Hydraulic Power Plants Including Solved Problems

Lecture Notes

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Preface

Hydraulic machines -hydraulic turbine, pumps (water and oil types) and reversible hydraulic machines (pump- turbine)- find application in hydro-electric power plants, and water supply system. Also, in thermal nuclear and pumped-storage station. In addition, pumps are widely used in construction of hydraulic structures, such as dams, canals, river and sea ports.

The book describes the construction of hydraulic power plants and treats the theory of the working process for each part, i.e. the kinematic and dynamic of the liquid flowing through hydraulic machine and systems, only in the scope necessary for understanding their operation conditions and basic calculation relationships.

The book contains a large number of drawings and charts. It also includes the most important specification and working examples and solved problems which can be applied in designing practice and maintenance of hydroelectric power plants, pumping stations and pump installation.

Authors

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Chapter One

The momentum equation and its application

1.1 Momentum and fluid dynamic force

Momentum = m.V

The partials of fluid stream posses' momentum and whenever the velocity of the stream is changed in magnitude or direction there will be corresponding change in the momentum of the fluid particles.

$\therefore$ Change of momentum $= mdV$

And

Rate of Change of momentum $= \dot{m}dV$

Where $\dot{m} = \frac{m}{t}\ mass\ flow\ rate = \rho\, Q\, \frac{kg}{s}$

According to Newton's 2nd law "the rate of change of momentum of a body is proportional to the force applied and takes place in the direction of action of that force"

i.e Dynamic force applied = Rate of Change of momentum

i.e $F = \dot{m}\Delta\vec{V}$

For a control volume with fluid entering with uniform velocity $\vec{V}_{in}$ and after time t with uniform velocity $\vec{V}_{out}$:

$$\therefore \sum F = \dot{m}\left(\vec{V}_{out} - \vec{V}_{in}\right) \dots\dots\dots (1)$$

$in\ x - direction$

$$\sum F_x = \dot{m}\left(\vec{V}_{2x} - \vec{V}_{1x}\right) \dots\dots\dots (2)$$

$in\ y - direction$

$$\sum F_y = \dot{m}(\vec{V}_{2y} - \vec{V}_{1y}) \ldots\ldots\ldots (3)$$

$$\sum F = F_1 + F_2 + F_3$$

F_1 =The force exerted by the boundary on the fluid.

$= -R$ The force exerted by the fluid on the boundary

$F_2 =$ Pressure force $= \Delta PA$

$F_3 =$ Gravitational force $= W \downarrow$

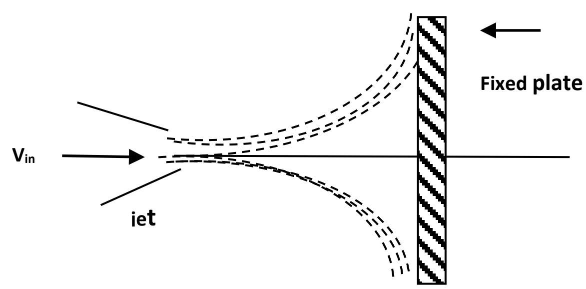
1.2 Application of momentum Equation

1.2.1 Force exerted by fluid jet on a flat plate

(a) Stationary flat plate: -

$F = \dot{m}(\vec{V}_{out} - \vec{V}_{in}) \ldots\ldots\ldots (1)$

$\vec{V}_{out} = V_2 = 0 \quad Stationary\ plate$



$\vec{V}_{in} = V_1$

$\therefore F = \dot{m}(0 - \vec{V}_{in})$

$F = -\dot{m}\vec{V}_{in}$

$since\ \dot{m} = \rho Q$

$Q = AV$

$$F = -\rho Q\vec{V}_{in}$$

$and\ R = -F$

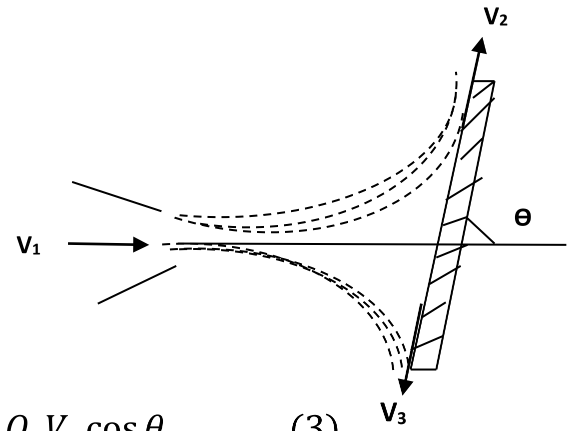
$\therefore R = \rho AV^2 \ldots\ldots\ldots (2)$

$A = cross - sectional\ area\ of\ the\ jet.$

(b) Inclined Stationary flat plate: -

$F = \dot{m}\left(\vec{V}_{out} - \vec{V}_{in}\right)$

$\sum F = \sum \dot{m}\vec{V}_{out} - \sum \dot{m}\vec{V}_{in}$



$\sum F_x = \rho\, Q_2 V_2 \cos\theta_2 + \rho\, Q_3\, V_3 \cos\theta_3 - \rho\, Q_1 V_1 \cos\theta_1 \dots\dots\dots (3)$

$\sum F_y = \rho\, Q_2 V_2 \sin\theta_2 + \rho\, Q_3\, V_3 \sin\theta_3 - \rho\, Q_1 V_1 \sin\theta_1 \dots\dots\dots (3a)$

(c) Moving plate

For moving plate $V \rightarrow V_r$

$V_r =$ Relative velocity $= V - U$

$U =$ Velocity of the plate.

$\vec{V}_{out} = 0 \qquad \theta_2 = 90° \qquad \theta_1 = 0$

$$\therefore \; F = -\dot{m}\vec{V}_{r1}$$

$R = \dot{m}\vec{V}_{r1} \qquad \dot{m} = \rho A(V-U)$ For single moving plate

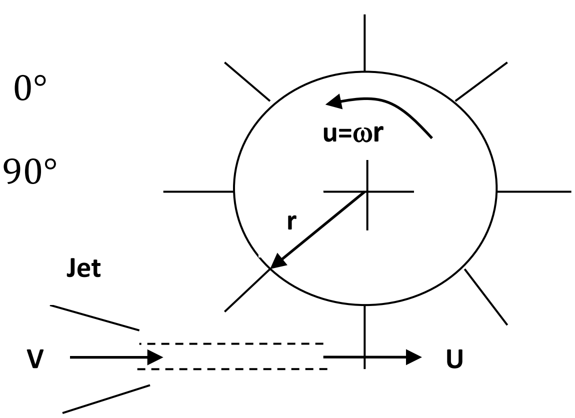
$\therefore R = \rho A(V-U)^2 \dots\dots\dots (4)$

For series moving plate

$$\vec{V}_{in} = V - U \qquad \theta = 0°$$

$$\vec{V}_{out} = 0 \qquad \theta_2 = 90°$$

$$\dot{m} = \rho A V_{in}$$

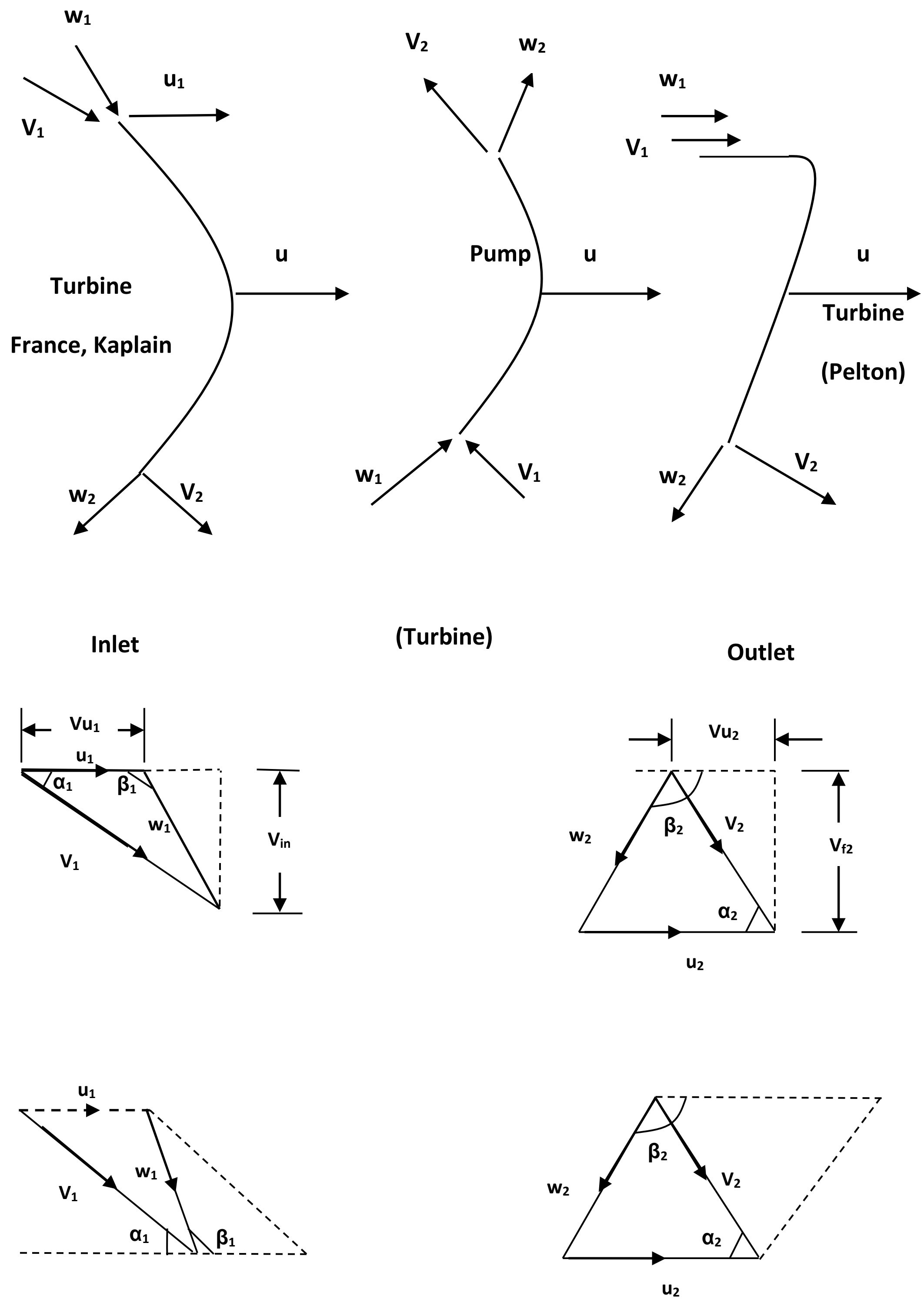


$\therefore F = -\rho\, A V_1 (V-U)$

$or \;\; R = \rho\, A V_1 (V-U) \dots\dots\dots (5)$

$Power = F.U$ Series of moving plates (vanes)

1.2.2 Velocity diagram (general)



V_1 , V_2 : The inlet and outlet velocity (absolute)

α_1 , $\propto_2$: The angles of the inlet and outlet velocities

U_1 , U_2 : The velocities of the (Vane, blade, tangential, perphrial) wheel.

$U_1 = U_2 = U$For implies turbine.

W_1 , W_2 : relative velocities at inlet and outlet. It is tangential to the blade at inlet and outlet.

β_1 , β_2 : The (blade, vane, wheel, relative velocities) angles at inlet and outlet.

$V_u = V\cos\alpha$(Whirl velocity) at inlet and outlet.

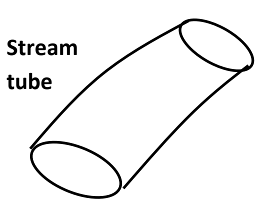
$V_f = V\sin\alpha$(Flow velocity) at inlet and outlet.

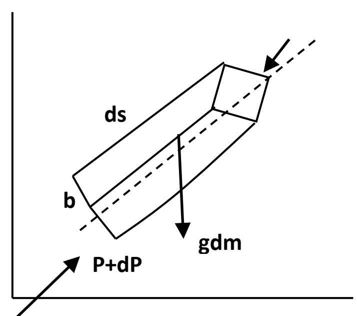
1.3 Bernoulli's Equation for Relative Motion

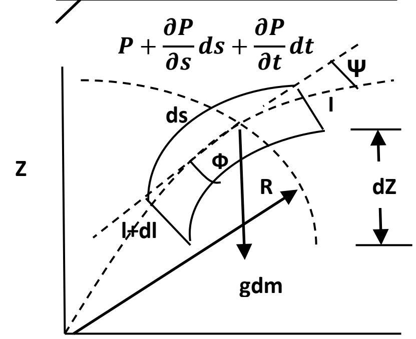
Consider the motion of fluid inside a turbine runner or an impeller of centrifugal pump. Let its angular velocity ω be constant and the relative velocity be W. consider a small element of fluid of length d_s

breadth b and thickness l and l+dl at two ends which is a part of stream tube.

Forces on the element are fluid pressure, gravitational and centrifugal force.







Φ: the angle between the tangent and the gravity force

Ψ: the angle between the tangent and the radians of curvature.

For steady stale $\frac{\partial}{\partial t} = 0$

Acceleration in the direction of relative velocity W

$\frac{dw}{dt} = \frac{\partial w}{\partial t} + w\frac{\partial w}{\partial s} = w\frac{\partial w}{\partial s}$ for steady flow.

Mass of fluid acceleration $= \frac{\gamma}{g}(blds)$

Forces acting on the element are

a) Weight,(gravitational force) $= \gamma blds$

b) Centrifugal force $= \frac{\gamma}{g} blds. R\omega^2$

c) Pressure difference force $= Pdl - (P + dP)b(l + dl)$

$$= -bldP(neglecting\ \frac{dl}{l}\ terms)$$

Now resultant force in the direction of stream line =mass × acceleration

$$\therefore \gamma blds \cos\emptyset - bldP - \frac{\gamma}{g} blds. R\omega^2 = \frac{\gamma}{g} bldsw\frac{dw}{ds}$$

$$w\frac{dw}{ds} = ds\cos\emptyset - \frac{dP}{\gamma} - \frac{R\omega^2}{g} ds\cos\emptyset$$

Substituting $ds\cos\emptyset = -dZ\ and\ ds\cos\varphi = -dR$

$$\therefore \frac{wdw}{g} = -dZ - \frac{dP}{\gamma} + \frac{R\omega^2}{g} dR$$

$$or\ w.\frac{dw}{g} + dZ + \frac{dP}{\gamma} - \frac{R^2\omega^2}{2g} dR = 0$$

$$\frac{w^2}{2g} + Z + \frac{P}{\gamma} - \frac{R^2\omega^2}{2g} = C \quad since\ U = R\omega$$

$$\therefore \frac{W^2}{2g} - \frac{U^2}{2g} + \frac{P}{\gamma} + Z = C$$ B.E for relative motion

$$\therefore \frac{{w_1}^2}{2g} - \frac{{U_1}^2}{2g} + \frac{P_1}{\gamma} + Z_1 = \frac{{w_2}^2}{2g} - \frac{{U_2}^2}{2g} + \frac{P_2}{\gamma} + Z_2 + losses_{1-2}$$

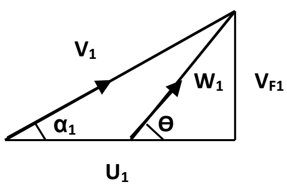
Ex.1 A jet of water, moving at 60 m/s is deflected by a vane moving at 25 m/s in a direction at 30° to the direction of the jet. The water leaves the blades normally to the motion of the vanes.

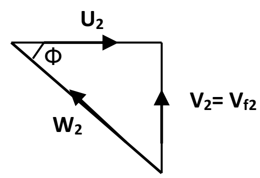
Draw inlet and outlet velocity triangle, and find the vane angles for no shock at entry and exit. Take relative velocity at outlet to be 0.85 of the relative velocity at inlet

Solution:-

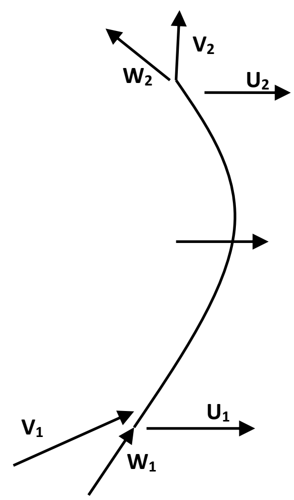
$$V_1 = 60\frac{m}{s}\ ;\ U_1 = 25\frac{m}{s}\ ;\alpha_1 = 30°$$

$w_2 = 0.85w_1\ ;\alpha_2 = 90°$





Outlet



$\beta_1 = 180 - \theta$

$\beta_2 = 180 - \emptyset$

From inlet velocity triangle

$$V_{u1} = V_1 \cos\alpha_1 = 60 \times 0.86 = 51.96\ m/s$$

$$V_{f1} = V_1 \sin\alpha_1 = 60 \times 0.5 = 30\ m/s$$

$$\tan\theta = \frac{V_{f1}}{V_{u1} - U} = \frac{30}{51.96 - 25}$$

$$\theta = 48.07° \quad \beta_1 = 180 - 48.07 = 132°$$

From inlet velocity triangle

$$w_1 = \frac{V_f}{\sin 48.07} \qquad \therefore\ w_1 = 40.34\ m/s$$

$$\therefore w_2 = 0.85 w_1 = 34.29\ m/s$$

$$and\ U_1 = U_2$$

From outlet velocity triangle

$$\cos \emptyset = \frac{U_2}{w_2} = \frac{25}{34.29}$$

$$\emptyset = 43.2 \qquad \beta_2 = 136.8°$$

Ex.2 A jet of water, having a velocity of 30 m/s impinges on a series of vanes with a velocity of 15 m/s. take jet makes an angle of 30° to the direction of motion of vanes when entering and leaves at an angle of 120°. Sketch velocity triangle at inlet and outlet and determine.

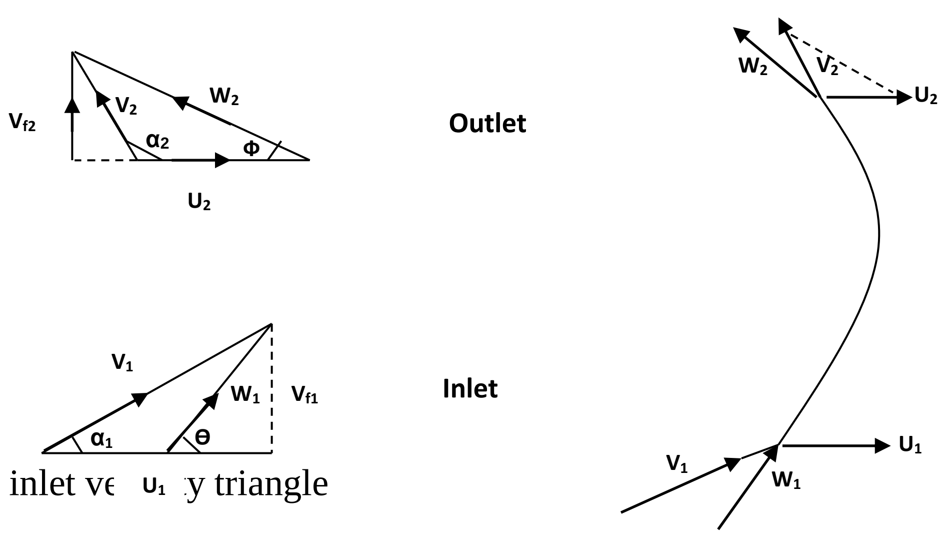
a) Angle of the vane at inlet and leaves without stock.

b) Work done per kg of water entering the vanes.

c) Efficiency.

Solution:-

$V_1 = 30\frac{m}{s}$; $U_1 = U_2 = 15\frac{m}{s}$; $\alpha_1 = 30°$; $\alpha_2 = 120°$



From inlet velocity triangle

$$V_{u1} = V_1 \cos\alpha_1 = 30 \times \cos 30 = 25.98\ m/s$$

$$V_f = V_1 \sin\alpha_1 = 30 \times \sin 30 = 15\ m/s$$

$$\therefore \tan\theta = \frac{V_{f1}}{V_{u1} - U_1} = \frac{15}{25.98 - 15}$$

$$\therefore \theta = 53.8°$$

$$\therefore \beta_1 = 126.2°$$

$$also \quad W_1 = \frac{V_f}{\sin 53.8°} = \frac{15}{\sin 53.8°} = 18.59\ m/s$$

From outlet velocity triangle

$$\frac{U_2}{\sin(60 - \emptyset)} = \frac{W_2}{\sin 120°}$$

$$\frac{15}{\sin(60 - \emptyset)} = \frac{18.59}{\sin 120°}$$

$$(60 - \emptyset) = 44.3° \quad \emptyset = 15.67°$$

$$\therefore \beta_2 = 124.3°$$

$$V_{u2} = W_2 \cos 15.67 - U_2$$

$$= 18.59 \times 0.963 - 15 = 2.9\ m/s$$

$$\therefore Work\ W = \frac{1}{g}(V_{u1}U_1 - V_{u2}U_2)$$

$$= \frac{U}{g}(V_{u1} - V_{u2}) = \frac{15(25.98 - 2.9)}{9.81}$$

$$= 346.2\ N.m$$

$$Efficiency = \frac{Work\ done\ per\ kg\ of\ water}{Energy\ of jet\ per\ kg\ of\ water}$$

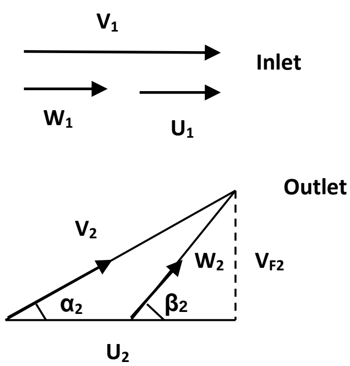
$$= \frac{346.2}{\frac{V^2}{2g} \times g} \qquad Where \ \frac{V^2}{2g} = kg.m \times g = N.m$$

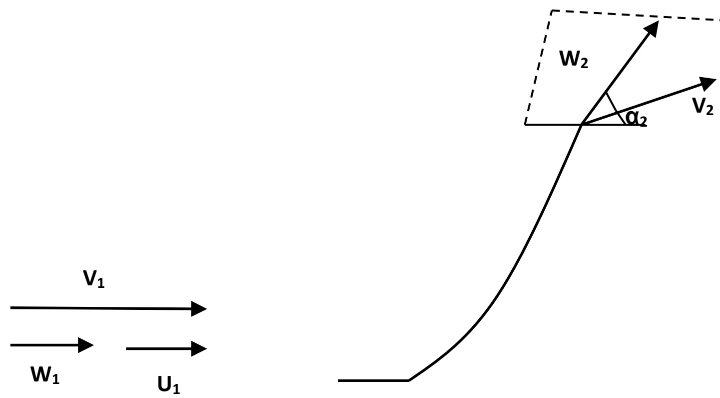
$$Efficiency = \frac{346.2}{450} = 77\%$$

