Mechanical Engineering Design (Design I)

Content:

1. Simple stresses and material selection.
2. Combined stresses.
3. Dynamic loading design.
4. Shafts and Forces on belts and Gears.
5. Keys, splines & couplings.
6. Rolling bearings selection.
7. Clutches and brakes.
8. Power screws, bolts and welding.
9. Pressure vessels.
10. Gears and belts

References:

1. Machine Elements in Mechanical Design by Robert L. Mott, P.E.
2. Mechanical Engineering Design by Shigley.
3. Machine Elements by Gustav Niemann.
4. Machine Design by Gupta.
5. Machine Design by Black and Adams.

LECTURE ONE & TWO

The Nature of Mechanical Design and Materials in Mechanical Design

(Note: See chapter 1 & 2 of Reference No.1)

There are many fields where mechanical products are designed and produced. For examples see figures (1-1 to 1-10).

FIGURE 1–1 Drill-powered band saw [Courtesy of Black & Decker (U.S.) Inc.]

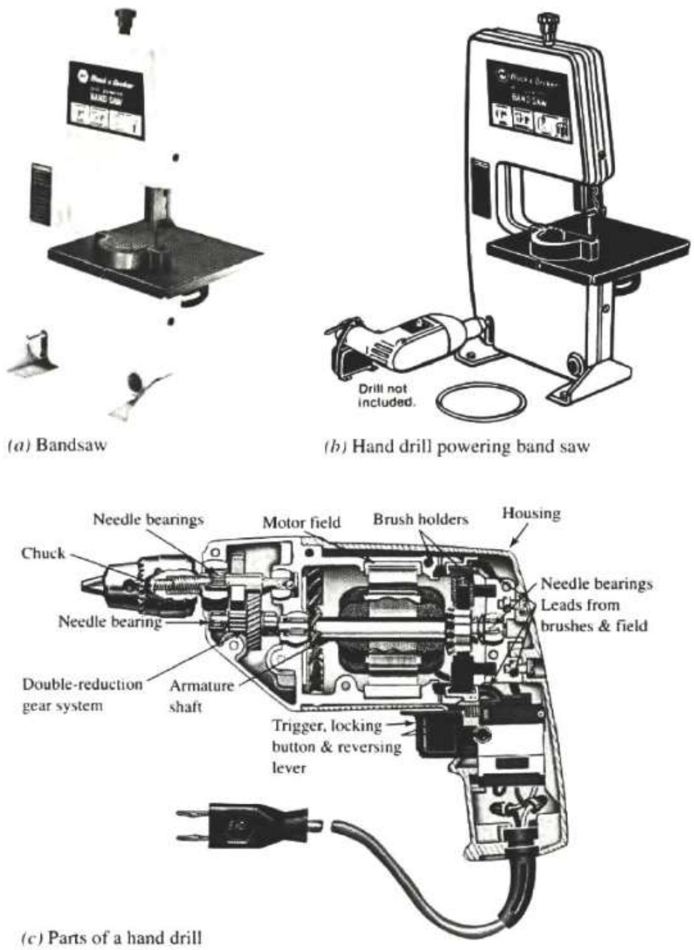
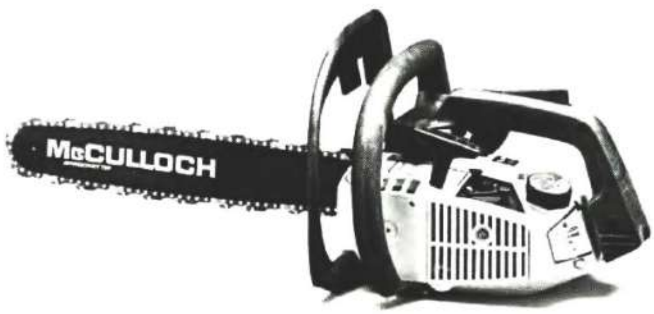
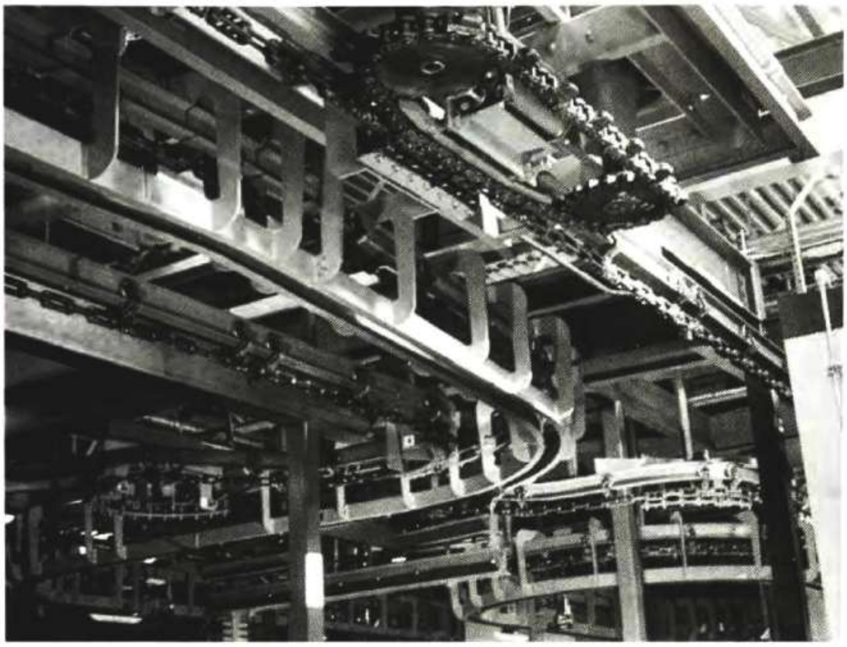


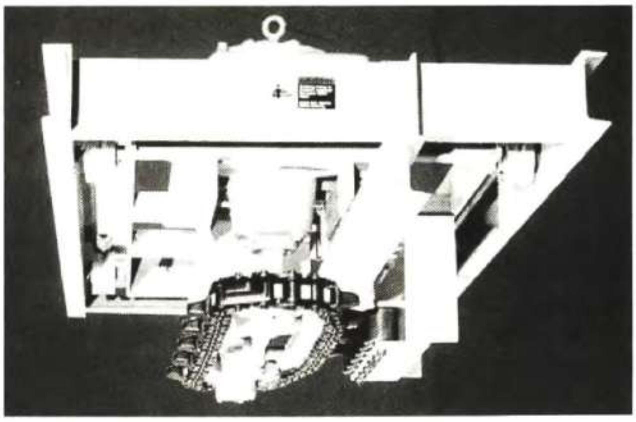
FIGURE 1–2 Chain saw (Copyright McCulloch Corporation, Los Angeles, CA)





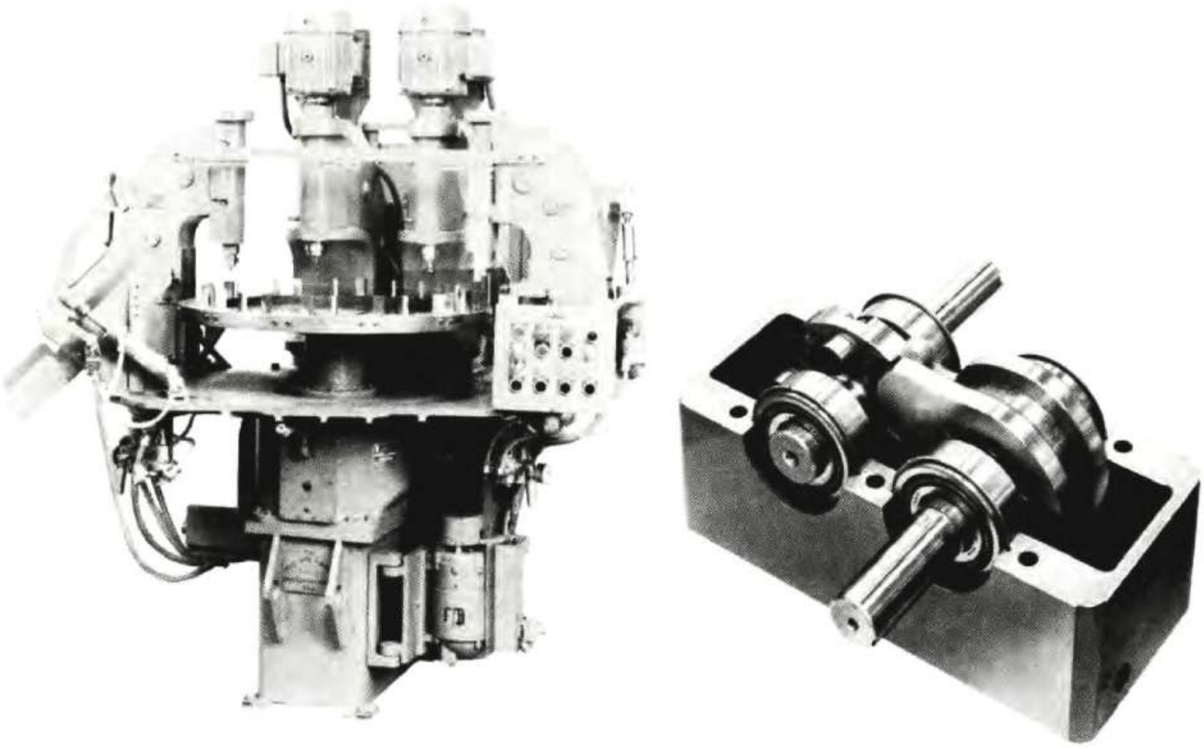
(a) Chain conveyor installation showing the drive system engaging the chain

(b) Chain and roller system supported on an I-beam



(c) Detail of the drive system and its structure

FIGURE 1–3 Chain conveyor system (Richards-Wilcox, Inc., Aurora, IL)



(a) Automatic assembly machine with indexing table

(b) Indexing drive mechanism

FIGURE 1–4 Machinery to automatically assemble automotive components (Industrial Motion Control, LLC, Wheeling, IL)

FIGURE 1–5 Industrial crane (Air Technical Industries, Mentor, OH)

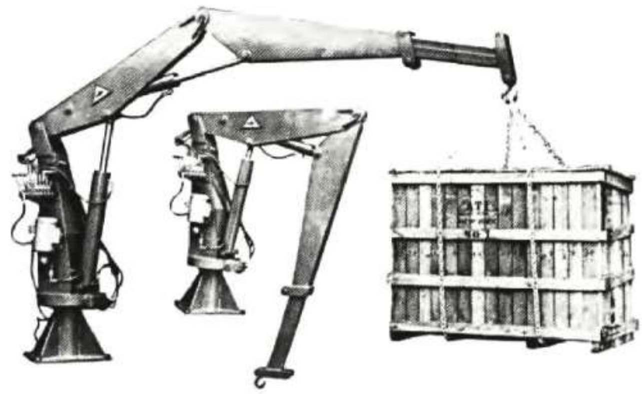
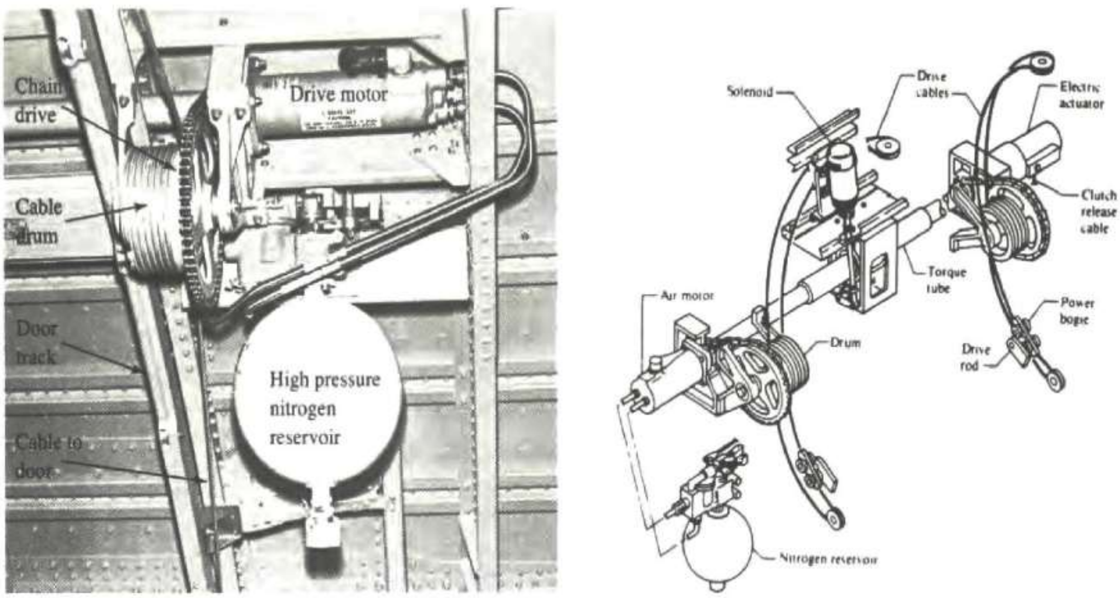


FIGURE 1–6 Tractor with a front-end-loader attachment (Case IH, Racine, WI)

FIGURE 1–7 Tractor pulling an implement (Case IH, Racine, WI)

FIGURE 1–8 Cutaway of a tractor (Case IH, Racine, WI)



(a) Photograph of installed mechanism *(b)* Cabin door drive mechanism

FIGURE 1–9 Aircraft door drive mechanism (The Boeing Company, Seattle, WA)

FIGURE 1–10
Aircraft landing gear assembly (The Boeing Company, Seattle, WA)

Machine elements must be compatible, must fit well together and must perform safely and efficiently. Figure (1-12) shows and example for the primary elements of the speed reducer, which consists: Gear, Shafts, Bearings, Keys, Housing and you can add many other items.

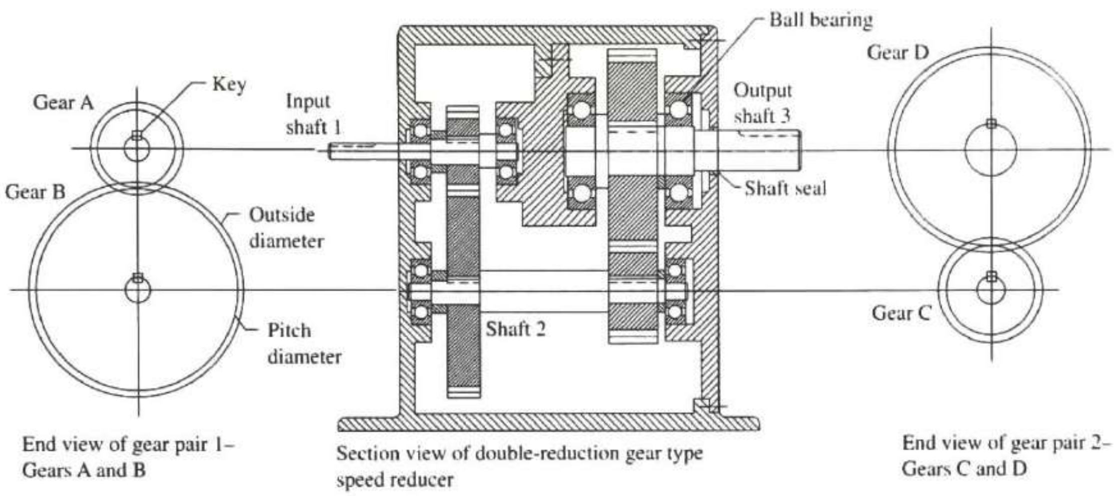
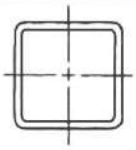
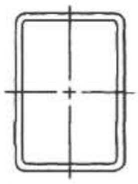


FIGURE 1–12 Conceptual design for a speed reducer

Steel structural shapes

Table (1-1 Page 21) shows standard angle (L-shapes) channels (C-shapes), etc. See Appendix 16; page (A-31 to A-38).

TABLE 1–1 Designations for steel and aluminum shapes

Name of shape	Shape	Symbol	Example designation and Appendix table
Angle		L	$L4 \times 3 \times \frac{1}{2}$ Table A16–1
Channel		C	$C15 \times 50$ Table A16–2
Wide-flange beam		W	$W14 \times 43$ Table A16–3
American Standard beam		S	$S10 \times 35$ Table A16–4
Structural tubing—square			$4 \times 4 \times \frac{1}{4}$ Table A16–5
Structural tubing—rectangular			$6 \times 4 \times \frac{1}{4}$ Table A16–5
Pipe			4-inch standard weight 4-inch Schedule 40 Table A16–6
Aluminum Association channel		C	$C4 \times 1.738$ Table A17–1
Aluminum Association I-beam		I	$I8 \times 6.181$ Table A17–2

Unit systems:

Table (A18-1) and (18-2) page A-39 shows conversation of U.S customary units to SI units.

Note: you should work with SI unit always.

APPENDIX 18 CONVERSION FACTORS

TABLE A18–1 Conversion of U.S. Customary units to SI units: basic quantities

Quantity	U.S. Customary unit		SI unit		Symbol	Equivalent units
Length	1 foot (ft)	=	0.3048	meter	m	
Mass	1 slug	=	14.59	kilogram	kg	
Time	1 second	=	1.0	second	s	
Force	1 pound (lb)	=	4.448	newton	N	kg·m/s²
Pressure	1 lb/in²	=	6895	pascal	Pa	N/m² or kg/m·s²
Energy	1 ft-lb	=	1.356	joule	J	N·m or kg·m²/s²
Power	1 ft-lb/s	=	1.356	watt	W	J/s

TABLE A18–2 Other convenient conversion factors

Quantity				
Length				
	1 ft	=	0.3048	m
	1 in	=	25.4	mm
	1 mi	=	5280	ft
	1 mi	=	1.609	km
	1 km	=	1000	m
	1 cm	=	10	mm
	1 m	=	1000	mm
Area				
	1 ft²	=	0.0929	m²
	1 in²	=	645.2	mm²
	1 m²	=	10.76	ft²
	1 m²	=	10^6	mm²
Volume				
	1 ft³	=	7.48	gal
	1 ft³	=	1728	in³
	1 ft³	=	0.0283	m³
	1 gal	=	0.00379	m³
	1 gal	=	3.785	L
	1 m³	=	1000	L
Volume flow rate				
	1 ft³/s	=	449	gal/min
	1 ft³/s	=	0.0283	m³/s
	1 gal/min	=	6.309×10^{-5}	m³/s
	1 gal/min	=	3.785	L/min
	1 L/min	=	16.67×10^{-6}	m³/s
Stress, pressure, or unit loading				
	1 lb/in²	=	6.895	kPa
	1 lb/ft²	=	0.0479	kPa
	1 kip/in²	=	6.895	MPa
Section modulus				
	1 in³	=	1.639×10^4	mm³
Moment of inertia				
	1 in⁴	=	4.162×10^5	mm⁴
Density				
	1 slug/ft³	=	515.4	kg/m³
Specific weight				
	1 lb/ft³	=	157.1	N/m³
Energy				
	1 ft·lb	=	1.356	J
	1 Btu	=	1.055	kJ
	1 W·h	=	3.600	kJ
Torque or moment				
	1 lb·in	=	0.1130	N·m
Power				
	1 hp	=	550	ft·lb/s
	1 hp	=	745.7	W
	1 ft·lb/s	=	1.356	W
	1 Btu/h	=	0.293	W
Temperature				
	T(°C)	=	$[T(°F) - 32]5/9$	
	T(°F)	=	$\frac{9}{5}[T(°C)] + 32$	

Example 1 (page 26), Ref.1:

Express the diameter of a shaft in (mm) if it is measured to be 2.755 in.

Sol:

Table A18 gives the conversation factor for length to be (1 in = 25.4 mm) then

$$Diameter = 2.755\ in * \frac{25.4\ mm}{1\ in} = 69.98\ mm$$

Note: see also example 1-2 page 26 (Solve problems on page 28 Prob. 15-16-17-18-19-20-22-23-25-26-28)

Materials in Mechanical Design:

- **Tensile Strength, S_u**

The peak of the stress-strain curve is considered the ultimate tensile strength (S_U) sometimes called the ultimate strength or simply the tensile strength. At this point during the test, the highest apparent stress on a test bar of the material is measured. As shown in Figures 2-1 and 2-2. The curve appears to drop off after the peak. However, notice that the instrumentation used to create the diagrams is actually plotting load versus deflection rather than true stress versus strain. The apparent stress is computed by dividing the load by the original cross-sectional area of the test bar. After the peak of the curve is reached, there is a pro-nounced decrease in the bar's diameter, referred to as necking down. Thus, the load acts over a smaller area, and the actual stress continues to increase until failure. It is very difficult to follow the reduction in diameter during the necking-down process, so it has become cus-tomary to use the peak of the curve as the tensile strength, although it is a more conservative value.

- **Yield Strength, S_Y**

That portion of the stress-strain diagram where there is a large increase in strain with little or no increase in stress is called the yield strength (**S_Y**). This property indicates that the material has, in fact, yielded or elongated plastically, permanently, and to a large degree. If the point of yielding is quite noticeable, as it is in Figure 2-1, the property is called the yield point rather than the yield strength. This is typical of plain carbon hot rolled steel. Figure 2-2 shows the stress-strain diagram form that is typical of a nonferrous metal such as aluminum or titanium or of certain high-strength steels. Notice that there is no pro-nounced yield point, but the material has actually yielded at or near the stress level indicated as S_Y. That point is determined by the offset method, in which a line is drawn parallel to the straight-line portion of the curve and is offset to the right by a set amount, usually 0.20% strain (0.002 in/in). The intersection of this line and the stress-strain curve defines the material's yield strength. In this book, the term yield strength will be used for S_Y regardless of whether the material exhibits a true yield point or whether the offset method is used.

FIGURE 2–1 Typical stress-strain diagram for steel

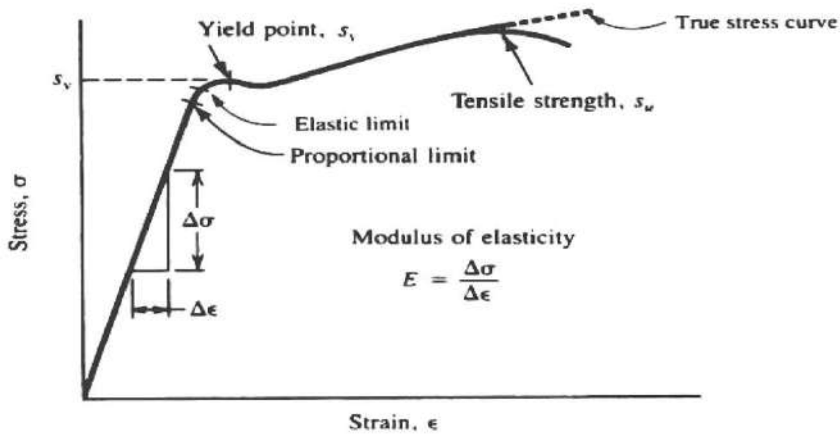
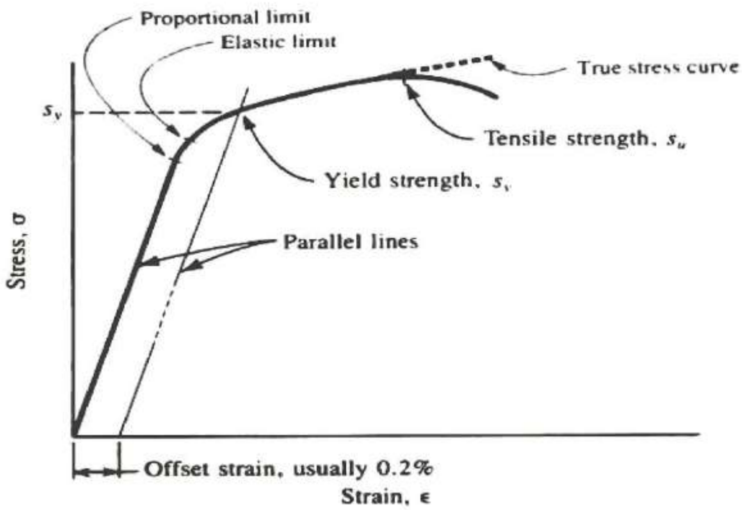


FIGURE 2–2 Typical stress-strain diagram for aluminum and other metals having no yield point



Where: S_u = ultimate tensile strength

S_Y = yield strength

E = modulus of Elasticity = $\frac{\sigma}{\epsilon}$

σ = stress

ϵ = strain

For shear strength

S_{YS} = yield strength in shear = $S_Y/2 = 0.5\ S_Y$

S_{US} = ultimate strength in shear = $0.75\ S_U$

Classification of metals and alloys

UNS : The Unified Numbering systems.

ASTM : The American Society for Testing and Materials

AA : The Aluminum Association.

AISI : The American Iron and Steel Institute.

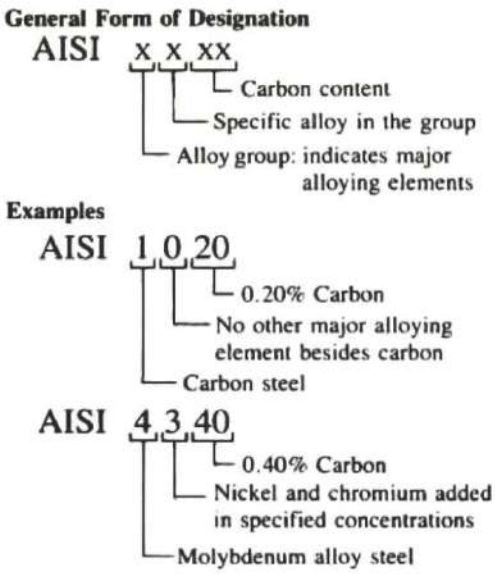
CDA : The Copper Development Association.

SAE : The Society of Automotive Engineers.

Steel Designation System:

Section 2–5 ■ Carbon and Alloy Steel

FIGURE 2–11 Steel designation system



Importance of Carbon:

As carbon content increase, strength and hardness also increased under the same conditions of processing and heat treatment. Since the ductility decreases with the increasing of carbon content, selecting suitable steel involves some compromise between strength and ductility.

As a rough classification scheme:

Low-carbon steel ≅ 0.3% carbon → (low strength + good formability)
Medium-carbon steel ≅ (0.3% - 0.5%) → (Moderate to high strength + fairly good ductility)
High-carbon steel ≅ (0.5% - 0.95%) → (suitable for durable cutting edges)

Properties of carburized steel, stainless steel, structural steel…

APPENDIX 3 DESIGN PROPERTIES OF CARBON AND ALLOY STEELS

Material designation (AISI number)	Condition	Tensile strength (ksi)	Tensile strength (MPa)	Yield strength (ksi)	Yield strength (MPa)	Ductility (percent elongation in 2 inches)	Brinell hardness (HB)
1020	Hot-rolled	55	379	30	207	25	111
1020	Cold-drawn	61	420	51	352	15	122
1020	Annealed	60	414	43	296	38	121
1040	Hot-rolled	72	496	42	290	18	144
1040	Cold-drawn	80	552	71	490	12	160
1040	OQT 1300	88	607	61	421	33	183
1040	OQT 400	113	779	87	600	19	262
1050	Hot-rolled	90	620	49	338	15	180
1050	Cold-drawn	100	690	84	579	10	200
1050	OQT 1300	96	662	61	421	30	192
1050	OQT 400	143	986	110	758	10	321
1117	Hot-rolled	62	427	34	234	33	124
1117	Cold-drawn	69	476	51	352	20	138
1117	WQT 350	89	614	50	345	22	178
1137	Hot-rolled	88	607	48	331	15	176
1137	Cold-drawn	98	676	82	565	10	196
1137	OQT 1300	87	600	60	414	28	174
1137	OQT 400	157	1083	136	938	5	352
1144	Hot-rolled	94	648	51	352	15	188
1144	Cold-drawn	100	690	90	621	10	200
1144	OQT 1300	96	662	68	469	25	200
1144	OQT 400	127	876	91	627	16	277
1213	Hot-rolled	55	379	33	228	25	110
1213	Cold-drawn	75	517	58	340	10	150
12L13	Hot-rolled	57	393	34	234	22	114
12L13	Cold-drawn	70	483	60	414	10	140
1340	Annealed	102	703	63	434	26	207
1340	OQT 1300	100	690	75	517	25	235
1340	OQT 1000	144	993	132	910	17	363
1340	OQT 700	221	1520	197	1360	10	444
1340	OQT 400	285	1960	234	1610	8	578
3140	Annealed	95	655	67	462	25	187
3140	OQT 1300	115	792	94	648	23	233
3140	OQT 1000	152	1050	133	920	17	311
3140	OQT 700	220	1520	200	1380	13	461
3140	OQT 400	280	1930	248	1710	11	555
4130	Annealed	81	558	52	359	28	156
4130	WQT 1300	98	676	89	614	28	202
4130	WQT 1000	143	986	132	910	16	302
4130	WQT 700	208	1430	180	1240	13	415
4130	WQT 400	234	1610	197	1360	12	461
4140	Annealed	95	655	60	414	26	197
4140	OQT 1300	117	807	100	690	23	235
4140	OQT 1000	168	1160	152	1050	17	341
4140	OQT 700	231	1590	212	1460	13	461
4140	OQT 400	290	2000	251	1730	11	578

APPENDIX 5 PROPERTIES OF CARBURIZED STEELS

Material designation (AISI number)	Condition	Core properties: Tensile strength (ksi)	Tensile strength (MPa)	Yield strength (ksi)	Yield strength (MPa)	Ductility (percent elongation in 2 inches)	Brinell hardness (HB)	Case hardness (HRC)
1015	SWQT 350	106	731	60	414	15	217	62
1020	SWQT 350	129	889	72	496	11	255	62
1022	SWQT 350	135	931	75	517	14	262	62
1117	SWQT 350	125	862	66	455	10	235	65
1118	SWQT 350	144	993	90	621	13	285	61
4118	SOQT 300	143	986	93	641	17	293	62
4118	DOQT 300	126	869	63	434	21	241	62
4118	SOQT 450	138	952	89	614	17	277	56
4118	DOQT 450	120	827	63	434	22	229	56
4320	SOQT 300	218	1500	178	1230	13	429	62
4320	DOQT 300	151	1040	97	669	19	302	62
4320	SOQT 450	211	1450	173	1190	12	415	59
4320	DOQT 450	145	1000	94	648	21	293	59
4620	SOQT 300	119	820	83	572	19	277	62
4620	DOQT 300	122	841	77	531	22	248	62
4620	SOQT 450	115	793	80	552	20	248	59
4620	DOQT 450	115	793	77	531	22	235	59
4820	SOQT 300	207	1430	167	1150	13	415	61
4820	DOQT 300	204	1405	165	1140	13	415	60
4820	SOQT 450	205	1410	184	1270	13	415	57
4820	DOQT 450	196	1350	171	1180	13	401	56
8620	SOQT 300	188	1300	149	1030	11	388	64
8620	DOQT 300	133	917	83	572	20	269	64
8620	SOQT 450	167	1150	120	827	14	341	61
8620	DOQT 450	130	896	77	531	22	262	61
E9310	SOQT 300	173	1190	135	931	15	363	62
E9310	DOQT 300	174	1200	139	958	15	363	60
E9310	SOQT 450	168	1160	137	945	15	341	59
E9310	DOQT 450	169	1170	138	952	15	352	58

Notes: Properties given are for a single set of tests on 1/2-in round bars.

SWQT: single water-quenched and tempered

SOQT: single oil-quenched and tempered

DOQT: double oil-quenched and tempered

300 and 450 are the tempering temperatures in °F. Steel was carburized for 8 h. Case depth ranged from 0.045 to 0.075 in.

APPENDIX 6 PROPERTIES OF STAINLESS STEELS

Material designation			Tensile strength		Yield strength		Ductility (percent elongation
AISI number	UNS	Condition	(ksi)	(MPa)	(ksi)	(MPa)	in 2 inches)
Austenitic steels							
201	S20100	Annealed	115	793	55	379	55
		1/4 hard	125	862	75	517	20
		1/2 hard	150	1030	110	758	10
		3/4 hard	175	1210	135	931	5
		Full hard	185	1280	140	966	4
301	S30100	Annealed	110	758	40	276	60
		1/4 hard	125	862	75	517	25
		1/2 hard	150	1030	110	758	15
		3/4 hard	175	1210	135	931	12
		Full hard	185	1280	140	966	8
304	S30400	Annealed	85	586	35	241	60
310	S31000	Annealed	95	655	45	310	45
316	S31600	Annealed	80	552	30	207	60
Ferritic steels							
405	S40500	Annealed	70	483	40	276	30
430	S43000	Annealed	75	517	40	276	30
446	S44600	Annealed	80	552	50	345	25
Martensitic steels							
410	S41000	Annealed	75	517	40	276	30
416	S41600	Q&T 600	180	1240	140	966	15
		Q&T 1000	145	1000	115	793	20
		Q&T 1400	90	621	60	414	30
431	S43100	Q&T 600	195	1344	150	1034	15
440A	S44002	Q&T 600	280	1930	270	1860	3
Precipitation-hardening steels							
17-4PH	S17400	H 900	200	1380	185	1280	14
		H 1150	145	1000	125	862	19
17-7PH	S17700	RH 950	200	1380	175	1210	10
		TH 1050	175	1210	155	1070	12

APPENDIX 7 PROPERTIES OF STRUCTURAL STEELS

Material designation (ASTM number)	Grade, product, or thickness	Tensile strength (ksi)	Tensile strength (MPa)	Yield strength (ksi)	Yield strength (MPa)	Ductility (percent elongation in 2 inches)
A36	$t \le 8$ in	58	400	36	250	21
A242	$t \le 3/4$ in	70	480	50	345	21
A242	$t \le 1\frac{1}{2}$ in	67	460	46	315	21
A242	$t \le 4$ in	63	435	42	290	21
A500	Cold-formed structural tubing, round or shaped					
	Round, Grade A	45	310	33	228	25
	Round, Grade B	58	400	42	290	23
	Round, Grade C	62	427	46	317	21
	Shaped, Grade A	45	310	39	269	25
	Shaped, Grade B	58	400	46	317	23
	Shaped, Grade C	62	427	50	345	21
A501	Hot-formed structural tubing, round or shaped	58	400	36	250	23
A514	Quenched and tempered, $t \le 2\frac{1}{2}$ in	110–130	760–895	100	690	18%
A572	42, $t \le 6$ in	60	415	42	290	24
A572	50, $t \le 4$ in	65	450	50	345	21
A572	60, $t \le 1\frac{1}{4}$ in	75	520	60	415	18
A572	65, $t \le 1\frac{1}{4}$ in	80	550	65	450	17
A588	$t \le 4$ in	70	485	50	345	21
A992	W-shapes	65	450	50	345	21

Note: ASTM A572 is one of the high-strength, low-alloy steels (HSLA) and has properties similar to those of the SAE J410b steel specified by the SAE.

APPENDIX 8 DESIGN PROPERTIES OF CAST IRON

Material designation (ASTM number)	Grade	Tensile strength (ksi)	Tensile strength (MPa)	Yield strength (ksi)	Yield strength (MPa)	Ductility (percent elongation in 2 inches)	Modulus of elasticity (10^6 psi)	Modulus of elasticity (GPa)
Gray iron								
A48-94a	20	20	138			<1	12	83
	25	25	172			<1	13	90
	30	30	207			<1	15	103
	40	40	276			<1	17	117
	50	50	345			<1	19	131
	60	60	414			<1	20	138
Malleable iron								
A47-99	32510	50	345	32	221	10	25	172
	35018	53	365	35	241	18	25	172
A220-99	40010	60	414	40	276	10	26	179
	45006	65	448	45	310	6	26	179
	50005	70	483	50	345	5	26	179
	70003	85	586	70	483	3	26	179
	90001	105	724	90	621	1	26	179
Ductile iron								
A536-84	60-40-18	60	414	40	276	18	22	152
	80-55-06	80	552	55	379	6	22	152
	100-70-03	100	689	70	483	3	22	152
	120-90-02	120	827	90	621	2	22	152
Austempered ductile iron								
ASTM 897-90	1	125	850	80	550	10	22	152
	2	150	1050	100	700	7	22	152
	3	175	1200	125	850	4	22	152
	4	200	1400	155	1100	1	22	152
	5	230	1600	185	1300	<1	22	152

Notes: Strength values are typical. Casting variables and section size affect final values. Modulus of elasticity may also vary. Density of cast irons ranges from 0.25 to 0.27 lb/in^3 (6920 to 7480 kg/m^3). Compressive strength ranges 3 to 5 times higher than tensile strength.

APPENDIX 9 TYPICAL PROPERTIES OF ALUMINUM

Alloy and temper	Tensile strength		Yield strength		Ductility (percent elongation in 2 inches)	Shearing strength		Endurance strength	
	(ksi)	(MPa)	(ksi)	(MPa)		(ksi)	(MPa)	(ksi)	(MPa)
1060-O	10	69	4	28	43	7	48	3	21
1060-H14	14	97	11	76	12	9	62	5	34
1060-H18	19	131	18	124	6	11	121	6	41
1350-O	12	83	4	28	28	8	55		
1350-H14	16	110	14	97		10	69		
1350-H19	27	186	24	165		15	103	7	48
2014-O	27	186	14	97	18	18	124	13	90
2014-T4	62	427	42	290	20	38	262	20	138
2014-T6	70	483	60	414	13	42	290	18	124
2024-O	27	186	11	76	22	18	124	13	90
2024-T4	68	469	47	324	19	41	283	20	138
2024-T361	72	496	57	393	12	42	290	18	124
2219-O	25	172	11	76	18				
2219-T62	60	414	42	290	10			15	103
2219-T87	69	476	57	393	10			15	103
3003-O	16	110	6	41	40	11	121	7	48
3003-H14	22	152	21	145	16	14	97	9	62
3003-H18	29	200	27	186	10	16	110	10	69
5052-O	28	193	13	90	30	18	124	16	110
5052-H34	38	262	31	214	14	21	145	18	124
5052-H38	42	290	37	255	8	24	165	20	138
6061-O	18	124	8	55	30	12	83	9	62
6061-T4	35	241	21	145	25	24	165	14	97
6061-T6	45	310	40	276	17	30	207	14	97
6063-O	13	90	7	48		10	69	8	55
6063-T4	25	172	13	90	22				
6063-T6	35	241	31	214	12	22	152	10	69
7001-O	37	255	22	152	14				
7001-T6	98	676	91	627	9			22	152
7075-O	33	228	15	103	16	22	152		
7075-T6	83	572	73	503	11	48	331	23	159

Note: Common properties:
Density: 0.095 to 0.102 lb/in^3 (2635 to 2829 kg/m^3)
Modulus of elasticity: 10 to 10.6 $\times$ 10^6 psi (69 to 73 GPa)
Endurance strength at 5 $\times$ 10^8 cycles

Problems:

Solve problems No. 20, 21, 22, 23, 25, and 26.

LECTURE THREE. FOUR &FIVE

STRESS AND DEFORMATION ANALYSIS

1- Direct stresses, tension and compression:

Tensile and compressive stresses, called *normal stresses,* are shown acting perpendicular to opposite faces of the stress element. Tensile stresses tend to pull on the element, whereas compressive stresses tend to crush it.

$$\sigma_d = \frac{P}{A}$$

$$\delta = \frac{FL}{AE}$$

Where: σ_d= Design tensile or compression stress
F= direct axial load
A= cross sectional area
δ= total deformation
L=original total length
E= Modulus of elasticity

$$\sigma_d = \frac{Strength\ of\ material\ from\ which\ the\ component\ made}{Design\ factor\ (or\ safety\ factor)(N)}$$

For ductile material:

N = 1.25 – 2 (for static loads)
N = 2 – 2.5 (for dynamic loads with average confidence)
N = 2.5 – 4 (for dynamic loads with uncertainly about load)
N = 4 or higher (for shock load or to desire extra safety)

For brittle material:

N = 3 – 4 (for static load with high level of confidence)
N = 4 – 8 (for dynamic loading with uncertainly about load)

$\sigma_d = \frac{S_{ut}}{N}$ (For tensile stress)
$\sigma_d = \frac{S_{uc}}{N}$ (For compression stress)

} Based on Ultimate tensile strength

OR

$\sigma_d = \frac{S_{yt}}{N}$ (For tensile stress)
$\sigma_d = \frac{S_{yc}}{N}$ (For compression stress)

} Based on Ultimate yield strength

Example:

A large electrical transformer is to be suspended from a roof truss of a building. The total weight of transformer is 142.33 KN. Design the means of support.

Selection:

Choose N=3
Choose material AISI 1040 cold-drawn steel
Choose two straight cylindrical rod to support the transformer
Sol:

From APPENDIX 3, S_Y = 489.54 MPa

$$\sigma_d = \frac{S_y}{N} = \frac{489.54}{3} = 163.18\, MPa$$

$$\sigma_d = \frac{F}{A} \rightarrow A = \frac{142330\, N}{163.18\, \frac{N}{mm^2}} = 871\, mm^2 = \frac{\pi d^2}{4} \rightarrow d = 33.3\, mm$$

Key:
A *key* is a machinery component placed at the interface between a shaft and the hub of a power-transmitting element for the purpose of transmitting torque (see figure 11-1). It is installed in an axial groove machined into the shaft, called a *keyseat*. Keyseat in shafts are usually machined with either an end mill or circular milling cutter, producing the profile or sled runner key seat.

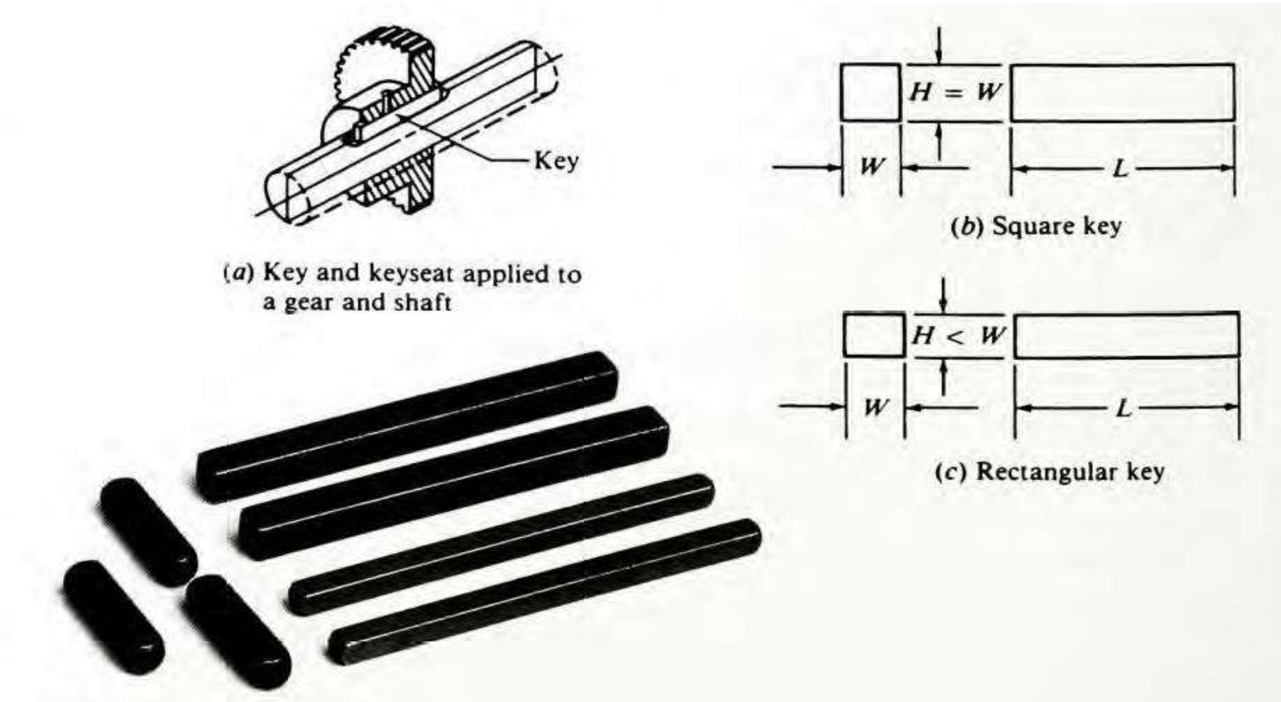
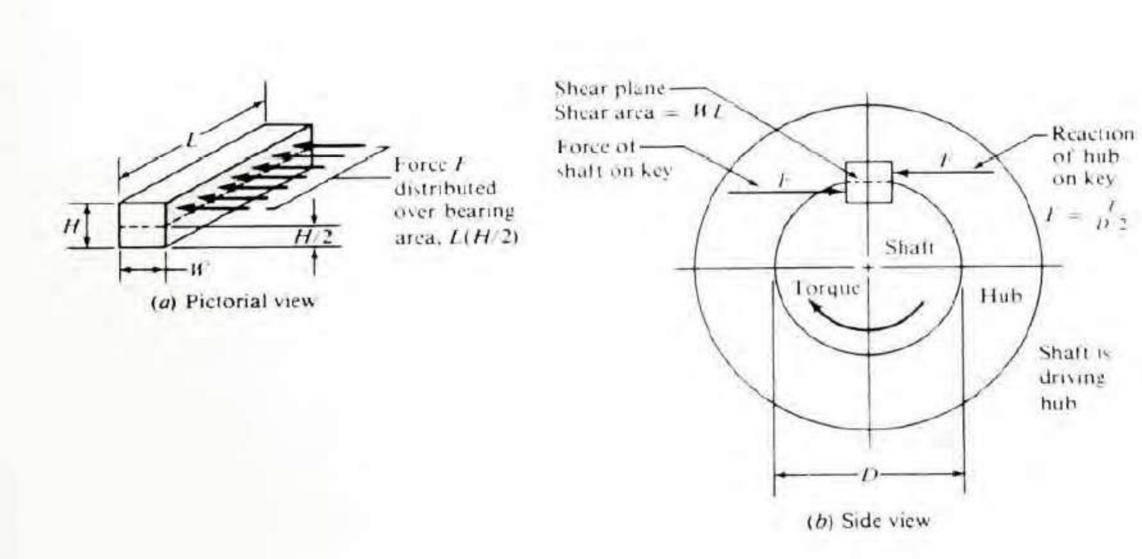


Figure (11-1) parallel keys



2- Direct shear stress:

Direct shear stress occurs when the applied force tends to cut through the member as scissors or shears do or when a punch and a die are used to punch a slug of material from a sheet. Another important example of direct shear in machine design is the tendency for a key to be sheared off at the section between the shaft and the hub of a machine element when transmitting torque.
Figure (3-7) shows the action.

$$\tau_d = \frac{F}{A}$$ Where: τ_d : Design shear stress

Example Problem (3-3), (page 93), [Ref. 1]:

Figure 3-7 shows a shaft carrying two sheaves that are keyed to the shaft. Part (b) shows that a force F is transmitted from the shaft to the hub of the sheave through a square key. The shaft is 57.15 mm in diameter and transmits a torque of 1589.119 N. The key has a square cross section, 12.7 mm on a side, and a length of 44.45 mm. compute the force on the key and the shear stress caused by this force.

Sol:

Torque = F * R → $F = \frac{T}{R} = \frac{1589.12*10^3\ N.mm}{28.575\ mm} = 55600\ N$

Area in shear = b*L = 12.7*44.45 = 564.55 mm^2

$$\tau = \frac{F}{A_s} = \frac{55600\ N}{564.55\ mm^2} = 98598\ kPa \quad (Ans.)$$

Note: see table 11-1 (Page 495) to find key size vs. shaft diameter.

FIGURE 3–7 Direct shear on a key

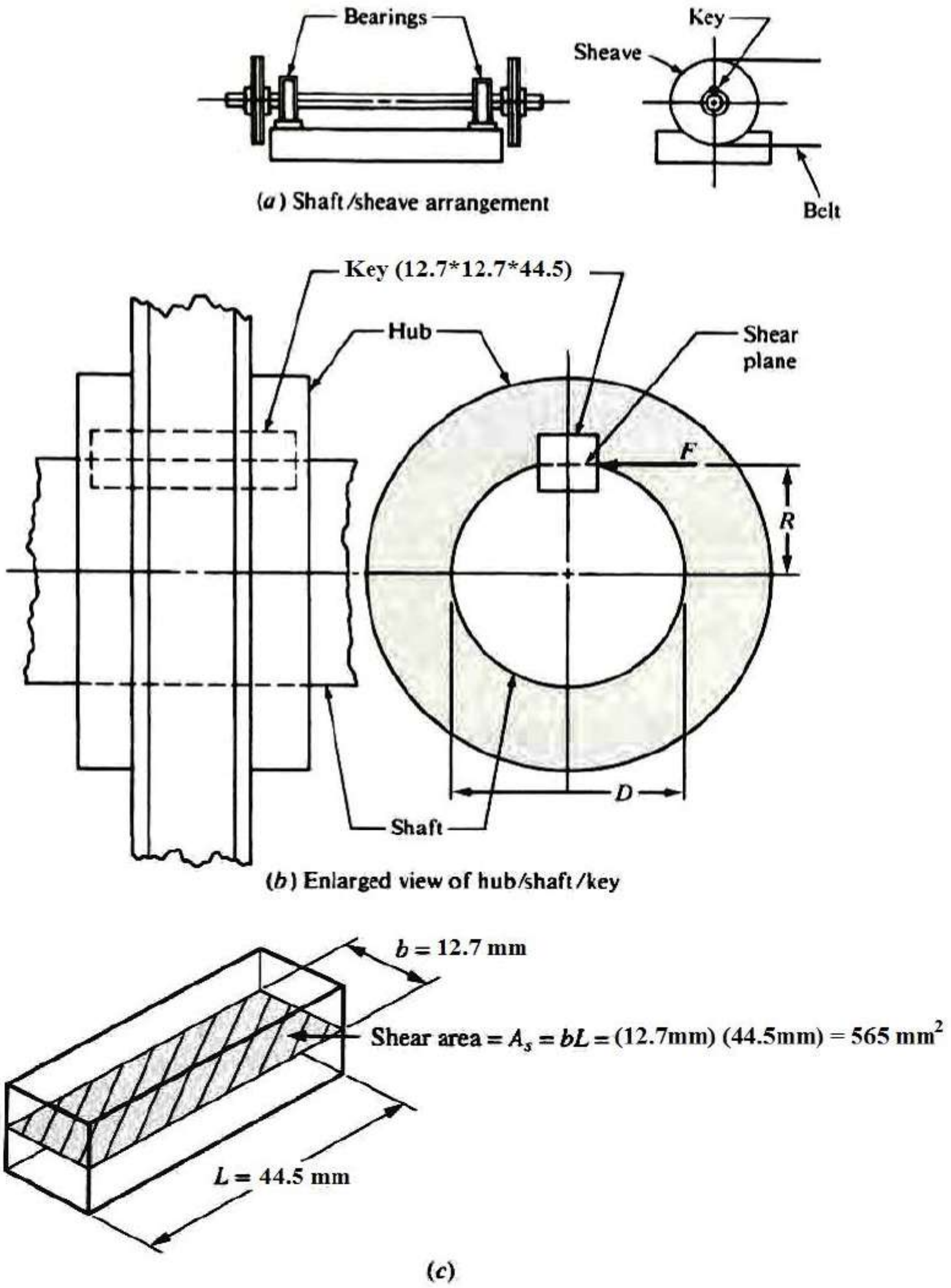


TABLE 11–1 Key size vs. shaft diameter

Nominal shaft diameter		Nominal key size		
			Height, *H*	
Over	To (incl.)	Width, *W*	Square	Rectangular
5/16	7/16	3/32	3/32	
7/16	9/16	1/8	1/8	3/32
9/16	7/8	3/16	3/16	1/8
7/8	1 1/4	1/4	1/4	3/16
1 1/4	1 3/8	5/16	5/16	1/4
1 3/8	1 3/4	3/8	3/8	1/4
1 3/4	2 1/4	1/2	1/2	3/8
2 1/4	2 3/4	5/8	5/8	7/16
2 3/4	3 1/4	3/4	3/4	1/2
3 1/4	3 3/4	7/8	7/8	5/8
3 3/4	4 1/2	1	1	3/4
4 1/2	5 1/2	1 1/4	1 1/4	7/8
5 1/2	6 1/2	1 1/2	1 1/2	1
6 1/2	7 1/2	1 3/4	1 3/4	1 1/2
7 1/2	9	2	2	1 1/2
9	11	2 1/2	2 1/2	1 3/4
11	13	3	3	2
13	15	3 1/2	3 1/2	2 1/2
15	18	4		3
18	22	5		3 1/2
22	26	6		4
26	30	7		5

3- <u>Bearing stress:</u>

A localized compressive stress at the surface of contact between two members of a machine part that is relatively at rest is known as bearing stress or crushing stress or bearing pressure.

<u>Example:</u>

Find the bearing stress caused by force in last example for the key.

<u>Sol:</u>

$$\sigma_{bearing} = \frac{F}{A} = \frac{55600\ N}{\frac{12.7}{2} * 44.45} = 197.2\ MPa$$ (Give your comments for this result)

Note:

$\sigma_{bearing}$ For CI Hub ≤ 50 MPa

$\sigma_{bearing}$ For steel Hub ≤ 90 MPa

} **From Reference 2**

Splines: A *spline* can be described as a series of axial keys machined into a shaft, with corresponding grooves machined into the bore of the mating part (gear, sheave, sprocket, and so on: see Figure 11-6). The splines perform the same function as a key in transmitting torque from the shaft to the mating element.

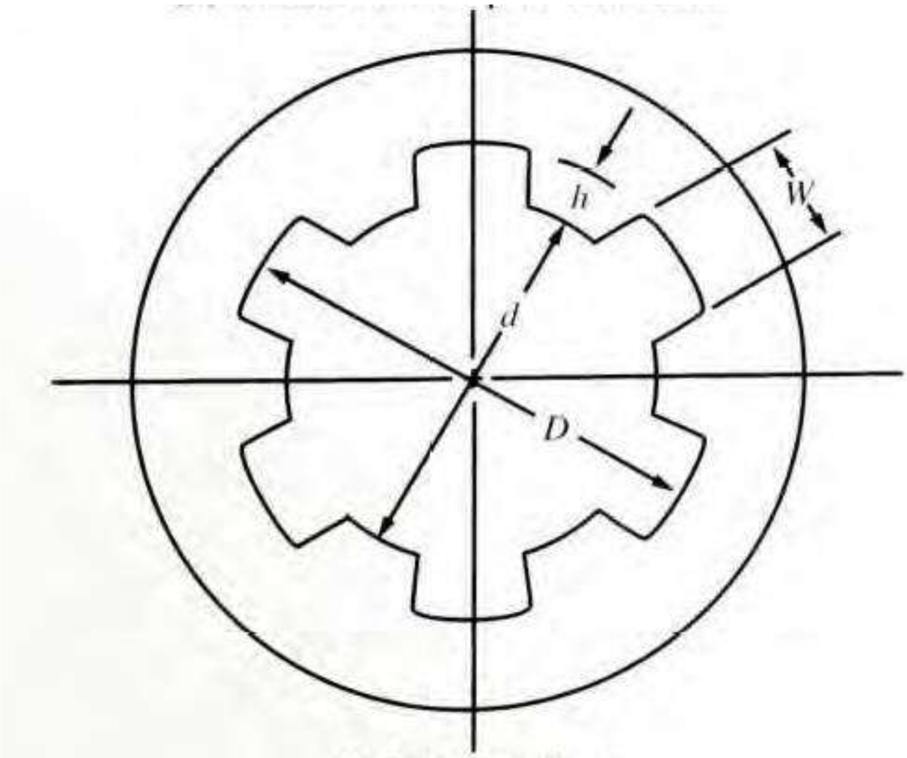


Figure (11-6) internal spline

TABLE 11–4 Formulas for SAE straight splines

No. of splines	W, for all fits	A: Permanent fit		B: To slide without load		C: To slide under load	
		h	*d*	*h*	*d*	*h*	*d*
Four	0.241D	0.075D	0.850D	0.125D	0.750D		
Six	0.250D	0.050D	0.900D	0.075D	0.850D	0.100D	0.800D
Ten	0.156D	0.045D	0.910D	0.070D	0.860D	0.095D	0.810D
Sixteen	0.098D	0.045D	0.910D	0.070D	0.860D	0.095D	0.810D

Note: These formulas give the maximum dimensions for *W*, *h*, and *d*.

Coupling:

The term *coupling* refers to a device used to connect two shafts together at their ends for the purpose of transmitting power. There are two general types of couplings: rigid and flexible.

Rigid Couplings:

Rigid couplings are designed to draw two shafts together tightly so that no relative motion can occur between them. This design is desirable for certain kinds of equipment in which precise alignment of two shafts is required and can be provided. In such cases, the coupling must be designed to be capable of transmitting the torque in the shafts.

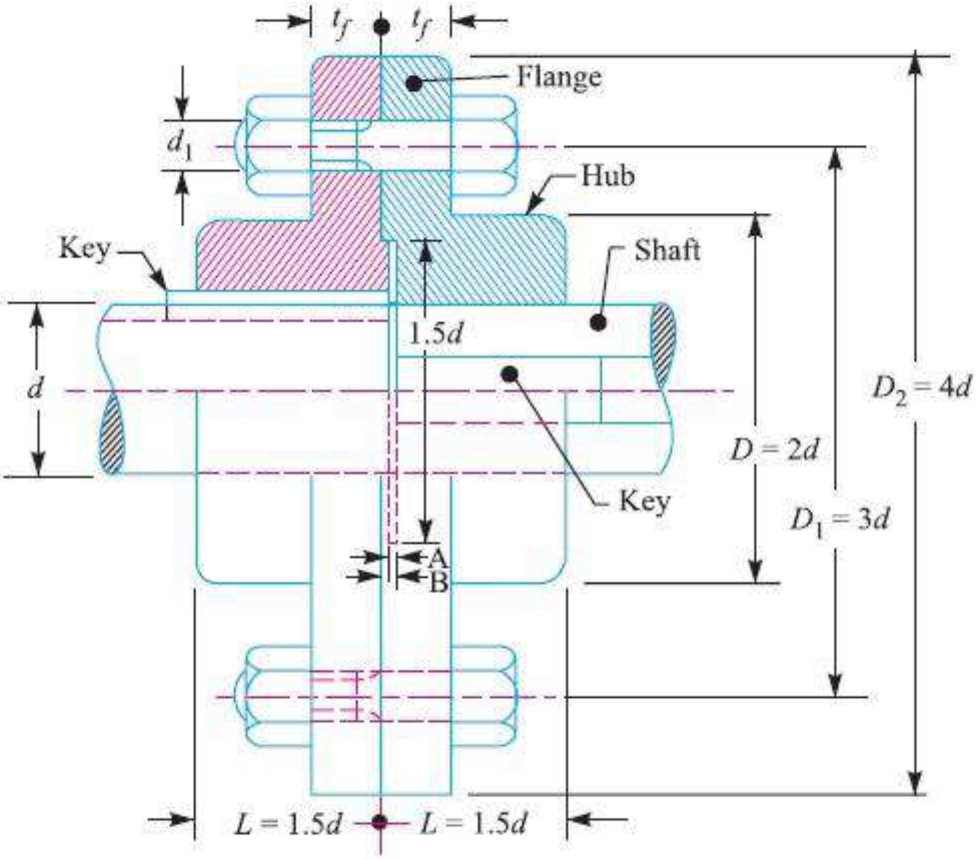


Figure (11-15) Unprotected type flange coupling

The usual proportions for an unprotected type cast iron flange couplings, as shown in figure (11-15) are as follows:

If (**d**) is the diameter of the shaft or inner diameter of the hub, then

Outside diameter of hub, **D=2 d**

Length of hub, **L=1.5 d**

Pitch circle diameter of bolts, $\mathbf{D_1=3\ d}$

Outside diameter of flange, $\mathbf{D_2=D_1+(D_1-D)=2\ D_1-D=4\ d}$

Thickness of flange, $\mathbf{t_f=0.5\ d}$

Number of bolts = 3, for **(d)** up to 40mm

= 4, for **(d)** up to 100mm

= 6, for **(d)** up to 180mm

4- <u>Torsional shear stress:</u>

When a torque, or twisting moment, is applied to a member, it tends to deform by twisting, causing a rotation of one part of the member relative to another. Such twisting causes a shear stress in the member. For a small element of the member, the nature of the stress is the same as that experienced under direct shear stress.

$$\tau_{max.} = \frac{T*C}{J} = \frac{T}{Z_P}$$

Where: C = radius of the shaft to outside surface

J = Polar moment of inertia

Zp = Section modulus (J/C)

Note: see APPENDIX 1 [Ref. 1] for formulas for J

$$\theta = \frac{T\,L}{G\,J}$$

Where : θ = angle of twist (in radian)

L = Length of shaft

G = Modulus of elasticity in shear (Modulus of Rigidity)

Example problem 3-6 (Page 96), [Ref. 1]:

Compute the maximum torsional shear stress in a shaft having a diameter of 10 mm when it carries a torque of 4.10 N.m.

Sol:

$$J = \frac{\pi D^4}{32} = \frac{\pi * 10^4}{32} = 982\ \text{mm}^4$$

$$\tau_{max} = \frac{(4.10\ \text{N.m})(5\ \text{mm})}{982\ \text{mm}^4} * \frac{10^3\ \text{mm}}{1\ \text{m}} = 20.9\frac{\text{N}}{\text{mm}^2} = 20.9\ \text{MPa}$$

Example problem (3-7), (Page 97), [Ref.1]:

Compute the angle of twist of a 10 mm-diameter shaft carrying 4.10 N.m of torque if it is 250 mm long and made of steel with G = 80 GPa. Express the result in both radians and degrees.

Solution Objective Compute the angle of twist in the shaft.

Given Torque = T = 4.10 N·m; length = L = 250 mm.
Shaft diameter = D = 10 mm; G = 80 GPa.

Analysis Use Equation (3–11). For consistency, let $T = 4.10 \times 10^3$ N·mm and $G = 80 \times 10^3$ N/mm^2. From Example Problem 3–6, $J = 982$ mm^4.

