Chapter one

MAINTENANCE MANAGEMENT

Objectives of Chapter:

It presents various aspects related to maintenance management and control, including

(i) Department functions and organizations,
(ii) Elements of effective management.

There is a definite need for effective asset management and maintenance practices that will positively influence critical success factors such as safety, product quality, and speed of innovation, price, profitability, and reliable delivery.

(1) Maintenance Terms and Definitions:

Maintenance: All actions appropriate for retaining an item/part/equipment in, or restoring it to, a given condition.

Maintenance engineering: The activity of equipment/item maintenance that develops concepts, criteria, and technical requirements in conceptional and acquisition phases to be used and maintained in a current status during the operating phase to assure effective maintenance support of equipment.

(2) Objectives of Maintenance and Maintenance Engineering

The **maintenance** is a function that must be performed under normally adverse circumstances and stress, and its main objective is to rapidly restore the equipment to its operational readiness state using available resources.

The objectives of **maintenance engineering** include: improve maintenance operations, reduce the amount and frequency of maintenance, reduce the effect of complexity, reduce the maintenance skills required, reduce the amount of supply support, establish optimum frequency and extent of preventive maintenance to be carried out, improve and ensure maximum utilization of maintenance facilities, and improve the maintenance organization.

(3) Maintenance Management:

Maintenance management may be described as the function of providing policy guidance for maintenance activities, in addition to exercising technical and management control of maintenance programs.

Generally, as the size of the maintenance activity and group increases, the need for better management and control become essential.

Functions of Maintenance Department:

A maintenance department is expected to perform a wide range of functions including:

- Planning and repairing equipment/facilities to acceptable standards
- Performing preventive maintenance; more specifically, developing and implementing a regularly scheduled work program for the purpose of maintaining satisfactory equipment / facility operation as well as preventing major problems
- Preparing realistic budgets that detail maintenance personnel and material needs
- Managing inventory to ensure that parts/materials necessary to conduct the maintenance tasks are readily available
- Keeping records on equipment, services, etc.
- Developing effective approaches to monitor the activities of maintenance staff
- Developing effective techniques for keeping operations personnel, upper-level management, and other concerned groups aware of maintenance activities
- Training maintenance staff and other concerned individuals to improve their skills and perform effectively
- Reviewing plans for new facilities, installation of new equipment, etc.
- Implementing methods to improve workplace safety and developing safety education-related programs for maintenance staff
- Developing contract specifications and inspecting work performed by contractors to ensure compliance with contractual requirements

Many factors determine the **place of maintenance** in the plant organization including size, complexity, and product produced.

Maintenance Organization

The four guidelines useful in **planning a maintenance organization** are: establish reasonably clear division of authority with minimal overlap, optimize number of persons reporting to an individual, fit the organization to the personalities involved, and keep vertical lines of authority and responsibility as short as possible.

One of the first considerations in planning a maintenance organization is to decide whether it is advantageous to have a **centralized or decentralized maintenance function**. Generally, centralized maintenance serves well in small- and medium-sized enterprises housed in one structure, or service buildings located in an immediate geographic area.
Some of the benefits and drawbacks of **centralized maintenance** are as follows:

Benefits

- More efficient compared to decentralized maintenance
- Fewer maintenance personnel required
- More effective line supervision

- Greater use of special equipment and specialized maintenance persons
- Permits procurement of more modern facilities
- Generally allows more effective on-the-job training

Drawbacks

- Requires more time getting to and from the work area or job
- No one individual becomes totally familiar with complex hardware or equipment
- More difficult supervision because of remoteness of maintenance site from the centralized headquarters
- Higher transportation cost due to remote maintenance work

In the case of **decentralized maintenance**, a maintenance group is assigned to a particular area or unit. Some important reasons for the decentralized maintenance are to reduce travel time to and from maintenance jobs, a spirit of cooperation between production and maintenance workers, usually closer supervision, and higher chances for maintenance personnel to become familiar with sophisticated equipment or facilities.

4) Program of Maintenance Management by Objectives:

Improving a maintenance management program is a continuous process that requires progressive attitudes and active involvement. A nine-step approach for managing a maintenance program effectively is presented below:

- *Identify existing deficiencies.* This can be accomplished through interviews with maintenance personnel and by examining in-house performance indicators.

- *Set maintenance goals.* These goals take into consideration existing deficiencies and identify targets for improvement.

- *Establish priorities.* List maintenance projects in order of savings or merit.

- *Establish performance measurement parameters.* Develop a quantifiable measurement for each set goal, for example, number of jobs completed per week and percentage of cost on repair.

- *Establish short- and long-range plans.* The short-range plan focuses on high-priority goals, usually within a one-year period. The long-range plan is more strategic in nature and identifies important goals to be reached within three to five years.

- *Document both long- and short-range plans and forward copies to all concerned individuals.*

- *Implement plan.*

- *Report status.* Preparing a brief report periodically, say semi-annually, and forward it to all involved individuals. The report contains for each objective identified in the short-range plan information on actual or potential slippage of the schedule and associated causes.

- *Examine progress annually.* Review progress at the end of each year with respect to stated goals. Develop a new short-range plan for the following year by considering the goals identified in the long-range plan and adjustment made to the previous year's planned schedule, resources, costs, and so on.

5) Principles of Critical Maintenance Management:

Over the years many maintenance management principles have been developed. Table 1.1 presents six critical maintenance management principles. These principles, if applied on a regular basis, can help make a maintenance department productive and successful.

Table 1.1: Important Maintenance Management Principles

No.	Principle	Brief Description
1	**Maximum productivity** results when each involved person in an organization has a defined task to perform in a definitive way and a definite time.	This principle of scientific management formulated by Frederick W. Taylor in the late nineteenth century remains an important factor in management.
2	**Schedule control** points effectively.	Schedule control points at intervals such that the problems are detected in time, thus the scheduled completion of the job is not delayed.
3	**Measurement** comes before control.	When an individual is given a definitive task to be accomplished using a good representative approach in a specified time, he/she becomes aware of management expectations. Control starts when managing supervisors compare the results against set goals.
4	**The customer service relationship** is the basis of an effective maintenance organization.	A good maintenance service is an important factor in maintaining facilities at an expected level effectively. The team approach fostered by the organizational setup is crucial to consistent, active control of maintenance activity.
5	**Job control** depends on definite, individual responsibility for each activity during the life span of a work order.	It is the responsibility of the maintenance department to develop, implement, and provide operating support for the planning and Scheduling of maintenance work. It is the responsibility of the supervisory individuals to ensure proper and complete use of the system within their sphere of control.
6	**The optimal crew size** is the minimum number that can perform an assigned task effectively.	Most tasks require only one individual.

6) Questions of Maintenance Program Effectiveness Evolution for Maintenance Managers:

The U.S. Energy Research and Development Administration conducted a study on maintenance management-related matters and formulated the following ten questions for maintenance managers to self-evaluate their maintenance effort:

1. Are you aware of how your craftpersons spend their time; i.e., travel, delays, etc.?

2. Are you aware of what facility/equipment and activity consume most of the maintenance money?

3. Are you aware if the craftpersons use proper tools and methods to perform their tasks?

4. Have you balanced your spare parts inventory with respect to carrying cost vs. anticipated downtime losses?

5. With respect to job costs, are you in a position to compare the "should" with the "what"?

6. Do you ensure that maintainability factors are considered properly during the design of new or modified facilities/equipment?

7. Are you aware of how much time your foreman spends at the desk and at the job site?

8. Do you have an effective base to perform productivity measurements, and is productivity improving?

9. Are you aware of whether safety practices are being followed?

10. Are you providing the craftpersons with correct quality and quantity of material when and where they need it?

If an unqualified "yes" is the answer to each of the above questions, then your maintenance program is on a sound footing to meet organizational objectives. Otherwise, appropriate corrective measures are required.