Solved Problems

Q.1 A circular jet delivers water at the rate of 60 l/s with a velocity of 24 m/s. The jet impinge tangentially on a vane moving in the direction of the jet, with a velocity of 12 m/s. The vane angle at outlet 45^o, through what angle will if deflect the jet

<u>Solution:-</u>





$$Q = 60\frac{l}{s} \ ; \ U_1 = U_2 = 12\frac{m}{s} \ ; \alpha_1 = 0 \ ; \beta_2 = 45°$$

$$\alpha_2 = ? \ ; \ w_1 = w_2$$

$$V_{u1} = V_1 \cos \alpha_1 = V_1 = 24 \ m/s$$

$$w_1 = V_{u1} - U_1 = 12 \ m/s$$

From velocity triangle, since $U_2 = w_2 = 12 \ m/s$

$$\therefore \alpha_2 = \frac{45}{2} = 22.5°$$

$$Work = \dot{m}(V_{u1}U_1 - V_{u2}U_2) \qquad U_1 = U_2$$

$$= \rho Q(V_{u1} - V_{u2})U$$

$$V_{u2} = U_2 + W_2 \cos 45° = 12 + 12 \cos 45° = 20.5 \; m/s$$

$$\therefore Work/s = 1000 \times 0.06(24 - 20.5) \times 15$$

$$= 3150 \; W$$

Q.2 A jet of water 100 mm in diameter, moving with a velocity of 25 m/s in the direction of the vanes, enters the vane moving with a velocity of 12.5 m/s. If the jet leaves the vane at an angle of 60^{o} with the direction of motion of the vanes, find

a) The force on the vanes in the direction of motion

b) The work done per second (Power)

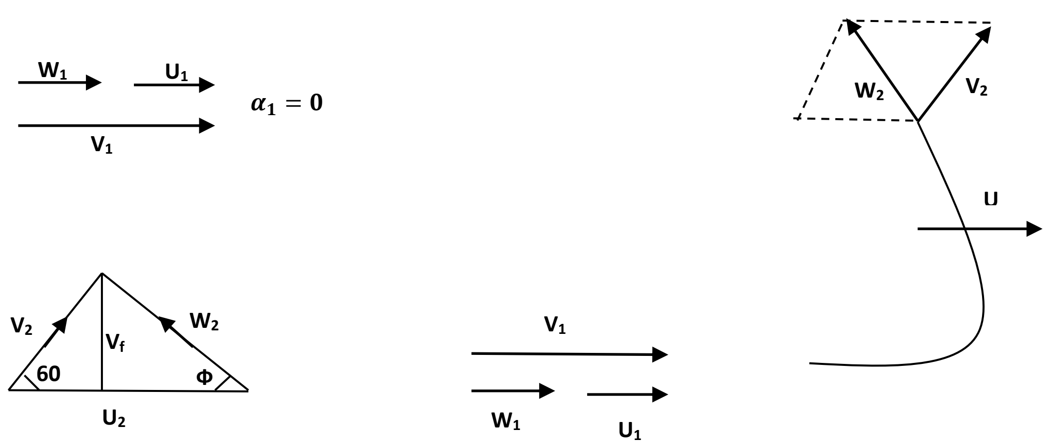
<u>Solution:-</u>

Dia. of jet $= 100 \; mm = 0.1 m$ $\qquad a = \frac{\pi}{4}(0.1)^2 = 0.00785 \; m^2$

$$V_1 = 25 \; m/s \, ; U_1 = U_2 = 12.5 \; m/s$$

$$\alpha_1 = 0 \, ; \alpha_2 = 60°$$

$$F = \rho Q (V_{u1} - V_{u2})$$



$$V_{u1} = V_1 \cos \alpha_1 = 25 \; m/s$$

$$w_1 = V_{u1} - U_1 = 25 - 12.5 = 12.5\ m/s$$

From outlet triangle $w_2 = U_2 \quad \therefore \emptyset = 60°$

$$\therefore \frac{V_2}{\sin 60°} = \frac{w_2}{\sin 60°}$$

$$\therefore V_2 = w_2 = 12.5$$

$$Then \quad V_{u2} = V_2 \cos 60° = 6.25\ m/s$$

$$F = \dot{m}(V_{u1} - V_{u2})U = 1000 \times 0.00785 \times 25(25 - 6.25)$$

$$= 3.68\ kN$$

$$Power = F.U = 3.68 \times 12.5 = 46\ kW$$

Q.3 A jet of water 50 mm in diameter impinges on a curved vane and is deflected through on angle of 135°. The vane moves in the same direction as that of the jet with a velocity of 5 m/s. if the rate of flow of the water is 30 l/s. Determine,

a) component of force on the vane in the direction of motion.

b) Power developed by the vane.

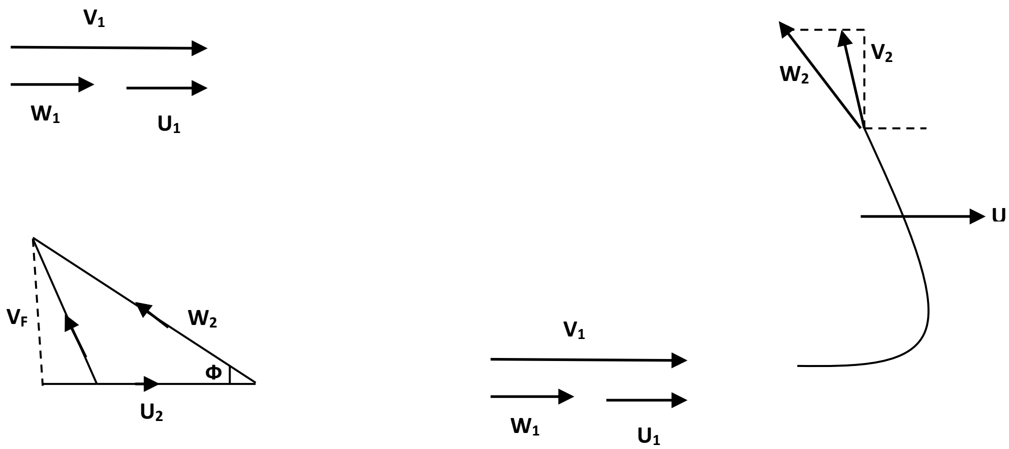
c) Efficiency.

Solution:-

$$d = 50\ mm \quad \therefore a = 0.00196\ m^2\ ; \beta_2 = 135°$$

$$\emptyset = 45°\ ;\ U_1 = U_2 = 5\ m/s\ ; Q = 30\ l/s$$

$$w_1 = w_2$$



$$V = \frac{Q}{a} = \frac{0.03}{0.00196} = 15.31\, m/s$$

$$V_{u1} = V_1 \cos\alpha_1 = 15.31\ \ m/s \qquad ;\ \alpha_1 = 0$$

$$w_1 = V_1 - U = 15.31 - 5 = 10.31\, m/s$$

$$V_{u2} = w_2 \cos 45° - U = (10.31 \times 0.707) - 5$$

$$= 2.29\, m/s$$

$$\therefore F = \rho\, Q(V_{u2} - V_{u1}) = 1000 \times 0.03(15.31 - 2.29)$$

$$= 390.6\, N$$

$$Power = F.U = 390.6\ \ \times 5 = 1953\, watt$$

$$also\ \ K.E/s = \frac{1}{2}\dot{m}V^2 = \frac{1}{2}\rho\, QV^2$$

$$= \frac{1}{2} \times 1000 \times 0.03 \times (15.31)^2 = 3516\, watt$$

$$\eta = \frac{1953}{3516} = 55.5\%$$

Chapter two

Hydraulic Machine

2.1 General

The_fluid machine can be divided into two gropes:

1. Rotodynamic group: Consists of the following

a) Turbines b) Pumps

c) Fans and blowers d) Compressors.

They are consists of a rotor carrying a numbers of vanes or blades and there is a transfer of energy between the fluid and the vanes or blades.

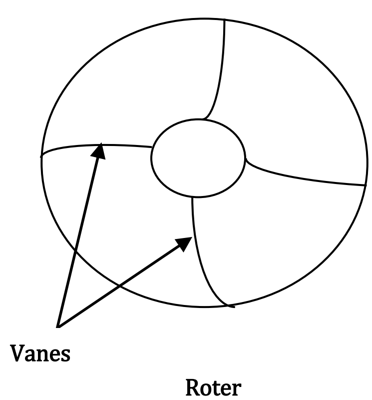
For greater rates of flow and lower Pressure, Rotodynamic machine are more satisfactory (except gear pumps).

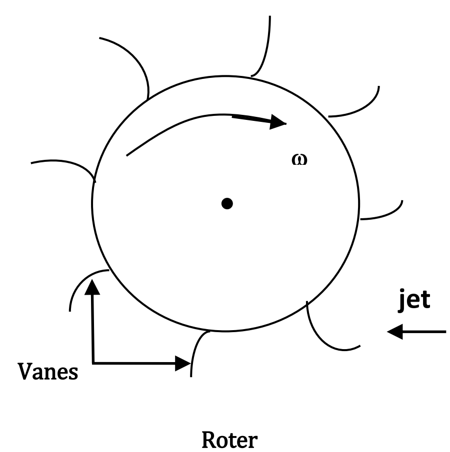
The machine may be classified according to main direction of the fluid path in the rotor:

a) Radial flow machine.

b) Axial flow machine.

c) Mixed flow machine.





2- Positive-displacement group: consists of the following.

a) Reciprocating pumps b) Engines (I.C.)

c) Diaphragm pumps d) air pumps

The function of the positive-displacement machine is deriving essentially from changes of the volume occupied by the fluid within the machine.

The reciprocating pumps are suitable for low rates of flow and high pressure.

2.1.1 Hydraulic Turbines

Are designed for service at hydroelectric power plants (HEPP) where they drive electric generators. In the turbine. the energy of water is converted into mechanical energy of the rotating shaft from which rotation is transferred to the rotor of the electric generator (hydraulic generator) where mechanical energy is converted into electric power that is supplied to consumers via high-voltage transmission lines.

In general there are two types of turbine

a) Impulse Turbines (Pelton turbine)

b) Reaction Turbines

1. Francis Turbine
2. Kaplain Turbines

2.1.2 Pumps

In general, there are two types

a) Centrifugal pumps:

All types of pumps depend on the change of momentum during the flow over the impeller across the vanes which called C.P.

The basic principle of the Centrifugal pump is the vanes or impellers rotating inside a closed fitting housing draw the liquid into the pump through a central inlet opening and by means of centrifugal force, or change in momentum, which means that changes the K.E head to pressure head.

b) Reciprocating pumps:

The basic operation principle of the pump is that it forces water above the atmospheric presser range, as distinguished from lift pump which elevates the water to flow from a spout.

2.2 Hydraulic Turbines and Hydroelectric Power Plant

1. Hydraulic Turbine: - classified as follows

- Impulse turbine (Pelton turbine)

- Reaction Turbines

-Radial flow, Francis Turbine

-Axial flow propeller (fixed blades) or Kaplian (variable pitch blades) turbines

- Reversible, pump- turbines

2. Hydroelectric power Plant:-

- Run –of the river, small amounts of storages little control of the flow through the plant.

- Storage, an artificial basin (created by a dam on a rive course) allows to store water and thus control the flow through the plant on a daily or seasonal basis.

-pumped storage, during off-peak hours water is pumped (by means of reversible pump- turbines or dedicated pump)from a lower reservoir to an upper reservoir, energy is thus stored for later production during peak hours Fig(1)

Gross head is the difference between hydraulic heads in the upstream and downstream reservoirs:

$$H_g = H_u - H_d =$$
$$z_u - z_d + \frac{p_u - p_d}{\rho_g} + \frac{c_u^2 - c_d^2}{2g}$$

Usually the only non-negligible contribution comes from the geodetic head:

$$H_g = z_u - z_d$$

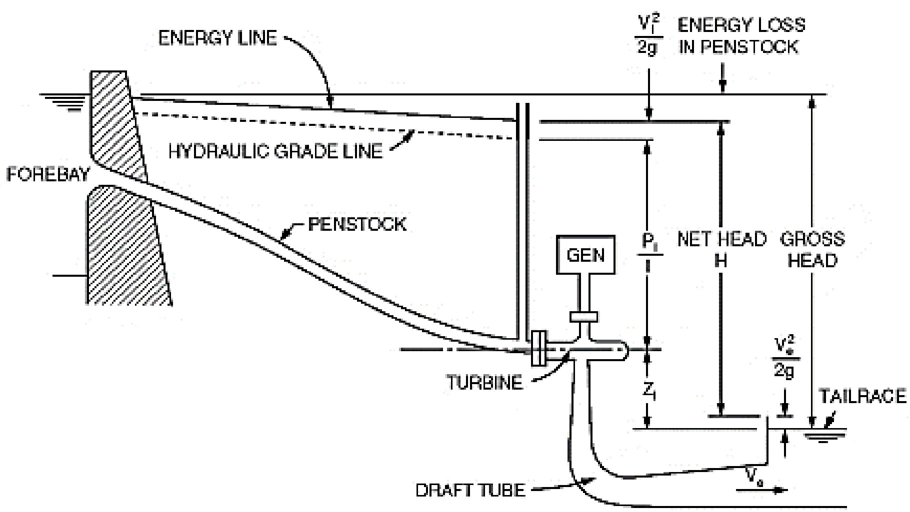
Net head is lower than gross head due to energy losses in the penstock:

$$H = H_g - Y$$

Penstock efficiency is the ratio of net and gross head:

$$\eta_p = \frac{H}{H_g} = 1 - \frac{Y}{H_g}$$

$$Where\ \ Y = \frac{{V_e}^2}{2_g} \qquad V_e = exit\ velocity\ of\ flow$$



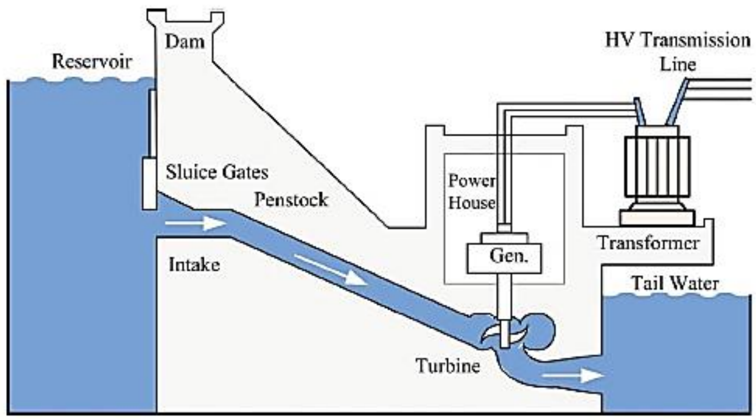


Fig. (1).

Chapter three

Impulse Turbine

3.1 Components of impulse turbine

It is a turbine, which runs by the impulse of water, e.g, (Pelton wheel). It is used for a high head of water (flow rate ≈ 0.5→20 m^3/s, Head ≈300→1500m, and net power up to≈ 200 MW) and has the following components.

1- Guide Mechanism: - it is controls the quantity of water passing through the nozzle and striking the buckets. It consists of the nozzle and the governor.

2. Buckets and Runner: - each bucket is divided vertically into two parts by a splitter which is a sharp edge at the centre giving the shape of double hemispherical cap.

3. Casting: - The casting of Pelton wheel has no hydraulic function to perform. It is necessary only to prevent splashing and to lead the water to the tail race, and also as a safeguard against accidents.

4. Hydraulic Brake: - the brake consists of small nozzle fitted in such a way that on bring opened it directs a jet on the back of the buckets to bring the revolving runner quickly to rest.

3.2 Theory of Pelton turbine

1. Turbine Power: -

Power input $P_a = \gamma Q H_T \quad kW \dots\dots\dots (1)$

$P_a =$ Power available in kW

H=net head acting at the turbine inlet (m)

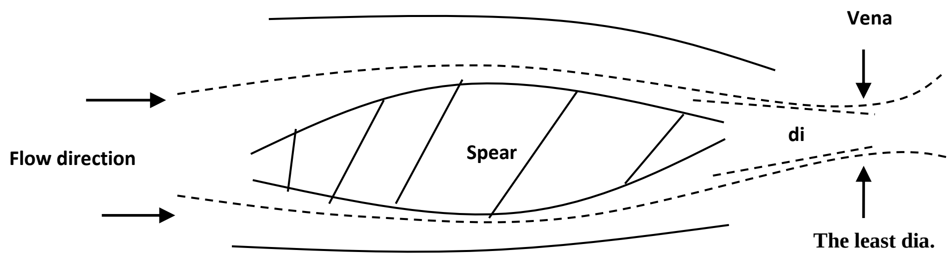
$P_t = P_a \eta_t \dots\dots\dots (2)$

$= Power\ supplied\ by\ the\ turbine\ in\ kW.$

2. Nozzle and Jet diameter

The least diameter of the jet which occurs at the Vena contraction which is given by the relation:-

$Q = \frac{\pi}{4} {d_1}^2 V_1 \dots \dots \dots (3)$



$d_i =$ The least diameter of the jet in (m)

$V_i =$ Velocity of the jet m/s

$Q =$ Discharge m^3/s

$$V_i = C_V \sqrt{2gH} \dots \dots \dots (4)$$

C_V=Velocity coefficient

$$\therefore d_i = \sqrt{\frac{4Q}{\pi C_V \sqrt{2gH}}} \dots \dots \dots (5)$$

3. Multi jets

The specific speed of Pelton turbine is calculated for a single jet and one runner (wheel).

i.e

$$N_s = \frac{N\sqrt{P_t}}{H^{\frac{5}{4}}} \dots \dots \dots (6)$$

If a single wheel two nozzle i.e two jets are provided.

$$Power = 2P_t \dots \dots \dots (7)$$

Maximum number of nozzle so far used in a turbine is <u>six</u> in vertical installation and <u>two</u> in a horizontal installation.

4. Mean diameter of Pelton wheel

$U = \emptyset\sqrt{2gH}$

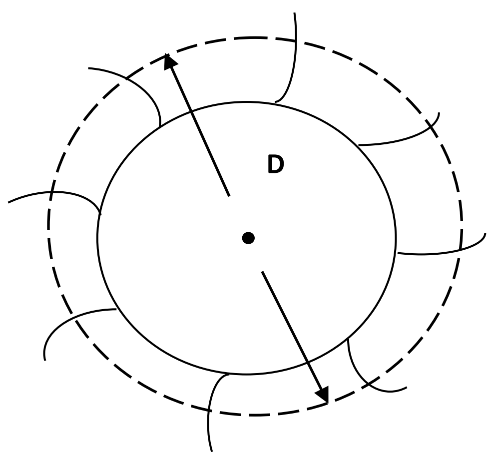
$\emptyset$ = Coefficient of speed ratio

In general $\emptyset \to 0.44 \to 0.46$

$$\text{i.e} \quad \emptyset = \frac{U}{\sqrt{2gH}} \qquad since\ U = \frac{\pi DN}{60}$$

$$\therefore \frac{\pi DN}{60} = \emptyset\sqrt{2gH}$$

$$\therefore D = \frac{60\emptyset\sqrt{2gH}}{\pi N} \ldots\ldots\ldots (8)$$

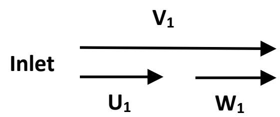


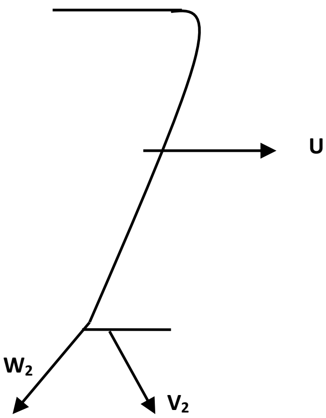
$$Jet\ ratio = \frac{mean\ dia.\ Of\ runner}{least\ dia.\ Of\ the\ jet}$$

$$i.e\ m = \frac{D}{d_i} \quad in\ general(11 \to 18)$$

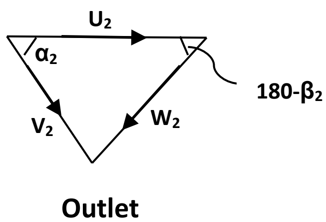
$$maximum\ efficiency\ \eta_{max}\ when \frac{d\eta_h}{du} = 0$$

5- Velocity diagram





For Pelton wheel $U_1 = U_2 = U$



Also, The relative vel. For open to atmosphere

$w_1 = w_2 = V_1 - U$

EX.1: The mean bucket speed of a Pelton turbine is 14 m/s. the rate of flow of water supplied by jet water at head of 45 m is 800 lit/s. if the jet is deflected by the bucket at an angle of 165°. Find the power and efficiency of the turbine.