Results
$$\theta = \frac{TL}{GJ} = \frac{(4.10 \times 10^3\ \text{N}\cdot\text{mm})(250\ \text{mm})}{(80 \times 10^3\ \text{N/mm}^2)(982\ \text{mm}^4)} = 0.013\ \text{rad}$$

Using π rad = 180°,

$$\theta = (0.013\ \text{rad})(180\ \text{deg}/\pi\ \text{rad}) = 0.75\ \text{deg}$$

Comment Over the length of 250 mm, the shaft twists 0.75 deg.

5- Vertical shearing stress:

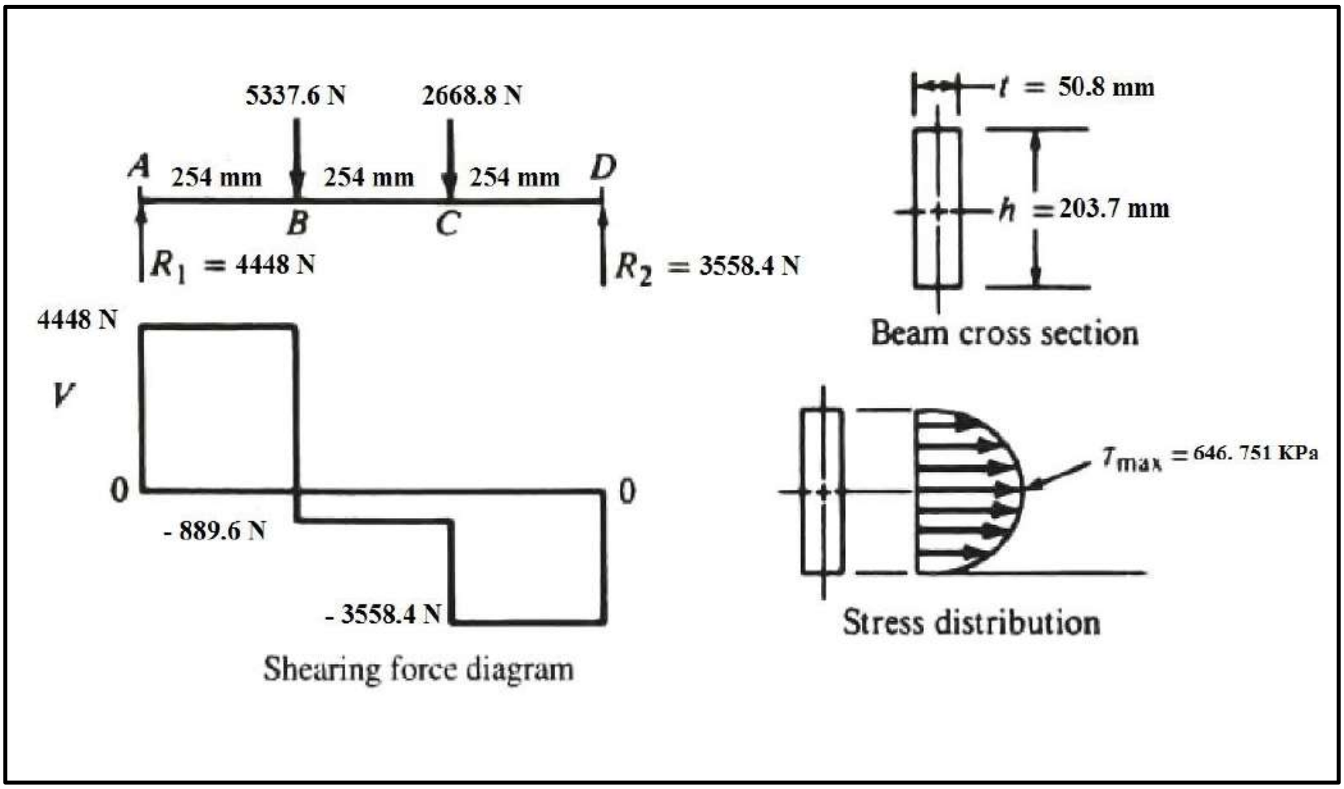
A beam carrying loads transverse to its axis will experience shearing forces denoted by V. In the analysis of beams, it is usual to draw the shearing force diagram. Then the resulting vertical shearing stress can be completed from:

$$\boldsymbol{\tau_{max} = \frac{3\ V}{2\ A}} \quad \textbf{(For rectangular section)}$$

$$\boldsymbol{\tau_{max} = \frac{4\ V}{3\ A}} \quad \textbf{(For circular section)}$$

Example problem (3-11), (Page 104), [Ref. 1]:

Compute the maximum shearing stress in the beam described below:



$$\tau_{max} = \frac{3\,V}{2\,A} = \frac{3 * 4448\,N}{2 * (50.8\,mm * 203.7\,mm)} = 646.7\,kPa$$

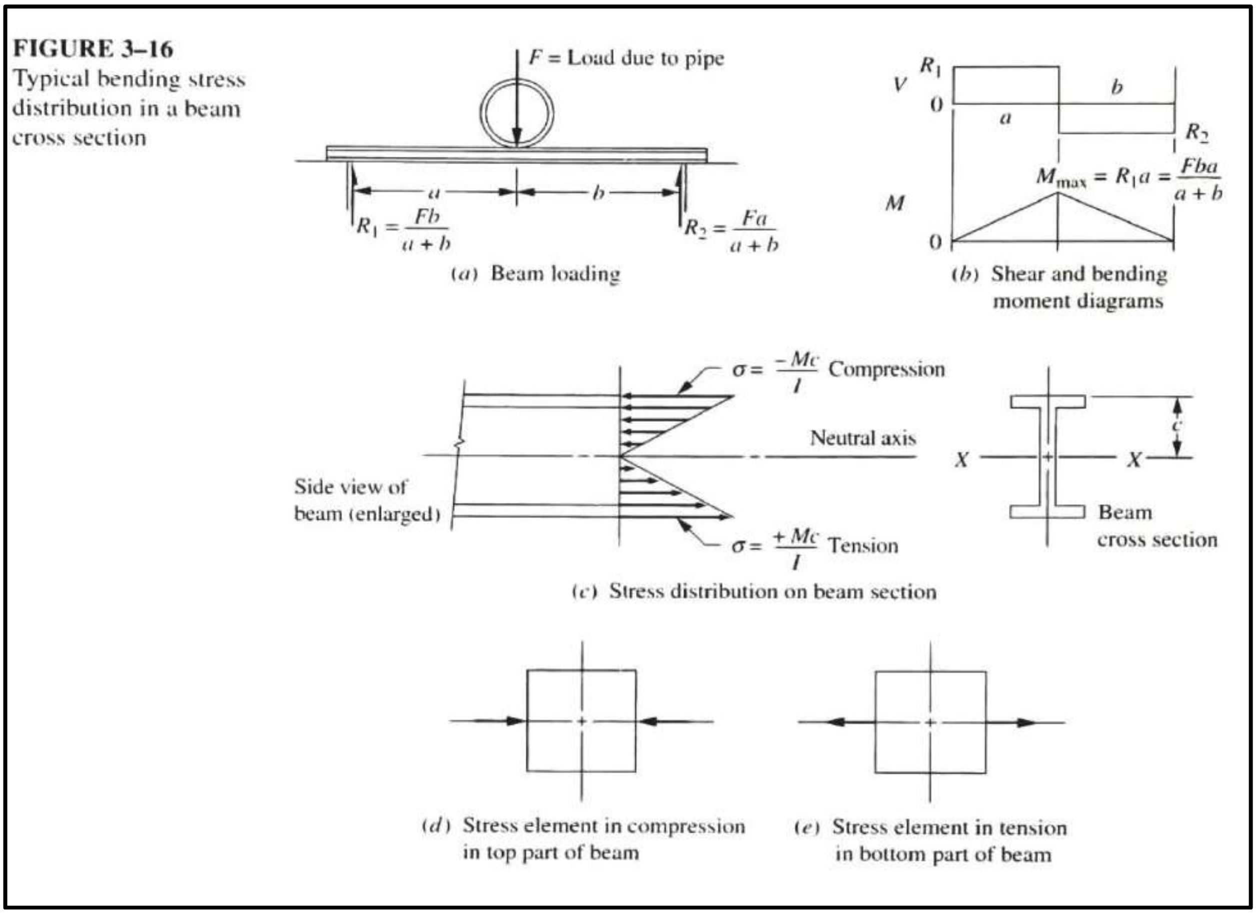
6-Stress due to Bending:

$$\boldsymbol{\sigma} = \frac{\mathbf{M\,C}}{\mathbf{I}} = \frac{\mathbf{M}}{\mathbf{S}}$$

Where: M= magnitude of the bending moment at the section
I = moment of inertia of the C.S. with respect to its neutral axis
C= distance from the neutral axis to the outer most fiber of the beam C.S.
S = section modulus = I/C

Example Problem (3-12), (Page 107) [Ref.1]

For the beam shown in Figure 3-16, the load F due to the pipe is 53376 N. The distances are a = 1.2192 m and b = 1.8288 m. Determine the required section modulus for the beam to limit the stress due to bending to 206850 kPa, the recommended design stress for a typical structural steel in static bending.



Sol:

$$\sigma = \frac{M}{S} \quad \rightarrow \quad S = \frac{M}{\sigma} = \frac{R_1 * a}{\sigma} = \frac{F * b}{a + b} * \frac{a}{\sigma} = \frac{53376\,(1.83)}{1.22 + 1.83} * \frac{1.22}{206850}$$

$$= 18.85 * 10^4 \ mm^3$$

Now from table A16-3 & A16-4 choose (W203*22.1) wide-flange shape with

S = 19.3 * 10^4 mm^3

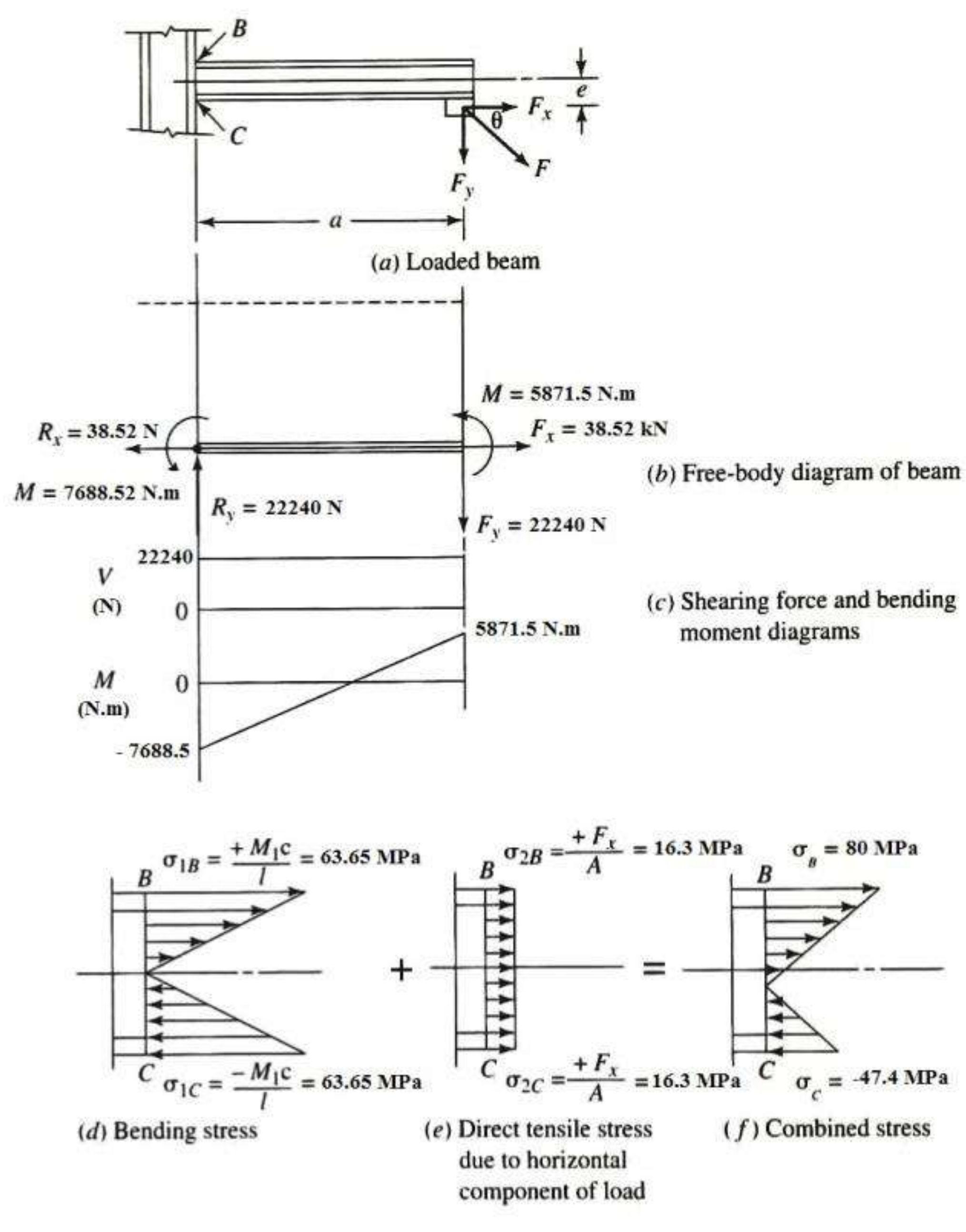
7-Stresses due to combined bending moment with axial load:

$$\sigma = \pm \frac{MC}{I} \pm \frac{F}{A}$$

Example problem (3-17), (Page 117), [Ref.1]:

The cantilever beam in Figure 3-24 is a steel American Standard beam. S6x12.5. The force F is 44480 N. and it acts at an angle of 30° below the horizontal, as shown. Use a = 609.6 mm and e = 152.4 mm. Draw the free-body diagram and the shearing force and bending moment diagrams for the beam. Then compute the maximum tensile and maximum compressive stresses in the beam and show where they occur.

FIGURE 3–24 Beam subjected to combined stresses



Sol:

From table A16-4 ($S = 12.079*10^4\ mm^4$ & $A=2367.884\ mm^2$)

$F_X = F \cos30 = 44480 \cos30 = 38520$ N

$F_Y = F \sin30 = 44480 \sin30 = 22240$ N

$M_1 = F_X$ (0.152 m) = 5871.5 N.m

M_{max} = 7688.5 N.m occurs at left end of beam

$$\sigma_1 = \pm \frac{M}{S} = \frac{7688.5}{12 * 10^4} = \pm 63654.6\ \text{kPa}$$

$$\sigma_2 = \frac{F_x}{A} = \frac{38520}{2307.9} = 16.27\ \text{MPa}$$

$\sigma_B = +\sigma_1 + \sigma_2 = 63654.6 \text{ kPa} + 16.27\text{MPa} = 79.93 \text{ MPa}$

$\sigma_C = -\sigma_1 + \sigma_2 = -63654.6 \text{ kPa} + 16.27\text{MPa} = -47.4 \text{ MPa}$

8- Stress Concentrations:

The above simple stresses are applicable for the geometry of a member is uniform throughout the section of interest. But if there is a fillet, holes, key seals, grooves, etc, will cause the actual max. stress, so defining stress concentration factors by which the actual max. stress exceeds the nominal stress.

$$\boldsymbol{\sigma_{max} = K\,\sigma_{nom.}} \quad \& \quad \boldsymbol{\tau_{max} = K\,\tau_{nom.}}$$

Where K can be found from **APPENDIX 15** [Ref.1]

Example Problem (3-18), (Page 121), [Ref.1]:

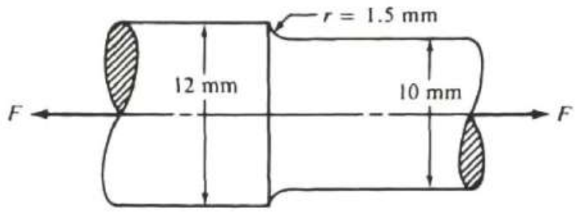
Compute the maximum stress in a round bar subjected to an axial tensile force of 9800 N. The geometry is shown in Figure 3-26.

Solution Objective Compute the maximum stress in the stepped bar shown in Figure 3–26.

Given The layout from Figure 3–26. Force = F = 9800 N.
The shaft has two diameters joined by a fillet with a radius of 1.5 mm.
Larger diameter = D = 12 mm; smaller diameter = d = 10 mm.

Analysis The presence of the change in diameter at the step causes a stress concentration to occur. The general situation is a round bar subjected to an axial tensile load. We will use the top

FIGURE 3–26
Stepped round bar subjected to axial tensile force



graph of Figure A15–1 to determine the stress concentration factor. That value is used in Equation (3–27) to determine the maximum stress.

Results Figure A15–1 indicates that the nominal stress is computed for the smaller of the two diameters of the bar. The stress concentration factor depends on the ratio of the two diameters and the ratio of the fillet radius to the smaller diameter.

$$D/d = 12\text{ mm}/10\text{ mm} = 1.20$$
$$r/d = 1.5\text{ mm}/10\text{ mm} = 0.15$$

From these values, we can find that K_t = 1.60. The stress is

$$\sigma_{nom} = F/A = (9800\text{ N})/[\pi(10\text{ mm})^2/4] = 124.8\text{ MPa}$$
$$\sigma_{max} = K_t\sigma_{nom} = (1.60)(124.8\text{ MPa}) = 199.6\text{ MPa}$$

Comments The maximum tensile stress of 199.6 MPa occurs in the fillet near the smaller diameter. This value is 1.60 times higher than the nominal stress that occurs in the 10-mm-diameter shaft. To the left of the shoulder, the stress reduces dramatically as the effect of the stress concentration diminishes and because the area is larger.

LECTURE FIVE

V-BELT DRIVES

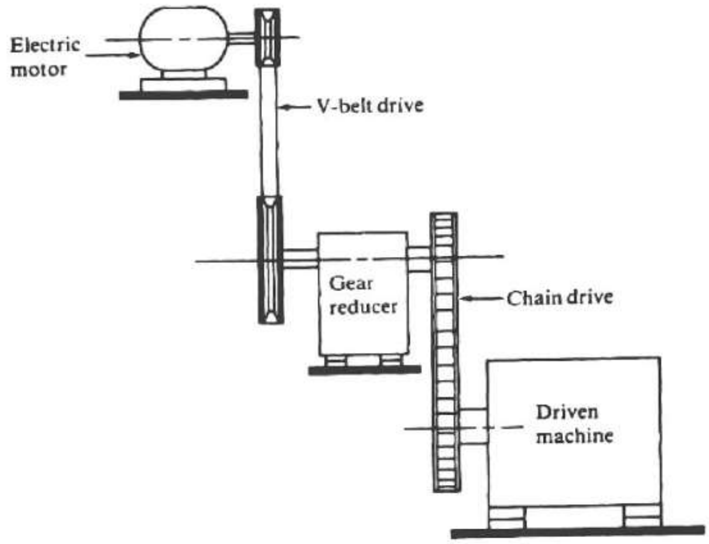
References:

Machine Elements in Mechanical Design by Robert L. Mott, P.E. (Chapter 7)

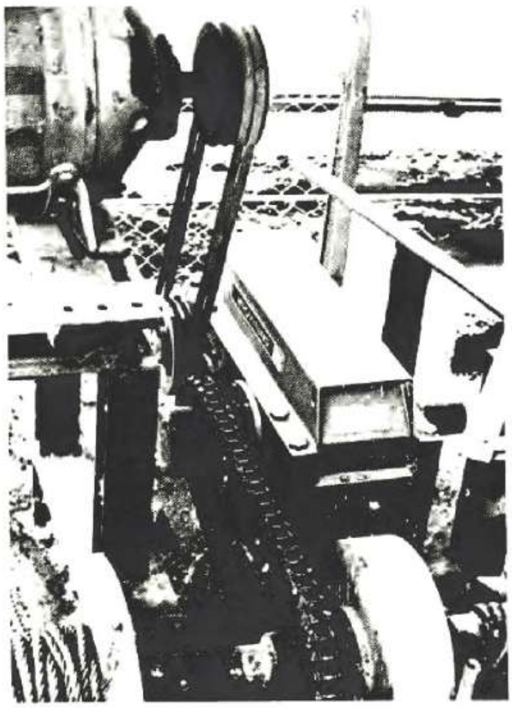
Introduction:

Belts represent the major types of flexible power transmission elements. Figure (7-1) shows a typical industrial application of this element.

FIGURE 7–1
Combination drive employing V-belts, a gear reducer, and a chain drive [Source for Part (*b*): Browning Mfg. Division, Emerson Electric Co., Maysville, KY]



(*a*) Sketch of combination drive


(*b*) Photograph of an actual drive installation. Note that guards have been removed from the belt and chain drives to show detail.

Basic belt drive:

FIGURE 7–2 Basic belt drive geometry

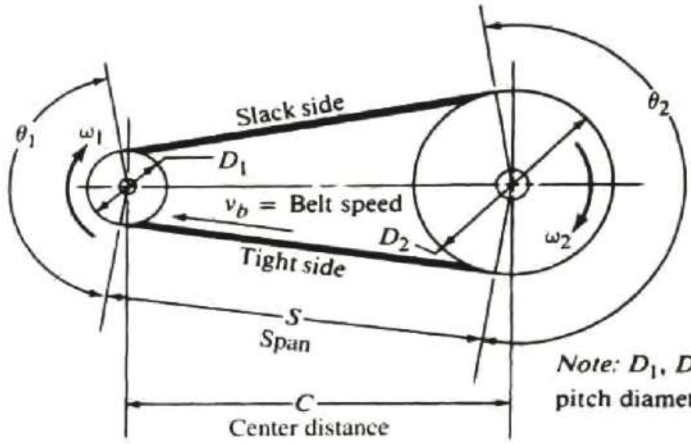
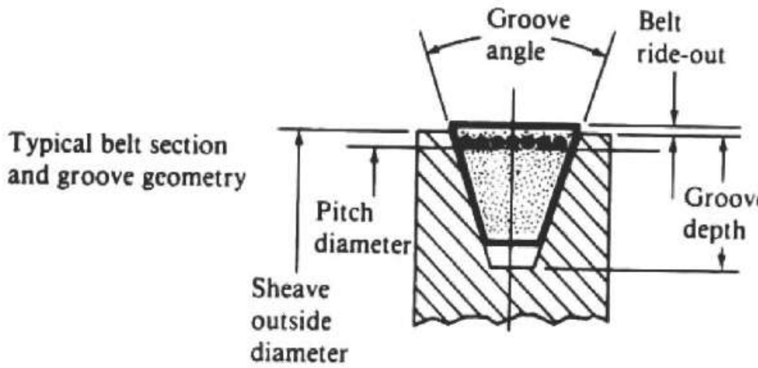


FIGURE 7–4 Cross section of V-belt and sheave groove



$$V_b = R_1W_1 = R_2W_2 \quad \text{Or} \quad V_b = \frac{D_1W_1}{2} = \frac{D_2W_2}{2} \qquad \text{.......... (7-1)}$$

$$\text{And} \quad \frac{W_1}{W_2} = \frac{D_2}{D_1} \qquad \text{.......... (7-2)}$$

$$L = \text{Pitch length} = 2C + 1.57\,(D_2 + D_1) + \frac{(D_2 - D_1)^2}{4C} \qquad \text{.......... (7-3)}$$

$$C = \text{Center distance} = \frac{B + \sqrt{B^2 - 32\,(D_2 - D_1)^2}}{16} \qquad \text{....... (7-4)} \qquad \text{Where } B = 4L - 6.28\,(D_2 + D_1)$$

$$\theta_1 = Angle\ of\ contact\ of\ belt\ on\ sheave\ 1 = 180^o - 2\sin^{-1}\left|\frac{D_2 - D_1}{2C}\right| \qquad \text{......... (7-5)}$$

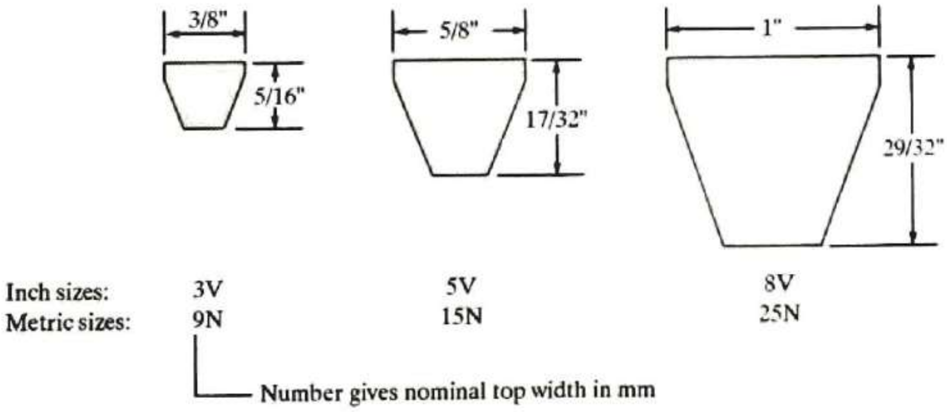
$$\theta_2 = Angle\ of\ contact\ of\ belt\ on\ sheave\ 2 = 180^o - 2\sin^{-1}\left|\frac{D_2 - D_1}{C}\right| \qquad \text{......... (7-6)}$$

$$S = \text{Length of span} = \sqrt{C^2 - \left|\frac{D_2 - D_1}{2}\right|^2}$$

Note: The design value of the ratio of tight side tension to the slack side tension $\cong$ 5.0, the actual value may be range as high as 10.

Standard Belt cross sections (Page 271)

FIGURE 7–6
Industrial narrow-section V-belts



Notes:

1. The basic data required for drive selection are as mentioned in page 272 section 7-4.

- The rated power of the driving motor or other prime mover
- The service factor based on the type of driver and driven load
- The center distance
- The power rating for one belt as a function of the size and speed of the smaller sheave
- The belt length
- The size of the driving and driven sheaves
- The correction factor for belt length
- The correction factor for the angle of wrap on the smaller sheave
- The number of belts
- The initial tension on the belt

2. The nominal range of center distance should be

$$D_2 < C < 3(D_2 + D_1) \quad \text{........ (7-8)}$$

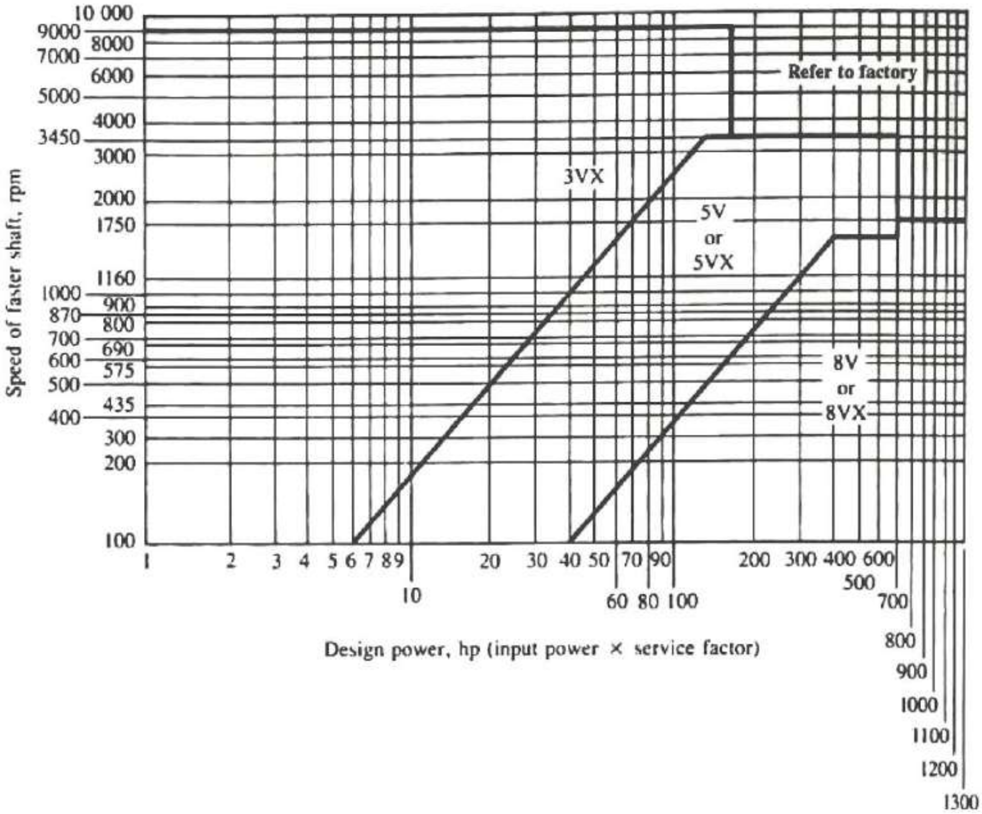
3. The angle of wrap on smaller sheave should be $> 120°$

4. Most commercially available sheaves are cast iron. This should be limited to (1981 m/min = 33 m/sec) belt speed.

5. Consider on alternative type of drive, such as a gear type or chain, if the belt speed is less than (304.8 m /min = 5 m/sec).

6. Figure (7-9) page 274 can be used to choose the basic size for belt cross section.

FIGURE 7–9
Selection chart for narrow-section industrial V-belts (Dayco Corp., Dayton, OH)



7. The service factor can be taken from table (7-1) page 274.

TABLE 7–1 V-belt service factors

Driven machine type	AC motors: Normal torque[a] DC motors: Shunt-wound Engines: Multiple-cylinder <6 h per day	6–15 h per day	>15 h per day	AC motors: High torque[b] DC motors: Series-wound, compound-wound Engines: 4-cylinder or less <6 h per day	6–15 h per day	>15 h per day
Agitators, blowers, fans, centrifugal pumps, light conveyors	1.0	1.1	1.2	1.1	1.2	1.3
Generators, machine tools, mixers, gravel conveyors	1.1	1.2	1.3	1.2	1.3	1.4
Bucket elevators, textile machines, hammer mills, heavy conveyors	1.2	1.3	1.4	1.4	1.5	1.6
Crushers, ball mills, hoists, rubber extruders	1.3	1.4	1.5	1.5	1.6	1.8
Any machine that can choke	2.0	2.0	2.0	2.0	2.0	2.0

[a]Synchronous, split-phase, three-phase with starting torque or breakdown torque less than 175% of full-load torque.
[b]Single-phase, three-phase with starting torque or breakdown torque greater than 175% of full-load torque.

8. Figures (7-10), (7-11) and (7-12) give rated power per belt for three cross sections.

FIGURE 7–10
Power rating: 3V belts

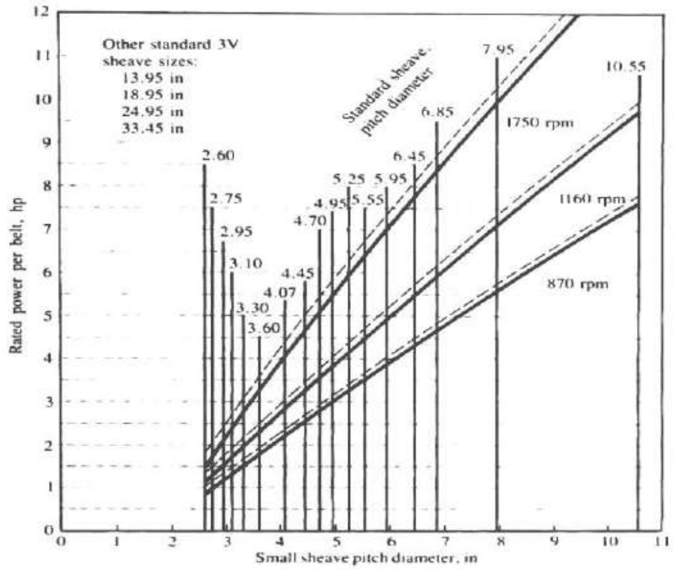


FIGURE 7–11
Power rating: 5V belts

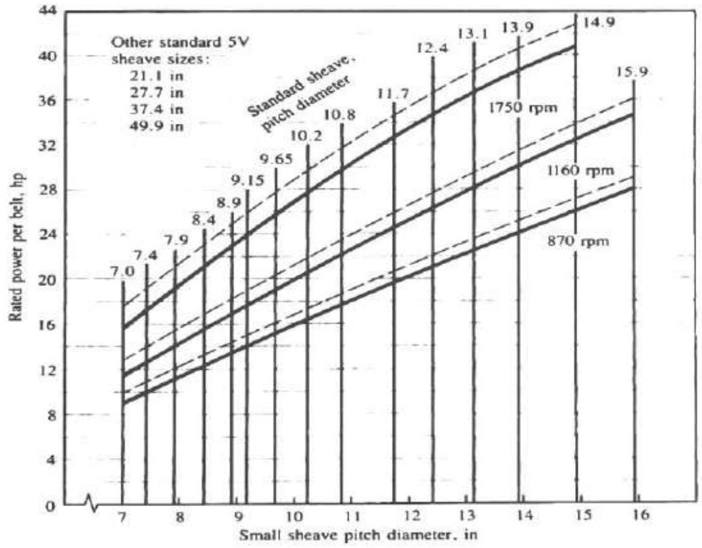
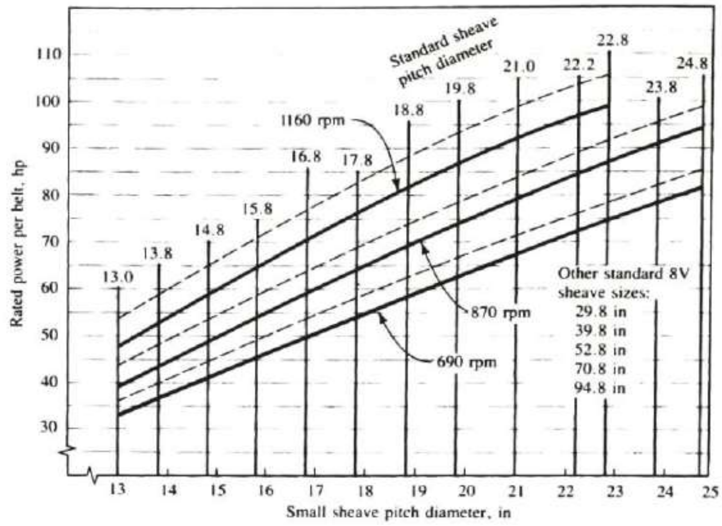
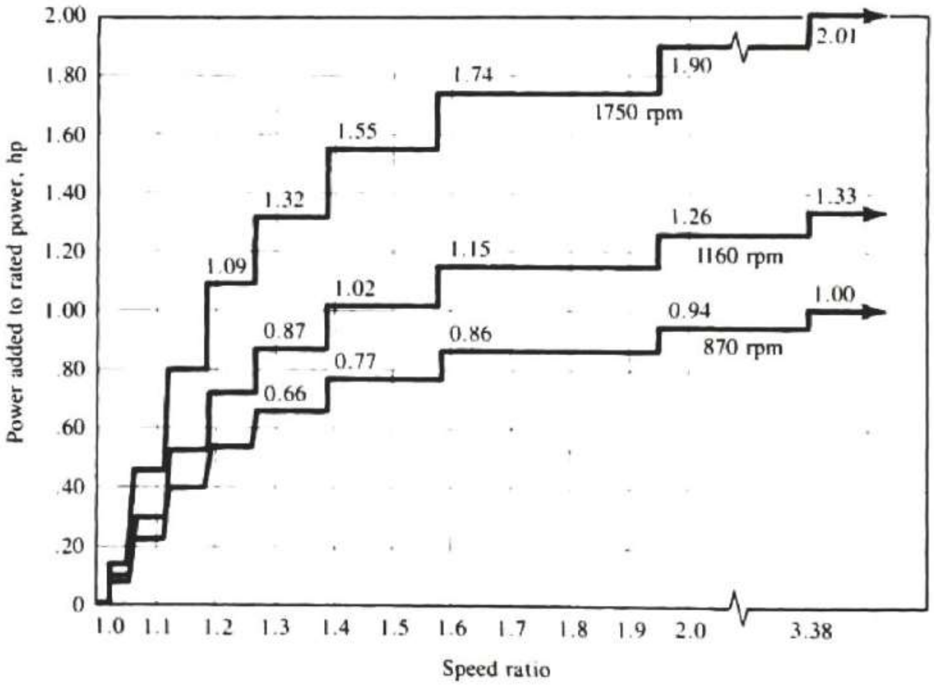


FIGURE 7–12
Power rating: 8V belts



9. The basic power rating for a speed ratio of 1 is given as solid curve; figure (7-13) is a plot of the added power to basic rating as a function of speed ration for SV belt size.

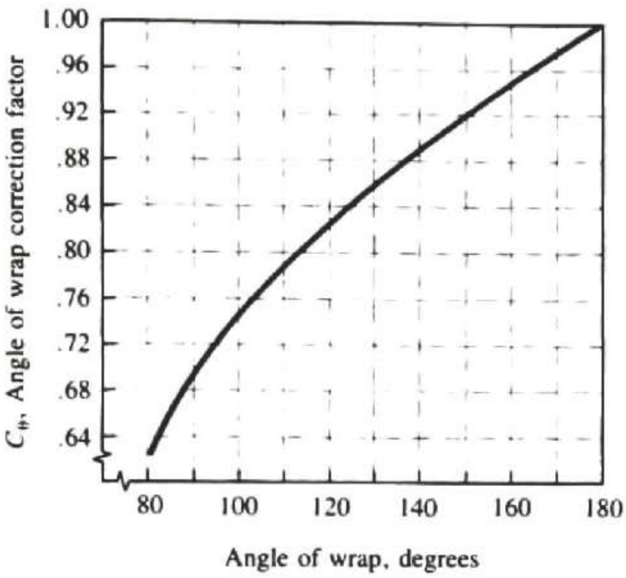
FIGURE 7–13 Power added versus speed ratio: 5V belts



10. For ratio above 3.38 was used. Draw dashed curves in fig. (7-10) , (7-11) & (7-12) (Make interpolation if possible).

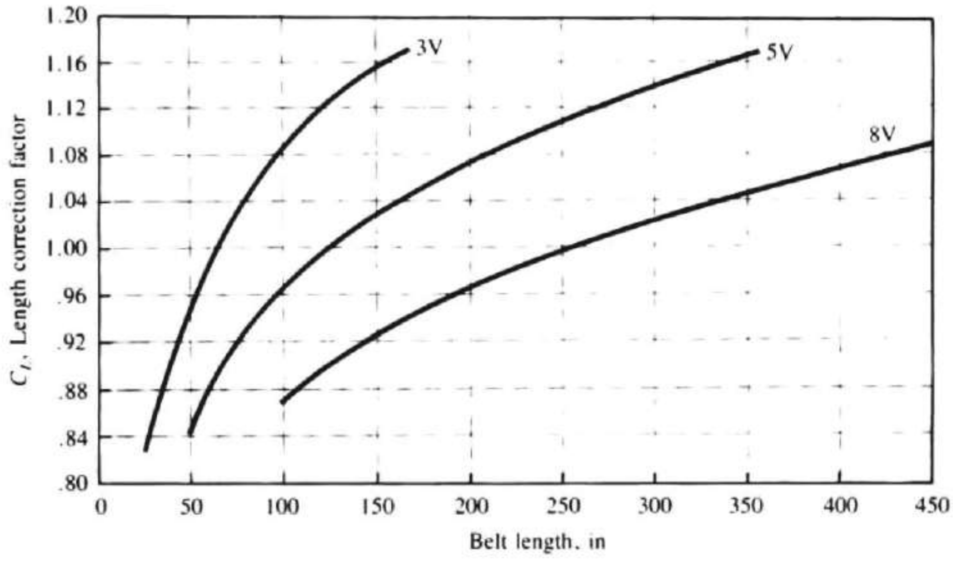
11. Figure (7-14) page 277 give value of correction factor C_θ as a function of angle of wrap of the belt on the small sheave.

FIGURE 7–14 Angle of wrap correction factor, C_θ



12. Figure (7-15) gives the value of correction factor C_L.

FIGURE 7–15 Belt length correction factor, C_L



13. Table (7-2) gives certain standard lengths are available.

TABLE 7–2 Standard belt lengths for 3V, 5V, and 8V belts (in)

3V only	3V and 5V	3V, 5V, and 8V	5V and 8V	8V only
25	50	100	150	375
26.5	53	106	160	400
28	56	112	170	425
30	60	118	180	450
31.5	63	125	190	475
33.5	67	132	200	500
35.5	71	140	212	
37.5	75		224	
40	80		236	
42.5	85		250	
45	90		265	
47.5	95		280	
			300	
165			315	
			335	
			355	

Example Problem 7–1 Design a V-belt drive that has the input sheave on the shaft of an electric motor (normal torque) rated at 50.0 hp at 1160-rpm, full-load speed. The drive is to a bucket elevator in a potash plant that is to be used 12 hours (h) daily at approximately 675 rpm.

Solution Objective Design the V-belt drive.

Given

Power transmitted = 50 hp to bucket elevator

Speed of motor = 1160 rpm; output speed = 675 rpm

Analysis Use the design data presented in this section. The solution procedure is developed within the Results section of the problem solution.

Results ***Step 1.*** Compute the design power. From Table 7–1, for a normal torque electric motor running 12 h daily driving a bucket elevator, the service factor is 1.30. Then the design power is 1.30(50.0 hp) = 65.0 hp.

Step 2. Select the belt section. From Figure 7–9, a 5V belt is recommended for 70.0 hp at 1160-rpm input speed.

Step 3. Compute the nominal speed ratio:

$$\text{Ratio} = 1160/675 = 1.72$$

Step 4. Compute the driving sheave size that would produce a belt speed of 4000 ft/min, as a guide to selecting a standard sheave:

$$\text{Belt speed} = v_b = \frac{\pi D_1 n_1}{12}\text{ft/min}$$

Then the required diameter to give v_b = 4000 ft/min is

$$D_1 = \frac{12\,v_b}{\pi n_1} = \frac{12(4000)}{\pi n_1} = \frac{15\,279}{n_1} = \frac{15\,279}{1160} = 13.17 \text{ in}$$

Step 5. Select trial sizes for the input sheave, and compute the desired size of the output sheave. Select a standard size for the output sheave, and compute the actual ratio and output speed.

For this problem, the trials are given in Table 7–3 (diameters are in inches).

The two trials in **boldface** in Table 7–3 give only about 1% variation from the desired output speed of 675 rpm, and the speed of a bucket elevator is not critical. Because no space limitations were given, let's choose the larger size.

Step 6. Determine the rated power from Figure 7–10, 7–11, or 7–12.

For the 5V belt that we have selected, Figure 7–11 is appropriate. For a 12.4-in sheave at 1160 rpm, the basic rated power is 26.4 hp. Multiple belts will be required. The ratio is relatively high, indicating that some added power rating can be used. This value can be estimated from Figure 7–11 or taken directly from Figure 7–13 for the 5V belt. Power added is 1.15 hp. Then the actual rated power is 26.4 + 1.15 = 27.55 hp.

Step 7. Specify a trial center distance.

We can use Equation (7–8) to determine a nominal acceptable range for C:

$$D_2 < C < 3(D_2 + D_1)$$
$$21.1 < C < 3(21.1 + 12.4)$$
$$21.1 < C < 100.5 \text{ in}$$

In the interest of conserving space, let's try C = 24.0 in.

TABLE 7–3 Trial sheave sizes for Example Problem 7–1

Standard driving sheave size, D_1	Approximate driven sheave size ($1.72D_1$)	Nearest standard sheave, D_2	Actual output speed (rpm)
13.10	22.5	21.1	720
12.4	**21.3**	**21.1**	**682**
11.7	20.1	21.1	643
10.8	18.6	21.1	594
10.2	17.5	15.9	744
9.65	16.6	15.9	704
9.15	**15.7**	**15.9**	**668**
8.9	15.3	14.9	693

Step 8. Compute the required belt length from Equation (7–3):

$$L = 2C + 1.57(D_2 + D_1) + \frac{(D_2 - D_1)^2}{4C}$$

$$L = 2(24.0) + 1.57(21.1 + 12.4) + \frac{(21.1 - 12.4)^2}{4(24.0)} = 101.4 \text{ in}$$

Step 9. Select a standard belt length from Table 7–2, and compute the resulting actual center distance from Equation (7–4).

In this problem, the nearest standard length is 100.0 in. Then, from Equation (7–4),

$$B = 4L - 6.28(D_2 + D_1) = 4(100) - 6.28\,(21.1 + 12.4) = 189.6$$

$$C = \frac{189.6 + \sqrt{(189.6)^2 - 32(21.1 - 12.4)^2}}{16} = 23.30 \text{ in}$$

Step 10. Compute the angle of wrap of the belt on the small sheave from Equation (7–5):

$$\theta_1 = 180^\circ - 2\sin^{-1}\left[\frac{D_2 - D_1}{2C}\right] = 180^\circ - 2\sin^{-1}\left[\frac{21.1 - 12.4}{2(23.30)}\right] = 158^\circ$$

Step 11. Determine the correction factors from Figures 7–14 and 7–15. For $\theta = 158^\circ$, $C_\theta = 0.94$. For $L = 100$ in, $C_L = 0.96$.

Step 12. Compute the corrected rated power per belt and the number of belts required to carry the design power:

$$\text{Corrected power} = C_\theta C_L P = (0.94)(0.96)(27.55 \text{ hp}) = 24.86 \text{ hp}$$

$$\text{Number of belts} = 65.0/24.86 = 2.61 \text{ belts (Use 3 belts.)}$$

Comments

Summary of Design

Input: Electric motor, 50.0 hp at 1160 rpm

Service factor: 1.4

Design power: 70.0 hp

Belt: 5V cross section, 100-in length, 3 belts

Sheaves: Driver, 12.4-in pitch diameter, 3 grooves, 5V. Driven, 21.1-in pitch diameter, 3 grooves, 5V

Actual output speed: 682 rpm

Center distance: 23.30 in

LECTURE SIX & SEVEN

COMBINE STRESSES, MOHR'S CIRCLE & DESIGN FOR DIFFERENT TYPE OF LOADING

1. Maximum normal stresses: Principle stresses, [Ref.1]:

The combination of the applied normal and shear stresses that produce the maximum normal stresses is called the max. & min. principle stresses σ_1 & σ_2 are:

$$\boldsymbol{\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

The angle of inclination of planes is:

$$\emptyset_\sigma = \frac{1}{2} \tan^{-1} \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

FIGURE 4–3
General two-dimensional stress element

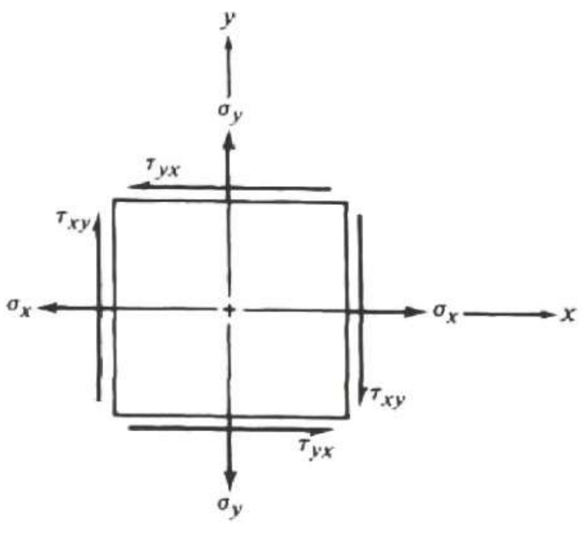
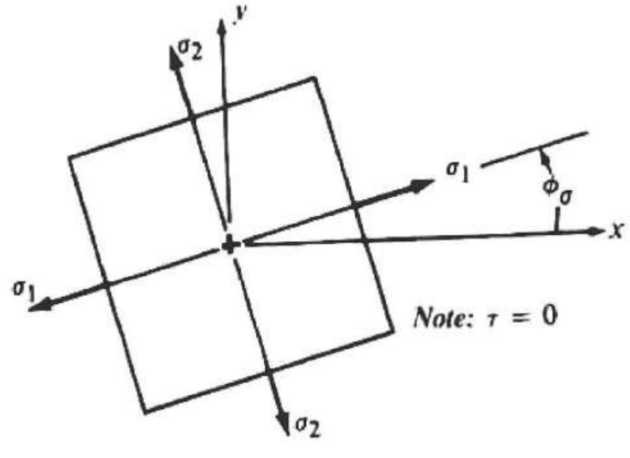


FIGURE 4–4
Principal stress element



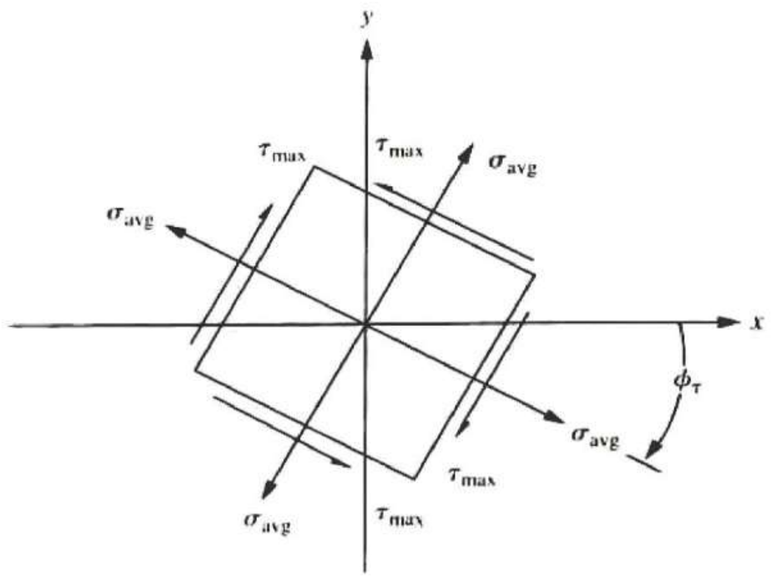
2. Maximum shear stress, [Ref.1]:

On different orientation of stress element, the maximum shear stress will occur. It magnitude can be computed from:

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} \quad ; \quad \emptyset_\tau = \frac{1}{2}\tan^{-1}\left(-\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right)$$

FIGURE 4–5
Maximum shear stress element



Note: the angle between principle stress element and the max. shear stress element is always 45 degree.

Example Problem 4-1, (page 141), [Ref.1]:

The shaft shown in Figure 4-7 is supported by two bearings and carries two V-belt sheaves. The tensions in the belts exert horizontal forces on the shaft, tending to bend it in the X-Z plane. Sheave B exerts a clockwise torque on the shaft when viewed toward the origin of the coordinate system along the X-axis. Sheave C exerts an equal but opposite torque on the shaft. For the loading condition shown, determine the principal stresses and the maximum shear stress on element K on the front surface of the shaft (on the positive Z-side) just to the right of sheave B, Follow the general procedure for analyzing combined stresses given in this section.

Sol:

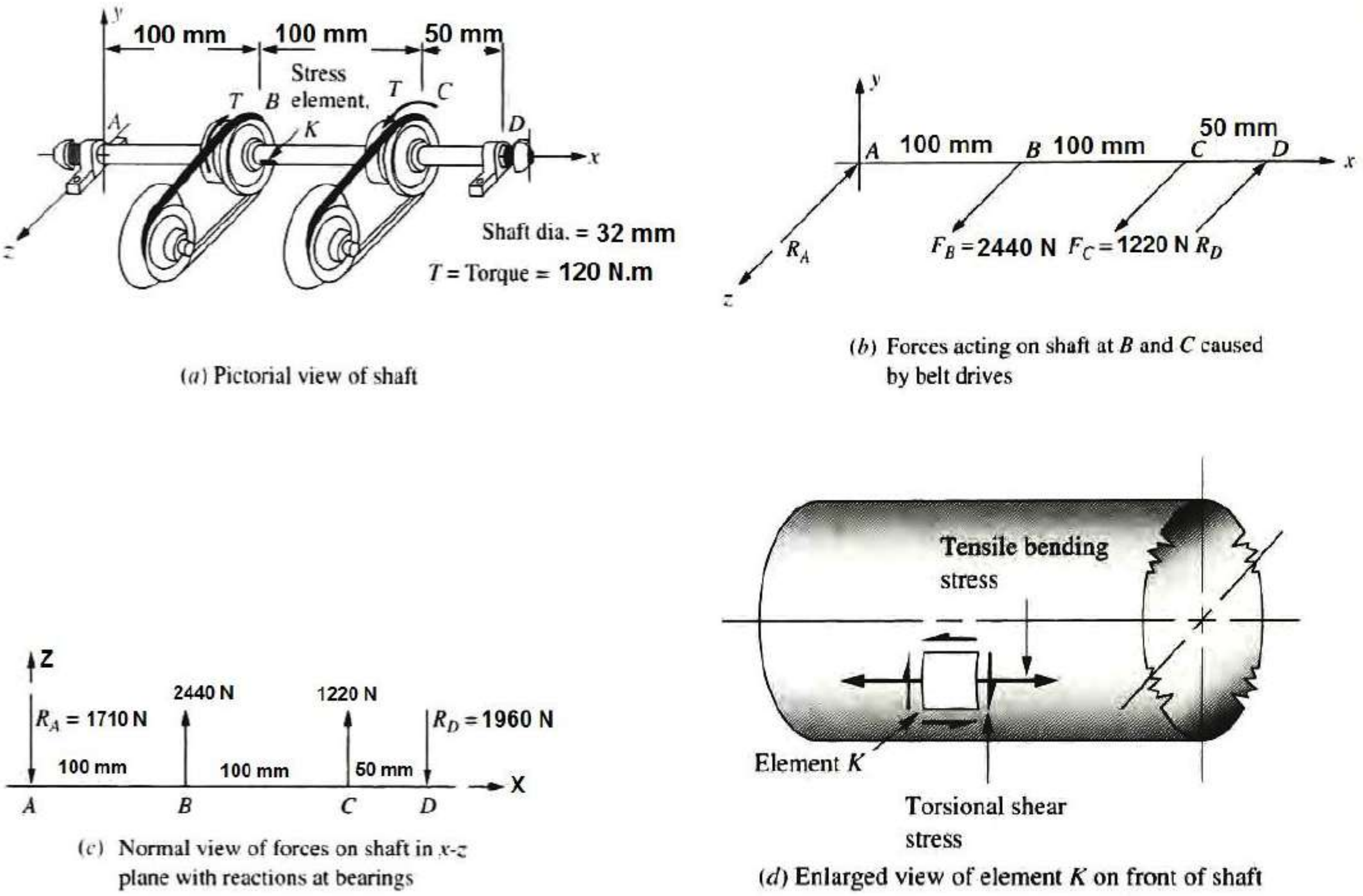


FIGURE 4–7 Shaft supported by two bearings and carrying two V-belt sheaves

FIGURE 4–8
Shearing force and bending moment diagrams for the shaft

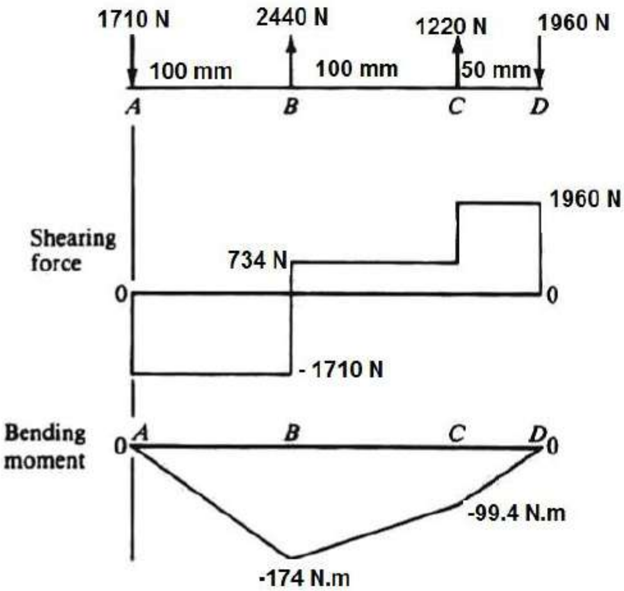
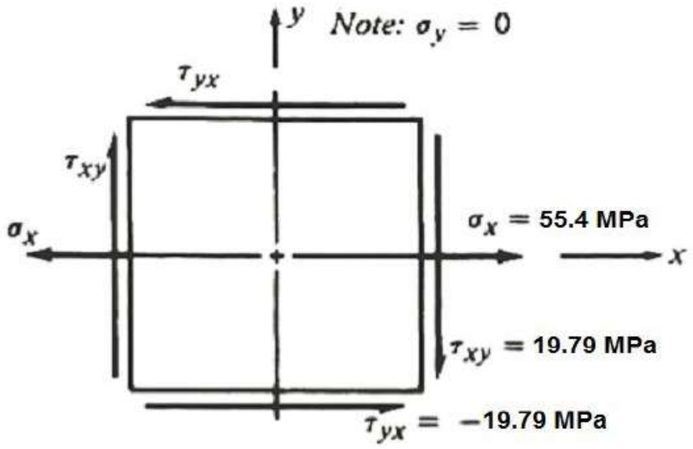


FIGURE 4–9
Stresses on element K



$$\sigma_x = \frac{M}{S} \quad ; \quad S = \frac{\pi D^3}{32} = \frac{\pi\,(31.75)^3}{32} = 3146.3\ mm^3$$

$$\sigma_x = \frac{174\ N.m}{3146.3\ mm^3} = 55.4\ MPa$$

$$\tau_{xy} = \frac{T}{Z_p} \quad ; \quad Z_p = \frac{\pi D^3}{16} = \frac{\pi\,(31.75)^3}{16} = 6276.2\ mm^3$$

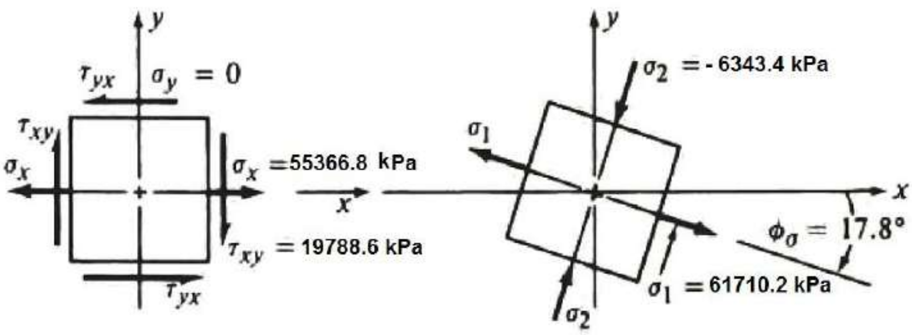
$$\tau_{xy} = \frac{120\ N.m}{6276.2\ mm^3} = 19.79\ MPa$$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \left(\frac{55.4}{2}\right) + \sqrt{\left(\frac{55.4}{2}\right)^2 + (19.79)^2} = 61.7\ MPa$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \left(\frac{55.4}{2}\right) - \sqrt{\left(\frac{55.4}{2}\right)^2 + (19.79)^2} = -\,6.3\ MPa$$

$$\emptyset_\sigma = \frac{1}{2}\,tan^{-1}\frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{1}{2}\,tan^{-1}\frac{2 * 19.79}{55.4} = 17.8°$$

FIGURE 4–10
Principal stress element



(a) Original stress element (b) Principal stress element

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{55366.8}{2}\right)^2 + (19788.6)^2} = \pm 34026.8\ kPa$$

$$\emptyset_\tau = \frac{1}{2}\tan^{-1}\left(-\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right) = \frac{1}{2}\tan^{-1}\left(-\frac{55366.8}{2 * 19788.6}\right) = -27.2°$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{55366.8}{2} = 27683.4\ kPa$$

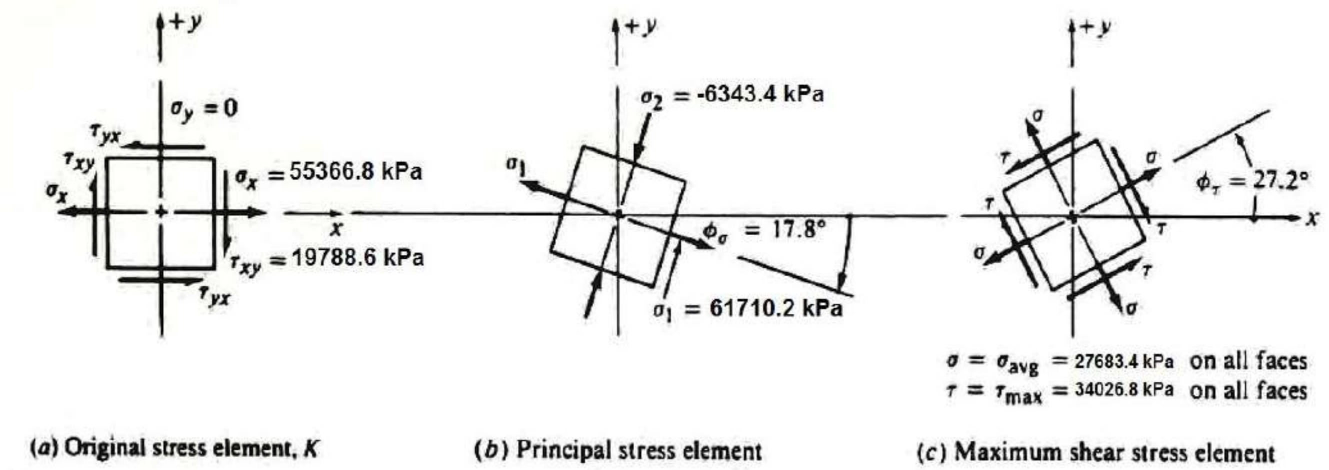


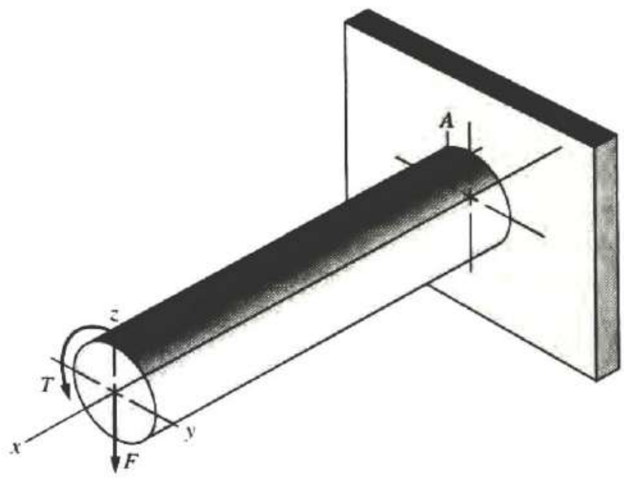
FIGURE 4–11 Relation of maximum shear stress element to the original stress element and the principal stress element

