto$ = angle of the force form with direction of motion

Δx = displacement from A_1 to A_2

Work of a Weight:

the work of the weight W of a body is obtained by substituting the components of W in the rectangular components with y-axis chosen upward we have.



$F_x = 0$, $F_y = -W$, $Fz = 0$

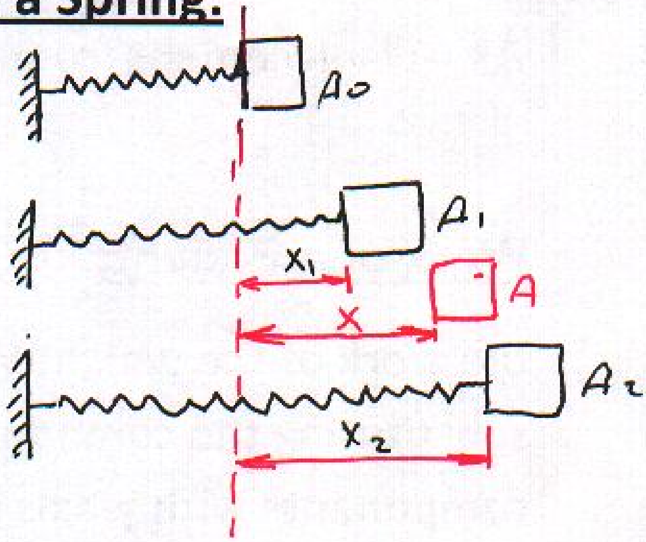
$du = -W\,dy$

$$U_{1-2} = -\int_{y_1}^{y_2} W\,dy = Wy_1 - Wy_2 \quad \text{or} \quad U_1\text{-}2 = -W(y_2 - y_1)$$

$$= -W\Delta y$$

The work (+ve) when Δy (-ve) i.e when the body moves down.

Work of the force Exerted by a Spring:



-A body attached to a fixed point (B) by a spring

-The ~~point~~ spring un deformed when the body at Ao

-Experimental shows that the magnitude of the force (F) exerted by the spring on body (A) is proportional to the deflection X of the spring measured the position Ao

$F = KX$

K is the spring constant (N/m)

The work of the force F exerted by the spring during a finite displacement of a body from $A_1(X = X_1)$ to $A_2(X = X_2)$

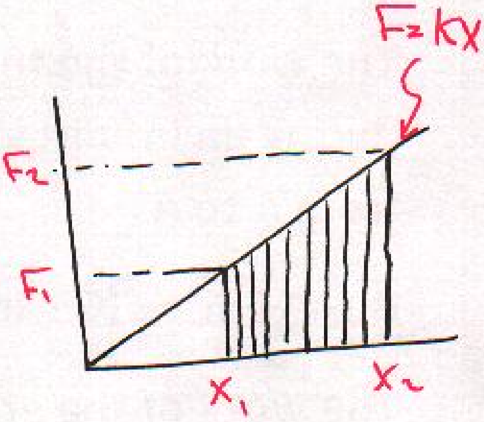
$du = -FdX = -KXdX$

$U_{1-2} = -\int_{x1}^{x2} KXdX = \frac{1}{2} KX_1^2 - \frac{1}{2} KX_2^2$

$= -\frac{1}{2} K(X_2^2 - X_1^2)$

$= -\frac{1}{2} K(X_2 - X_1)(X_2 + X_1)$

~~$= -\frac{1}{2} K(F_1 + F_2)\Delta X$~~



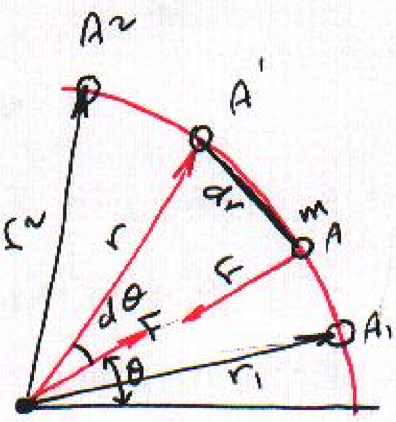
$U_{1-2} = -\frac{1}{2} (F_1 + F_2) \Delta X$

The work of the force (F) exerted by the spring on the body is positive when $X_2 < X_1$ i.e when the spring is returning to its un deformed position.

<u>Work of a Gravitational Force:</u>

Each two particles at a distance (r) from each other and respectively of mass (M) and (m) attract each other with equal and opposite forces (F) and (-F) directed a long the line joining the particles and of magnitude.

$F = G\,Mm/r^2$



Let us assume that the particle

M occupies a fixed position (O)

M = moves along the path

The work of the force (F) exerted on the particle (m) during an infinitesimal displacement of the particle from A to A`

$du = -Fdr = -(G\,Mm/r^2)\,dr$

The work of the gravitational force (F) during a finite displacement from $A_1(r = r_1)$ to $A_2(r = r_2)$

$U_{1\text{-}2} = -\int_{r1}^{r2} GMm/r^2\,dr = GMm/r_2 - GMm/r_1$

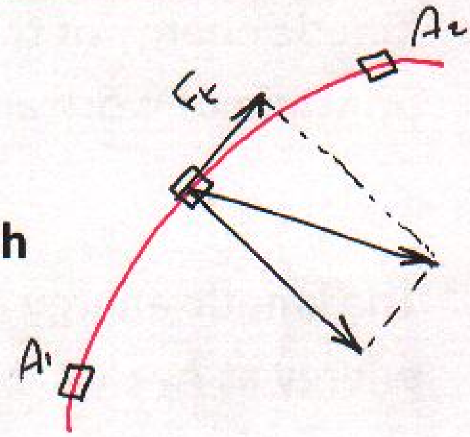
$G = (6.673 \mp 0.003)\times 10^{-11}\ m^3/Kg.s^2$ (from experiments)

Radius of the earth $K = 6.37\times 10^6 m$

= 3960mi

The formula obtained may be used to determine the work of the force exerted by the earth on a body of mass (m) at a distance (r) from the center of the earth.

Kinetic Energy of a particle .Principle of work and Energy:



A particle of mass (m) acted upon by a force (F) and moving along a path which is either rectilinear or curved. Applying Newton's 2nd law in terms of the tangential components of the force and the acceleration.

$F_t = ma_t$ or $F_t = m\ dv/dt$

$F_t = m\ dv/ds\ .\ ds/dt = mv\ dv/ds$

$F_t\ ds = mvdv$

Integration from A_1 where $S = S_1$ an $dv = v_1$ to A_2 where $S = S_2$ and $V = V_2$

$$\int_{S1}^{S2} Fds = m\int_{V1}^{V2} vdv = ½\ mv^2_2 - ½\ mv^2_1$$

The left hand member $\int_{S1}^{S2} Fds = U_{1\text{-}2}$ (scalar quantity) the expression ½ mv^2 is defined as the Kinetic energy it is also scalar quantity and is denoted by (T)

$T = \frac{1}{2} mv^2$ $U_1\text{-}2 = T_2 - T_1$

The work of the force (F) is equal to the change in the Kinetic energy of the particle. This known as the principle of work and energy.

$T_1 + U_{1\text{-}2} = T_2$

The kinetic energy at A_2 obtained by adding the kinetic energy at A_1 to A_2 by the force (F).