7) Elements of Effective Maintenance Management:

The following are the main elements of effective maintenance management whose effectiveness is the key to the overall success of the maintenance activity:

a) Maintenance policy
It is essential for continuity of operations and a clear understanding of the maintenance management program. Usually, maintenance organizations have manuals containing items such as policies, programs, objectives, responsibilities, and authorities for all levels of supervision, reporting requirements, useful methods and techniques, and performance measurement indices.

b) Material control

Past experience indicates that, on average, material costs account for approximately 30 to 40% of total direct maintenance costs. Material problems can lead to false starts, excess travel time, delays, unmet due dates, etc.

c) Work order system

A work order authorizes and directs an individual or a group to perform a given task. a work order should at least contain information such as requested and planned completion dates, work description and its reasons, planned start date, labor and material costs, item or items to be affected, work category (preventive maintenance, repair, installation, etc.), and appropriate approval signatures.

d) Equipment records

Usually, equipment records are grouped under four classifications:

- Maintenance work performed category contains chronological documentation of all repairs and preventive maintenance (PM) performed during the item's service life to date.

- Maintenance cost category contains historical profiles and accumulations of labor and material costs by item.

- Usually, information on inventory is provided by the stores or accounting department. The inventory category contains information such as property number, size and type, procurement cost, date manufactured or acquired, manufacturer, and location of the equipment/item.

- The files category includes operating and service manuals, warranties, drawings, and so on.

e) Preventive and corrective maintenance

The basic purpose of performing PM is to keep facility/equipment in satisfactory condition through inspection and correction of early-stage deficiencies. Three principle factors shape the requirement and scope of the PM effort: process reliability, economics, and standards compliance.

A major proportion of a maintenance organization's effort is spent on corrective maintenance (CM). Thus, CM is an important factor in the effectiveness of maintenance organization.

f) Job planning and scheduling

A number of tasks may have to be performed prior to commencement of a maintenance job; for example, procurement of parts, tools, and materials, coordination and delivery of parts, tools, and materials, identification of methods and sequencing, coordination with other departments, and securing safety permits.

Maintenance scheduling is as important as job planning. Schedule effectiveness is based on the reliability of the planning function. For large jobs, in particular those requiring multi-craft coordination, serious consideration must be given to using methods such as Program Evaluation and Review Technique (PERT) and Critical Path Method (CPM) to assure effective overall control.

g) Backlog control and priority system

Identification of backlogs is important to balance manpower and workload requirements. Furthermore, decisions concerning overtime, hiring, subcontracting, shop assignments, etc., are largely based on backlog information. Management makes use of various indices to make backlog related decisions.

In assigning job priorities, it is important to consider factors such as importance of the item or system, the type of maintenance, required due dates, and the length of time the job awaiting scheduling will take.

h) Performance measurement

Successful maintenance organizations regularly measure their performance through various means. Performance analyses contribute to maintenance department efficiency and are essential to revealing the downtime of equipment, peculiarities in operational behavior of the concerned organization, developing plans for future maintenance, and so on.

Chapter Two

MAINTENANCE PROJECT CONTROL METHODS

Ojectives of Chapter:

It presents:

(i) Project Control Methods
(ii) Maintenance Management Control Indices

(1) Project Control Methods:

Two widely used maintenance project control methods are:

(i) Program Evaluation and Review Technique (**PERT**)
(ii) Critical Path Method (**CPM**)

In maintenance and other projects, three important factors of concern are time, cost, and resource availability. CPM and PERT deal with these factors individually and in combination.

PERT and CPM are similar. The major difference between the two is that when the completion times of activities of the project are uncertain, PERT is used and with the certainty of completion times, CPM is employed.

PERT and CPM steps:

• Break a project into individual jobs or tasks.
• Arrange these jobs/tasks into a logical network.
• Determine duration time of each job/task.
• Develop a schedule.
• Identify jobs/tasks that control the completion of project.
• Redistribute resources or funds to improve schedule.

ACTIVITY EXPECTED DURATION TIME ESTIMATION

The PERT scheme calls for three estimates of activity duration time using the following formula to calculate the final time:

$$\mathrm{Ta} = \frac{\mathrm{OT} + 4(\mathrm{MT}) + \mathrm{PT}}{6} \quad \ldots\ldots\ldots\ldots..(2.1)$$

Where,

Ta = activity expected duration time,
OT = optimistic or minimum time an activity will require for completion,
PT = pessimistic or maximum time an activity will require for completion,
MT = most likely time an activity will require for completion. This is the time used for CPM activities.

Example 2.1:

Assume that we have the following time estimates to accomplish an activity when OT = 55 days, PT = 80 days and MT = 60 days. Calculate the activity expected duration time.

Solution:

Substituting the given data into Eq. (2.1), we get

$$Ta = \frac{55+4(60)+80}{6} = 62.5\ days$$

The expected duration time for the activity is 62.5 days.

CRITICAL PATH M ETHOD (CPM)

Four symbols used to construct a CPM network are shown in Fig. 2.1. The **circle** denotes an **event.** Specifically, it represents an unambiguous point in the life of a project. An event could be the start or completion of an activity or activities, and usually the events are labeled by number.

A circle shown with three divisions in Fig. 2.1(b) is also denotes an event. Its **top half** labels the event with a number, and the **bottom portions** indicate **latest event time (LET)** and **earliest event time (EET)**. LET may be described as the latest time in which an event can be reached without delaying project completion. EET is the earliest time in which an activity can be accomplished or an event could be reached.

The **continuous arrow** represents an activity that consumes time, money, and manpower. This arrow always **starts at a circle and ends at a circle**. The **dotted arrow** denotes a **dummy activity** or a **restraint**. Specifically, this is an imaginary activity that does not consume time, money, or manpower. Figure 2.2 depicts an application of a dummy activity. It shows that activities **L** and M must be accomplished before activity **N** can start. However, only activity M must be completed prior to starting activity **O**.

Path: It represents a series of successive arrows beginning with the start event and ending with the end event, each path is usually distinguished by the numbers of events that pass, and **the path that takes the longest time is called Critical Path (CP).**

Critical time: The activities of this path are critical ones, since any delay occurs during implementing any of its activities leads to delay the implementation of the work. Therefore, the time of critical path determines the time (**critical time**) required to complete the work.

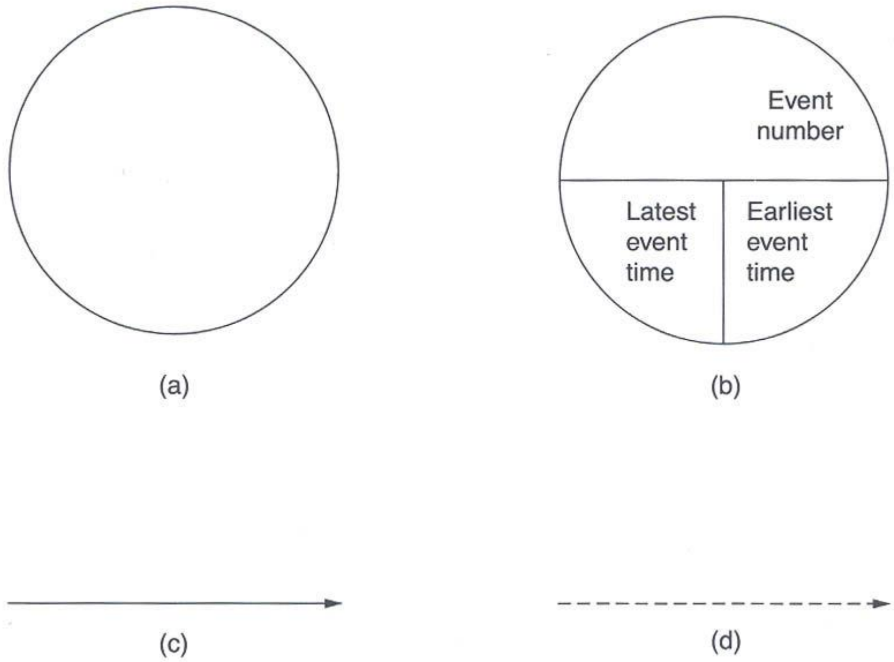


FIGURE 2.1: CPM symbols: **(a)** circle, **(b)** circle with divisions, **(c)** continuous arrow, and **(d)** dotted arrow.

FIGURE 2.2: A portion of a CPM network with a dummy activity.