Assume $C_V = 0.985$

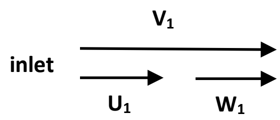
Solution:-

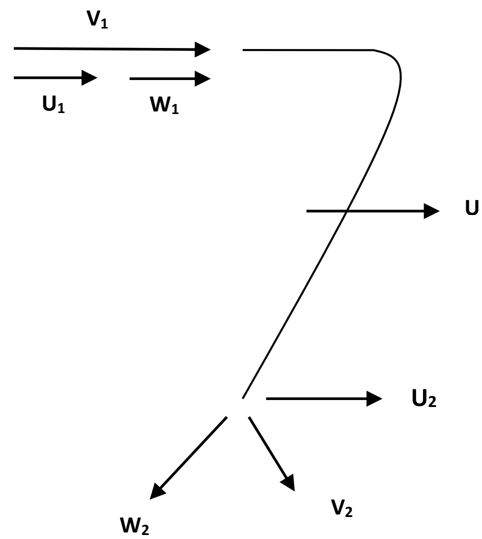
$$\alpha_1 = 0 \ ; \ U_1 = U_2 = 14\frac{m}{s} \ ; Q = 800\frac{l}{s}$$

$$H = 45\,m \ ; \ \beta_2 = 165° \ ; C_V = 0.985$$

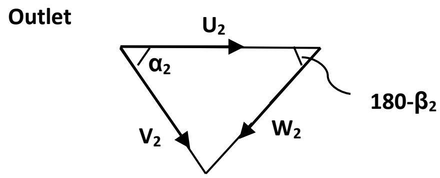
$$V_1 = C_V\sqrt{2gH} = 0.985\sqrt{2 \times 9.81 \times 45}$$

$$= 29.2\ m/s$$





$$Pelton\,wheel \begin{cases} U_1 = U_2 = U \\ w_1 = w_2 \end{cases}$$



From the inlet velocity triangle

$$V_{u1} = V_1 \cos\alpha_1 = U_1 + w_1 \cos\beta_1$$

$$\therefore V_1 = U_1 + w_1 \qquad \alpha_1 = 0 \qquad \beta_1 = 0$$

$$\therefore w_1 = V_1 - U_1 = 29.4 - 14 = 15.2\, m/s = w_2$$

From outlet velocity triangle

$$w_2 = 15.2\, m/s \quad U = 14\, m/s$$

$$\therefore V_{u2} = V_2 \cos\alpha_2 = U + w_2 \cos(180 - \beta_2)$$

$$= 14 - 15.2 \cos 15°$$

$$= -0.7$$

$$\therefore power\ P_t = F.U = \rho\, Q(V_{u1} - V_{u2})U$$

$$= 1000 \times 0.8(V_1 \cos\alpha_1 - V_2 \cos\alpha_2)U$$

$$= 1000 \times 0.8(29.2 + 0.7)14$$

$$= 334.88\, kW$$

$$\eta_t = \frac{P_{out}}{P_{in}} = \frac{334.88}{\gamma QH} = \frac{334.88}{9.81 \times 0.8 \times 45}$$

$$= 94.8\%$$

To find α_2:

$$\tan(180 - \beta_2) = \frac{V_2 \sin \alpha_2}{U - V_2 \cos \alpha_2}$$

$$\tan 15 = \frac{V_2 \sin \alpha_2}{14 + 0.7}$$

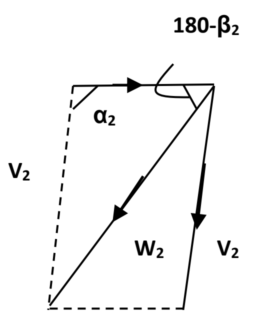
$$V_{f2} = V_2 \sin \alpha_2 = 3.94$$

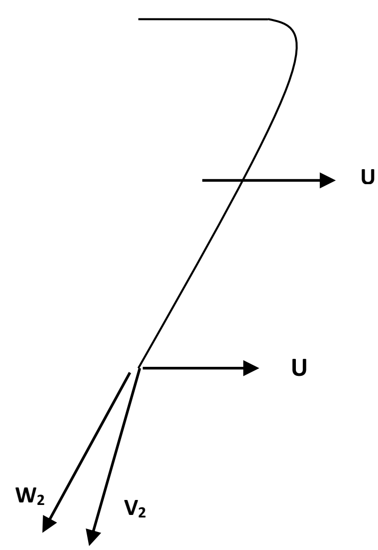
$$\therefore \tan \alpha_2 = \frac{V_2 \sin \alpha_2}{V_2 \cos \alpha_2} = \frac{3.94}{-0.7}$$

$$\alpha_2 \approx -80°$$

$$or \;\; \alpha_2 = 180 - 80 = 100°$$

This means the velocity triangle as follows at outlet





Another solution

$F = \rho Q(\overrightarrow{w_1} - \overrightarrow{w_2})$

$P_t = F.U = 1000 \times 0.8(w_1 \cos\beta_1 - w_2 \cos\beta_2)U$

$w_1 = w_2 = V_1 - U = 29.2 - 14 = 15.2\, m/s$

$\therefore P_t = 1000 \times 0.8 \times 15.2\big(1 - (-0.966)\big)14$

$= 334.8\, kW$

$$\eta_t = \frac{P_{out}}{P_{in}} = 94.8\%$$

EX.2: A single jet Pelton turbine is required to drive a generator to develop 10000 kW- The available head at the nozzle is 760 m. Assuming electric generator efficiency 95%, Pelton wheel efficiency 87%, coefficient of velocity for nozzle 0.97, mean bucket velocity 0.46 of jet velocity, outlet angle of the bucket 165° and the relative velocity of the water leaving the bucket 0.85 of that at inlet find:

a) diameter of the jet

b) the force exerted by the jet on the bucket.

If the ratio of the mean bucket circle diameter to the jet diameter is not to be less than 10, find the speed?

<u>Solution:-</u>

$$P_t = 10000\, kW \;\; ; H = 760\, m \; ; \eta_g = 0.95$$

$$C_V = 0.97 \; ; \emptyset = 0.46 \; ; \eta_t = 0.87$$

$$\beta_1 = 0 \; ; \alpha_1 = 0 \; ; w_2 = 0.85 w_1 \;\; ; m = 10$$

$$\text{Output of the turbine } P_t = \frac{P_g}{\eta_g} = \frac{10000}{0.95} = 10526\, kW$$

$$\eta_t = \frac{P_{out}}{P_{in}} = \frac{10526}{9.81 \times Q \times 760} = 0.87$$

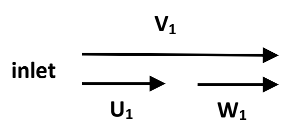
$$Q = 1.62\ m^3/s$$

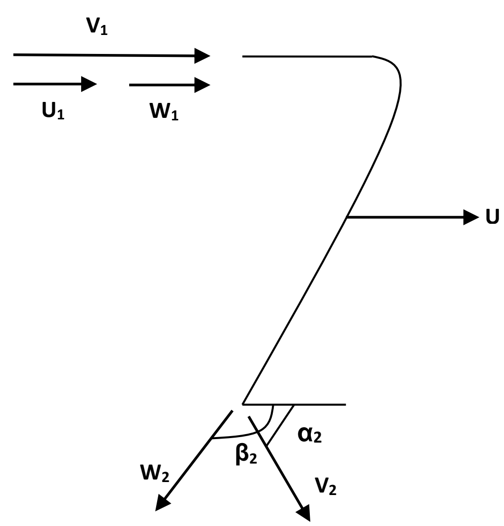
$$\therefore d_i = \sqrt{\frac{4Q}{\pi \times C_V\sqrt{2gH}}} = \sqrt{\frac{4 \times 1.62}{\pi \times 0.97\sqrt{2 \times 9.81 \times 760}}}$$

$$= 0.132\ m$$

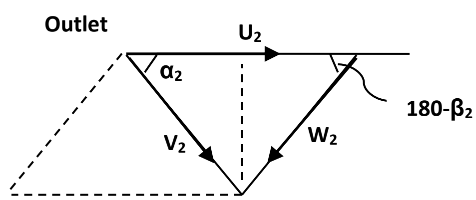
Force exerted by the jet

$F = \rho Q (V_{u1} - V_{u2})$





$U_1 = U_2 = U$
$w_1 = w_2$



$$V_1 = C_V\sqrt{2gH} = 0.97\sqrt{2 \times 9.81 \times 760} = 118.5\ m/s$$

$$U_1 = \emptyset = \sqrt{2gH} = 0.46\sqrt{2 \times 9.81 \times 760} = 54.5\ m/s$$

From velocity triangle at inlet $\alpha_1 = 0 \qquad \beta_1 = 0$

$$w_1 = V_1 - U = 118.5 - 54.5 = 64\ m/s$$

$$w_2 = 0.85 w_1 = 0.85 \times 64 = 54.6\ m/s$$

$$and \quad V_{u1} = V_1 \cos\alpha_1 = 118.5\ m/s$$

From outlet velocity triangle :

$$V_{u2} = U_2 - w_2 \cos(180 - \beta_2)$$

$$= 54.5 - 54.6\, cos15$$

$$= 1.83\ m/s$$

$$\therefore F = 1000 \times 1.629118.5 - 1.83)$$

$$= 189\ kN$$

$$m = \frac{D_1}{d_1} = 10 \qquad \therefore D_1 = 1.32\ m$$

$$U = \frac{\pi D_1 N}{60} \qquad or\ \ N = \frac{60 \times 54.5}{\pi \times 1.32}$$

$$N = 787\ rpm$$

3.3 Power Regulation Mechanisms

The rate of flow through the Pelton turbine and turbine power are regulated by shifting the needles changing thereby, the flow area of the nozzles. Formerly, use was made, as a rule, of the needle- operating gear shown schematically in Fig(1). Needle 2 is fixed on the end of the long rod 3 connected to the piston of hydraulic servomotor 4. The needle is aligned and its radial displacement is prevented by a guide sleeve and spacer ribe 5. This design requires a rather sharp turn of the water supply conduite directly upstream of nozzle 1, and this, due to the origination of revers flows at the turn, diminishes the density of the jet at the nozzle outlet and impairs the power-generating charaeteristics of the turbine.

In the last few decades use has been made of the so-called straigh-flow nozzle I , Fig (1.a). Needle 2 is fasted on short rod 3 connected to the piston of servomotor 4. The servomotor is made in the form of a buld secure in the center of a tube by ribs 5. The buld is exposed to water flow and this make it unneccessary to turn water supply conduit Fig(2).

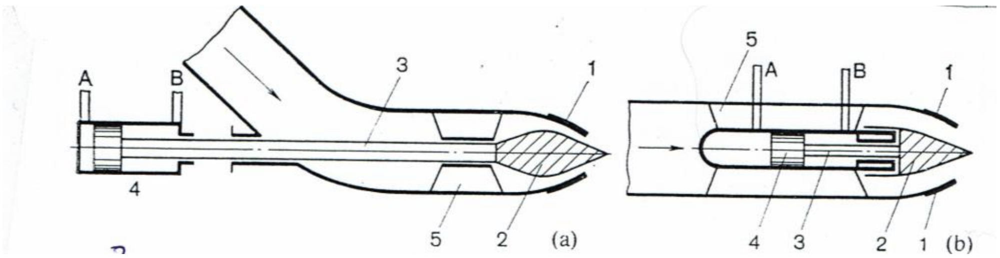


Fig. (1) Schematic diagram of Pelton turbine requlating needle operating gear.

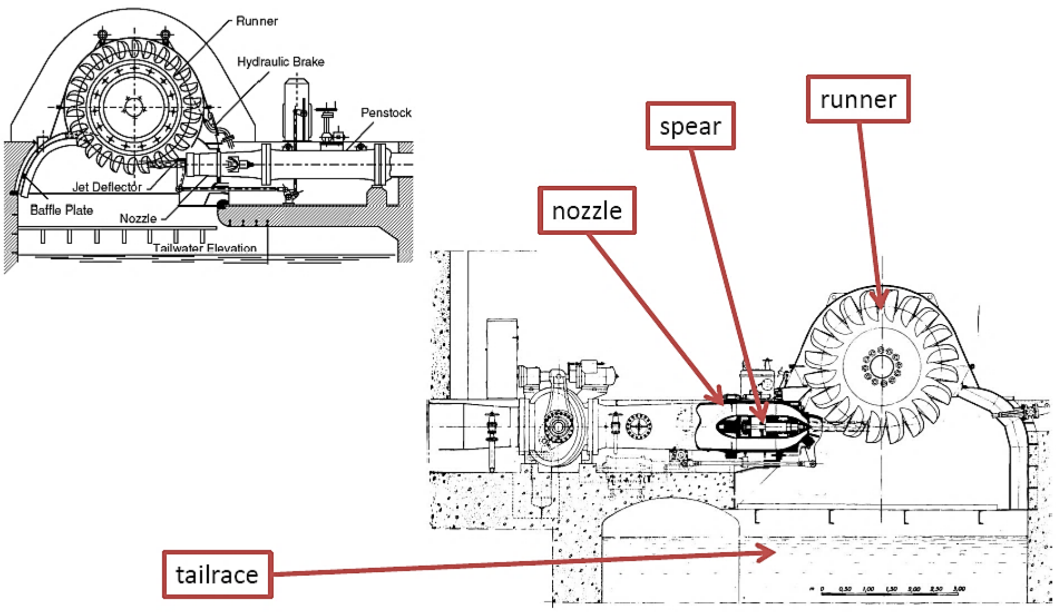


Fig. (2) Horizontal axis 1-jet turbine.

3.4 Supply and discharge system

The water is brough in the penstock ending in a single or multi nozzle. The whole pressure energy of water is the from of a rree jet is made to strike on a series of buckets mounted on the periphery of a wheel. Therefore the sasing of an impulse turbine has no hydraulic function to perform. It is necessary only to prevent splashing and lead the water to the tail race, and also act as a safeguard against accident the shape and size of turbine case and the penstek must be agreement with loyout of the house of the HEPP. Fig(3).

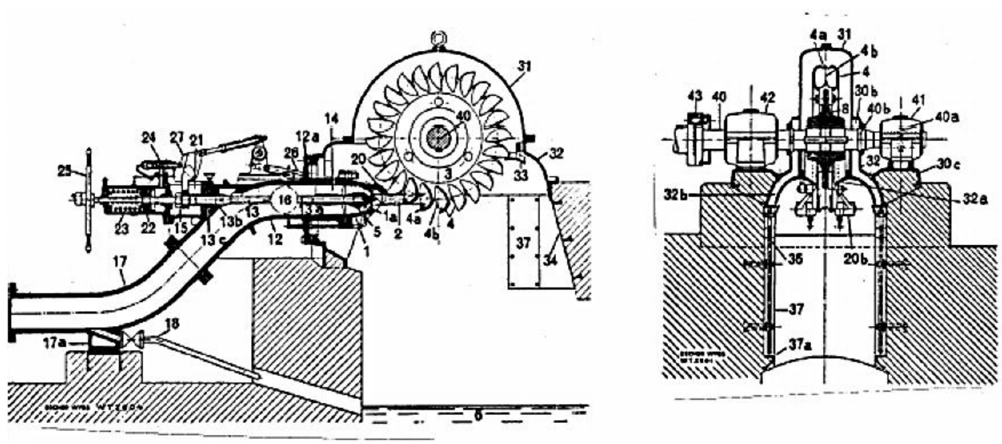


Fig. (3) Horizontal Arrangement of a Pelton Turbine.

Horizontal arrangement is found only in medium and small sized turbines with usually one or two jets. In some designs, up to four jets have been used. The flow passes through the inlet bend to the nozzle outlet, where it flows out as a compact through atmospheric air on to the heel buckets. From the outlet of the buckets the water falls through the pit down into the tail water canal.

Part of Pelton Turbine and different types of jet distribution shown in Fig. (4, 5, 6, 7)

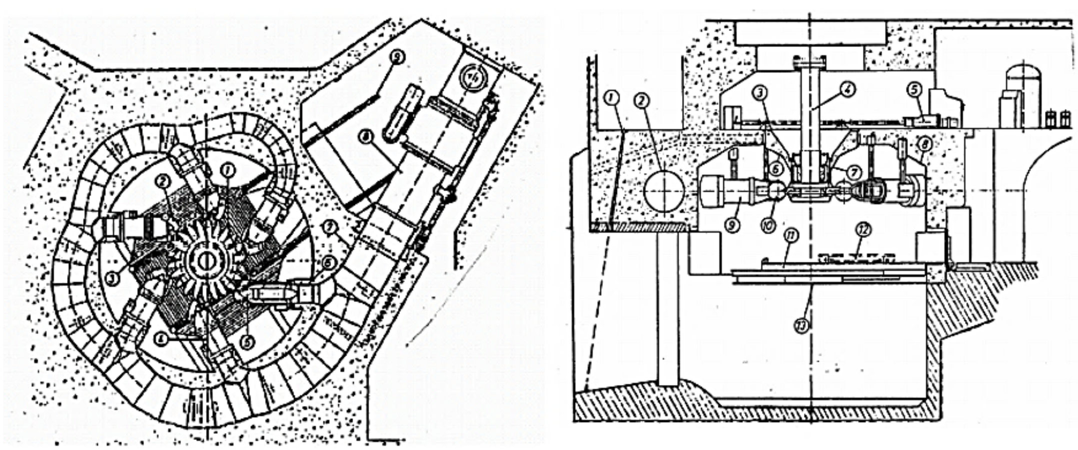


Fig. (4) Vertical Arrangement of a Pelton Turbine.

Large Pelton turbines with many jets are normally arranged with vertical shaft. The jets are symmetrically distributed around the runner to balance the jet forces. The figure shows the vertical and horizontal sections of the arrangement of a six jet vertical Pelton turbine.

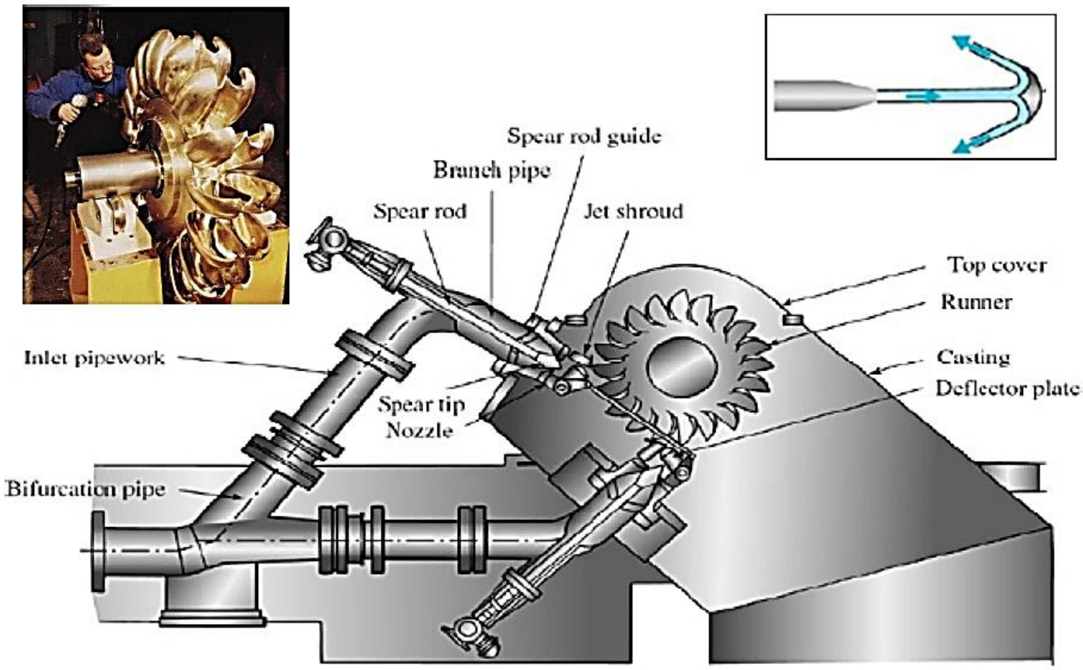
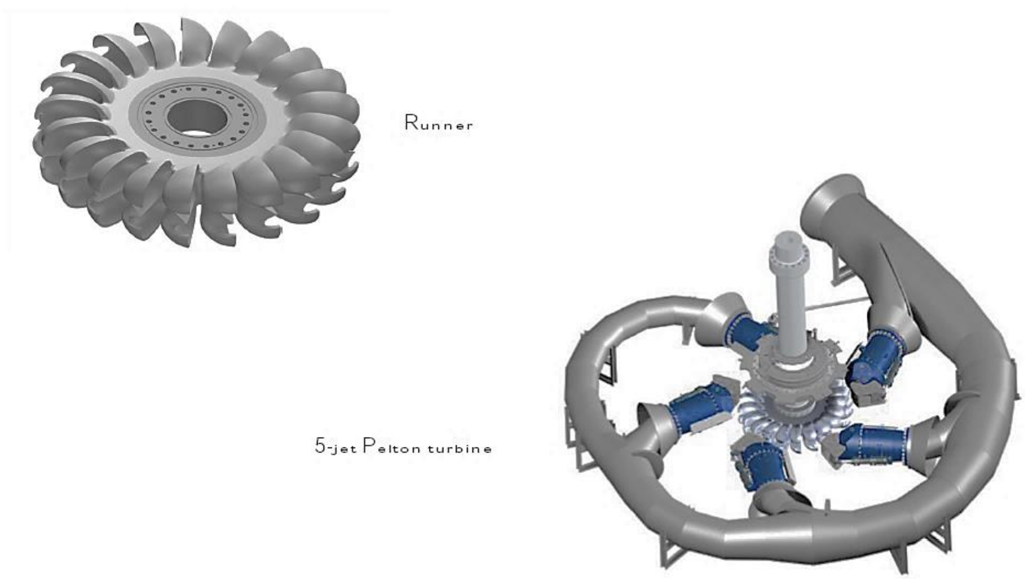


Fig. (5) Parts of a Pelton Turbine.



Source: Voith-Siemens

Fig. (6) Pelton Turbine Components.

Fig. (7) Pelton Turbine 5-jet turbine.