Power and Efficiency:

Average power = $\frac{\Delta u}{\Delta t}$

As Δt approach Zero, we obtained at the limit

Power = du/dt

But du = scalar product F.dr

:. Power = F.dr/dt = F.v (v = dr = dt)

The mechanical efficiency is defined as the ratio of the output work to the input work:

η = output work/input work

η = power output/power input

The power should be expressed in J/s = watt

1W = 1 J/s = 1 N.m/s

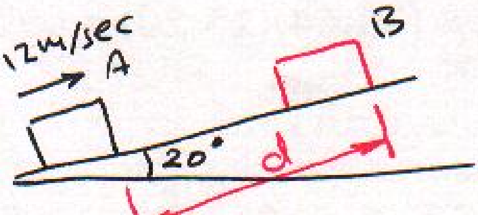
1 hp = 550 ft.lb/s

1 ft.lb/s = 1.356 J/s = 1.356 W

1 hp = 550(1.356W) = 746 W = 0.746 Kw

Ex: A 20 kg package is projected up a 20 incline with an initial velocity of 12 m.sec. the coefficient of friction between the incline and the package is 0.15 determine (a) the maximum distance that package will move up the incline

(b) the velocity of the package when it returns to its original position.

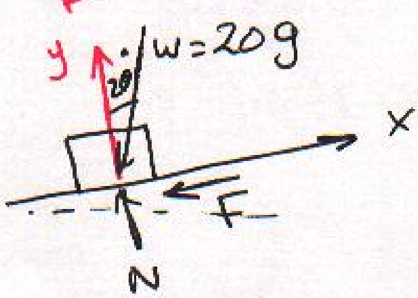


$\therefore T_1 = \frac{1}{2} m v_1^2 = \frac{1}{2}(20)(12)^2$

$= 1440$ J

<u>At B</u>: $T_2 = 0$

$\sum F_y = 0$



$N - (196.2)\cos 20 = 0$

$N = 184.4\ N$

$F = \mu N = 0.15(184.4) = 27.66\ N$

$U_{1\text{-}2} = -(W\sin 20 + F)d = -(196.2\sin 20° + 27.66)d$

$= -\ 94.76d$

$T_1 + U_1\text{-}2 = T_2$

$1440 + (-94.76d) = 0$

$d = 15.20m$

<u>At B</u>: $T_2 = 0$

Back at A $T_3 = ½\, mv^2 = ½\,(20)(v^2) = 10v^2$

$U_{2\text{-}3} = (w \sin 20 - F)d$

$= (196.2 \sin 20 - 27.66)(15.20)$

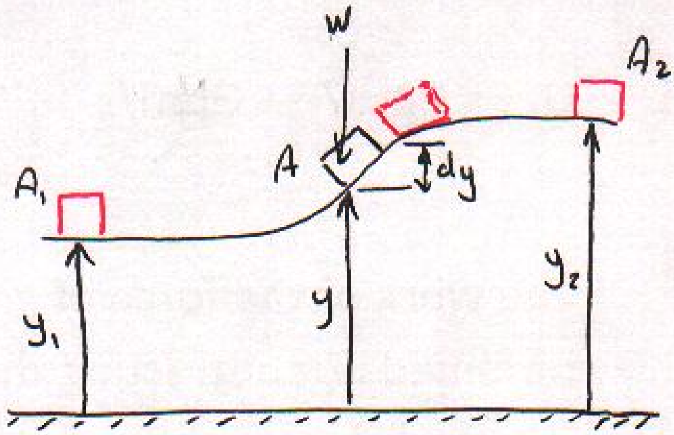
$U_{2\text{-}3} = (39.44)(15.20) = 599.6\ J$

$T_2 + U_{2\text{-}3} = T_3$

$0 + 599.6 = 10v^2 \rightarrow v^2 = 59.96$

$V = 7.74$ m/sec.

Potential Energy:



$U_{1-2} = Wy_1 - Wy_2$

1.The work of (W) may be obtained by subtracting the value of the function Wy corresponding to second position from its value corresponding to first position. the work (W) is independent of the actual path followed.

:.Wy is called the potential energy of the body with respect to the force of gravity (W) and is denoted by (V_y)

.:. $U_{1-2} = (V_y)_1 - (V_y)_2$ (with $V_y = Wy$)

If $(V_y)_2 > (V_y)_1$ the potential energy increases during the displacement, the work U_{1-2} (-ve) If the work of (w) is (+ve) the potential energy decreases.

2.In the case of space vehicle, the variation of the force of gravity with the distance (r) from the center of the earth.

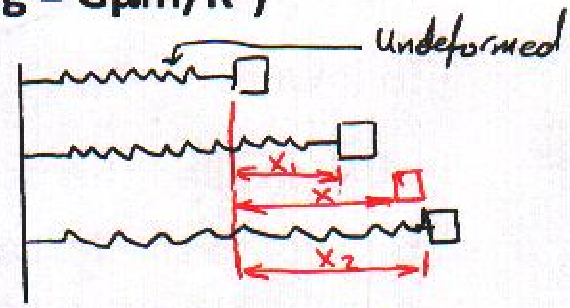
$U_{1-2} = G\mu m/r_2 - G\mu m/r_1$

The work of the force of gravity may therefore be obtained by subtracting the value of the function $-G\mu m/r$ corresponding to the second position from its value corresponding to the first position of the body.

The expression should be used for the potential energy

$V_g = -G\mu m/r$ or $V_g = -WR^2/r$ $(W = mg = G\mu m/R^2)$

$U_{1-2} = ½ KX^2_1 - ½ KX^2_2$



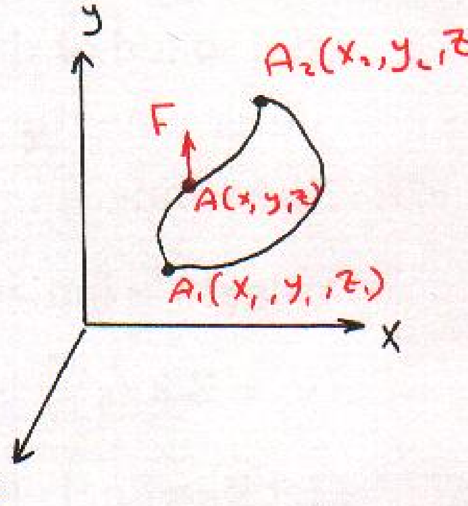
3.The work of the elastic force is obtained by subtracting the value of the function ½ KX^2 corresponding to the second position of the body from its value corresponding to the first position of the body, The function ½ KX^2 is called the potential energy of the body w.r.t the elastic force (F) and is denoted by V_e

.:. $U_{1-2} = (V_e)_1 - (V_e)_2$ (with $V_e = ½ KX^2$)

The work of the force (F) exerted by a spring on the body is (-ve) and the potential energy (ve) increases.

Conservative Force:

A Force (F) acting on a particle (A) is said to be conservative if its work (U_1-2) is independent of the path followed by the particle (A) as it moves from A_1 to A_2.



$U_1\text{-}2 = V(x_{1\,1}y_{1\,1}z_1) - V(x_{2\,1}y_{2\,1}z_2)$ or

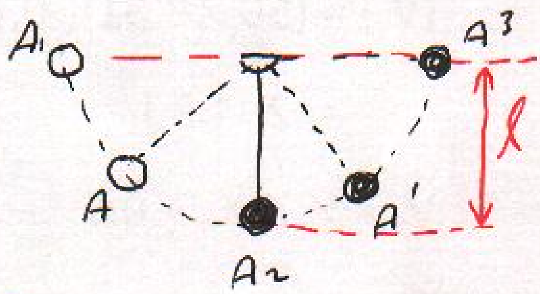
$U_1\text{-}2 = V_1 - V_2$

Conservative of Energy:

$V_1 - V_2 = T_2 - T_1 = U_1\text{-}2$

$\underline{T_1 + V_1 = T_2 + V_2}$

T + r = E = total mechanical energy.