Example 2.2:

Draw the network planning for the following projects:

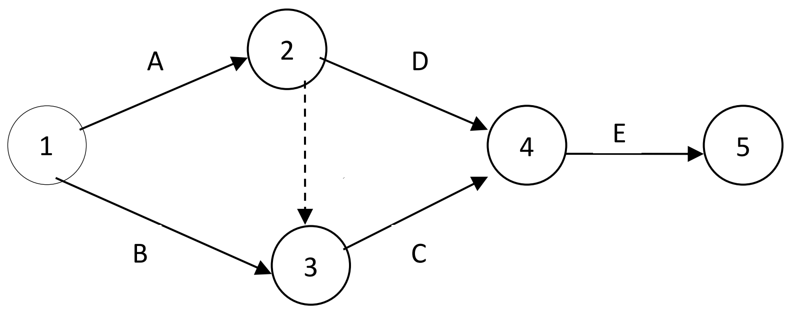
a)

Act.	**Pre-activity**
A	**----**
B	**----**
C	**A,B**
D	**A**
E	**C,D**

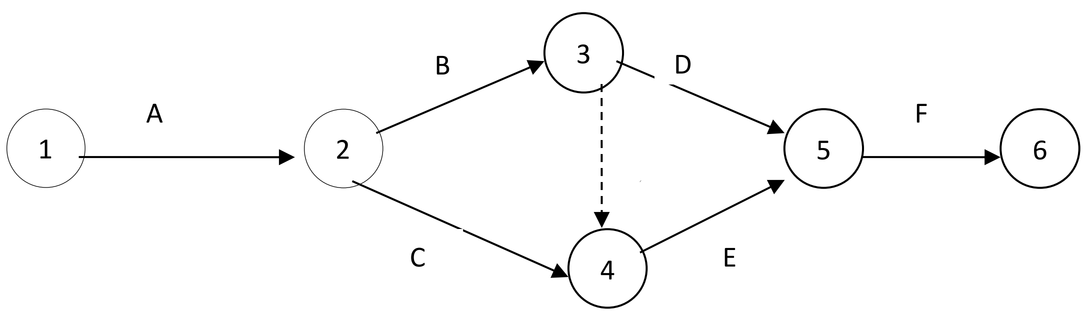
b)

Act.	**Pre-activity**
A	**----**
B	**A**
C	**A**
D	**B**
E	**B,C**
F	**D,E**

(a)



(b)



Example 2.3

A maintenance project was broken down into a set of seven activities, after which Table 3.2 was prepared. **(i) Prepare a CPM network** using Fig. 2.1 symbols and Table 2.2 data, and **(ii) determine the critical path** associated with the network. A CPM network for given data in Table 2.2 is presented in Fig. 2.3.

Table 2.2: Maintenance Project Activities' Associated Data

Activity Identification	Immediate Predecessor Activity or Activities	Expected Duration in Days
L	-	12
M	-	2
N	L, M	2

O	**L**	**6**
P	**O**	**3**
S	**N, P**	**9**
T	**S**	**15**

FIGURE 3.3: A CPM network for Table 2.2 data.

Solution:

In this figure, the following paths originate and terminate at events 1 and 7, respectively:
(M–N–S–T (2+2+9+15 = 28 days)
(L–X–N–S–T (12+0+2+9+15 = 38 days)
(L–O–P–S–T (12+6+3+9+15 = 45 days)

The quantities in parentheses above show the total time in days for each path. The dummy activity consumes zero time. By definition, the longest path through the network is the critical path. Inspection of the above three values shows those **45** days is the largest time. Specifically, it will take 45 days from **event 1 to reach event 7**. Thus, this is the **critical path**. The word "**critical**" is used because any delay in the completion of activities along the critical path will result in delay of completion of the maintenance project.

Critical Path Determination Approach

For simple and straightforward CPM networks, the critical path can easily be identified in a manner discussed above. For complex networks, a more systematic approach is required. This section presents one such approach with the aid of Fig. 2.4. The symbols used in the figure are defined below.

FIGURE 2.4: A single activity CPM network.

EET(*i*) = earliest event time of event ***i***
EET(*j*) = earliest event time of event ***j***
LET(*i*) = latest event time of event ***i***
LET(*j*) = latest event time of event ***j***
D(i, j) = expected duration time of the activity between events ***i*** and ***j***

The following steps are associated with the approach:
1. Construct **CPM** network.
2. Calculate **EET** of each event by making a **forward pass** of the network and using: For any event ***j***,

$$\textbf{EET}(j) = \textbf{Maximum for all preceding } i \textbf{ of } [\textbf{EET}(i) + D(i, j)] \qquad ..... (2.2)$$

Also,

$$\textbf{EET (first event)} = 0 \qquad ...... (2.3)$$

3. Calculate **LET** of each event by making a **backward pass** of the network and using: For any event ***i*,**

$$\textbf{LET}(i) = \textbf{Minimum for all succeeding j of } [\textbf{LET}(j) + D(i, j)] \qquad .....(2.4)$$

Also,

$$\textbf{LET (last event)} = \textbf{EET (last event)} \qquad .... (2.5)$$

If **LET** of all events of the network in question was calculated correctly, we should get

$$\textbf{LET (first event)} = 0 \qquad ..... (2.6)$$

4. Select network events with equal **EET** and **LET**. If the network results in only one path, i.e., from the first event to the last event, with **EET = LET**, this path is critical. Otherwise, go to next step.

5. Calculate the total float for each activity on each of the paths with **EET = LET**. The critical path is the path that results in the least sum of the total floats. The total float for any activity can be calculated using the following equation:

$$\textbf{Total float} = \textbf{LET}(j) - \textbf{EET}(i) - D(i, j) \qquad ...... (2.7)$$

<u>Example 2.4:</u>

Determine the critical path by calculating **EET** and **LET** of each event of the network shown in Fig. 2.3.

<u>Solution:</u>

Using Eq. (2.2) we obtain EET of events 1, 2, 3, 4, 5, 6, and 7 as 0, 12, 12, 18, 21, 30, and 45, respectively. Similarly, with the aid of Eq. (2.4) the LET of events 1, 2, 3, 4, 5, 6, and 7 are 0, 19, 12, 18, 21, 30, and 45, respectively.

Figure 2.5 shows a redrawn Fig. 2.3 CPM network with these **EETs** and **LETs.** The lower left quarter of each circle in Fig. 2.5 shows **LET** and the right quarter the **EET**. The activities marked CP in Fig. 2.5 indicate the critical path as all the events that fall on this path have **EET = LET**, and it is the only path whose events have equal EETs and LETs. In other two paths the EET and LET of all events encountered are not equal.

FIGURE 2.5: Redrawn Fig. 2.3 network with EETs and LETs.

Calculation of the probability of Maintanence project implementation:

$$Ta = \frac{OT + 4(MT) + PT}{6}$$

While, the **Variance (V)** for each activity is calculated from the relationship:

$$V = \frac{PT - OT}{6} \quad \ldots \ldots (2.9)$$

Therefore, the probability of implementing the project on time will be:

$$Pr\left(Z \leq \frac{ST_i - CT_i}{\sqrt{V(\mu_i)}} \right) \quad \ldots \ldots (2.10)$$

Where,

ST_i represents the limited time to complete the project.
CT_i represents the critical time of the project.
V (μ) represents the sum of variances of the critical activities of the project.
The probability value above can be found from the **normal distribution table**.
Note that the **standard deviation** is the square root of the **variance**.

Example 2.5:

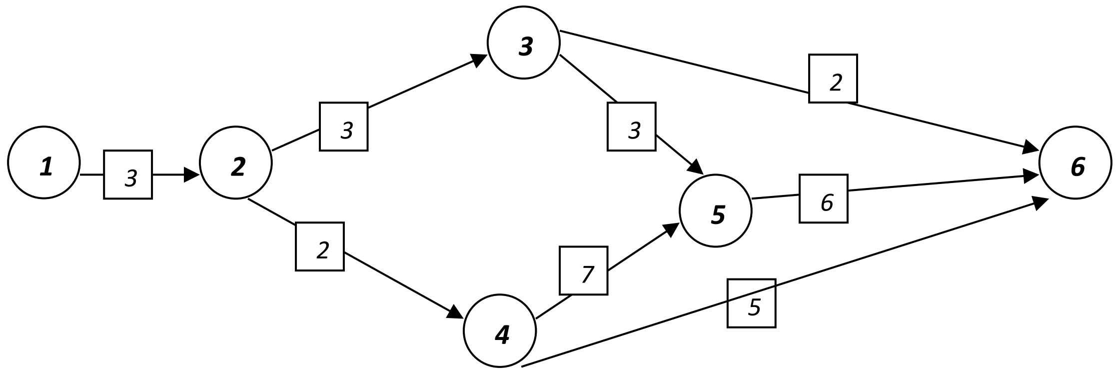
The following data show the implementation times of each activity of an industrial project. Calculate the probability of maintenance project implementation within 20 months.