3. <u>Mohr's circle for different stress conditions:</u>

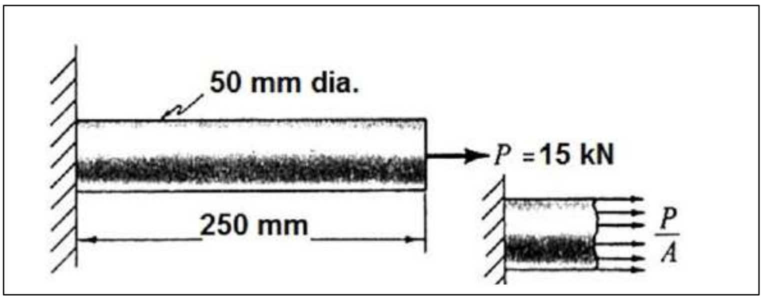
Use the Mohr's circle module from M-design to complete the following cases:

Example: A hypothetical machine member 50mm diameter by 250mm long and supported at one end as a cantilever will be used to demonstrate how numerical tensile, compressive, and shear stresses are determined for various types of uniaxial loading. In this example not that $(\boldsymbol{\sigma})_y$ ***= 0*** for all arrangements, at the critical points. Compute the following cases:

FIGURE 4–33
Circular bar in bending and torsion



a) Axial load only

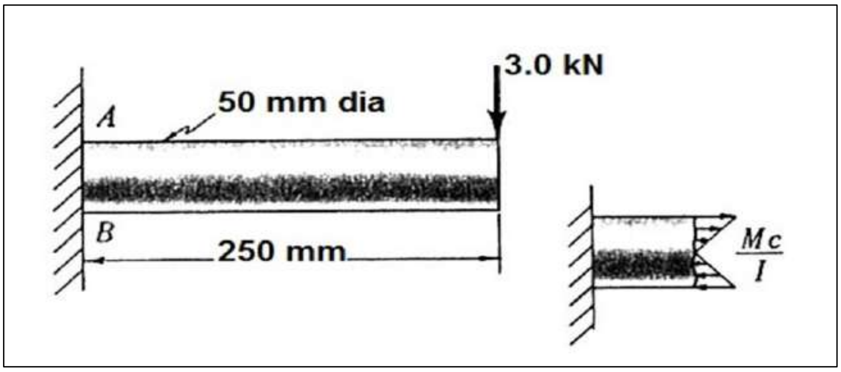


$$\sigma_x = \frac{P}{A} = \frac{15000\ N}{\frac{\pi}{4}(50)^2} = 7.65\ MPa$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{7.65 + 0}{2} \pm \sqrt{\left(\frac{7.65 - 0}{2}\right)^2 + 0}$$

$\sigma_1 = 7.65\ MPa$; $\sigma_2 = 0$; $\tau_{xy} = 0$; $\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 3.83\ MPa$

b) Bending only



$\sigma_x = +\frac{MC}{I}$ For point A

$\sigma_x = -\frac{MC}{I}$ For point B

$\tau_{xy} = 0\ at\ points\ A\ \&\ B$

$\sigma_x = +\frac{3*10^3*250*10^{-3}*25*10^{-3}*64}{\pi(50*10^{-3})^4} = 61.1\ MPa$ (For point A)

$\sigma_x = -61.1\ MPa$ (For point B)

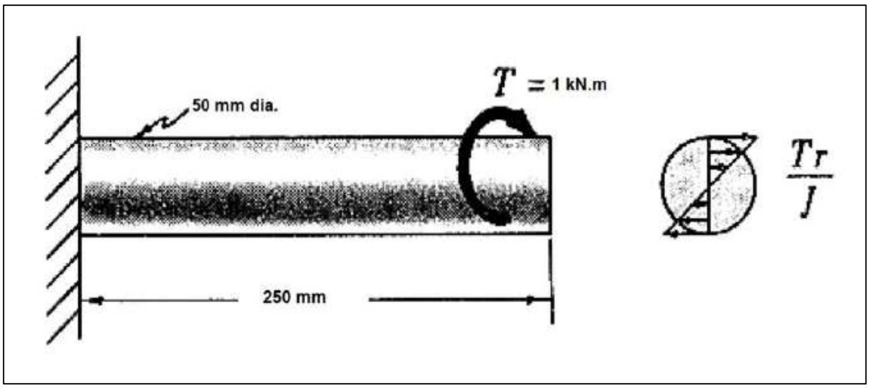
$$\boldsymbol{\sigma_{1,2}} = \frac{\boldsymbol{\sigma_x + \sigma_y}}{\mathbf{2}} \pm \sqrt{\left(\frac{\boldsymbol{\sigma_x - \sigma_y}}{\mathbf{2}}\right)^2 + \boldsymbol{\tau_{xy}^2}}$$

$\therefore\ \sigma_1 = 61.1\ MPa$ & $\sigma_2 = 0$ *at point A*

$\sigma_1 = 0$ & $\sigma_2 = -61.1\ MPa$ *at point B*

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{61.1}{2} = 30.6\ MPa \quad at\ point\ A\ \&\ B$$

c) Torsion only



$\sigma_x = 0$

$$\tau_{xy} = \frac{T*r}{J}$$

$$\tau_{xy} = \frac{1*10^3*25*10^{-3}*32}{\pi(50*10^{-3})^4}$$

$\tau_{xy} = 40.7\ MPa$

$$\boldsymbol{\sigma_{1,2}} = \frac{\boldsymbol{\sigma_x + \sigma_y}}{\mathbf{2}} \pm \sqrt{\left(\frac{\boldsymbol{\sigma_x - \sigma_y}}{\mathbf{2}}\right)^{\mathbf{2}} + \boldsymbol{\tau_{xy}^2}} \quad \& \quad \boldsymbol{\tau_{max}} = \sqrt{\left(\frac{\boldsymbol{\sigma_x - \sigma_y}}{\mathbf{2}}\right)^{\mathbf{2}} + \boldsymbol{\tau_{xy}^2}}$$

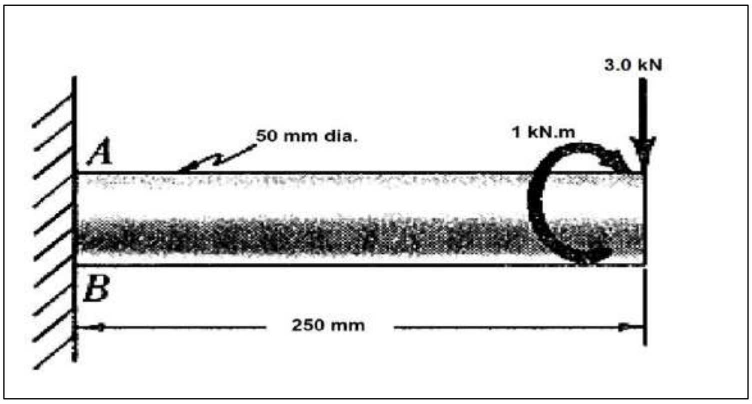
$\therefore \sigma_1 = 40.7\ MPa\ (Tension)\ \&\ \sigma_2 = -40.7\ MPa\ (Compression)\ \&\ \tau_{max} = 40.7\ MPa$

d) Bending and torsion

$$\sigma_x = \frac{M * C}{I} = +61.1\ MPa\ (at\ A)$$

$$\sigma_x = -61.1\ MPa\quad (at\ B)$$

$$\tau_{xy} = \frac{T * r}{J} = 40.7\ MPa\ (at\ A\ \&\ B)$$



$$\boldsymbol{\sigma_{1,2}} = \frac{\boldsymbol{\sigma_x + \sigma_y}}{\mathbf{2}} \pm \sqrt{\left(\frac{\boldsymbol{\sigma_x - \sigma_y}}{\mathbf{2}}\right)^{\mathbf{2}} + \boldsymbol{\tau_{xy}^2}} \quad \& \quad \boldsymbol{\tau_{max}} = \sqrt{\left(\frac{\boldsymbol{\sigma_x - \sigma_y}}{\mathbf{2}}\right)^{\mathbf{2}} + \boldsymbol{\tau_{xy}^2}}$$

$\therefore \sigma_1 = 81.4\ MPa\quad \&\quad \sigma_2 = -20.3\ MPa\quad \&\quad \tau_{max} = +50.9\ MPa\qquad (at\ point\ A)$

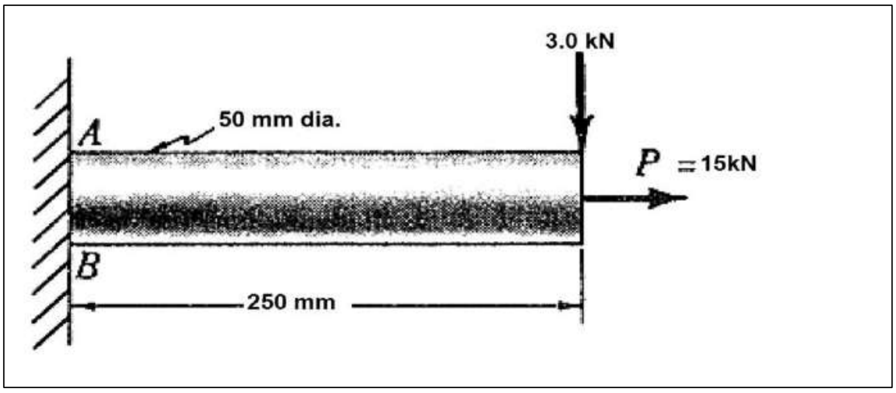
$\sigma_1 = 20.3\ MPa\quad \&\quad \sigma_2 = -81.4\ MPa\quad \&\quad \tau_{max} = -50.9\ MPa\qquad (at\ point\ B)$

e) Bending and axial load

At point A

$$\sigma_x = +\frac{P}{A} + \frac{M * c}{I}$$

$$= 68.8\ MPa\quad (Tension)$$



$$\boldsymbol{\sigma_{1,2}} = \frac{\boldsymbol{\sigma_x + \sigma_y}}{\mathbf{2}} \pm \sqrt{\left(\frac{\boldsymbol{\sigma_x - \sigma_y}}{\mathbf{2}}\right)^{\mathbf{2}} + \boldsymbol{\tau_{xy}^2}} \quad \& \quad \boldsymbol{\tau_{max}} = \sqrt{\left(\frac{\boldsymbol{\sigma_x - \sigma_y}}{\mathbf{2}}\right)^{\mathbf{2}} + \boldsymbol{\tau_{xy}^2}}$$

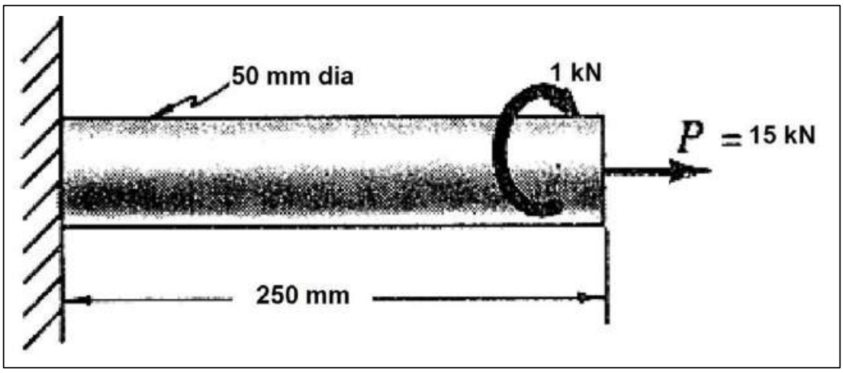
$\therefore \sigma_1 = \sigma_x = 68.8\ MPa\quad \&\quad \sigma_2 = 0\quad \&\quad \tau_{max} = \frac{\sigma_x}{2} = 34.4\ MPa\qquad (\boldsymbol{at\ point\ A})$

At point B

$$\sigma_x = +\frac{P}{A} - \frac{M * c}{I} = -53.5\ MPa\quad (Compression)$$

$\therefore \sigma_1 = 0\quad \&\quad \sigma_2 = -53.5\ MPa\quad \&\quad \tau_{max} = \frac{\sigma_x}{2} = -26.7\ MPa\qquad (\boldsymbol{at\ point\ B})$

f) Torsion and axial load



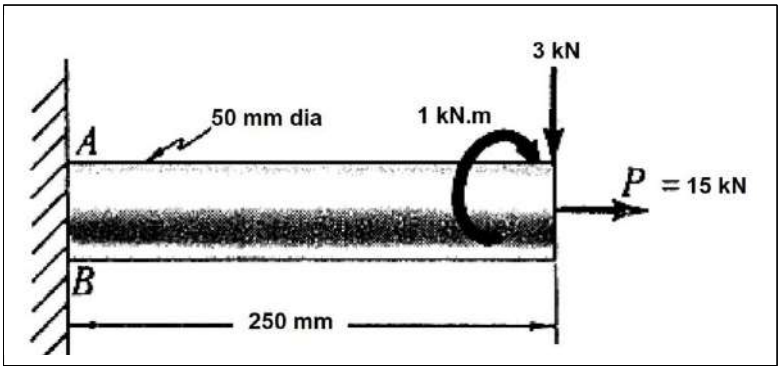
$$\sigma_x = \frac{P}{A} = 7.65\ MPa$$

$$\tau_{xy} = \frac{T*r}{J} = 40.7\ MPa$$

$$\boldsymbol{\sigma_{1,2}} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \& \quad \tau_{max} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$\therefore \sigma_1 = 44.7\ MPa\ (tension)$ & $\sigma_2 = -37.1\ MPa\ (comp.)$ & $\tau_{max} = 40.9\ MPa$

g) Bending, axial load and torsion



At point A

$$\sigma_x = \frac{M*c}{I} + \frac{P}{A}$$

$$= 61.1 + 7.65 = 68.8\ MPa$$

$$\tau_{xy} = \frac{T*r}{J} = 40.7\ MPa$$

$$\boldsymbol{\sigma_{1,2}} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \& \quad \tau_{max} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$\therefore \sigma_1 = 87.7\ MPa\ (tension)$ & $\sigma_2 = -19\ MPa\ (comp.)$ & $\tau_{max} = 53.3\ MPa$

At point B

$$\sigma_x = -\frac{M*c}{I} + \frac{P}{A} = -61.1 + 7.65 = -53.5\ MPa$$

$$\tau_{xy} = \frac{T*r}{J} = 40.7\ MPa$$

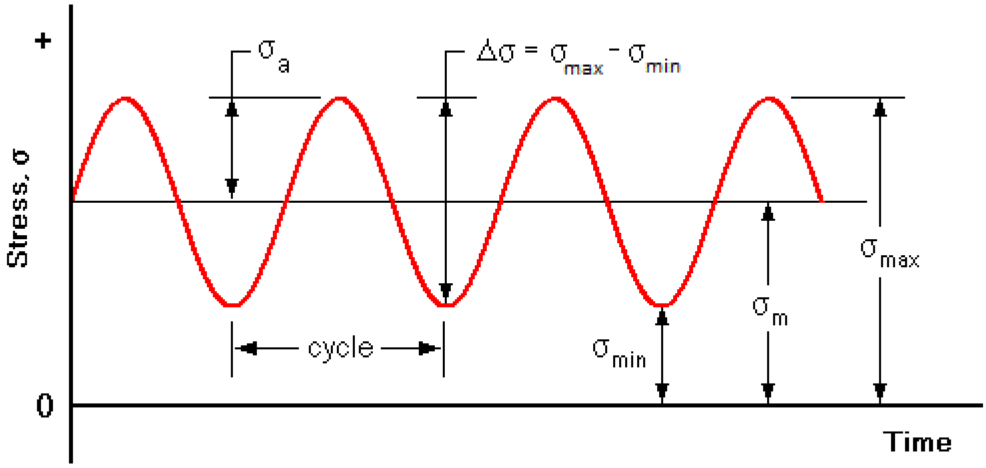
$$\boldsymbol{\sigma_{1,2}} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \& \quad \tau_{max} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$\therefore \sigma_1 = 21.9\ MPa\ (tension)$ & $\sigma_2 = -75.5\ MPa\ (comp.)$ & $\tau_{max} = 48.7\ MPa$

LECTURES EIGHT, NINE & TEN

DESIGN FOR DIFFERENT TYPE OF LOADING

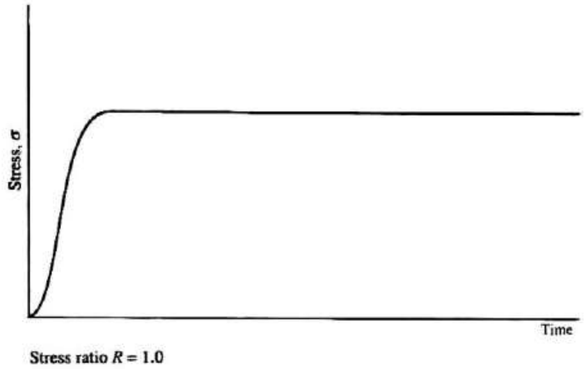
Reference: "Machine Elements in Mechanical Design" 4th Edition in SI units, By: Robert L. Mott, Chapter 5.



Loading Types

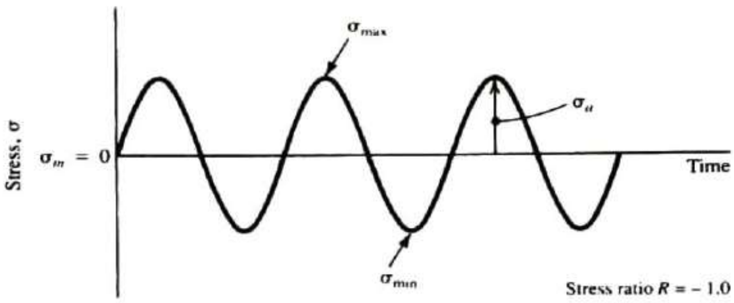
1. Static

- Load applied slowly
- $\sigma_{max.} = \sigma_{min.} = \sigma$
- Stress ratio (R) = 1



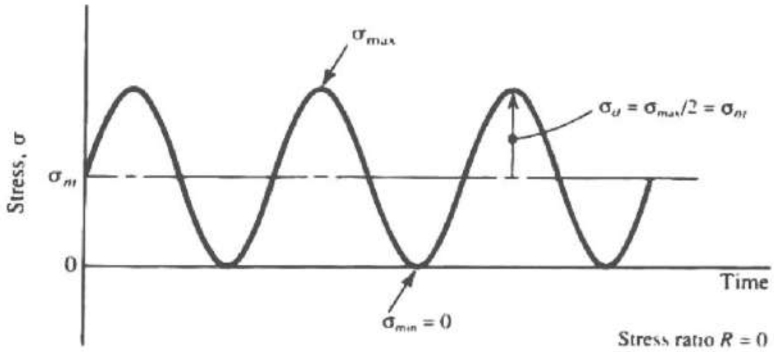
2. Reversed

- $\sigma_m = 0$
- $\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$
- $\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$
- R = -1



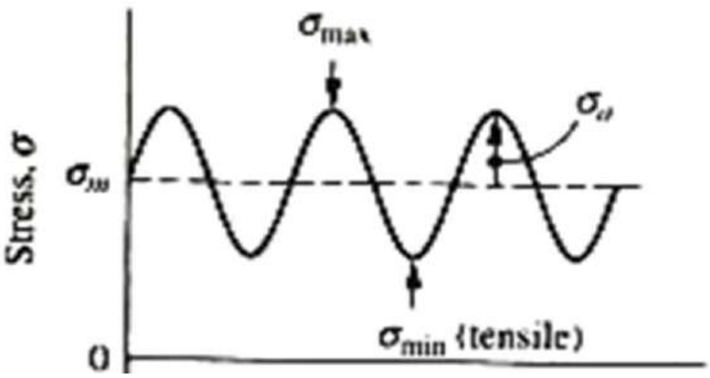
3. Repeated

- $\sigma_a = \sigma_m = \frac{\sigma_{max}}{2}$
- $\sigma_{min} = 0$
- R = 0



4. Fluctuating

- $\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$
- $\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$
- Stress ratio (0 < R < 1)



5. Shock or Impact

- Load applied suddenly & rapidly

6. Random

- When load not regular in their amplitude

Where:

σ_{max} = Maximum Stress & σ_{min} = Minimum Stress

σ_m = Mean (average) Stress

σ_a = Amplitude Stress (Alternating stress)

R = Stress ratio = $\frac{\text{min. stress } (\sigma_{min})}{\text{max. stress } (\sigma_{max})}$

A = Stress ratio = $\frac{\sigma_a}{\sigma_m}$

Repeated and Reversed Stress:

An important example in machine design is a rotating circular shaft loaded in bending such as that shown in Figure (5-3). All parts of the shaft that are in bending see repeated, reversed stress. This is a description of the classical loading case of reversed bending. This machine is called a standard R.R. Moore fatigue test device.

FIGURE 5–2
Repeated, reversed stress

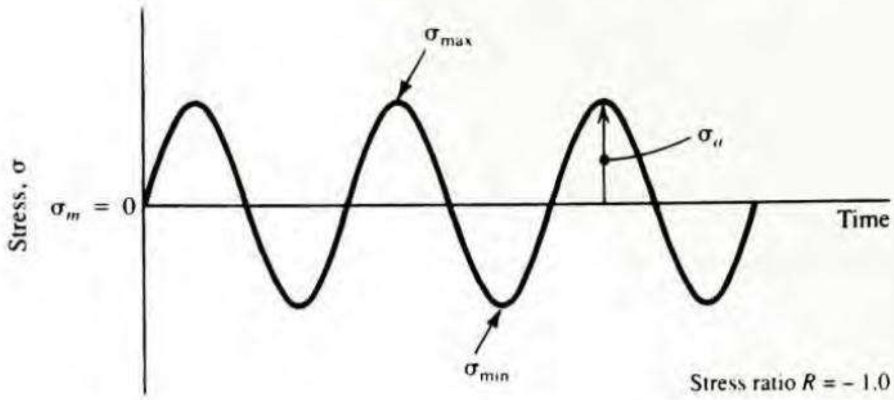
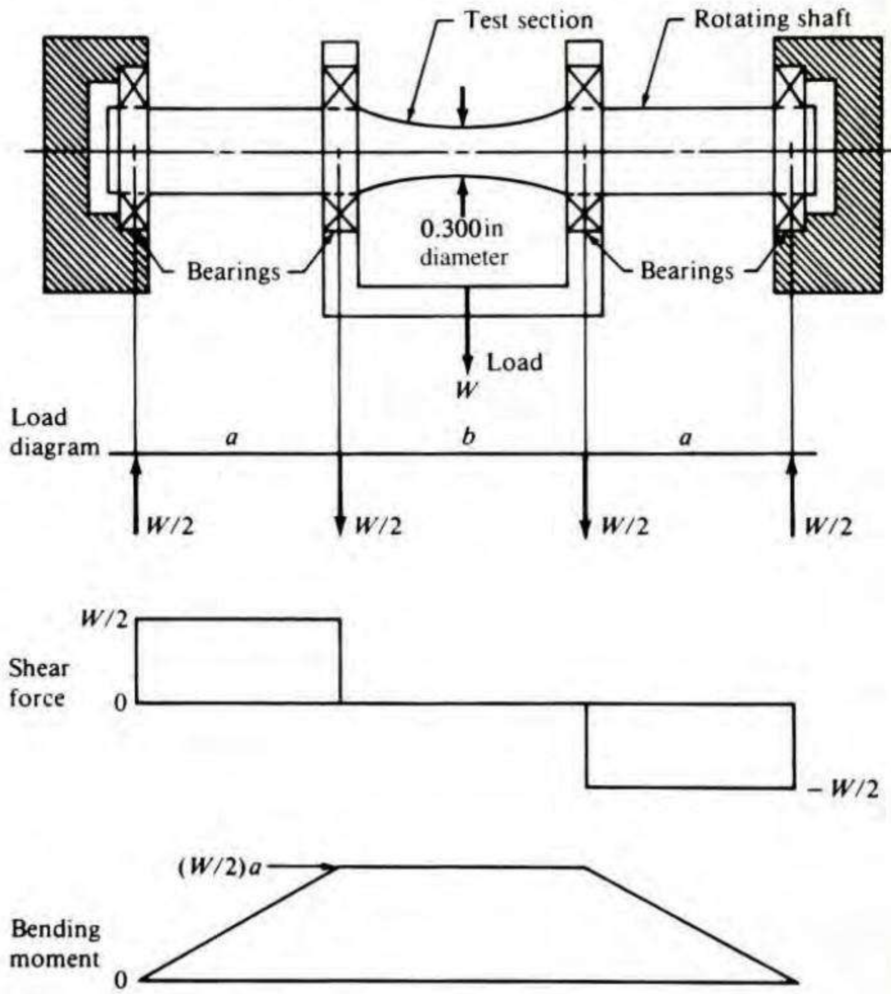


FIGURE 5–3
R. R. Moore fatigue test device



Endurance Strength:

Its ability to withstand fatigue loads. In general, it is the stress level that Material can survive for given number of cycles of loading. If the number of cycles is infinite, the stress level is called the endurance strength. Figure (5-7) (page 173), (Ref. 1), is called the S-N diagram.

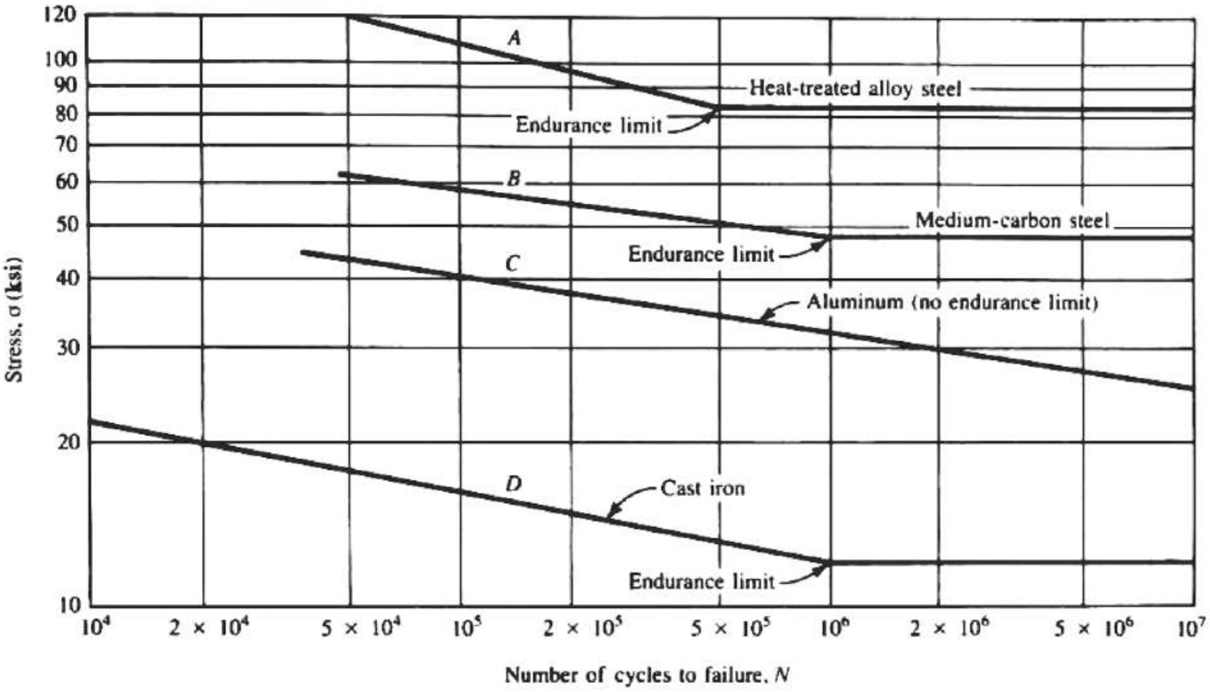


Figure (5-7) Representative endurance strengths.

The approximate value for endurance strength for wrought steel:
Endurance Strength = 0.5 (Ultimate tensile strength)

$$S_n = 0.5\ S_u$$

OR
See figure (5-8) page 175, (Ref. 1), for various surface conditions.

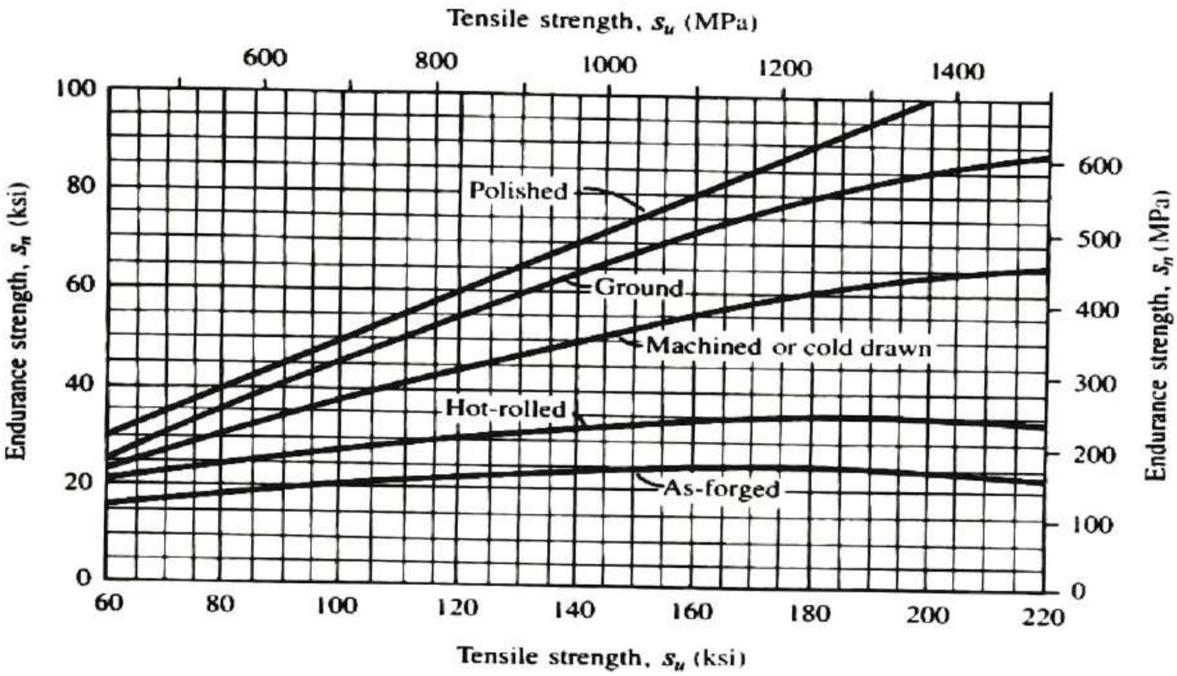


Figure (5-8) Endurance strength S_n versus tensile strength S_u for wrought steel for various surface conditions.

Estimated Actual Endurance Strength (S'_n):

1. Specify the material for the part and determine its ultimate tensile strength, $\mathbf{S_u}$, considering its condition, as it will be used in service.
2. Specify the manufacturing process used to produce the part with special attention to the condition of the surface in the most highly stressed area.
3. Use Figure 5-8 to estimate the endurance strength, S_n
4. Apply a material factor, $\mathbf{C_m}$, from the following list.

Wrought steel: $\mathbf{C_m}$ = 1.00 ; Malleable cast iron: $\mathbf{C_m}$ = 0.80

Cast steel: $\mathbf{C_m}$ = 0.80 ; Gray cast iron: $\mathbf{C_m}$ = 0.70

Powdered steel: $\mathbf{C_m}$ = 0.76 ; Ductile cast iron: $\mathbf{C_m}$ = 0.66

5. Apply a type-of-stress factor: $\mathbf{C_{st}}$ = 1.0 for bending stress; $\mathbf{C_{st}}$ = 0.80 for axial tension.
6. Apply a reliability factor, $\mathbf{C_R}$, from Table 5-1.
7. Apply a size factor, $\mathbf{C_S}$ using Figure 5-9 and Table 5-2 as guides.
8. Compute the estimated actual endurance strength, S'_n from

$$S'_n = S_n\,(C_m)\,(C_{St})\,(C_R)\,(C_S) \quad \text{......... (5-4) Page 174}$$

TABLE 5–1
Approximate reliability factors, C_R

Desired reliability	C_R
0.50	1.0
0.90	0.90
0.99	0.81
0.999	0.75

Figure (5-9) Size Factor

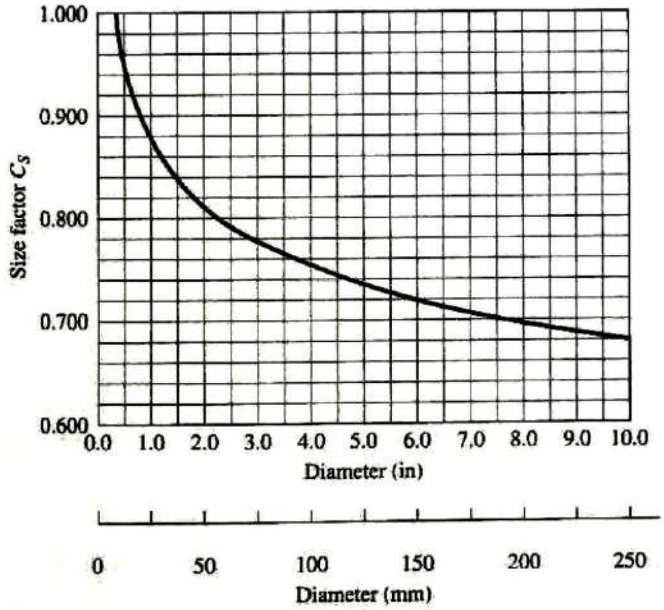
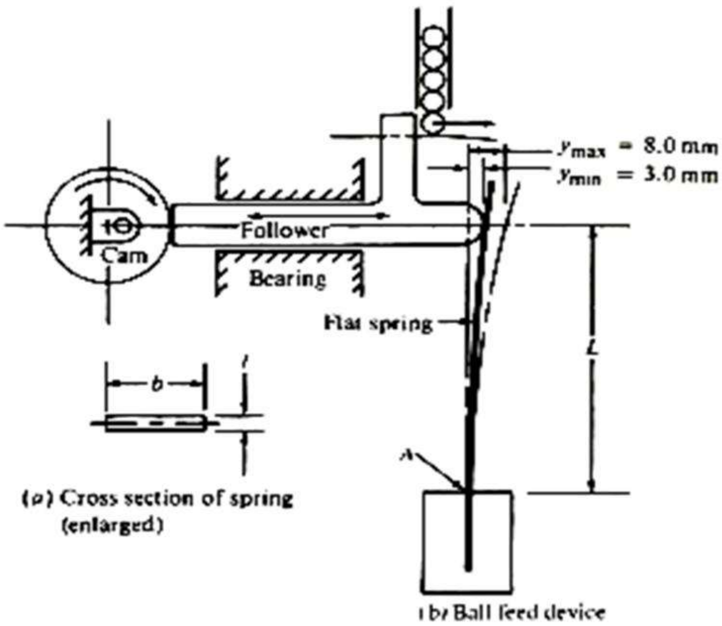


TABLE 5–2 Size factors

Size Range	For D in inches
U.S. customary units	
$D \le 0.30$	$C_S = 1.0$
$0.30 < D \le 2.0$	$C_S = (D/0.3)^{-0.11}$
$2.0 < D < 10.0$	$C_S = 0.859\text{-}0.02125D$
SI units	
Size Range	For D in mm
$D \le 7.62$	$C_S = 1.0$
$7.62 < D \le 50$	$C_S = (D/7.62)^{-0.11}$
$50 < D < 250$	$C_S = 0.859\text{-}0.000837D$

Example 5-1 (Page 170). (Ref. 1): For the flat steel spring shown below, compute the maximum stress, the minimum stress, the mean stress, the alternating stress and the stress ratio R. The length L is 65 mm. The dimensions of the spring cross section are t = 0.80 mm and b=6.0 mm. If E= 207 GPa.



Deflection $(Y) = \frac{PL^3}{3EI} \rightarrow P = \frac{3YEI}{L^3}$; $I = \frac{bt^3}{12} = \frac{6*0.8^3}{12}$ $0.256\ mm^4$

$P_{min} = \frac{3\,(207*10^9)\,(0.256)\,(Y=3)}{65^3} * \frac{1}{10^6} = 1.74\ N$

$P_{max} = \frac{3\,(207*10^9)\,(0.256)\,(Y=8)}{65^3} * \frac{1}{10^6} = 4.63\ N$

Bending Moment at point (A) = P*L

$M_{min} = P_{min} * L = 1.74*65 = 113\ N.mm$; $M_{max} = P_{max} * L = 4.63*65 = 301\ N.mm$

$\sigma = \frac{M\,C}{I}$

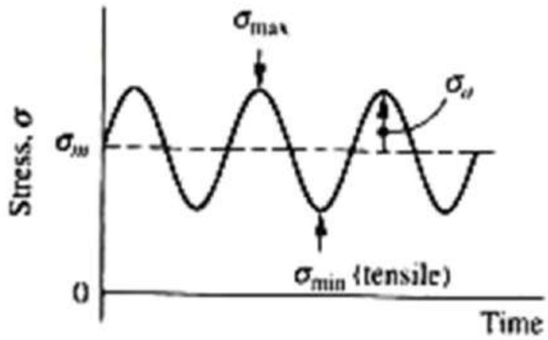
$\sigma_{max} = \frac{301*0.4}{0.256} = 470\ \frac{N}{mm^2}$; $\sigma_{min} = \frac{113*0.4}{0.256} = 176\ N/mm^2$

$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = (470+176)/2 = 323\ MPa$

$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = (470-176)/2 = 147\ MPa$

Stress ratio $R = \frac{\sigma_{min}}{\sigma_{max}}$ $176/470 = 0.37$

Type of loading is Fluctuating.



Example 5-2 (Page 181), Ref.1:

Estimate the actual endurance strength of AISI 1050 cold-drawn steel when used in a circular shaft subjected to rotating bending only. The .shaft will be machined to a diameter of approximately 44.45 mm.

Solution:

S_u *from Appendix 3 = 689.5 MPa* ⟶ $S_n = 262\ MPa$

From figure 5-8 (curve machine or cold drawn).

C_m ⟶ *for wrought steel = 1*

C_R ⟶ *(Design decision) for (Reliability = 0.99) = 0.81*

C_{st} ⟶ *reversed Bending = 1*

C_S ⟶ *from figure 5-9 at D= 44.45 mm,* $C_s = 0.83$

$S'_n = S_n\,(C_m)\,(C_{St})\,(C_R)\,(C_S) = 262\,(1)\,(1)\,(0.81)\,(0.83) = 176\ MPa$

Predictions of Failure:

See section 5-8 (page 186) (For the application of some of the followings theories, see Design Example 5-1 to 5-4 (Page 201 to Page 213)), Ref.1.

1. Maximum normal stress (uniaxial static stress on Brittle Materials):

For tensile stress $Kt\ \sigma < \sigma_d = S_{ut}/N$ .... *(5-9)* or

For compressive stress $Kt\ \sigma < \sigma_d = S_{uc}/N$ ...... *(5-10)*

2. Modified Mohr (Biaxial Static Stress on Brittle materials).

3. Yield strength (uniaxial static stress on Ductile materials). Equations (5-11) & (5-12), this theory is called "**Rankine**".

For tensile stress: $\sigma < \sigma_d = S_{yt}/N$ .... *(5-11)*

For compressive stress: $\sigma < \sigma_d = S_{uc}/N$ .... *(5-12)*

For most wrought ductile metals, $\boldsymbol{S_{ut} = S_{uc}}$

4. Maximum shear stress (Biaxial static stress on ductile material) (Mod. Cons.), this theory is called "**Tresca**".

$$\zeta_{max} < \zeta_d = S_{sy}/N = 0.5S_y/N \quad \text{......... (5-13)}$$

5. Distortion energy (Biaxial or Triaxial stress on ductile material) (Good Predictor). (**Von-Mises theory**)

$$\sigma' = \text{Von Mises stress} = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2} \quad \text{......... (5-14)}$$

$$\sigma' < \sigma_d = S_y/N \quad \text{......... (5-15)} \quad \text{or}$$

$$\sigma' = \sqrt{\sigma_x^2 + 3\tau_{xy}^2} \quad \text{.........(5-16)}$$

6. Goodman (Fluctuating stress on ductile material) (Slightly cons.)

$$\frac{\sigma_a}{S_n'} + \frac{\sigma_m}{S_u} = 1 \quad \text{.......... (5-19)}$$

Or (Design equation) $\frac{K_t\ \sigma_a}{S_n'} + \frac{\sigma_m}{S_u} = \frac{1}{N}$ .... *(5-20)*

7. Gerber (Fluctuating stress on ductile material) (Good Predictor)

$$\frac{\sigma_a}{S_n'} + [\frac{\sigma_m}{S_u}]^2 = 1 \quad \text{........... (5-23)}$$

Or

$$\frac{K_t\ \sigma_a}{S_n'} + [\frac{\sigma_m}{S_u}]^2 = \frac{1}{N}$$

8. Soderberg (Fluctuating stress on Ductile material) (Mod. Cons.)

$$\frac{K_t\,\sigma_a}{S'_n} + \frac{\sigma_m}{S_y} = 1 \quad \text{..........} \ (5\text{-}24)$$

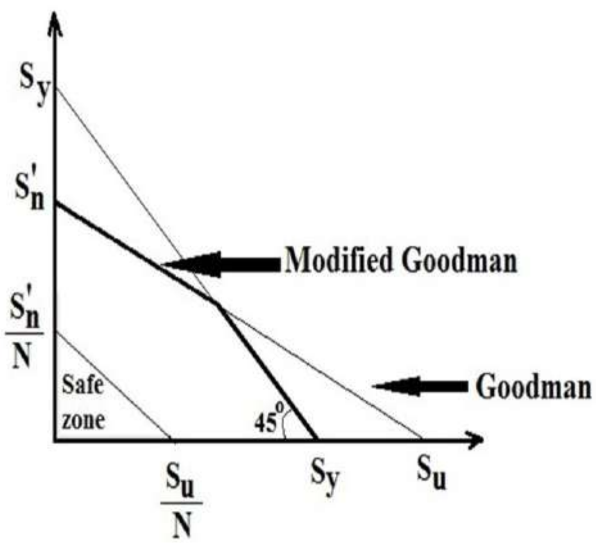
Or

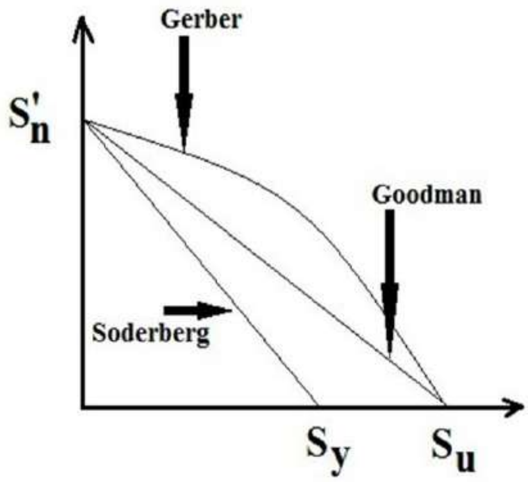
$$\frac{K_t\,\sigma_a}{S'_n} + \frac{\sigma_m}{S_y} = \frac{1}{N}$$

Note: There are many recommended methods for design analysis based on:

1. Material (Brittle or Ductile)
2. Nature of load (Static or Cyclic)
3. Type of stress (Uniaxial or Biaxial).

So there are 16 cases should be discussed (see figure 5-17 to end of page 197)



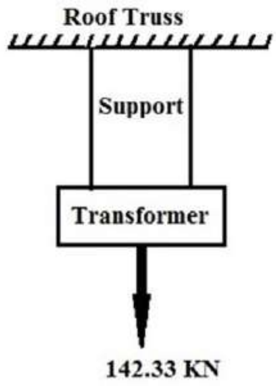


	Type of loading	Tension or Bending	Compression	Torsion
Static ultimate strength	Static	S_u or S_{ut}	S_{uc}	S_{su}
Yield strength	Static	S_y	S_{yc}	S_{sy}
Endurance strength	General or Fluctuating	S_n	S_n	S_{sn}
Endurance strength under actual condition	General or Fluctuating	S'_n	S'_n	S'_{sn}
Mean stresses Amplitude stresses	General or Fluctuating	σ_{xm} σ_{xa}	σ_{-xm} σ_{-xa}	τ_{xym} τ_{xya}

	S_n	S_{sn}	S_{su}	S_{sy}
Wrought steel	**0.5 Su**	**0.577 Sn**	**0.75 Su**	**0.577 Sy**

Design Example 5-1 (Page 201). Ref.1:

A large electrical transformer is to be suspended from a roof truss of a building. The total weight of the transformer is. 32 000 lb. Design the means of support.



- The load is static
- Two rod was assumed
- The end of rod will be threaded
- Only one rod assumed to carry the load during instillation.
- From Appendix 3, AISI 1040 cold-drawn steel assumed as a material of rod.

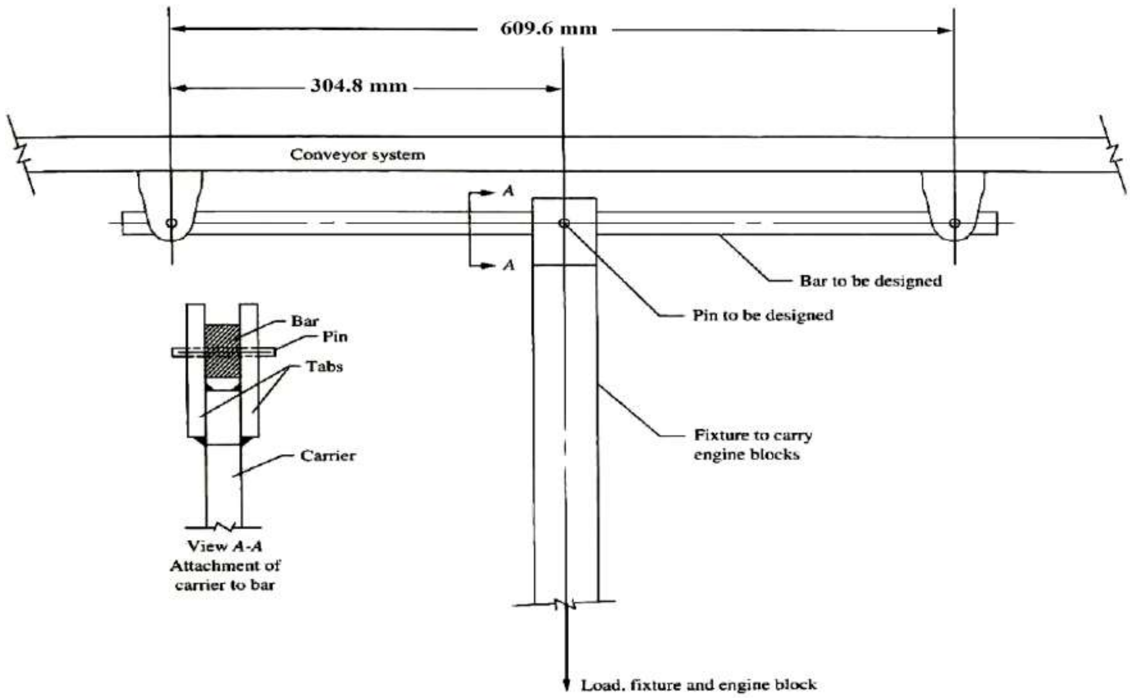
[S_y = 489.54 MPa]

- Critical place of stress in the threaded part of rod.
- $\sigma_{design} = \frac{S_{yt}}{N}$ & σ *should be* ≤ σ_d …….. *(5-11)*

$$\sigma_d = \frac{489.54}{3} = 163.18\ MPa\ , and\ \ \sigma_d = \frac{F}{A} \rightarrow A = \frac{142000}{163.18} = 871\ mm^2$$

- Find standard diameter of thread with higher tensile stress area.
- Stress concentration (K_t) is neglected in this case (give your comments).

Design Example 5-2 (Page 202). Ref.1:



A part of a conveyor system for a production operation is shown in Figure 5-18. Design the pin that connects the horizontal bar to the fixture. The empty fixture weighs 378 N. A cast iron engine block weighing 1000.8 N is hung on the fixture to carry it from one process to another where it is then removed. It is expected that the system will experience many thousands of cycles of loading and unloading of the engine blocks

- Weight of empty fixture = 378 N
- Weight of C.I. engine block = 1000.8 N
- Many thousands of Cycles of loading & unloading.
- To design a pin then:

$\tau = \frac{F}{2A}$ & $F_{min} = 378\ N$, $F_{max} = 1379\ N$

$F_{mean} = \frac{F_{max} + F_{min}}{2} = 879\ N$ & $F_a = \frac{F_{max} - F_{min}}{2} = 501\ N$

$$\tau_m = \frac{F_m}{2A} \quad \& \quad \tau_a = \frac{F_a}{2A} \quad \& \quad \frac{K_t\,\tau_a}{S'_{sn}} + \frac{\tau_m}{S_{su}} = \frac{1}{N}$$

- **Note:** comment for using equation $\frac{K_t\,\tau_a}{S'_{sn}} + \frac{\tau_m}{S_{su}} = \frac{1}{N}$
- From figure if Goodman equation are used.
- If there is shear so *use* $\frac{K_t\,\tau_a}{S'_{sn}} + \frac{\tau_m}{S_{su}} = \frac{1}{N}$
- Choose material for pin AISI 1020 cold-drawn steel. From **APPENDIX 3** Sy = 351.64 & Su = 420.59 MPa
- In the absence of shear strength data use estimates:

$S'_{sn} = 0.577\,S'_n$ & $S_{su} = 0.75\,S_u$

$S_{su} = 0.75\,S_u = 0.75 * 420.5 = 315.4\ MPa$

From figure (5-8), S_n*=144.9 MPa and find data for (*C_m, C_{st}, C_R *and* C_s*)*

$S'_{sn} = 0.577\,S'_n = (0.577) * [\,S_n * C_m * C_{st} * C_R * C_s]$

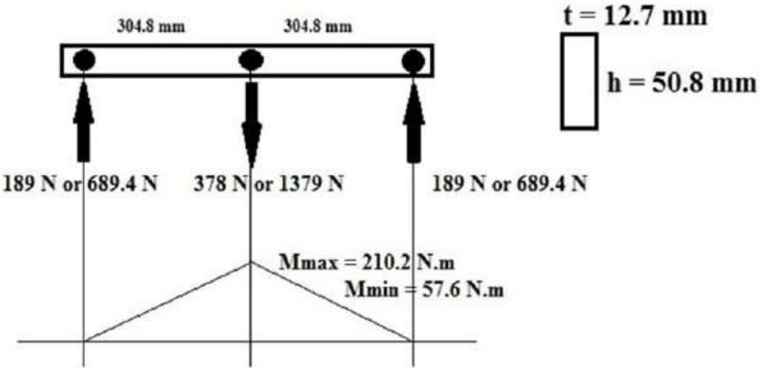
*= 0.577*144.8*1*1*0.75*1 = 62.6 MPa*

$$\frac{K_t\,\tau_a}{S'_{sn}} + \frac{\tau_m}{S_{su}} = \frac{1}{N}$$

$\frac{1}{(N=4)} = \frac{(K_t=1) * \tau_a}{62.6} + \frac{\tau_m}{315.4}$ $(N = 4\ for\ mild\ shock)$, the pin will be uniform diameter, K_t = 1.

A = 21.579 mm² & D = 5.24 mm (we should choose higher diameter D=12.7 mm)

Design example 5-3, (Page 204)Ref.1: Find the safety factor (N) of the horizontal arm in Ex.5-2 it is propose to make the bar from steel in the form of rectangular bar. Assume pin in the middle = 12.7 mm and at end are 9.52 mm, use material, from Appendix 3, AISI1020 hot-rolled with S_y = 206.85 MPa & S_u = 379.2 MPa.



$\sigma = \frac{M}{S} = \frac{M\,C}{I}$,

$M_{mean} = \frac{M_{max} + M_{min}}{2} = 133.9\ N.m$

$M_a = \frac{M_{max} - M_{min}}{2} = 76.3\ N.m$

Use Goodman line.

$$\frac{1}{N} = \frac{\sigma_m}{S_u} + \frac{K_t\,\sigma_a}{S_n'} \quad , \quad \sigma_m = \frac{M_m}{S} \quad \& \quad \sigma_a = \frac{M_a}{S} \quad , \quad S = Section\ Modulus$$

$S_n' = S_n\,(C_m)\,(C_{St})\,(C_R)\,(C_S)$

C_m = 1 for wrought-hot rolled steel

C_{st} = 1 for repeated bending stress

C_R = let = 0.75 to achieve a reliability of 0.999 (table 5-1)

C_s = size factor (fig. (5-9) page 175 [curve between CS & diameter]

If it is not round then diameter = 0.808 $\sqrt{ht}$

C_S = 0.9 from figure (5-8) at D = 0.808 $\sqrt{50.8 * 12.7}$ = 20.5 mm

S_n = 137.9 MPa from figure 5-8 for hot rolled steel & Su = 379 MPa

S_n' = 137.9*1*1*0.75*0.9 = 93.08 MPa

$$\frac{1}{N} = \frac{\sigma_m}{S_u} + \frac{K_t\,\sigma_a}{S_n'}$$

$$\frac{1}{N} = \frac{M_m}{(S)(S_u)} + \frac{K_t\,M_a}{S\,(S_n')} = \frac{1}{S}\left[\frac{M_m}{S_u} + \frac{K_t\,M_a}{S_n'}\right] \quad \rightarrow \quad N \cong 4\ \ at\ K_t = 1$$

Because the ratio of d/h < 0.5 from Appendix (15),

$S = \frac{t\,(h^3 - d^3)}{6h} = 0.46 * 10^4\ \ mm^4$ *And t = 12.7 mm, then h =46.99 mm*

EX.1: A bar of steel has S_{ut} = 700 MPa, Sy= 500 MPa & S'_n= 200 MPa. Find safety factor (N) against static and fatigue failures for:

a. $\tau_m = 140\ MPa$ ***b.*** $\tau_m = 140\ MPa$, $\tau_a = 70\ MPa$

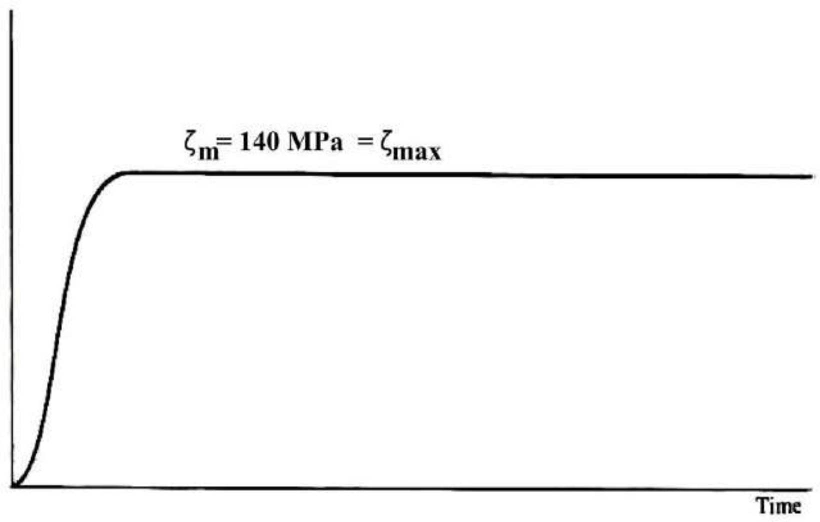
c. $\tau_{xym} = 100\ MPa$, $\sigma_{xa} = 80\ MPa$

d. $\sigma_{xm} = 60\ MPa$, $\sigma_{xa} = 80\ MPa$, $\tau_{xym} = 70\ MPa$, $\tau_{xya} = 35\ MPa$

Solution:

a) $S_{Sy} = 0.577\ S_y = 0.577*500 = 288\ MPa$

$N = \frac{S_{Sy}}{\tau_m} = \frac{288}{140} = 2.06$



ζm= 140 MPa = ζmax = ζmin , ζa =0

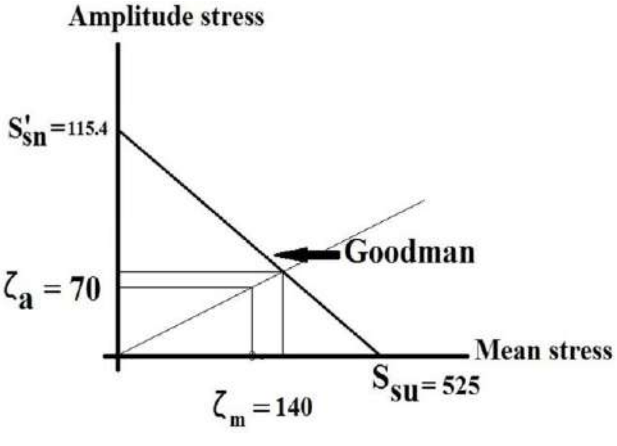
b) $\tau_{max} = \tau_m + \tau_a = 140 + 70 = 210\ MPa$

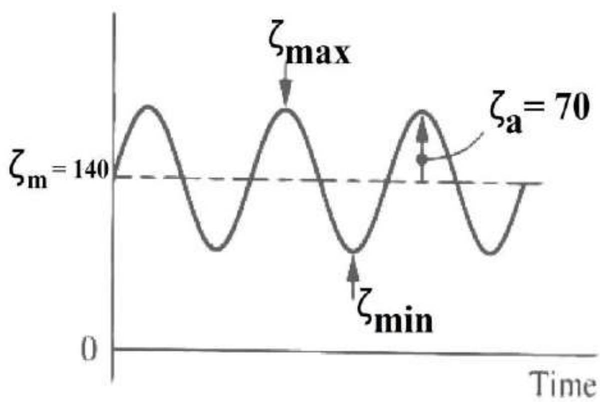
$N\ (static) = \frac{S_{sy}}{\tau_{max}} = \frac{288}{210} = 1.37$

$S'_n = 200\ MPa$ $S'_{sn} = 0.577 * S'_n = 0.577*200 = 115.4\ MPa$

$S_{su} = 0.75 S_u = 0.75*700 = 525\ MPa$

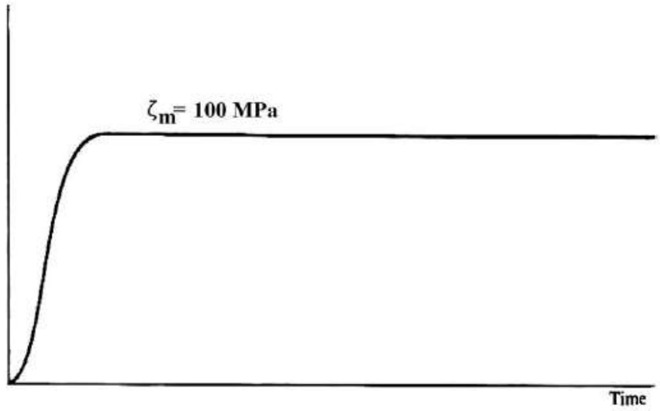
$$\frac{\tau_a}{S'_{sn}} + \frac{\tau_m}{S_{su}} = \frac{1}{N} \quad \rightarrow \quad \frac{70}{115.4} + \frac{140}{525} = \frac{1}{N} \quad \rightarrow \quad N = 1.145$$





c) Use Distortion energy theory (equation 5-16):

$\sigma' < \sigma_d = \frac{S_y}{N}$ & $\sigma' = \sqrt{\sigma_x^2 + 3\tau_{xy}^2}$



$\sigma' = \sqrt{(80)^2 + 3(100)^2} = 191\ MPa$

$N\ (static) = \frac{S_y}{\sigma'} = \frac{500}{191} = 2.62$

$N\ (fatigue) = \frac{S_m}{\sigma'_m}$ or $= \frac{S_a}{\sigma'_a}$

$\sigma'_m = \sqrt{\sigma_{xm}^2 + 3\tau_{xym}^2} = \sqrt{0 + 3(100)^2} = 173\ MPa$

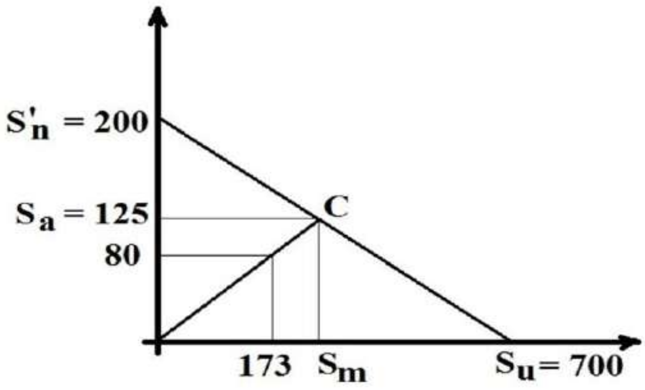
$\sigma'_a = \sqrt{\sigma_{xa}^2 + 3\tau_{xya}^2} = \sqrt{80^2 + 0} = 80\ MPa$

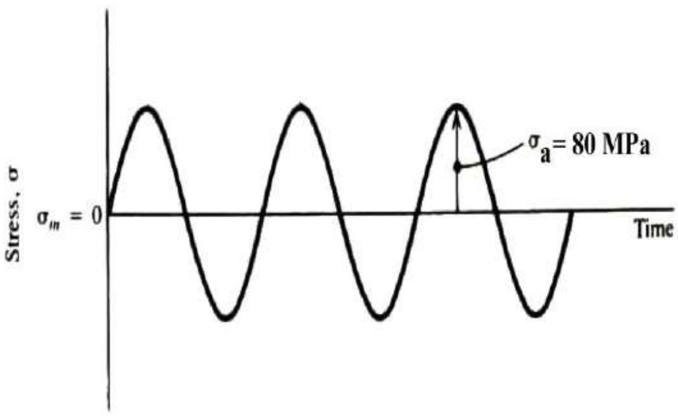
Now, from figure Sm = 270 MPa & Sa = 125 MPa

N (fatigue) = 270/173 = 125/80 = 1.56

OR

$\frac{1}{N} = \frac{\sigma'_m}{S_u} + \frac{\sigma'_a}{S'_n} \rightarrow \frac{1}{N} = \frac{173}{700} + \frac{80}{200} \rightarrow N = 1.55$



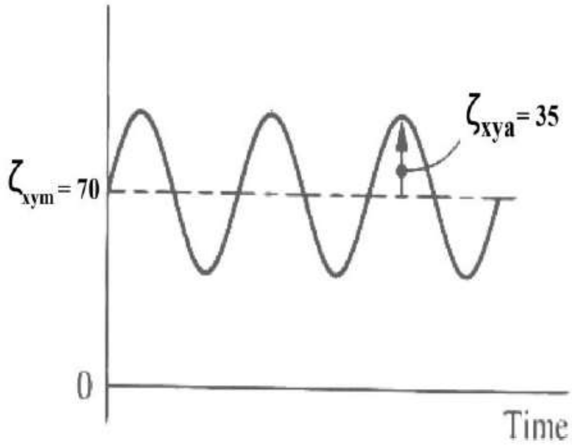


d) From figure $\sigma_{x(max)} = \ 60 + 80 = 140\ MPa$

$\tau_{xy(max)} \ = 70 + 35 = 105\ MPa$

$\sigma'_{max} = \sqrt{140^2 \ + 3(105)^2} \ = \ 229\ MPa$

$N\ (fatigue) = \frac{S_y}{\sigma'_{max}} \ = \ \frac{500}{229} \ = \ 2.18$



For fatigue (as before):

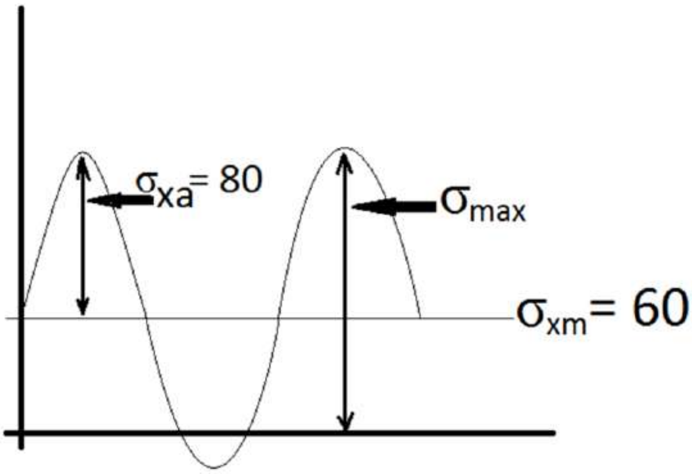
$\sigma'_m \ = \ \sqrt{60^2 \ + \ 3(70)^2} \ = \ 135\ \ MPa$

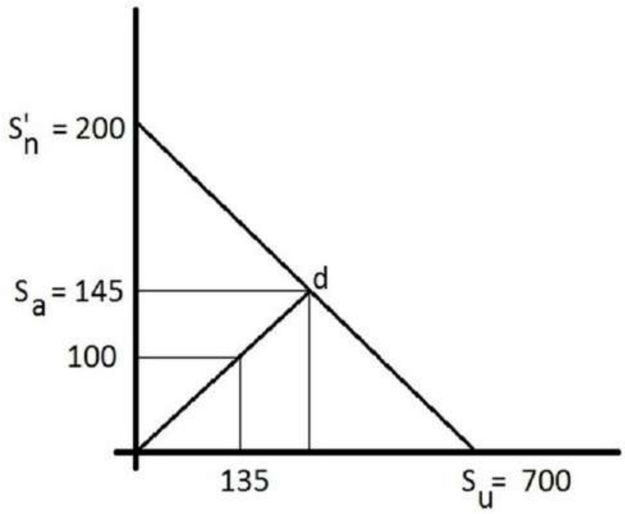
$\sigma'_a \ = \ \sqrt{80^2 \ + \ 3(35)^2} \ = \ 100\ MPa$

$N\ (fatigue) = \frac{S_m}{\sigma'_m} \ = \ \frac{S_a}{\sigma'_a} \ = \ \frac{145}{100} \ = \ 1.45$

Or

$\frac{1}{N} \ = \ \frac{\sigma'_m}{S_u} \ + \ \frac{\sigma'_a}{S'_n} \quad \rightarrow \quad \frac{1}{N} \ = \ \frac{135}{700} \ + \ \frac{100}{200} \quad \rightarrow \quad N = 1.45$

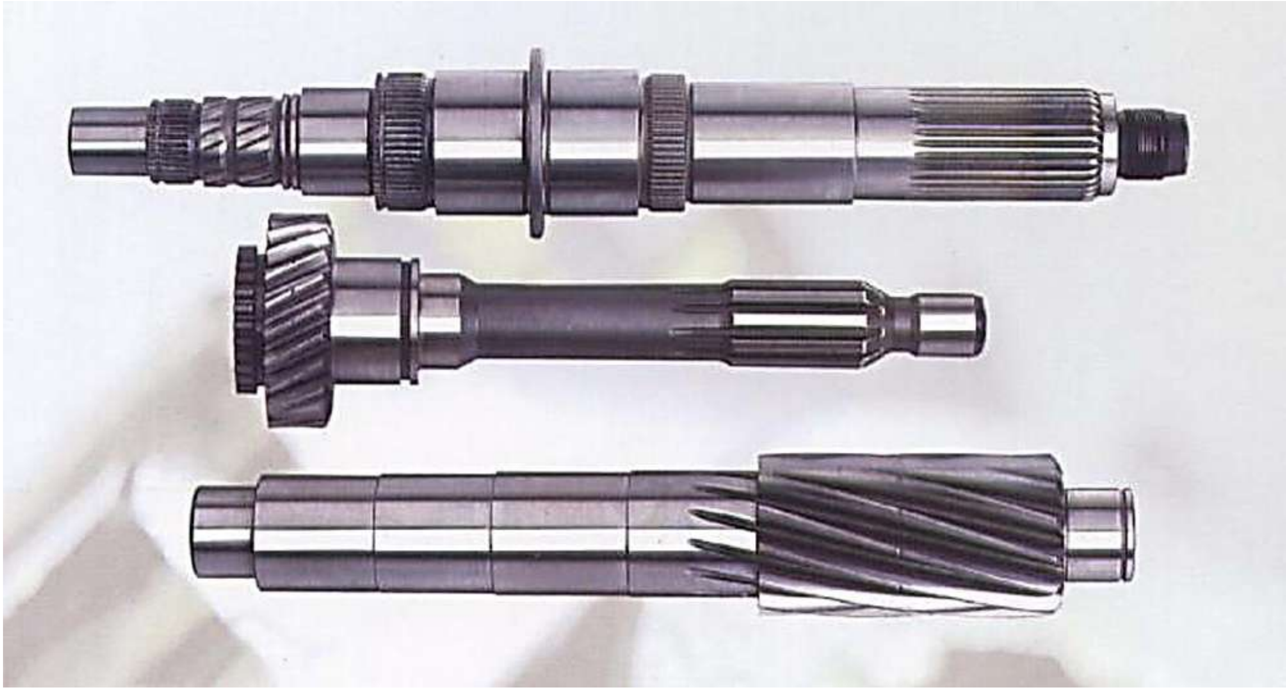




1-Solve problems in chapter 5 (Page 219), [Q1 – Q8 – Q9 –Q11 – Q19 – Q28 – Q30 – Q42 – Q67 – Q77].