Solved Problems

Q.1: A power Hose is equipped with impulse turbine of pelton type. Each turbine delivers a maximum power of 14.2 MW when working under a head of 855 m and running at 600 rpm. Find the least diameter of the jet and the mean diameter of the wheel. What would be the approximate diameter of orifice of the nozzle tip? Determine the value of the jet ratio and state if it is within the limits. Specify the number of buckets for the wheel. Take the overall efficiency of the turbine as 89.2% and coefficient of speed ratio Φ=0.45. And velocity coefficient C_V=0.988

Solution:-

$$P_t = 14.2\, MW \;\; ; H = 855\, m$$

$$N = 600\, rpm \;\; ; \eta_t = 0.892$$

a) Least diameter of the jet d_1:

$$P_t = \gamma Q H \eta_t$$

$$14200 = 9.81 \times 855 \times 0.892$$

$$Q = 1.895\ m^3/s$$

Further

$$Q = a_1 V_1 = \frac{\pi}{4} {d_1}^2 . C_V \sqrt{2gH}$$

$$1.895 = \frac{\pi}{4} {d_1}^2 \times 0.988\sqrt{2 \times 9.81 \times 855}$$

$$d_1 = 0.1372\ m = 137.2\ mm$$

b) Mean diameter of wheel D_1

$$U_1 = \emptyset\sqrt{2gH} = \frac{\pi D_1 N}{60}$$

$$D_1 = \frac{60 \times 0.45 \times \sqrt{2 \times 9.81 \times 855}}{\pi \times 600}$$

$$D_1 = 1.85\ m \quad or\ 1850\ mm$$

c) Diameter of the nozzle $d_0 = 1.25 d_1 = 1.25 \times 13.72$

$$= 17.2\ cm$$

d) Jet ratio $m = \frac{D_1}{d_1} = \frac{1.85}{0.1372} = 13.5$

Q.2: A double overhung Pelton turbine unit is to operate at 30 MW generator under an effective head of 300 m at the base of the nozzle. Find the size of jet mean diameter of runner, synchronous speed and specific speed of each wheel. Assume generator efficiency 93%, Pelton wheel efficiency 85%, coefficient of nozzle velocity 0.97, speed ratio 0.46 and jet ratio 12.7

Solution:-

There are two runner keyed on the two ends of the shaft, and the generator lies between them each runner it to be taken as one complete turbine. Thus the generator is fed by two Pelton turbines

$\therefore$ The power developed by each turbine:

$$P_t = \frac{30000}{2 \times 0.93} = 16130\ kW$$

Available power of each turbine

$$P_a = \frac{16130}{0.85} = 18975\ kW$$

$$= \gamma Q H_n \qquad \therefore Q = \frac{18975}{9.81 \times 30}$$

$$Q = 6.47\ m^3/s$$

Velocity of the jet $\quad V_1 = C_V\sqrt{2gH} = 0.97\sqrt{2 \times 9.81 \times 300}$

$$V_1 = 74.3\ m/s$$

$$and\ \ Q = a_1 V_1 = \frac{\pi}{4} {d_1}^2 \times 74.3$$

$$\therefore\ d_1 = 0.333\ m$$

Jet ratio $\quad m = \frac{D_1}{d_1} = 12 \qquad \therefore\ \ D_1 = 12 \times 0.333 = 4\ m$

$\therefore U_1 = \emptyset\sqrt{2gH} = \frac{\pi D_1 N}{60}$ Peripheral velocity of the wheel

$$\therefore U_1 = 0.46 \times \sqrt{2 \times 9.81 \times 300} = \frac{\pi D_1 N}{60} = 35.3$$

$$N = \frac{60 \times 35.3}{\pi \times 4} = 168.5\ rpm$$

$$f = \frac{P.N}{60}\ \ where\ P\ No.of\ Poles = 18\ (assume)$$

$$\therefore N_{syn} = \frac{60 \times f}{P} = \frac{3000}{18} = 166.7\ rpm$$

Revised Diameter of the wheel

$$D_1 = \frac{168.5 \times 4}{166.7} = 4.05\ m$$

Specific speed $\quad N_s = \frac{N\sqrt{P_t}}{H^{\frac{5}{4}}} = \frac{166.7\sqrt{16130}}{(300)^{\frac{5}{4}}}$

$$= N_s = 17\ m - kWunit$$

Q.3: A Pelton wheel is to be designed to develop 750 kW at 400 rpm. It is to be supplied with water from a reservoir whose level is 250 m above the wheel through a pipe 900 m long. The pipe line losses are to be 5% of gross head. The coefficient of friction is 0.02. The bucket speed 0.46 of the jet speed and efficiency of wheel is 85%. Calculate the pipe line diameter, jet diameter and wheel diameter, C_V=0.985(pipe=440 mm, jet=84 mm, wheel=1.5 m)

<u>Solution:-</u>

$$P_t = 750\, kW \;\; ; N = 100\, rpm \;\; ; H_{gress} = 250\, m$$

$$L = 900\, m \;\; ; \; h_f = 5\%\; of\;\; gress\; head \; ; f = 0.02$$

$$\emptyset = 0.46 \;\; ; \; \eta_t = 0.85 \;\; ; \; C_V = 0.985$$

$$a) \qquad h_f = f.\frac{L}{D}.\frac{V^2}{2g} = \frac{8fLQ^2}{\pi^2 g D^5} = 0.05 \times 250 = 12.5\, m$$

$$but\;\; Q = \frac{P_t}{\gamma H \eta_t} = \frac{750}{9.81(0.95 \times 250) \times 0.85}$$

$$= 0.379\, m^3/s$$

$$\therefore 12.5 = \frac{8 \times 0.02 \times 900 \times (0.379)^2}{\pi^2 \times 9.81 \times D^5}$$

$$\therefore D = 0.44 \quad or\; 440\, mm$$

b) Jet ratio

$$d_1 = \sqrt{\frac{Q}{\frac{\pi}{4}(C_V\sqrt{2gH})}}$$

$$d_1 = \sqrt{\frac{0.379}{\frac{\pi}{4} 0.985\sqrt{2 \times 9.81 \times 0.95 \times 250}}} = 0.0847\, m \; = 84.7\, mm$$

c) Wheel diameter D_1:

$$U_1 = \emptyset\sqrt{2gH}$$

$$= 0.46\sqrt{2 \times 9.81 \times 237.5}$$

$$= 31.4\, m/s$$

$$and\ \ U_1 = \frac{\pi D_1 N}{60}$$

$$\therefore\ D_1 = \frac{60 \times 31.4}{\pi \times 400} = 1.5\,m$$

Q.4: Determine the discharge, least jet diameter, mean runner diameter jet ratio and number of buckets of the following Pelton turbine:

Power =127 kW   Head =300 m   speed =600 rpm

Assume the following constants

C_V=0.98   Φ =0.45 η_t =0.75

(57.3 l/s , 31 mm , 1100 mm , 35.3 , 33)

<u>Solution:-</u>

$$P_t = \gamma Q H \eta_t$$

a)Discharge $\quad Q = \frac{P_t}{\gamma H \eta_t} = \frac{127}{9.81 \times 300 \times 0.75} = 0.0575\,m^3/s$

b) $\quad Q = \frac{\pi}{4} d_1{}^2.\left(C_V\sqrt{2gH}\right)\ \ where\ \ V_1 = \left(C_V\sqrt{2gH}\right)$

$$\therefore d_1 = \sqrt{\frac{0.0575}{\frac{\pi}{4} \times 0.98 \times \sqrt{2 \times 9.81 \times 300}}} = 0.0312\,m\ = 31.2\,mm$$

c) $\quad U_1 = \frac{\pi D_1 N}{60} = \emptyset\sqrt{2gH}$

$$\therefore\ \frac{\pi D_1 \times 600}{60} = 0.45\sqrt{2 \times 9.81 \times 300}$$

$$D_1 = 1.1\,m$$

d) Jet ratio $\quad m = \frac{D_1}{d_1} = \frac{1.1}{0.0312} = 35.3$

e) Bucket Number $Z = 0.5 \times m + 15 = 0.5 \times 35.3 + 15 = 33$

Q.5: Show that in Pelton wheel, where the bucket deflect the water through an angle of (180^0- β_2) the hydraulic efficiency of the wheel is given by

$$\eta_h = \frac{2U(V-U)(1+\cos\theta)}{V^2}$$

Where V is the velocity of jet and U the velocity of the wheel at the pitch radius, If the bucket speed is 30 m/s, β_2=160° and the jet diameter is 12.5 cm, find the jet velocity for maximum efficiency and the corresponding water power in kW of the turbine (60 m/s , 1325 kW)

<u>Solution:-</u>

$\alpha_1 = 0 \;;\; \beta_1 = 0$

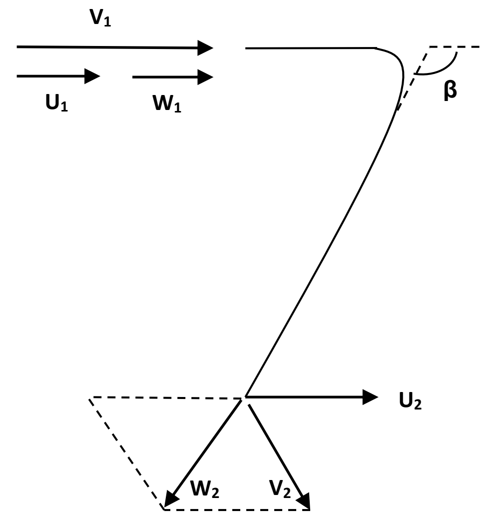
For Pelton turbine

$U_1 = U_2$

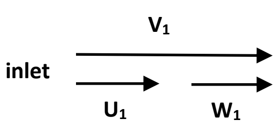
$W_1 = W_2$

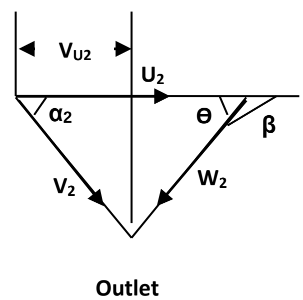
$and\;\; W_1 = V_1 - U_1$

$V_{u1} = V_1 cos\alpha_1 = V_1$



Velocity diagram





$V_{u2} = U_2 cos\theta$

$= U_1 - (V_1 - U_1)cos\theta$

Hydraulic power $P_h = \rho\, Q(V_{u1} - V_{u2})U_1$

$$= \rho\, Q(V_1 - (U_1 - (V_1 - U_1)cos\theta)U_1$$

$$= \rho\, Q((V_1 - U_1) + (V_1 - U_1)cos\theta)U_1$$

$$P_h = \rho\, Q(V_1 - U_1)(1 + cos\theta)U_1 \ldots\ldots\ldots (1)$$

Available power $P_a = \gamma QH = \rho\, gQ.\dfrac{{V_1}^2}{2g} = \rho\, Q\dfrac{{V_1}^2}{2} \ldots\ldots\ldots (2)$

$$\eta_h = \frac{P_h}{P_a} = \frac{\rho Q(V_1 - U_1)(1 + cos\theta)U_1}{\rho Q \frac{{V_1}^2}{2}}$$

$$\eta_h = \frac{2U(V - U)(1 + cos\theta)}{V^2}$$

b) for maximum efficiency $\frac{d\eta_h}{du} = 0 \quad \therefore \frac{V_1}{U_1} = 2$

$$\therefore\ V_1 = 2U_1 = 2 \times 30 = 60\ m/s$$

c) Available power $P_a = \gamma QH = \gamma Q.\frac{{V_1}^2}{2g}$

$$Q = \frac{\pi}{4}{d_1}^2{V_1}^2 = \frac{\pi}{4}(0.125)^2 \times 60 = 0.736\ m^3/s$$

$$\therefore\ P_a = \rho Q\frac{{V_1}^2}{2} = 1 \times 0.736 \times \frac{60^2}{2} = 1325\ kW$$

Q.6: A Pelton wheel supplied with water in the nozzle box at a pressure head of 150 m. The velocity coefficient for the nozzle is 0.96. The relative velocity of water while in contact with the bucket is turbine through 150^o, and is also reduced by 15% owing to friction losses. The required ratio of bucket to jet speed is 0.47 and 0.8 of the energy imparted to the wheel as determined by the velocity diagram, is available at outlet shaft. The bucket circle diameter is ten times the jet diameter. Find the jet and wheel diameters, and the speed of rotation to develop 150 kW under these conditions.

(62 mm, 620 mm, 754 rpm)

Solution:-

$$H = 150\ m\ ;\ C_V = 0.96\ ;\ \beta_2 = 150°\ ;\ w_2 = 0.85w_1$$

$$\frac{U}{V} = 0.47 \quad or \quad \emptyset = 0.96 \times 0.47 = 0.45$$

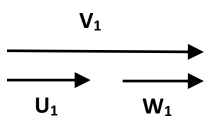
$$\eta_{mech} = 0.8\ ;\ m = \frac{D_1}{d_1} = 10\ ;\ P_t = 150\ kW$$

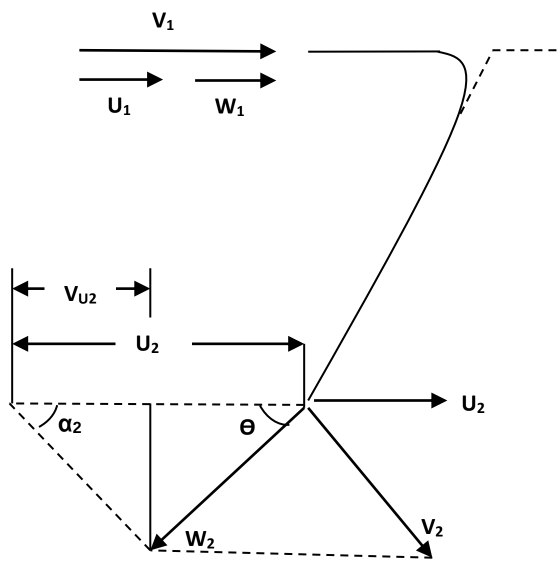
a) jet dia d_1 $Q = a_1 V_1 = \frac{\pi}{4} {d_1}^2 V_1$

$$V_1 = C_V \sqrt{2gH} = 0.96\sqrt{2 \times 9.81 \times 150} = 52.1\, m/s$$

$$\therefore U_1 = 0.47 \times 52.1 = 24.5\, m/s = U_2$$

For velocity diagram





$V_{u1} = V_1 cos\alpha_1 = V_1$

$\alpha_1 = 0$

$w_1 = V_1 - U_1 = 27.6\, m/s$

$V_{u2} = U_2 - W_2 cos\theta$

$w_2 = 0.85 w_1$

$= 0.85(V_1 - U_1)$

$= 0.85(52.1 - 24.6)$

$= 23.46\, m/s$

$\therefore V_{u2} = 24.5 - 23.46 \cos 30 = 4.2\, m/s$

Hydraulic power $P_h = \rho Q (V_{u1} - V_{u2}) U_1$

$$= 1000 \times Q(52.1 - 4.2)24.5$$

$$= 1173.55\, Q\, kW$$

$$since\ \eta_{mech} = 0.8 \quad \therefore P_t = 0.8\, P_h = 938.84\, Q\, kW$$

$$or\ \ 150 = 938.34\, Q$$

$$or\ Q = 0.16\, m^3/s$$

$$\therefore\ d_1 = \sqrt{\frac{0.16}{\frac{\pi}{4}(52.1)}} = 0.062\, m = 62\, mm$$

b) $D_1 = 10\, d_1 = 620\, mm$

c) $U_1 = \frac{\pi D_1 N}{60} = 24.5$

$$\therefore N = \frac{60 \times 24.5}{\pi \times 0.62}$$

$$N = 755\, rpm$$

Q.7: A Pelton turbine has a mean bucket speed of 40 m/s and is supplied with water at the rate of 0.55 m^3/s, the head of water behind the nozzle being 400 m. if the jet is deflected by head of water through 170°, find the power developed and the efficiency of the wheel. Assume β_1=0.

(2064 kW, 95.6%).

Solution:-

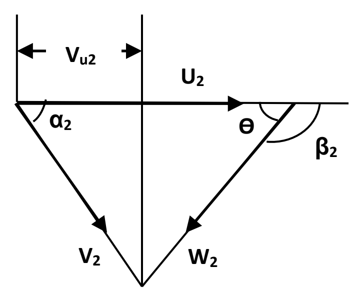
$$U_1 = U_2 = 40\, m/s \quad ; Q = 0.55\, m^3/s$$

$$H = 400\, m \ ; \ \beta_2 = 170° \ ; C_V = 0.985$$

$$V_1 = C_V\sqrt{2gH} = 0.985\sqrt{2 \times 9.81 \times 400} = 87.27\, m/s$$

$$and \ \ U_1 = U_2 \quad ; \quad w_1 = w_2$$

$V_{u1} = V_1 cos\alpha_1 = V_1 \qquad \alpha_1 = 0$



And from velocity triangle

$V_{u2} = U_2 - W_2 \cos\theta$

$= 40 - (87.27 - 40)\cos 10$

$= -6.55\, m/s$

Power developed by the wheel $P_h = \rho\, Q(V_{u1} - V_{u2})U_1$

$$= 1000 \times 0.55(87.27 + 6.55) \times 40$$

$$= 2064\, kW$$

Available power $P_a = \gamma QH = 9.81 \times 0.55 \times 400 = 2158\, kW$

$$\eta_h = \frac{P_h}{P_a} = \frac{2064}{2158} = 95.6\%$$

Q.8: A Pelton turbine has a mean speed of 12.2 m/s and supplied with water at the rate of 1370 l/s under ahead of 30.5m. If the bucket deflects the jet through an angle of 160°. Find the outlet power and the efficiency of the wheel. C_V=0.985, α_1=0. (386 kW, 94.1%)

Note number of bucket Z=0.5 m+15

Solution:-

$$U_1 = U_2 = 12.2\ m/s \quad ; Q = 1.37\ m^3/s$$

$$H = 30.5\ m\ ;\ \beta_2 = 160°\ ; \alpha_1 = 0\ ;\ C_V = 0.985$$

$W_1 = W_2 = V_1 - U_1$

$and\ \ V_1 = 0.985\sqrt{2 \times 9.81 \times 30.5}$

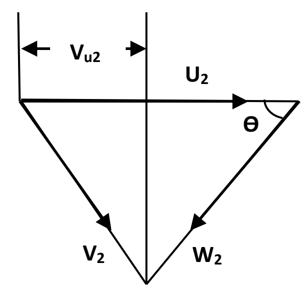
$V_1 = 24.1\ m/s \quad and\ \ W_1 = 11.9\ m/s$

$also\ \ V_{u1} = V_1 cos\alpha_1 = 24.1$

$V_{u2} = U_2 - W_2 \cos\theta$

$= 12.2 - 11.9 \cos 20$

$= 1\ m/s$



$$\therefore \text{Available power}\ \ P_a = \gamma Q H$$

$$= 9.81 \times 1.37 \times 30.5 = 410\ kW$$

$$\eta_h = \frac{386}{410} = 94.1\%$$

Chapter four

Reaction turbines

4.1 Type of Reaction Turbine

In general, there are two type of reaction turbine, Francis and Kaplan turbines according to the direction of flow. The water enters the runner under pressure and flows over the vanes, the pressure head of water while flowing over the vanes, is converted into velocity head and finally reduced to the atmospheric pressure.

1. Francis turbine: - it is an inward flow reaction turbine having radial discharge at outlet. It is operating under medium head and required medium quantity of water

Flow rate ≈ 2 → 800 m^3/s

Head ≈ 50 → 400 m

Net power up to ≈ 800 MW

2. Kaplan turbine: - it is an axial flow reaction turbine in which the flow of water is parallel to shaft. A Kaplan turbine used where a large quantity of water is available at low head.

Flow rate up to ≈ 1000 m^3/s

Head ≈ 40 m

Net power up to ≈ 200 MW

All parts of Kaplan turbine such as spiral casing, guide mechanism and draft tube are similar as in Francis turbine system except,

The runner: the runner has two major differences. In Francis runner, the water enters radially while in Kaplan type it strikes the blades axially. Number of vanes in Francis turbines is 16 → 24 while in Kaplan it is only

3→ 6 vanes or at most 8 in exceptional case. Thee RPM more than twice than that of Francis turbine

Kaplan turbines have taken the place of Francis turbine for certain medium head installation.

4.2 Construction of Reaction Turbine

The turbine systems have the following component: -

1. Penstock: it is a waterway to carry water from the reservoir to the turbine casing. The Penstock section were manufactured in quarters and welded at the site (e.g. the penstock of Hydropower station in Venezuela is 7.5 m in diameters).
2. Spiral casing or scroll casing: - to avoid losses of efficiency, the scroll casing is designed with a cross-sectional area reducing uniformly around the circumference, maximum at the entrance and nearly zero at the tip. This gives a spiral shape and hence the casing named as spiral casing.
3. Guide mechanisms: - the Guide vanes or wicket gates, are fixed between two rings known as guide wheel. The guide vane can be closed or opened to allowing a variable quantity of water according to the needs. The guide mechanism parts are: - Guide vanes, Guide wheel, regulating shaft, and Governor.
4. Runner and turbine main shaft: - the aim of the runner is to guide the flow inside the turbine. The width of the runner depends upon the specific speed. The runner may be classified as (i) slow (ii) medium (iii) fast, depending upon the specific speed. The runner is keyed to the shaft which may be vertical or horizontal.

5. Draft tube: - the water after passing through the runner flows down through a tube called draft tube. It is generally drowned a proximately 1 m below the tail rase level. The advantage of it:

a) It increases the head of water by an amount equal to the height of the runner outlet above the tail race.

b) It increases the efficiency of the turbine.

4.3 Theory of Reaction Turbine

From momentum equation,

$$\sum F = \dot{m}\left(\vec{V}_{out} - \vec{V}_{in}\right)$$

And for the turbine the force exerted the fluid on the runner (reaction force according to Newton 3rd law)

$$-F = R$$

$$\therefore R = \rho Q\left(\vec{V}_{in} - \vec{V}_{out}\right)$$

$$or \;\; F = \rho Q(V_{u1} - V_{u2})$$

Where $V_u = V\cos\alpha$ the whirl velocity of flow.

$\therefore \;\; Power \;\; P_t = \rho Q(V_{u1}U_1 - V_{u2}U_2)$

Where $\quad U_1 \neq U_2 \qquad U = wr \qquad w = \frac{\pi DN}{60}$

The power developed by the turbine or may be named hydraulic power then:

For a given rate of flow the minimum value of V_2 occurs when V_2 perpendicular to the tangential velocity U_2

$$i.e \;\; V_{u2} = 0 \quad where \; \alpha_2 = 90°$$

$$\therefore P_t = \rho Q V_{u1} U_1 = P_H \quad hydraulic\ power.$$

$$the\ Work\ done/\sec \quad per \;\; kN = \frac{V_{u1}U_1}{g}$$

4.4 Efficiency of Reaction Turbines

1. Head efficiency $\eta_H = \frac{H-\Delta H}{H}$

H: operating head (net head)

ΔH: losses (friction, kinetic head ($\frac{V^2}{2g}$) ….. etc

Since

$$P_H = P_a \eta_H = \gamma Q H \eta_H$$

Or

$$\rho\, Q(V_{u1}U_1 - V_{u2}U_2) = \gamma Q H \eta_H$$

$$\therefore \quad \eta_H = \frac{V_{u1}U_1 - V_{u2}U_2}{gH} \quad (general)$$

Or

$\eta_H = \frac{V_{u1}U_1}{gH}$ For radial flow at outlet $\alpha_2 = 90°$

2. Hydraulic Efficiency

Total hydraulic loss in turbine made up of total head loss and Q leakage.

i.e

$$P_h = \gamma(Q - \Delta Q)(H - \Delta H)$$

$$P_a = \gamma Q H$$

$$\therefore \quad \eta_h = \frac{\gamma(Q - \Delta Q)(H - \Delta H)}{\gamma Q H} = \eta_Q \eta_h$$

3. Volumetric Efficiency

$$\eta_Q = \frac{Q - \Delta Q}{Q}$$

If the volumetric losses neglected

Then $\eta_H = \eta_h = \frac{V_{u1}U_1}{gH}$ it is the general formula.

4. Mechanical efficiency

$$\eta_{mech} = \frac{P_h - \Delta P_{mech}}{P_h}$$

$$= 1 - \frac{\Delta P_{mech}}{P_h}$$

Where the power developed by the turbine or (brake power)

$$P_t = P_h \eta_{mech} \quad kW$$

5. Overall efficiency

$$\eta_t = \frac{P_t}{P_a}$$

And $$\eta_t = \eta_Q \eta_h \eta_{mech}$$

4.5 Flow-rate through Reaction Turbine

1. Francis turbine

$$Q = \pi D B V_f \times contraction \text{ factor}$$

$$or \quad Q = \pi D_1 B_1 V_{f1} \times contraction \text{ factor}$$

$$Q = \pi D_2 B_2 V_{f2} \times contraction \text{ factor}$$

Where

$$B = width\ or\ depth\ or\ breadth\ of\ the\ runner.$$

$$For\ exact\ solution$$

$$Q = (\pi D - Zt) B V_F$$

$$Z = number\ of\ vanes\ or\ blades\ in\ the\ runner$$

$$t = thickness\ of\ the\ vanes.$$

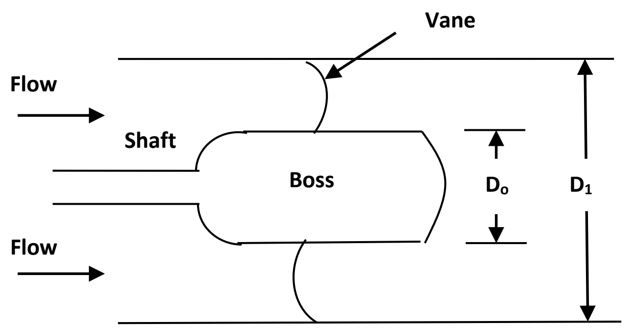
$$V_f = V \sin\alpha\ flow\ velocity.$$

2. Kaplan turbine

$$Q = \frac{\pi}{4}({D_1}^2 - {D_0}^2)V_F$$

$$D_1 = Outlet\ diameter\ of\ the\ runner.$$

$$D_0 = Diameter\ of\ the\ shaft(Boss)$$



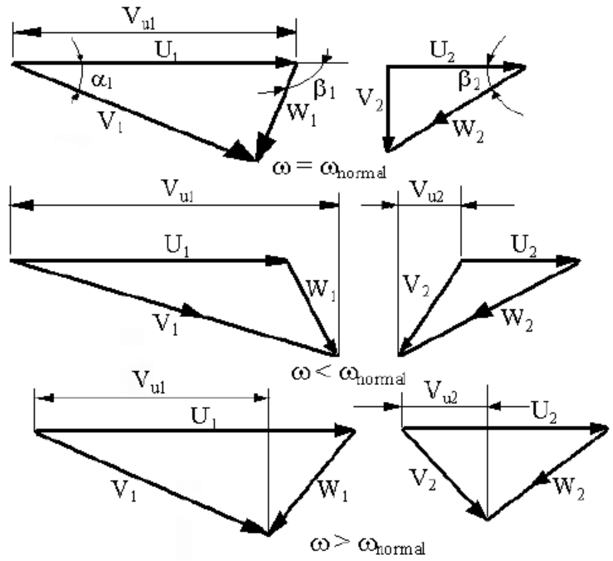
4.6 Velocity Triangle for Reaction Turbine

The absolute velocity at exit of the runner is such that there is no whirl at the outlet i.e $V_{u2} = 0\ ;\ \alpha_2 = 90°$ then the power

$$P_t = \rho Q(U_1V_{u1} - U_2V_{u2})$$

w=w normal as shown in following figures are the rotational speed for which the turbine give the lowest energy loss at outlet represented mainly by $\frac{{V_2}^2}{2}$ for given α of the guide vane canal.