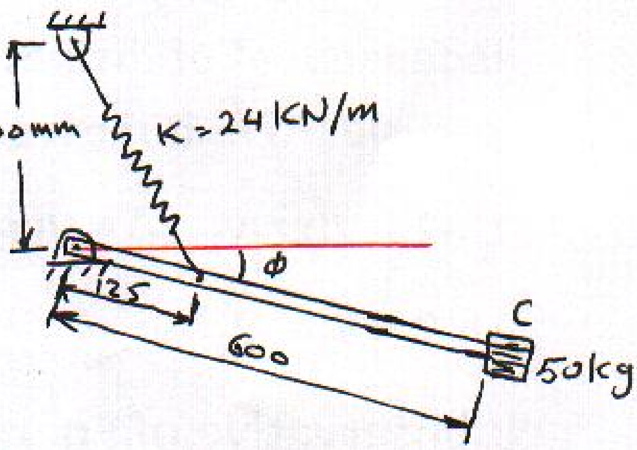
$T_1 = 0$, $V_1 = wl$, $T_1 + V_1 = wl$

at A_2 , $v_2 = \sqrt{2gl}$

$T_2 = ½\, m v_2^2 = ½\, w/g\,(2gl) = wl$, $V_2 = 0$

$T_2 + V_2 = wl$

Ex: The 50kg block is released from rest when Ø = 0 if the speed of the block when Ø = 90° is to be 2.5m/sec., determine the required value of the initial tension in the spring.



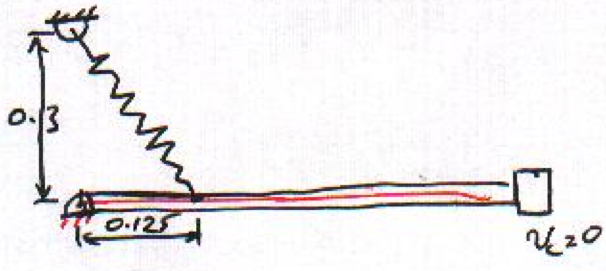
.: let X_o = initial elongation

$V_o = ½ KX_o^2$, $T_o = 0$

let X_1 = final elongation

$X_1 = X_o + 0.425 – 0.325$

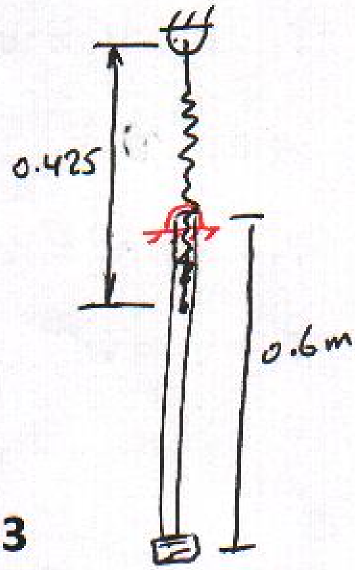
$X_1 = X_o + 0.1m$



$(V_e)_{spr.} = ½ KX^2_1 = K/2 (X_o + 0.1)^2$

$(V_y) = (50 \times 9.81)(-0.6)$

$= -294.3$ J



$V_1 = ½ K(X_o + 0.1)^2 – 294.3$

$T_1 = ½ mv^2_o = ½ (50)(2.5)^2$

$T_1 = 156.25$ J

$T_o + V_o = T_1 + V_1$

$0 + ½ KX^2_o = 156.25 + ½ K(X_o + 0.1)^2 – 294.3$

$\frac{1}{2} KX^2_0 = \frac{1}{2} KX^2_0 + 0.1 KX_0 + 0.005 k - 138.05$

$KX_0 = 1380.5 - 0.05 k$

But KX_0 = initial tension

Impulse and momentum of particles:

Consider a particle of mass (m) acted upon by a force (F) Newton's 2nd law may be expressed in the form:

$\sum F = ma = d/dt\ (mv)$

Where (mv) is the linear momentum of the particle

$Fdt = d\ (mv)$

Integrating from t_1 to t_2

$\int_{t1}^{t2} Fdt = mv_2 - mv_1$

Now:

$mv_1 + \int_{t1}^{t2} Fdt = mv_2$

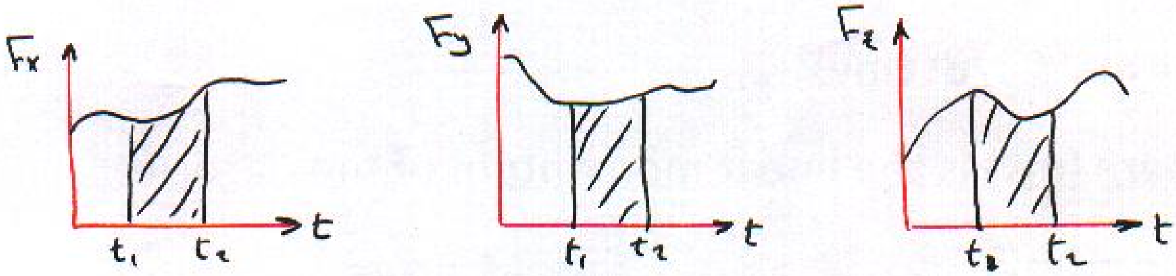
$\int_{t1}^{t2} Fdt$ it is call the linear impulse or impulse of the force (F) during the interval of time considered.

If we resolve (F) into rectangular components we have,

$Imp_{1\text{-}2} = \int_{t1}^{t2} F dt$

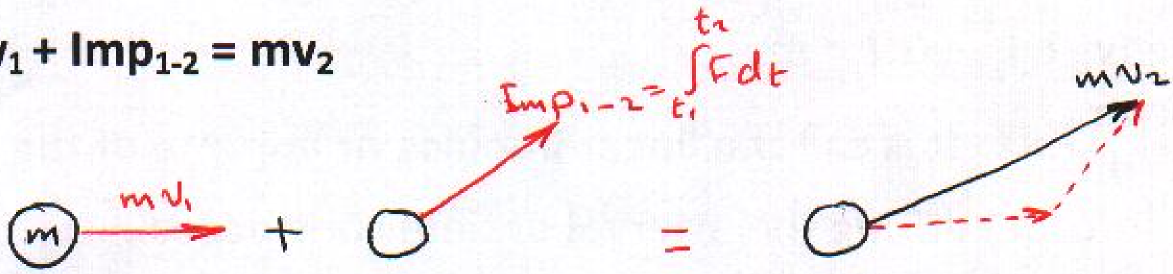
$= i \int_{t1}^{t2} Fx\, dt + j \int_{t1}^{t2} Fy\, dt + k \int_{t1}^{t2} Fz\, dt$

Note that the components of impulse of the force (F) are respectively, equal to the areas under the curves obtained by plotting the components F_x , F_y and F_z against (t)



The final momentum (mv_2) of a particle may be obtained by adding vectorially its initial momentum (mv_1) and the impulse of the force (F) during the time interval.

$mv_1 + Imp_{1\text{-}2} = mv_2$



while kinetic energy and work are scalar quantity, momentum and impulse are vector quantities.

To obtain an analytic solution, it is thus necessary to replace the equation of impulse and momentum by its equivalent components equation.

$(mv_x)_1 + \int_{t1}^{t2} Fxdt = (mv_x)_2$

$(mv_y)_1 + \int_{t1}^{t2} Fydt = (mv_y)_2$

$(mv_z)_1 + \int_{t1}^{t2} Fzdt = (mv_z)_2$

When several forces act on a particle, the impulse of each of the forces must be considered.

$mv_1 + \sum Imp_{1\to 2} = mv_2$

When there are two or more particles, each may consider separately and the equation of impulse and momentum may be written for each particle.

We may also add vectorially the momentum of all the particles and the impulse of all the forces involved.