LECTURES ELEVEN, TWELVE &THIRTEEN

DESIGN OF SHAFTS



References:

Machine Elements in Mechanical Design by Robert L. Mott, P.E. (Chapter 12)

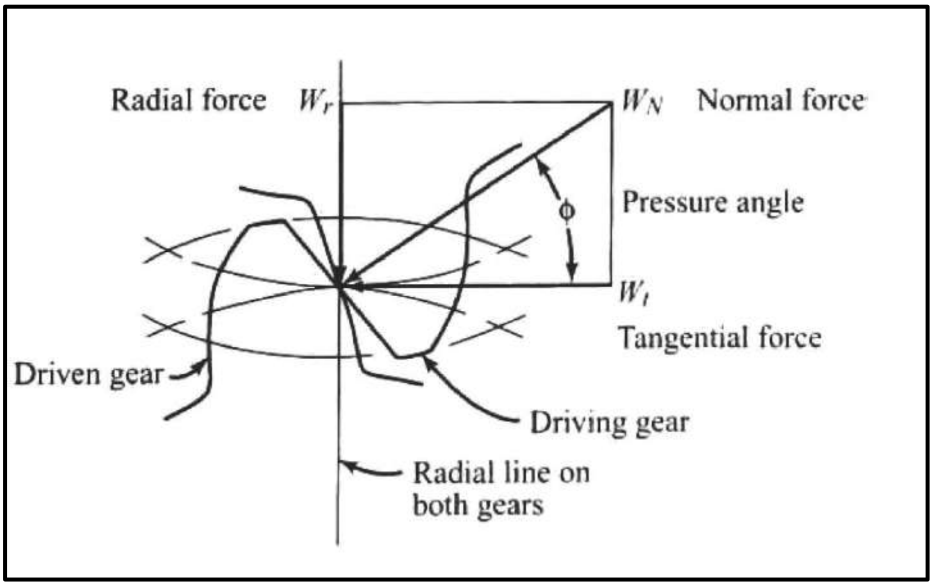
Note: Read section (12-1) objective of this chapter (Page 532)

Shaft Design Procedure (Sec. 12-2, Page 532)

1. Determine rpm of shaft
2. Determine power or torque
3. Determine part mounted on shaft (gears, belts…)
4. Determine location of bearing
5. Determine location of (key, fillets …)
6. Draw torque diagram
7. Determine forces on shaft
8. Resolve components of forces in horizontal & vertical plan
9. Solve the reaction on bearing
10. Produce S.F. Diagram & B.M. diagram
11. Select material and specify its condition (Suggested material is : AISI 1040, 4140, 4340, 4640, 5150, 6150, and 8650). Then determine S_Y & S_{ut}
12. Determine appropriate design stress, considering the manner of loading (smooth, shock…)
13. Analyze each critical point of shaft then decide which point is safer
14. Specify the final dimensions

Forces exerted on shafts (Sec. 12-3 Page 535)

1. **Spur Gears:**



$$Power = \frac{2\pi\, n\, T}{60} \quad \ldots (S.I.) \quad or \quad Power = \frac{T\, n}{63000} \quad \ldots .(British)$$

$$W_t = \frac{2\,T}{D} = Tangentional\ Force \quad \ldots\ldots .(12-2)$$

$$W_r = \ W_t\ tan\emptyset\ = Radial\ load \quad \ldots\ldots .(12-3)$$

where: $P = power\ being\ transmitted$

n = Rotational speed

T = Torque

D = pitch diameter of gear = m N

m = Module

N = No. of teeth

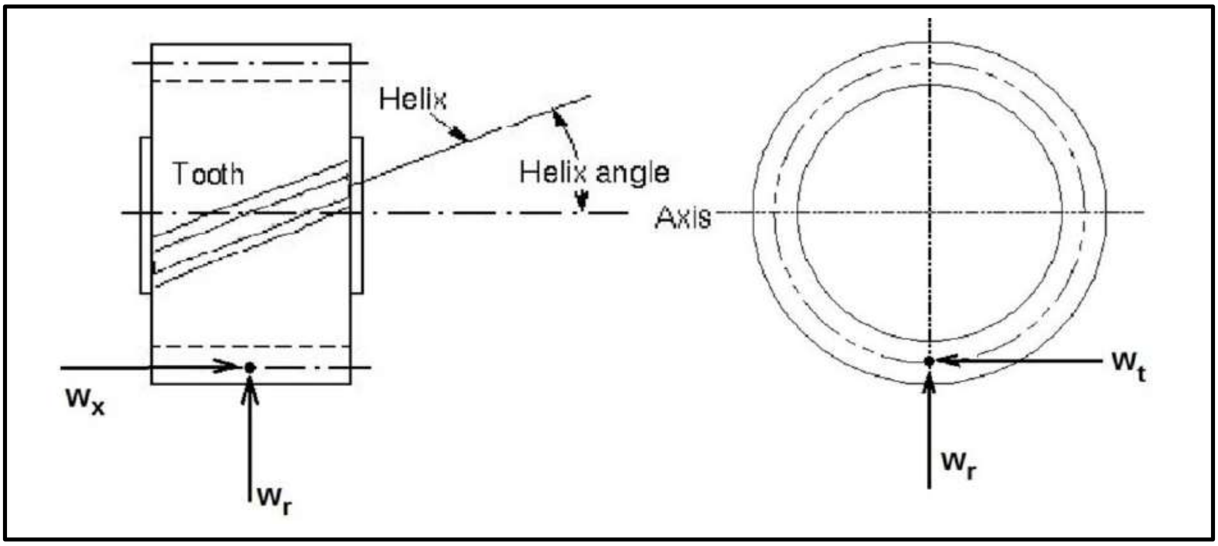
$\emptyset = pressure\ angle$

2. Helical Gears:

$$W_r = \frac{W_t * tan\emptyset_n}{cos\varphi} \quad \ldots\ldots (12-4)$$

$$W_x = axial\ load = W_t\ tan\varphi \quad \ldots\ldots (12-5)$$

Where $\varphi = Helix\ angle$



3. **Bevel Gears:**

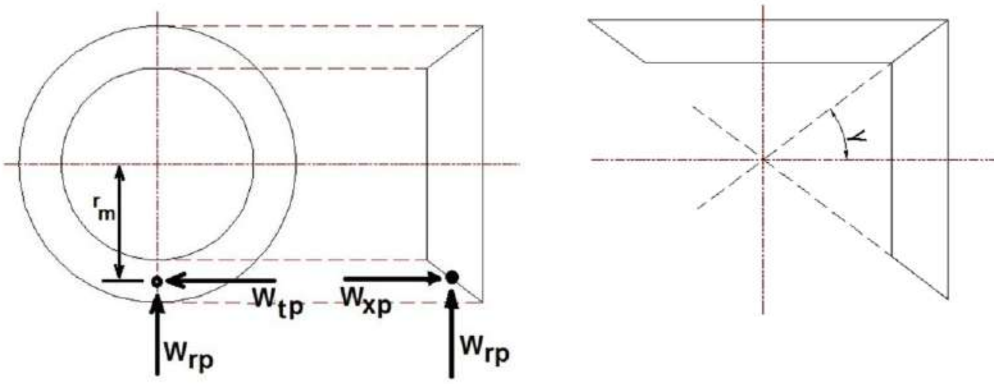
$$W_{t\,p} = W_{t\,G}$$

$$W_{x\,p} = W_{r\,G}$$

$$W_{r\,p} = W_{x\,G}$$

$$r_m = \frac{d}{2} - \frac{F}{2}\sin\gamma$$

$Where\ F = Face\ width$

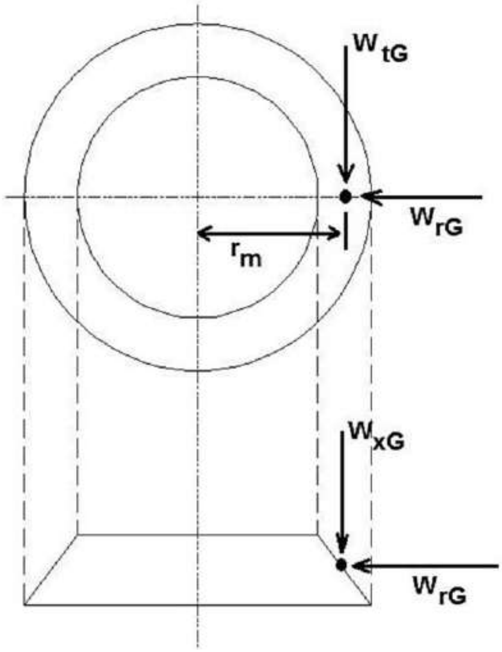


$$W_{r\,p} = W_t\ tan\emptyset\ cos\gamma$$

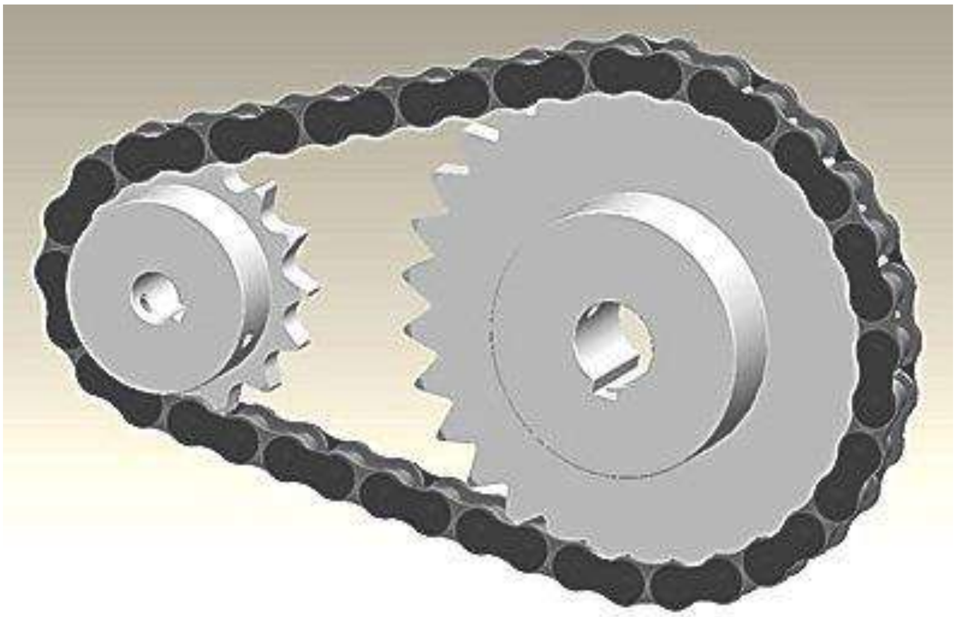
$$W_{x\,p} = W_t\ tan\emptyset\ sin\gamma$$

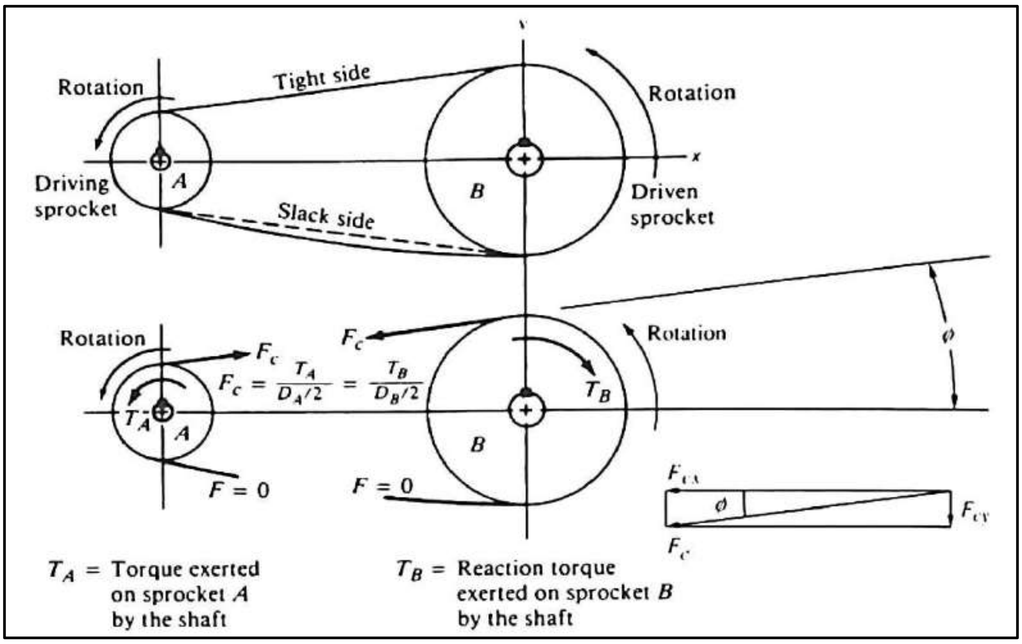
$$W_{t\,p} = \frac{T}{r_m} \quad (r_m = mean\ raduis\ of\ pinion)$$

$$\gamma = Pitch\ cone\ angle\ for\ pinion = tan^{-1}\left(\frac{N_p}{N_G}\right)$$



4. Chain Sprocket:

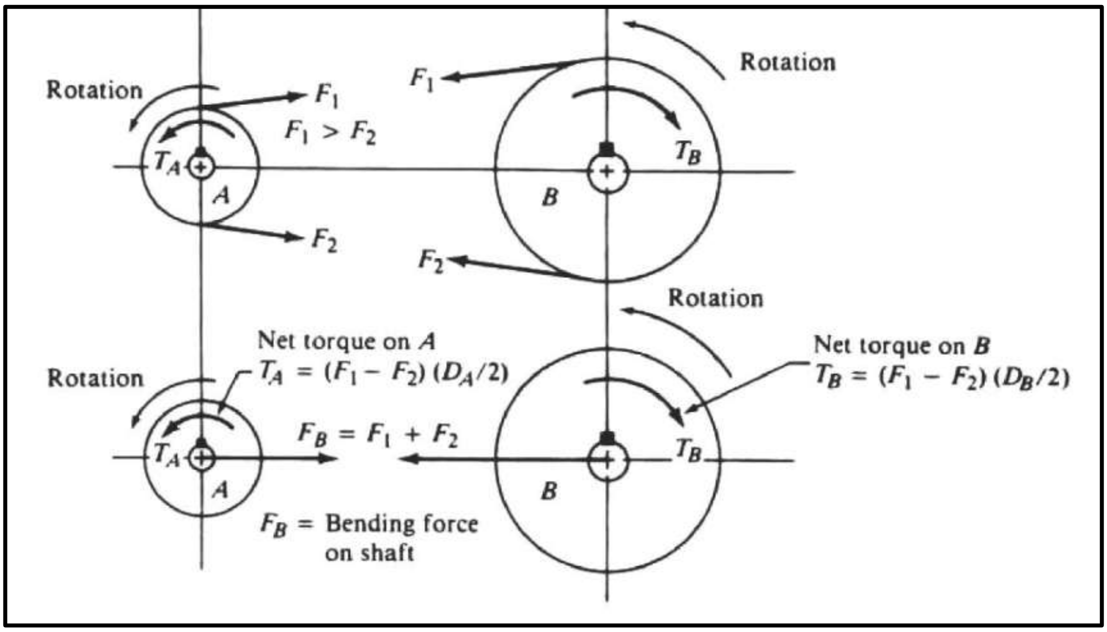


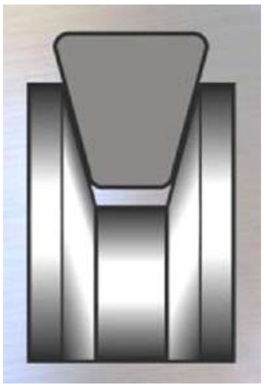


$$F_c = \frac{2\,T}{D} \quad \ldots\ldots (12-6) \qquad ; \; where\ D = pitch\ diameter\ of\ sprocket$$

If $\emptyset = small = 0$ $\quad \therefore\ F_{cx} = F_c \quad \& \quad F_{cy} = 0$

5. V-Belt sheaves:





$$F_1 * \frac{D}{2} - F_2 * \frac{D}{2} = T_B \quad \Rightarrow \quad F_1 - F_2 = \frac{2\,T_B}{D} \quad \ldots\ldots(1)$$

Assume a ratio of $\left(\frac{F_1}{F_2}\right) = 5\ (If\ not\ given)\ \ \ldots\ldots(2)$

From eq. (1) & eq. (2) Find F_1 & F_2 $then\ load\ on\ shaft = F_1 + F_2$

6. Flat-Belt sheaves:

Same as in V-Belt but assume $\left(\frac{F_1}{F_2}\right) = 3$ (if not given) then load on shaft = $F_1 + F_2$

Stress Concentration in Shafts (sec. 12-4, Page 540):

- Keyseats → $K_t = 2$ $for\ profile\ keyseats$
  → $K_t = 1.6$ $for\ sled\ runner\ keyseats$
- Shoulder fillets → $K_t = 2.5$ $for\ sharp\ fillet$
  → $K_t = 1.5$ $for\ well-round\ fillet$
- Retaining ring grooves → $K_t = 3$ $or\ increase\ diameter\ by\ 6\%$

When shaft designed according to strength the following cases can be considered:

a) Shaft subjected to bending only (Axle)

$$\sigma_x = \frac{M*C}{I} = \frac{M*{}^{d}/_{2}}{\pi d^4/64} = \frac{32M}{\pi d^3}$$

b) Shaft subjected to torsion only

$$\tau_{xy} = \frac{T*C}{J} = \frac{T*{}^{d}/_{2}}{\pi d^4/_{32}} = \frac{16\,T}{\pi\, d^3}$$

c) Shaft subjected to torsion & bending

$$\tau_{max} = \frac{16}{\pi d^3}\sqrt{M^2 + T^2} \qquad (Max.\,Shear\ stress\ theory)$$

$$\sigma_{max} = \frac{32}{\pi d^3}\sqrt{M^2 + \frac{3}{4}T^2} \qquad (Von - Mises\ theory)$$

$$\sigma_{max} = \frac{16}{\pi d^3}\left(M + \sqrt{M^2 + T^2}\right) \qquad (max.\,normal\ stress\ theory)$$

d) Shaft subjected to torsion, bending and axial load

$$\sigma_x = \frac{M*C}{I} + \frac{F_a}{A} = \frac{32M}{\pi d^3} + \frac{4F_a}{\pi d^2} = \frac{32}{\pi d^3}\left(M + \frac{F_a*d}{8}\right)$$

$$\tau_{max} = \frac{16}{\pi d^3}\sqrt{\left(M + \frac{F_a*d}{8}\right)^2 + T^2} \quad \ldots\ldots Max.\,Shear\ stress\ theory$$

$$\sigma_{max} = \frac{32}{\pi d^3}\sqrt{\left(M + \frac{F_a*d}{8}\right)^2 + \frac{3}{4}T^2} \quad \ldots\ldots Von - Mises\ theory$$

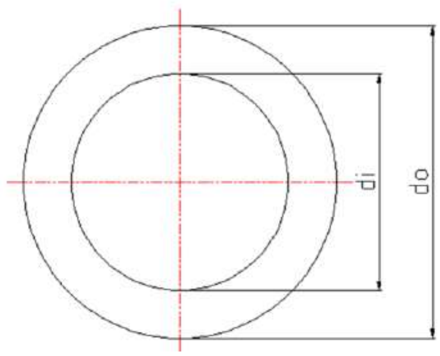
$$\sigma_{max} = \frac{16}{\pi d^3}\left[\left(M + \frac{F_a*d}{8}\right) + \sqrt{\left(M + \frac{F_a*d}{8}\right)^2 + T^2}\right] \ldots Max.\,Normal\ stress\ theory$$

- **For Hollow shaft and combined factors:**

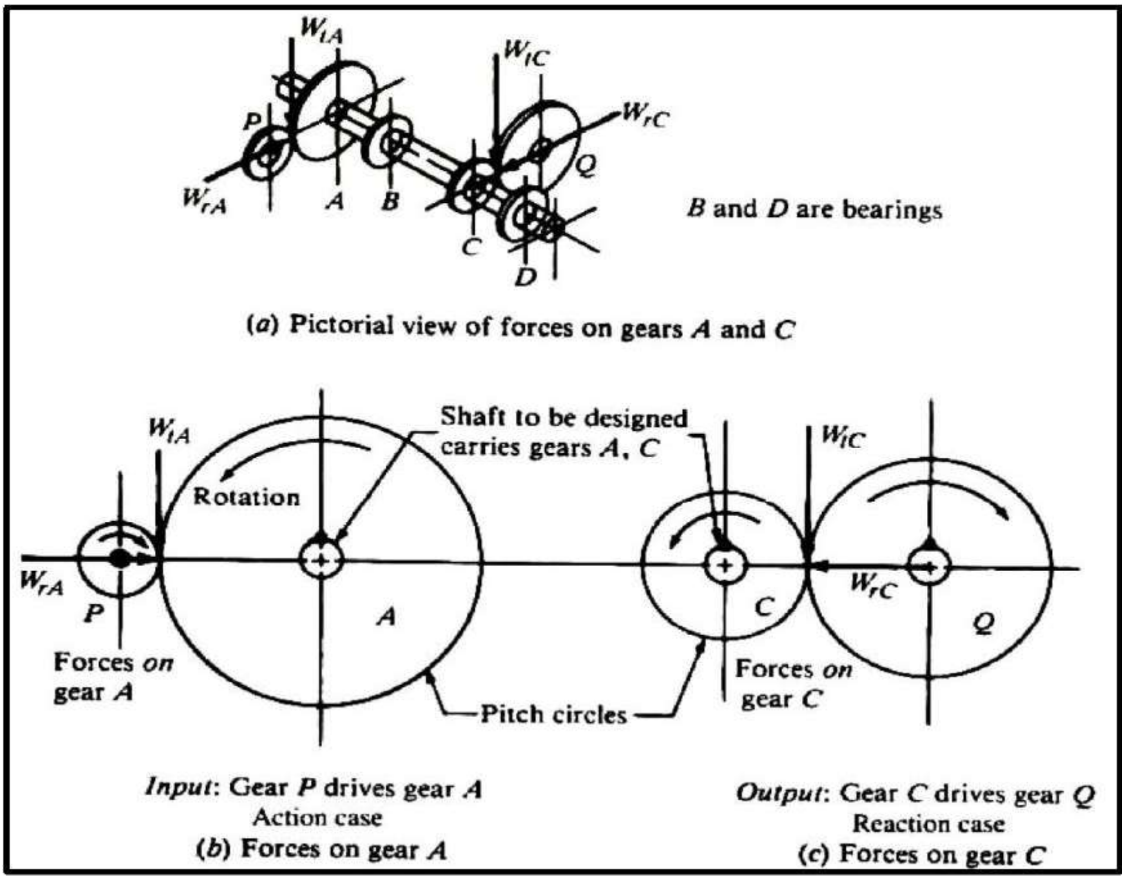
$$A = \frac{\pi}{4}\left(d_o^2 - d_i^2\right) = \frac{\pi d_o^2}{4}\left(1 - K^2\right)\ ;\ where\ K = \frac{d_i}{d_o}$$

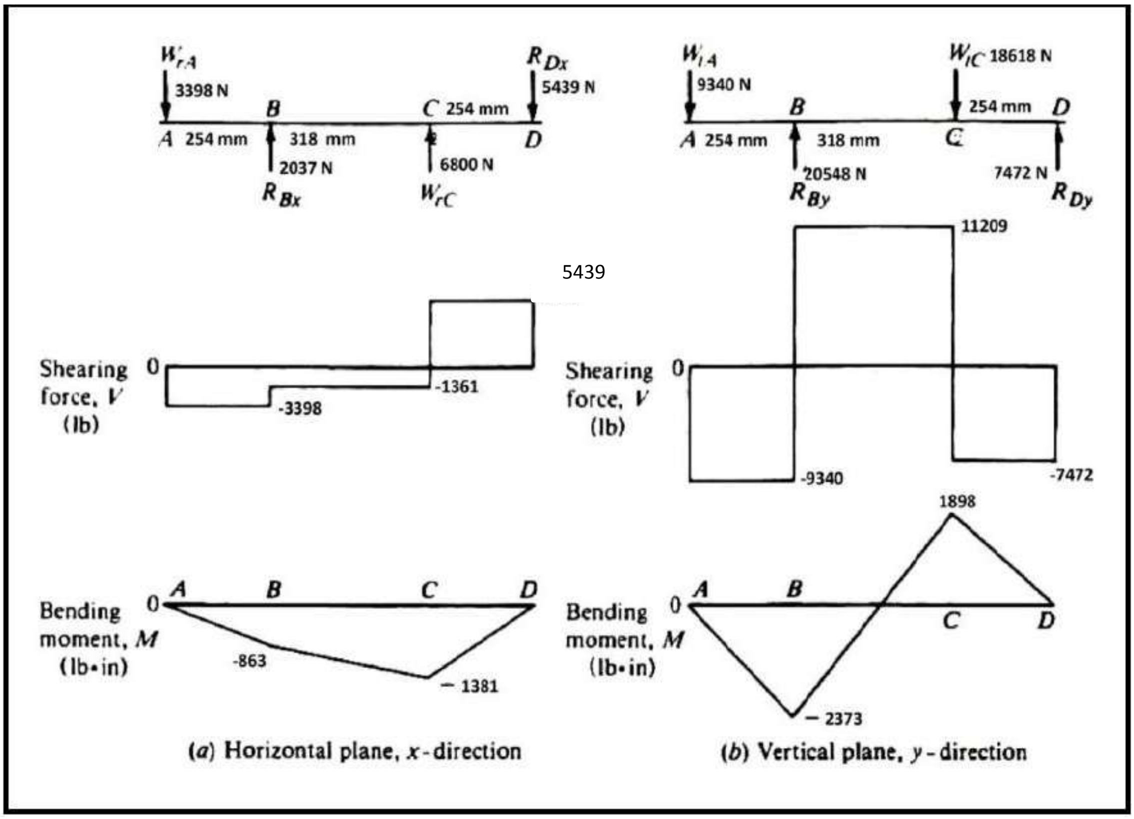
$$I = \frac{\pi}{64}\left(d_o^4 - d_i^4\right) = \frac{\pi d_o^4}{64}\left(1 - K^4\right)$$

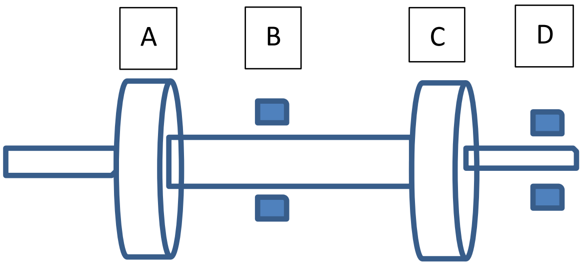
$$J = \frac{\pi}{32}\left(d_o^4 - d_i^4\right) = \frac{\pi d_o^4}{32}\left(1 - K^4\right)$$



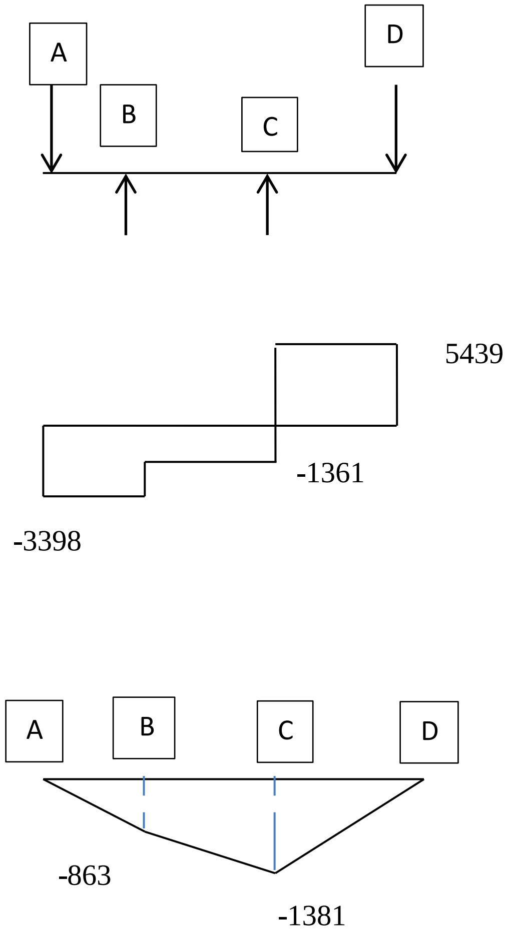
Example: Design the shaft shown in the figure below, if the diameter of shaft at points A, B, C and D is: (A=42mm, B=84mm, C=90mm and D=28mm).



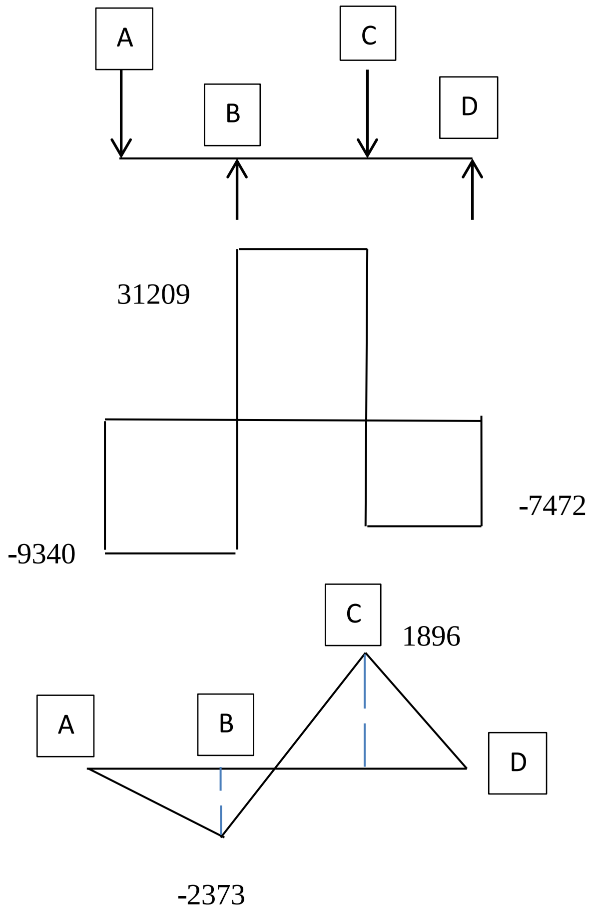




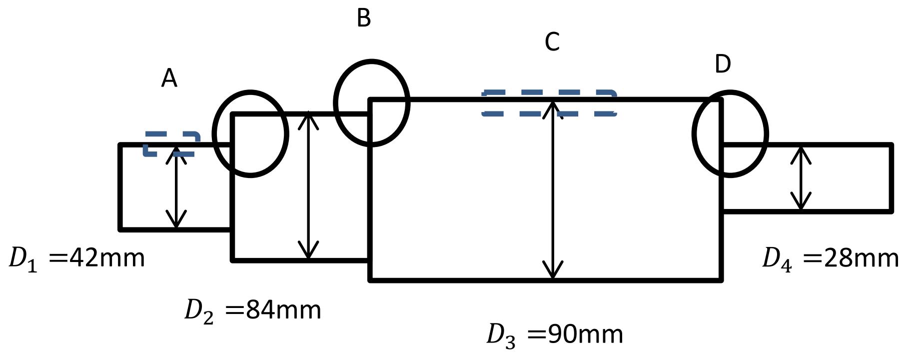
Horizontal plane



Vertical plane



K_t For key way =1.6



At point (D): assume well rounded fillet $K_t)_{bend} = 1.5$

$K_t)_{tor} = 1.5$

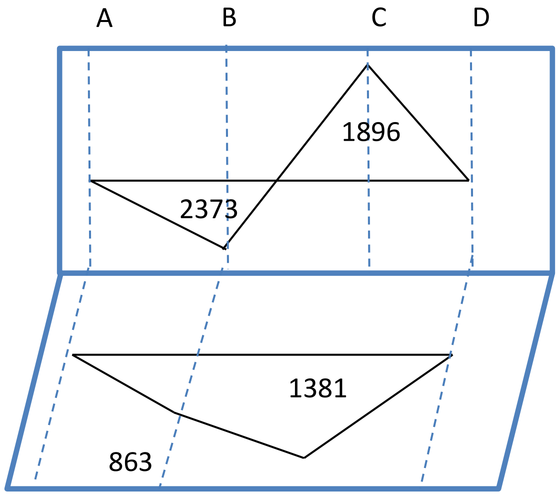
At points (A&C): assume sled-runner key way $K_t)_{key\ way} = 1.6$

At points (B&C): assume well rounded fillet $K_t)_{bend} = 1.5$

$K_t)_{tor} = 1.5$

$$\sigma_1 = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} \quad \text{--------} (1)$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} \quad \text{--------} (2)$$



At point A:

At this point $M = 0$ & $T = 2373$ N.m

$\sigma_x = 0$ & $\tau_{XY} = \frac{T\,c}{J} = (2373 * 1000 * 2)/(\pi*(42)^3/32) = 163.2\ N/mm^2$

$\sigma_1 = 0 + \sqrt{0 + (163.2)^2} = 163.2\ MPa$

At point B:

At this point $M_B = \sqrt{(M_{bx})^2 + \left(M_{by}\right)^2} = \sqrt{(863)^2 + (2373)^2} = 2525.5$ N.m

Also, $T = 2373$ N.m

$\sigma_x = \frac{M_b\,C}{I} = \frac{2525.5*1000*32}{\pi*(84)3} = 43.40\ MPa$

$\tau_{xy} = \frac{T\,r}{J} = \frac{2373*1000*16}{\pi*(84)3} = 20.40\ MPa$

$\sigma_1 = \frac{43.2}{2} + \sqrt{\left(\frac{43.2}{2}\right)^2 + (20.4)^2} = 51.5\ MPa$

$\tau_{max} = \sqrt{\left(\frac{43.2}{2}\right)^2 + (20.4)^2} = 29.8\ MPa$

At point C:

At this point $M_C = \sqrt{(M_{Cx})^2 + \left(M_{Cy}\right)^2} = \sqrt{(1381)^2 + (1896)^2} = 2348$ N.m

Also, $T = 2373$ N.m

$\sigma_x = \frac{M_C\,C}{I} = \frac{2348*1000*32}{\pi*(90)3} = 32.8\ MPa$

$\tau_{xy} = \frac{T\,r}{J} = \frac{2373*1000*16}{\pi*(90)3} = 16.5\ MPa$

$\sigma_1 = \frac{32.8}{2} + \sqrt{\left(\frac{32.8}{2}\right)^2 + (16.5)^2} = 39.7\ MPa$

$\tau_{max} = \sqrt{\left(\frac{32.8}{2}\right)^2 + (16.5)^2} = 23.3\ MPa$

At point D:

At this point $M_D = 0$

Also, $T = 0$

There is a vertical shearing force in x-direction (horizontal plane) and y-direction (vertical plane).

$R_D = \sqrt{(5440)^2 + (7473)^2} = 9243\ N$

$\sigma_x = 0$

$\tau_{xy} = \frac{4V}{3A} = \frac{(4*9243)}{3*(\pi*\frac{d^2}{4})} = \frac{(4*9243)}{3*(\pi*\frac{28^2}{4})} = 62.9\ MPa$

$\sigma_1 = 0 + \sqrt{0 + (62.9)^2} = 62.9\ MPa$

$\tau_{max} = \sqrt{0 + (62.9)^2} = 62.9\ MPa$

So you can see from the above results, that the worst case at point A

$\tau_{max} = 163.2\ N/mm^2$

If we assume that $K_t = 1.6$

$\tau_{design} = K_t\ \tau_{max} = 1.6*163.2 = 261.12\ MPa = S_{sy}/N$

$S_{sy} = 522.24\ MPa$, *if we assume* $(N=2)$ *and* $S_{sy} = 0.5S_y$

$S_y = 1044.48\ MPa$

From (appendix-3) use carbon and alloy steel (P. A6) AISI 4140 0QT 1000

$S_y = 1050$ MPa

Or any material that you see advisable

So you can apply

$K_t\ \sigma \le \sigma_d = S_{ut}/N$

Or

$K_t\ \tau_{max} \le \tau_d = S_{sy}/N$

If we apply Von-Mises theory for the previous example:

$\sigma` = \sqrt{\sigma_x{}^2 + 3\tau_{xy}^2}$

At point A:

$\sigma_x = 0$ $\tau_{xy} = 163.2$ *MPa*

$\sigma` = \sqrt{0 + 3 * (163.2)^2} = 282.7$ *MPa*

At point B:

$\sigma` = \sqrt{(43.4)^2 + 3 * (20.4)^2} = 56$ *MPa*

At point C:

$\sigma` = \sqrt{(32.8)^2 + 3 * (16.5)^2} = 43.5$ *MPa*

At Point D:

$\sigma` = \sqrt{0 + 3 * (62.9)^2} = 108.25$ *MPa*

Also the worst case is at point A

$\sigma = 282.7$ *MPa*

$\sigma_d = K_t \sigma = 1.6*282.7 = 452.32$ *MPa* $= S_{ut}/N$

$S_{ut} = 1357$ *MPa* at $N = 3$

From (appendix 3), P. A6

Say material chooses in AISI 4150 OQT 1000

Example (12-1), p.548:

Design the shaft shown in the figures 12-1, 12-2, 12-11 and 12-12. It is to be machined from AISI 1144 OQT 1000 steel. Gear A receives 150 KW from gear P. Gear C delivers power to gear Q. The shaft rotates at 62.8 rad/s.

Notes:

1- Use the bending moments and shear forces diagrams as shown in the previous example.

2- If you want to find the forces on the gears, you can do the followings:

*T=P/ω = 150 KW*1000/62.8 rad/s = 2373 N.m*

$W_{tA}=T_A/(D_A/2)$ = 2373*2/0.508 = 9341 N ↓

$W_{rA}= W_{tA} \tan \phi$ = 9341 tan 20 = 3398 N ⟶

$W_{tC}=T_C/(D_C/2)$ = 2373*2/0.254 = 18680 N ↓

$W_{rC}= W_{tC} \tan \phi$ = 18580 tan 20 = 6800 N ⟵

Now, solve the previous example by using fatigue equation:

Solution:

From A4-2 ⟶ S_y= 572.28 MPa & S_u=813.61 MPa
From fig. 5-8 ⟶ S_n = 289.6 MPa & $S_n' = S_n\,(C_m)\,(C_{st})(C_R)(C_s)$
C_m=1 *(material factor =1 for wrought steel)*
C_{st}= 1 *(type of stress factor =1 for bending stresses)*
C_R= 0.81 *(for design reliability of 0.99)*
C_s = 0.75 *(assumed from fig. 5-9 because the size of shaft is not available in this stage)*

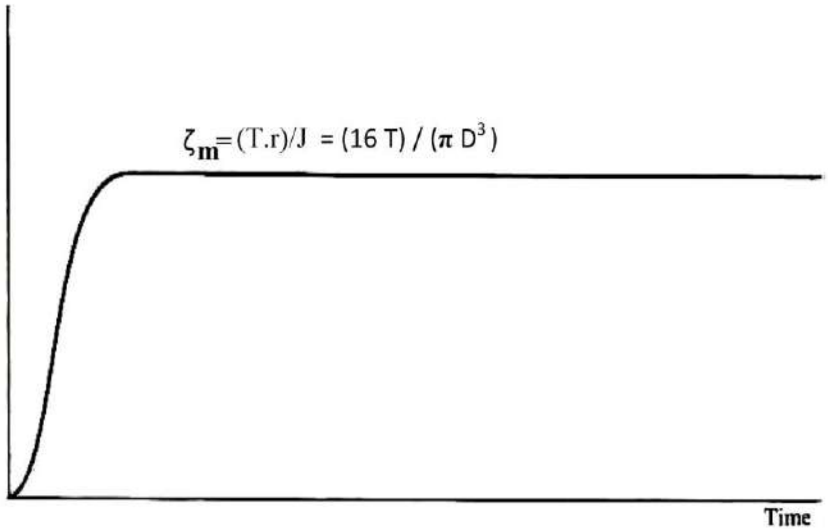
$S_n' = 289.6 * 0.75 * 0.81 = 175.82\ MPa$

$\sigma' = Von\ Miseses\ stress = \sqrt{\sigma_x^2 + 3\tau_{xy}^2}$

1. **Point A:**

$$\sigma_m' = \sqrt{\sigma_{xm}^2 + 3\tau_{xym}^2} = \sqrt{0 + 3\tau_{xym}^2} = \sqrt{3}\ \frac{T.r}{J} = \sqrt{3}\ \frac{16T}{\pi D_1^3}$$

For static loading $\sigma_m' = \frac{S_y}{N} = \frac{572.3}{2} = \sqrt{3} * \frac{16*2372}{\pi D_1^3} \rightarrow D_1 = 41.9\ mm$



2. **Point B:**

$$\sigma'_m = \sqrt{\sigma_{xm}^2 + 3\tau_{xym}^2} = \sqrt{0 + 3(\frac{16T}{\pi D^3})^2} = \sqrt{\frac{3}{4}(\frac{32T}{\pi D^3})^2}$$

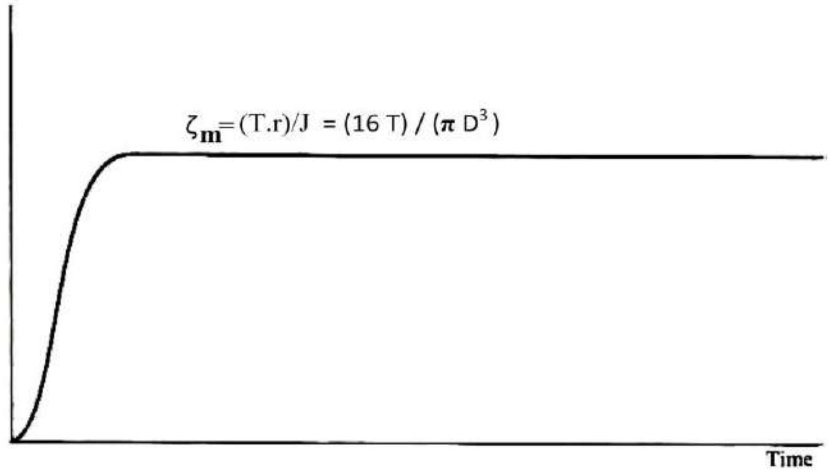
$$\sigma'_a = \sqrt{\sigma_{xa}^2 + 3\tau_{xya}^2} = \sqrt{(\frac{32M}{\pi D^3})^2 + 0} = \sqrt{(\frac{32M}{\pi D^3})^2}$$

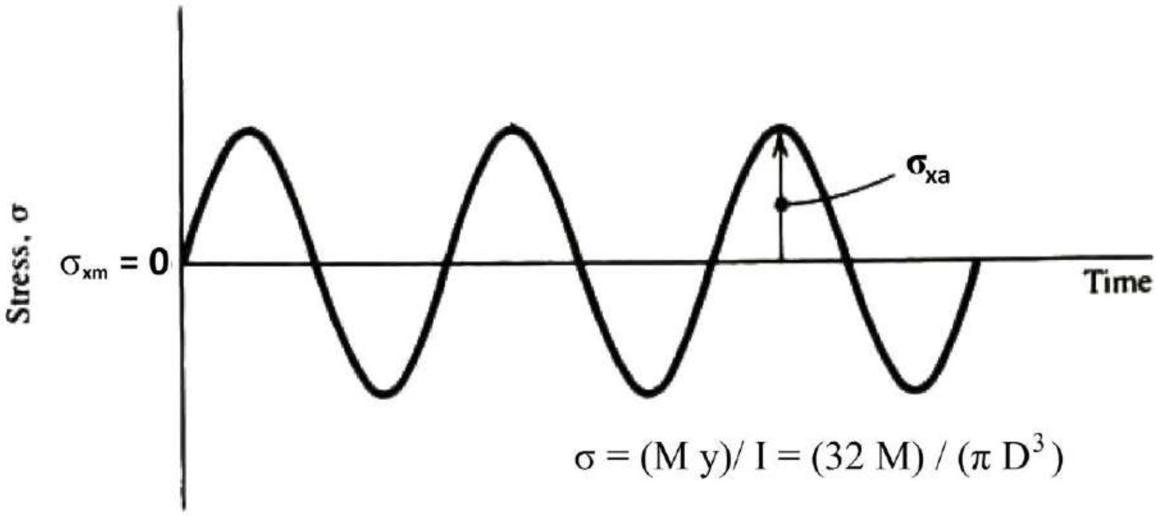
$$\frac{1}{N} = \frac{\sigma'_m}{S_y} + \frac{K_t\,\sigma'_a}{S'_n} \quad \text{(Soderberg equation)}$$

To solve the equation above for the shaft diameter (D), use this equation:

$$D = \left[\frac{32N}{\pi}\sqrt{\left[\frac{K_t\,M}{S'_n}\right]^2 + \frac{3}{4}\left[\frac{T}{S_y}\right]^2}\right]^{\frac{1}{3}}$$

$$D_2 = \left[\frac{32*2}{\pi}\sqrt{\left[\frac{1.5*2525.5}{175.8*10^3}\right]^2 + \frac{3}{4}\left[\frac{2373}{572*10^3}\right]^2}\right]^{\frac{1}{3}} = 83.82 \text{ mm}$$



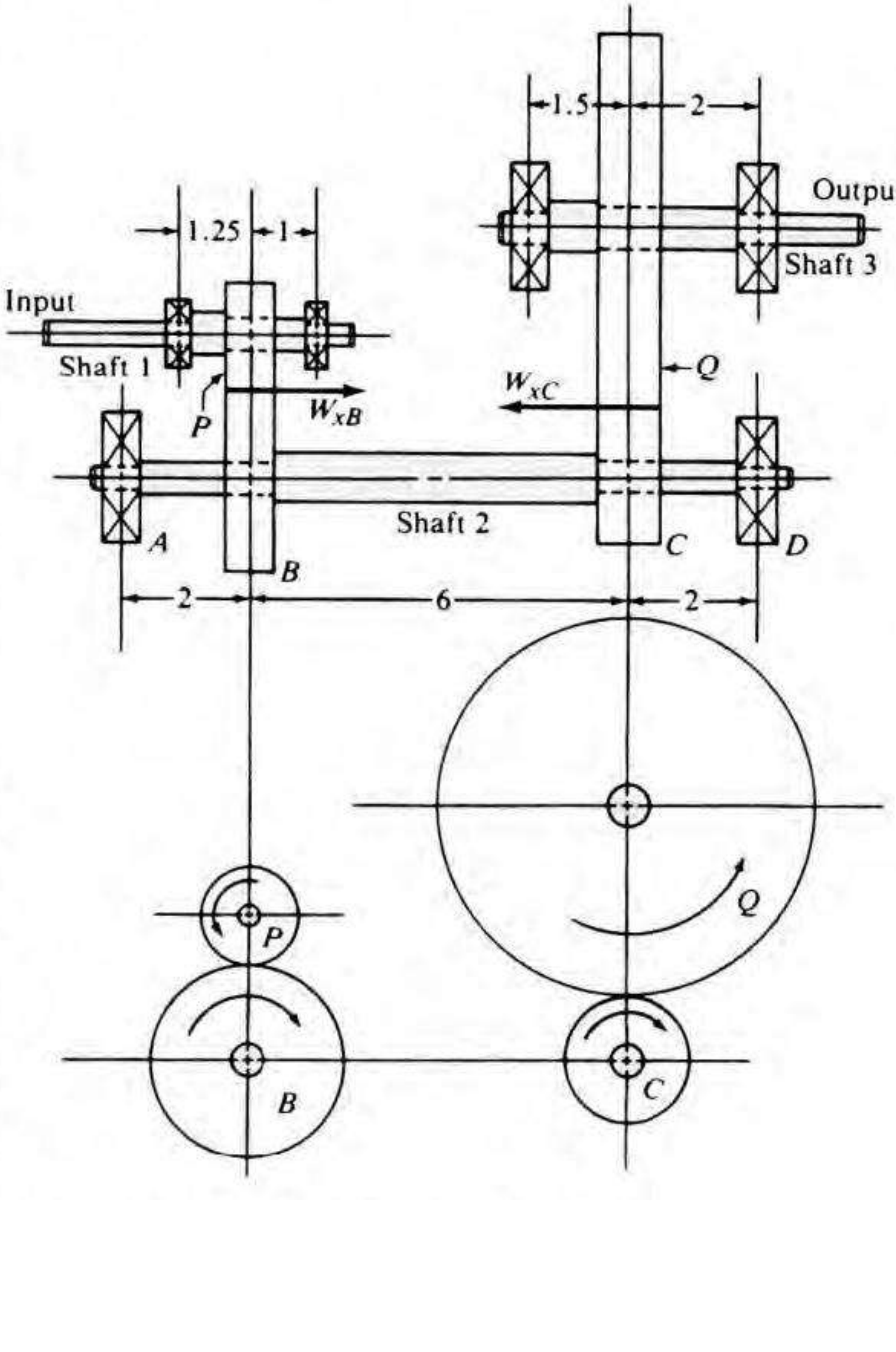


3. **Point C:** (you can use the same equation above with different value of stress concentration K_t & bending moment with same torque).

Exercises

1- Solve problems (Page 571), [Q35 and Q37] and draw complete construction for each of the above two questions.

Q35: The double-reduction, helical gear reducer shown in Figure PI 2-35 transmits 5.0 hp. Shaft 1 is the input, rotating at 1800 rpm and receiving power directly from an electric motor through a flexible coupling. Shaft 2 rotates at 900 rpm. Shaft 3 is the output, rotating at 300 rpm. A chain sprocket is mounted on the output shaft as shown and delivers the power upward. The data for the gears are given in Table below.



Gear	Module mm (m)	Pitch Diameter (m N) mm	No.of Teeth (N)	Face Width (F) mm
P	3	36	12	19
B	3	72	24	19
C	4	48	12	25
Q	4	144	36	25

Power= 5hp = 3728.5 watt.