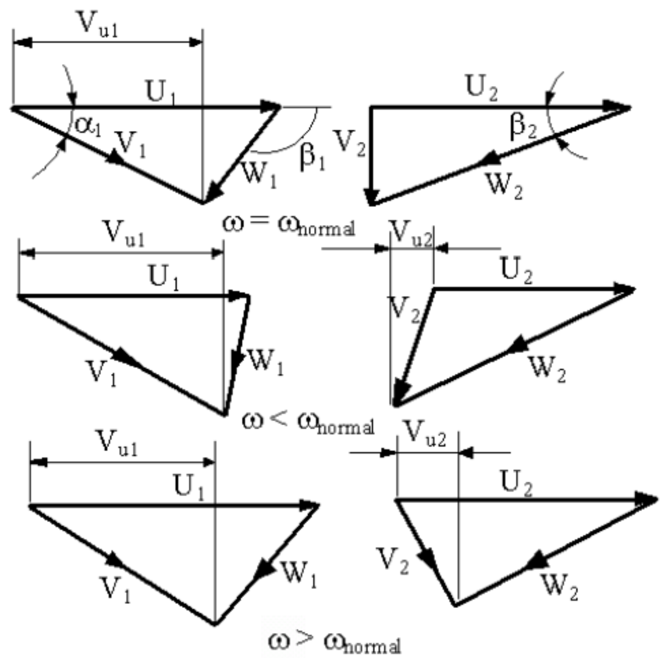
The velocity triangle for Francis turbine as follows



Velocity triangle for three angular velocities.

w=w normal mean the rotational speed for which the turbine gives lowest energy loss at outlet represented mainly by $\frac{V_2{}^2}{2}$ and highest hydraulic efficiency for given angle α of the guide vane canal.

The velocity triangles for Kaplan turbine as follows:



Velocity triangle for three angular velocities.

For w = w $_{normal}$ the same as Frances turbine.

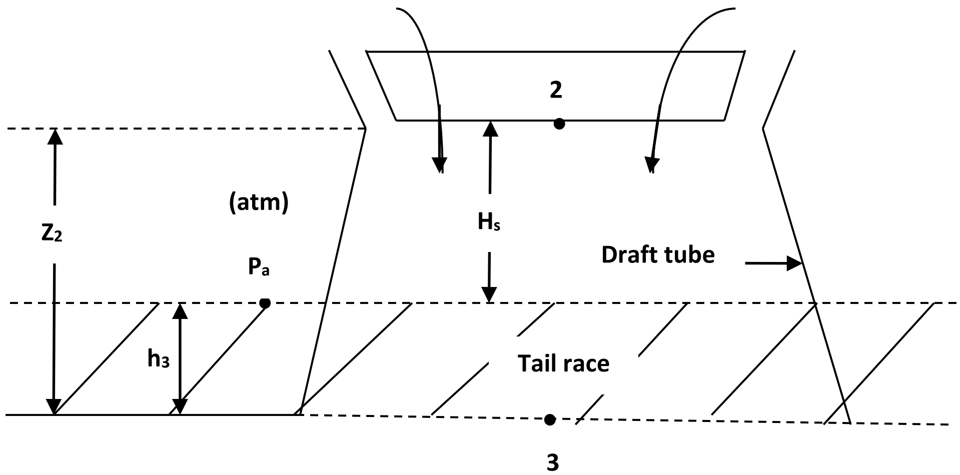
4.7 Draft Tube

-To operate properly, reaction turbines must have a submerged discharge.
-The aim of the draft tube is also to convert the main part of the kinetic energy at the runner outlet to pressure energy at the draft tube outlet.
-This is achieved by increasing the cross section area of the draft tube in the flow direction.
- In an intermediate part of the bend, however, the draft tube cross sections are decreased in the flow direction to prevent separation and loss the efficiency. Fig (1)

If there is no draft tube, the kinetic head $\frac{{V_2}^2}{2g}$ would have been entirely lost.

$$\text{i.e} \quad \Delta H = \frac{{V_2}^2}{2g}$$

Therefore, the kinetic head thus saved by draft tube.



Appling Bernoulli's equation between 2, 3 we get.

$$\frac{P_2}{\gamma} + \frac{{V_2}^2}{2g} + Z_2 = \frac{P_3}{\gamma} + \frac{{V_3}^2}{2g} + Z_3 + h_f \ \ldots\ldots\ldots (1)$$

$$\therefore \frac{P_2}{\gamma} = \frac{P_3}{\gamma} + (Z_3 - Z_2) + \frac{{V_3}^2 - {V_2}^2}{2g} + h_f$$

$$But \ \ \frac{P_3}{\gamma} = \frac{P_a}{\gamma} + h_3$$

$$where \ \ P_a = atmospheric \ pressure$$

$$And \ \ Z_2 - Z_3 - h_3 = H_s$$

$$(height \ of \ runner \ outlet \ above \ the \ tail \ race \ level).$$

$$\therefore \frac{P_2}{\gamma} = \frac{P_a}{\gamma} - \left(H_s + \frac{{V_2}^2 - {V_3}^2}{2g}\right) + h_f \ \ldots\ldots\ldots (2)$$

$$And\ \frac{{V_2}^2 - {V_3}^2}{2g} = dynamic\ suction\ head$$

Minimum value of $\frac{P_2}{\gamma}$ is vapor pressure

If we considering friction in draft tube as h_f which is in general expressed as a friction of dynamic suction head.

$$i.e \quad h_f = K\frac{{V_2}^2 - {V_3}^2}{2g}$$

$$\therefore \frac{P_2}{\gamma} = \frac{P_a}{\gamma} - \left(H_s + (1-k)\frac{{V_2}^2 - {V_3}^2}{2g}\right) \ldots\ldots\ldots (3)$$

$$if \quad \frac{P_2}{\gamma} < \frac{P_a}{\gamma} \ldots\ldots\ldots\ldots\ldots\ldots Cavitation \text{ occur}$$

4.8 Net Head

It is the total difference in elevation minus all losses Fig (1)

$$H_n = H_B - H_c$$

$$H_n = \frac{P_B}{\gamma} + \frac{{V_B}^2}{2g} + Z_B - \frac{{V_c}^2}{2g} \qquad Net\ head$$

$$V_c = the\ velocity\ at\ draft\ tube \text{ exit}$$

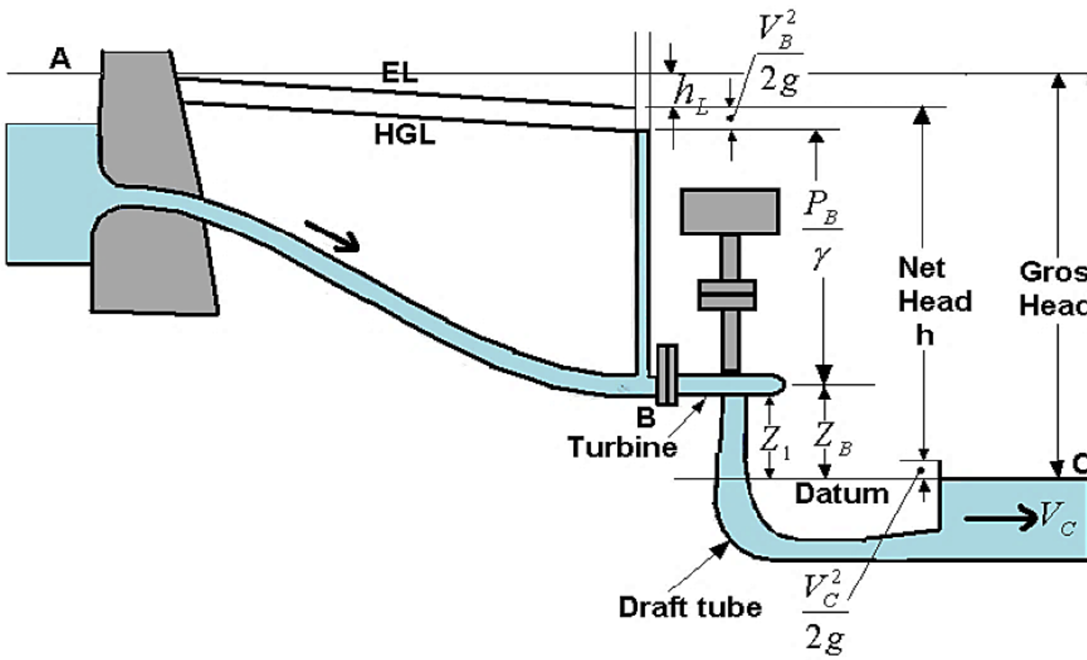


Fig. (1) Schematic diagram of hydraulic power plant.

Ex: A Franeis turbine is fitted with veritical shaped draft tube. The top and bottom diameters are equal to 60 cm and 90 cm respectively. The tube is running full with water flowing downwords and it has a vertical height of 6 m out of which 1.5 m is drowned in the tail race water. Assume friction losses of head between the top and the bottom points as 0.3 times the kinetic head at draft tube exit. The velocity at exit is 1.2 m/s.Determine

a) the preesure head at the top point of the draft tube in m of water.

b) the total head at the same point with referance to the tail race as a datum.

c) the total head at the bottom point with referance to the tail race as a datum.

d) the power in the water at the top of the tube.

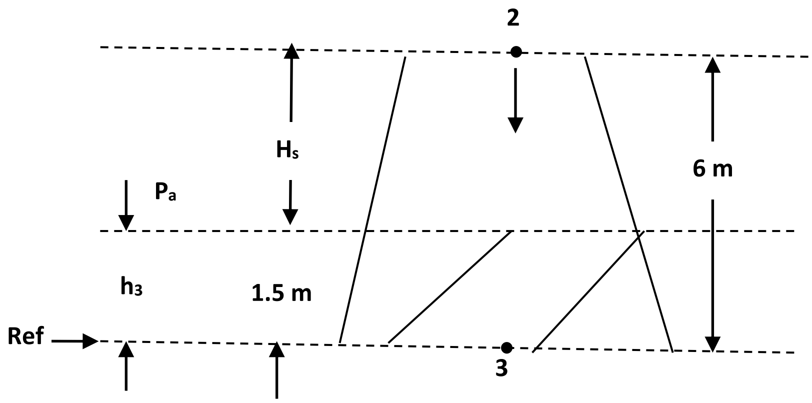
e)) the power in the water at the bottom of the tube.

f) the efficiency of the draft tube.

Solution:-

$$D_2 = 60\ cm\ ;\ D_3 = 90\ cm\ ;H = 6\ m$$

$$H_L = 0.3\frac{{V_3}^2}{2g} \qquad ;V_3 = 1.5\frac{m}{s}$$



E.E. between 2,3

$$\frac{P_2}{\gamma} + \frac{{V_2}^2}{2g} + Z_2 = \frac{P_3}{\gamma} + \frac{{V_3}^2}{2g} + Z_3 + H_L$$

$$\frac{P_2}{\gamma} = \frac{P_3}{\gamma} - (Z_2 - Z_3) - \left(\frac{{V_2}^2 - {V_3}^2}{2g}\right) + 0.3\frac{{V_3}^2}{2g}$$

$$\frac{P_2}{\gamma} = \frac{P_a}{\gamma} + h_3$$

$$\frac{P_2}{\gamma} = \frac{P_a}{\gamma} - H_s - \left(\frac{{V_2}^2 - {V_3}^2}{2g}\right) + 0.3\frac{{V_3}^2}{2g}$$

From continuitg equation

$$A_2V_2 = A_3V_3 \qquad \frac{\pi}{4}(0.6)^2 \times V_2 = \frac{\pi}{4}(0.6)^2 \times 1.5$$

$$V_2 = 3.38\ m/s$$

$$\therefore \frac{P_2}{\gamma} = 10 - 4.5 - \left(\frac{(3.38)^2 - (1.5)^2}{2 \times 9.81}\right) + 0.3\frac{(1.5)^2}{2 \times 9.81}$$

$$\frac{P_2}{\gamma} = 5.0654\, m\ of\ water\ absolute$$

$$or\ \frac{P_2}{\gamma} = -4.9346\, m\ of\ water$$

b) total head at (2)

$$H_2 = \frac{P_2}{\gamma} + \frac{{V_2}^2}{2g} + H_s = -4.9346 + \frac{(3.38)^2}{2 \times 9.81} + 4.5$$

$$H_2 = 0.1464\, m$$

c) at the bottom (3)

$$H_3 = \frac{P_3}{\gamma} + \frac{{V_3}^2}{2g} + Z_3 = H_2 - H_L$$

$$= 0.1464 - 0.3\frac{(1.5)^2}{2 \times 9.81} = 0.1112\, m$$

d) $P_2 = \gamma Q H_2 = 9.81 \times \left(\frac{\pi}{4}(0.6)^2 \times 3.38\right) \times 0.1464$

$$= 1.27\ kW$$

e) $P_3 = \gamma Q H_3 = 1.047\ kW$

f) $\eta_{draft} = \frac{P_{out}}{P_{in}} = \frac{P_3}{P_2} = \frac{1.047}{1.27}$

$$\eta_d = 82\%$$

4.9 Working Prooerties of Reaction Turbines

1. Francies turbine:

$-$ Speed ratio $= \frac{U}{\sqrt{2gH}}$ $\quad its\ value \quad Renerally\ between\ 0.6 \ \rightarrow\ 0.9$

$-$ Flow ratio $= \frac{V_f}{\sqrt{2gH}}$ $\quad its\ value \quad 0.15 \rightarrow 0.3$

$-$ Breadth ratio $= \frac{B}{D}$ $\quad its\ value \quad 0.15 \rightarrow 0.4$

2. Kaplan turbines:

Speed ratio $upto \rightarrow 2.09$

Flow ratio $upto \rightarrow 0.69$

Breadth ratio $upto \rightarrow 0.35 \rightarrow 0.6$

EX. 1: A reaction turbine has outer and inner diameters of the wheel as (1 and 0.5) meters respectively. The vanes are radial at inlet and the discharge is radial at outlet and the enters the vanes at an angle of 10^0. Assuming the velocity of flow as constant and equal to 3 m/s. find the speed of the wheel and the vane angle at outlet.

Solution:-

$D_1 = 1\,m\ ;\ D_2 = 0.5\ ;\ \beta_1 = 90°\ \ (vane\ radial)$

$\alpha_2 = 90°(discharge\ radial\ ;\ \alpha_1 = 10°\ ;\ V_{f1} = V_{f2} = 3\,m/s$

The velocity diagram at inlet and outlet

$$U_1 = \frac{V_{f1}}{\tan\alpha_1} = \frac{3}{\tan 10°} = 17\,m/s$$

$$also\ \ U_1 = \frac{\pi D_1 N}{60}$$

$$\therefore N = \frac{17 \times 60}{\pi \times 1}$$

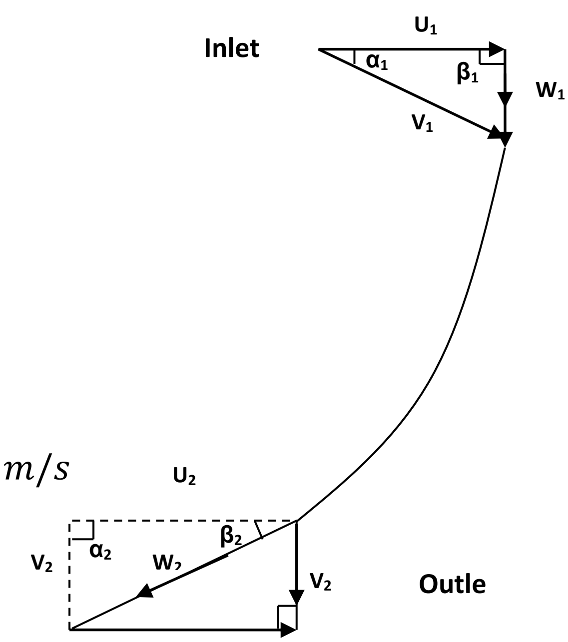
$$N = 325\,rpm$$

At outlet

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.5 \times 325}{60} = 8.5\,m/s$$

Then $\tan\beta_2 = \frac{V_{f2}}{U_2} = \frac{3}{8.5} = 0.353$

$or\ \beta_2 = 19.27°$



EX. 2: A reaction turbine with a supply of 550 l/s under a head of 15 m develops 73.6 kW at 375 rpm. The inner and the outer diameter of the runner are 50 cm and 75 cm respectively. The velocity of water at exit is 3 m/s. Assuming that the discharge is radial and that the width of the wheel is constant. Fig the actual and the theoretical hydraulic efficiency of the turbine and the inlet angles of the guide and wheel vanes.

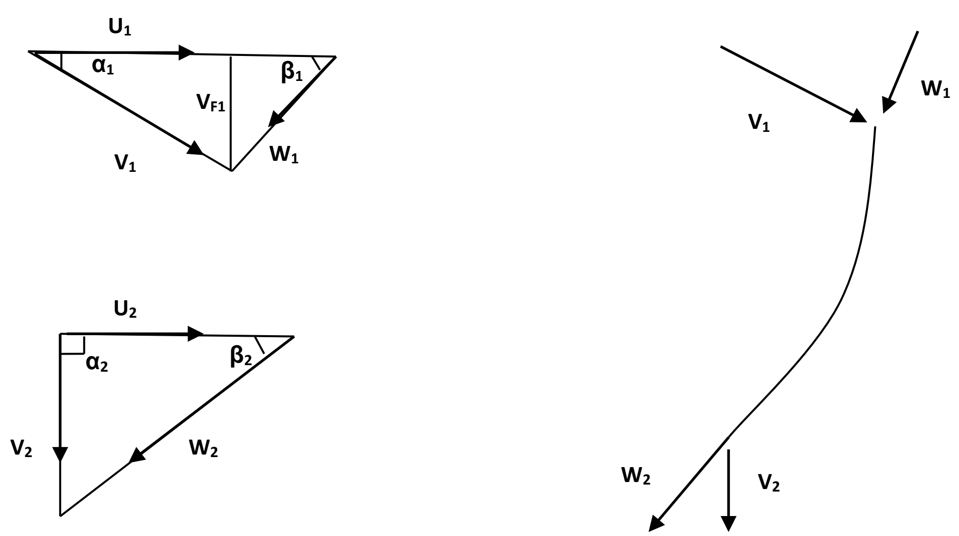
Solution:-

$Q = 0.55\, m^3/s\ ; D_1 = 75\, cm\ ;\ D_2 = 50\, cm$

$H = 15\, m\ ; P_t = 73.6\, kW\ ;\ V_{f2} = 3\, m/s\ ; N = 375\, rpm$

$\alpha_2 = 90°\ (radial\ discharge)\ \ B_1 = B_2\ width\ of\ runner.$

a) $U_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.75 \times 375}{60} = 14.75\, m/s$



From velocity triangle at exit

$$V_{f2} = 3\, m/s = V_2 \quad ; \quad \alpha_2 = 90°$$

Work done /sec by the turbine per kg of water $= \frac{V_{u1} U_1}{g}$

And this is equal to the head utilized by the turbine.

$\frac{V_{u1}U_1}{g} = H - \frac{{V_2}^2}{2g}$ (Assuming no pressure head loss at outlet)

$$Or\ \frac{V_{u1} \times 14.75}{9.81} = 15 - \frac{3^2}{19.62}$$

$$\therefore\ V_{u1} = 9.7\ m/s$$

$$\therefore \text{ The power developed by the turbine runner } = \rho Q V_{u1} U_1$$

$$= 1000 \times 0.55 \times 9.7 \times 14.75$$

$$= 78.7\ kW$$

$$\text{The available power of water } = \gamma Q H = 9.81 \times 0.55 \times 15$$

$$= 80.93\ kW$$

$$\therefore\ overall\ efficiency\ \eta_o = \frac{P_{out}}{P_{in}} = \frac{73.6}{80.93} = 90.9\%$$

$$and\ hydraulic\ ficiency\ \eta_h = \frac{78.7}{80.73} = 96.82\%$$

b) $Q = (\pi D_1 B_1) V_{F1} = \pi D_2 B_2 V_{F2}$

$$\therefore\ V_{f1} = V_{f2} \times \frac{D_2}{D_1} = 3 \times \frac{0.5}{0.75} = 2\ m/s$$

From inlet velocity triangle

$$\tan \beta_1 = \frac{V_{F1}}{U_2 - V_{u1}} = \frac{2}{14.75 - 9.7}$$

$$\beta_1 = 21.6°$$

$$Then\ \ V_1 = \sqrt{{V_{u1}}^2 + {V_{F1}}^2} = \sqrt{(9.7)^2 + (2)^2} = 9.9\ m/s$$

$$and\ V_{u1} = V_1 \cos \alpha_1 \quad \therefore\ \cos \alpha_1 = \frac{V_{u1}}{V_1} = \frac{9.7}{9.9}$$

$$\alpha_1 = 10.9°$$

EX.3: A reaction turbine rotates at 370 rpm. The wheel vanes are radial at inlet and $D_1=2D_2$. the constant velocity of flow in the wheel 3 m/s. water enters the wheel at an angle of (10.07^0) to the tangent of the wheel at inlet. The breadth of the wheel at inlet is 75 mm and the area of flow blocked by the vanes is 5% of the gross area of flow at inlet. Find

a) The outer and inner diameters of the wheel.

b) The net available head.

c) The wheel vane angle at outlet.

d) The theoretical power developed by the wheel.