Activity	OT	PT	MT
1,2	2	8	2
2,3	1	11	1.5
2,4	0.5	7.5	1
3,5	1	7	2.5
3,6	1	3	2

4,5	**6**	**8**	**7**
4,6	**3**	**11**	**4**
5,6	**4**	**8**	**6**

Solution:

Activity	*Ta*	**V**
1,2	**3**	**1**
2,3	**3**	**---**
2,4	**2**	**1.36**
3,5	**3**	**----**
3,6	**2**	**----**
4,5	**7**	**0.11**
4,6	**5**	**----**
5,6	**6**	**0.44**
	V(μ)	**2.91**



Thus, the critical path is represented by the activities: **(1,2), (2,4), (4,5), (5.6),** and the critical time: **CT = 18**.

$$Pr\left(Z_i \leq \frac{20-18}{\sqrt{2.91}} \right) = Pr(Z \leq 1.17) = 0.879$$

In other words, the probability of completing the project in **20** months is approximately **88%**.

(2) Maintenance Management Control Indices:

Management employs various approaches to measure effectiveness of the maintenance function. Often it uses indices to manage and control maintenance. These indices show trends by using past data as a reference point. Usually, a maintenance organization employs various indices to measure maintenance effectiveness, as there is no single index that can accurately reflect the overall performance of the maintenance activity. The main objective of these indices is to encourage maintenance management to improve on past performance.

This section presents a number of **broad and specific indices**. The **broad indices** indicate the overall performance of the organization with respect to maintenance, and the **specific indices** indicate the performance in particular areas of the maintenance function. The values of all these indices are plotted periodically to show trends.

i) BROAD INDICATORS:

This section presents three such indicators.

Index I

$$\mathbf{I1} = \frac{\mathbf{TMC}}{\mathbf{TS}} \quad \ldots\ldots(\mathbf{2.8})$$

Where,
TMC = total maintenance cost,
TS = total sales,
$\mathbf{I_1}$ = index parameter

Past experience indicates that average expenditure for maintenance for all industry was around 5% of sales. However, there was a wide variation among industries. For example, the average values of $\mathbf{I_1}$ for steel and chemical industries were 12.8 and 6.8%, respectively.

Index II

This is expressed by

$$\boldsymbol{I2} = \frac{\mathbf{TMC}}{\mathbf{TO}} \quad \ldots\ldots(\mathbf{2.9})$$

Where,
TO = total output expressed in gallons, tons, megawatts, etc.,
$\mathbf{I_2}$ = index parameter

Index III

This is defined as follows:

$$\boldsymbol{I3} = \frac{\mathbf{TMC}}{\mathbf{TIPE}} \quad \ldots\ldots(\mathbf{2.10})$$

Where,
$\boldsymbol{I_3}$ = index parameter,
TIPE = total investment in plant and equipment

This index relates the total maintenance cost to the total investment in plant and equipment. The approximate average figures for $\boldsymbol{I_3}$ in the steel and chemical industries are 8.6 and 3.8%, respectively.

(ii) SPECIFIC INDICATORS:

This section presents twelve such indicators.

Index IV

This is a useful index to control **preventive maintenance** activity within a maintenance organization and is defined by

$$\boldsymbol{I4 = \frac{TTPM}{TTEM}} \quad \dots\dots(\mathbf{2.11})$$

Where,
I_4 = index parameter,
TTPM = total time spent in performing preventive maintenance,
TTEP = total time spent for the entire maintenance function.

As per the past experience, the value of I_4 should be kept within 20 and 40% limits.

Index V

This index relates maintenance cost to manufacturing cost and is defined by

$$\boldsymbol{I5 = \frac{TMC}{TMFC}} \quad \dots\dots(\mathbf{2.12})$$

Where,
I_5 = index parameter,
TMFC = total manufacturing cost.

Index VI

This index relates maintenance cost to man-hours worked and is expressed by

$$\boldsymbol{I6 = \frac{TMC}{TNMW}} \quad \dots\dots(\mathbf{2.13})$$

Where,
I_6 = index parameter,
TNMW = total number of man-hours worked.

<u>Homework:</u> The following table represents the requirements for the maintenance project was broken down into a set of nine activities. Find: **(i)** the critical path by calculating **EET** and **LET** of each event of the network and **(ii)** critical time to this project.

Activity	***1-2***	***2-3***	***2-4***	***3-4***	***3-5***	***3-6***	***4-5***	***4-6***	***5-6***

D_{ij}	***3***	***3***	***2***	***0***	***3***	***2***	***7***	***5***	***6***

Chapter 3:

Replacement and Maintenance models

All devices, equipment and machines are subject to failure, downtime or loss of efficiency over their lifetime, which requires reducing the time when these devices are out of service as little as possible, as well as minimizing the final cost to the minimum possible. That requires the determination of the **proper time** to inspect and check the devices carefully and probably renew them as a protective measure and repair or replacement of the idle devices.

1. Replacement models:

All companies and laboratories resort to replace certain units (machines or equipment) when they encounter one or both of the following problems:

- The existing units do not function efficiently.
- The existence of new units that perform the same function but in less time or in less cost or higher quality level.

Replacement shall be made when one or more of the following technical or economic reasons for which the replacement is required:

Technical reasons - The defects caused by the current unit or technical modifications to the production system or because of the risk caused by the current unit during the operation or the emergence of a new unit to perform additional works besides the original work or other advantages.

Economic reasons - Maintenance and operation costs, the damage rate, number of the units produced, number of the workers required, the effort made in use, etc.

The first model: It is noted in this model that the cost of maintenance and repair increases over time. In this model, the **total cost rate** of the machine is used as a criterion for decision-making regarding the period of replacement.

Assuming that **C** represents the cost of purchasing the new equipment,
S represents the resale value of the old equipment,
n is the number of time periods for the equipment service,

TC represents the sum of total costs,
T= TC/n represents the total cost rate, and
f(t) represents the maintenance cost in time **t**.

So,

$$TC = C + \int_0^n f(t)dt - S$$

$$T = \frac{TC}{n} = \frac{1}{n}\left(C - S + \int_0^n f(t)dt\right)$$

The total cost rate will be as low as possible when:

$$g(n) = \frac{1}{n}\left(C - S + \int_0^n f(t)dt\right)$$ If the time **t** is a variable continuous

$$g(n) = \frac{1}{n}\left(C - S + \sum_{t=0}^{n} f(t)\right)$$ If time **t** is a discrete variable

Example 1: If the cost of maintenance and resale value of a machine within 8 years is shown in the table below:

Year (t)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Maintenance f(t)	**900**	**1200**	**1600**	**2100**	**2800**	**3700**	**4700**	**5900**
Resale value (S)	**4000**	**2000**	**1200**	**600**	**500**	**400**	**400**	**400**

In which year it is preferable to replace the machine when the price of purchasing the new machine is 7000 $?

Solution:

C = 7000 $

N	S	C - S	f(t)	∑f(t)	g(n)
1	4000	3000	900	900	3900
2	2000	5000	1200	2100	3550
3	1200	5800	1600	3700	3166.7
4	600	6400	2100	5800	3050
5	500	6500	2800	8600	3020 →mini.
6	400	6600	3700	12300	3150
7	400	6600	4700	17000	3371.4
8	400	6600	5900	22900	3687.5

Therefore, it is therefore best to replace the machine at the **end of the fifth year**.

Example 2: The cost of purchasing a new machine is 9000 $. The operating cost in the first year is 200 $ and this cost increases by 2000 $ per year. Determine the best life of the machine to replace it, assuming that this machine does not have a selling price when it is replaced.