Shaft (1) n_1=1800 rpm $\omega_1 = 188.5\ \frac{rad}{sec}$

Shaft (2) n_2=900 rpm $\omega_2 = 94.25\ \frac{rad}{sec}$

Shaft (3) n_3=300 rpm $\omega_3 = 31.4\ \frac{rad}{sec}$

$\phi = 14.5^o$ $\Psi = 45^o$ $\emptyset_n = 20^o$

Material AISI 4140 OQT 1200

a. **Determine the magnitude of torque in shafts at all points.**

Shaft (1) $T_1 = \frac{power}{\omega} = \frac{3728.5}{188.5} = 19.8\ N.m$

Shaft (2) $T_2 = \frac{power}{\omega} = \frac{3728.5}{94.25} = 39.6\ N.m$ $(Assuming\ \eta = 100\%)$

Shaft (3) $T_3 = \frac{power}{\omega} = \frac{3728.5}{31.4} = 118.79\ N.m$ $(Assuming\ \eta = 100\%)$

b. **Compute the forces on shafts and on bearings**

Shaft No.1

$$W_{tp} = \frac{2T_1}{D_p} = \frac{2 * 19.8 * 1000}{36} = 1100\ N$$

$$W_{rp} = W_t \frac{tan\emptyset}{cos\varphi} = 1100 * \frac{tan14.5}{cos45} = 402\ N$$

$$W_{xp} = W_t tan\varphi = 1100\ N.$$

$$W_{E)V} * 57 - W_{xp} * 18 - W_{rp} * 25 = 0 \quad (M_F = 0)$$

$$\therefore W_{E)V} = \frac{1100 * 18 + 402 * 25}{57} = 523.7\ N$$

$$\therefore W_{F)V} = 402 - 523.7 = -121.7N$$

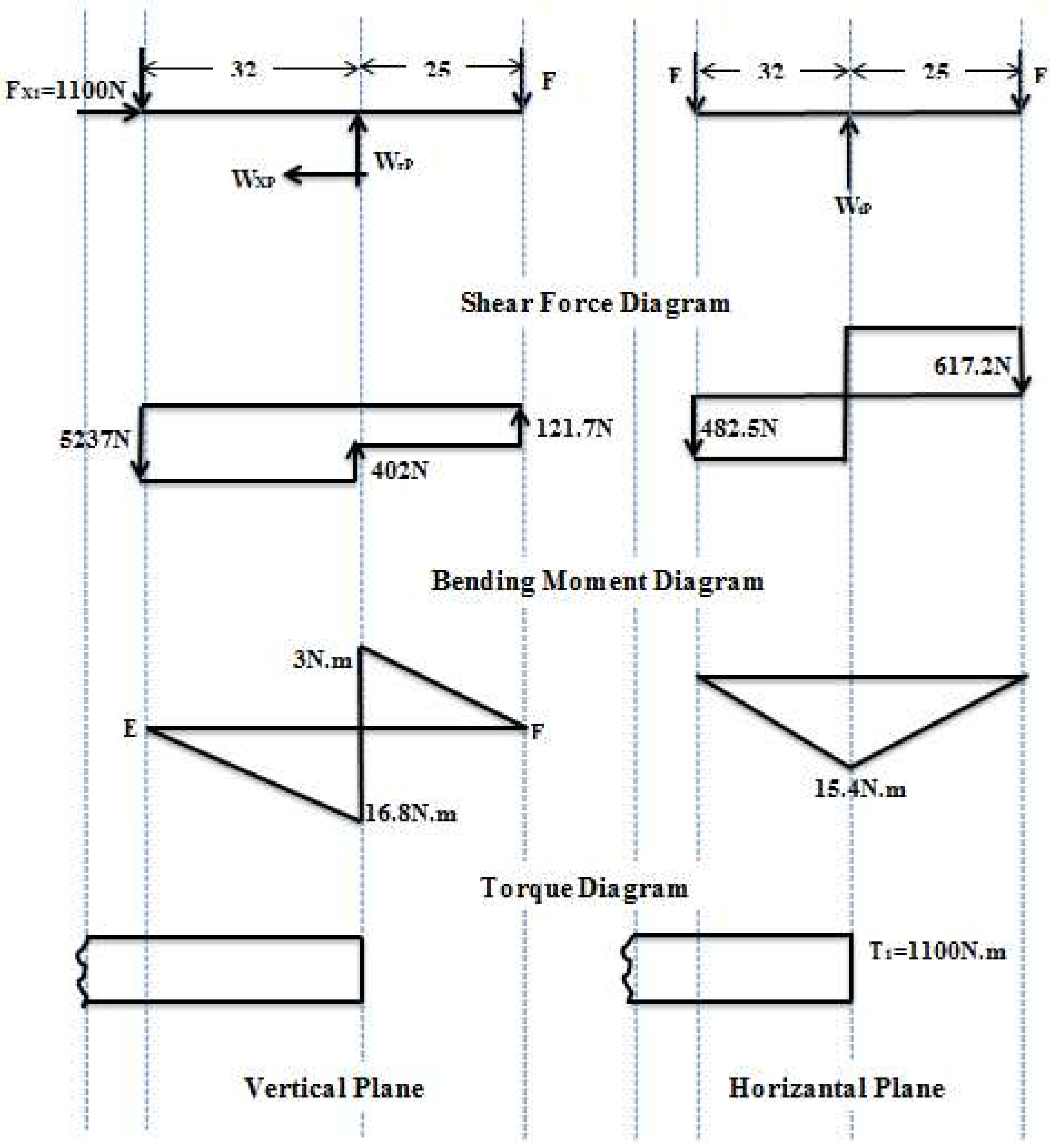
$$W_{E)H} * 57 - W_{tp} * 25 = 0 \quad (M_F = 0)$$

$$\therefore W_{E)H} = \frac{1100 * 25}{57} = 482.5\ N$$

$$\therefore W_{F)H} = 1100 - 482.5 = 617.5N$$

$$M_{P)V} = -W_{E)V} * 32 = 523.7 * 32 = 16.8\ N.m$$

$$M_{P)H} = -W_{E)H} * 32 = 482.5 * 32 = 15.4\ N.m$$



Shaft No.2

$$W_{tc} = \frac{2T_2}{D_c} = \frac{2*39.6*1000}{48} = 1650\ N$$

$$W_{rc} = W_{tc}\frac{tan\emptyset}{cos\varphi} = 1650*\frac{tan14.5}{cos45} = 603.5\ N$$

$$W_{xc} = W_{tc}tan\varphi = 1650\ N.$$

M_D=0 [Note : (W_{rp}= W_{rB})(W_{xp}= W_{xB})]

$$W_{A)V}*250+1100*36-402*200-1650*24-603.2*50=0$$

$$\therefore W_{A)V} = 442\ N$$

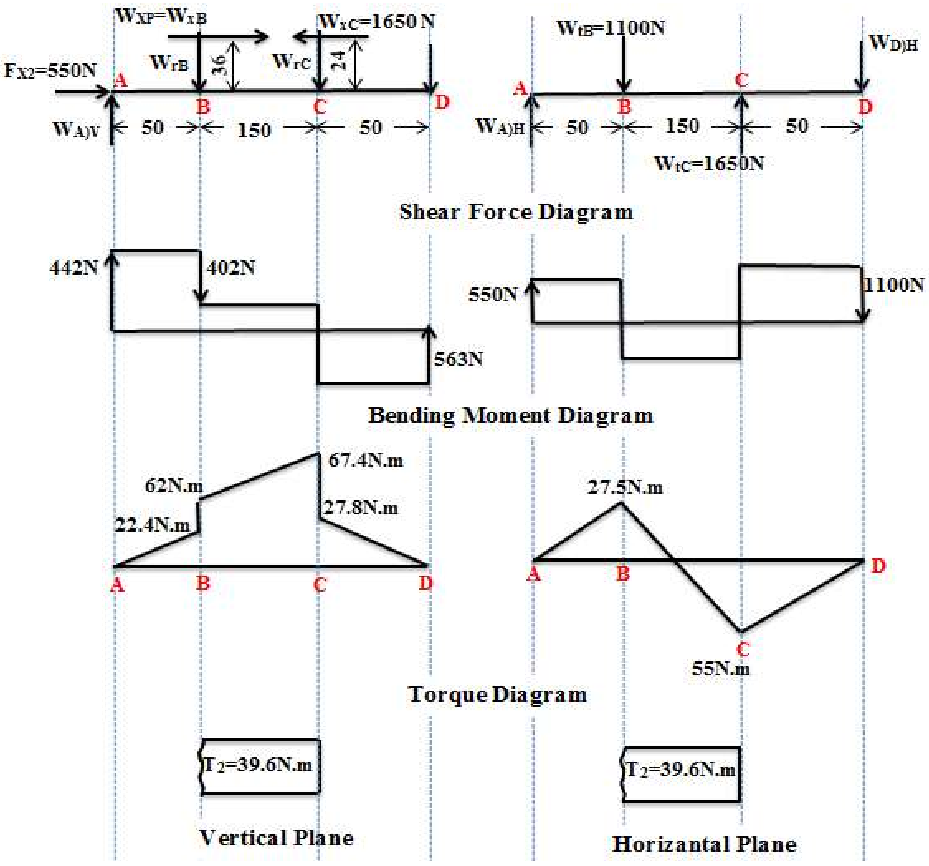
$$\therefore W_{D)V} = 402+603.5-442 = 563N$$

M_D=0 [Note : (W_{tp}= W_{tB})]

$$W_{A)H}*250-1100*200+1650*50=0$$

$$\therefore W_{A)H} = 550\ N$$

$$\therefore W_{D)H} = 550-1100 = 1650 = 1100N$$



Shaft No.3

$$W_{tQ} = \frac{2T_3}{D_Q} = \frac{2 * 118.5 * 1000}{144} = 1646\ N$$

$$W_{rQ} = W_{tQ}\frac{tan\emptyset}{cos\varphi} = 1646 * \frac{tan14.5}{cos45} = 603.2\ N$$

$$W_{xQ} = W_{tQ}tan\varphi = 1646\ N.$$

Assume Pitch dia. Of sprocket =288 mm

$$F_{chain} = \frac{2T_3}{D_{chain}} = \frac{2 * 118.5 * 1000}{288} = 823\ N$$

M_G=0 [Note : (W_{rc}= W_{rQ})(W_{xc}= W_{xQ})]

$$\therefore 823 * 138 - W_{s)V} * 88 + W_{rQ} * 38 + W_{xQ} * 72 = 0$$

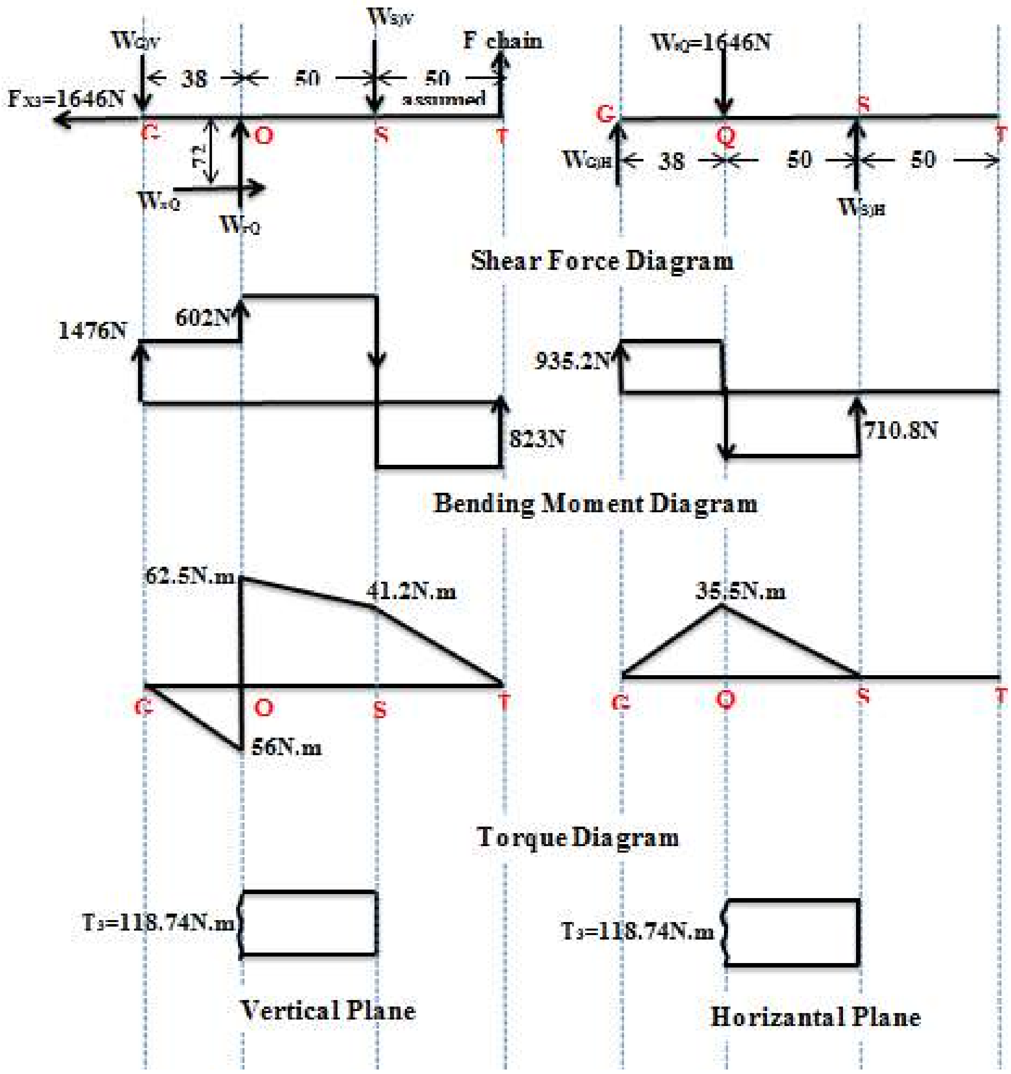
$$\therefore W_{s)V} = \frac{823 * 138 + 603.2 * 38 + 1650 * 72}{88} = 2901\ N$$

$$\therefore W_{G)V} = -1476\ N$$

M_S=0 [Note : (W_{tc}= W_{tQ})]

$$\therefore W_{G)H} = \frac{1646 * 50}{88} = 935.2\ N$$

$$\therefore W_{S)H} = 1646 - 935.2 = 710.8N$$


W_{G)V}
W_{S)V}
F chain
assumed
38
50
50
Fx3=1646N
G
O
S
T
72
WxQ
WrQ
WtQ=1646N
G
Q
S
T
WG)H
38
50
50
WS)H
Shear Force Diagram
1476N
602N
823N
935.2N
710.8N
Bending Moment Diagram
62.5N.m
41.2N.m
56N.m
35.5N.m
G
O
S
T
G
O
S
T
Torque Diagram
T3=118.74N.m
T3=118.74N.m
Vertical Plane
Horizantal Plane

c. Material of all shafts:

AISI 4140 OQT 1200

From figure (A4-4) P.A-9 for AISI 4140 oil quenched and tempered at 1200 F ∴ S_y=785 Mpa and S_u=896 Mpa.

From figure (5-8) Sn=335 Mpa.

$$\acute{S_n} = S_n * C_m * C_{st} * C_R * C_S$$

=335*1*1*0.81 (for design reliability of 0.99)*0.75(assumed because the size of shaft not available).

=203.5 Mpa.

d. Design Factor (N)

There are many factors influence the design factor (N) discussed before so choose a nominal value of N in our case =2 for general machine design.

e. Minimum allowable shaft diameters

The min. allowable shaft diameter is now computed at several sections along the shaft. Table below summarizes the data necessary for computed diameter of shaft.

Points	Torque(N.m)	Shearing Forces		Bending Moments		Axial Force (N)	Stress Concentration factor Kt	Loading Condition
		Vy (N)	Vx (N)	My (N.m)	Mx (N.m)			
E	19.8	524	483	0	0	1100	Fillet Kt=1.5	Static torsion static axial load
P	19.8	402	1100	16.8	15.4	1100	Key Kt=1.6	Static torsion reversed B.M static axial load
F	0	122	617	0	0	0	Fillet Kt=1.5	static shear load
A	0	442	550	0	0	550	Fillet Kt=1.5	static axial load
B	39.6	402	1100	62	27.2	(550) (1650)	Key Kt=1.6	Static torsion reversed B.M static axial load
C	39.6	603	1650	67.2	55	1650	Key Kt=1.6	Static torsion reversed B.M static axial load
D	0	563	1100	0	0	0	Fillet Kt=1.5	static shear load
G	0	1476	935	0	0	1646	Fillet Kt=1.5	static axial load
Q	118.74	603	1646	62.5	35.5	1646	Key Kt=1.6	Static torsion reversed B.M static axial load
S	118.74	2901	711	41.2	0	0	Fillet Kt=1.5	Static torsion reversed B.M static axial load
T	118.74	823	0	0	0	0	Key Kt=1.6	Static torsion

Now from table above you can see the complete analysis for each point on shafts. So to take all factors and stress will make the design complicated, then the following notes will help us for finding the shaft diameters.

Notes:

1- Neglect the stress concentration for static loading and the material is ductile.
2- Neglect the axial load and shear loads on shaft if loading is torsion only, bending moment only or combined bending and torsion.
3- When you neglect certain items, increase the shaft diameter then find design factor N for checking or find new allowable stress and should be less or equal the actual allowable stress.

Examples:

In point E (F_a*d_E/8) was neglected. Then d_E=7.64mm.

This value was checked $\boldsymbol{\sigma_{max} = 342\ Mpa\ < 392.5\ Mpa}$

In point P also (F_a*d_E/8) was neglected. Then d_p=15.9mm and increasd to 17mm, in this case check N=2.4>2.

4- Use appendix 2 page A-3 to choose the preferred basic sizes.

EXAMPLE:

In point E instead of d_E=7.64mm_choose the preferred basic size =8mm or you should choose at this point suitable bearing then from page 607, the smaller diameter of shaft=10mm. So at last the minimum diameter d_E=10mm. which_give you more safety.

1- Point E

$$\sigma_{all} = \frac{S_y}{N} = \frac{785}{2} = 392.5\ MPa$$

$$\sigma_{max.} = \frac{32}{\pi d_E^3}\sqrt{\left(M + \frac{F_a * d_E}{8}\right)^2 + \frac{3}{4}T^2}$$

M=0 and neglect (F_a*d_E/8) temporarily.

$$\therefore 392.5 = \frac{32}{\pi d_E^3} * \frac{\sqrt{3}}{2} T_1^2$$

$$\therefore d_E^3 = \frac{32}{\pi(392.5)} * \frac{\sqrt{3}}{2} * (19800)^2$$

∴dE=7.64mm

Now say d_E=8mm and check for stress.

$$\sigma_{max.} = \frac{32}{\pi 8^3}\sqrt{\left(\frac{1100*8}{8}\right)^2 + \frac{3}{4}(19800)^2} = 342\ MPa\ < 392.5\ MPa$$

∴dE=8mm is O.K.

2- Point P

$$\sigma_{max.} = \frac{32}{\pi d_P^3}\sqrt{\left(K_t M + \frac{F_a * d_P}{8}\right)^2 + \frac{3}{4}T^2}$$

$$\sigma_m = \frac{32}{\pi d_P^3}\sqrt{\left(\frac{1100 * d_P}{8}\right)^2 + \frac{3}{4}(19800)^2}$$

Neglect (F_a*d_E/8)

$$\therefore \sigma_m = \frac{32}{\pi d_P^3} * \frac{\sqrt{3}}{4} * (19800) \quad = \frac{151.338}{d_P^3}$$

$$\sigma_a = \frac{32}{\pi d_P^3}\sqrt{(K_t M)^2} = \frac{32*1.6}{\pi d_P^3}\sqrt{(16.8)^2 + (15.4)^2} * 1000 = \frac{371.613}{d_P^3}$$

$$\frac{1}{N} = \frac{\sigma_a}{\acute{S}_n} + \frac{\sigma_m}{S_y}$$

$$\therefore \frac{1}{2} = \frac{371.613}{d_P^3 * 203.5} + \frac{151.338}{d_P^3 * 785}$$

$$\therefore d_P^3 = 3652.2 + 385.6$$

∴dP=15.9mm Say dP=17mm

Then, to check N

$$\sigma_m = \frac{32}{\pi (17)^3}\sqrt{\left(\frac{1100 * (17)}{8}\right)^2 + \frac{3}{4}(19800)^2} = 35.9MPa$$

$$\therefore \sigma_a = \frac{371613}{(17)^3} = 75.6\ MPa.$$

$$\therefore \frac{1}{N} = \frac{75.6}{203.5} + \frac{35.9}{785} \qquad \therefore N = 2.4 > 2 \qquad \therefore \text{diameter is O.K.}$$

3- Point F

At this point there is only vertical shearing force $=\sqrt{(121.7)^2 + (617.2)^2} = 629\ \mathrm{N}$

$$\therefore \tau_{max.} = \frac{4 * V}{3 * A} = \frac{4 * 629 * 4}{3 * \pi * d_F^2}$$

$$\tau_{max.} = 0.577 * \sigma_{max.} = 0.577 * 392.5 = 226.5\ MPa$$

$$d_F = \sqrt{\frac{16 * 629}{3 * \pi * 226.5}} = 2.17mm$$

So this value is very small and the minimum inside diameter of roller bearings in text book =10mm

So say d_F=10mm

4- Point A

At this point there are axial load and vertical shearing force

$$\sigma_{max.} = \sqrt{\sigma_x^2 + 3\tau_{xy}^2}$$

$$\sigma_x = \frac{F_a}{A} = \frac{4 * 550}{\pi d_A^2} = \frac{700}{d_A^2}$$

$$\tau_{max.} = \frac{4 * V}{3 * A} = \frac{16\sqrt{(449)^2 + (550)^2}}{3 * \pi * d_A^2} = \frac{1206}{d_A^2}$$

$$\therefore 392.5 = \sqrt{\left(\frac{700}{d_A^2}\right)^2 + 3\left(\frac{1206}{d_A^2}\right)^2} = \frac{1}{d_A^2}\sqrt{(700)^2 + 3(1206)^2}$$

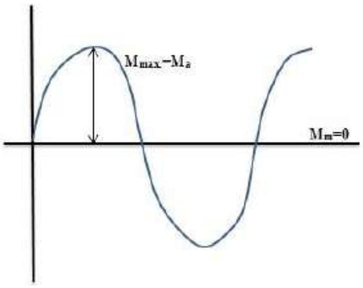
$\therefore$ d_A=2.4mm so say d_A=10mm (as before).

5- Point B

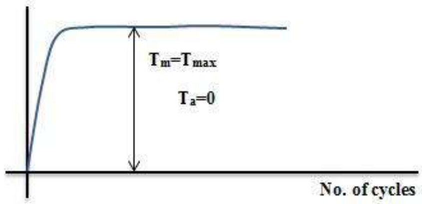
At this point there are torsion, bending moment, vertical shearing force and axial force.

Now neglect vertical shearing force and axial force.

∴ Use the following equations.



$$\sigma_{max.} = \frac{32}{\pi d^3}\sqrt{(K_t M)^2 + \frac{3}{4}T^2} \quad and \quad \frac{1}{N} = \frac{\sigma_a}{\acute{S}_n} + \frac{\sigma_m}{S_y}$$



$$\sigma_m = \frac{32}{\pi d_B^3} * \frac{\sqrt{3}}{2} * T_2 \qquad (M_m = Mean\ B.M. = 0)$$

$$\sigma_m = \frac{349500}{d_B^3}$$

$$\sigma_a = \frac{32}{\pi d_B^3}\left(1.6 * \sqrt{(62)^2 + (27.2)^2}\right) * 1000 = \frac{1104000}{d_B^3}$$

$$\therefore \frac{1}{2} = \frac{1104000}{203.5 d_B^3} + \frac{349500}{785 d_B^3}$$

∴d_B=22.7mm ***say*** ***d_B=25mm (APP.2) P. A-3.***

Also you can check as on point (P).

6- *Point C*

$$\sigma_m = \frac{349500}{d_C^3}$$

$$\sigma_a = \frac{32}{\pi d_C^3}\left(1.6 * \sqrt{(67.2)^2 + (55)^2}\right) * 1000 = \frac{1416000}{d_C^3}$$

$$\therefore \frac{1}{2} = \frac{1416000}{203.5 d_C^3} + \frac{349500}{785 d_C^3}$$

∴d_C=24.56mm *say d_C=30mm*

Also you can check as on point (P).

Finally: you can solve other diameters for D, G, Q, S and T as before.

Now the complete construction of the gearbox may be similar to construction below:

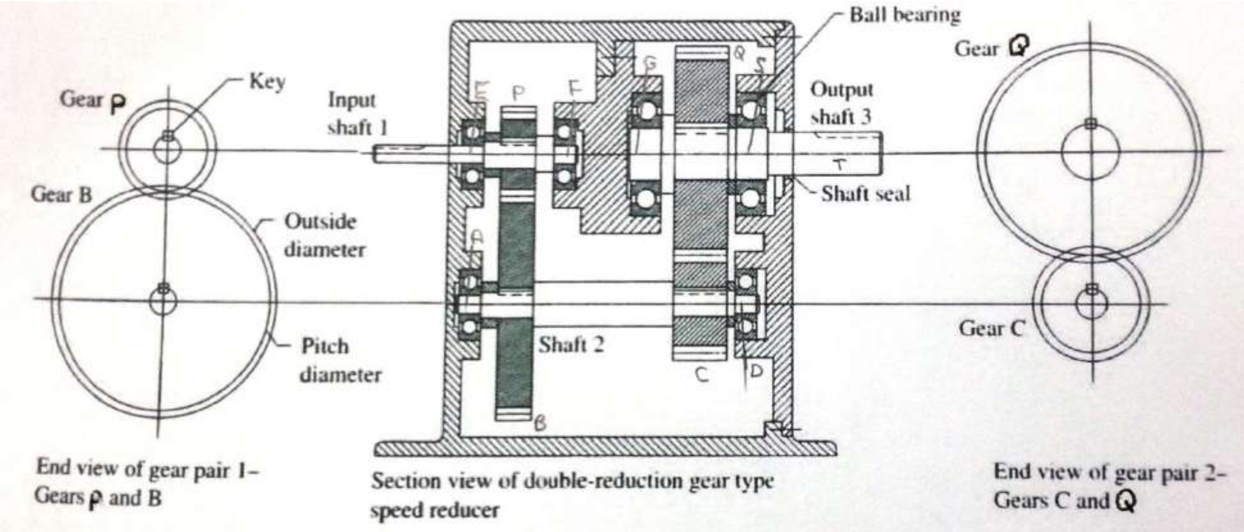
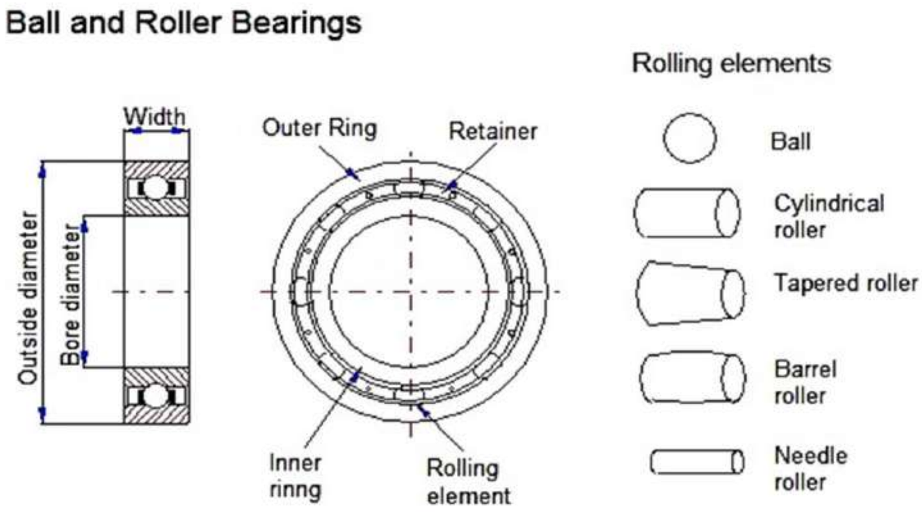


FIGURE 8–3 Conceptual design for a speed reducer

LECTURES FOURTEEN & FIFTEEN

ROLLING CONTACT BEARINGS



Reference: "Machine Elements in Mechanical Design" 4th Edition in SI units, By: Robert L. Mott, Chapter 14.

Introduction:

- See all sections on this chapter i.e.

14-1 Objectives of this chapter (Page 600)
14-2 Types of rolling contact bearings (Page 600 to 604)
.

- See also tables and figures on this chapter.
- See the CD for MDesign which give all types of rolling contact bearings, fixation, and calculations.

Summary of design procedure for rolling contact bearings:

1. From analysis of forces on shaft take reaction on bearing and speed.
2. Specify equivalent load (P) as below:

P= VR ………. (14-5) for radial load only (see page 613)

P= VXR + YT ……….. (14-6) for axial & radial load (see page 614)

$$\mathbf{P_A = 0.4F_{rA} + 0.5\left(\frac{Y_A}{Y_B}\right) F_{rB} + Y_A T_A}$$

$$\mathbf{P_B = F_{rB}}$$

(14-8) for tapered rolling bearing (See page 618)

Where:

V: Rotation parameter (V=1 if inner race rotating; V=1.2 if outer race rotating)
R: Radial load; T: Thrust load; X: Radial factor = 0.56;
Y: Thrust factor (see table 14.5)

Note: for small thrust load X=1 & Y=0 as in equation (14-5)

P_A: Equivalent radial load on bearing A
P_B: Equivalent radial load on bearing B
F_{rA}: Applied radial load on bearing A
F_{rB}: Applied radial load on bearing B
T_A: Thrust load on bearing A
Y_A: Thrust factor for bearing A (from table 14.7 page 614)
Y_B: Thrust factor for bearing B (from table 14.7 page 614)

3. Assume recommended life from table 14-4 (page 612) and find suitable capacity & bearing or choose the bearing then find its life.

$$C = P_d \frac{F_L}{F_n} \qquad \text{........ (14-4) page 612}$$

Where:
C: Dynamic load rating
You can also use the following equation for C:
load (P) & life (L) relationship

$$\frac{L_2}{L_1} = \left(\frac{P_1}{P_2}\right)^K \quad \text{........} \qquad (14\text{-}1)$$

$$C = P_1/(L_d/10^6)^{1/k} \qquad (14\text{-}3)$$

F_L: Life factor......... see figure (14-12)
F_n: Speed factor .... see figure (14-12)
K: factor (K=3 for ball bearing & K=3.33 for roller bearing)
$L_2 = L_d$ = Design life at design load ($P_2=P_d$)
$L_1=L_{10}$= Life at load C ($L_{10}= 10^6$ revolution at P_1=C)
$L_2=L_d$= (hours) (rpm) (60 min/hr)

Ex: 14-1 (Page 611), Ref. 1:

A catalog lists the basic dynamic load rating for a ball bearing to be (31358 N) for a rated life of 1 million rev. What would be the expected L_{10} life of the bearing if it were subjected to a load of 15568 N)?

Sol:

P_1 = C = 31358 N ; $P_2 = P_d$ = 15568 N ; $L_1=L_{10}=10^6$ rev. ; K=3
Find: L_2=design life at design load P_2

$L_2=L_d= L_1 (P_1/P_2)^K = 10^6 (31356/15568)^3 = 8.17*10^6$ rev.
$L_2=8.17*10^6$ rev. at P_d= 15568 N

Ex: 14-2 (Page 611), Ref. 1:

Compute the required basic dynamic load rating, C for a ball bearing to carry a radial load of (2891.2 N) from a shaft rotating at 600 rpm that is part of an assembly conveyor in a manufacturing plant.

Sol:

C=? ; P_d= 2891.2 N ; n=600 rpm ; Application: conveyor
From eq.(14-3) $C= P_d (L_d/10^6)^{1/K}$ K=3

L_d= {30000 hr (from table 14-4, page 612)}* 600 rpm * (60 min/hr)

L_d= 1.08 *10^9 rev.

C= 2891.2 $(1.08*10^9/10^6)^{1/3}$ = 29668 N

Or, you can solve this example as follows:

From figure 14-12 page 612:

F_n= 0.381 at 600 rpm (62.82 rad/sec)

F_L= 3.9 at 30000 hr life

C= P_d (F_L/F_n) ….. Equation (14-4)

C = 2891 (3.9/0.381) = 29596 N

Ex: 14-3 (Page 614), Ref.1:

Select a single-row, deep-groove ball bearing to carry (2891 N) of pure radial load from a shaft that rotates at 600 rpm. The design life is to be 30 000 h. The bearing is to be mounted on a shaft with a minimum acceptable diameter of 40 mm.

Sol:

R= 2891 N ; N=62.82 rad/sec = 600 rpm ; life = 30000 hr ; d= 40 mm

Select a single-row deep groove ball bearing

V= 1 for inner race rotating

Use table 14-3 page 607 to page 609 at d=40 mm; choose:

A-series 6200 ⟶ C=22.46 KN at d=40 mm type 6208

B-series 6300 ⟶ C=31.36 KN at d= 40 mm type 6308

But from Ex: 14-2 (page 611) choose type 6308, C= 31.36 KN

Which is greater than 29.7 KN?

Ex: 14-4 (Page 615), Ref.1:

Select a single-row, deep-groove ball bearing from Table 14-3 to carry a radial load of (8220 N) and a thrust load of (3002 N). The shaft is to rotate at 1150 rpm, and a design life of 20 000 h is desired. The minimum acceptable diameter for the shaft is 78.74 mm.

Sol:

R= 8228 N; T=3002 N; N=120.4 rad/sec; L_d= 20000 hr; d≥ 78.74 mm

Select a single-row deep groove ball bearing?

P= VXR + YT ……… (14-6)

Assume Y= 1.5; V=1 (inner race rotates); X=0.56

P= (1*0.56*8228) + (1.5*3002) = 9113 N

F_n = 0.3 & F_L= 3.41 ⟶ C= P*F_L /F_n = 9113 * 3.41/0.3 =103.64 KN=23300 Ib

From table 14-3 ⟶ choose 6222 or 6318 , so take 6318 with d= 90 mm,

So at this bearing the following information are:

Bearing No.	d	D	B	C_0	C
6318	90 mm	190 mm	43 mm	100.1 KN	109.8 KN

Now we should check the suitable Y and we will make the new calculation:

T/C_0 = 3002/100100 = 0.03 & from table 14-5 (Page 614) ⟶ Y=1.97

P= VXR + YT = (1*0.56*8228) + (1.97*3002) = 10523 N

& C = 10523 (3.41/0.3) = 119651 N = 119.6 KN = 29700 Ib

(from table 14-3), 119.6 KN(29700 Ib) < 133.44 KN (30000 Ib)

This bearing is satisfactory.

Bearing No.	d	D	B	C_0	C
6320	100 mm	215 mm	47 mm	132.6 KN	133.4 KN

Ex: 14-5 (Page 619), Ref.1:

The shaft shown in Figure 14-15 carries a transverse load of (30246 N) and a thrust load of (11120 N). The thrust is resisted by bearing A. The shaft rotates at 350 rpm and is to be used in a piece of agricultural equipment. Specify suitable tapered roller bearings for the shaft.

Sol:

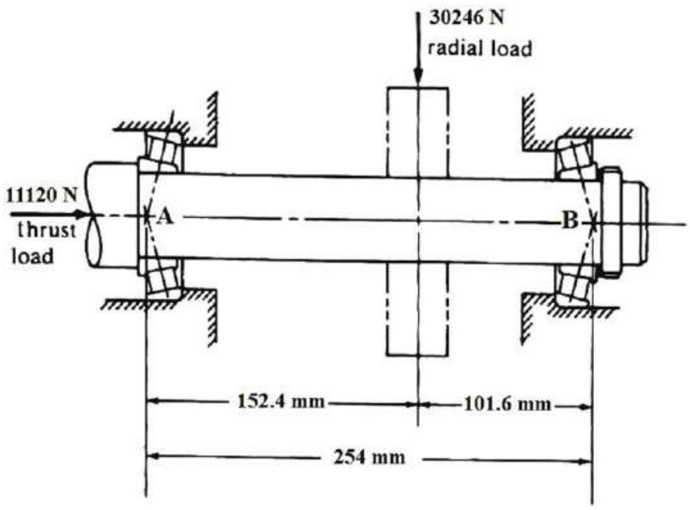
The thrust resisted by bearing A

N= 36.64 rad/sec

Application:

Agricultural Equipment

Select suitable tapered rolling bearing contact for shaft?



F_{rA} = 30246 * (101.6/254) = 1209

F_{rB} = 20246 * (152.4/254) = 18147 N

T_A = 11120 N

Assume $Y_A = Y_B = 1.75$

Now use equation 14-8 ⟶ $P_A = 0.4F_{rA} + 0.5\frac{Y_A}{Y_B}F_{rB} + Y_AT_A$

P_A = 0.4 (12098) + 0.5(1.75/1.75) (18147) + 1.75 (11120) = 33373 N

$P_B = F_{rB}$ = 18147 N

From table 14-4 ⟶ choose L_d = 4000h * (350 rpm) * 60 min/h

$L_d = 8.4 * 10^7$ rev.

From eq. 14-3 ⟶ $C_A = P_A (L_d/10^6)^{1/K} = 33373 (8.4*10^7/10^6)^{1/3.33}$

C_A = 126323 N=28400 Ib

& $C_B = P_B (L_d/10^6)^{1/K} = 18147 (8.4*10^7/10^6)^{1/3.33}$ = 68499 N=15400 Ib

From table 14-7 (page 619) ⟶ choose the following bearings:

Bearing A ⟶ d= 63.5 mm ; D= 127 mm ; Y_A= 1.65 ; C= 130326 N

Bearing B ⟶ d= 44.45 mm ; D= 106.6 mm ; Y_B = 1.5 ; C = 95187 N

Now we should check our calculation as follows:

P_A= 0.4(12098) + 0.5(1.65/1.5) (18147) + 1.65 (11120) = 33168 N

& $P_B = F_{rB}$ = 18147 N

C_A = 125433 N & C_B = 68499 N

They are still satisfactory for the selected bearings.

Note: if $P_A < F_{rA}$ then let $P_A = F_{rA}$ and compute P_B

$P_B = 0.4 F_{rB} = 0.5 (Y_B/Y_A) (F_{rA}) - (Y_B T_A)$ ………. 14-10 (Page 620)

Note: follow the same procedure for Angular contact ball bearing.

7-3: Life predication under varying loads, (Page 625):

Mean effective load (F_m) = $\left\{\frac{\sum i\ (Fi)^P\ Ni}{N}\right\}^{\frac{1}{P}}$

Where: Fi = individual load among a series of i loads

Ni = No. of revolutions at which Fi operates

N = total No. of revolutions in complete cycle

P = 3 for ball bearings; P = 10/3 for roller bearings

Ni = No. of minutes of operation at Fi

N = the sum of no. of minutes in the total cycle = N1 + N2 + …….. + Ni

Expected life in millions of revolutions is:

$$L = (C/F_m)^P \quad \text{………} \ 14\text{-}12$$

Ex: 14-6 (Page 626), Ref.1:

A single-row, deep-groove ball bearing number 6308 is subjected to the following set of loads for the given times

Condition	Fi	Time
1	2891 N	30 min.
2	3336 N	10 min.
3	1112 N	20 min.

This cycle of 60 min is repeated continuously throughout the life of the bearing. The shaft carried by the bearing rotates at 600 rpm (62.82 rad/sec). Estimate the total life of the bearing.

Sol:

$F_m = \left\{\frac{\sum i\ (Fi)^P\ Ni}{N}\right\}^{\frac{1}{P}}$

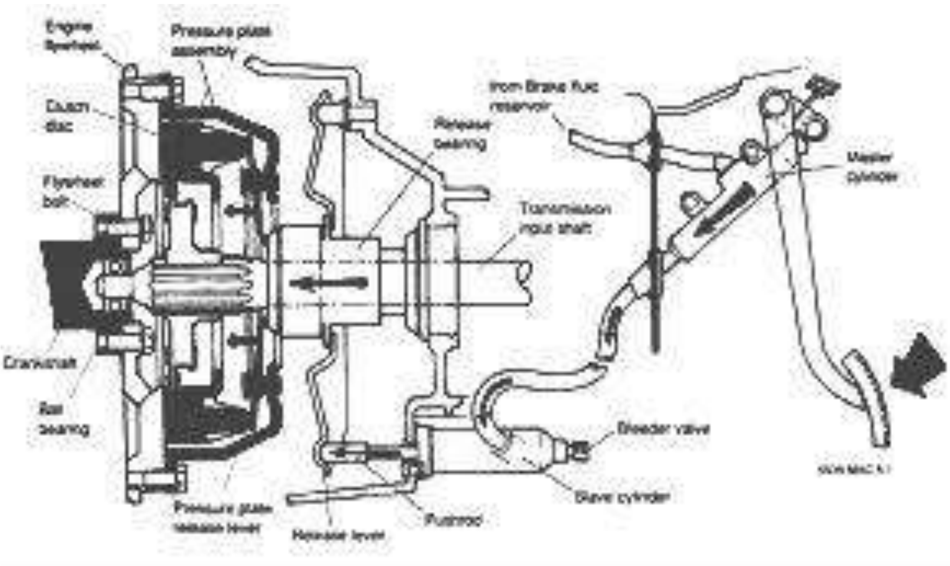
$= \left\{\frac{30\ (2891)^3+10\ (3336)^3+20\ (1112)^3}{30+10+20}\right\}^{1/3} = 2655\ N$

$L = (C/F_m)^P$ = {(31358 (tab. 14-3 for No. 6308)) / 2655}^{3} = 1647 million rev.

$L = (1647*10^6/1)$ * (min / 600 rev.) * (h / 60 min.) = 4574 hours

LECTURE SIXTEEN & SEVENTEEN

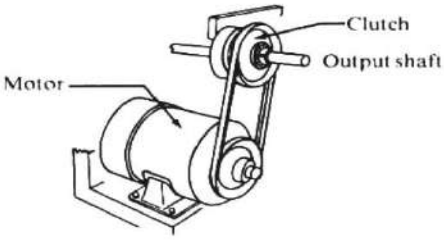
MOTION CONTROL "CLUTCHES AND BRAKES"



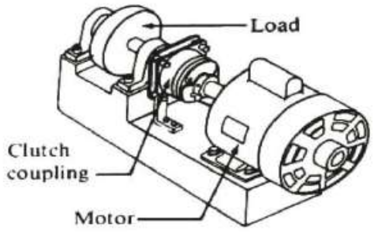
Reference: "Machine Elements in Mechanical Design" 4th Edition in SI units, By: Robert L. Mott, Chapter 22.

Introduction

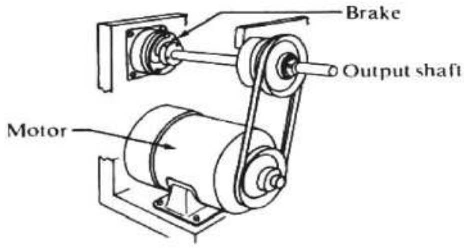
- A brake is a device used to bring a moving system to rest, to slow its speed or to control its speed to a certain value under varying condition.
- A clutch is a device used to connect or disconnect a driven component from the prime mover of the system.
- See section 22-2 (Page 833) and figure 22-1:



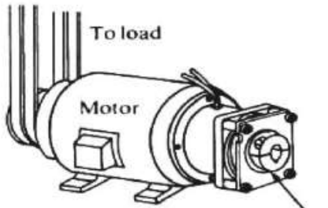
(*a*) Clutch: Transmits rotary motion to a parallel shaft only when coil is energized, by using sheaves, sprockets, gears, or timing pulleys.



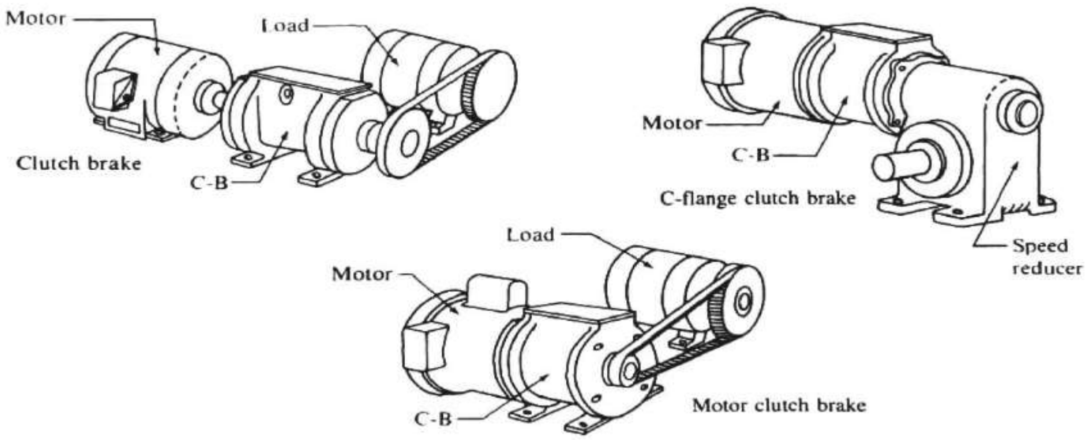
(*b*) Clutch coupling: Transmits rotary motion to an in-line shaft only when coil is energized. Split shaft applications.



(*c*) Brake: Stops (brakes) load when coil is energized. Panel shows clutch imparting rotary motion to the output (load) shaft while brake is de-energized. Conversely, de-energizing clutch coil and energizing brake coil causes load to stop.



(*d*) Fail-safe brake: Stops load by de-energization of coil; power off—brake on.

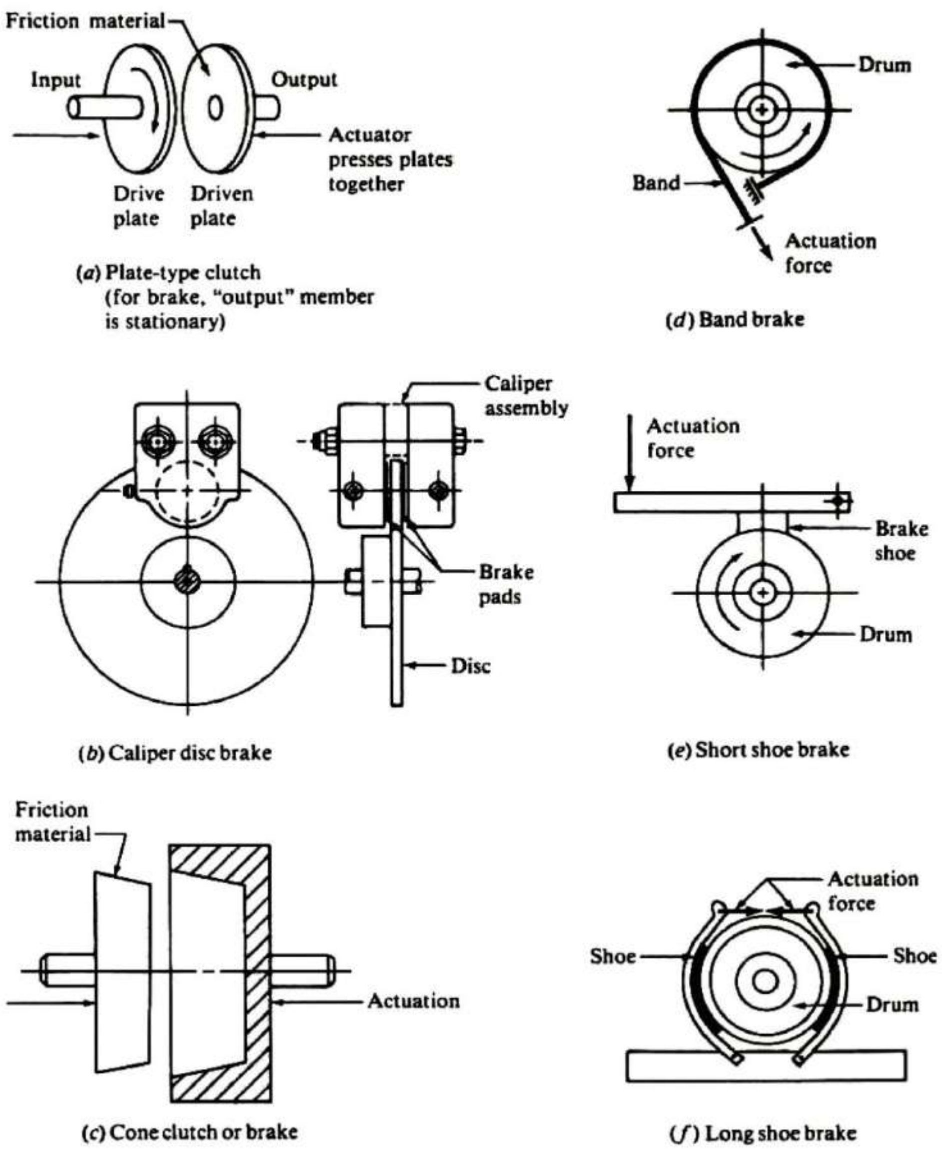


(*e*) Three types of mountings for clutch-brake modules: *Clutch brake* combines functions of clutch and brake in a complete preassembled package with input and output shafts. *C-flange clutch brake* performs same function but for use between NEMA "C" flange motor and speed reducer. *Motor clutch brake* is a preassembled module for mounting to a NEMA "C" flange motor and presenting an output shaft for connection to load.

FIGURE 22–1 Typical applications of clutches and brakes (Electroid Company, Springfield, NJ)

- Types of friction clutches & brakes (section 22-2 , page 835):

FIGURE 22–2 Types of friction clutches and brakes [(b), Tol-O-Matic, Hamel, MN]



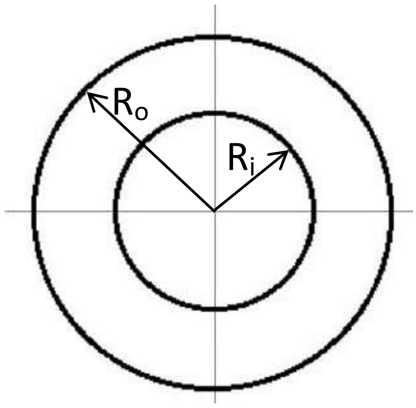
- Friction material and coefficient of friction (table 22-2, page 850):

TABLE 22–2 Coefficients of friction

Friction material	Dynamic friction coefficient		Pressure range	
	Dry	In oil	(psi)	(kPa)
Molded compounds	0.25–0.45	0.06–0.10	150–300	1035–2070
Woven materials	0.25–0.45	0.08–0.10	50–100	345–690
Sintered metal	0.15–0.45	0.05–0.08	150–300	1035–2070
Cork	0.30–0.50	0.15–0.25	8–15	55–100
Wood	0.20–0.45	0.12–0.16	50–90	345–620
Cast iron	0.15–0.25	0.03–0.06	100–250	690–1725
Paper-based		0.10–0.15		
Graphite/resin		0.10–0.14		

Plate-type clutch or brake (section 22-11, Page 851)

$$R_m = \frac{R_o + R_i}{2}$$



$$T_f = f * N * R_m \quad ....\ (22\text{-}8)\ (\text{page } 851)$$

Where:

T_f : Friction torque
f : Coefficient of friction
N : Axial force which pressed plates together

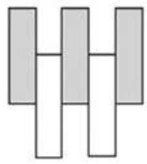
Notes:

1. For multiple disc clutch or brake let (n) be the number of pairs of contact surfaces:

$$T_f = n * f * N * R_m$$

2. If there are (n_{1d}) No. of discs on driving shafts and (n_{2d}) No. of discs on driven shaft then No. of pairs of contacts surface are:

$$n_p = n_{1d} + n_{2d} - 1$$



3. A reasonable for the ratio (R_O/R_i) is from (1.2 – 2.5).

4. The required torque capacity is usually expressed as:

$$T_f = \frac{C * P_f * K}{\omega} \quad \ldots\ldots (22-1)$$

Where:

C : Conversion factor for units
P_f: Power
K : Service factor base application
K = 1 for average condition; 1.5 for moderate duty; 3 for heavy duty
C = 1 if P: in watt and n: in rad/sec and T: in N.m

5. Judge the suitability of wear rating (WR)

$$WR = \frac{P_f}{A} \ldots\ldots (22-9)$$

$$P_f = T_f * \omega \ldots\ldots (22-10)$$

WR = 0.046 W/mm^2 for frequent application (conservation rating)
= 0.116 W/mm^2 for average service
= 0.46 W/mm^2 for infrequently use

Example Problem 22-6:

Compute the dimensions of an annular plate-type brake to produce a braking torque of 33.9 N.m. Springs will provide a normal force of 1423.4 N between the friction surfaces. The coefficient of friction is 0.25. The brake will be used in average industrial service, stopping a load from 78.5 rad/sec.

Solution:

1- Compute the required mean radius from equation (22-8).

$$\boldsymbol{R_m = \frac{T_f}{fN} = \frac{33.9\ N.m}{(0.25)*(1423.4N)} = 95.25\ mm}$$

2- Specify a desired ratio of R_o/R_i and solve for the dimensions. A reasonable value for the ratio is approximately 1.50. The range can be from 1.2 to about 2.5, at the designer's choice. Using 1.50, $R_o = 1.50R_i$ and

$$\boldsymbol{R_m = \frac{R_o + R_i}{2} = \frac{1.5R_i + R_i}{2} = 1.25R_i}$$

Then,

$$\boldsymbol{R_i = \frac{R_m}{1.25} = \frac{95.25\ mm}{1.25} = 76.2\ mm}$$

$$\boldsymbol{R_o = 1.5R_i = 1.5(76.2) = 114.3\ mm}$$

3- Compute the area of the friction surface:

$$\boldsymbol{A = \pi({R_o}^2 - {R_i}^2) = \pi((114.3)^2 - (76.2)^2) = 0.02278\ m^2}$$

4- Compute the frictional power absorbed:

$$\boldsymbol{P_f = T_f * \omega} = (33.9\ N.m)\left(78.5\frac{rad}{\sec}\right) = 2661\ W$$

5- Compute the wear ratio:

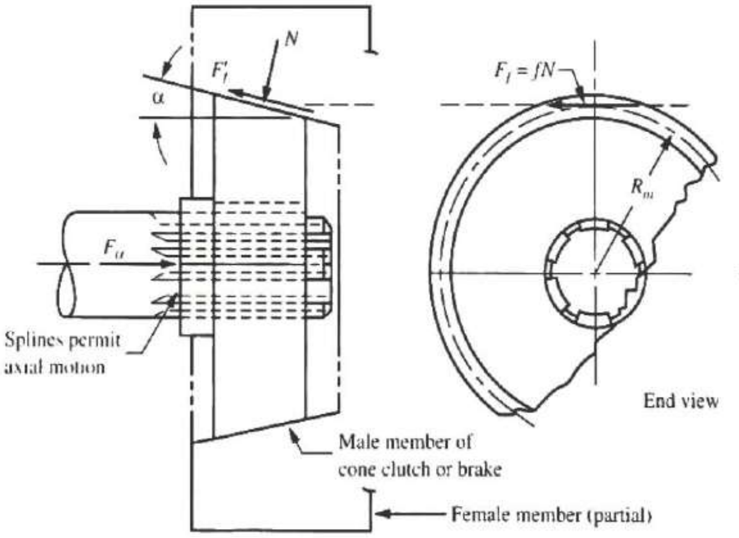
$$\boldsymbol{WR = \frac{P_f}{A} = \frac{2661\ W}{22.7*10^3\ mm^2} = 0.116\ W/mm^2}$$

6- Judge the suitability of *WR.* If *WR* is too high, return to Step 2 and increase the ratio. If *WR* is too low, decrease the ratio. In this example, WR is acceptable.

Cone clutch or Brake (section 22-13), Page 854

$$T_f = F_f * R_m = f * N * R_m$$

...... (22-8)



In addition to tangential friction force, there is a friction force along surface of cone = $F_f' = f$N

$$F_a = N \sin\alpha + F_{f'} \cos\alpha$$

$$F_a = N\,(\sin\alpha + f\cos\alpha)$$

$$N = \frac{F_a}{\sin\alpha + f\cos\alpha} \quad \dots\dots\dots(22-12)$$

$$T_f = \frac{f * R_m * F_a}{\sin\alpha + f\cos\alpha} \quad \dots\dots\dots(22-13)$$

Example Problem 22-7:

Compute the axial force required for a cone brake if it is to exert a braking torque of 67.8 N.m. The mean radius of the cone is 127mm Use f = 0.25. Try cone angles of 10°, 12°, and 15°.

Solution: We can solve Equation (22-13) for the axial force F_a:

$$F_a = \frac{T_f(\sin\alpha + f\cos\alpha)}{f * R_m} = \frac{(67.8\ \mathrm{N.m})(\sin\alpha + 0.25\cos\alpha)}{(0.25)(0.127)m}$$

$$F_a = 2135(\sin\alpha + 0.25\cos\alpha)\ N$$

Then the values of (F_a) as a function of the cone angle are as follows:

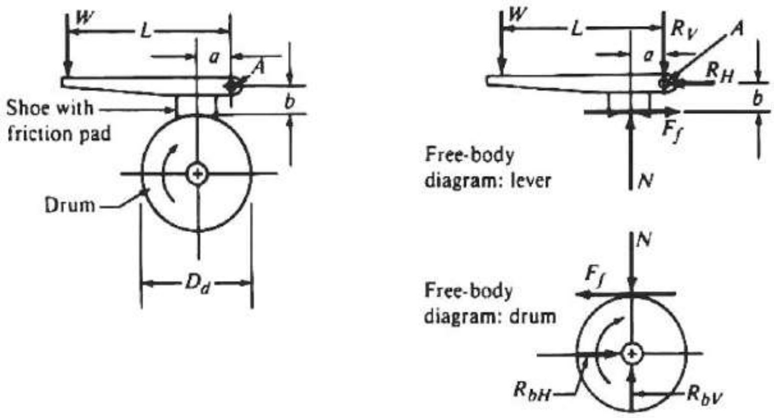
For α = 10°
F_a =898.5 N

For α = 12°
F_a =965.2 N

For α = 15°
F_a =1067.5 N

Short shoe drum brakes (section 22-14), Page 855:

$\sum M_A = 0$



$$0 = W * L - N * a + F_f * b$$

But note that: $F_f = f * N$

Or: $N = \frac{F_f}{f}$ then,

$$0 = WL - \frac{F_f\, a}{f} + F_f b = WL - F_f\left(\left(\frac{a}{f}\right) - b\right)$$

Solving for W gives:

$$W = \frac{F_f\left(\frac{a}{f} - b\right)}{L} \quad \dots\dots\dots\dots (22-15)$$

Solving for F_f gives:

$$F_f = \frac{WL}{\left(\frac{a}{f} - b\right)} \quad \dots\dots\dots\dots\dots (22-16)$$

$$T_f = F_f * \frac{D_d}{2} = \frac{W * L * D_d}{2\left(\frac{a}{f} - b\right)} \quad \dots\dots\dots\dots (22-17)$$

Note 1 : for alternate position of pivot, see part (b) & (c) on fig. (22-17) Page 856
Note 2 : see example 22-8 (Page 857)
Note 3 : if the angle of contact is greater than 45°, in such case $T_f = F_f' * D_d/2$
Where f' : Equivalent coefficient of friction

$$f' = \frac{4f\,\sin\theta}{2\theta + \sin 2\theta} \quad ; \quad \textbf{where } 2\theta \textbf{ : angle of contact}$$

Example Problem 22-8:

Compute the actuation force required for the short shoe drum brake of the figure above to produce a friction torque of 67.8 N.m. Use a drum diameter of 254 mm, a = 76.2 mm, and L =381 mm. Consider values of (f) of 0.25, 0.50, and 0.75, and different points of location of pivot A such that b ranges from 0 to 152.4 mm.

Solution:

The required friction force can be found from Equation (22-17):

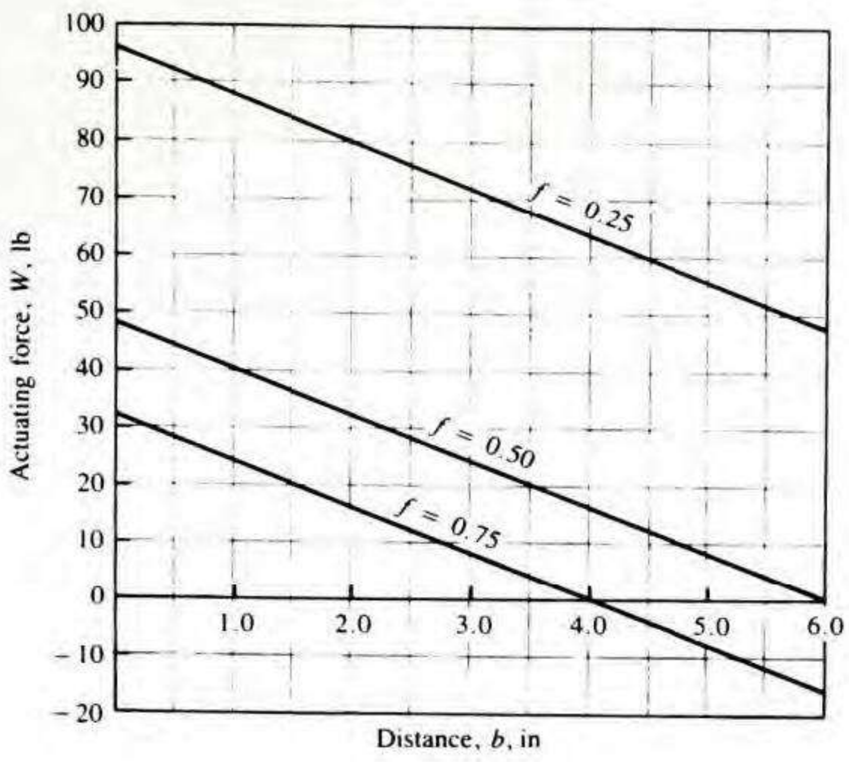
$$F_f = \frac{2T_f}{D_d} = \frac{(2)(67.8\ N.m)}{0.254\ m} = 534\ N$$

In Equation (22-15), we can substitute for a, L, and F_f.

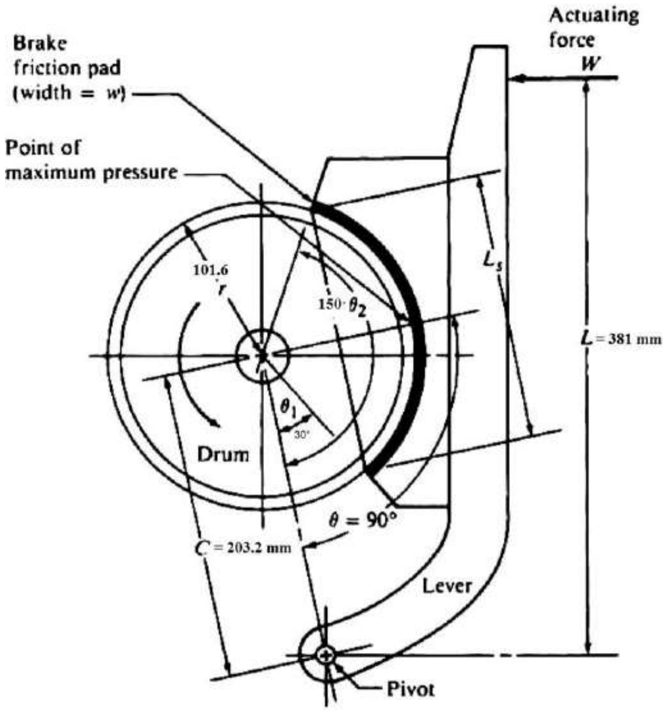
$$W = \frac{F_f\left(\frac{a}{f} - b\right)}{L} = \frac{534\ N * \left(\frac{76.2\ mm}{f} - b\right)}{381\ mm} = 1.402 N/mm\left(\frac{76.2\ mm}{f} - b\right) N$$

We can substitute the varying values of f and b into this last equation to compute the data for the curves of Figure 22-18, showing the actuating force versus the distance b for different values of (f), Note that for some combinations, the value of W is *negative.* This means that the brake is *self-actuating* and that an upward force on the lever would be required to release the brake.

FIGURE 22–18
Results: actuating load force vs. distance b



Example:



$f = 0.25$, $P_{max} = 496.6$ KPa

$T_f = 84.75$ N.m , ω= 12.564 rad/sec

Design shoe drum brake?

$$f' = \frac{4f\ sin\theta}{2\theta + sin2\theta} = \frac{4*0.25\ sin60}{120*\pi/180 + sin120}$$

f' = 0.293

$$T_f = F_f * r$$

$$F_f = 84750/101.6 = 834.2\ N$$

$$F_f = f' * N$$

$$N = 834.2 / 0.293 = 2847\ N$$

$$W * L + F_f * r = N * C \longrightarrow W = (2847*203.2 - 834.2*101.6) / 381 = 1296\ N$$

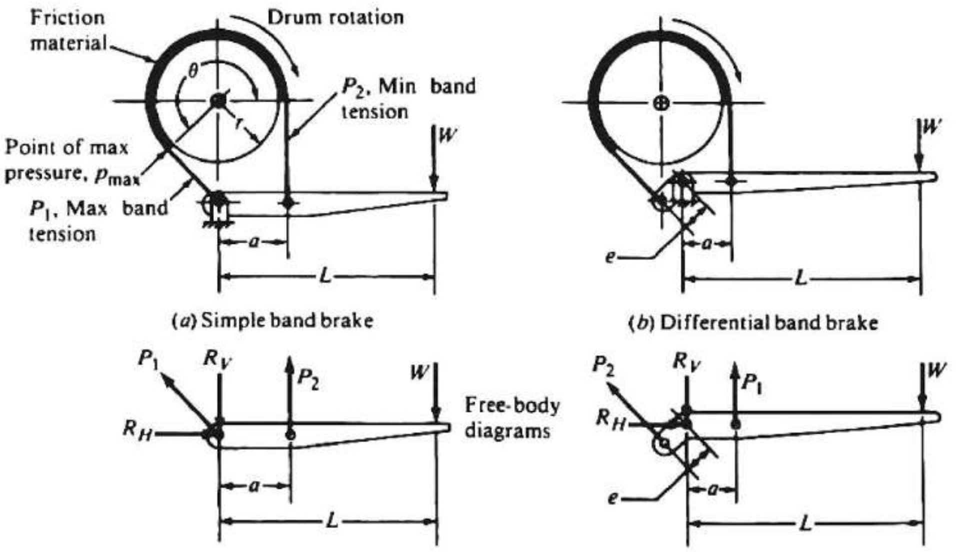
$$A = (2r\ sin\ \theta) * w$$

$$P_{max} = N/A = 2847 / (2*101.6*w*sin60) \longrightarrow w = 32.6\ mm$$

Note: you can use equations from 22-18 to 22-24 for solving the example (page 858).