<u>Solution:-</u>

$$N = 370\, rpm \;;\; D_1 = 2D_2 \;;\; V_{f1} = V_{f2} = 2\, m/s$$

$\alpha_1 = 10.07° \;;\; B_1 = 75\, mm$ Contraction coefficient K=1-0.5=0.95

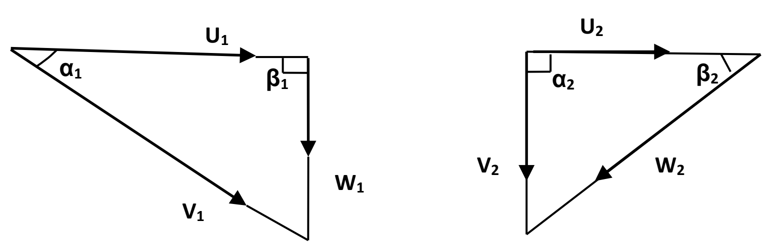
$\beta_1 = 90°$ (Vanes are radial at inlet)

a) $V_{f1} = V_1 \sin\alpha_1 \qquad \therefore\; V_1 = \frac{V_{f1}}{\sin\alpha_1}$

$$V_1 = 11.43\, m/s$$

$$V_{u1} = V_1 \operatorname{coc}\alpha_1 = 11.25\, m/s$$

$$Assume\ \beta_1 = 90° \qquad \therefore\; V_{u1} = U_1$$



$$\text{And}\quad U_1 = \frac{\pi D_1 N}{60} = \frac{\pi D_1 \times 370}{60} = 11.25$$

$$\therefore\ D_1 = 0.58\,m \quad and\ \ D_2 = 0.29\,m$$

b) Net available head at the wheel =Work done /sec per unit mass of water

$$H = \frac{V_{u1}U_1 - V_{u2}U_2}{g}$$

$$or\ H = \frac{V_{u1}U_1}{g}\ for\ radial\ outlet\ flow\ \alpha_2 = 90°$$

$$H = \frac{11.25 \times 11.25}{9.81} = 12.9$$

c) $$U_2 = \frac{\pi \times 0.29 \times 370}{60} = 5.63\,m/s$$

$$V_{F2} = 2\,m/s$$

$$\therefore\ \tan\beta_2 = \frac{V_{F2}}{U_2} = \frac{2}{5.63}\ m/s$$

$$\beta_2 = 19.5°$$

d) $Q = \pi D_1 B_1 V_F$ Contraction coefficient (K)

$$= \pi \times 0.58 \times 0.075 \times 2 \times 0.95$$

$$= 0.26\,m^3/s$$

$$\therefore available\ power\ P_a = \gamma QH$$

$$= 9.81 \times 0.26 \times 12.9$$

$$= 32.9\ kW$$

EX. 4: A Francis turbine is required to developed 3680 kW when operating under a net of 30 m and the specific speed to be about 231.55 m-kW unit assuming guide vane angle at full gate opening 30^0 hydraulic efficiency 90%, overall efficiency 87%, radial velocity of flow at inlet $0.3\sqrt{2gH}$, blade thickens coefficient 5% draw the inlet velocity diagram and find: -

a) The nearest synchronous speed to drive an alternator to give a frequency of 50 cycles per sec.

b) The diameter and the width of the runner at inlet.

c) The theoretical inlet angle of the runner vanes.

Solution:-

$$P_t = 3680\ kW\ ;\ \alpha_1 = 30°\ ; H = 30\ m$$

$$V_{F1} = 0.3\sqrt{2gH}\ ;\ N_s = 231.55\ ; K = 0.95$$

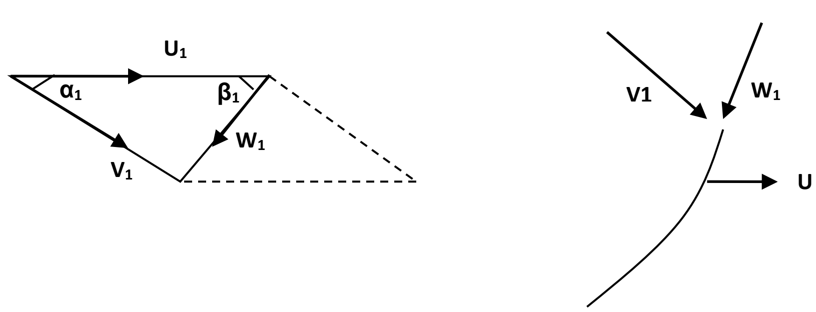
$$\eta_h = 0.9\ ; f = 50\ cycles/sec\ ;\ \eta_t = 0.87$$

a) $N_s = \frac{N\sqrt{P_t}}{H^{\frac{5}{4}}} = \frac{N\sqrt{3680}}{(30)^{\frac{5}{4}}} = 231.55 \therefore N = 268\ rpm$

Assuming No. of poles 11

$\therefore$ Nearest synchronous speed $= \frac{3000}{11} = 272.8\ rpm$

b) Velocity of flow $V_{F1} = 0.3\sqrt{2 \times 9.81 \times 30} = 7.26\ m/s$



$$V_{F1} = V_1 \sin\alpha_1 \qquad \therefore V_1 = \frac{7.26}{\sin 30} = 14.52\ m/s$$

$$and\ V_{u1} = V_1 \cos\alpha_1 = 14.52 \times \cos 30 = 12.6\frac{m}{s}$$

$$\therefore \eta_h = \frac{V_{u1}U_1}{gH} \qquad \therefore H = \frac{V_{u1}U_1}{g\eta_h}$$

$$U_1 = \frac{30 \times 0.9 \times 9.81}{12.6} = 21\ m/s$$

$$and\ \ U_1 = \frac{\pi D_1 N}{60} \qquad N = \frac{60 \times 21}{\pi \times 272.8} = 1.47\ m$$

$$Now\ \ P_t = \gamma Q H \qquad \therefore Q = \frac{3680}{9.81 \times 30} = 14.4\ m^3/s$$

$$Q = \pi D_1 B_1 V_{F1}.K$$

$$14.4 = \pi \times 1.47 \times B_1 \times 7.26 \times 0.95$$

$$B = 0.453\ m\ \ the\ width\ of\ the\ runner$$

c) $\tan\beta_1 = \frac{V_{F1}}{U_1 - V_{u1}} = \frac{7.26}{21 - 12.6} = 0.865$

$$\therefore \beta_1 = 40.6°\ the\ inlet\ angle\ of\ the\ vane$$

EX. 5: A reaction turbine, the peripheral velocity of the wheel at inlet is given by $U_1 = \emptyset\sqrt{2gH}$ and the radial velocity of flow $V_F = \varphi\sqrt{2gH}$

The breadth of the wheel at inlet is n' time the diameter of the runner at inlet. If the turbine efficiency is 85% and the area taken up by vanes at inlet 5% of the peripheral area at inlet. prove that the specific speed of the turbine is equal to$(888.2\emptyset\sqrt{\varphi n})$. A runner of the above type having a specific speed of 188.7 m-kW unit required to develop 6625 kW under a head 85 m. taken φ as 0.18 and n=0.2, calculate the rpm and the diameter of the runner.

Solution :-

$$\varphi = 0.18\ ;\ B_1 = nD_1\ \ \therefore\ \ B_1 = 0.2D_1$$

$$\eta_t = 0.85\ ;\ P_t = 6625\ kW\ ;\ N_s = 188.7$$

$$H = 83\ m\ ; K = 0.95$$

a) $Q = 0.95 \times \pi \times D_1 \times B_1 \times V_{F1}$

$$= 0.95 \times \pi \times D_1 \times nD_1 \times \varphi\sqrt{2gH}$$

$$= 121.88 \times n \times {D_1}^2 \times \varphi \ldots\ldots\ldots (1)$$

$$U_1 = \frac{\pi D_1 N}{60} = \emptyset\sqrt{2gH}$$

$$N = \frac{60 \times \emptyset\sqrt{2 \times 9.81 \times 85}}{\pi D_1} = \frac{780\emptyset}{D_1} \ldots\ldots\ldots (2)$$

$$Now\ N_s = \frac{N\sqrt{P_t}}{H^{\frac{5}{4}}} \qquad P_t = \gamma QH\eta_t$$

$$= 9.81 \times 121.88 \times n{D_1}^2\varphi \times 0.85 \times 85$$

$$\therefore P_t = 1016.3\, n{D_1}^2\varphi \ldots\ldots\ldots (3)$$

$$\therefore N_s = \frac{\frac{780\emptyset}{D_1} \times \sqrt{86385.2n{D_1}^2\varphi}}{(85)^{\frac{5}{4}}}$$

$$N_s = \frac{\frac{780\emptyset}{D_1} \times D_1 \times 294\sqrt{n\varphi}}{258.1} = (888.2\emptyset\sqrt{\varphi n})$$

$$\therefore N_s = (888.2\emptyset\sqrt{\varphi n})$$

b) $N_s = \frac{N\sqrt{P_t}}{H^{\frac{5}{4}}}$

$$188.7 = \frac{N \times \sqrt{6625}}{85^{1.25}}$$

$$N = 600\ rpm$$

$$\therefore Q = 0.95 \times \pi \times D_1 \times B \times V_F$$

$$Q = 0.95 \times \pi \times {D_1}^2 \times n \times \varphi\sqrt{2gH}$$

$$= \frac{P_t}{\gamma H \eta_t}$$

$$since\ n = 0.2 \quad \varphi = 0.18$$

$$\therefore\ 0.95 \times \pi \times {D_1}^2 \times 0.2 \times 0.18 \times \sqrt{2 \times 9.81 \times 85}$$

$$= \frac{6625}{2 \times 9.81 \times 85}$$

$$D_1 = 1.46\ m$$

EX. 6: A Kaplan turbine develops 5900 kW under an effective head of 5 m. Its speed ratio is 2 and flow ratio is 0.6 and the diameter of the boss=0.35 times the external diameter of the runner. Mechanical efficiency of the turbine is 90%. Calculate the diameter of the runner and also the specific speed.

Solution:-

$$P_t = 5900\ kW\ ; H = 5\ m\ ; \eta_t = 90\%$$

$$\emptyset = 2\ ; \varphi = 0.6\ ; d = 0.35 D_1$$

$$\eta_t = \frac{P_{out}}{P_{in}} = \frac{P_t}{\gamma Q H} = \frac{5900}{9.81 \times Q \times 5} = 0.9$$

$$Q = 133.3\ m^3/s$$

$$V_F = \varphi\sqrt{2gH} = 0.6\sqrt{2 \times 9.81 \times 5}$$

$$= 5.92\ m/s$$

$$Q = AV_F \qquad \therefore A = \frac{Q}{V_F}$$

$$area\ of\ flow = \frac{133.3}{5.92} = 22.5\ m^2$$

$$= \frac{\pi}{4}{D_1}^2 - \frac{\pi}{4}{d_1}^2 = \frac{\pi}{4}({D_1}^2 - (0.35D_1)^2)$$

$$\therefore 22.5 = \frac{\pi}{4} {D_1}^2 (1 - (0.35)^2)$$

$$\therefore \; D_1 = 5.7 \, m$$

$$and \; U_1 = \frac{\pi D_1 N}{60} = \sqrt{2gH}$$

$$\therefore \; \frac{\pi \times 5.7 \times N}{60} = 2\sqrt{2 \times 9.81 \times 5}$$

$$N = 66.4 \, rpm$$

$$\therefore N_s = \frac{N\sqrt{P_t}}{H^{\frac{5}{4}}}$$

$$= \frac{66.4\sqrt{5900}}{6^{\frac{5}{4}}}$$

$$N_s = 682 \, m - kW \; units$$

4.10 Power Regulating Mechanisms

1- Guide –vane operating gear: -

The operating gear must ensure setting of all guide vanes at the same angle (equal values of α_1) with any openings of the wicket gate.

The schematic of the most widely employed guide: - Vane operating gears is show in Fig (2). The levers 1 set on the upper pivot of the guide vanes are connected to the regulating ring 3 by means of shackles and pull rods 2. These three elements constitute the main part of the guide -vane operating gear. Figure (2.a) shows the guide-vane operating gear in the full closing position. If the regulating ring turns counterclockwise, all levers will turn through the same angle, as well as the guide vanes and the turbine will be opened. Fig (2.b). It follows that to change the turbine power it is necessary to turn the regulating ring. Servomotors of the wicket gate are designed for turning the regulating ring and changing the opening of the guide vanes so as to regulate turbine power. The servomotors are operated by high pressure oil and they find use in all regulating system of high-power turbine.

The servomotors may be operating the guide vanes of the turbine as presented schematically in Fig (3) with two cylinders. There are many designed for servomotors application as shown in Figures (4, 5, 6)

2- Runner- Blade Operating Gear of Adjustable-Blade: -

The runner operating gear must ensure a change in the blade angle and accurate setting of the blades as required without stopping the turbine, i.e with a rotating runner. The blade operating rings must be able to overcome tremendous forces caused by water pressure, centrifugal and frictional forces in the blade pin bearings, they must arrange in very confined space in the runner hub and feature an exceptionally high reliability, since their repair requires complete disassembly of units.

The schematic diagram of a runner blade operating gear is shown in Fig. (6). Blade pivot 2, having two supports in housing 3 and 4 is connected to the blade flange 1. The lever 5 set on pivot 2 is connected by pull rod 6 to piston 7 of the runner servomotor. Displacement of the piston 7 through pull rod 6 results in turning of the pivot 2 and blade 1.

There exist a great number of designs of the above considered runner blade operating, mainly differing in the way in which the servomotor piston is connected to the levers. Two examples are shown in figures (7) and Fig (8) for high-head and law-head axial flow turbines.

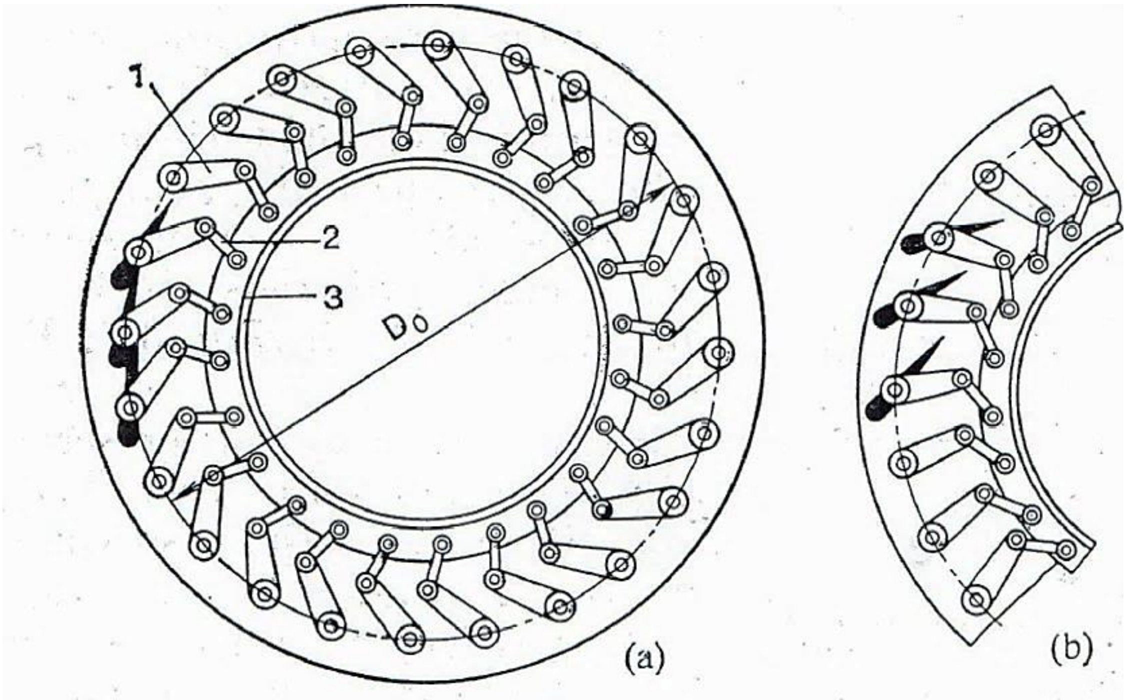


Fig. (2) Guide vane operating mechanism. Schematic diagram.

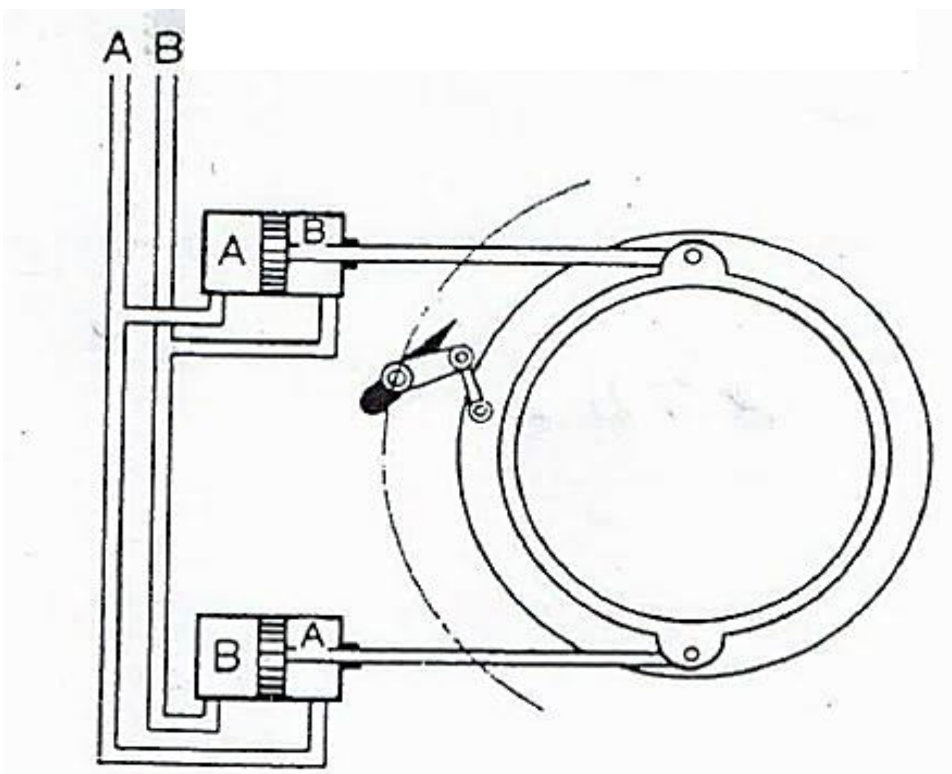


Fig. (3). Schematic diagram of guide vane operating gear with two cylindrical servomotors.

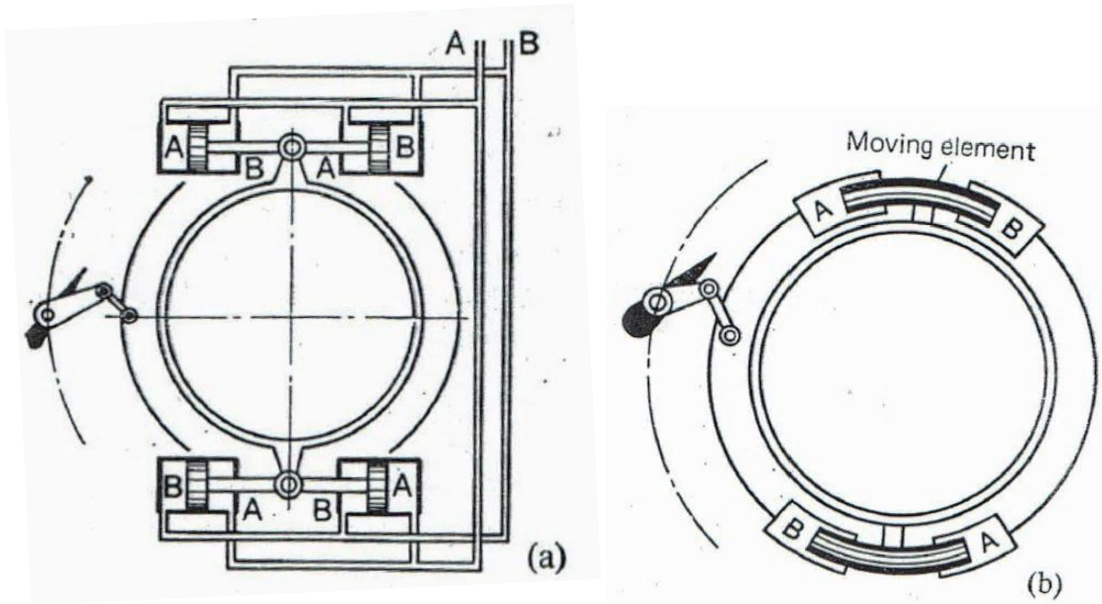
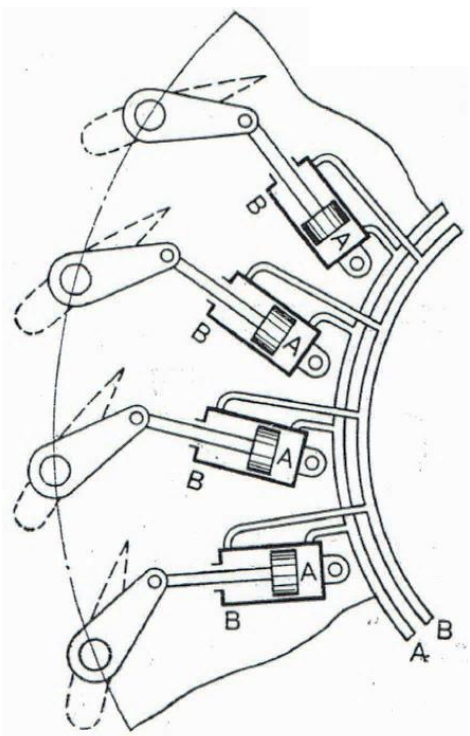


Fig. (4) Schematic diagram of guide vane operating gear with two twinned (a) and anchor-ring (b) servomotors.



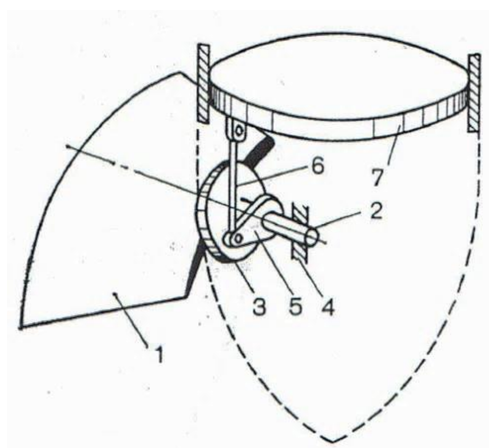


Fig. (5) Schematic diagram of guide vane operating gear with individual servomotors.

Fig. (6) Schematic diagram of adjustable-blade runner operating gear.