$\sum mv_1 + \sum Imp_{1\to 2} = \sum mv_2$

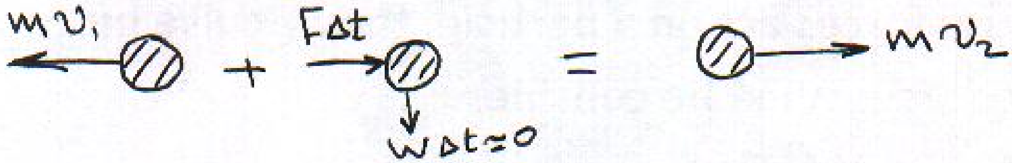
If the sum of the external forces is zero, then the last equation reduce to:

$\sum mv_1 = \sum mv_2$

which expresses that the total momentum of the particle is conserved.

Impulsive Motion:

Sometimes a very large force may act during a very short time interval on a particle and produce a definite change in momentum such a force is called an impulsive force and the resulting motion an impulsive motion.

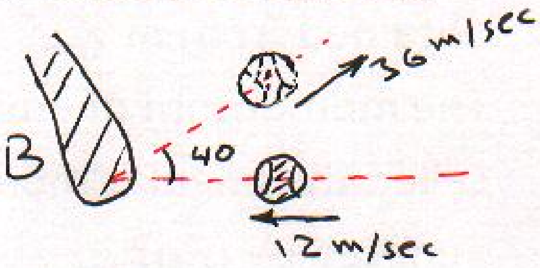


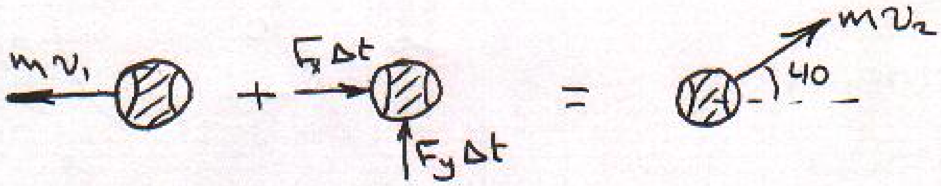
$mv_1 + \sum F\Delta t = mv_2$

in the case of several particles

$\sum mv_1 + \sum F\Delta t = \sum mv_2$

EX: A 2-N baseball is thrown with a vel. of 12m/sec. toward a batter. After the ball is hit by the bat (B), it has a vel. of 36m/sec. in the direction shown in Fig. if the ball and the bat are in contact 0.025sec. , determine the average impulsive force exerted by the bat on the ball during the impact.





$mv_1 + \sum F\Delta t = mv_2$

($\overset{+}{\rightarrow}$ X comp.) $-(2/9.81)(12) + F_x(0.025) = (2/9.81)(36\cos 40)$

$-2.45 + 6.025F_x = 5.62$

$\therefore F_x = 8.072/0.025 = 323\ N \rightarrow$

($+\uparrow$ Y comp.) $0 + F_y(0.025) = 2/9.81\ (36\sin 40)$

$\therefore F_y = 4.717/0.025 = 188 N \nearrow$

$\tan\theta = 188/323 \rightarrow \theta = 30.3°$

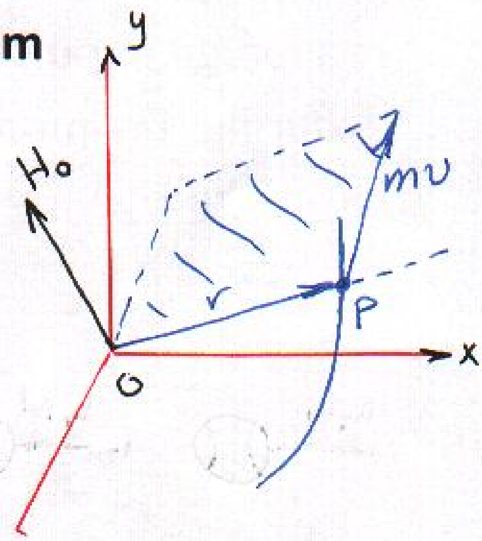
$F = 188/\sin 30.3 = 373.7\ N \rightarrow \therefore F = 373.7\ N$

Angular Momentum of a particle. Rate of change of Angular Momentum:

P:particle of mass (m)

$m\vec{v}$: vector defined the linear momentum obtained from $m \times \vec{v}$

The momentum of $m\vec{v}$ about (o) is called the angular moment and is denoted by (Ho)



$\vec{r}$: position vector of (P)

$\vec{Ho} = r \times mv$

$\vec{Ho}$: is a vector perpendicular to the plane containing (r) and mv, and of magnitude

Ho:

$Ho = rmv \sin\phi$

ϕ: is the angle between r and mv

Ho: is obtained from the sense of mv by the right hand rule

The units of Ho is: (m)(Kg.m/sec.) = $Kg.m^2/sec.$

Now resolving the vectors r and mv into components

$$Ho = \begin{vmatrix} i & j & k \\ x & y & z \\ mv_x & mv_y & mv_z \end{vmatrix}$$

$H_x = m(Yv_z - Zv_y)$

$H_y = m(Zv_x - Xv_z)$

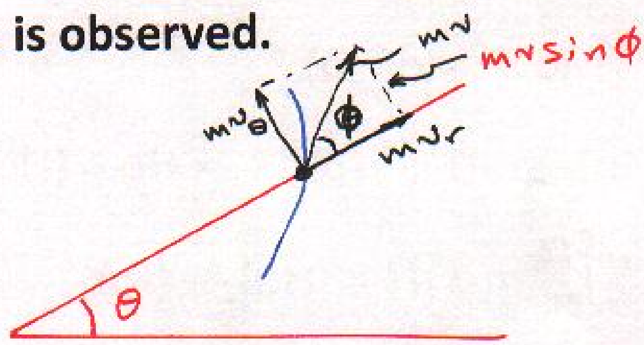
$H_z = m(Xv_y - Yv_x)$

In case of particle moving in (x_y) plane

$Z = V_z = o$ and H_x and H_y reduce to zero

.:. $Ho = H_z = m(Xv_y - Yv_x)$

Ho is positive(+ve) or negative (-ve), according to the sense in which the particle is observed.



$Ho = m\, v\, r \sin\phi = r\, m\, v_\theta$

$v_\theta = r_\theta$

.:. $Ho = mr^2_\theta$

Now the derivative of the angular momentum of the particle (P) is:

$dHo/dt = d/dt\ (r\ m\ v)$

$\overset{\circ}{H}o = \overset{\circ}{r}xmv + rx\overset{\circ}{mv} = rxmv + rxma$

$\vec{v}$ and $\vec{mv}$ vectors are collinear, so the first term is zero, and ma equal to ∑F (Newton's 2nd law).

∑F = the forces acting on the particle (P)

r x ∑F = ∑Mo (the sum of moments about (o) of these forces)

.:.∑Mo = $\overset{\circ}{H}o$

The sum of the moments about (o) of the forces acting on the particle is equal to the rate of change of the angular momentum (Ho).