Band brakes (section 22-15), (Page 860):

FIGURE 22–20
Band brake design



$$T_f = (P_1 - P_2) * r \quad \dots\dots\dots (22-25), page\ 860$$

$$\frac{P_1}{P_2} = e^{f\theta} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots . (22-26)$$

$$P_1 = P_{max} * r * w \quad \dots\dots\dots . (22-27)$$

$$W = \frac{P_2 * a}{L} \dots\dots\dots\dots\dots\dots\dots\dots\dots . (22-28)\ for\ fig. (a)$$

$$W = \frac{P_2 * a - P_1 * e}{L} \dots\dots\dots\dots . (22-28)\ for\ fig. (b)$$

Example Problem (22-10):

Design a band brake to exert a braking torque of 81.36 N.m, while slowing the drum from120 rpm (12.564 rad/sec).

Solution

1- Select a material and specify a design value for the maximum pressure. A woven friction material is desirable to facilitate the conformance to the cylindrical drum shape. Let's use p_{max} = 172.4*10^3 N/m^2 and a design value of *f* = 0.25. See Section 22-10.
2- Specify trial geometry: r, θ, w: For this problem, let's try r = 0.152 m, θ =225°, and w=0.051 m. Note that 225°= 3.93 rad.
3- Compute the maximum band tension. P_1, from equation (22-27)

$$P_1 = P_{max} * r * w = (172 * 10^3\ N/m^2)(0.152\ m)(0.051\ m) = 1333\ N$$

4- Compute tension P_2 from Equation (22-26):

$$P_2 = \frac{P_1}{e^{f\theta}} = \frac{1333\ \text{N}}{e^{(0.25*3.93)}} = 498\ N$$

5- Compute the friction torque, $\mathbf{T}_f$:

$$T_f = (P_1 - P_2) * r = (1333\ N - 498\ N)(152.4\ mm) = 127\ N.m$$

Note: Repeat Steps 2-5 until you achieve a satisfactory geometry and friction torque. Let's try a smaller design, say, r =0.127 m:

$$P_1 = (172.4 * 10^3\ N/m^2)(0.127\ m)(0.051\ m) = 1110\ N$$

$$P_2 = \frac{1110\ \text{N}}{e^{(0.25*3.93)}} = 417\ N$$

$$T_f = (1110\ N - 417\ N)(0.127\ m) = 88\ N.m \ \text{(Okay)}$$

6- Specify the geometry of the lever, and compute the required actuation force. Let's use a = 0.127 m and L = 0.381 m. Then:

$$W = \frac{P_2 * a}{L} = (417\ N) * \frac{0.127\ m}{0.381\ m} = 139\ N$$

7- Compute the average wear ratio from WR= P_f/A:

$$A = 2\pi r w\left(\frac{\theta}{360}\right) = 2\pi * (0.127\ m) * (0.051\ m)\left(\frac{225}{360}\right) = 0.025\ m^2$$

$$P_f = T_f * \omega = (88\ N.m)(12.564\ rad/sec) = 1110\ W$$

$$WR = \frac{P_f}{A} = \frac{1110\ W}{0.025\ m^2} = 44.4\frac{W}{m^2}$$

This should be conservative for average service.

Tedata

Program : MDESIGN	User :	Customer :
Version : 1.1.2	Date : 19.12.2013	Proj. Nr :

Plate-Type Clutch or Brake

Input data:

Plate-Type Clutch or Brake

Braking torque	T =	33.9	N.m
Normal force	N =	1423.424	N
Coefficient of friction	f =	0.25	
Rotational speed	n =	750	rpm
Reasonable value for the ratio Ro/Ri	rv =	1.5	

Results

Required mean radius	Rm	=	95.250	mm
Inside radius	Ri	=	76.200	mm
Outside radius	Ro	=	114.300	mm
Area of the friction surface	A	=	22796.182	mm²
Frictional power absorbed	Pf	=	2.663	kW
Wear ratio	WR	=	0.101	hp/in²

Tedata

Program : MDESIGN	User :	Customer :
Version : 1.1.2	Date : 19.12.2013	Proj. Nr :

Cone Clutch or Brake

Input data:

Cone Clutch or Brake

Braking torque	T =	5.65	N.m
Mean radius	Rm =	127	mm
Coefficient of friction	f =	0.25	
Cone angle	α =	10	°

Results

Axial force required for a cone brake	Fa	=	74.703	N

Tedata

Program : MDESIGN	User :	Customer :
Version : 1.1.2	Date : 19.12.2013	Proj. Nr :

Short Shoe Drum Brakes

Input data:

Short Shoe Drum Brakes

Friction torque	T = 5.65	N.m
Drum diameter	D = 254	mm
Pivot to pin centerline dimension	a = 76.2	mm
Pivot to contact dimension	b = 114.3	mm
Length	L = 381	mm
Coefficient of friction	f = 0.25	

Results

Friction force	F	=	44.482	N
Actuation force required for the short shoe drum brake	W	=	22.241	N

Tedata		
Program : MDESIGN Version : 1.1.2	User : Date : 19.12.2013	Customer : Proj. Nr :

Long Shoe Drum Brake

Input data:

Long Shoe Drum Brake

Friction torque	T =	84.75	N.m
Rotational speed	n =	120	rpm
Approximate maximum pressure	Pmax =	0.517107	N/mm²
Radius	r =	101.6	mm
Parameter	C =	203.2	mm
Length	L =	381	mm
Angle	θ1 =	30	°
Angle	θ2 =	150	°
Coefficient of friction	f =	0.25	

Results

Required width	w	=	36.662	mm
Let's specify	w	=	38.100	mm
Maximum pressure	pmax	=	0.498	N/mm²
Moment of the normal force	Mn	=	579.419	N.m
Moment of the friction force	Mf	=	84.750	N.m
Required actuation force	W	=	1298.162	N
Frictional power	Pf	=	1.065	kW
Projected area	A	=	6703.037	mm²
Wear ratio	WR	=	0.137	hp/in²

Tedata

Program : MDESIGN	User :	Customer :
Version : 1.1.2	Date : 19.12.2013	Proj. Nr :

Band Brakes

Input data:

Band Brakes

Braking torque	T =	81.36	N.m
Rotationall speed	n =	120	rpm
Maximum pressure	pmax =	0.172369	N/mm²
Coefficient of friction	f =	0.25	
Trial geometry			
Radius	r =	152.4	mm
Angle	θ =	225	°
Band width	w =	50.8	mm
Band distance	a =	127	mm
Length	L =	381	mm

Results

Maximum band tension	P1	=	1334.460	N
Tension	P2	=	499.963	N
Friction torque	Tf	=	127.195	N.m
Required actuation force	W	=	166.654	N
Friction area	A	=	30394.909	mm²
Friction power	Pf	=	1.599	kW
Average wear ratio	WR	=	0.045	hp/in²

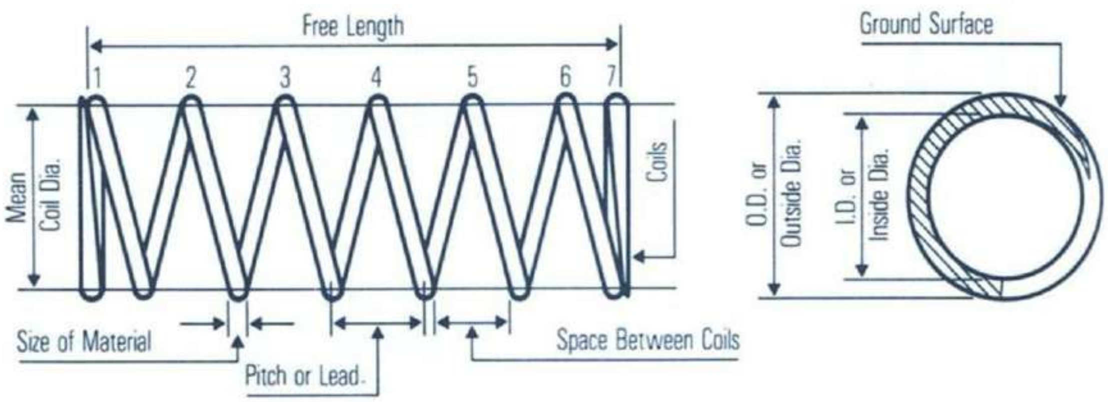
University of Technology
Mechanical Engineering Department

Machine Design I

Third Class for All Branches

LECTURES EIGHTTEEN & NINETEEN

SPRINGS



Reference: "Machine Elements in Mechanical Design" 4th Edition in SI units, By: Robert L. Mott, Chapter 19.

Introduction:

- A spring is a flexible element used to exert a force or torque and at the same time to store energy.
- Objective of this chapter (see section 19-1, page 732)

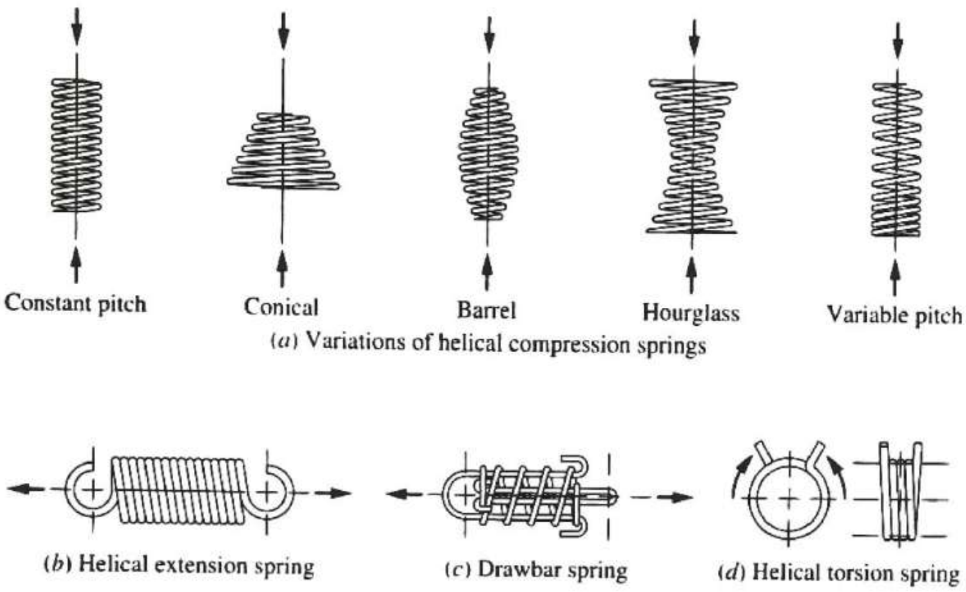
Kinds of springs (section 19-2, page 732):

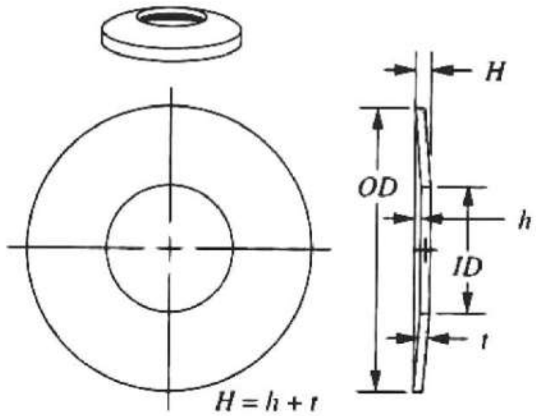
See table (19-1) and Figure (19-2)

TABLE 19–1 Types of springs

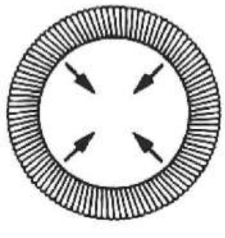
Uses	Types of springs
Push	Helical compression spring
	Belleville spring
	Torsion spring: force acting at the end of the torque arm
	Flat spring, such as a cantilever or leaf spring
Pull	Helical extension spring
	Torsion spring: force acting at the end of the torque arm
	Flat spring, such as a cantilever or leaf spring
	Drawbar spring (special case of the compression spring)
	Constant-force spring
Radial	Garter spring, elastomeric band, spring clamp
Torque	Torsion spring, power spring

FIGURE 19–2
Several types of springs

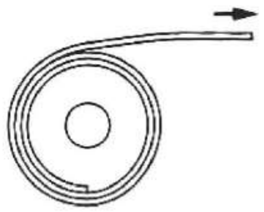




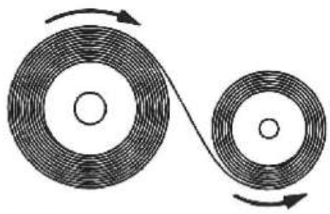
(e) Belleville spring



(f) Garter spring



(g) Constant force spring

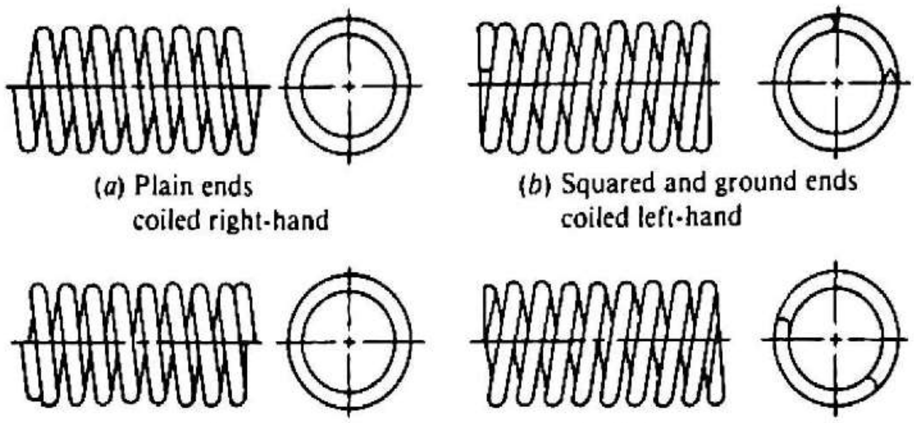


(h) Constant force spring motor

Helical compression springs (section 19-3, page 735):

In the most common form of helical compression spring, round wire is wrapped into a cylindrical form with a constant pitch between adjacent coils. This basic form is completed by a variety of end treatments as shown in fig. 19-3 (b, c, and d), page 734.

FIGURE 19–3
Appearance of helical compression springs showing end treatments



(a) Plain ends coiled right-hand

(b) Squared and ground ends coiled left-hand

(c) Squared or closed ends not ground, coiled right-hand

(d) Plain ends ground coiled left-hand

Note:

Figure 19-3 (b) is using for medium to large-size springs. Figure 19-3 (c) is using for springs with smaller wire. Figure 19-3 (d) is using for springs with unusual cases.

Diameters:

OD: outer diameter = $D_m + D_w$
ID: inside diameter = $D_m - D_w$
Where:
D_m: mean diameter of coil
D_w: wire diameter

Tables (19-2), page 736 shows Standard wire diameters

FIGURE 19–5
Notation for diameters

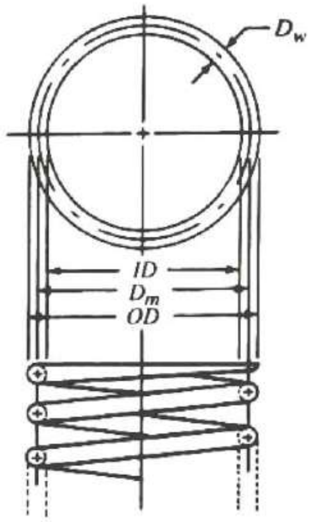


TABLE 19–2 Wire gages and diameters for springs

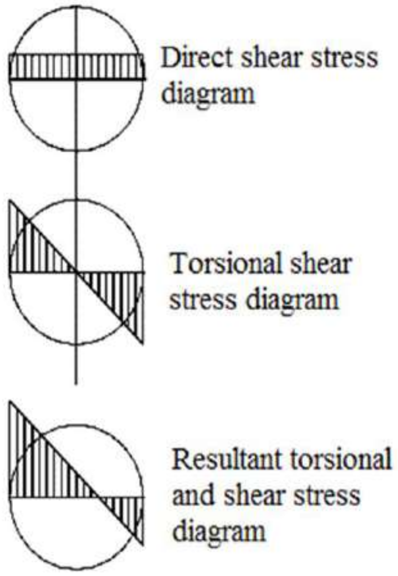
Gage no.	U.S. Steel Wire Gage $(in)^a$	Music Wire Gage $(in)^b$	Brown & Sharpe Gage $(in)^c$	Preferred Metric Diameters $(mm)^d$
7/0	0.4900			13.0
6/0	0.4615	0.004	0.5800	12.0
5/0	0.4305	0.005	0.5165	11.0
4/0	0.3938	0.006	0.4600	10.0
3/0	0.3625	0.007	0.4096	9.0
2/0	0.3310	0.008	0.3648	8.5
0	0.3065	0.009	0.3249	8.0
1	0.2830	0.010	0.2893	7.0
2	0.2625	0.011	0.2576	6.5
3	0.2437	0.012	0.2294	6.0
4	0.2253	0.013	0.2043	5.5
5	0.2070	0.014	0.1819	5.0
6	0.1920	0.016	0.1620	4.8
7	0.1770	0.018	0.1443	4.5
8	0.1620	0.020	0.1285	4.0
9	0.1483	0.022	0.1144	3.8
10	0.1350	0.024	0.1019	3.5
11	0.1205	0.026	0.0907	3.0
12	0.1055	0.029	0.0808	2.8
13	0.0915	0.031	0.0720	2.5
14	0.0800	0.033	0.0641	2.0
15	0.0720	0.035	0.0571	1.8
16	0.0625	0.037	0.0508	1.6
17	0.0540	0.039	0.0453	1.4
18	0.0475	0.041	0.0403	1.2
19	0.0410	0.043	0.0359	1.0
20	0.0348	0.045	0.0320	0.90
21	0.0317	0.047	0.0285	0.80
22	0.0286	0.049	0.0253	0.70

TABLE 19–2 *(continued)*

Gage no.	U.S. Steel Wire Gage (in)[a]	Music Wire Gage (in)[b]	Brown & Sharpe Gage (in)[c]	Preferred metric diameters (mm)[d]
23	0.0258	0.051	0.0226	0.65
24	0.0230	0.055	0.0201	0.60 or 0.55
25	0.0204	0.059	0.0179	0.50 or 0.55
26	0.0181	0.063	0.0159	0.45
27	0.0173	0.067	0.0142	0.45
28	0.0162	0.071	0.0126	0.40
29	0.0150	0.075	0.0113	0.40
30	0.0140	0.080	0.0100	0.35
31	0.0132	0.085	0.00893	0.35
32	0.0128	0.090	0.00795	0.30 or 0.35
33	0.0118	0.095	0.00708	0.30
34	0.0104	0.100	0.00630	0.28
35	0.0095	0.106	0.00501	0.25
36	0.0090	0.112	0.00500	0.22
37	0.0085	0.118	0.00445	0.22
38	0.0080	0.124	0.00396	0.20
39	0.0075	0.130	0.00353	0.20
40	0.0070	0.138	0.00314	0.18

Stresses on wire diameter of spring:

$\zeta_1 = \frac{F}{A}$ ……… (a)



$\zeta_2 = \frac{T.c}{J}$ ……….. (b)

$\zeta_3 = \zeta_1 + \zeta_2$

Lengths:

FIGURE 19–6
Notation for lengths and forces

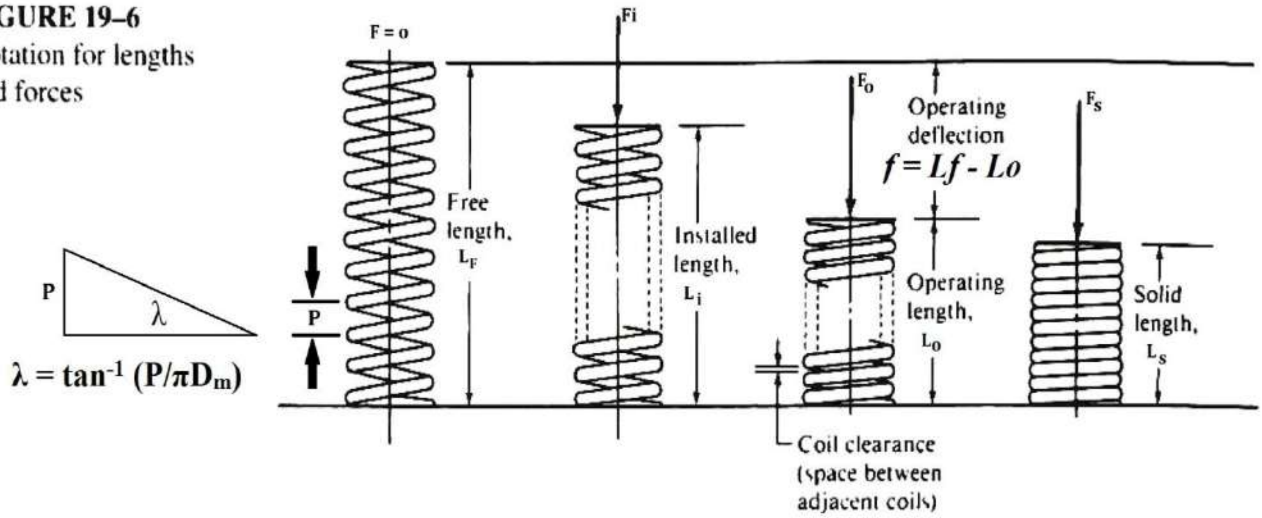
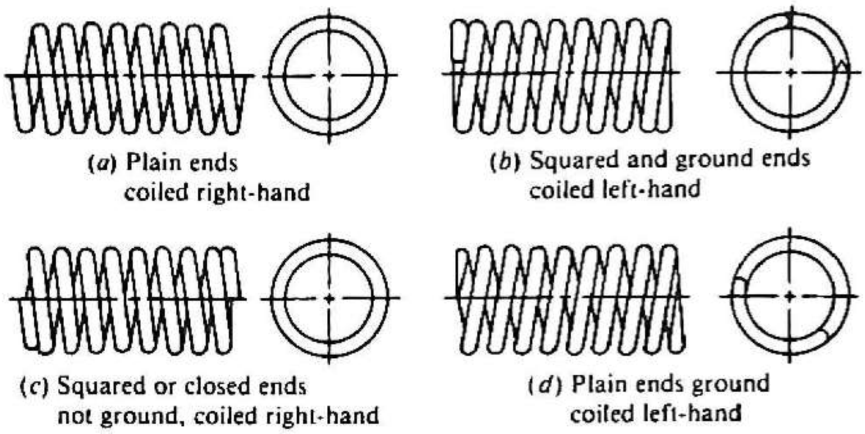


FIGURE 19–3
Appearance of helical compression springs showing end treatments



For above fig. (a) ⟶ $N_a = N$; $L_F = PN_a + D_w$

Fig. (b) ⟶ $N_a = N - 2$; $L_F = PN_a + 2D_w$

Fig. (c) ⟶ $N_a = N - 2$; $L_F = PN_a + 3D_w$

Fig. (d) ⟶ $N_a = N - 1$; $L_F = P(N_a + 1)$

Notes:

- **Forces:** see figure 19-6, page 737
- **Spring rate:** The relationship between force and its deflection (K)

$$K = \frac{\Delta F}{\Delta L} = \frac{F_O - F_i}{L_i - L_O} = \frac{F_O}{L_F - L_i} = \frac{Fi}{LF - Li} \text{ .... } (19\text{-}1)$$

- **Spring index:** $C = \frac{D_m}{D_w}$
- **Number of coils :** N
- **Number of active coil:** N_a
- **Pitch (P):** Axial distance from one coil to adjacent coil.
- **Pitch angle :** $\lambda = \tan^{-1}\left(\frac{P}{\pi D_m}\right)$
- **Materials used for springs:** see page 740 and table (19-3), page 741.
- **Types of loading and allowable stresses:**
  - Light service: for static load or up to 1000 cycles
  - Average service: for moderate load or up to 10^6 cycles
  - Severe service: for impact load or over 10^6 cycles

TABLE 19–3 Spring materials

Material type	ASTM no.	Relative cost	Temperature limits, °F
A. *High-carbon steels*			
Hard-drawn	A227	1.0	0–250
General-purpose steel with 0.60%–0.70% carbon; low cost			
Music wire	A228	2.6	0–250
High-quality steel with 0.80%–0.95% carbon; very high strength; excellent surface finish; hard-drawn; good fatigue performance; used mostly in smaller sizes up to 0.125 in			
Oil-tempered	A229	1.3	0–350
General-purpose steel with 0.60%–0.70% carbon; used mostly in larger sizes above 0.125 in; not good for shock or impact			
B. *Alloy steels*			
Chromium-vanadium	A231	3.1	0–425
Good strength, fatigue resistance, impact strength, high-temperature performance; valve-spring quality			
Chromium-silicon	A401	4.0	0–475
Very high strength and good fatigue and shock resistance			
C. *Stainless steels*			
Type 302	A313(302)	7.6	<0–550
Very good corrosion resistance and high-temperature performance; nearly nonmagnetic; cold-drawn; types 304 and 316 also fall under this ASTM class and have improved workability but lower strength			
Type 17-7 PH	A313(631)	11.0	0–600
Good high-temperature performance			
D. *Copper alloys:* All have good corrosion resistance and electrical conductivity.			
Spring brass	B134	High	0–150
Phosphor bronze	B159	8.0	<0–212
Beryllium copper	B197	27.0	0–300
E. *Nickel-base alloys:* All are corrosion-resistant, have good high- and low-temperature properties, and are nonmagnetic or nearly nonmagnetic (trade names of the International Nickel Company).			
Monel™			−100–425
K-Monel™			−100–450
Inconel™			Up to 700
Inconel-X™		44.0	Up to 850

Source: Associated Spring, Barnes Group, Inc. *Engineering Guide to Spring Design.* Bristol, CT, 1987. Carlson, Harold. *Spring Designer's Handbook.* New York: Marcel Dekker, 1978. Oberg, E., et al. *Machinery's Handbook.* 26th ed. New York: Industrial Press, 2000.

FIGURE 19–8
Design shear stresses for ASTM A227 steel wire, hard-drawn (Reprinted from Harold Carlson, *Spring Designer's Handbook*, p. 144, by courtesy of Marcel Dekker, Inc.)

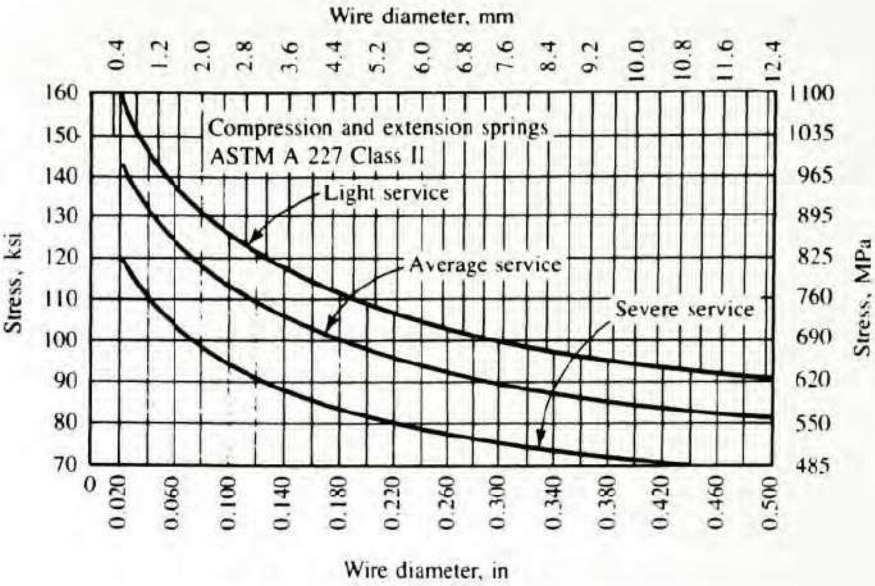


FIGURE 19–9
Design shear stresses for ASTM A228 steel wire (music wire) (Reprinted from Harold Carlson, *Spring Designer's Handbook*, p. 143, by courtesy of Marcel Dekker, Inc.)

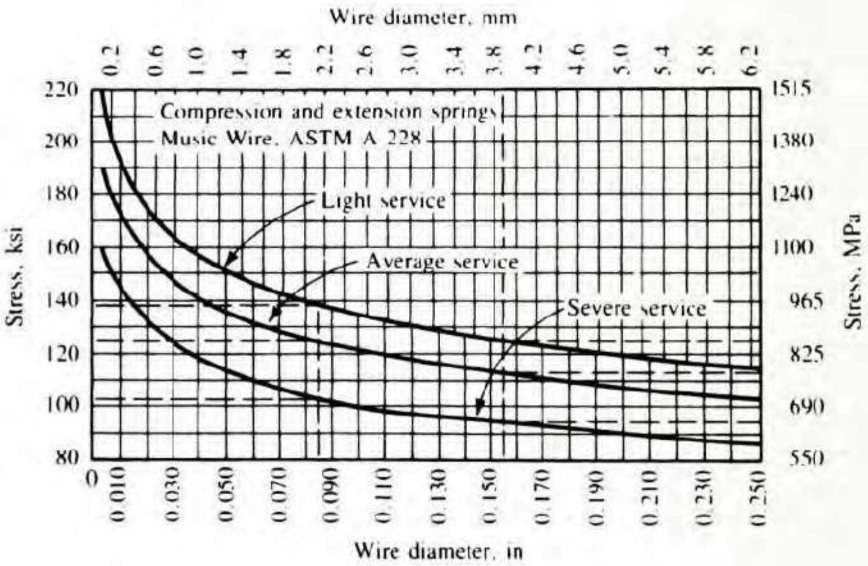


FIGURE 19–10
Design shear stresses for ASTM A229 steel wire, oil-tempered (Reprinted from Harold Carlson, *Spring Designer's Handbook*, p. 146, by courtesy of Marcel Dekker, Inc.)

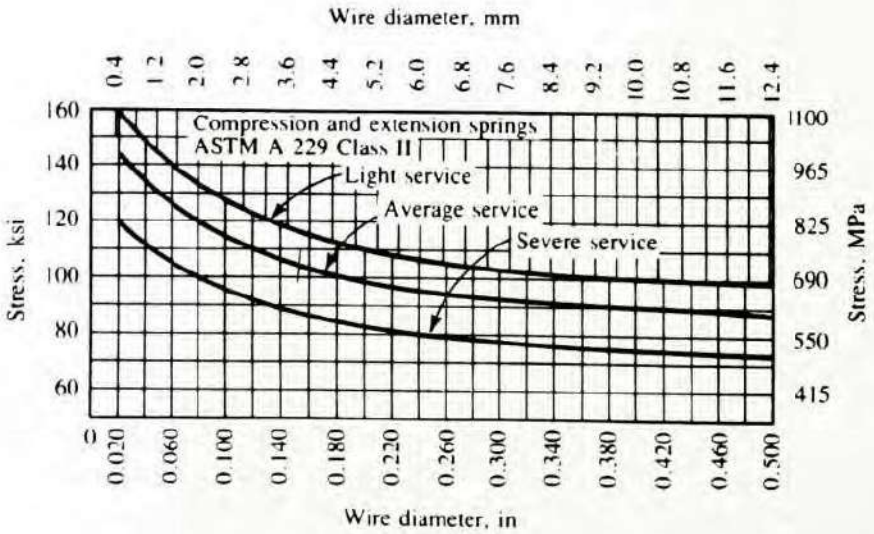


FIGURE 19–11
Design shear stresses for ASTM A231 steel wire, chromium-vanadium alloy, valve-spring quality (Reprinted from Harold Carlson, *Spring Designer's Handbook*, p. 147, by courtesy of Marcel Dekker, Inc.)

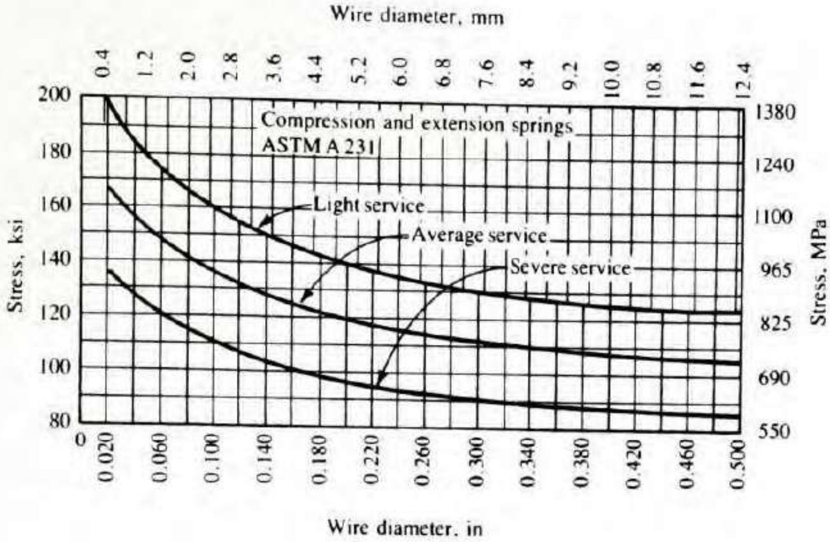


FIGURE 19–12
Design shear stresses for ASTM A401 steel wire, chromium-silicon alloy, oil-tempered (Reprinted from Harold Carlson, *Spring Designer's Handbook*, p. 148, by courtesy of Marcel Dekker, Inc.)

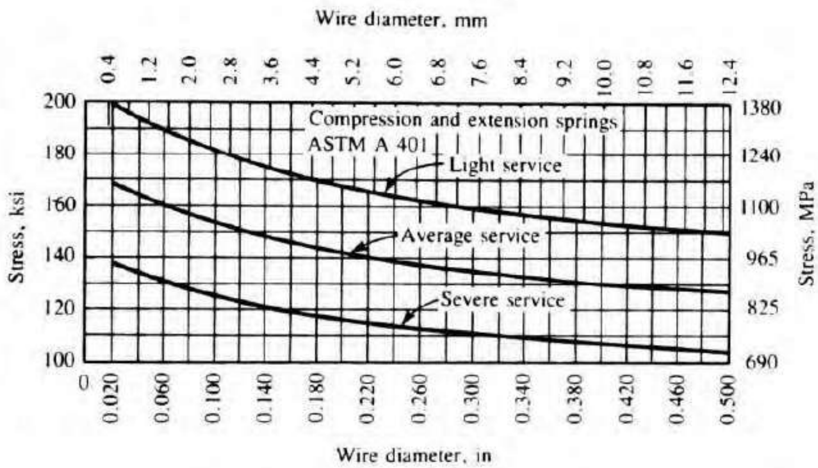
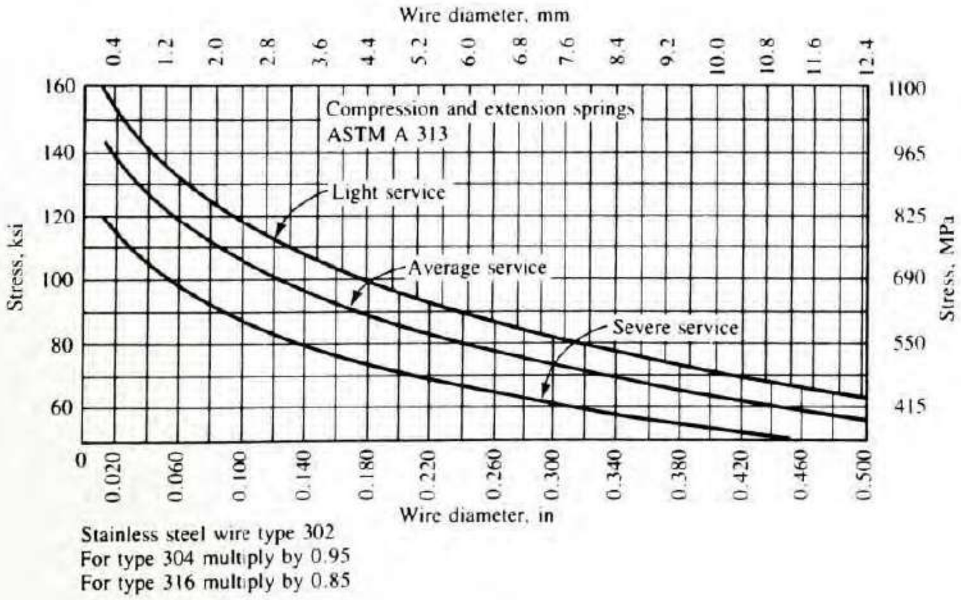


FIGURE 19–13
Design shear stresses for ASTM A313 corrosion-resistant stainless steel wire (Reprinted from Harold Carlson, *Spring Designer's Handbook*, p. 150, by courtesy of Marcel Dekker, Inc.)



Figures (19-8, 19-9, 19-10, 19-11, 19-12, and 19-13) are used for different materials.

Stresses and deflection for helical springs (section 19-4, page 744):

As a compression spring is compressed under an axial load the wire is twisted. Therefore the stress developed is Torsional shear stress $\left\{\zeta 1 = \frac{T*c}{J}\right\}$ combined with direct shear stress $\left(\zeta 2 = \frac{F}{A}\right)$then ($\zeta = \zeta_1 + \zeta_2$).

$$\zeta = \left\{\frac{\left(F * \frac{D_m}{2} * \frac{D_W}{2}\right)}{\frac{\pi}{32} D_W^4}\right\} + \frac{F}{\left(\frac{\pi D_W^2}{4}\right)}$$

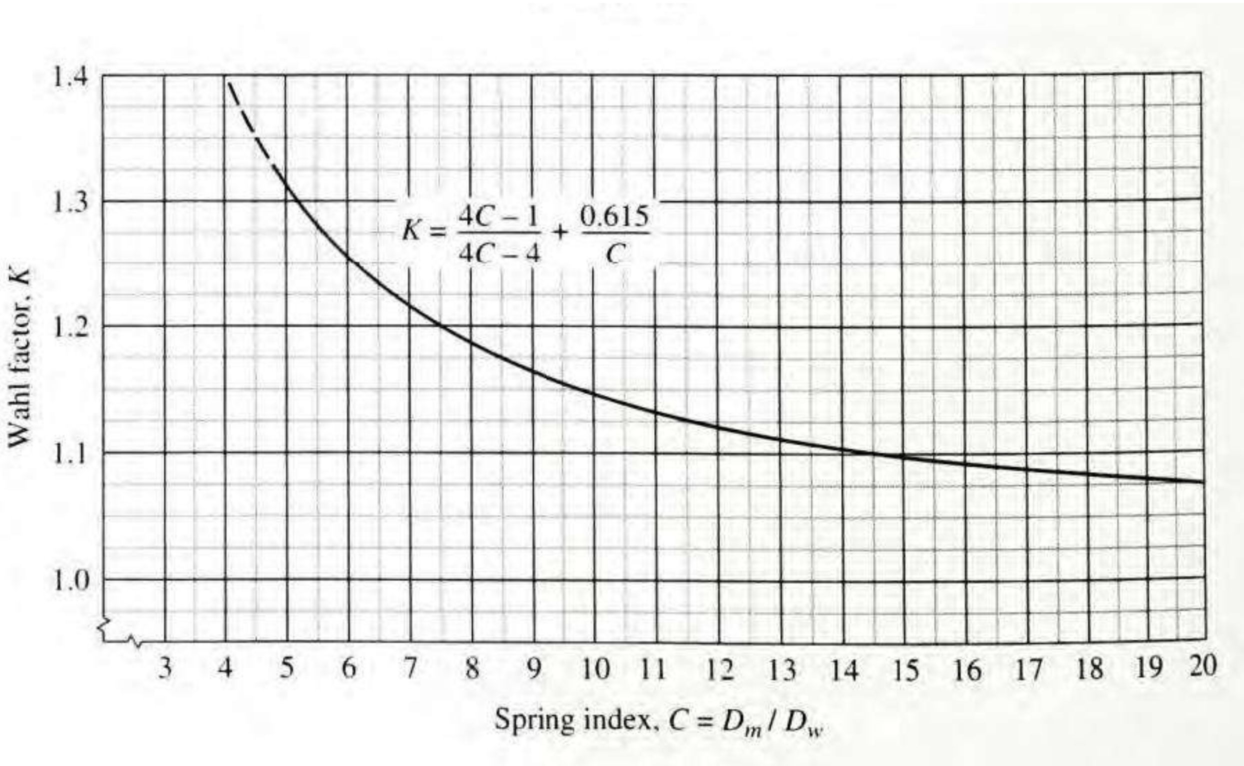
$$= \left(\frac{8FD_m}{\pi D_W^3}\right)\left(1 + \frac{D_W}{2D_m}\right) = \left(\frac{8FD_m}{\pi D_W^3}\right)\left(1 + \frac{1}{2C}\right)$$

$$\zeta = \left(\frac{8FD_m}{\pi D_W^3}\right)(K) = \left(\frac{8KFC}{\pi D_W^2}\right) \quad \text{……… (19-4)}$$

Where: C = spring index ; K = Wahl factor = $\frac{4C-1}{4C-4} + \frac{0.615}{C}$

Note:

1. Wahl factor is the term that account for the curvature of the wire and stress concentration.
2. See fig. (19-14), page 744 to find Wahl factor.
3. Recommended C is 12 > C > 5



Deflection:

$\theta = \frac{T*L}{G*J}$ & f = Deflection = $\theta * \frac{Dm}{2}$

$$f = \frac{T*L}{G*J} * \frac{D_m}{2} = \frac{\left\{\left(F*\frac{Dm}{2}\right)*(\pi D_m * N_a)\right\}}{\frac{G*\pi D_W^4}{32}} * \frac{D_m}{2}$$

$$\boldsymbol{f = \frac{(8F * D_m{}^3 * Na)}{G*D_w{}^4} = \frac{(8F * C^3 * Na)}{G*D_w}} \qquad \text{.......... (19-6)}$$

Where:

θ: Angle of twist in radians.

T: Applied torque = F * $\frac{D_m}{2}$

L: Length of wire = $\pi D_m * N_a$

G: Modulus of elasticity in shear (see table 19-4, page 745)

J: polar moment of inertia for wire =$\frac{\pi D_W^4}{32}$.

TABLE 19–4 Spring wire modulus of elasticity in shear (G) and tension (E)

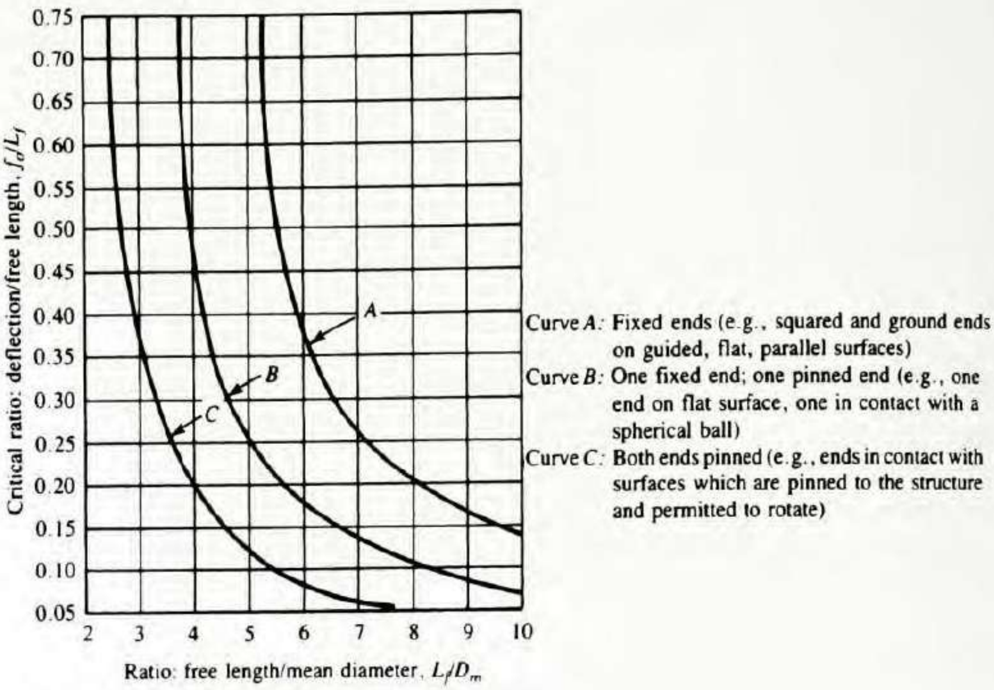
Material and ASTM no.	Shear modulus, G (psi)	Shear modulus, G (GPa)	Tension modulus, E (psi)	Tension modulus, E (GPa)
Hard-drawn steel: A227	11.5×10^6	79.3	28.6×10^6	197
Music wire: A228	11.85×10^6	81.7	29.0×10^6	200
Oil-tempered: A229	11.2×10^6	77.2	28.5×10^6	196
Chromium-vanadium: A231	11.2×10^6	77.2	28.5×10^6	196
Chromium-silicon: A401	11.2×10^6	77.2	29.5×10^6	203
Stainless steels: A313				
Types 302, 304, 316	10.0×10^6	69.0	28.0×10^6	193
Type 17-7 PH	10.5×10^6	72.4	29.5×10^6	203
Spring brass: B134	5.0×10^6	34.5	15.0×10^6	103
Phosphor bronze: B159	6.0×10^6	41.4	15.0×10^6	103
Beryllium copper: B197	7.0×10^6	48.3	17.0×10^6	117
Monel and K-Monel	9.5×10^6	65.5	26.0×10^6	179
Inconel and Inconel-X	10.5×10^6	72.4	31.0×10^6	214

Note: Data are average values. Slight variations with wire size and treatment may occur.

Buckling:

Figure (19-15) shows buckling criteria (page 746). First compute $\frac{L_f}{D_m}$ then find $\frac{f_O}{L_f}$ after that check the buckling from figure (19-15).

FIGURE 19–15
Spring buckling criteria. If the actual ratio of f_o/L_f is greater than the critical ratio, the spring will buckle at operating deflection.



Analysis of spring characteristics and design:

See example problem (19-1) to example (19-3).

Note:

- The following formula can be used for OD at the solid length condition:

$$OD_S = \sqrt{D_m^2 + \frac{P^2 - D_w^2}{\pi^2}} + D_w \quad \text{……….} \ (19\text{-}3)$$

- An initial diametric clearance of one- tenth of the wire diameter is recommended for springs having a diameter of 12 mm or greater.
- Coil clearance $C_C = (L_O - L_S)/N_a$
- Check that $(L_O - L_S) > 0.15(L_F - L_S)$

Example Problem 19-1, (Page 796): A spring is known to be made from music wire, ASTM A228 steel, but no other data are known. You are able to measure the following features using simple measurement tools:

Free length = L_f = 44.45 mm
Outside diameter = OD = 14.25 mm
Wire diameter = D_w= 1.4 mm
The ends are squared and ground.

The total number of coils (N) = 10.0.

Load=62.27 N at ≈300000 cycles.

Sol:

1. From table 19-2 the wire is 17-gage music wire,
   $D_m = OD - D_w = 14.25-1.4 = 12.85$ mm ;
   $ID = D_m - D_w = 12.85-1.4 = 11.45$ mm
   $C = D_m/D_w = 12.85/1.4 = 9.2$;
   $K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = 1.158$

2. $F = F_0 = 62.27$ N
   $\zeta = \left(\frac{8KFC}{\pi D_w^2}\right) = \frac{8\,(1.158)\,(62.27)(9.2)}{\pi\,(1.4)^2} = 865.1$ MPa

3. $f = \frac{(8F * C^3 * Na)}{G * Dw} = \frac{8\,(62.27)(9.2^3)\,(8)}{81.7 * 10^9 * 1.4} = 27.2\text{ mm} = f_o$

   Na = N – 2 = 10 – 2 = 8 (for squared & ground) ; G= 81.7 GPa from table (19-4)

4. $L_0 = L_F - f_0 = 44.45 - 27.2 = 17.25$ mm ; $L_S = D_w * N = 1.4*10 = 14$ mm ;
   $K = \Delta F/\Delta L = F_o/(L_F - L_o) = F_0/f_0 = 62.27/27.2 = 2.29$ N/mm
5. $F_S = K\,(L_F - L_S) = 2.29(44.45-14) = 69.79$ N ;
   $\zeta = \frac{8\,F\,F_S\,C}{\pi\,D_w^2} = 970$ MPa
6. From fig. 19-9 for ASTM A228 steel for average service
   $\zeta_d = 930.8$ MPa at $D_w = 14$ mm > $\zeta_o = 865.7$ MPa **satisfactory**

7. $\zeta_S > \zeta_d$ so use light service ζ_d= 1034 MPa at D_w = 1.4 mm (in this case $\zeta_d > \zeta_S$ **satisfactory**)
8. $\frac{L_F}{D_m}$ = 49.5/12.85 = 3.46, so from fig. 19-15 use curve A (No Buckling)
9. $D_{hole} > OD + \frac{D_w}{10} = 14.4$ mm

Tedata

Program : MDESIGN	User :	Customer :
Version : 1.1.2	Date : 22.12.2013	Proj. Nr :

Helical Compression Springs

Example: 19-1 (Page 746)

Input data:

Helical Compression Springs

Spring Geometry		
Installed length	Li = 44.45	mm
Operating length	Lo = 17.25	mm
Trial mean diameter	Dm = 12.85	mm
Ends type	Squared and ground	
Forces and Stresses		
Type of spring wire	= Music wire: A228	
Shear modulus of elasticity of spring wire	G = 81702.906	N/mm²
Maximum operating force	Fo = 62.27	N
Installed force	Fi = 0.0001	N
Load type	Average service	
Initial design stress	τid = 970	N/mm²

Results

Spring rate	k	=	2.289N/mm
Free length	Lf	=	44.450mm
Actual design stress	τd	=	926.366N/mm²
Maximum allowable stress	τmax	=	1032.697N/mm²
Computed wire diameter	Dw	=	1.372mm
Spring index	C	=	9.369
Actual expected stress due to operating force	τo	=	3.361e+006mm
Number of active coils	Na	=	7.441
Solid length	Ls	=	12.949mm
Force at solid length	Fs	=	72.116N
Stress at solid length	τs	=	1056.510N/mm²

Note
The Stress at solid length τs should be greater than the Maximum allowable stress τmax

Outside diameter	OD	=	14.222mm
Inside diameter	ID	=	11.478mm
Buckling ratio	Lf/Dm	=	3.459
Coil clearance	cc	=	0.578mm
If installed in hole, minimum hole diameter			14.359mm

12/22/2013 11:58:54 Page 1/1

Tedata

Program : MDESIGN	User :	Customer :
Version : 1.1.2	Date : 22.12.2013	Proj. Nr :

Helical Compression Springs

Example: 19-2 (Page 749)

Input data:

Helical Compression Springs

Spring Geometry			
Installed length	Li =	44.45	mm
Operating length	Lo =	31.75	mm
Trial mean diameter	Dm =	8.71	mm
Ends type	Squared and ground		
Forces and Stresses			
Type of spring wire	= Chromium-vanadium: A231		
Shear modulus of elasticity of spring wire	G =	77221.312	N/mm²
Maximum operating force	Fo =	53.38	N
Installed force	Fi =	35.6	N
Load type	Light service		
Initial design stress	τid =	992.9	N/mm²

Results

Spring rate	k	=	1.400N/mm
Free length	Lf	=	69.879mm
Actual design stress	τd	=	1231.283N/mm²
Maximum allowable stress	τmax	=	1231.283N/mm²
Computed wire diameter	Dw	=	1.206mm
Spring index	C	=	7.219
Actual expected stress due to operating force	τo	=	2.995e+006mm
Number of active coils	Na	=	22.109
Solid length	Ls	=	29.088mm
Force at solid length	Fs	=	57.107N
Stress at solid length	τs	=	869.639N/mm²
Outside diameter	OD	=	9.916mm
Inside diameter	ID	=	7.503mm
Buckling ratio	Lf/Dm	=	8.023
Coil clearance	cc	=	0.120mm
If installed in hole, minimum hole diameter			10.037mm

12/22/2013 12:26:49 Page 1/1

Tedata

Program : MDESIGN	User :	Customer :
Version : 1.1.2	Date : 22.12.2013	Proj. Nr :

Helical Compression Springs

Example: 19-2 (Page 749)

Input data:

Helical Compression Springs

Spring Geometry			
Installed length	L_i =	44.45	mm
Operating length	L_o =	31.75	mm
Trial mean diameter	D_m =	8.71	mm
Ends type	Squared and ground		
Forces and Stresses			
Type of spring wire	=	Chromium-vanadium: A231	
Shear modulus of elasticity of spring wire	G =	77221.312	N/mm²
Maximum operating force	F_o =	53.38	N
Installed force	F_i =	35.6	N
Load type	Light service		
Initial design stress	τ_{id} =	992.9	N/mm²

Results

Spring rate	k	=	1.400N/mm
Free length	Lf	=	69.879mm
Actual design stress	τd	=	1231.283N/mm²
Maximum allowable stress	τmax	=	1231.283N/mm²
Computed wire diameter	Dw	=	1.206mm
Spring index	C	=	7.219
Actual expected stress due to operating force	τo	=	2.995e+006mm
Number of active coils	Na	=	22.109
Solid length	Ls	=	29.088mm
Force at solid length	Fs	=	57.107N
Stress at solid length	τs	=	869.639N/mm²
Outside diameter	OD	=	9.916mm
Inside diameter	ID	=	7.503mm
Buckling ratio	Lf/Dm	=	8.023
Coil clearance	cc	=	0.120mm
If installed in hole, minimum hole diameter			10.037mm

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Tedata

Program : MDESIGN	User :	Customer :
Version : 1.1.2	Date : 22.12.2013	Proj. Nr :

Helical Compression Springs

Example: 19-3 (Page 754)

Input data:

Helical Compression Springs

Spring Geometry			
Installed length	Li =	44.45	mm
Operating length	Lo =	31.75	mm
Trial mean diameter	Dm =	15.24	mm
Ends type	Squared and ground		
Forces and Stresses			
Type of spring wire	= Chromium-vanadium: A231		
Shear modulus of elasticity of spring wire	G =	77221.312	N/mm²
Maximum operating force	Fo =	53.38	N
Installed force	Fi =	35.6	N
Load type	Light service		
Initial design stress	τid =	896.35	N/mm²

Results

Spring rate	k	=	1.400N/mm
Free length	Lf	=	69.879mm
Actual design stress	τd	=	1183.293N/mm²
Maximum allowable stress	τmax	=	1183.293N/mm²
Computed wire diameter	Dw	=	1.587mm
Spring index	C	=	9.600
Actual expected stress due to operating force	τo	=	2.196e+006mm
Number of active coils	Na	=	12.371
Solid length	Ls	=	22.815mm
Force at solid length	Fs	=	65.890N
Stress at solid length	τs	=	735.837N/mm²
Outside diameter	OD	=	16.827mm
Inside diameter	ID	=	13.652mm
Buckling ratio	Lf/Dm	=	4.585
Coil clearance	cc	=	0.722mm
If installed in hole, minimum hole diameter			16.986mm

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University of Technology
Mechanical Engineering Department

Machine Design I

Third Class for All Branches

LECTURES TWENTY &TWENTY ONE

FASTENERS

Reference: "Machine Elements in Mechanical Design" 4th Edition in SI units, By: Robert L. Mott, Chapter 18 & 20.

"A Textbook of Machine Design" 14th Edition in SI units, By: R. S. Khurmi & J. K. Gupta.

Introduction:

A fastener: is any device used to connect or join two or more components. Literature hundreds of fastener types and variation are available. The most common are threaded fastener referred to by many names, among them bolts, screws, nut, and studs ….etc. see figures below.

A bolt: is a threaded fastener designed to pass through holes in the mating members and to be second by tightening a nut from the end opposite threaded of the bolt, see figure (18-1- a).

A screw: is a threaded fastener designed to be inserted through a hole in one member and to be joined and in to a threaded hole in the mating member, see figure (18-1- b).

FIGURE 18–1 Comparison of a bolt with a screw (R. P. Hoelscher et al., *Graphics for Engineers*, New York: John Wiley & Sons, 1968)

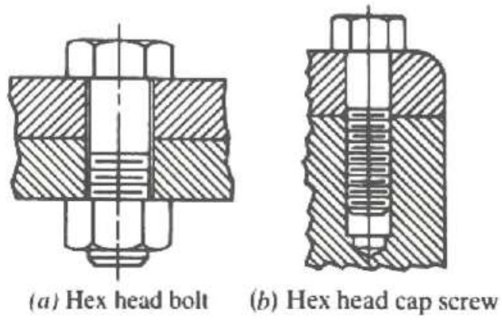


FIGURE 18–2 Bolt styles. See also the hex head bolt in Figure 18–1. (R. P Hoelscher et al., *Graphics for Engineers*, New York: John Wiley & Sons, 1968)

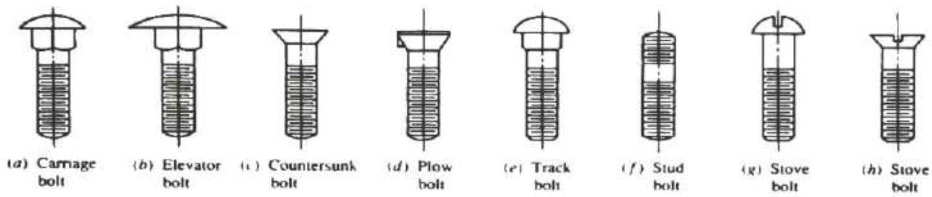


FIGURE 18–3 Cap screws or machine screws. See also the hex head cap screw in Figure 18–1. (R. P. Hoelscher et al., *Graphics for Engineers*, New York: John Wiley & Sons, 1968)

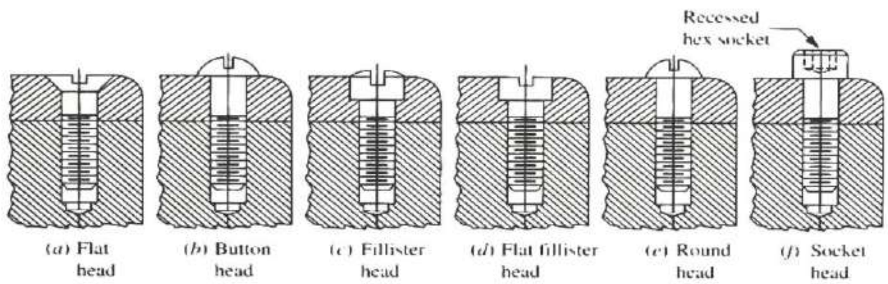


FIGURE 18–4 Sheet-metal and lag screws (R. P. Hoelscher et al., *Graphics for Engineers*, New York: John Wiley & Sons, 1968)

Bolt Materials and Strength (Section 18-2, Page 714)

Table 18-1 SAE grades of steels for fasteners

Grade No.	Bolt size		Tensile Strength		Yield Strength		Proof Strength		Head marking
	(in)	(mm)	(Ksi)	(GPa)	(Ksi)	(GPa)	(Ksi)	(GPa)	
1	$\frac{1}{4}$ - $1\frac{1}{2}$	6.3-38.1	60	0.41	36	0.25	33	0.23	None
2	$\frac{1}{4}$ - $\frac{3}{4}$	6.3-19.1	74	0.51	57	0.39	55	0.38	None
	$>\frac{3}{4}$ - $1\frac{1}{2}$	> 19.1-39.1	60	0.41	36	0.25	33	0.23	
4	$\frac{1}{4}$ - $1\frac{1}{2}$	6.3-38.1	115	0.79	100	0.69	65	0.45	None
5	$\frac{1}{4}$ - 1	6.3-25.4	120	0.83	92	0.63	85	0.59	
	> 1 - $1\frac{1}{2}$	>25.4-38.1	105	0.72	81	0.56	74	0.51	
7	$\frac{1}{4}$ - $1\frac{1}{2}$	6.3-38.1	133	0.92	115	0.79	105	0.72	
8	$\frac{1}{4}$ - $1\frac{1}{2}$	6.3-38.1	150	1.03	130	0.5	120	0.83	

Notes:

1. In machine design, most fasteners are made from steel because of its high strength, high stiffness, good ductility, and good machinability and formability.
2. Aluminum, Brass, Copper, bronze, Nickel and its alloys, plastic and stainless steel are also used for fasteners.
3. Proof strength is similar to elastic limit $\cong (\mathbf{0.9-0.95})\boldsymbol{S_y}$

Thread designations and stress area (Section 18.3, Page 717)

See table (18-4) for American standard thread dimensions. Below is table (18-5) for metric thread dimensions for coarse and fine thread (Page 718).