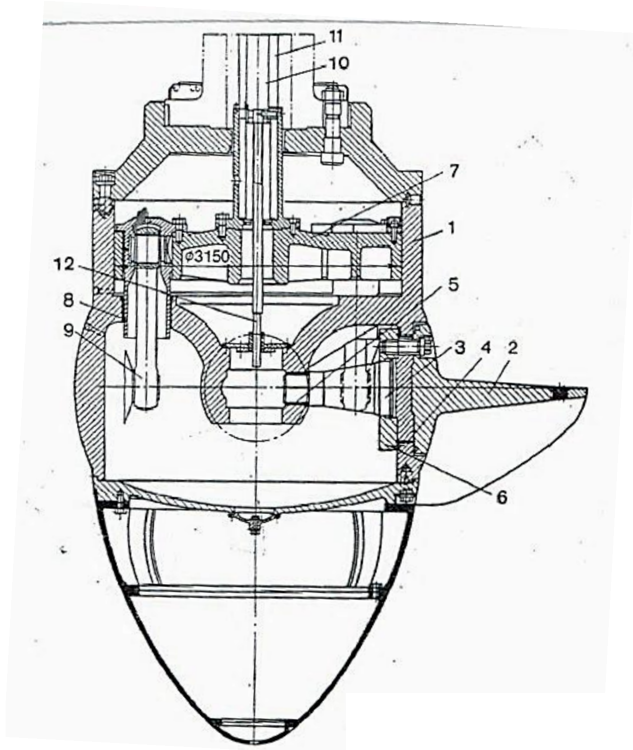


Fig. (7) Blade operating gear of high-head turbine run.

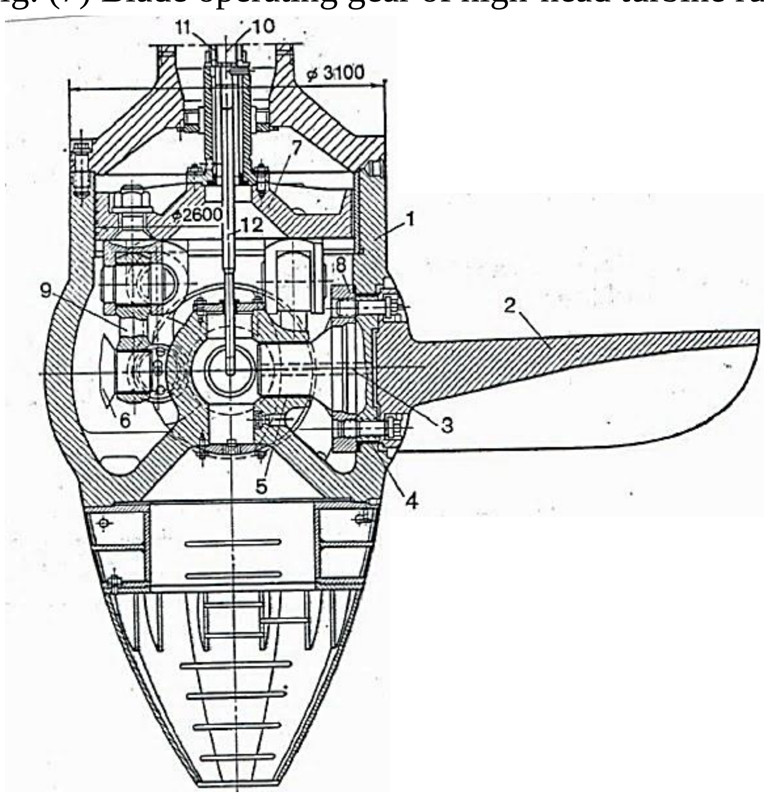


Fig. (8) Blade operating gear of low-head turbine runner.

4.11 Supply and discharge systems

1- Penstock: - in determining the number of penstocks for any particular installation various factors have to be considered. Let us compare by a single penstock and by a system of n realized by selecting diameters either, Fig (8)

a) for identical flow velocities.

b) for identical friction losses.

Let Q: Discharge conveyed in a single penstock.

D: Diameter of the penstock.

V: flow velocity.

h_c: Head loss.

e: wall thickness.

G: weight of the penstock.

a) Identical flow velocities

Dividing the discharge Q among n conduits, the diameter of each pipe should be determined to ensure an identical flow velocity V. with each penstock discharging.

$$Q_n = \frac{Q}{n}$$

$$for\ identical\ velocity\quad V = \frac{Q}{\frac{\pi D^2}{4}} = \frac{Q_n}{\frac{\pi {D_n}^2}{4}} \ldots\ldots\ldots (1)$$

Where D_n is the diameter of any penstock:

From eq. (1)

$$\therefore D_n = D\sqrt{\frac{Q_n}{Q}} = \frac{D}{\sqrt{n}} \ldots\ldots\ldots (2)$$

The head loss due to friction in case of single penstock installation:

$$h_f = f.\frac{L}{D}.\frac{V^2}{2g} \qquad since\ Q = AV$$

$$or\ h_f = \frac{8fL}{g\pi}.\frac{Q^2}{D^5} \dots\dots\dots (3)$$

$$let\ \frac{8fL}{g\pi} = a_1 = 0.26\ fL$$

$$\therefore h_f = a_1 \frac{Q^2}{D^5} \dots\dots\dots (4)$$

And for n penstocks

$$h_{fn} = a_1 \frac{\left(\frac{Q}{n}\right)^2}{\left(\frac{D}{\sqrt{n}}\right)^5} = a_1 \frac{Q^2\sqrt{n}}{D^5} \dots\dots\dots (5)$$

$$\therefore h_{fn} = h_f\sqrt{n} \dots\dots\dots (6)$$

The wall thickness in case of single penstock arrangement:

$$e = \frac{PD}{2\sigma_{steel}} \rightarrow\ a_2 = \frac{P}{2\sigma_{steel}}$$

$$e = a_2 D \dots\dots\dots (7)$$

$$P = static\ + water\ hammer\ pressure$$

$$\sigma = tensile\ stress\ of\ the\ steel$$

$$For\ n\ penstocks,$$

$$e_n = a_2 D_n = a_2 \frac{D}{\sqrt{n}}$$

$$\therefore e_n = \frac{e}{\sqrt{n}} \dots\dots\dots (8)$$

The total penstock weight in case of single penstock installation

$$G = \gamma_{steel} \pi De \qquad a_3 = \gamma_{steel} \pi$$

$$\therefore G = a_3 De \dots \dots \dots (9)$$

Also for n penstocks

$$G_n = \frac{G}{n} \dots \dots \dots (10)$$

Or the total weight of n penstocks

$$nG_n = G$$

b) Identical friction head losses

for determining the diameter D_n ensuring a head loss identical with that in the single penstock,

$$h_f = a_1 \frac{Q^2}{D^5} = a_1 \frac{\left(\frac{Q}{n}\right)^2}{{D_n}^5}$$

$$D_n = \frac{D}{\sqrt[5]{n^2}} \dots \dots \dots (11)$$

Also wall thickness of the n penstocks

$$e_n = \frac{e}{\sqrt[5]{n^2}} \dots \dots \dots (12)$$

And the weight $G_n = \frac{G}{\sqrt[5]{n^4}} \dots \dots \dots (13)$

As can be seen the theoretical weight increase for several penstocks with $n^{1/5}$ on the other hand it is more safety of operation when use n penstocks.

Large power penstocks subject to heads of several hundred meters may be constructed of banded steel pipe.

Simple steel pipes are used for

$$PD < 10000 \ (kg/cm)$$

Banded steel pipe for

$$PD < 10000\ (kg/cm)$$

$$Where\ P: kg/cm^2 \quad D: pipe\ diameter\ (cm)$$

Practical empirical equations used to find out the diameter of a penstock will give Fig. (9).

For maximum velocity in the penstock may be V_{max}=6 m/s

$$h_f = \frac{V^2 L' n^2}{R^{\frac{4}{3}}} \leq 0.05\ H_{gross}\ \dots\dots\dots(14)$$

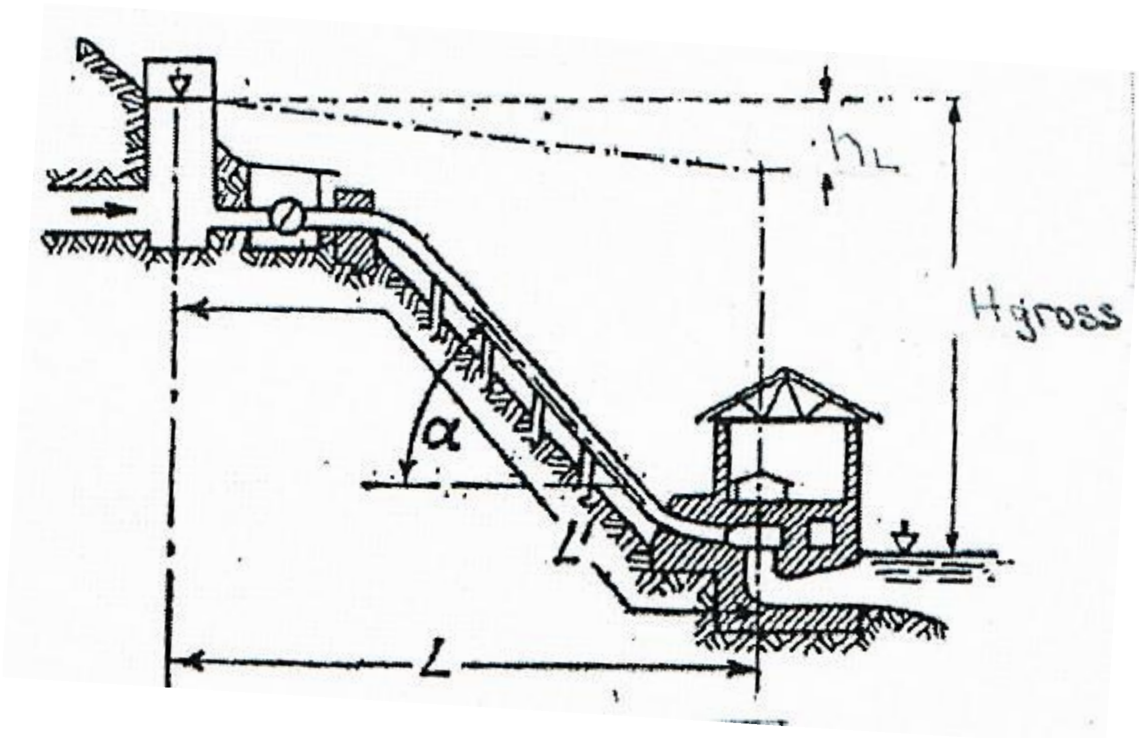
$$and\ H_{gross} < 100\ m \quad D = \sqrt[7]{0.05Q^3}\ m \dots\dots\dots(15)$$

$$H_{gross} > 100\ m \quad D = \sqrt[7]{\frac{5.2Q^3}{H_{gross}}}\ m \dots\dots\dots(16)$$

$$R = hydraulic\ radius$$

$$n = manning\ coefficient$$

$$L' = penstock\ length\ (m)$$



$$L' \cong \sqrt{{H_{gross}}^2 + L^2}$$

Fig. (9)

2- Turbine case

The turbine case serves to supply water to the guide vanes (wicket gate) of a reaction turbine.

Turbine cases must satisfy the following requirements:

1- They must ensure uniform supply of water to the guide vanes over the entire perimeter of the wicket gate.

2- They must ensure minimum hydraulic losses in the case proper, in the stayring and where water flow enters the guide vanes.

3- The shape and size of a turbine case must be in agreement with the layout of the power house of the HEPP.

3- Draft tubes

In reaction turbine water is discharged from the runner into a draft tube through which the water is discharged into tail water pool. The draft tube affects materially the power-generating properties of turbines, especially of low-head ones. In addition, the draft tube determines the dimensions of the lower part of the power house of HEPP and the elevation of the base. In this convection, great attention is devoted to the definition of the form and dimension of draft tube in designing HEPP.

Two types of draft tube are distinguished straight and elbow type.

a- Straight draft tube:

The simplest is the straight-type conical draft tube Fig. (10) that possesses good power-generating characteristics; however, it must be of a considerable required length issue. For high-power vertical turbine this makes it necessary to construct the base at a considerable depth and results in a greater cost at the HEPP. In this connection, draft tubes of this type find application only in small-power turbines.

b- Elbow-type draft tubes:

It is used in almost all HEPP where high-power vertical turbines operate Fig. (11,12)

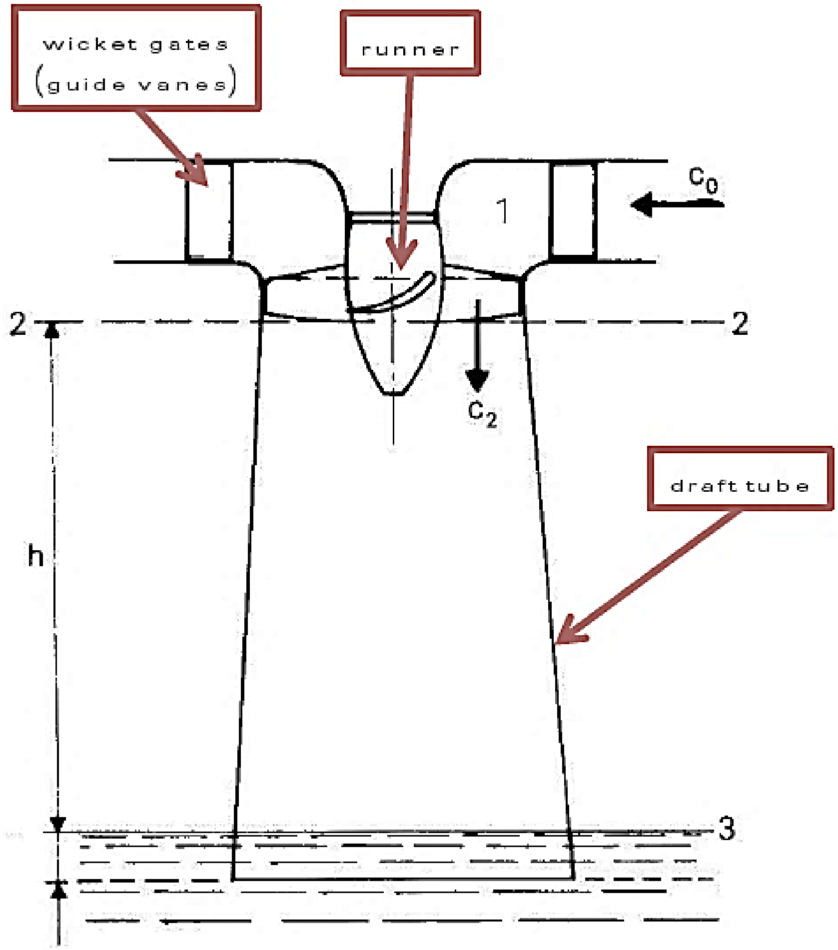


Fig. (10) Straight draft tube.

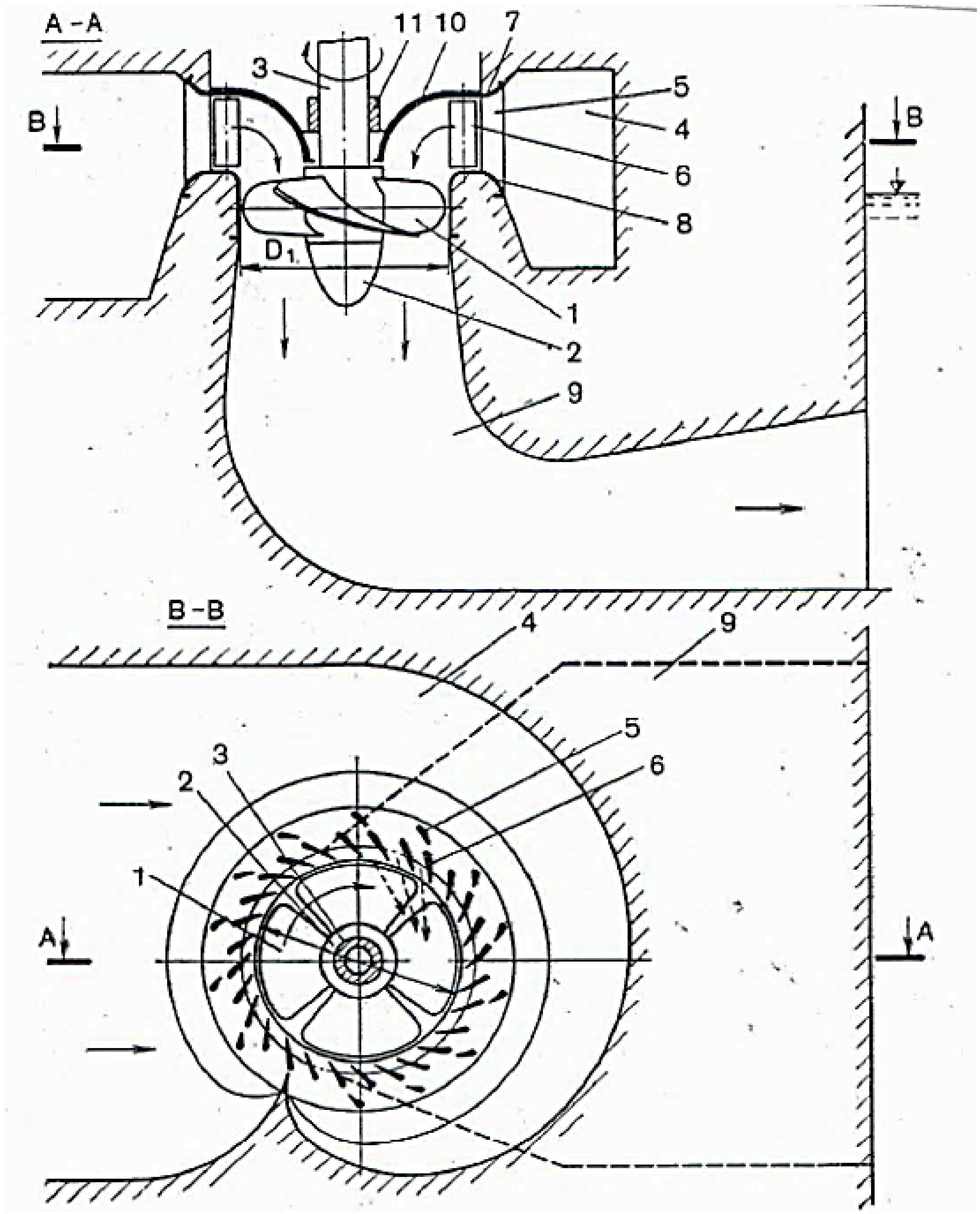


Fig. (11) Schematic diagram of axial-flow turbine with draft tube.

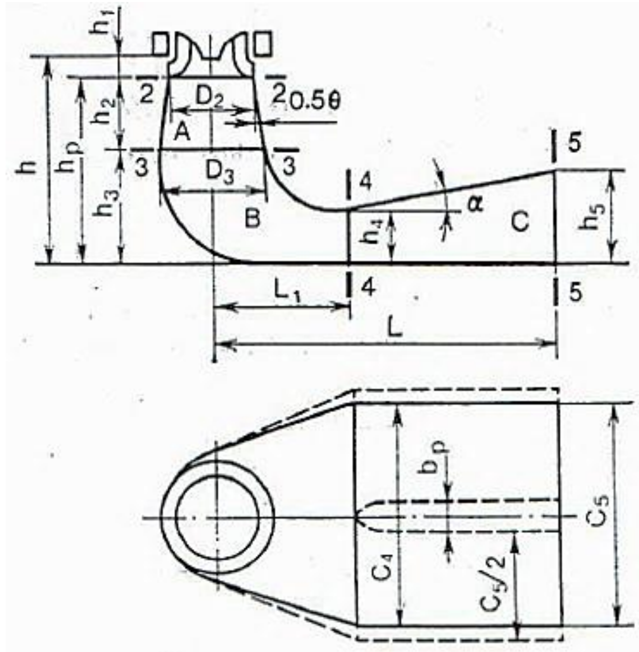


Fig. (12 a) Elbow-type draft tube.

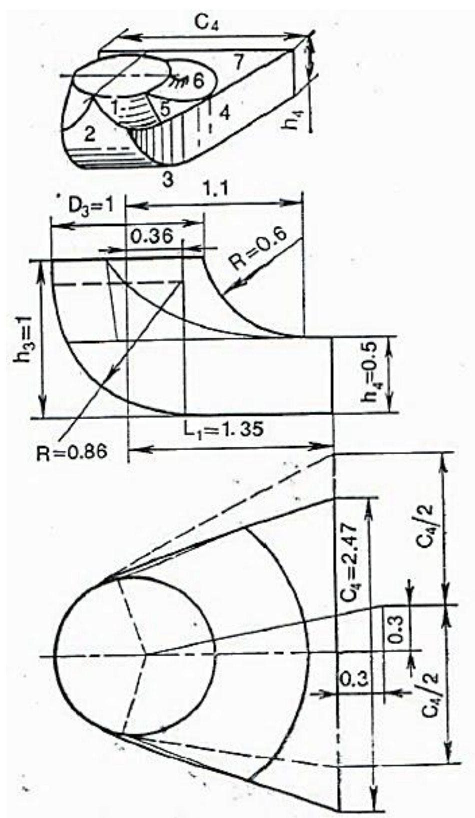


Fig. (12 b) Standard elbow - type draft tube.

A comparison between impulse and reaction turbine shown in the following table.

Impulse turbine	Reaction turbine
The entire available energy of the water is Converted into kinetic energy.	Only a portion of the energy is converted into kinetic energy before the fluid enters the turbine runner.
The work is done only by the change in the Kinetic energy of the jet.	The work is done partly by the change in the velocity head, but almost entirely by the change in pressure head.
Flow regulation is possible without loss.	It is possible to regulate the flow without loss.
Unit is installed above the tailrace.	Unit is entirely submerged in water below the tailrace.
Casing has no hydraulic function to perform, Because the jet is unconfined and is it Atmospheric. Thus, casing serves only To prevent splashing of water.	Casing is absolutely necessary, because the pressure at inlet to the turbine is much higher than the pressure at outlet. Unit has to be sealed from atmospheric pressure.
It is not essential that the wheel should run full And air has free access to the buckets.	Water completely fills the vane passage.

Chapter five

Similarity laws for turbine specific speed and cavitations

5.1 Similarity laws

It is required to predict the performance of a prototype hydraulic machine before it is manufactured

1- Specific speed: - it is the speed of identical turbine (geometrically similar and having same blade angle) working under unit head and delivery unit power.