Solution:

N	C - S	f(t)	∑f(t)	g(n)
1	9000	200	200	9200
2	9000	2200	2400	5700
3	9000	4200	6600	5200 → mini.
4	9000	6200	12800	5450
5	9000	8200	21000	6100

So, the best replacement for the machine is at the **end of the third year**.

The second model: In this model, the units or parts that are suddenly damaged are replaced by calculating the individual replacement cost rate and the grouped replacement cost rate for each period. The optimal replacement policy is determined by choosing the lowest total cost rate and \as follow:

Number of replaced units during the period **(i)** is N_i.

- Average life of items (**AL**):

$$AL = \sum_{i=1}^{n} iP_i$$

- Average failure per period (**AF**):

$$AF = \frac{N_0}{AL}$$

- Cost of individual replacement (**CIR**):

$$CIR = C_1 * AF$$

- Average cost group replacement per period **(i)**:

$$ACGR_i = \frac{C_2 * N_0 + C_1 * \sum_{j=1}^{i} N_j}{i}$$

Accordingly, the cost of the individual replacement (**CIR)** is compared with the grouped replacement cost rate (**$ACGR_i$)**, and <u>the lower cost determines the type of replacement (individual or grouped)</u>. Also from $ACGR_i$ values, the optimum period **(i)** is determined for the grouped replacement.

Note that **Pi** represents the probability of failure of the new units over the time period **(i)**.

C_1 represents the cost of individual replacement per unit.

C_2 represents the cost of grouped replacement per unit.

N_0 represents the total number of units used at the beginning of the period.
i = 1, 2, 3, ... , n

Example 3: The probability of failure to certain production units before operating for the period **n** is shown in the table below with the number of replaced units during per week:

End of week (i)	1	2	3	4	5	6	7	8	9	10	11
Prob. of failure (P_i)	0.01	0.03	0.05	0.07	0.10	0.15	0.20	0.15	0.11	0.08	0.05

If the individual replacement cost is 1.25 $ and the grouped replacement cost is 0.5 $ per unit of production, select the optimal replacement policy when the number of used units is 1000.

Solution:

$$AL = \sum_{i=1}^{11} i * P_i = 1*0.01 + 2*0.03 + 3*0.05 + 4*0.07 + 5*0.1 + 6*0.15 + 7*0.2 + 8*0.15$$
$$+ 9*0.11 + 10*0.08 + 11*0.05 = 6.84$$

$$AF = \frac{N_0}{AL} = \frac{1000}{6.84} = 146.2 \quad and \quad CIR = C_1 * AF = 1.25 * 146.2 = 182.75$$

End of week (i)	$ACGR_i = \frac{C_2 * N_0 + C_1 * \sum_{j=1}^{i} N_j}{i}$
1	$\frac{1000*0.5 + 10*1.25}{1} = 512.5$
2	$\frac{1000*0.5 + (10 + 30.1)*1.25}{2} = 275.06$
3	$\frac{1000*0.5 + (40.1 + 50.6)*1.25}{3} = 204.46$
4	$\frac{1000*0.5 + (90.7 + 71.9)*1.25}{4} = 175.81$
5	$\frac{1000*0.5 + (162.6 + 104.4)*1.25}{5} = 166.75 \Rightarrow \min i.$
6	$\frac{1000*0.5 + (267 + 158.8)*1.25}{6} = 172.04$
7	$\frac{1000*0.5 + (425.8 + 216.4)*1.25}{7} = 186.11$
8	$\frac{1000*0.5 + (642.2 + 178.8)*1.25}{8} = 190.78$
9	$\frac{1000*0.5 + (821 + 155.8)*1.25}{9} = 191.22$

10	$\frac{1000*0.5+(976.8+145.8)*1.25}{10}=190.33$
11	$\frac{1000*0.5+(1122.6+139.1)*1.25}{11}=188.83$

Because of the lower cost of grouped replacement (**$ACGR_5$ = 166.76**) **<** individual replacement cost (**CIR = 182.75**), it is therefore preferable to make the **grouped replacement at the end of the fifth week**.

2. Maintenance Models:

We have noted in the previous models that all the decisions taken were to replace the production equipment to achieve the highest benefit (lowest cost) possible, but it is possible to repair the equipment rather than replacing them through inspecting carefully and periodically before any critical stage of its operation.

In these models, the expected costs or the expected failure are used to compare the different policies for the maintenance of the equipment provided that the probability of failure of the equipment during their lifetime is known.

Cost of maintenance (repair) (**CM**):

$$CM=\frac{Maintenance\ Cost\ \ per\ \ unit\ \ (MC)}{Expected\ \ life\ \ per\ \ unit\ \ (EL)}*N_0$$

Example 4: The cost of replacing any of the keys in the keyboard with high specifications of the terminal plant of the electronic calculator is $ 3, and the probability distribution of the life of these keys is given in the following table with the number of replaced units during the year:

End of year (i)	1	2	3	4
Prob. of failure (P_i)	0.20	0.25	0.42	0.13
Number of replaced units per year (N_i)	9.6	13.92	25.344	18.821

If the keyboard has 48 keys and the grouped replacement cost is $ 45, the key can be repaired instead of replacing it with a new key. The cost of repairing the key is $ 4, and the key provides an expected lifetime of 3.1 years. Which of the three possible cases (individual replacement, grouped replacement or maintenance) is more feasible in terms of cost and expected time?

Solution:

a) The individual replacement:

$$AL = \sum_{i=1}^{4} i * P_i = 1 * 0.20 + 2 * 0.25 + 3 * 0.42 + 4 * 0.13 = 2.48$$

$$AF = \frac{N_0}{AL} = \frac{48}{2.48} = 19.355 \quad \text{and} \quad CIR = C_1 * AF = 3 * 19.355 = 58.065$$

b) The grouped replacement:

N_0 = 48 keys

End of year (i)	$ACGR_i = \dfrac{C_2 * N_0 + C_1 * \sum_{j=1}^{i} N_j}{i}$
1	$\dfrac{45 + 9.6 * 3}{1} = 73.8$
2	$\dfrac{45 + (9.6 + 13.92) * 3}{2} = 57.78 \Rightarrow \min i.$
3	$\dfrac{45 + (23.52 + 25.344) * 3}{3} = 63.864$
4	$\dfrac{45 + (48.864 + 18.821) * 3}{4} = 62.014$

c) Maintenance (repair) of key:

$$CM = \frac{MC}{EL} * N_0 = \frac{4}{3.1} * 48 = 61.92$$

From the above we note that:

- The cost of the individual replacement of keys annually = **58.065 $**

- The lowest annual cost of the grouped replacement of keys at the end of the second year = **57.78 $**
- The cost of repairing the keys annually = **61.92 $**

It is therefore preferable to make the **grouped replacement of keys at the end of the second year.**

Homework

Replacement and maintenance costs for a particular type of truck were as follows:

Year	**1**	**2**	**3**	**4**	**5**	**6**
Maintenance cost / year	**200**	**450**	**680**	**850**	**1300**	**1600**
Resale value	**10000**	**8000**	**7000**	**5000**	**2000**	**1000**

If the cost of purchasing a new truck is 15000, determine the optimal time to own the truck.

Chapter 4:

Replacement and Maintenance models

All devices, equipment and machines are subject to failure, downtime or loss of efficiency over their lifetime, which requires reducing the time when these devices are out of service as little as possible, as well as minimizing the final cost to the minimum possible. That requires the determination of the **proper time** to inspect and check the devices carefully and probably renew them as a protective measure and repair or replacement of the idle devices.

1. Replacement models:

All companies and laboratories resort to replace certain units (machines or equipment) when they encounter one or both of the following problems:

- The existing units do not function efficiently.
- The existence of new units that perform the same function but in less time or in less cost or higher quality level.

Replacement shall be made when one or more of the following technical or economic reasons for which the replacement is required:

Technical reasons - The defects caused by the current unit or technical modifications to the production system or because of the risk caused by the current unit during the operation or the emergence of a new unit to perform additional works besides the original work or other advantages.

Economic reasons - Maintenance and operation costs, the damage rate, number of the units produced, number of the workers required, the effort made in use, etc.