Angular Impulse and Momentum principles:

∑Mo dt = d(Ho)

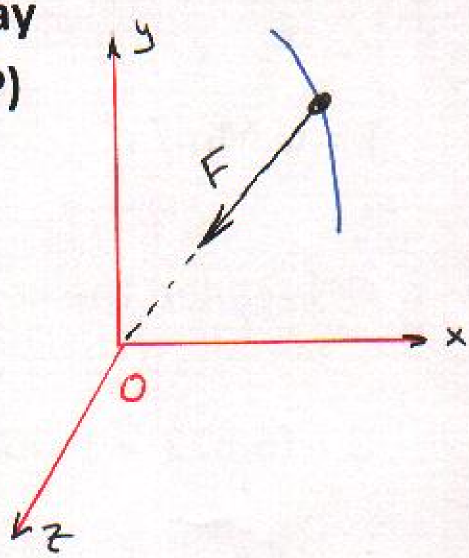
Integrating from t_1 = Ho = $(Ho)_1$ and at t_2 , Ho = $(Ho)_2$

$\sum\int_{t1}^{t2} mo\, dt = (Ho)_2 - (Ho)_1$

$(Ho)_1 + \sum\int_{t1}^{t2} mo\, dt = (Ho)_2$

Motion under a central Force. Conservation of Angular Momentum:

F: Central force directed toward or away from a fixed point (o) on the particle (P) (o) referred to the center of force (F) since (F) passes through (o):



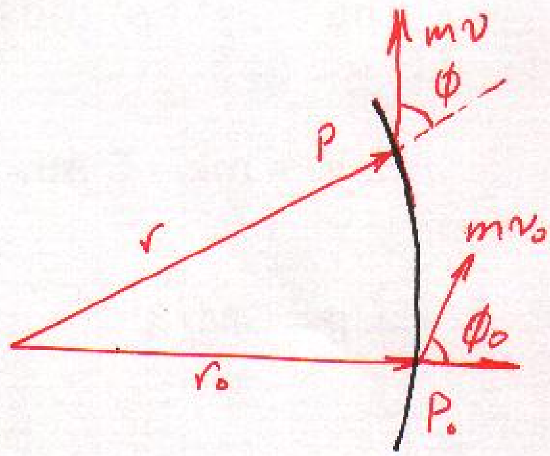
$\sum Mo = 0$ at any given instant

$\sum mo = H°o = 0$

.:. Ho = constant

The angular momentum of a particle moving under a central force is constant in magnitude and direction.

.:. Ho = r x mv = constant



Particle under a central force moves in fixed plane ⊥ r to (Ho)

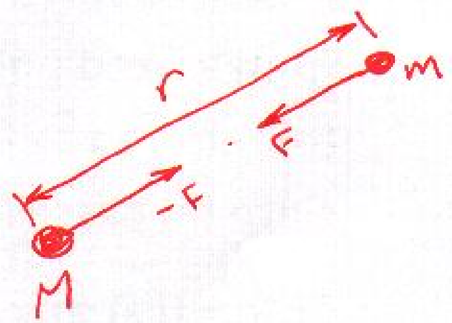
$rmv \sin\phi = r_o m v_o \sin\phi_o$

$Ho = constant = mr^2\theta$

$h = Ho/m = r^2\theta$

Newton's law of Gravitation (Newton's 3rd law)

$F = G\,Mm/r^2$



G: is called the const. of gravitation

$G = (6.673 \mp 0.003)\times10^{-11}\ m^3/Kg.s^2$

For a body mass (m) located on its surface.

The weight of the body W = mg = Force

$\therefore W = mg = G\,mM/R^2$ (R = radius of the earth)

$\therefore g = GM/R^2$

EX: A satellite is launched in a direction parallel to the surface of the earth with a vel. of 18.820/h from an altitude of 240km. Determine the vel. of the satellite as it reaches its max. altitude of 2340km. It is recalled that the radius of the earth is 6336km.

Since the satellite is moving under a central force directed toward the central (o) of the earth, its angular momentum (Ho) is constant .

rmv sinØ = Ho = const.
which shows that v is minimum at (B) where both (r) and sinØ are max.

Conservation of Angular momentum:

$r_A m v_A = r_B m v_B$

.:. $N_B = N_A\, r_A/r_B$ = (18820km/h)(6336 + 240/6336 + 2340)

.:. N_B = 14264 km/h

<u>**Impact:**</u>

_A collision between two bodies which occurs in a very small interval of time, and during which the two bodies exert on each other relative large forces is called impact.

_The common normal to the surface in contact during the impact is called the line of impact.

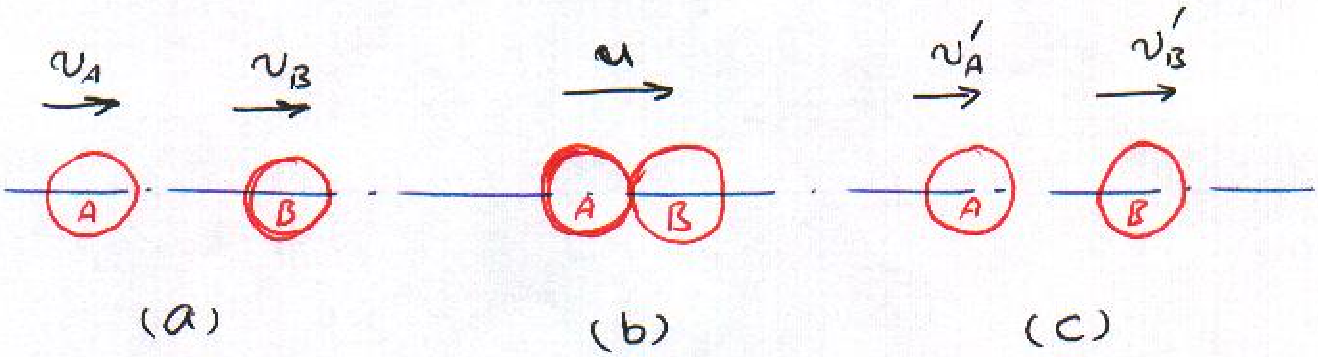
<u>**Direct Central Impact:**</u>

_Two particles (A) and (B) of masses (m_A) and (m_B)

_$N_A > N_B$ and particle (A) will strike particle (B)

_At the period of max. deformation, they will have same velocity (U)

_A period of restitution will take place at the end of max. deformation, the velocities (N'_A) and (N'_B) occurs at the end of restitution period.



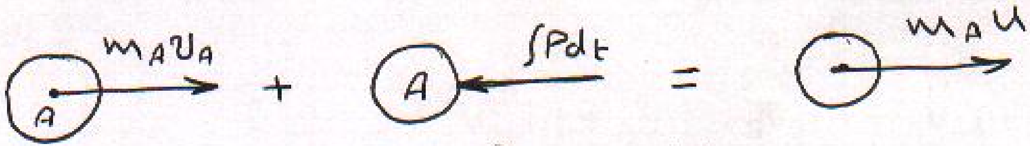
a) Before Impact
b) At the max. deformation
c) After impact

The total momentum of the two particles is conserved

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B \quad ....(1)$$

The impulse force acting on (A) during the max. deformation period is the force (F) exerted by (B).

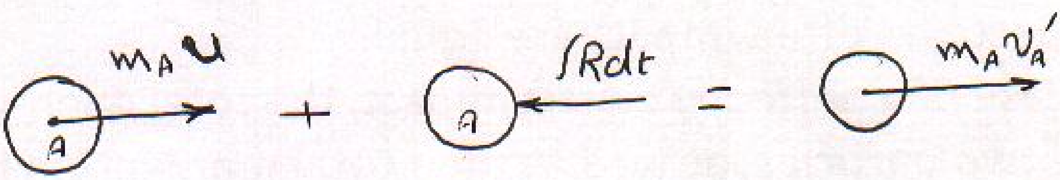
$$m_A v_A - \int p\,dt = m_A u....(2)$$



Period of max. deformation

The force exerted by (B) on (A) during the period of restitution is R.

$$m_A u - \int R\,dt = m_A v'_A \quad ....(3)$$



period of restitution

The ratio of the magnitudes of impulse corresponding respectively to the period of restitution and to the period of deformation is called the coefficient of restitution and is denoted by (e).

$$e = (V'_B - V'_A)/(V_A - V_B) \rightarrow V'_B - V'_A = e(V_A - V_B) \ldots(4)$$

Two particular cases of impact are of special interest :