TABLE 18–4 American Standard thread dimensions

A. Numbered sizes

Size	Basic major diameter (in)	Coarse threads: UNC Threads per in	Coarse threads: UNC Tensile stress area (in^2)	Fine threads: UNF Threads per in	Fine threads: UNF Tensile stress area (in^2)
0	0.0600			80	0.001 80
1	0.0730	64	0.00263	72	0.002 78
2	0.0860	56	0.00370	64	0.003 94
3	0.0990	48	0.00487	56	0.005 23
4	0.1120	40	0.00604	48	0.006 61
5	0.1250	40	0.00796	44	0.008 30
6	0.1380	32	0.00909	40	0.010 15
8	0.1640	32	0.0140	36	0.014 74
10	0.1900	24	0.0175	32	0.0200
12	0.2160	24	0.0242	28	0.0258
B. Fractional sizes					
1/4	0.2500	20	0.0318	28	0.0364
5/16	0.3125	18	0.0524	24	0.0580
3/8	0.3750	16	0.0775	24	0.0878
7/16	0.4375	14	0.1063	20	0.1187
1/2	0.5000	13	0.1419	20	0.1599
9/16	0.5625	12	0.182	18	0.203
5/8	0.6250	11	0.226	18	0.256
3/4	0.7500	10	0.334	16	0.373
7/8	0.8750	9	0.462	14	0.509
1	1.000	8	0.606	12	0.663
$1\frac{1}{8}$	1.125	7	0.763	12	0.856
$1\frac{1}{4}$	1.250	7	0.969	12	1.073
$1\frac{3}{8}$	1.375	6	1.155	12	1.315
$1\frac{1}{2}$	1.500	6	1.405	12	1.581
$1\frac{3}{4}$	1.750	5	1.90		
2	2.000	$4\frac{1}{2}$	2.50		

American standard designation

10-24 UNC ; 1/2-13 UNC 10-32 UNF ; 1/2-20 UNF

$10 \rightarrow size$; $24 \rightarrow No. of\ thread\ per\ inch$

$10 \rightarrow size$; $32 \rightarrow No. of\ thread\ per\ inch$

UNC : Coarse thread ; UNF: Fine thread

TABLE 18–5 Metric thread dimensions

Basic major diameter (mm)	Coarse threads		Fine threads	
	Pitch (mm)	Tensile stress area (mm^2)	Pitch (mm)	Tensile stress area (mm^2)
1	0.25	0.460		
1.6	0.35	1.27	0.20	1.57
2	0.4	2.07	0.25	2.45
2.5	0.45	3.39	0.35	3.70
3	0.5	5.03	0.35	5.61
4	0.7	8.78	0.5	9.79
5	0.8	14.2	0.5	16.1
6	1	20.1	0.75	22.0
8	1.25	36.6	1	39.2
10	1.5	58.0	1.25	61.2
12	1.75	84.3	1.25	92.1
16	2	157	1.5	167
20	2.5	245	1.5	272
24	3	353	2	384
30	3.5	561	2	621
36	4	817	3	865
42	4.5	1121		
48	5	1473		

Matric designation

M3*0.5 M3*0.35

M→ $Metric$ 3 → $basic\ major\ diameter$ 0.5 or 0.35 → $Pitch\ in\ mm$

Clamping Load (Section 18.4, Page 719)

$$P = \frac{F}{n} \quad \dots\dots\dots (1)$$

$$\sigma_a = K\,[\sigma] \quad \dots\dots\dots (2)$$

$$A_t = \frac{P}{\sigma_a} \quad \dots\dots\dots (3)$$

$$T = K_1 * D * P \quad \dots\dots\dots (4)$$

Where:

P : Clamping load on one bolt

F : Overall clamping load on bolts

n : No. of bolts

σ_a : Allowable stress

$[\sigma]$: Proof strength

A_t **:** Required tensile stress area

T: Required tightening torque

D : Nominal outside diameter of threads

K = 0.75 { The clamping load is often taken to be 0.75 times the proof load. Where: the proof load is = [**σ**] ∗ tensile stress area (A_t)} . Also this factor called demand factor in MDesign.

K_1 = Constant depends on lubricant present

= 0.15 Lubricant at all is present

= 0.2 If thread well cleaned and dried.

Example (18-1) Page 719

A set of three bolts is to be used to provide a clamping force of (12000 Ib) 53370 N between two components of a machine. The load is shared equally among the three bolts. Specify suitable bolts, including the grade of the material, if each is to be stressed to *75%* of its proof strength. Then compute the required tightening torque.

Solution:

Choose SAE grade from table (18-1) say grade no.5

$[\boldsymbol{\sigma}] = 85000\ Psi = 590\ MPa$

P = clamping load = $\frac{F}{n} = \frac{53370}{3} = 17.79\ KN$

$\sigma_a = K\ [\sigma] = 0.75 * 586 = 439.6\ MPa$

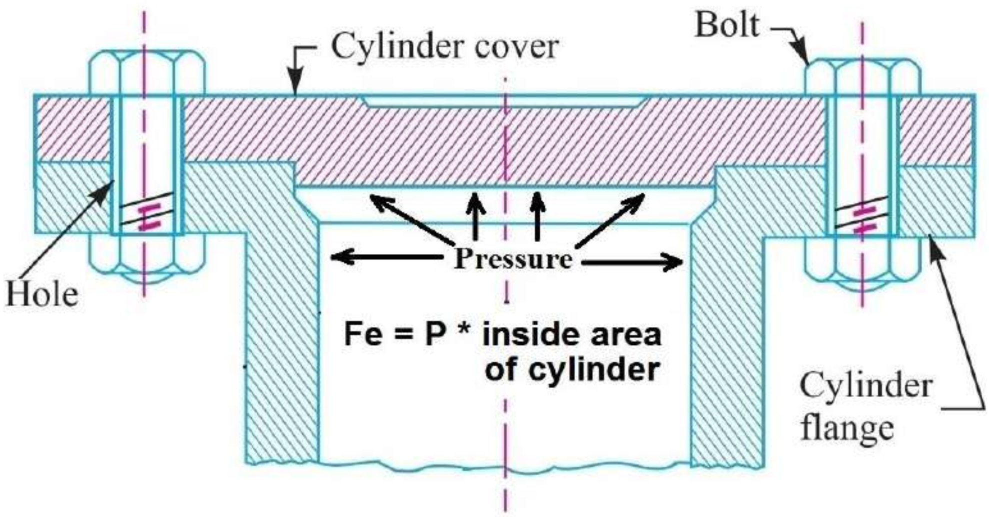
$$A_t = \frac{P}{\sigma_a} = \frac{17.79 * 10^3 N}{439.6\ MPa} = 40.45\ mm^2$$

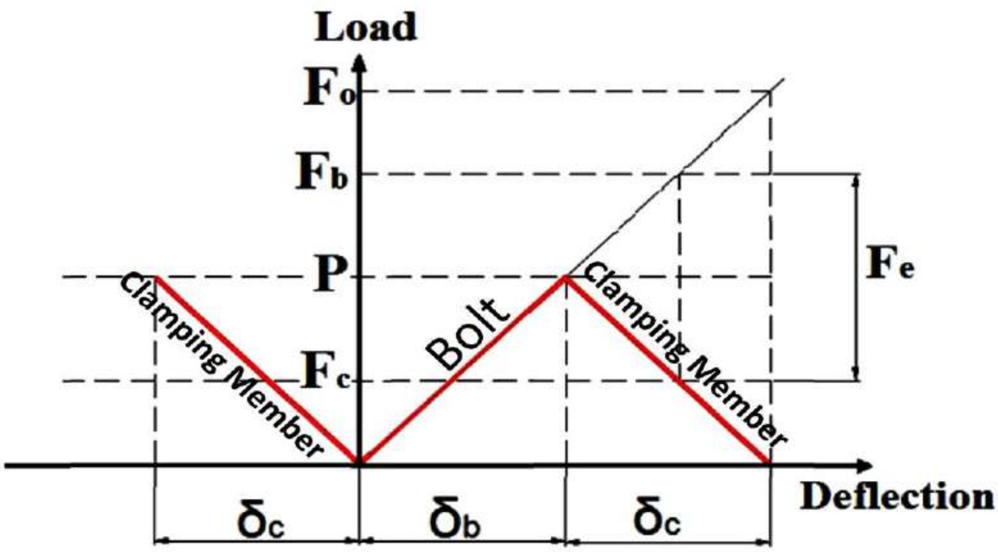
From table (18-5) choose M10*1.5 (A_t = 58 mm^2)

$T = K_t * D * P = 0.15 * 10 * 10^{-3} * 17.79 * 10^3 = 26.7\ N.m$

Externally Applied force on a bolted joint (Section 18.5, Page 722)

The above previous example (18-1) is for bolts under static conditions, now if there is an external load on the bolts (Fe) as shown below, when the cover of pressure vessels is fixed by bolts.





$$F_b = P + \frac{K_b}{K_b + K_c} F_e \qquad \dots\dots\dots (18-8)$$

$$F_c = P - \frac{K_c}{K_b + K_c} F_e \qquad \dots\dots\dots (18-9)$$

For more than two clamping members:

$$\frac{1}{K_c} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + \quad \dots\dots\dots\dots$$

Where:

F_e : externally applied load
P : initial clamping load
F_b : Final force on bolt
F_c : Final force on clamping member
K_b : Stiffness of bolt = $\frac{P}{\delta_b}$
K_c : Stiffness of clamping member = $\frac{P}{\delta_c}$
F_o : Load to open the connection.

<u>Example (18-2) Page 722</u>

Assume that the joint described in Example Problem 18-1 was subjected to an additional external load of 3000 Ib (13.344 KN) after the initial clamping load of 4000 lb (17.792 KN) was applied. Also assume that the stiffness of the clamped members is three times that of the bolt. Compute the force in the bolt, the force in the clamped members, and the final stress in the bolt after the external load is applied.

Solution: $\boldsymbol{K_c = 3\,K_b}$

$$\mathrm{F_b = P + \frac{K_b}{K_b + K_c}\, F_e = P + \frac{K_b}{4K_b}\, F_e = P + \frac{1}{4}\, F_e} = 17.792 + \frac{13.344}{4} = 21.128\ \mathrm{KN}$$

$$\mathrm{F_c = P - \frac{K_c}{K_b + K_c}\, F_e = P - \frac{3}{4}\, F_e} = 17.79 - \frac{3}{4}(13.34) = 7.784\ \mathrm{KN}$$

Fc $>$ 0 $\rightarrow$ the joint is still $\rightarrow$ $tight$

Now for M10*1.5 ; A_t = 58 mm^2

$$A_t = \frac{P}{\sigma_a} \quad \rightarrow \sigma_a = \frac{P}{A_t} = \frac{21128}{58} = 364.3\ MPa$$

$$now\ [\sigma] = proof\ stress = 590\ MPa$$

$$\text{Demand factor} = \frac{\sigma_a}{[\sigma]} = \frac{364.3}{590} = 61.7\% < 75\% \quad \{\text{the select bolt is still safe}\}$$

Example (18-3) Page 723

Solve Example Problem 18-2 again, but assume that the joint has a flexible elastomeric gasket separating the clamping members and that the stiffness of the bolt is then 10 times that of the joint.

Solution: $\boldsymbol{K_b = 10\ K_c}$

$$F_b = P + \frac{K_b}{K_b + K_c} F_e = P + \frac{10}{11} F_e = 17.79 + \frac{10}{11} * 13.34 = 29.922 \text{ KN}$$

$$F_c = P - \frac{K_c}{K_b + K_c} F_e = P - \frac{1}{11} F_e = 17.79 - \frac{1}{11} * 13.34 = 16.578 \text{ KN}$$

$$\sigma = \frac{29.922 \text{ KN} * 1000}{58 \text{ mm}^2} = 517.1 \text{ MPa}$$

Demand factor = $\frac{517.1}{590} = 87.6\% > 75\%$

{The selected bolt is dangerous close to proof strength and yield strength}.

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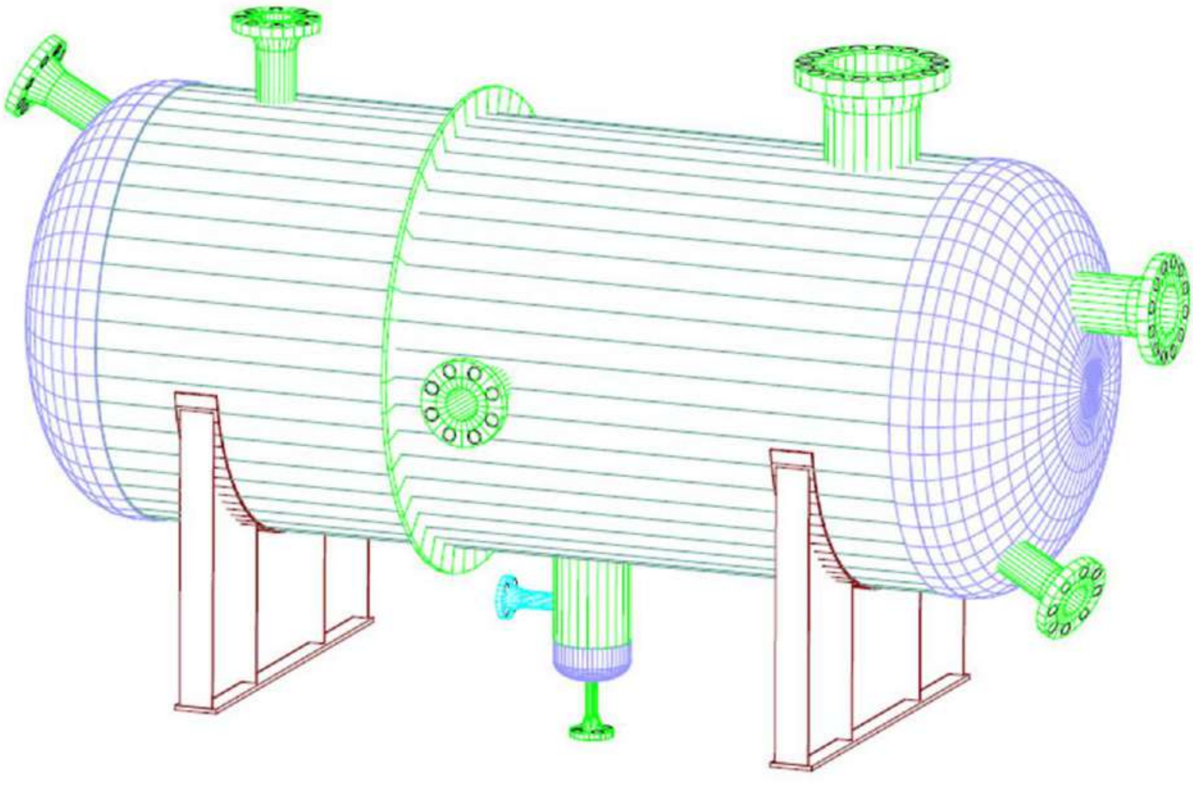
Machine Design I

Third Class for All Branches

LECTURES TWENTY TWO

PRESSURE CYLINDERS



Introduction:

Internally pressurized cylinders have a variety of uses in mechanical equipment. They may be classified variously as below:

1. According to Dimensions:

a) Thin $\left\{\frac{t}{d} \leq 0.1\right\}$

b) Thick $\left\{\frac{t}{d} > 0.1\right\}$

2. According to end construction:

a) Open end

b) Closed end

3. According to material:

a) Brittle material

b) Ductile material

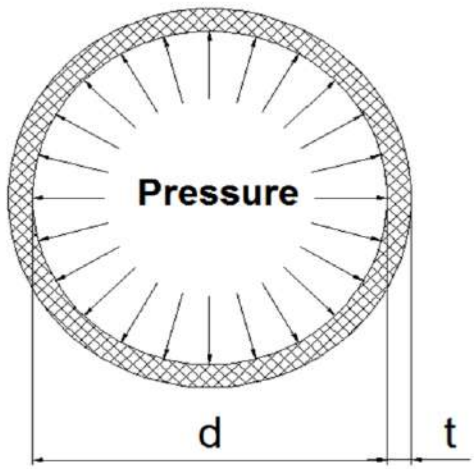
4. According to service:

a) Pressure

b) Temperature

c) Environment

Thin walled cylinder



σ_h = Circumferntial stress

σ_L = Longitudinal stress

P = Internal Pressure

d = Internal diameter

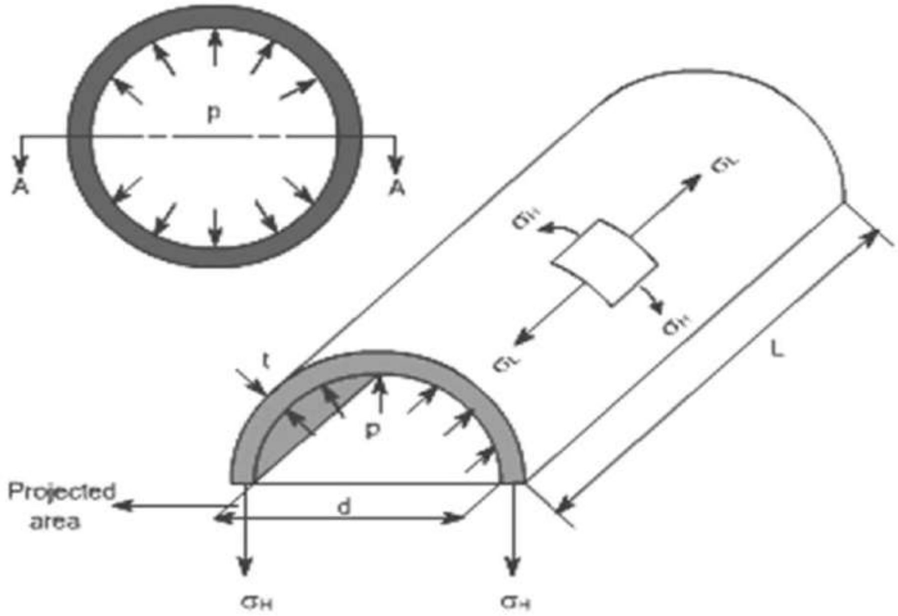
t = Wall thickness

Circumferential tensile stress

$P * d * L = \sigma_h * 2 * t * L \qquad \therefore \quad \left\{\frac{t}{d} = \frac{P}{2\sigma_h}\right\}$ …….. (1)

Longitudinal tensile stress

$P * \frac{\pi d^2}{4} = \sigma_L * \pi d_m * t \qquad \therefore \quad \left\{\frac{t}{d} = \frac{P}{4\sigma_L}\right\}$ …….. (2)



Thick cylinders

When the wall thickness of cylinder is large relative to its diameter the following equations may be used:

Note: *The detail of deriving the equations when the consideration of a cross section of a cylinder perpendicular to its axis will not take in our analysis.*

1. Lame's equations:

a) For Brittle material

$$\boldsymbol{t = \frac{d}{2}\left(\sqrt{\frac{[\sigma] + P}{[\sigma] - P}} - 1\right)} \quad \dots\dots\dots (\mathbf{3})$$

- Based on Normal stress theory.
- Used for open and closed ends.

b) For Ductile material

$$\boldsymbol{t = \frac{d}{2}\left(\sqrt{\frac{[\tau]}{[\tau] - P}} - 1\right)} \quad \dots\dots\dots(4)$$

- Based on maximum shear stress theory.
- Used for open and closed ends.

2. Birnie's equation

$$\boldsymbol{t = \frac{d}{2}\left(\sqrt{\frac{[\sigma] + (1+\upsilon)\,P}{[\sigma] - (1+\upsilon)\,P}} - 1\right)} \quad \dots\dots\dots(5)$$

- Based on maximum strain theory
- For open end cylinder
- Used for Ductile material

3. Clavarino's equation

$$\boldsymbol{t = \frac{d}{2}\left(\sqrt{\frac{[\sigma] + (1-2\upsilon)P}{[\sigma] - (1+\upsilon)P}} - 1\right)} \quad \dots\dots\dots(6)$$

- Based on maximum strain theory
- For closed end cylinder
- Used for Ductile material

Where: $\boldsymbol{\upsilon}$ = Poisson's ratio

$\boldsymbol{\upsilon} = (0.28 - 0.3)$ for steel
$\boldsymbol{\upsilon} = 0.26$ for Cast Iron
$\boldsymbol{\upsilon} = 0.36$ for Bronze
$\boldsymbol{\upsilon} = 0.33$ for Alminum

Cylinder heads thickness

$$h = d\sqrt{\frac{C*P}{[\sigma]}} \quad = \text{Cover thickness}$$

Where:

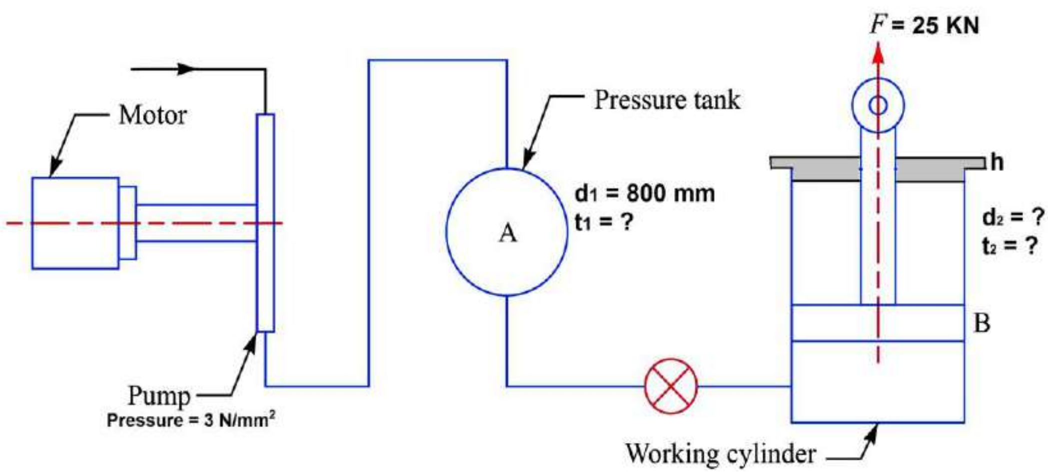
C = 0.162 for bolted cover and if it's integral with cylinder
= 0.2 for riveted joints
= (0.25-0.3) for welded joint

[σ] = Permissible stress

Example

A hydraulic control for a straight line motion, as shown in Fig. below, utilizes a spherical pressure tank 'A' connected to a working cylinder B. The pump maintains a pressure of 3 N/mm2 in the tank.

1. If the diameter of pressure tank is 800 mm, determine its thickness for 100% efficiency of the joint. Assume the allowable tensile stress as 50 MPa.
2. Determine the diameter of a cast iron cylinder and its thickness to produce an operating force F = 25 kN. Assume (i) an allowance of 10 per cent of operating force F for friction in the cylinder and packing, and (ii) a pressure drop of 0.2 N/mm2 between the tank and cylinder. Take safe stress for cast iron as 30 MPa.
3. Determine the power output of the cylinder, if the stroke of the piston is 450 mm and the time required for the working stroke is 5 seconds.
4. Find the power of the motor, if the working cycle repeats after every 30 seconds and the efficiency of the hydraulic control are 80 percent and that of pump 60 percent.



Solution:

- **To find t_1 :** check the sphere thin or thick $\left\{\frac{t_1}{d_1} = \frac{P}{4\sigma}\right\}$

Now use equation (2) because on sphere there is longitudinal tensile stress

$\frac{t_1}{d_1} = \frac{3}{4*50} = 0.015 < 0.1 \quad \therefore \ cylinder \ is \ thin$

$$\therefore \ t_1 = \frac{1.5 * 800}{100} = 12 \text{ mm}$$

- **To find t_2 & d_2 :** check the sphere thin or thick $\left\{\frac{t_2}{d_2} = \frac{P}{2\sigma}\right\}$

$$\frac{t_2}{d_2} = \frac{(3 - 0.2)}{2 * 30} = 0.0467 < 0.1 \quad \therefore cylinder \ is \ thin$$

F $_{Total}$ = P * Area $\therefore \ (25000 * 1.1) = 2.8 * \frac{\pi d_2^2}{4} \rightarrow d_2 = 112 \ mm$

- **To find the power:**

Power = $\frac{Work}{time} = \frac{(25000*1.1)*0.45}{55} = 2475 \ Watt$

- **To find the power of motor** = $\frac{2475}{0.6*0.8} = 5156.25 \ Watt$

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Machine Design I

Third Class for All Branches

LECTURES TWENTY THREE & TWENTY FOUR

MACHINE FRAMES, BOLTED CONNECTIONS AND WELDED JOINTS

Figure (20) shows an example of an eccentrically loaded bolted joint. The motor on the extended bracket places the bolts in shear because its weight acts directly downward. But there also exists a moment equal *to (P*a)* that must be resisted. The moment tends to rotate the bracket and thus to shear the bolts. The basic approach to the analysis and design of eccentrically loaded joints is to determine the forces that act on each bolt because of all the applied loads. Then, by a process of superposition, the loads are combined vectorially to determine which bolt carries the greatest load. That bolt is then sized. The method will be illustrated in Example Problem 20-1.

The American Institute of Steel Construction (AISC) lists allowable stresses for bolts made from ASTM grade steels, as shown in Table 20-1. These data are for bolts used in standard-sized holes, (1.5875 mm) larger than the bolt.

In the design of bolted joints, you should ensure that there are no threads in the plane where shear occurs. The body of the bolt will then have a diameter equal to the major diameter of the thread. You can use the tables in Chapter 18 to select the standard size for a bolt.

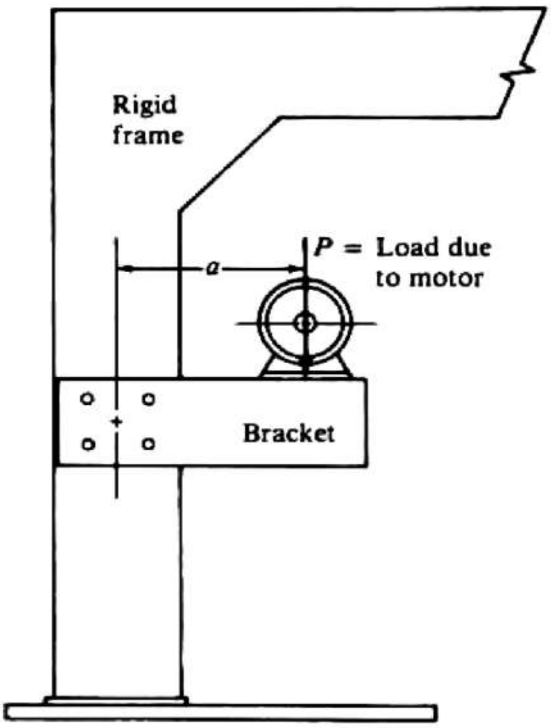


FIGURE 20–4
Eccentrically loaded bolted joint

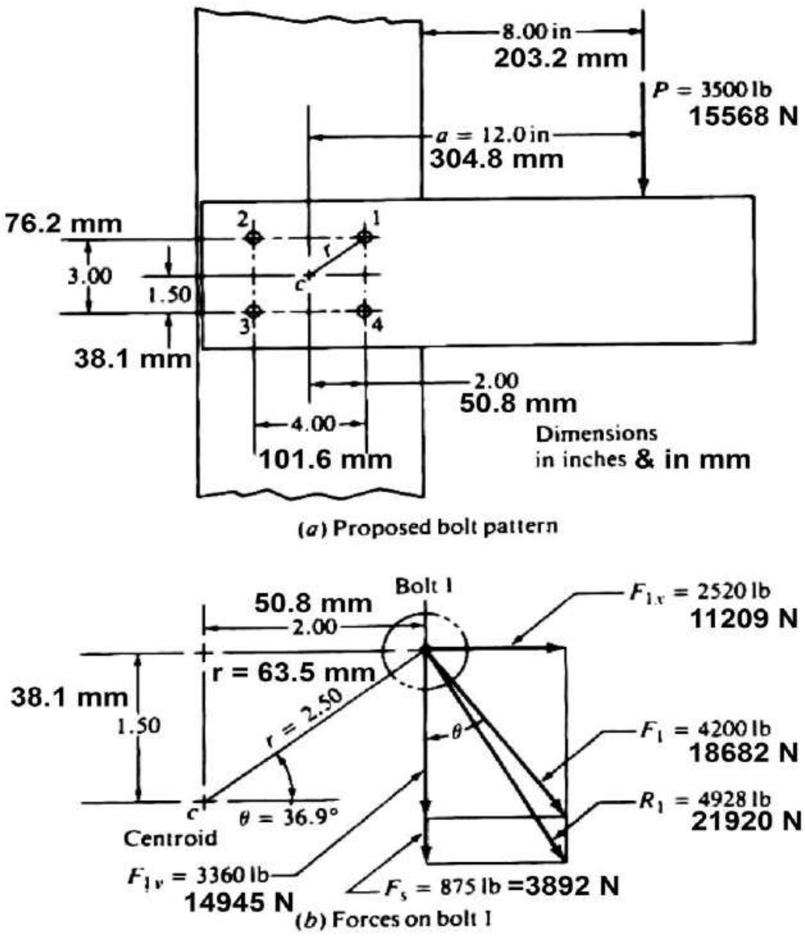


FIGURE 20–5
Geometry of bolted joint and forces on bolt 1

TABLE 20–1 Allowable stresses for bolts

ASTM grade	Allowable shear stress	Allowable tensile stress
A307	10 ksi (69 MPa)	20 ksi (138 MPa)
A325 and A449	17.5 ksi (121 MPa)	44 ksi (303 MPa)
A490	22 ksi (152 MPa)	54 ksi (372 MPa)

Example Problem 20-1 (page 781)

For the bracket in Figure (20-4), assume that the total force (P) is 3500 lb (15568 N) and the distance (a) is 12 in. (304.8 mm). Design the bolted joint, including the location and number of bolts, the material (see table 20-1), and the diameter.

Solution:

The solution shown is an outline of procedure that can be used to analyze similar joints. The data of this problem illustrate the procedure:

1. Propose the number of bolts and the pattern. This is a design decision, based on your judgment and the geometry of the connected parts. Assume the No. of bolts and their orientation, so assume (4 bolts) and fixed as shown in figure (20-5).
2. Determine the direct shear force: $\left\{F_s = \frac{P}{No.of\ bolts}\right\}$. Load per bolt = $F_s = \frac{P}{4}$

$$\Rightarrow \ \mathrm{F_s} = \frac{15568}{4} = 3892 \text{ N} \downarrow$$

3. Compute the **Moment** to be resisted by bolts pattern: the product of the overhanging load and the distance to the **centroid** of the bolt pattern. In this problem: (M=P*a = 15568*304.8 = 4.745 KN.m)
4. Compute the radial distance from the centroid of the bolt pattern to the center of each bolt. In this problem, each bolt has a radial distance of:

$$\left\{r = \sqrt{(38.1)^2 + (50.8)^2} = 63.5\ mm\right\}$$

5. Compute the sum of the squares of all radial distances to all bolts. In this problem, all the four bolts have the same (r), then

$$\sum r^2 = r_1^2 + r_2^2 + r_3^2 \; \ldots\ldots\ldots = 4\,(63.5)^2 = 16130\, mm^2$$

6. Compute the force on each bolt required to resist the bending moment from the relation:

M = P*a = $F_1\, r_1 + F_2 r_2 + F_3 r_3 + \ldots\ldots\ldots$

$$F_i \;\alpha\; r_i \qquad \rightarrow \qquad \frac{F_i}{r_i} = constant = \frac{F_1}{r_1} = \frac{F_2}{r_2} = \frac{F_3}{r_3} = \quad \ldots\ldots\ldots$$

$$M = \frac{F_1}{r_1} * r_1^2 + \frac{F_2}{r_2} * r_2^2 + \frac{F_3}{r_3} * r_3^2 + \ldots\ldots = \frac{F_i}{r_i}\,(r_1^2 + r_2^2 + r_3^2 + \ldots\ldots)$$

$$F_i = \frac{M\, r_i}{r_1^2 + r_2^2 + r_3^2 + r_4^2} = \frac{M\, r_i}{\sum r^2} \quad \ldots\ldots\ldots (20-3)$$

r_i = radial distance from the centriod of the bolt pattern to the i^{th} bolt

F_i = Force on the i^{th} bolt due to moment. The force acts perpendicular to the reduis.

In this problem, all such forces are equal. For example, for bolt 1:

$$F_1 = \frac{4745 \text{ N.m} * 63.5 \text{ mm}}{16130} = 18682 \text{ N}$$

7. Determine the resultant of all forces acting on each bolt. A vector summation can be performed either analytically or graphically, or each force can be resolved into horizontal and vertical components. The components can be summed and then the resultant can be computed.
Let us use the letter approach for this problem. The shear force acts directly downward, in the y- direction. The x- and y- components of F_i are:

$$F_{1x} = F_1 \sin\theta = 18682 \sin 36.9 = 11209 \text{ N}$$

$$F_{1y} = F_1 \cos\theta = 18682 \cos 36.9 = 14945 \text{ N}$$

Total force in Y-direction

$$F_{1T)y} = F_{1y} + F_s = 14945 + 3892 = 18837 \text{ N}$$

R_1 = Resultant force on bolt 1 = $\sqrt{(11209)^2 + (18837)^2} = 21920\, N$

8. Specify the bolt material; compute the required area for the bolt; and select an appropriate size. For this problem, let us specify ASTM A325 steel for the bolts having an allowable shear stress of (121 MPa) from table (20-1). Then the required area for the bolt is:

Allowable shear stress = $\tau_a = 121\ MPa = \frac{R_1}{A_s} = \frac{21920}{\frac{\pi D^2}{4}} \quad \rightarrow \quad D = 15.215\ mm$

Now choose 4 bolts with M16*2 from table 18-5 or use table 18-4 choose size

5/8 = 0.625 inch = 15.875 mm

WELDED JOINTS

References:

Machine Elements in Mechanical Design by Robert L. Mott, P.E. (Chapter 20)

Note: Read section (20-4) (Page 783)

Introduction:

The design of welded joints requires:

- Manner of loading on the joint.
- The type of materials in weld and in the member to be joined.
- The geometry of the joint itself.

Table (20-2) below shows material of weld and allowable stresses:

TABLE 20–2 Allowable shear stresses on fillet welds

A. Steel

Electrode type	Typical metals joined (ASTM grade)	Allowable shear stress
E60	A36, A500	18 ksi (124 MPa)
E70	A242, A441	21 ksi (145 MPa)
E80	A572, Grade 65	24 ksi (165 MPa)
E90		27 ksi (186 MPa)
E100		30 ksi (207 MPa)
E110		33 ksi (228 MPa)

B. Aluminum

	Filler Alloy							
	1100		4043		5356		5556	
	Allowable Shear Stress							
Metal joined	ksi	MPa	ksi	MPa	ksi	MPa	ksi	MPa
1100	3.2	22	4.8	33				
3003	3.2	22	5.0	34				
6061			5.0	34	7.0	48	8.5	59
6063			5.0	34	6.5	45	6.5	45

For steel welded by electric arc method, the type of electrode is an indication of the tensile strength for filler metal. For example, E70 electrode has Su ≥ 70 Ksi (483 MPa). More information can be found from:

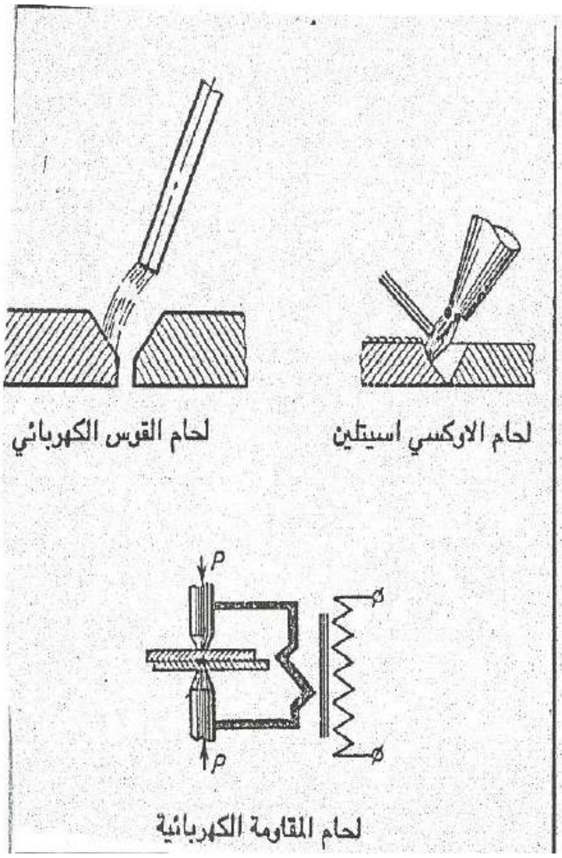
American Welding Society (AWS)
American Institute for Steel Construction (AISC)
Aluminum Association (AA)

See internal sites 3 & 7 in Robert L. Mott (Ch. 20)

<u>Type of Joints, Type of Welds, and size of weld:</u>

في العام الماضي قد تعلمت في موضوع الرسم الميكانيكي وفي الفصل الثامن عن اللحام مايلي:

نبذة مختصرة عن اللحام ، تمثيل اللحام على الرسم ، الرموز الاساسية ، وضع الابعاد ، وغيرها من المواضيع وفيما يلي بعض فقط للتذكرة تم اخذها من كتاب الاستاذ عبد الرسول الخفاف.



لقد تزايد استعمال اللحام في السنوات الاخيرة لربط الاجزاء بدلا من البراغي والبراشيم وغيرها . ويستعمل اللحام في تصنيع ابدان المكائن واجزائها . ويستعمل بنطاق واسع في بناء السفن وانشاء الهياكل الحديدية المستعمله في البناء الحديدي .

تتميز المنتجات الملحومة عن المنتجات المسبوكة برخص كلفتها في حالة انتاج وحدات قليلة ومحدودة لان اللحام لا يتطلب التجهيزات اللازمة للسباكة مثل قوالب الصب وغيرها .

توجد ثلاثة طرق رئيسية للحام وهي :

1 - لحام الاوكسي اسيتلين ويعرف بـ لحام الغاز (Gas welding) .

ان حرق غاز الاسيتيلين مع الاوكسجين يولد لهب ذو درجة حرارية عالية تكفي لصهر المعادن ولحامها . ويمكن استعمال هذه الحرارة ايضا لقطع المعادن .

2 - لحام القوس الكهربائي ويعرف بـ لحام القوس (Arc welding) .

تستخدم الحرارة الناتجه من القوس الكهربائي لغرض صهر المعادن ولحامها .

3 - لحام المقاومة الكهربائية ويعرف بـ لحام المقاومة (Resistance welding) .

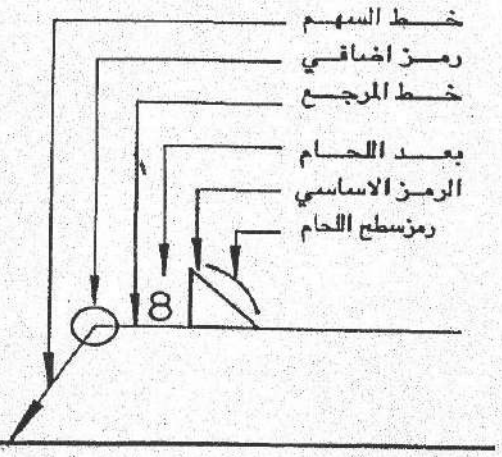
في لحام المقاومة تمسك قطعتان من المعدن تحت ضغط معين ثم يتم امرار كمية كبيرة من التيار الكهربائي خلال الجزئين حيث تنتج مقاومة المعدن لامرار التيار الكهربائي حرارة عاليه في مناطق الاتصال مما تسبب لحام القطعتين مع بعضهما .

تمثيل اللحام على الرسم

ليس من السهل رسم الشكل الحقيقي للحام وتوضيح معالمه ، لذا يجب استعمال رموز خاصه بذلك . ينبغي ان تعطي الرموز جميع البيانات الضرورية التي تخص اللحام المطلوب تنفيذه بصورة مبسطه وواضحة دون الحاجه الى استعمال الملاحظات الكثيرة او رسم مساقط اضافيه ، وقد وضعت هيئة المواصفات الدولية مواصفه خاصه برموز اللحام (ISO 2253).

تشمل الرموز ما يلي :

- خط المرجع
- خط السهم
- الرمز الاساس
- رمز سطح اللحام
- ابعاد اللحام
- رموز اضافية

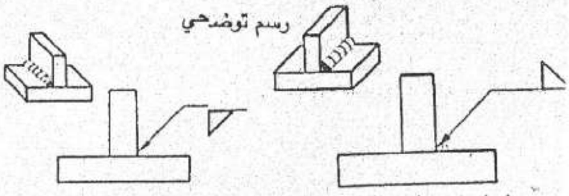


يبين هذا الشكل العناصر الاساسية التي تستعمل عند وضع بيانات اللحام.

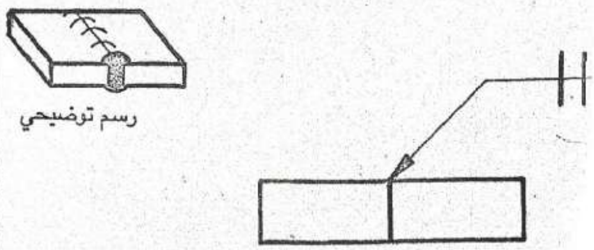
موقع الرمز نسبة الى خط المرجع

يوضع الرمز مع خط المرجع كما يلي :

- فوق خط المرجع اذا كان اللحام في جانب السهم (لحام ظاهر)
- تحت خط المرجع اذا كان اللحام في الجانب الاخر (لحام مخفي)
- عبر خط المرجع اذا كان اللحام في كلا الجنبين



لحام في الجانب الآخر من السهم   لحام في جانب السهم



رسم توضيحي

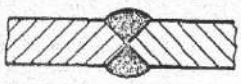
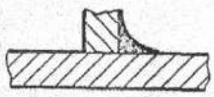
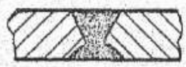
لحام في الجانبين

رموز سطح اللحام

تبين رموز سطح اللحام شكل السطح الخارجي للحام . الجدول التالي يبين هذه الرموز .

شكل سطح اللحام	الرمز
مسطح	
محدب	
مقعر	

· امثلة لاستعمال رموز اللحام

التفسير	رسم توضيحي	الرمز
لحام الحرف V مسطح		
لحام الحرف V في الجانبين محدب		
لحام مثلث مقعر		
لحام الحرف V في الجانب الأول ولحام الظهارة في الجانب الآخر مسطح في الجانبين .		

Figures (20-6) and (20-7) shows types and size of welded joints.

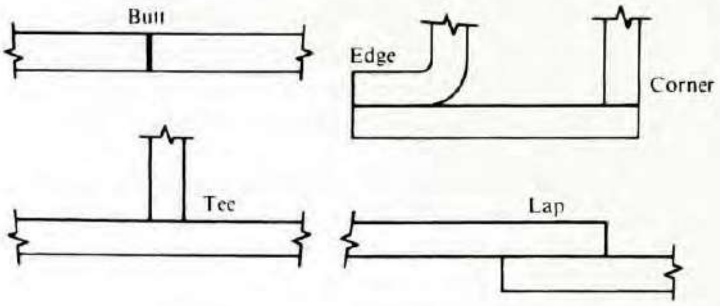


FIGURE 20–6 Types of weld joints

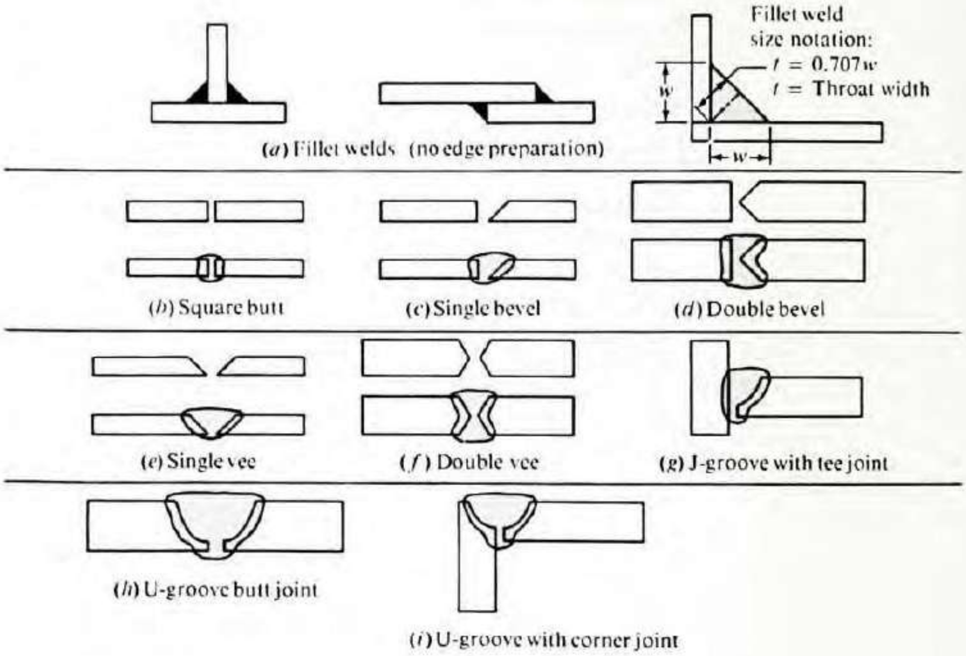


FIGURE 20–7 Some types of welds showing edge preparation

- The length of line from root of the weld = 0.707 W (W: leg dimension).
- The objective of the design of fillet welded joint is to specify the length of fillet, so the weld is treated as a line having no thickness.
- Fig. (20-3) gives the allowable force per (inch or mm) of leg.

TABLE 20–3 Allowable shear stresses and forces on welds

Base metal ASTM grade	Electrode	Allowable shear stress	Allowable force per inch of leg
Building-type structures:			
A36, A441	E60	13 600 psi	9600 lb/in
A36, A441	E70	15 800 psi	11 200 lb/in
Bridge-type structures:			
A36	E60	12 400 psi	8800 lb/in
A441, A242	E70	14 700 psi	10 400 lb/in

- Four different types of loading are considered here when the weld treated as a line:

The relationships used are summarized next:

Type of Loading	*Formula (and Equation Number) for Force per Inch of Weld*	
Direct tension or compression	$f = P/A_w$	**(20-4)**
Direct vertical shear	$f = V/A_w$	**(20-5)**
Bending	$f = M/S_w$	**(20-6)**
Twisting	$f = Tc/J_w$	**(20-7)**

- Fig. (20-8) Page 786 shows geometry factors for weld analysis.

Example: use class 5 in fig. (20-8) with b=101.6 mm, d=152.4 mm. find A_w, J_w , $\bar{X}$

Sol: A_w = d +2b = 152.4+ (2*101.6) = 355.6 mm (with no thickness)

$$J_w = \frac{(2b+d)^3}{12} - \frac{b^2\,(b+d)}{(2b+d)} = \frac{355.6^3}{12} - \frac{10323\,(254)^2}{355.6} = 1.876 * 10^{-3}\,m^3$$

$$\bar{X} = \frac{b^2}{2b+d} = \frac{10323}{355.6} = 28.956\,mm$$

$$S_w = bd + \frac{d^2}{6} = (101.6)(152.4) + \frac{152.4^2}{6} = 1963\,mm^2$$

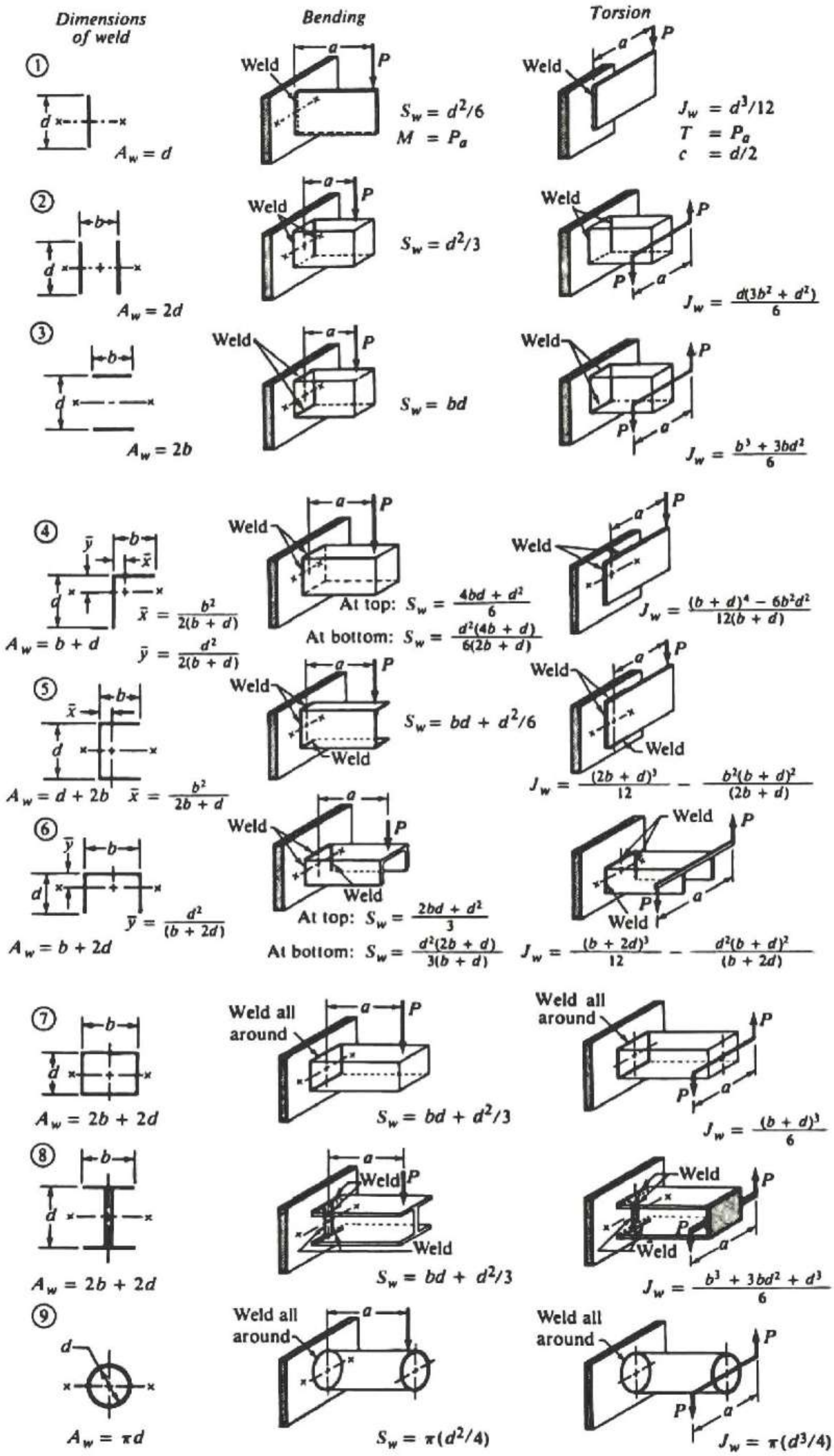


FIGURE 20–8
Geometry factors for weld analysis

General Procedure for Designing Welded Joints:

1. Propose the geometry of the joint and the design of the members to be joined.
2. Identify the types of stresses to which the joint is subjected (bending, twisting, vertical shear, direct tension, or compression).
3. Analyze the joint to determine the magnitude and the direction of the force on the weld due to each type of load.
4. Combine the forces vectorially at the point or points of the weld where the forces appear to be maximum.
5. Divide the maximum force on the weld by the allowable force from Table 20-3 to determine the required leg size for the weld. Note that when thick plates are welded, there are minimum acceptable sizes for the welds as listed in Table 20-4.

TABLE 20–4 Minimum weld sizes for thick plates

Plate thickness (in)	Minimum leg size for fillet weld (in)
≤1/2	3/16
>1/2–3/4	1/4
>3/4–1½	5/16
>1½–2¼	3/8
>2¼–6	1/2
>6	5/8

Example Problem 20–2 Design a bracket similar to that in Figure 20-4, but use welding to attach the bracket to the column. The bracket is 6.00 in high and is made from ASTM A36 steel having a thickness of 1/2 in. The column is also made from A36 steel and is 8.00 in wide.

Solution *Step 1.* The proposed geometry is a design decision and may have to be subjected to some iteration to achieve an optimum design. For a first trial, let's use the C-shaped weld pattern shown in Figure 20–9.

Step 2. The weld will be subjected to direct vertical shear and twisting caused by the 3500-lb load on the bracket.

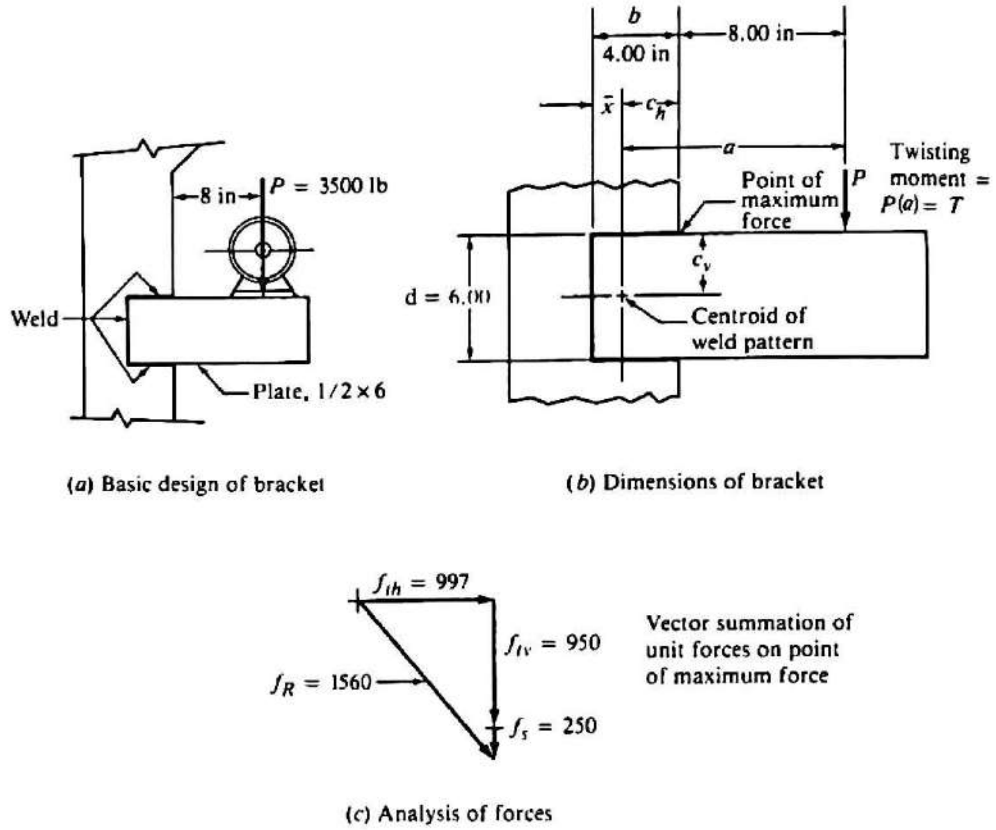
Step 3. To compute the forces on the weld, we must know the geometry factors A_w and J_w. Also, the location of the centroid of the weld pattern must be computed [see Figure 20–9(b)]. Use Case 5 in Figure 20–8.

$$A_w = 2b + d = 2(4) + 6 = 14 \text{ in}$$

$$J_w = \frac{(2b+d)^3}{12} - \frac{b^2(b+d)^2}{(2b+d)} = \frac{(14)^3}{12} - \frac{16(10)^2}{14} = 114.4 \text{ in}^3$$

$$\bar{x} = \frac{b^2}{2b+d} = \frac{16}{14} = 1.14 \text{ in}$$

FIGURE 20–9
C-shaped weld bracket



(*a*) Basic design of bracket (*b*) Dimensions of bracket

(*c*) Analysis of forces

Force Due to Vertical Shear

$$V = P = 3500 \text{ lb}$$
$$f_s = P/A_w = (3500 \text{ lb})/14 \text{ in} = 250 \text{ lb/in}$$

This force acts vertically downward on all parts of the weld.

Forces Due to the Twisting Moment

$$T = P[8.00 + (b - \bar{x})] = 3500[8.00 + (4.00 - 1.14)]$$
$$T = 3500(10.86) = 38\,010 \text{ lb}\cdot\text{in}$$

The twisting moment causes a force to be exerted on the weld that is perpendicular to a radial line from the centroid of the weld pattern to the point of interest. In this case, the end of the weld to the upper right experiences the greatest force. It is most convenient to break the force down into horizontal and vertical components and then subsequently recombine all such components to compute the resultant force:

$$f_{th} = \frac{Tc_v}{J_w} = \frac{(38\,010)(3.00)}{114.4} = 997 \text{ lb/in}$$
$$f_{tv} = \frac{Tc_h}{J_w} = \frac{(38\,010)(2.86)}{114.4} = 950 \text{ lb/in}$$

Step 4. The vectorial combination of the forces on the weld is shown in Figure 20–9(c). Thus, the maximum force is 1560 lb/in.

Step 5. Selecting an E60 electrode for the welding, we find that the allowable force per inch of weld leg size is 9600 lb/in (Table 20–3). Then the required weld leg size is

$$w = \frac{1560 \text{ lb/in}}{9600 \text{ lb/in per in of leg}} = 0.163 \text{ in}$$

Table 20–4 shows that the minimum size weld for a 1/2-in plate is 3/16 in (0.188 in). That size should be specified.

Example Problem 20–3 A steel strap, 1/4 in thick, is to be welded to a rigid frame to carry a dead load of 12 500 lb, as shown in Figure 20–10. Design the strap and its weld.

Solution The basic objectives of the design are to specify a suitable material for the strap, the welding electrode, the size of the weld, and the dimensions W and h, as shown in Figure 20–10.

Let's specify that the strap is to be made from ASTM A441 structural steel and that it is to be welded with an E70 electrode, using the minimum size weld, 3/16 in. Appendix 7 gives the yield strength of the A441 steel as 42 000 psi. Using a design factor of 2, we can compute an allowable stress of

$$\sigma_a = 42\,000/2 = 21\,000 \text{ psi}$$

Then the required area of the strap is

$$A = \frac{P}{\sigma_a} = \frac{12\,500 \text{ lb}}{21\,000 \text{ lb/in}^2} = 0.595 \text{ in}^2$$

But the area is $W \times t$, where $t = 0.25$ in. Then the required width W is

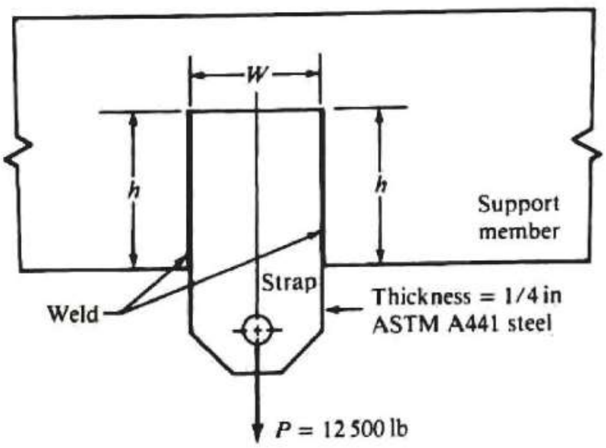
$$W = A/t = 0.595/0.25 = 2.38 \text{ in}$$

Let's specify that $W = 2.50$ in.

To compute the required length of the weld h, we need the allowable force on the 3/16-in weld. Table 20–3 indicates the allowable force on the A441 steel welded with an E70 electrode to be 11 200 lb/in per in of leg size. Then

$$f_a = \frac{11\,200 \text{ lb/in}}{1.0 \text{ in leg}} \times 0.188 \text{ in leg} = 2100 \text{ lb/in}$$

FIGURE 20–10
Steel strap



The actual force on the weld is

$$f_a = P/A_w = P/2h$$

Then solving for h gives

$$h = \frac{P}{2(f_a)} = \frac{12\,500\text{ lb}}{2(2100\text{ lb/in})} = 2.98\text{ in}$$

Let's specify $h = 3.00$ in.

Example Problem 20–4 Evaluate the design shown in Figure 20–11 with regard to stress in the welds. All parts of the assembly are made of ASTM A36 structural steel and are welded with an E60 electrode. The 2500-lb load is a dead load.

Solution The critical point would be the weld at the top of the tube where it is joined to the vertical surface. At this point, there is a three-dimensional force system acting on the weld as shown in Figure 20–12. The offset location of the load causes a twisting on the weld that produces a force f_t on the weld toward the left in the y-direction. The bending produces a force f_b acting outward along the x-axis. The vertical shear force f_s acts downward along the z-axis.

FIGURE 20–11
Bracket assembly

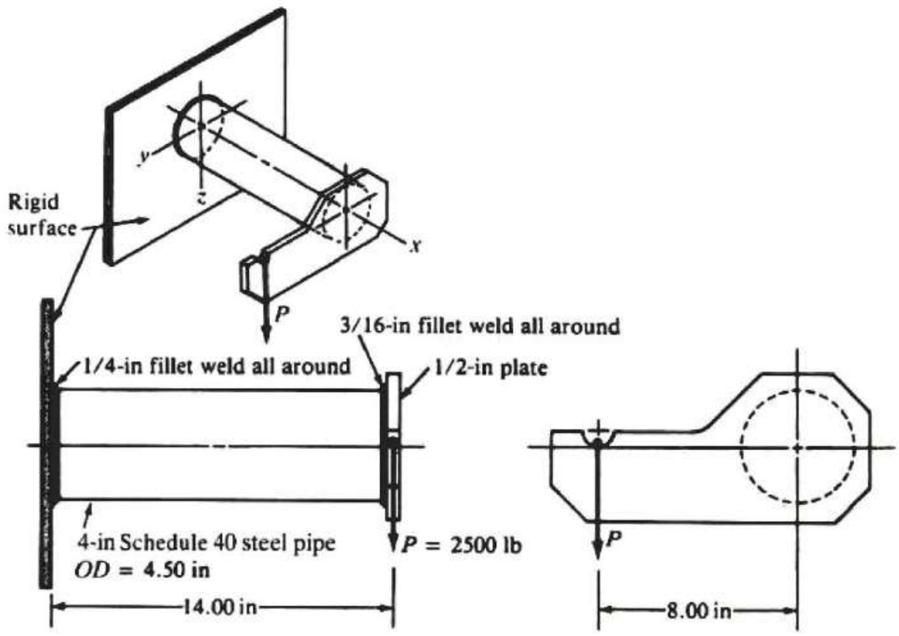
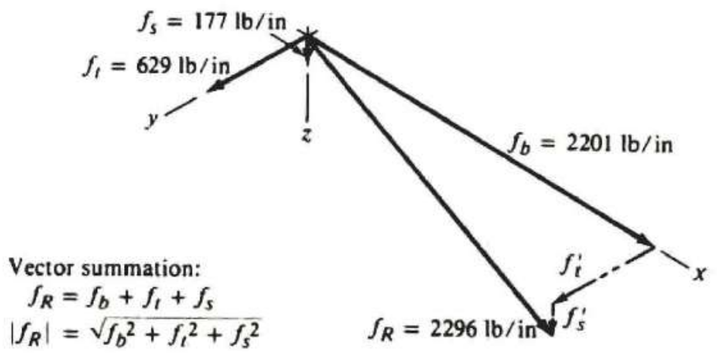


FIGURE 20–12
Force vectors



From statics, the resultant of the three force components would be

$$f_R = \sqrt{f_t^2 + f_b^2 + f_s^2}$$

Now each component force on the weld will be computed.