The first model: It is noted in this model that the cost of maintenance and repair increases over time. In this model, the **total cost rate** of the machine is used as a criterion for decision-making regarding the period of replacement.

Assuming that **C** represents the cost of purchasing the new equipment,
S represents the resale value of the old equipment,
n is the number of time periods for the equipment service,

TC represents the sum of total costs,
T= TC/n represents the total cost rate, and
f(t) represents the maintenance cost in time **t**.

So, $$TC = C + \int_0^n f(t)dt - S$$

$$T = \frac{TC}{n} = \frac{1}{n}\left(C - S + \int_0^n f(t)dt\right)$$

The total cost rate will be as low as possible when:

$$g(n) = \frac{1}{n}\left(C - S + \int_0^n f(t)dt\right)$$ If the time **t** is a variable continuous

$$g(n) = \frac{1}{n}\left(C - S + \sum_{t=0}^{n} f(t)\right)$$ If time **t** is a discrete variable

Example 1: If the cost of maintenance and resale value of a machine within 8 years is shown in the table below:

Year (t)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Maintenance f(t)	**900**	**1200**	**1600**	**2100**	**2800**	**3700**	**4700**	**5900**
Resale value (S)	**4000**	**2000**	**1200**	**600**	**500**	**400**	**400**	**400**

In which year it is preferable to replace the machine when the price of purchasing the new machine is 7000 $?

Solution:

C = 7000 $

N	S	C - S	f(t)	∑f(t)	g(n)
1	4000	3000	900	900	3900
2	2000	5000	1200	2100	3550
3	1200	5800	1600	3700	3166.7
4	600	6400	2100	5800	3050
5	500	6500	2800	8600	3020 →mini.
6	400	6600	3700	12300	3150
7	400	6600	4700	17000	3371.4
8	400	6600	5900	22900	3687.5

Therefore, it is therefore best to replace the machine at the **end of the fifth year**.

Example 2: The cost of purchasing a new machine is 9000 $. The operating cost in the first year is 200 $ and this cost increases by 2000 $ per year. Determine the best life of the machine to replace it, assuming that this machine does not have a selling price when it is replaced.

Solution:

N	C - S	f(t)	∑f(t)	g(n)
1	9000	200	200	9200
2	9000	2200	2400	5700
3	9000	4200	6600	5200 → mini.
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So, the best replacement for the machine is at the **end of the third year**.

The second model: In this model, the units or parts that are suddenly damaged are replaced by calculating the individual replacement cost rate and the grouped replacement cost rate for each period. The optimal replacement policy is determined by choosing the lowest total cost rate and \as follow:

Number of replaced units during the period **(i)** is N_i.

- Average life of items (**AL**):

$$AL = \sum_{i=1}^{n} iP_i$$

- Average failure per period (**AF**):

$$AF = \frac{N_0}{AL}$$

- Cost of individual replacement (**CIR**):

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$$ACGR_i = \frac{C_2 * N_0 + C_1 * \sum_{j=1}^{i} N_j}{i}$$

Accordingly, the cost of the individual replacement (**CIR)** is compared with the grouped replacement cost rate (**$ACGR_i$)**, and the lower cost determines the type of replacement (individual or grouped). Also from $ACGR_i$ values, the optimum period **(i)** is determined for the grouped replacement.

Note that **Pi** represents the probability of failure of the new units over the time period **(i)**.

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i = 1, 2, 3, ... , n

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$$AL = \sum_{i=1}^{11} i * P_i = 1*0.01 + 2*0.03 + 3*0.05 + 4*0.07 + 5*0.1 + 6*0.15 + 7*0.2 + 8*0.15 + 9*0.11 + 10*0.08 + 11*0.05 = 6.84$$

$$AF = \frac{N_0}{AL} = \frac{1000}{6.84} = 146.2 \quad and \quad CIR = C_1 * AF = 1.25 * 146.2 = 182.75$$

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Because of the lower cost of grouped replacement (**$ACGR_5$ = 166.76**) **<** individual replacement cost (**CIR = 182.75**), it is therefore preferable to make the **grouped replacement at the end of the fifth week**.

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End of year (i)	**1**	**2**	**3**	**4**
Prob. of failure (P_i)	**0.20**	**0.25**	**0.42**	**0.13**
Number of replaced units per year (N_i)	**9.6**	**13.92**	**25.344**	**18.821**

If the keyboard has 48 keys and the grouped replacement cost is $ 45, the key can be repaired instead of replacing it with a new key. The cost of repairing the key is $ 4, and the key provides an expected lifetime of 3.1 years. Which of the three possible cases (individual replacement, grouped replacement or maintenance) is more feasible in terms of cost and expected time?

Solution:

a) The individual replacement:

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N_0 = 48 keys

End of year (i)	$ACGR_i = \frac{C_2 * N_0 + C_1 * \sum_{j=1}^{i} N_j}{i}$
1	$\frac{45 + 9.6 * 3}{1} = 73.8$
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c) Maintenance (repair) of key:

$$CM = \frac{MC}{EL} * N_0 = \frac{4}{3.1} * 48 = 61.92$$

From the above we note that:

- The cost of the individual replacement of keys annually = **58.065 $**

- The lowest annual cost of the grouped replacement of keys at the end of the second year = **57.78 $**
- The cost of repairing the keys annually = **61.92 $**
It is therefore preferable to make the **grouped replacement of keys at the end of the second year.**

Homework

Replacement and maintenance costs for a particular type of truck were as follows:

Year	**1**	**2**	**3**	**4**	**5**	**6**
Maintenance cost / year	**200**	**450**	**680**	**850**	**1300**	**1600**
Resale value	**10000**	**8000**	**7000**	**5000**	**2000**	**1000**

If the cost of purchasing a new truck is 15000, determine the optimal time to own the truck.

Chapter 5:

Sequencing Models

Sequencing (consecutive) models generally aim to find the optimal sequence to implement various jobs as they pass through **m** machines (**m** = 1, 2, 3,...) in addition to obtain the least total implementation time and find (idle time) for each machine.

The general assumptions that the sequencing models rely on are:
1. Each job has a beginning and an end.
2. Only one job can be performed on a particular machine at a specific time.
3. The job must be completed before starting the next job.
4- There is only one machine of every kind.
5. The job must be fully prepared when the time of implementation starts.
6. The time required to transfer the job from machine to machine can be neglected.
7 - It is assumed that there is no failure that would disrupt or stop the work such as maintenance or change in work shifts or lack of any of the factors of production.

So, these models will take the following cases:

1- Processing n jobs through one machine:

In this case, **n** jobs are performed through only one machine under the following algorithm:
(A) Arranging the jobs according to the time taken ascending or descending.
(B) We find the shortest processing time (S.p.t.) by dividing the sum of the jobs finishing times for the **ascending order** by the number of jobs.
(C) We find the longest processing time (L.p.t) by dividing the sum of the jobs finishing times for the **descending order** by the number of jobs.

Example 1: Six jobs are performed on one machine and their taken times (hours) for each job are:

Jobs	A	B	C	D	E	F
Time	8	6	2	7	10	4

Find the least time to perform all jobs according to the two measures:

(**a)** The shortest processing time (S.p.t.)

(b) The longest processing time (L.p.t.)

Solution:

(a) According to the ascending order:

Sequence	Jobs	Time	Processing	
			Start	Finish
1	C	2	0	2
2	F	4	2	6
3	B	6	6	12
4	D	7	12	19
5	A	8	19	27
6	E	10	27	37
∑				103

S.p.t = 103/6 = 17.16 hrs.

(b) According to the descending order:

Sequence	Jobs	Time	Processing	
			Start	Finish
1	E	10	0	10
2	A	8	10	18
3	D	7	18	25
4	B	6	25	31
5	F	4	31	35
6	C	2	35	37
∑				156

L.p.t = 156/6 = 26 hrs

Note: The optimal sequence of jobs can be found if there are different weights for each job by finding the modified time ($\bar{t}$) by dividing the time taken for each job $\mathbf{t_i}$ by the weights corresponding to that job $\mathbf{W_i}$, and the **ascending order** of the modified time is the optimal sequence.