(1) e = 0 , perfectly plastic impact. When e = 0 , yields $V'_B = V'_A$. There is no period of restitution and both particles stay together after impact.
sub. $V'_B = V'_A = V'$ in the total momentum of particles

$$m_A v_A + m_B v_B = (m_A + m_B)V'$$

(2) e = 1 , perfectly elastic impact, when e = 1

$$V'_B - V'_A = V_A - V_B \ldots(5)$$

This equation express that the relative velocities before impact and often impact are equal.
In case of a perfectly elastic impact, the total energy of the two particles, as well as their total momentum is conserved. eq.(1) and eq.(5) may be written as follows

$m_A(V_A - V'_A) = m_B(V'_B - V_B)$....eq..(1)

$V_A + V'_A \quad = V_B + V'_B$...eq..(5)

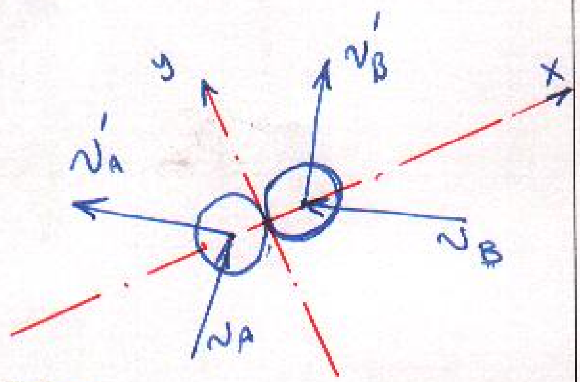
Multiplying eq.(1) and eq.(5) member by member we have

$m_A(V_A - V'_A)(V_A + V'_A) = m_B(V'_B - V_B)(V'_B + V_B)$

$m_A V^2_A - m_A(V'_A)^2 = m_B(V'_B)^2 - m_B V^2_B$

Rearranging the terms in the equation obtained, and multiplying by (1/2)

$\frac{1}{2} m_A V^2_A + \frac{1}{2} m_B V^2_B = \frac{1}{2} m_A(V'_A)^2 + \frac{1}{2} (V'_B)^2 m_B$



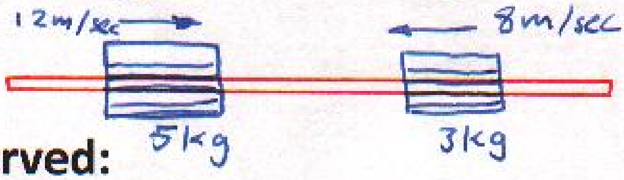
Oblique Central Impact:

Oblique central impact take place when the velocity of the two colliding particles are not directed along the line of the impact. The velocities of particles V'_A and V'_B are unknown in direction as well as in magnitudes, we choose X and Y axes respectively, along the line of impact and along the common tangent to the surface in contact. Assuming that the particles are perfectly smooth particles during the impact are interval forces directed along the X-axis , we may therefore express that:

1. The Y-component of the momentum of particle (A) is conserved.

2. The Y-component of the momentum of particle (B) is conserved.
3. The X-component of the total momentum of the particles is conserved.
4. The X-component of the relative of the two particles after impact is obtained by multiplying the X-component of their relative before impact by the coefficient of restitution.

<u>EX:</u> **The coefficient of restitution between the two collars is known to be 0.75 ; determine (a) their velocities after impact. (b) the energy losses during impact.**



(a) Total momentum conserved:

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B$$

$$5(12) + 3(-8) = 5v'_B + 3v'_B$$

$$36 = 5v'_A + 3v'_B \text{....(1)}$$

Relative velocities:

$$v'_B - v'_A = e(v_A - v_B)$$

$$v'_B - v'_A = 0.75(12 + 8) = 15\text{....(2)}$$

solving eq.(1) and eq.(2) simultaneously

$$v'_A = -1.125 = 1.125 \text{ m/sec} \leftarrow$$

$$v'_B = 13.875 = 13.875 \text{ m/sec} \rightarrow$$

(b) Before impact:

$T_1 = ½\, m_A v^2_A + ½\, m_B v^2_B$

$= ½\,(5)(12)^2 + ½\,(3)(-8)^2 = 456\ J$

After impact:

$T_2 = ½\, m_A(v'_A) + ½\, m_B(v'_B)^2$

$= ½\,(5)(-1.125)^2 + ½\,(3)(13.875)^2$

$= 291.9\ J$

Energy loss $= T_1 - T_2$

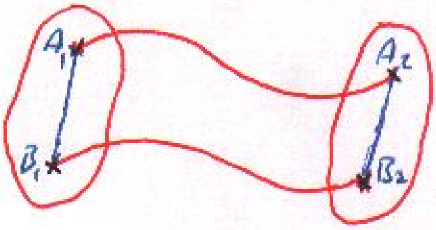
$= 456 - 291.9 = 164.1\ J$

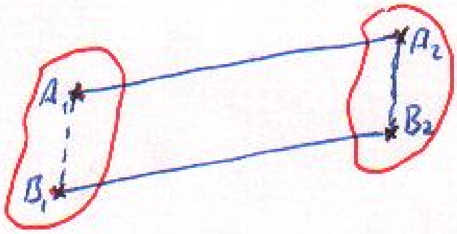
Kinetics of Rigid Bodies:

_A studying the relation existing between the time, the position, the velocities and accelerations of various particles forming a rigid body. The rigid body motion may be grouped as follows:

(1) **Translation:**

A motion is said to be translation if any straight line inside the body keeps the same direction during the motion the particles forming the body moving along parallel path. If these paths are straight lines, the motion is said to be a rectilinear translation, if the paths are curved lines, the motion is a curvilinear translation as shown in Fig.



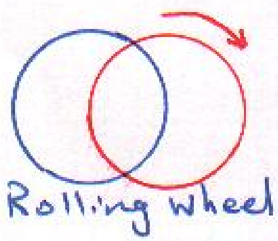


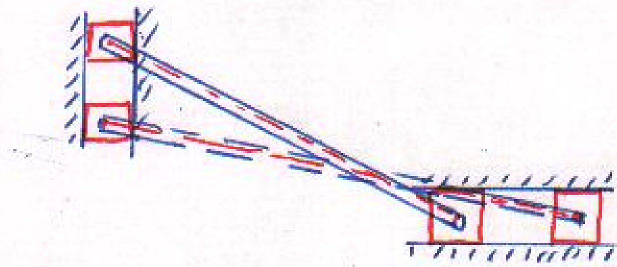
(2) **Rotation about Fixed Axis:**

In this motion particles forming the rigid body move in parallel planes along circles centered on the same fixed axis, this axis is called the axis of rotation as shown in Fig. the particles located on the axis of rotation have zero velocity.

(3) <u>General plane Motion:</u>

The particles forming the rigid body move in parallel planes. Any plane motion which is neither a rotation nor a translation is refered to as general plane motion as shown in Fig.