Twisting Force, f_t

$$f_t = \frac{Tc}{J_w}$$

$$T = (2500\ \text{lb})(8.00\ \text{in}) = 20\,000\ \text{lb}\cdot\text{in}$$

$$c = OD/2 = 4.500/2 = 2.25\ \text{in}$$

$$J_w = (\pi)(OD)^3/4 = (\pi)(4.500)^3/4 = 71.57\ \text{in}^3$$

Then

$$f_t = \frac{Tc}{J_w} = \frac{(20\,000)(2.25)}{71.57} = 629\ \text{lb/in}$$

Bending Force, f_b

$$f_b = \frac{M}{S_w}$$

$$M = (2500\ \text{lb})(14.00\ \text{in}) = 35\,000\ \text{lb}\cdot\text{in}$$

$$S_w = (\pi)(OD)^2/4 = (\pi)(4.500)^2/4 = 15.90\ \text{in}^2$$

Then

$$f_b = \frac{M}{S_w} = \frac{35\,000}{15.90} = 2201\ \text{lb/in}$$

Vertical Shear Force, f_s

$$f_s = \frac{P}{A_w}$$

$$A_w = (\pi)(OD) = (\pi)(4.500\ \text{in}) = 14.14\ \text{in}$$

$$f_s = \frac{P}{A_w} = \frac{2500}{14.14} = 177\ \text{lb/in}$$

Now the resultant can be computed:

$$f_R = \sqrt{f_1^2 + f_b^2 + f_s^2}$$

$$f_R = \sqrt{629^2 + 2201^2 + 177^2} = 2296\ \text{lb/in}$$

Comparing this with the allowable force on a 1.0-in weld gives

$$w = \frac{2296\ \text{lb/in}}{9600\ \text{lb/in per in of leg size}} = 0.239\ \text{in}$$

The 1/4-in fillet specified in Figure 20–11 is satisfactory.

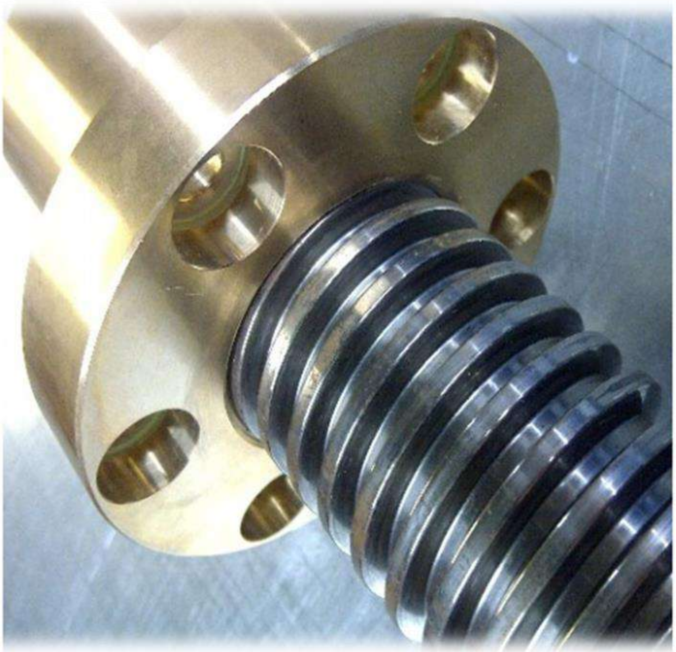
University of Technology
Mechanical Engineering Department

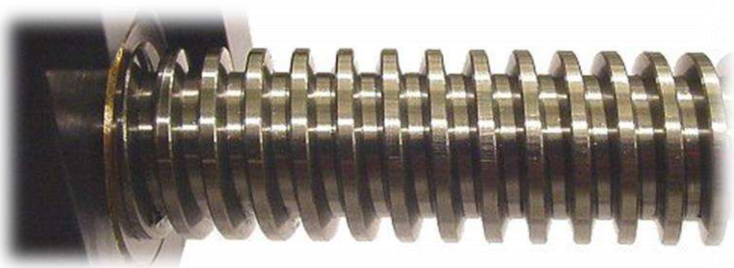
Machine Design I

Third Class for All Branches

LECTURES TWENTY FIVE & TWENTY SIX

LINEAR MOTION ELEMENTS

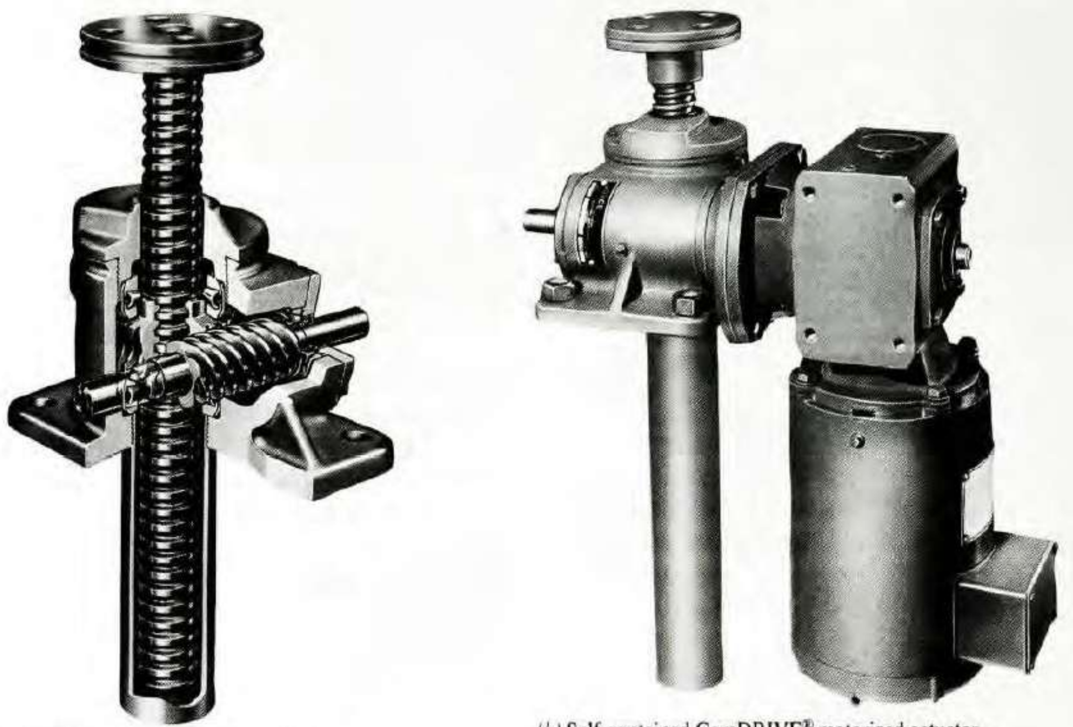




Reference: "Machine Elements in Mechanical Design" 4th Edition, by: Robert L. Mott.

- ❖ Many kinds of mechanical devices produce linear motion for machines such as automation equipment, packaging systems and machine tools.
- ❖ Power screws, jacks and ball screws are designed to convert rotary motion to linear motion or to convert linear motion to rotary motion to exert the necessary force to move a machine element a long a desired path. They use the principle of screw thread and it's mating nut.
- ❖ The linear motion also can be achieved by using the pressure vessels.
- ❖ Some examples of components and systems that facilitate linear motion are: Power screws, Ball screws, Jacks, Fluid power cylinders, Linear actuators, linear slides, Screw vice, Lead screw, Screw drills,……..

  Form Fig. 17-1 as an example.



(a) Cutaway of a machine screw jack

(b) Self-contained ComDRIVE® motorized actuator

FIGURE 17–1 Examples of linear motion machine elements (Joyce/Dayton Corporation, Dayton, OH)

Forms of power screw threads: (see Fig. 17-2 P.697)

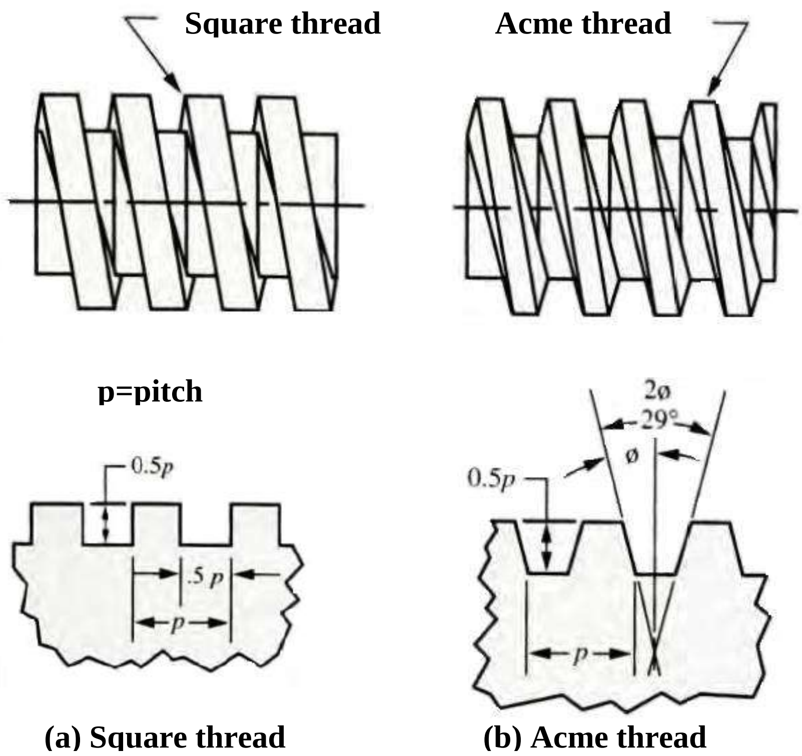
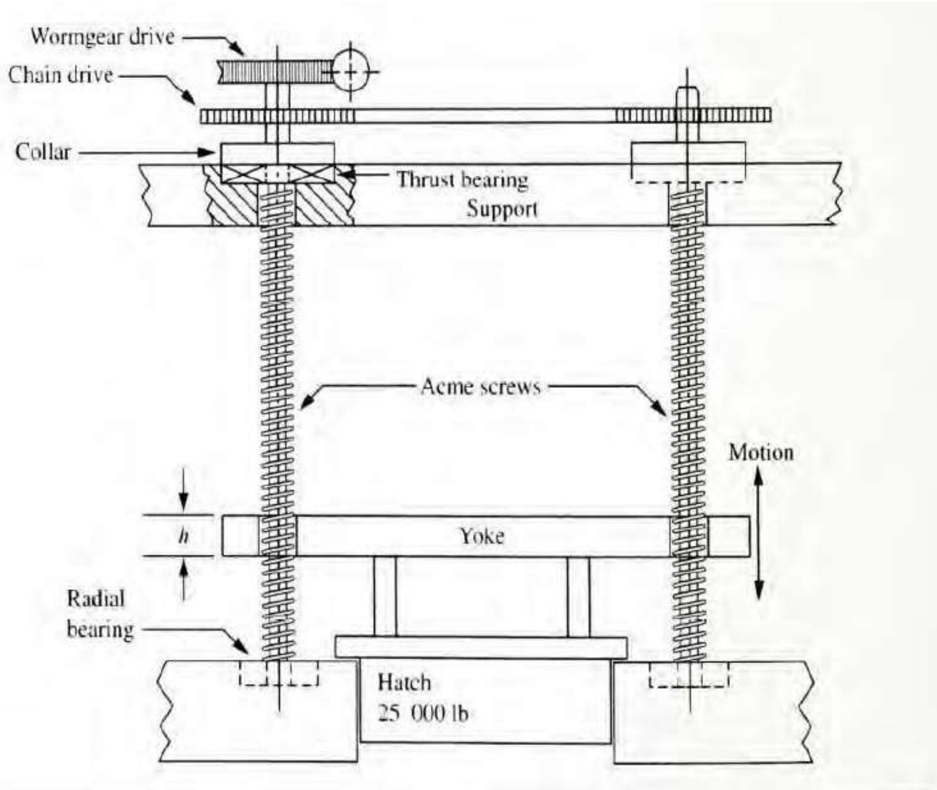


FIGURE 17-2 Forms of power screw threads

See Fig. 17-4 P.698 to see an example for using an ACME (Trapezoidal thread) screw-driven system for rasing a hatch.

FIGURE 17–4 An Acme screw-driven system for raising a hatch



Requirements:

(a) Study the design carefully.

(b) Draw complete construction for the system showing all details for trapezoidal screw, nut, bearings, chain, worm gears, supports…

2. See section 17-1 (objective of this chapter) p.698.

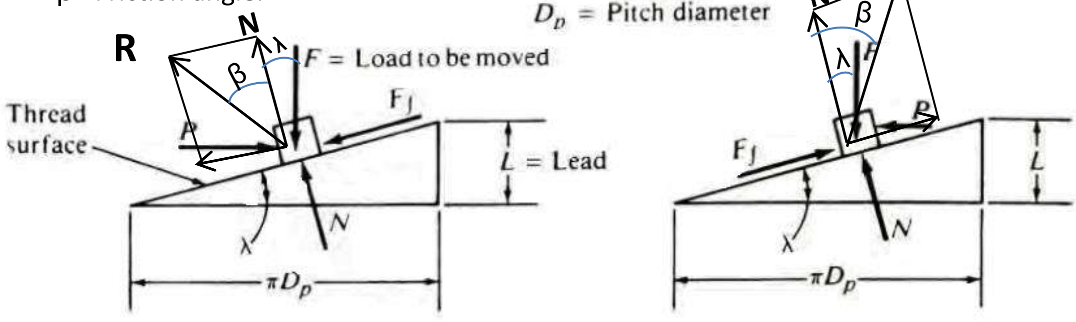
17-2 Power Screw p.699:

D_r= Minor or Root dia.

D= Nominal major dia.

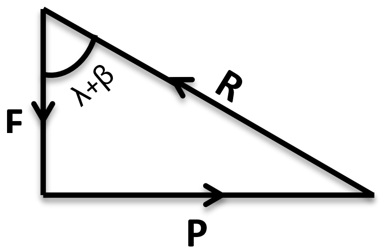
β= Friction angle.

P = Force required to move the load
F_f = Friction force
N = Normal force
λ = Lead angle
D_p = Pitch diameter



(a) Force exerted *up* the plane (b) Force exerted *down* the plane

(a) Force exerted up the plane



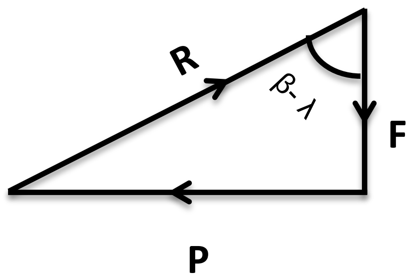
$$\tan(\lambda+\beta) = \frac{P}{F} \;\&\; T_u = P * \frac{D_P}{2}$$

$$\therefore T_u = F * \frac{D_P}{2} * \tan(\lambda+\beta) \;\&\; f = \tan\beta$$

$$T_u = F * \frac{D_P}{2} * \frac{\tan\lambda + f}{1 - f\tan\lambda} \;\&\; \tan\lambda = \frac{L}{\pi D_P}$$

$$\therefore T_u = \frac{FD_p}{2}\left[\frac{L+\pi f D_P}{\pi D_P - fL}\right] \ldots.(17-2)$$

(b) Force exerted down the plane



$$\tan(\beta-\lambda) = \frac{P}{F} \;\&\; T_d = P * \frac{D_P}{2}$$

$$\therefore T_d = F * \frac{D_P}{2} * \tan(\beta - \lambda) \ \& \ f = \tan\beta$$

$$\therefore T_d = F * \frac{D_P}{2} * \frac{f - \tan\lambda}{1 + f\tan\lambda}$$

$$\therefore T_d = \frac{FD_p}{2}\left[\frac{\pi f D_P - L}{\pi D_P + fL}\right] \ldots.(17-4)$$

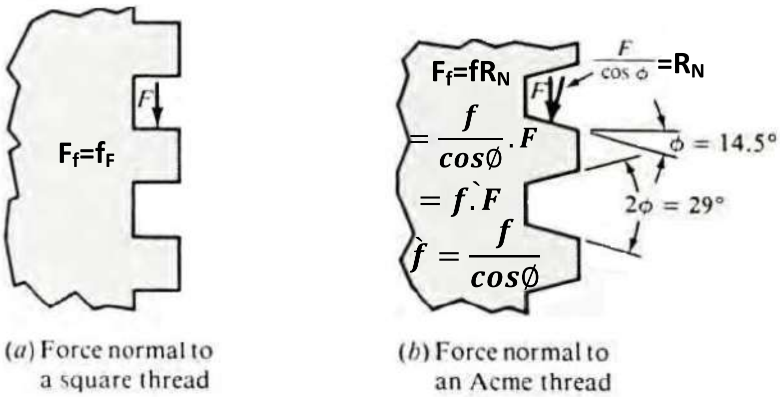
Note:

1- The condition that must be met for self-locking is:

$$f > \tan\lambda \quad \ldots.(17\text{-}5)$$

2- The above equations for square thread, but the adjustment for Acme threads is.

FIGURE 17–6 Force on an Acme thread



So from fig. above

$$T_u = F * \frac{D_P}{2} * \frac{\tan\lambda + \grave{f}}{1 - \grave{f}\tan\lambda}$$

$$T_u = F * \frac{D_P}{2} * \frac{\cos\emptyset\tan\lambda + f}{\cos\emptyset - f\tan\lambda} \ldots\ldots(17-10)$$

$$T_u = F * \frac{D_P}{2} * \frac{f - \cos\emptyset\tan\lambda}{\cos\emptyset + f\tan\lambda} \ldots\ldots(17-11)$$

3- Lead =L = pitch for single start.

= 2*pitch for double start.

= 3*pitch for triple start.

4- f =0.1 - 0.15 for running coefficient of friction.

f =0.14 - 0.21 for staring coefficient of friction.

5- Use table 17-1 P.699 for standard proportion of preferred Acme screw thread.

TABLE 17–1 Preferred Acme screw threads

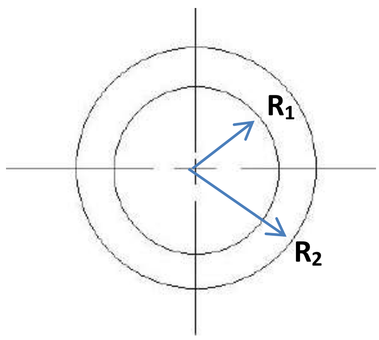
Nominal major diameter, D (in)	Threads per in. n	Pitch, $p = 1/n$ (in)	Minimum minor diameter, D_r (in)	Minimum pitch diameter, D_p (in)	Tensile stress area, A_t (in^2)	Shear stress area, A_s (in^2)[a]
1/4	16	0.0625	0.1618	0.2043	0.026 32	0.3355
5/16	14	0.0714	0.2140	0.2614	0.044 38	0.4344
3/8	12	0.0833	0.2632	0.3161	0.065 89	0.5276
7/16	12	0.0833	0.3253	0.3783	0.097 20	0.6396
1/2	10	0.1000	0.3594	0.4306	0.1225	0.7278
5/8	8	0.1250	0.4570	0.5408	0.1955	0.9180
3/4	6	0.1667	0.5371	0.6424	0.2732	1.084
7/8	6	0.1667	0.6615	0.7663	0.4003	1.313
1	5	0.2000	0.7509	0.8726	0.5175	1.493
$1\frac{1}{8}$	5	0.2000	0.8753	0.9967	0.6881	1.722
$1\frac{1}{4}$	5	0.2000	0.9998	1.1210	0.8831	1.952
$1\frac{3}{8}$	4	0.2500	1.0719	1.2188	1.030	2.110
$1\frac{1}{2}$	4	0.2500	1.1965	1.3429	1.266	2.341
$1\frac{3}{4}$	4	0.2500	1.4456	1.5916	1.811	2.803
2	4	0.2500	1.6948	1.8402	2.454	3.262
$2\frac{1}{4}$	3	0.3333	1.8572	2.0450	2.982	3.610
$2\frac{1}{2}$	3	0.3333	2.1065	2.2939	3.802	4.075
$2\frac{3}{4}$	3	0.3333	2.3558	2.5427	4.711	4.538
3	2	0.5000	2.4326	2.7044	5.181	4.757
$3\frac{1}{2}$	2	0.5000	2.9314	3.2026	7.388	5.700
4	2	0.5000	3.4302	3.7008	9.985	6.640
$4\frac{1}{2}$	2	0.5000	3.9291	4.1991	12.972	7.577
5	2	0.5000	4.4281	4.6973	16.351	8.511

[a]Per inch of length of engagement.

6- Use T_c = Torque to overcome collar friction:

$$T_c = f_1 . F . R_m$$

$$R_m = \frac{R_1 + R_2}{2}$$



$$total\ torque = T_{tot} = T_u + T_c$$

7- f_1= coefficient of friction when thrust collar are used =0.06-0.12 for running coefficient of friction.

=0.08-0.17 for staring coefficient of friction.

8- **Efficiency of a power screw:**

$$\mathbf{e = Efficiency = \frac{Torque\ without\ friction}{Torque\ with\ friction} = \frac{\grave{T}}{T_{tot}} = \frac{FL/2\pi}{T_{tot}}}$$

Example Problem 17-1:

Two Acme-threaded power screws are to be used to raise a heavy access hatch, as sketched in Figure 17-4. The total weight of the hatch is 111.2 kN, divided equally between the two screws. Select a satisfactory screw from table 17-1 on the basis of tensile strength, limiting the tensile stress to 68.95 MPa. Then determine the required thickness of the yoke that acts as the nut on the screw to limit the shear stress in the threads to 34.475 MPa. For the screw thus designed, compute the lead angle, the torque required to raise the load, the efficiency of the screw, and the torque required to lower the load. Use a coefficient of friction of 0.15.

Solution:

The load to be lifted places each screw in direct tension. Therefore, the required tensile stress area is:

$$A_t = \frac{F}{\sigma_d} = \frac{55.6\ KN}{68.95 * 10^6 N/m^2} = 806.5\ mm^2$$

From Table 17-1, a 38.1 mm-diameter Acme thread screw with four threads per 25.4 mm would provide a tensile stress area of 816.8 mm^2

For this screw, each inch of length of a nut would provide 1510.4 mm^2 of shear stress area in the threads. The required shear area is then

$$A_s = \frac{F}{\tau_d} = \frac{55.6\ KN}{34.475 * 10^6 N/m^2} = 1613\ mm^2$$

Then the required length of the yoke would be

$$h = 1613\ mm^2 \left(\frac{25.4\ mm}{1510.4\ mm^2}\right) = 27.18\ \ mm$$

For convenience, let's specify $h = 31.75\ mm$

The lead angle is (remember that *L = p = 1/n* = 1/4 = 6.35 mm)

$$\lambda = tan^{-1}\left(\frac{L}{\pi D_p}\right) = tan^{-1}\left(\frac{6.35}{\pi(34.11)}\right) = 3.39^\circ$$

The torque required to raise the load can be computed from equation (17-10):

$$T_u = \frac{FD_p}{2}\left(\frac{(cos\ \phi\ tan\ \lambda + f)}{(cos\ \phi - f tan\ \lambda)}\right)$$

Using: cos ϕ = cos (14.5°) = 0.968, and tan λ = tan (3.39°) = 0.0592, we have

$$T_u = \frac{(55.6\ kN)(34.11\ mm)}{2}\left(\frac{(0.968)(0.0592) + (0.15)}{(0.968) - (0.15)(0.0592)}\right) = 204.4\ N.m$$

The efficiency can be computed from Equation (17-7):

$$e = Efficiency\ = \frac{FL}{2\pi T_u} = \frac{(55.6\ KN)(6.35\ mm)}{2(\pi)(204.4\ N.m)} = 0.275\ or\ 27.5\ \%$$

The torque required to lower the load can be computed from Equation (17-11):

$$T_d = F * \frac{D_P}{2} * \frac{f - cos\ \emptyset\ tan\lambda}{cos\ \emptyset + f\ tan\lambda} \dots \dots (17 - 11)$$

$$T_d = (55.6\ kN) * \frac{(34.11\ mm)}{2} * \frac{(0.15) - (0.968)(0.0592)}{(0.968) - (0.15)(0.0592)} = 89.95\ N.m$$

Example Problem 17-2:

It is desired to raise the hatch in Figure 17-4 a total of (381 mm) in no more than (12.0 s). Compute the required rotational speed for the screws and the power required.

Solution:

The screw selected in the solution for Example Problem 17-1 was a (38.1) mm Acme-threaded. Screw with four threads per 25.4 mm. Thus, the load would be moved (6.35 mm) with each revolution. The linear speed required is:

$$\mathbf{V} = \frac{\mathbf{381\ mm}}{\mathbf{12.0\ s}} = \mathbf{31.75}\ \boldsymbol{mm/s}$$

The required rotational speed would be

$$\boldsymbol{\omega} = \frac{\mathbf{31.75\ mm}}{\boldsymbol{s}} * \frac{\mathbf{1}\ \boldsymbol{rev}}{\mathbf{6.35}\ \boldsymbol{mm}} * \frac{\mathbf{2}\boldsymbol{\pi}}{\mathbf{1}\ \boldsymbol{rev}} = \mathbf{31.415}\ \boldsymbol{rad/sec}$$

Then the power required to drive each screw would be

$$\boldsymbol{P} = \boldsymbol{T} * \boldsymbol{\omega} = (\mathbf{204.4}\ \boldsymbol{N.m})\left(\mathbf{31.415}\frac{\boldsymbol{rad}}{\boldsymbol{s}}\right) = \mathbf{6421.41}\ \boldsymbol{W}$$

LECTURE TWENTY EIGHT

SPUR GEAR DESIGN

References:

Machine Elements in Mechanical Design by Robert L. Mott, P.E. (Chapter 9)

Note: Read chapter 9 (Page 364-448)

Introduction:

In last yaer you studied the difinision of many parameters for spur year in mechanical engineering drawings and as shown on page 2.

Also in this yaer you studied the theory of spur gears in theory of machine and you found the forces on spur gears in machine design in first simister.

Procedure of Desgining a spur gear drive

Follow the steps in example problem (9-5) page 410 with some assumption to make the procedure very simple and the steps are as fallows:

Power transimited = 2.2 KW = 3 hp to pinion
Pinion rotates at = 183.225 rad/sec = 1750 rpm
Gear rotates netween = (48.162 – 48.64) rad/sec = (460 – 465) rpm

Step 1

Choose K_O = overload factor from table (9-5) page 389.

TABLE 9–5 Suggested overload factors, K_o

	Driven Machine			
Power source	Uniform	Light shock	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.50	1.75
Light shock	1.20	1.40	1.75	2.25
Moderate shock	1.30	1.70	2.00	2.75

Consider uniform driver with heavy shock $K_0 = 1.75$

Deign power = 1.75*2.2 = 3.9 KW = 5.25 hp

Now from fig. 9.27 (page 409) determine trial value for diametral pitch P_d or (m)

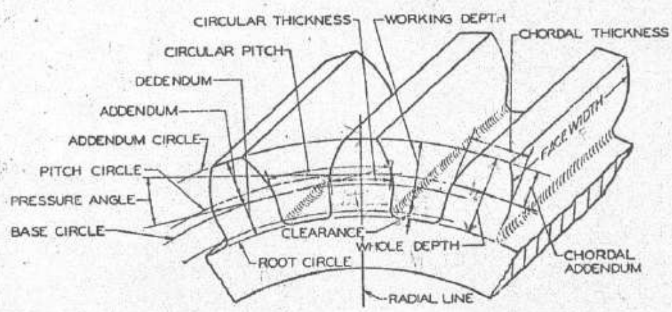
Try $P_d = 12$ or m = 2 (initial design)

Pinion diameter = $D_p = m\,N_p = \frac{N_p}{P_d}$ $\quad or \quad P_d = \frac{N_p}{D_p} = \frac{N_G}{D_G}$

Circular pitch = $P = \frac{\pi D}{N}$

تعاريف الترس الاسطواني العدل

لقد اصبحت تناسبات واشكال التروس مثبته بموجب مواصفات خاصه . والتعاريف المبينه في الجدول والموضحه في الشكل التالي هي عامه لجميع التروس الاسطوانيه العدله .
الابعاد التي تخص ارتفاع السن هي لسن الانفوليوت الذي زاوية الضغط فيه هي 14.5° او 20° .

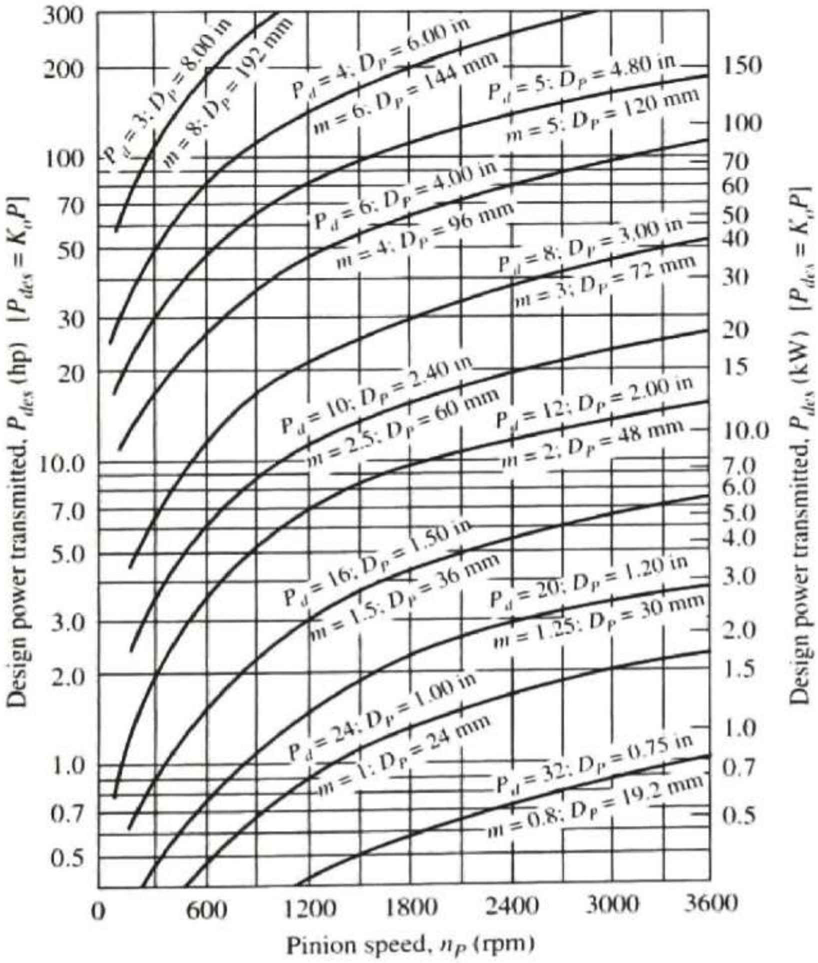


التعريف	الرمز	المصطلح
دائرة وهمية يتم عليها تصميم سن الترس عندما يتعشق ترسان بصورة مرضية فان دائرتي الخطوة فيهما تكونان مماستين .	D	دائرة الخطوة (Pitch circle) D = m N
الدائرة التي تمر خلال قمة الاسنان .	D_o	دائرة طرف السن (Addendum circle) $D_o = D + 2a$
الدائرة التي تمر خلال قاعدة الاسنان .	D_R	دائرة الجذر (Root circle) $D_R = D - 2b$
عدد الاسنان الموجودة حول المحيط الكامل للترس .	N	عدد الاسنان (Number of teeth) N = D/m
وحدة لقياس اسنان التروس . وهي الكمية الرئيسية في تعين حجم السن . الموديل عبارة عن حاصل قسمة قطر دائرة الخطوة على عدد الاسنان .	m	الموديل (Modul) $m = D/N = P/\pi$
المسافة من نقطة على سن الى النقطة المناظرة لها على سن مجاور مقاسه على دائرة الخطوة . اي ترسين متعشقين لهما نفس الخطوة الدائرية .	p	الخطوة الدائرية (Circular pitch) $p = \pi D/N = \pi m$

المعادلة	التعريف	الرمز	المصطلح
$t_c = D \sin(90° / N)$	سمك سن مقاس على وتر دائرة الخطوة .	t_c	السمك الوتري (Chordal thickness)
	عرض وجه السن .	F	عرض الوجه (Face width)
20° او 14.5°	الزاوية المحصورة بين اتجاه الضغط بين الاسنان المتعشقة وخط مماس لدائرة الخطوة . زاوية الضغط تحدد مماس دائرة الاساس التي يتولد منها المنحني المنشأ (Envolute) .	α	زاوية الضغط (Pressure angle)
$R = n_p / n_G = D_G / D_p = N_G / N_p$	النسبة بين عدد دورات الترس القائد وعدد دورات الترس المقاد .	R	نسبة السرع (Gear ratio)
$n_p = R \cdot n_G$ $N_G = n_p / R$	عدد الدورات التي يدورها الترس في الدقيقة الواحدة .	n	عدد الدورات في الدقيقة (Revolution per minute)
	اصغر الترسين المتعشقين .	P	الترس الصغير (Pinion)
	اكبر الترسين المتعشقين .	G	الترس الكبير (Gear)

المعادلة	التعريف	الرمز	المصطلح
a = m	المسافة الشعاعية من قمة السن الى دائرة الخطوة .	a	طرف السن (Addendum)
b = 1.12 m	المسافة الشعاعية من دائرة الخطوة الى قاعدة السن .	b	عمق جذر السن (Dedendum)
	المسافة الشعاعية من قمة السن الى وتر دائرة الخطوة .	ac	طرف السن الوتري (Chordal addendum)
h = a + b	الارتفاع الكلي للسن .	h	العمق الكلي (Whole depth)
hw = h - c	المسافة التي يسقطها سن الترس على فراغ سن الترس الاخر المعشق معه .	hw	العمق الشغال (Working depth)
c = h - hw	المسافة بين قمة سن وقعر فراغ السن المعشق معه .	c	الخلوص (Clearance)
تستنتج من الرسم	هي الدائرة التي ينشأ او يتولد منها المنحني المنشأ (Envolute) .	Db	دائرة الاساس (Base circle)
$t = P/2 = \pi m/2$	سمك مقاس على دائرة الخطوة .	t	السمك الدائري (Circular thickness)

FIGURE 9–27
Design power transmitted vs. pinion speed for spur gears with different pitches and diameters



For all curves: 20° full depth teeth:
$N_P = 24$; $N_G = 96$; $m_G = 4.00$; $F = 12/P_d$; $Q_v = 6$
Steel gears, HB 300; $s_{at} = 36000$ psi; $s_{ac} = 126000$ psi

Step 2

Specify N_p = No. of teeth in pinion = 17 to 20 (in this problem as a start choose N_p=18)

Step 3

$$V_R = velocity\ ratio = \frac{n_p}{n_g} = \frac{Rotation\ of\ pinion}{Rotation\ of\ gear} = \frac{1750}{462.5} = 3.78$$

Step 4

$$V_R = \frac{N_G}{N_p} \quad \therefore\ N_G = 3.78 * 18 = 68.04 \quad \rightarrow\ N_G = 68\ (Integer)$$

Step 5

Actual $V_R = \frac{68}{18} = 3.778$

Step 6

Actual $n_g = n_p \left[\frac{N_p}{N_G}\right] = 18.225 \frac{rad}{sec} * \frac{18}{68} = 48.49 \quad rad/sec$

The above value within the limit given in example ∴ our assumption is **OK**

Step 7

Compute $D_P\ (pitch\ diameter\ of\ pinion) = \frac{N_P}{P_d} = m\ N_p = 1.5\ inch = 38.1\ mm$

$D_G\ (pitch\ diameter\ of\ gear) = \frac{N_G}{P_d} = m\ N_G = 5.667\ inch = 143.9\ mm$

C = center distance = $\frac{N_P + N_G}{Z\ P_d} = \frac{m}{2}\left(N_p + N_G\right) = \frac{18+68}{24} = 3.58\ in = 91\ mm$

$\vartheta_t = pitch\ line\ speed = \pi\ D_P \frac{n_p}{12} = \frac{[\pi*1.5*1750]}{12} = 687 \frac{ft}{min} = 3.49\ m/sec$

$W_t = transmited\ load = \frac{P}{\vartheta_t} = \frac{2.2*1000}{3.49} = 640.5\ N$

$= \frac{33000P}{\vartheta_t} = \frac{33000*3hp}{687} = 144\ Ib$

The above value seems to be acceptable.

Step 8

F = Face width should be between $\frac{8}{P_d} < F < \frac{16}{P_d}$

By using equation (9-28) or $8m < F < 16m$

16.94 mm (0.667in) < F < 33.85 mm (1.33in)

Use this value: F = 25.4 mm (1 inch)

Step 9

Specify C_p = Elastic coefficient from table 4-9 (page 400) for both pinion and gear is steel

∴ $\boldsymbol{C_p = 191\ MPa = 2300\ Ib/in^2}$

TABLE 9–9 Elastic coefficient, C_p

Pinion material	Modulus of elasticity, E_P, lb/in² (MPa)	Gear material and modulus of elasticity, E_G, lb/in² (MPa): Steel 30×10^6 (2×10^5)	Malleable iron 25×10^6 (1.7×10^5)	Nodular iron 24×10^6 (1.7×10^5)	Cast iron 22×10^6 (1.5×10^5)	Aluminum bronze 17.5×10^6 (1.2×10^5)	Tin bronze 16×10^6 (1.1×10^5)
Steel	30×10^6 (2×10^5)	2300 (191)	2180 (181)	2160 (179)	2100 (174)	1950 (162)	1900 (158)
Mall. iron	25×10^6 (1.7×10^5)	2180 (181)	2090 (174)	2070 (172)	2020 (168)	1900 (158)	1850 (154)
Nod. iron	24×10^6 (1.7×10^5)	2160 (179)	2070 (172)	2050 (170)	2000 (166)	1880 (156)	1830 (152)
Cast iron	22×10^6 (1.5×10^5)	2100 (174)	2020 (168)	2000 (166)	1960 (163)	1850 (154)	1800 (149)
Al. bronze	17.5×10^6 (1.2×10^5)	1950 (162)	1900 (158)	1880 (156)	1850 (154)	1750 (145)	1700 (141)
Tin bronze	16×10^6 (1.1×10^5)	1900 (158)	1850 (154)	1830 (152)	1800 (149)	1700 (141)	1650 (137)

Source: Extracted from AGMA Standard 2001-C95, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, with the permission of the publisher. American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314.
Note: Poisson's ratio = 0.30; units for C_p are $(\text{lb/in}^2)^{0.5}$ or $(\text{MPa})^{0.5}$.

Step 10

$K_V = dynamic\ factor$

$Q_V = Quality\ number$

From table 9-2 for certain applications find Q_V, or from pitch line velocity. Then find K_V from fig. 9-21 Page 393.

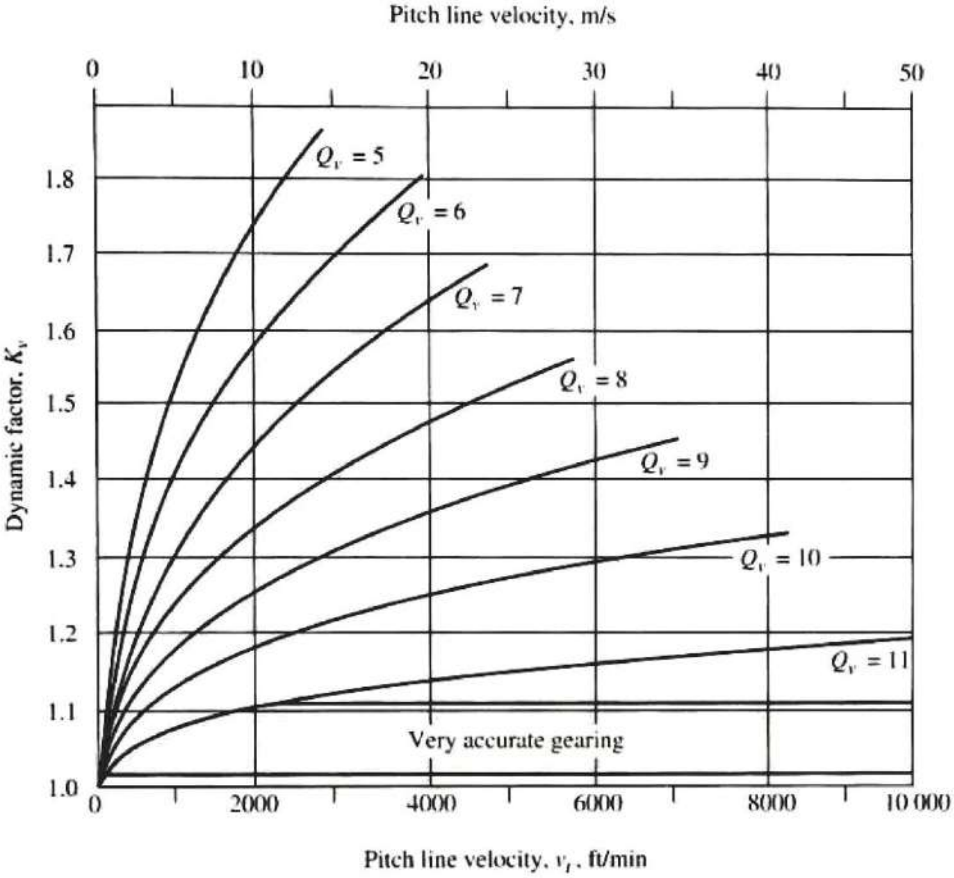
TABLE 9–2 Recommended AGMA quality numbers

Application	Quality number	Application	Quality number
Cement mixer drum drive	3–5	Small power drill	7–9
Cement kiln	5–6	Clothes washing machine	8–10
Steel mill drives	5–6	Printing press	9–11
Grain harvester	5–7	Computing mechanism	10–11
Cranes	5–7	Automotive transmission	10–11
Punch press	5–7	Radar antenna drive	10–12
Mining conveyor	5–7	Marine propulsion drive	10–12
Paper-box-making machine	6–8	Aircraft engine drive	10–13
Gas meter mechanism	7–9	Gyroscope	12–14

Machine tool drives and drives for other high-quality mechanical systems

Pitch line speed (fpm)	Quality number	Pitch line speed (m/s)
0–800	6–8	0–4
800–2000	8–10	4–11
2000–4000	10–12	11–22
Over 4000	12–14	Over 22

FIGURE 9–21
Dynamic factor, K_V
(Extracted from AGMA 2001-C95 Standard *Fundamental, Rating Factors and Calculation Methods for Involute Spur and Helical Gear, Teeth.* with the permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314)



Step 11

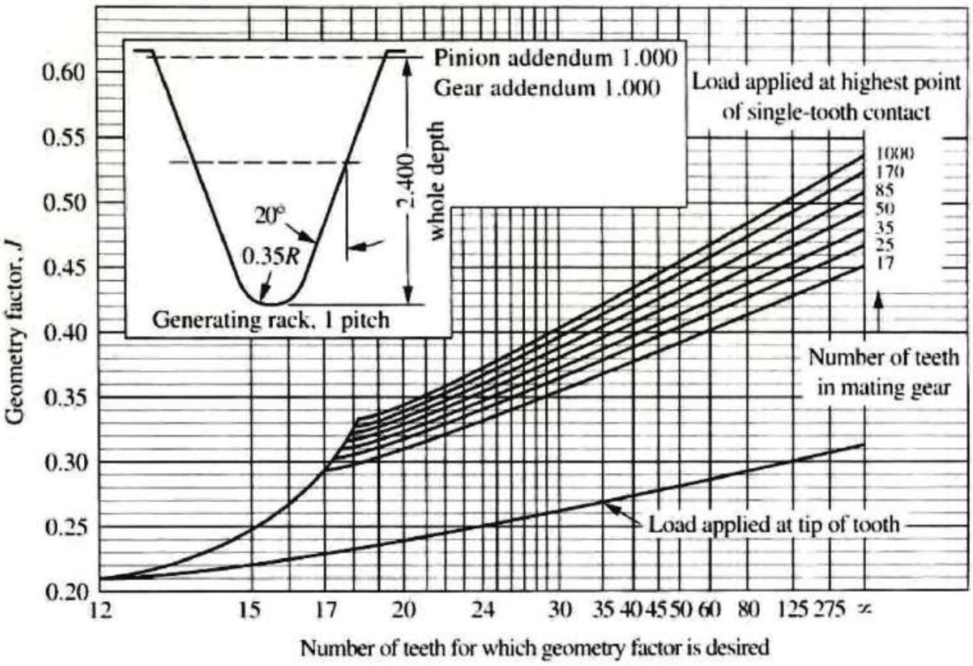
J_p = Bending geometry factor for pinion (should be found from fig. 9-17 (Page 387))
J_G = Bending geometry factor for gear (should be found from fig. 9-17 (Page 387))
I = pitting geometry factor for both pinion & gear (fig. 9-23, Page 402)

Now for this problem:

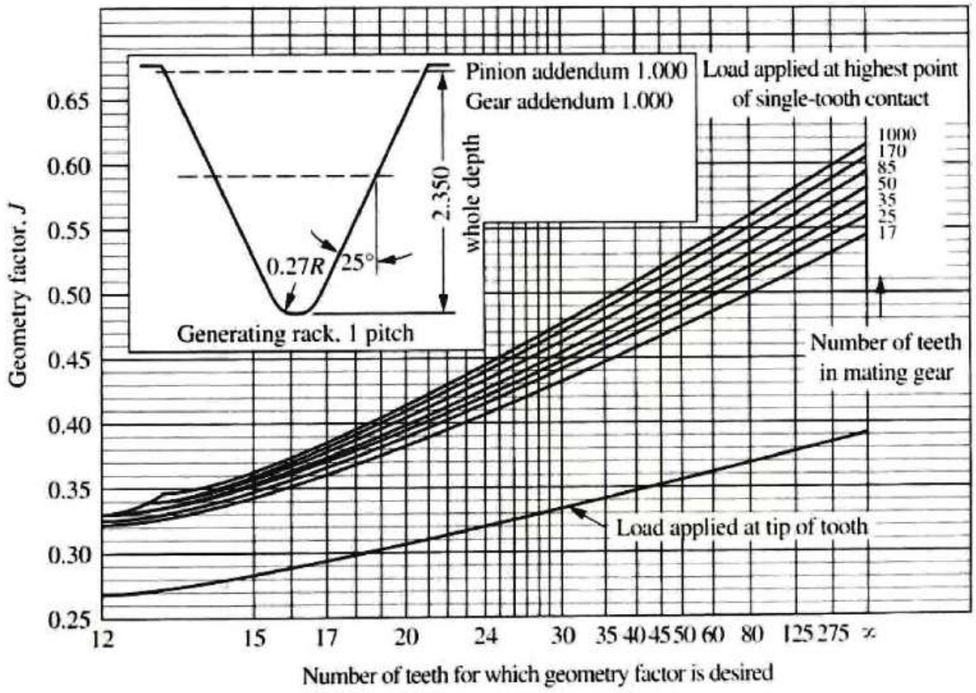
J_p = 0.325 , J_G = 0.41 , I = 0.104

For 20° full depth teeth,

FIGURE 9–17
Geometry factor, *J* (Extract from AGMA 218.01 Standard, *Rating the Pitting Resistance and Bending Strength of Spur and Helical Involute Gear Teeth,* with the permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314)

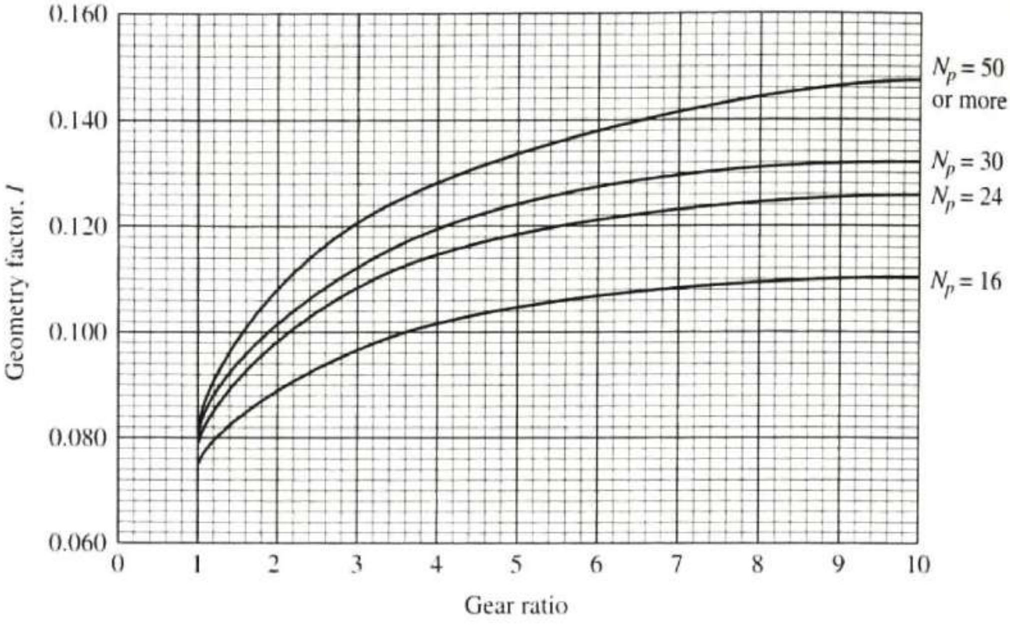


(*a*) 20° spur gear: standard addendum

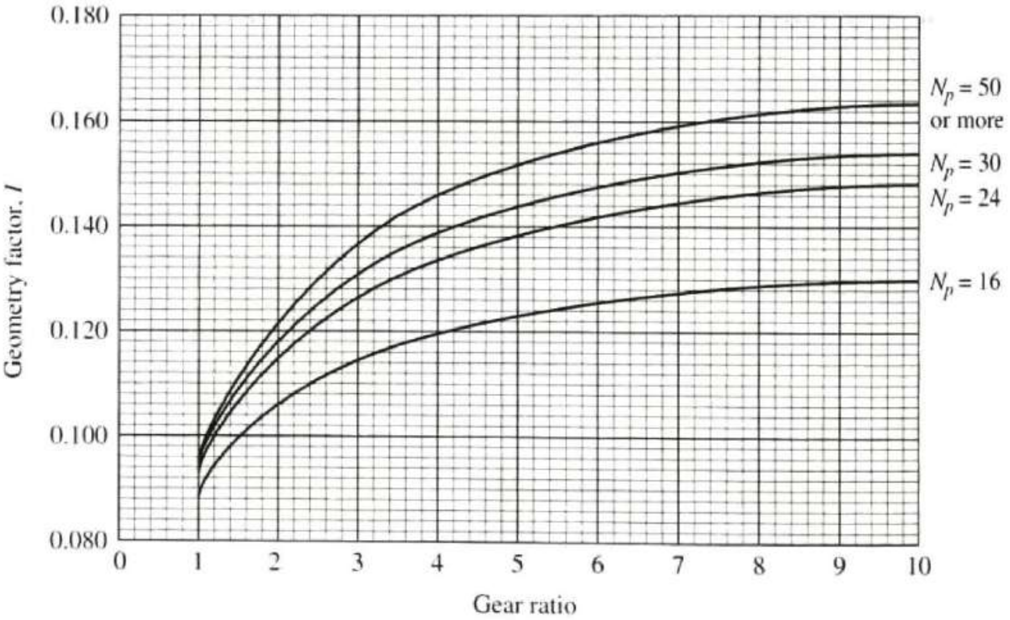


(*b*) 25° spur gear: standard addendum

FIGURE 9–23
External spur pinion geometry factor, *I*, for standard center distances. All curves area for the lowest point of single-tooth contact on the pinion. (Extracted from AGMA Standard 218.01, *Rating the Pitting Resistance and Bending Strength of Spur and Helical Involute Gear Teeth*, with the permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314)



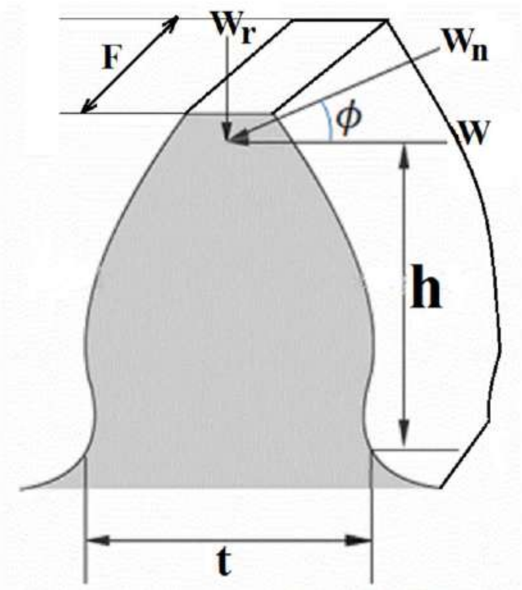
(*a*) 20° pressure angle, full-depth teeth (standard addendum = $1/P_d$)



(*b*) 25° pressure angle, full-depth teeth (standard addendum = $1/P_d$)

Step 12

Compute the expected bending stresses in pinion & gear



W = tangential force

F = face width

∅ = pressure angle

S_{t_1} = bending stress = $\frac{M*C}{I} = \frac{W*h*\frac{t}{2}}{F*\frac{t^3}{12}} = \frac{6Wh}{F\,t^2}$

Now Lewise factor Y = $\frac{t^2}{6hP_c}$

$P_c = circular\ pitch = \pi\, m = \frac{\pi}{P_d}$

$\therefore\ S_{t_1} = \frac{W}{F*Y*P_c}$

Now if K_t = stress concentration

$\therefore\ S_{t_2} = K_t\ \frac{W}{F*Y*P_c} = \frac{W}{F*\left(\frac{\pi}{P_d}\right)*\left(\frac{Y}{K_t}\right)} = \frac{W*P_d}{F*\left(\frac{\pi Y}{K_c}\right)} = \frac{W*P_d}{F*J_p}$

Where J = bending geometry factor = $\frac{\pi*Y}{K_c}$

Now the modified equation for bending stress is:

$$S_{at} = \frac{W_t*P_d}{F*J} * K_O * K_V$$

$$\therefore\ S_{at)P} = \frac{144*12}{1*0.325} * 1.75 * 1.35 = 83.8\ MPa = 12.15\ Ksi$$

$$S_{at)G} = S_{at)P} * \left(\frac{J_P}{J_G}\right) = 66.4\ MPa$$

Step 13

Compute $S_{ac} = Expected\ contact\ stress\ in\ pinion\ \&\ gear$

$$= C_p\sqrt{\frac{W_t*K_O*K_V}{F*D_p*I}} = 2300\sqrt{\frac{144*1.75*1.35}{1*1.5*0.104}} = 730.5\ MPa = 110\ Ksi$$

Step 14

From step 12 & step 13 choose the material of pinion & gear. From fig. 9-11 choose HB 250 for both pinion & gear,

((At 110*1000 Ib/in^2 from fig. Brinel hardness No. ≅ 250))

FIGURE 9–10
Allowable bending stress number for through-hardened steel gears, s_{at} (Extracted from AGMA 2001-C95 Standard, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, with permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314)

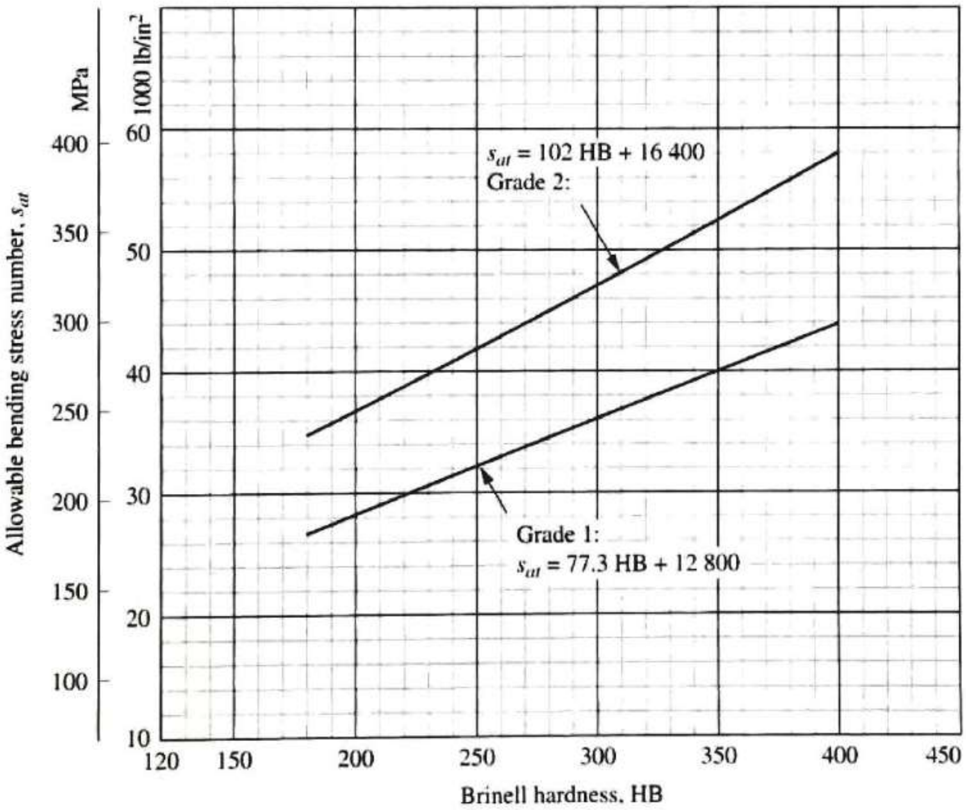


FIGURE 9–11
Allowable contact stress number for through-hardened steel gears, s_{ac} (Extracted from AGMA 2001-C95 Standard, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, with permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314)

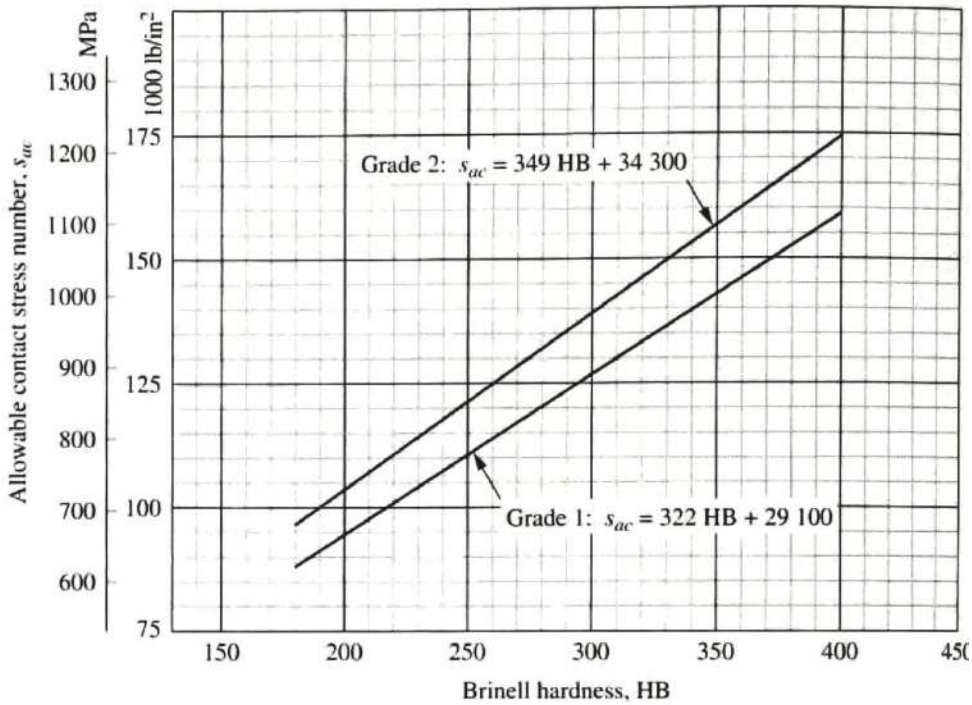


TABLE 9–3 Allowable stress numbers for case-hardened steel gear materials

	Allowable bending stress number, s_{at} (ksi)			Allowable contact stress number, s_{ac} (ksi)		
Hardness at surface	Grade 1	Grade 2	Grade 3	Grade 1	Grade 2	Grade 3
Flame- or induction-hardened:						
50 HRC	45	55		170	190	
54 HRC	45	55		175	195	
Carburized and case-hardened:						
55–64 HRC	55			180		
58–64 HRC	55	65	75	180	225	275
Nitrided, through-hardened steels:						
83.5 HR15N		See Figure 9–14.		150	163	175
84.5 HR15N		See Figure 9–14.		155	168	180
Nitrided, nitralloy 135M:[a]						
87.5 HR15N		See Figure 9–15.				
90.0 HR15N		See Figure 9–15.		170	183	195
Nitrided, nitralloy N:[a]						
87.5 HR15N		See Figure 9–15.				
90.0 HR15N		See Figure 9–15.		172	188	205
Nitrided, 2.5% chrome (no aluminum):						
87.5 HR15N		See Figure 9–15.		155	172	189
90.0 HR15N		See Figure 9–15.		176	196	216

Source: Extracted from AGMA Standard 2001-C95. *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, with the permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314.

[a]Nitralloy is a proprietary family of steels containing approximately 1.0% aluminum which enhances the formation of hard nitrides.