Example 2: Find the optimal sequence of the following jobs performed on one machine and the operating times (hours) are:

Jobs	A	B	C	D	E	F
Time (t_i)	10	6	5	4	2	8
Weight (W_i)	5	10	5	1	3	5

Solution:

The modified time is:

Modified time ($\bar{t}$)	Jobs
10/5 = 2	A
6/10 = 0.6	B
5/5 = 1	C
4/1 = 4	D
2/3 = 0.67	E
8/5 = 1.6	F

Therefore, the optimal sequence is: B - E - C - F - A – D

2- Processing n jobs through two machines:

This case takes the following algorithm:

1. Determine the minimum time of each job.
2. In the sequence of jobs, start with the **ascending sequence** of the first machine (i.e. from the lower time to the higher time). In the case of the least two equal times, first select the time that has a **greater difference** with the other time of the second machine or select the time that has the **smallest difference** with the other time of the first machine.
3. Continue the sequence of jobs according to the **descending sequence** of the second machine (i.e. from the higher time to the lower time).
4. Based on the sequence of jobs, find the starting and finishing time of each job of the first machine.
5. For the same sequence of jobs, find the starting and finishing time of each for the second machine. The starting time depends on the greater value

between the finishing time of the previous job on the second machine and the finishing time of the present job on the first machine.

6. Calculate the least total time required to perform all jobs on the two machines which is the time of performing the final job on the second machine.
7. The lost (**idle time**) of the first machine is the difference between the finishing times on both machines. While, the idle time of the second machine is the sum of the differences between the starting and finishing time of each job on the second machine.

Example 3: Six jobs are performed on two machines **A** and **B,** the sequence of work is **A** and then **B**, and the taken time (hour) for each job is:

Jobs	**1**	**2**	**3**	**4**	**5**	**6**
Mach. A	**3**	**12**	**5**	**2**	**9**	**11**
Mach. B	**8**	**10**	**9**	**6**	**3**	**1**

Required: Find **(a)** the optimal sequence, **(b)** the least total time required to perform all the jobs and **(c)** the idle time for both machines.

Solution:

1	**2**	**3**	**4**	**5**	**6**
2 **3**	**12**	3 **5**	1 **2**	**9**	**11**
8	4 **10**	**9**	**6**	5 **3**	6 **1**

(a) The optimal sequence is: **4 – 1 – 3 – 2 – 5 – 6**.

Jobs	**Mach. A**			**Mach. B**			
	Time	**Start**	**Finish**	**Time**	**Start**	**Finish**	**Idle time**
4	**2**	**0**	**2**	**6**	**2**	**8**	**2**
1	**3**	**2**	**5**	**8**	**8**	**16**	**0**
3	**5**	**5**	**10**	**9**	**16**	**25**	**0**
2	**12**	**10**	**22**	**10**	**25**	**35**	**0**
5	**9**	**22**	**31**	**3**	**35**	**38**	**0**
6	**11**	**31**	**42**	**1**	**42**	**43**	**4**
$\sum$							**6**

(b) The least total time required to perform all the jobs is **43** hours.
(c) The idle time for machine **A** is: **43 - 42** = **1** hr.
The idle time for machine **B** is: **6** hrs.

Example 4: Seven jobs are performed on two machines **A** and then **B**, and the taken time (hour) for each job is:

Jobs	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Mach. A	**3**	**12**	**15**	**6**	**10**	**11**	**9**
Mach. B	**8**	**10**	**10**	**6**	**12**	**1**	**3**

Required: Find **(a)** the optimal sequence, **(b)** the least total time required to perform all the jobs and **(c)** the idle time for both machines.

Solution:

1	**2**	**3**	**4**	**5**	**6**	**7**
1 <u>**3**</u>	**12**	**15**	2 <u>**6**</u>	3 <u>**10**</u>	**11**	**9**
8	5 <u>**10**</u>	4 <u>**10**</u>	**6**	**12**	7 <u>**1**</u>	6 <u>**3**</u>

(a) The optimal sequence is: **1 – 4 – 5 – 3 – 2 – 7 – 6**.

Jobs	**Mach. A**			**Mach. B**			
	Time	**Start**	**Finish**	**Time**	**Start**	**Finish**	**Idle time**
1	**3**	**0**	**3**	**8**	**3**	**11**	**3**
4	**6**	**3**	**9**	**6**	**11**	**17**	**0**
5	**10**	**9**	**19**	**12**	**19**	**31**	**2**
3	**15**	**19**	**34**	**10**	**34**	**44**	**3**
2	**12**	**34**	**46**	**10**	**46**	**56**	**2**
7	**9**	**46**	**55**	**3**	**56**	**59**	**0**
6	**11**	**55**	**66**	**1**	**66**	**67**	**7**
$\sum$							**17**

(b) The least total time required to perform all the jobs is **67** hours.
(c) The idle time for machine **A** is: **67 - 66** = **1** hr.
The idle time for machine **B** is: **17** hrs.

3. Processing n jobs through 3 machines:

In this case, you must check at least one of the two conditions:

(a) The least time on the first machine ≥ The greatest time on the second machine.
Or
(b) The least time on the third machine ≥ The greatest time on the second machine.

The solution algorithm will be:
(1) Convert the three machines into two imaginary machines **G** and **H**, and their operating times are:

$$\mathbf{G_i = A_i + B_i \quad , \quad H_i = B_i + C_i}$$

(2) Find the optimal sequence of the two machines **H** and **G**.
(3) Use the sequence of jobs according to the optimal sequence, and find the starting and finishing time of each process for each machine from the original machines according to the previous method.

(4) The idle time for both the first and third machines (**A** and **C**) is calculated in the same previous way, but the difference is in the calculation of the idle time on machine **B**, as it is calculated from the relationship:

Finishing time of the final job (according to the optimal sequence of jobs) on the third machine **-** Finishing time of the final job on the second machine **+** the idle time calculated for the second machine.

Example 5: Six jobs are performed on three machines **A, B, C**, according to the **ABC** sequence. Find **(a)** the optimal sequence for performing all the jobs, **(b)** the least total time and **(c)** idle time for each machine. The time taken (**hour**) for each operation on each machine is:

Jobs	1	2	3	4	5	6
Mach. A	3	12	5	2	9	11
Mach. B	8	6	4	6	3	1
Mach. C	13	14	9	12	8	13

Solution:

Check the second condition: the least time on the third machine ≥ the largest time on the second machine.

So, we can solve the model using the above algorithm:

Assuming that: $G_i = A_i + B_i$, $H_i = B_i + C_i$

Jobs	1	2	3	4	5	6
Mach. G	11 3	18 5	9 2	8 1	12	12 4
Mach. H	21	20	13	18	11 6	14

(a) The optimal sequence is**: 4 – 3 – 1 – 6 – 2 – 5**.

	Mach. A			Mach. B				Mach. C			
Jobs	T.	S.	F.	T.	S.	F.	I.	T.	S.	F.	I.
4	2	0	2	6	2	8	2	12	8	20	8
3	5	2	7	4	8	12	0	9	20	29	0
1	3	7	10	8	12	20	0	13	29	42	0
6	11	10	21	1	21	22	1	13	42	55	0
2	12	21	33	6	33	39	11	14	55	69	0
5	9	33	42	3	42	45	3	8	69	77	0
∑							17				8

(b) The least total time required to perform all the jobs is **77** hrs.

(c) The idle time for machine **A** is: **77 - 42 = 35** hrs. .

The idle time for machine **B** is: **77 - 45 +17 = 49** hrs.

The idle time for machine **C** is: **8** hrs.

Homework:

Five jobs are performed on three machines **A, B, C**, according to the **ABC** sequence. Find:-

(a) The optimal sequence for performing the jobs.

(b) The least total time to perform all the jobs.

(c) The idle time for each machine.

The time taken (**hour**) for each operation on each machine is:

Job	1	2	3	4	5
Mach. A	3	8	7	5	4
Mach. B	4	5	1	2	3
Mach. C	7	9	5	6	10

Chapter 6

Inventory Models

Introduction:

Inventory (or stock) control and its management are the most important challenges facing the economic firms in the present time. They effectively depend on the number of goods, the demand for these goods and the time between the receipts of orders. This results in either maximizing the profits or minimizing the cost of maintaining inventory. Where, the efficient control should not hold excess quantities from the current or expected need for inventory because this leads to undue costs.