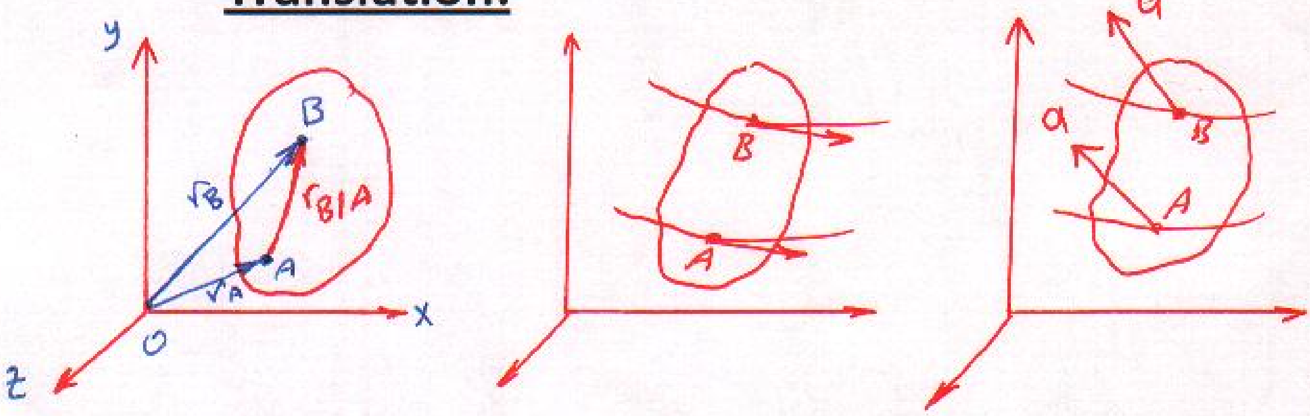
(4) <u>Motion about Fixed Point:</u>

This is the three _ dimensional motion of a rigid body attached at a fixed point (o) as shown in Fig.

(5) <u>General Motion:</u>

Any motion of a rigid body which
does not fall in any of the above categories is refered to as a general motion.

Translation:



$r_B = r_A + r_{B/A}$

$dr_B/dt = dr_A/dt + dr_{B/A}/dt$

$v_B = v_A + o$ (since $r_{B/A}$ must be const. in magnitude and direction)

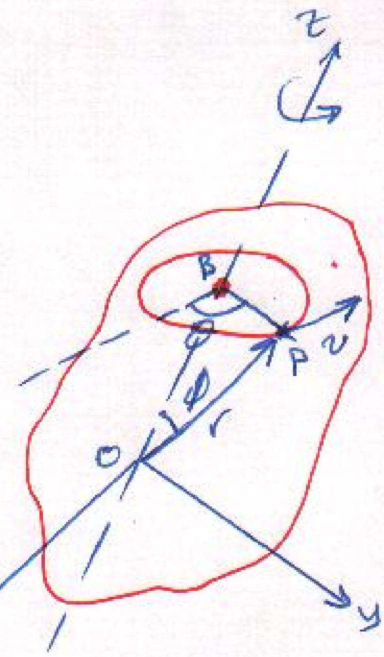
.:. $v_A = v_B$

$dv_B/dt = dv_A/dt$

$a_B = a_A$

Thus: when a rigid body in translation, all the points of the body have the same velocity and the same acc. at any given instant. In case of curvilinear translation the velocity and acc. of the particles changes in magnitude as well as in direction. In case of rectilinear motion all the particles keep the same vel. and acc. during entire motion.

Rotation about a Fixed Axis:



1 rev = 2πrad / = 360°

$v = dr/dt = \omega r$

$\omega = \omega k = \dot{\theta} K$

$a = \alpha r + \omega \times (\omega \times r)$

$\alpha = \alpha k = \dot{\omega} k = \ddot{\theta} k$

$a_t = \alpha k \times r \qquad a_t = r\alpha$

$a_n = -\omega^2 r \qquad a_n = r\omega^2$

Equation Defining the Rotation of a Rigid Body about a Fixed Axis:

$\omega = d\theta/dt$

$\alpha = d\omega/dt = d^2\theta/dt^2$

$\alpha = \omega\, d\omega/d\theta$

Uniform Rotation:

$\theta = \theta_o + \omega t$

ω = const.

$\alpha = 0$

Uniform Accelerated Rotation:

$\omega = \omega_o + \alpha t$

$\theta = \theta_o + \omega_o t + ½ \alpha t^2$

$\omega^2 = \omega^2_o + 2\alpha(\theta - \theta_o)$

<u>EX:</u> **The motion of a cam is defined by the relation $\theta = t^3 - 2t^2 - 4t + 10$ (θ is in radians and t in seconds). Determine the angular coordinates, the angular velocity and the angular acceleration of the cam when (a) t = 0 , (b) t = 3sec.**

$\theta = t^3 - 2t^2 - 4t + 10$

$w = d\theta/dt = 3t^2 - 4t - 4$

$\propto = dw/dt = 6t - 4$

(a) t = 0

$\theta = 10$ rad

$w = -4$ rad/sec.

$\propto = -4$ rad/sec^2

(b) t = 3 sec.

$\theta = (3)^3 - 2(3)^2 - 4(3) + 10$

$\theta = 7$ rad

$w = 3(3)^2 - 4(3) - 4$

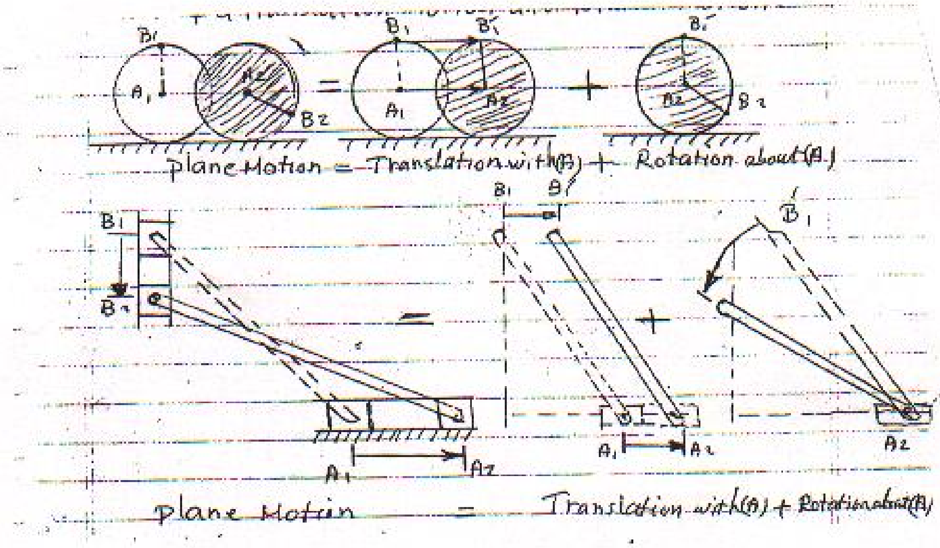
$= 11$ rad/sec.

$\propto = 6(3) - 4$

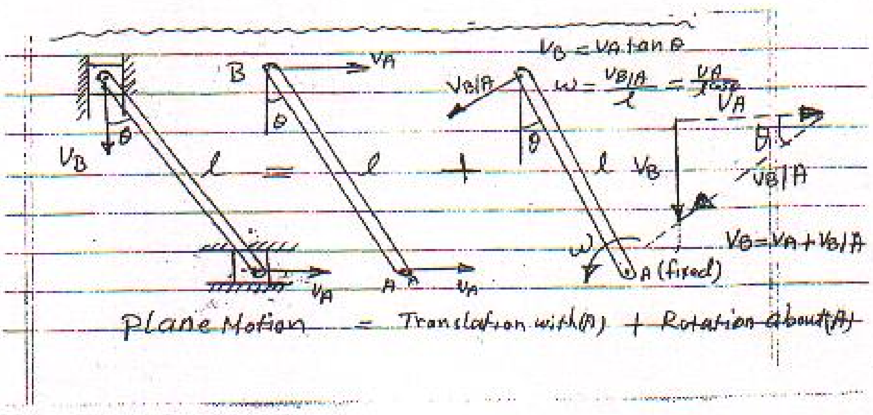
$= 14$ rad/sec^2

General Plane Motion:

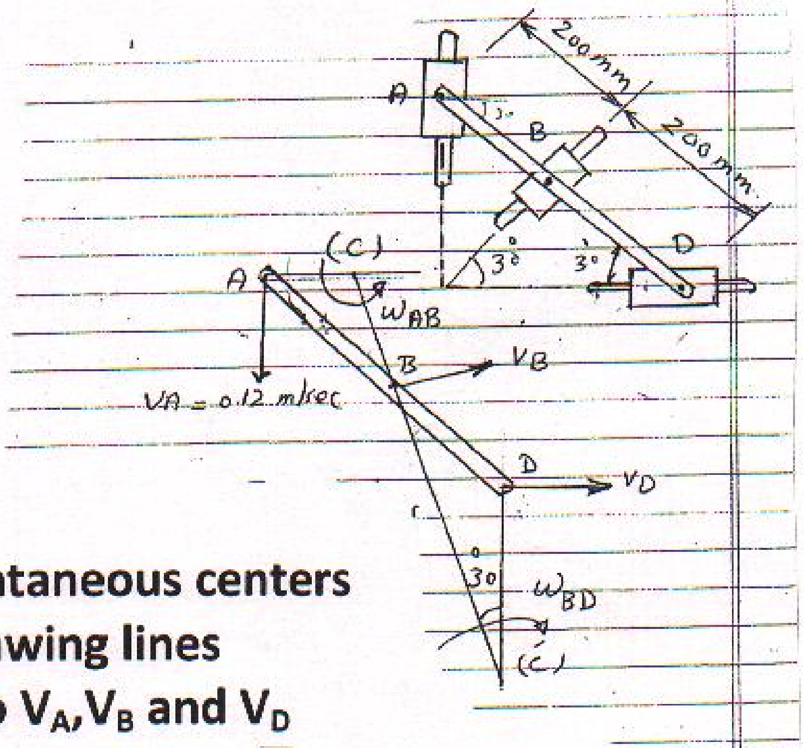
A general plane motion may always be considered as the sum of a translation motion and rotation motion.



Absolute and Relative velocity in Plane Motion:



<u>EX:</u> Two rods (AB) and (BD) are connecting to three collars as shown. Knowing that collar (A) moves downward with a constant vel. of 120 mm/sec., determine at the instant shown <u>(a)</u> the angular vel. of rods (AB) and (BD), <u>(b)</u> the vel. of collar (D).



>we locate instantaneous centers (c) and (c`) by drawing lines perpendiculars to V_A,V_B and V_D

$Ac = Bc = 0.1 / \cos 30 = 0.2 / \sqrt{3}$ m

$Dc` = 0.2$ m

$Bc` = 2(0.2)\cos 30 = 0.2\sqrt{3}$ m

(a) $w_{AB} = V_A/V_C = 0.12/0.2\sqrt{3} = 1.039$ rad/sec

$V_B = (B_C)(w_{AB}) = B_C(V_A/A_C) \rightarrow V_B = V_A = 0.12$m/sec.

$W_{BD} = V_B/B_{C`} = 0.12/0.2\sqrt{3} = 0.346$ rad/sec

(b) $V_D = (D_{C`})(w_{BD}) = (0.2)(0.346) = 69.3$ mm/sec

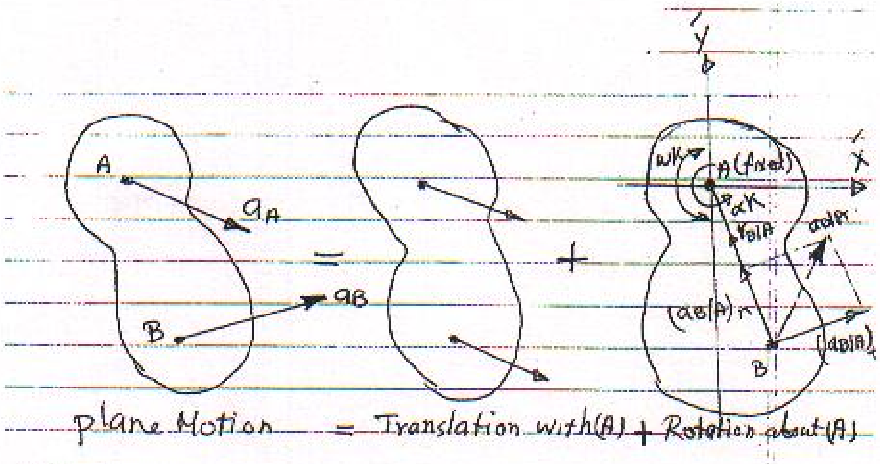
Absolute and Relative Acceleration in Plane Motion:

$a_B = a_A + a_B/a_A$

$a_{B/A}$: is associated with rotation of the body about (A) and is measured with respect to axes central at (A) and of fixed orientation.

$a_{B/A}$: may be resolved into two components, a tangential component $(a_{B/A})_t \perp$ to the line (AB), and a normal component $(a_{B/A})_n$ directed toward (A)

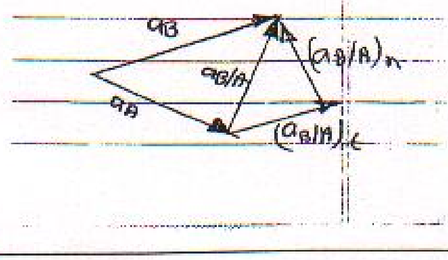
$r_{B/A}$: the position vector of B relative to (A)



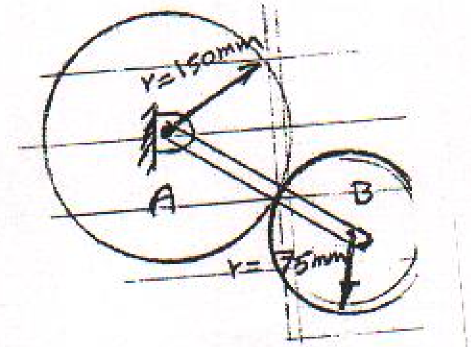
Plane Motion = Translation with(A) + Rotation about(A)

$a_B = a_A + a_{B/A}$

$a_B = a_A + \alpha r - w^2 r$



EX: Gear (A) rotates clockwise with a constant angular velocity of 60 rpm. Knowing that at the same time the arm (AB) rotates counter-clockwise with a constant angular velocity of 30 rpm, determine the angular velocity of gear (B)

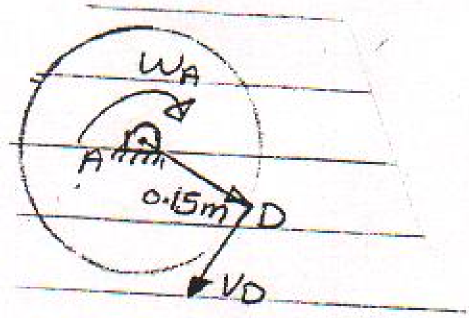


Gear (A)

w_A = 60 rpm = 6.28 rad/sec.

$V_D = rw = (0.15)(6.28)$

$V_D = 0.9425$

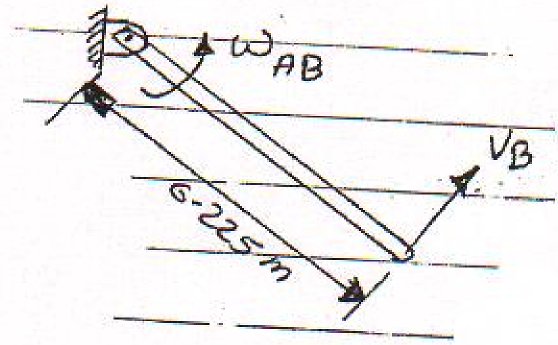


Arm (AB)

wAB = 30 rpm = 3.14 rad/sec.

$V_B = rw_{AB} = (0.225)(3.14)$

V_B = 0.7069 m/sec.



Gear (B)

$V_D = V_B + V_{D/B}$

$0.9425 = 0.7069 + 0.075w_B$

w_B = 22 rad/sec

= 210 rpm