Main cost elements for storing:

1- Cost of unit purchase or production

The concept of the cost of the unit varies between the merchant and the producer. The merchant represents the price paid to the financier, including the freight, while the producer represents the amount of money spent on this unit. And, this cost may be fixed or linked to the size of the order and thus the merchant benefits from the deduction of quantity or product of the savings of production.

2- Cost of setup or ordering

This cost is fixed and independent of the quantity required.

3- Cost of holing

This cost represents the various costs for holding the good inside the store, such as storage, insurance, damage, electricity, etc.

4- Cost of penalty or shortage

This cost occurs when the store is unable to meet the customers' requests, leading to the customer's acceptance to postpone the request or reject the order. In the case of purchases, this cost represents the loss resulting from the loss of customer confidence plus other administrative costs. While in the case of sales, this cost represents the lost profit due to the loss of demand in addition to the loss resulting from the loss of customer’s confidence.

Accordingly, the general cost of inventory becomes:

The total cost of inventory = Cost of unit purchase or production + Cost of setup or ordering + Cost of holing + Cost of penalty or shortage

Some economic concepts related to inventory:

1- Quantity ordered

It is the number of units of inventory that are received and placed in the store.

2- Order cycle

It is a period of time that separates two consecutive orders and is measured by day, month, or year.

3- Lead time

It is the period between the issuance of the purchase order and the arrival of the shipment to the stores and its receipt.

4- Reorder point (time)

It is the time in which the re-ordered must be done when the stock finishes.

5- Safety stock

It is the reserved inventory in anticipation for unusual circumstances and be extra quantity for the company's need.

6- Order cost

It is the cost of preparing an order for purchase, which includes all the costs related to the reorder and do not often depend on the size of the order.

7- Purchase price

It represents the price of the purchased material, i.e. **the unit price x the number of purchased units.**

8- Buffer stock

It is the amount of material in the stores as a reserve in case of depletion of stock and delay of the order.

Invetory Models:

First model: Economic Order Quantity (EOQ):

This model is the mostly famous one in the inventory models and the mostly used as well as the oldest model. It is used in the following cases:

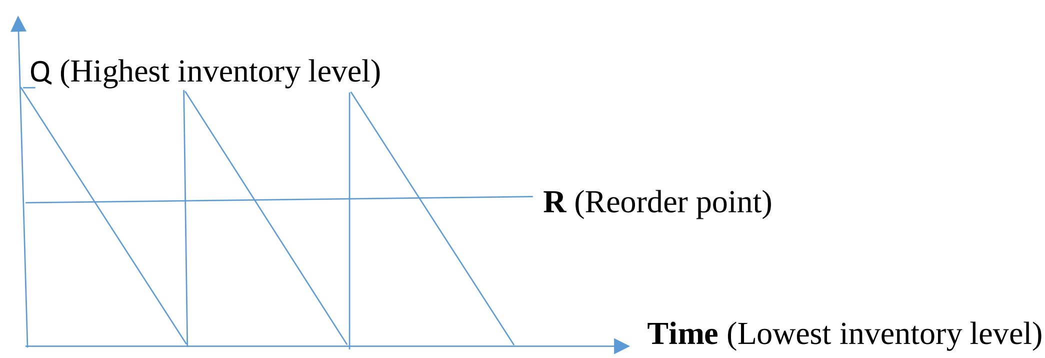
(a) The order is known, fixed, and constant over time.

(b) Inventory shortage is not allowed.

(c) Fixing the waiting times to receive the orders.

(d) Receiving the ordered quantity at the time of order.

Economic Order Quantity is the quantity at which the cost of holding the inventory and the cost of the order decrease as shown in the following figure:



This figure represents the economic order quantity cycle, where the new order is completed when the inventory reaches the reorder point (**R**) and the order is delivered to the store upon reaching the lowest inventory level, and the time period between the order and the receipt for each order is considered constant.

The economic order quantity (**EOQ**) is computed using the following calculations:

(a) Determination of the annual total cost of the inventory (TC):

(i) Calculation of annual ordering cost (AOC):

$$\mathbf{AOC} = \frac{\mathbf{Co \times D}}{\mathbf{Q}} \qquad \ldots\ldots\ldots\ldots\ldots (1)$$

Where,

C_o = Cost of order

D = Annual order quantity

Q = Size of order

(ii) Calculation of annual caring cost (ACC)

$$\mathbf{ACC} = \frac{\mathbf{Cc\ x\ Q}}{\mathbf{2}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Where, **Cc** is the annual caring cost for one unit.

Then, the annual total cost (**TC**) of the inventory is:

TC = AOC + ACC ………..….. (3)

(b) Determination of the economic order quantity (EOQ):

$$\mathbf{EOQ} = \sqrt{\left(\frac{\mathbf{2Co\ x\ D}}{\mathbf{Cc}}\right)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

(c) Determination of the minimum annual total cost of the inventory (TC_{min}):

By substituting the economic order quantity (**EOQ**) into equation (3), the minimum total cost is:

$$\mathbf{TC_{min}} = \frac{\mathbf{Co\ x\ D}}{\mathbf{EOQ}} + \frac{\mathbf{Cc(EOQ)}}{\mathbf{2}} \qquad \ldots\ldots\ldots (5)$$

(d) Determination of the Order cycle period (t):

The period between the order and the other one (**order cycle t**) is:

$$\mathbf{t} = \frac{\mathbf{T}}{\mathbf{n}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Where,

T = Number of days of the year.

n = Number of orders per year.

So that **Number of orders per year** (**n**) is:

$$\mathbf{n} = \frac{\mathbf{Annual\ order\ quantity}}{\mathbf{Economic\ order\ quantity}}$$

$$\mathbf{n} = \frac{D}{\mathbf{EOQ}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Example:

A good has an annual order of (120,000) unit. The unit price is ($ 5). The unit storing cost is (20%) of its price. The cost of processing the order is ($ 25). Calculate:

(a) The minimum annual total cost of the inventory (**EOQ**).

(b) The minimum annual total cost of the inventory (**TC_{min}**).

(c) The order cycle period (**t**) if the actual number of days of the year is (260).

Solution:

(a) $EOQ = \sqrt{\left(\frac{2Co \times D}{Cc}\right)}$

$Cc = 5 \times 0.20 = 1$

$EOQ = \sqrt{\left(\frac{2 \times 25 \times 120000}{1}\right)} = 2449.4 \approx 2450$ **unit**

(b) $TC_{min} = \frac{Co \times D}{EOQ} + \frac{Cc(EOQ)}{2}$

$= \frac{25 \times 120000}{2450} + \frac{1 \times 2450}{2} = 1224.5 + 1225 = 2449.5\ \$$

(c) $n = \frac{D}{EOQ}$

$= \frac{120000}{2450} = 48.98 = 49$ **order**

$t = \frac{T}{n}$

$= \frac{260}{49} = 5.31$ **day**

Second model: Reorder point (R):

It is the point at which the inventory arrives so as we ask for another order, i.e. the point at which the inventory reaches the safety stock, and this point is:

$R = d \times L$ ………………….. (8)

$d = \frac{D}{t}$ ………………... (9)

Where,

R is the reorder point.

d is the size of order per unit time.

L is the waiting period to receive the order.

D is the annual order quantity

t is the number of actual days of the year.

<u>Example:</u>

If the production company has the following information:

- Average of actual days of the year is (250) day.

- The annual order quantity is (8000) unit.

- Waiting period to receive the order is (20) day.

What is the reorder point?

<u>Solution:</u>

$$\mathbf{d} = \frac{\mathbf{D}}{\mathbf{t}} = \frac{\mathbf{8000}}{\mathbf{250}} = \mathbf{32}$$

R = d x L = 32 x 20 = 640 unit

This means when the inventory reaches to (640) unit, we must do the reorder.

<u>Homework</u>

A company for import and export found that its annual sales volume of small cars is 8000 cars. The cost of purchasing one car is $ 10000. The cost of each order prepared by the company is $ 1500. The cost of holding one car is 4% of its value.

<u>Required:</u>

(a) Determine the economic order quantity.

(b) The annual total cost of the inventory.

(c) The optimal number of orders per year and the order cycle period.