It has been shown previously that the tangential velocity at the runner is

$$U_1 = \frac{\pi D_1 N}{60} \quad or\ U_1 \propto D_1$$

And

$$U_1 = \sqrt{2gH}\ \ or\ U_1 \propto \sqrt{H}$$

$$\therefore\ \ N_1 D_1 \propto \sqrt{H}\ \rightarrow\ D_1 \propto \frac{\sqrt{H}}{N} \ldots\ldots\ldots (1)$$

Also the power developed by the turbine is

$$P_t = \gamma Q H_n \quad\ or\ \ P_t\ \alpha\ Q H_n$$

And

$$Q = AV \quad and\ V = \sqrt{2gH}$$

$$\therefore\ \ Q \propto\ {D_1}^2 \sqrt{H}$$

$$or\ \ P_t\ \alpha\ {D_1}^2 \sqrt{H}.H$$

$$or\ \ P_t\ \alpha\ {D_1}^2 H^{3/2} \ldots\ldots\ldots (2)$$

From 1, 2 substituting for D_1 we get

$$P_t \, \alpha \, \frac{H}{N^2} H^{\frac{3}{2}} \quad i.e \quad P_t \, \alpha \, \frac{H^{\frac{5}{2}}}{N^2}$$

$$or \; N \propto \sqrt{\frac{H^{\frac{5}{2}}}{P_t}} \quad or \;\; N = N_s \sqrt{\frac{H^{\frac{5}{2}}}{P_t}}$$

$$\therefore N_s = \frac{N\sqrt{P_t}}{H^{\frac{5}{4}}} \; \dots\dots\dots (3)$$

$If \;\; P_t = 1 \; kW \; and \; H = 1 \; m \; then \; numerically$

$N_s = N \; Which \; called \; unit \; speed$

$P_t = power \; developed \; by \; the \; turbine \; kW$

$N = speed \; of \; the \; runner \; in \; rpm$

$H = \; net \; head \;\; m$

During the design of turbine, the specific speed should be the same for the model and prototype therefore:

$N_{sm} = N_{sp}$

$m = model \;\; ; p = prototype$

$\therefore \; from \; equ. 3,2 \; for \; P_{tm} \; \& \; P_{tp}$

$$\frac{N_m}{N_p} = \frac{D_p}{D_m} \times \sqrt{\frac{H_m}{H_p}} \dots\dots\dots (4)$$

b) Specific flow:-

For reaction turbine

$$Q = \pi D B V_f$$

The dimensions D and B generally have linear relations with D_1 the runner diameter at inlet and therefore, since

$$V_f \propto \sqrt{H}$$

$$Q \propto D_1{}^2\sqrt{H}$$

$$or\ Q = Q_s\, D_1{}^2\sqrt{H}$$

$$\therefore\ Q_s = \frac{Q}{D_1{}^2\sqrt{H}} \dots \dots \dots (5)$$

$$D_1 = the\ outlet\ diameter\ of\ the\ runner\ m$$

$$H = net\ head\ \ m$$

$$Q = flow\ rate\ m^3/s$$

Also for Pelton turbine

$$Q_s = \frac{Q}{d_1{}^2\sqrt{H}} \dots \dots \dots (6)$$

$$d_1 = diameter\ of\ the\ jet\ (m)$$

Then for similarity low

$$Q_{sm} = Q_{sp}$$

$$or\quad \frac{Q_m}{Q_p} = \left(\frac{D_m}{D_p}\right)^2 \times \sqrt{\frac{H_m}{H_p}} \dots \dots \dots (7)$$

c) Specific Power:-

$$Since\ P_t = \gamma QH$$

Therefore, for reaction turbine

$$P_{ts} = \frac{P_t}{D_1{}^2 H^{3/2}} \dots \dots \dots (8)$$

And for Pelton turbine

$$P_{ts} = \frac{P_t}{{d_1}^2 H^{3/2}}$$

And for similarity low

$$\frac{P_m}{P_p} = \left(\frac{D_m}{D_p}\right)^2 \times \left(\frac{H_m}{H_p}\right)^{3/2} \dots\dots\dots (9)$$

Since the power P=Tw where T is the torque, therefore

$$\frac{T_m}{T_p} = \left(\frac{D_m}{D_p}\right)^2 \times \frac{H_m}{H_p} \dots\dots\dots (10)$$

$$Where\ w \propto \sqrt{H}$$

The scale ratio or model ratio: it is the ratio of the runner diameters of the prototype to the diameter of the model i.e

$$n = \frac{D_p}{D_m} \ \ or\ \frac{d_p}{d_m}$$

5.2 Cavitation in turbines

- Cavitation is a term used to describe a process which includes nucleation growth and implosion of vapor or gas filled cavities. These cavities are formed into a liquid when the static pressure of the liquid for one reason or another is reduced below its vapor presser at the prevailing temperature. When cavities are caved to high-pressure region, they implode violently.
- Cavitation is an undesirable effect that results in pitting, mechanical vibration and loss of efficiency.
- If the nozzle and buckets are not properly shaped in impulse turbine, flow separation from the boundaries may occur at same operating conditions that may cause regions of low pressure and result in cavitation.
- The turbine parts exposed to cavitation are the runner, draft tube cones for the Francis and Kaplan turbine and the needles, nozzle and the runner buckets of the Pelton turbines

- Measures for compating and damage under cavitation conditions include improvements in hydraulic design and production of components with erosion resistant materials and arrangement of the turbine for operations within good range of acceptable cavitation conditions.

Fig (1, a, b, c) show traveling bubble cavitations and damages in turbines

From the draft tube theory, it is found that the cavitation depends on the following:-

a) Vapor pressure, H_v which is a function of temperature of flowing water
b) Absolute pressure, $(H_a - H_v)$ or barometric pressure due to the location of turbine above mean sea level
c) Suction pressure, H_s which is the high of the runner outlet above the tailrace level.
d) Effective dynamic suction head and absolute of water at runner exit and V_2 respectively.

The dynamic head $=\eta_d\left(\frac{{V_2}^2-{V_3}^2}{2g}\right)$referring to draft tube.

According to Prof. Thoms cavitation can be avoided if the values of σ are not less than the critical value given by the manufacturer. i.e.

$$\sigma \leq \sigma_{critiol}$$

$$\sigma_{critiol} = \frac{\Delta H_t}{H} = \frac{\text{Theoretical head of the turbine}}{Working\ head\ of\ turbine\ (m)}$$

$$\Delta H_t = \frac{\Delta P}{\gamma} + \frac{{V_2}^2}{2g} + Z_2 + losses$$

Then the cavitation empirical formula as follows for Francis turbine

$$\sigma = 0.625\left(\frac{N_s}{100}\right)^2$$

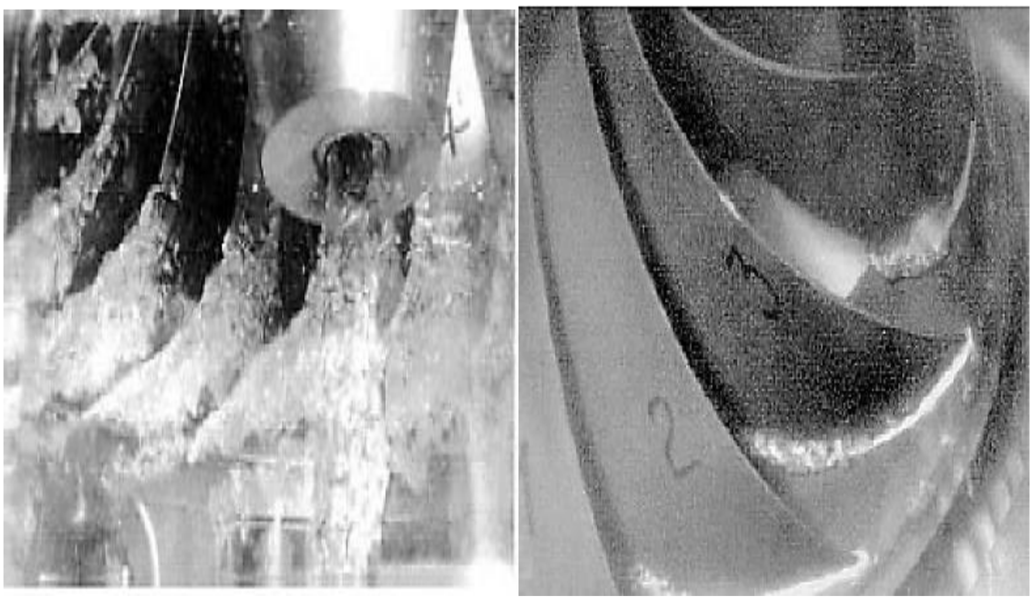
Kaplan turbine

$$\sigma = 0.28 + \frac{1}{7.5}\left(\frac{N_s}{100}\right)^2$$

General

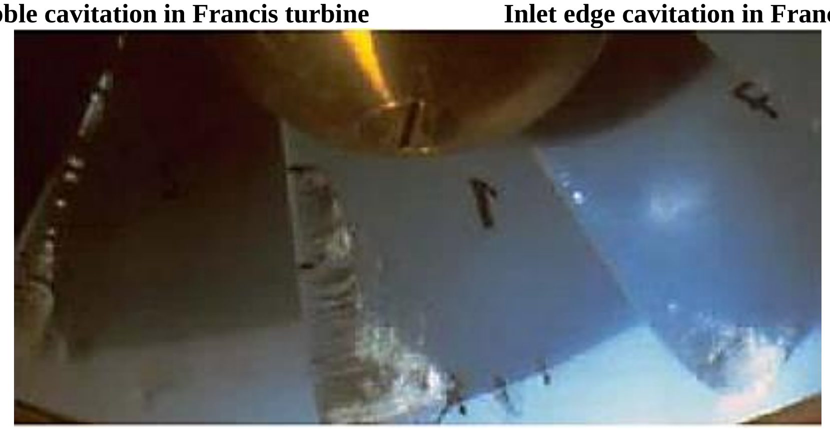
$$\sigma = \frac{H_a - H_v - H_s}{H_n}$$

Fig (2) show the relation between cavitation parameter and specific speed for the reaction turbines.



(a)
Traveling bubble cavitation in Francis turbine

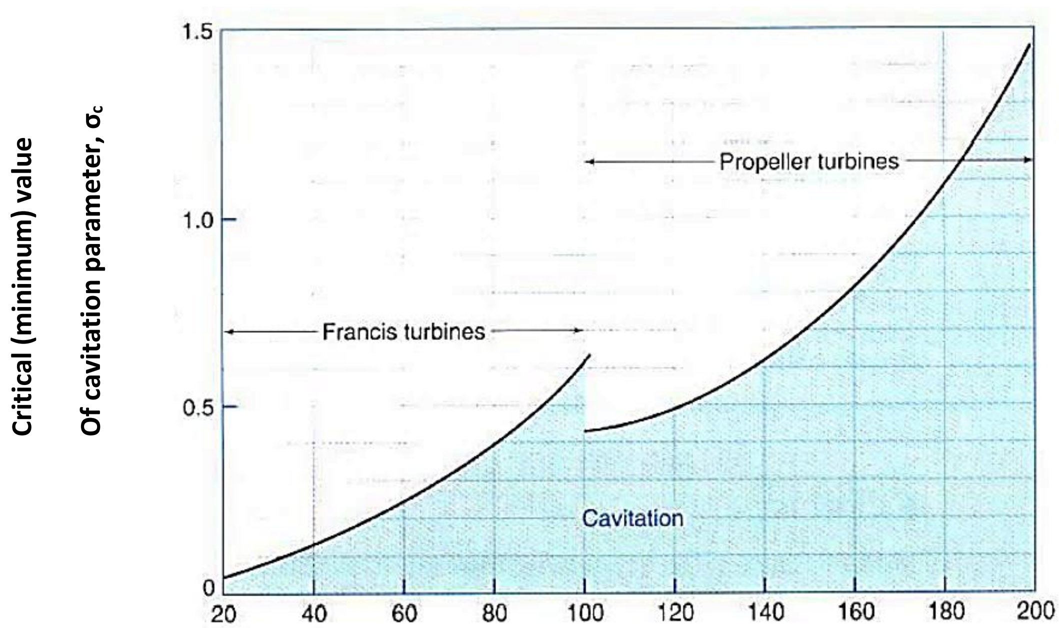
(b)
Inlet edge cavitation in Francis turbine



(c)
Leading edge cavitation damage in Francis turbine

Fig. (1) Cavitation in Turbine.

• The value of σ, at which cavitation will occur, is the critical value.
• Typical values of the critical cavitation parameter for reaction turbine are shown.



$$N_s = \frac{n_c\sqrt{bhp}}{H^{5/4}}$$

Fig. (2) Critical Value of Cavitation Parameter.

EX: A quarter scale turbine model is tested under a head of 36 m. the full scale turbine is required to work under a head of 100 m and to run at 428 rpm. At what speed must the model be run and if it develops 100 kW and uses 0.324 m^3/s of water at this speed. What power will be admitted from the fall scale turbine assuming that its efficiency is 3% better than that of the model?

<u>Solution :-</u>

$$H_m = 36\,m \;;\; H_p = 100\,m \;;\; N_p = 428\,rpm$$

$$Q_m = 0.324\,m^3/s \;;\; P_t = 100\,kW$$

$$scale\ ratio\ \ m = \frac{D_p}{D_m} = 4$$

$$Q_{sm} = Q_{sp}$$

$$\frac{Q_m}{{D_m}^2\sqrt{H_m}} = \frac{Q_p}{{D_p}^2\sqrt{H_p}}$$

$$or\ \ \ Q_p = Q_m\left(\frac{D_p}{D_m}\right)^2\left(\frac{H_p}{H_m}\right)^2$$

$$Q_p = 0.324(4)^2\left(\frac{100}{36}\right)^{\frac{1}{2}} = 8.65\frac{m^3}{s}$$

$$P_{tp} = \gamma Q_p H_p \mathfrak{y}_p$$

$$= 9.81 \times 8.65 \times 100 \times \mathfrak{y}_p$$

$$Since\ \ \mathfrak{y}_m = \frac{P_{tm}}{\gamma Q_m H_m} = \frac{100}{9.81 \times 0.324 \times 36} = 0.87$$

$$\mathfrak{y}_p = 0.87 + 0.03 = 0.9$$

$$\therefore\ \ P_{tp} = 7637\ kW$$

$$N_{sp} = N_{sm}$$

$$\frac{N_p\sqrt{P_{tp}}}{{H_p}^{\frac{5}{4}}} = \frac{N_m\sqrt{P_{tm}}}{{H_m}^{\frac{5}{4}}}$$

$$N_m = N_p \times \left(\frac{P_{tp}}{P_{tm}}\right)^{\frac{1}{2}} \times \left(\frac{H_m}{H_p}\right)^{\frac{5}{4}}$$

$$= 428 \times \left(\frac{7637}{100}\right)^{\frac{1}{2}} \times \left(\frac{36}{100}\right)^{\frac{5}{4}}$$

$$= 1027\ rpm$$

$$\therefore\ N_{sm} = 116.44$$

EX: A 11.77 MW reaction water turbine has effective head of 25 m. The runner is 3 m above the tail race. The turbine is installed at an elevation of 300 m above sea level when the barometric head is 9.64 m of water. The temperature of water is 27^0C. The following table give values if the plant sigma σ (Thoma's cavitation factor) against specific speed the turbine is coupled to a 50 cycles alternator. Calculate the synchronous speed of the turbine?

σ	0.05	0.10	0.16	0.228
N_s	116	155	193	230

Solution:-

$$P_t = 11.77\ kW\ ;\ H_n = 25\ m\ ;\ H_s = 3\ m\ ;\ H_b = 9.64$$

$$\therefore\ \sigma = \frac{H_b - H_s}{H_n} = \frac{9.64 - 3}{25} = 0.2646$$

The value of specific speed N_s corresponding to σ is found from the following curve drawn with help of given table. Thus $N_s \leqq 240$. It is clear from the curve that if $N_s > 240$, then cavitation appear.

$$Now\ \ N_s = \frac{N\sqrt{P_t}}{H^{\frac{5}{4}}} = \frac{N\sqrt{11770}}{(25)^{\frac{5}{4}}} = 240$$

$$or\ \ N = 123.5\ rpm$$

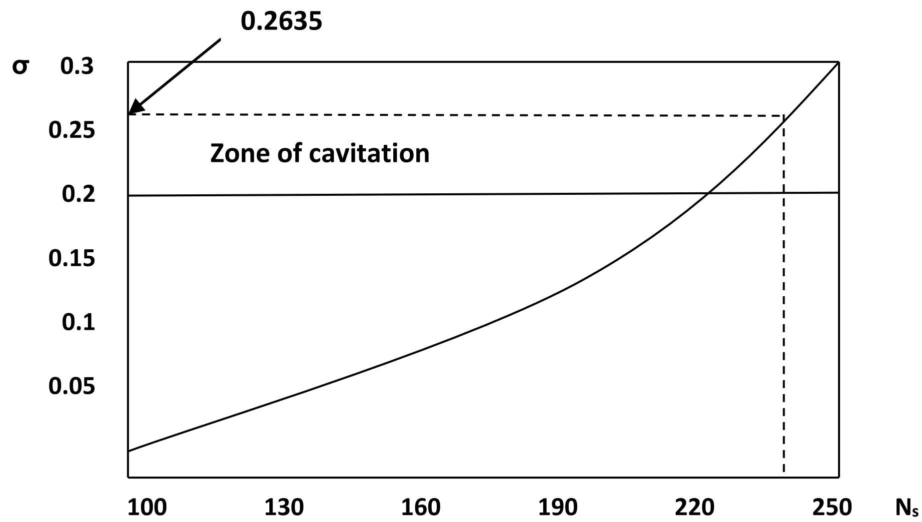
Assuming P, the number of pair poles for the alternator as 24, the synchronous speed can be found with help of

$$f = \frac{P.N}{60}$$

$$or\ \ 50 \times 60 = 24 \times N$$

$$\therefore\ \ N = 125\ rpm$$

Which is the nearest synchronous speed to 123.5 rpm.



σ Versus N_s

EX: In a hydroelectric station water is available at the rate of 175 m^3/s under a head of 18 m. if the available turbine run at a speed of 150 rpm. With overall efficiency of 82%. Find the minimum number of turbine of the same size required in case of

(i) Francis turbine with max specific speed of 260

(ii) Kaplan turbine of N_s =350

Solution:-

$$Q = 175 \frac{m^3}{s} \;\; ; \; H_n = 18\, m \;\; ; N = 150 \;\; ; \; \eta_0 = 0.82$$

$$\therefore \eta_0 = \frac{P_{out}}{P_{in}} = \frac{P_t}{\gamma Q H_n}$$

$$P_t = 0.82 \times 9.81 \times 175 \times 18 = 25339\, kW$$

i) No. of Francis turbine

$$\therefore\ N_s = \frac{N\sqrt{P_t}}{H^{\frac{5}{4}}} = 260 = \frac{150\sqrt{P}}{(18)^{\frac{5}{4}}}$$

$$\therefore\ P_t = 4129\ kW$$

$$\therefore No. of\ turbine = \frac{25339}{4129} = 6.135 \approx 7$$

Also for Kaplan turbine

$$N_s = 350 = \frac{150\sqrt{P}}{(18)^{\frac{5}{4}}}$$

$$\therefore\ P_t = 7484\ kW$$

$$\therefore No. of\ turbine = \frac{25339}{7484} = 3.38 \approx 4$$

5.3 Turbine selection

The selection of turbine is an essential stage in designing a HEPP when operating conditions of the turbine (head, power) must be taken into account, especially the layout of the power house, the supply and tail water conduits, construction and service conditions, engineering and cost data.

Some turbine elements are of such great size, that almost always they prescribe the dimension and layout of the power house of HEPP. The interrelation between the turbine and the building structure acquires a greater importance

A HEPP unit consists of two machines- a turbine and a generator, they have a common system of bearings.

The turbine runner and generator rotor are rigidly connected by common shaft, the rotate with an equal speed.

The synchronous speed n_{syn} is determined by two formulae

For 50 H_z n_{syn}=6000/P

For 60 H_z n_{syn}=7200/P

Where P is the number of generator poles that must be even, and at P>24 it is desirable to be multiple to four. Thus, a number of n_s values is defined and intermediate values are impossible.

5.4 Marking Types of Turbine

The nomenclatures permit the selection of turbines when designing a HEPP. Fig (3) shows the nomenclature (range chart) of large adjustable blade (axial flow) and radial turbine. It is clear that the nomenclature corresponds to heads ranging from 3-4 to 500 m, and powers from 1-2 MW to 300-800 MW. Fig (4) demonstrates the previously used nomenclature of small turbine corresponding to heads ranging from 1.5 to 100 m and powers within 10-2500 MW.

An abbreviated marking of turbine is often used with the mark including four indices.

1. The kind of a turbine is denoted by letters:

II JT- adjustable – blade axial turbine (AB in fig 3)

II or II JT TS- adjustable – blade mixed-flow turbine (ABM)

II JT K- adjustable – blade bulb-type turbine (aBB)

PO- radial-axial turbine (RA in Fig 3)

II P- Propeller-type wheel axial turbine (PA)

II P A- Propeller-type mixed-flow turbine (PM)

K- Pelton turbine (P)

2. The type of turbine is mainly determined by the head. Several types of turbine of the same head are available, differing only by the form of the water passage. Each type of turbine is assigned a serial No. that is given.

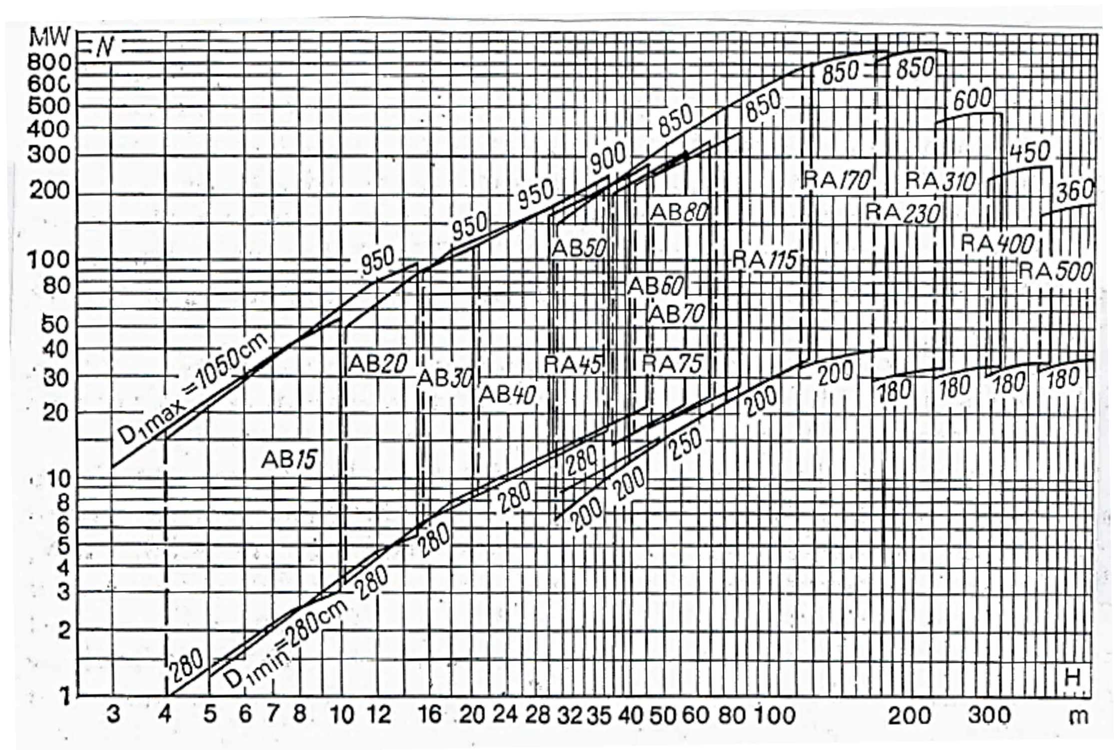


Fig. (3) Nomenclature of axial-flow adjustable – blade and radial-axial flow turbines.

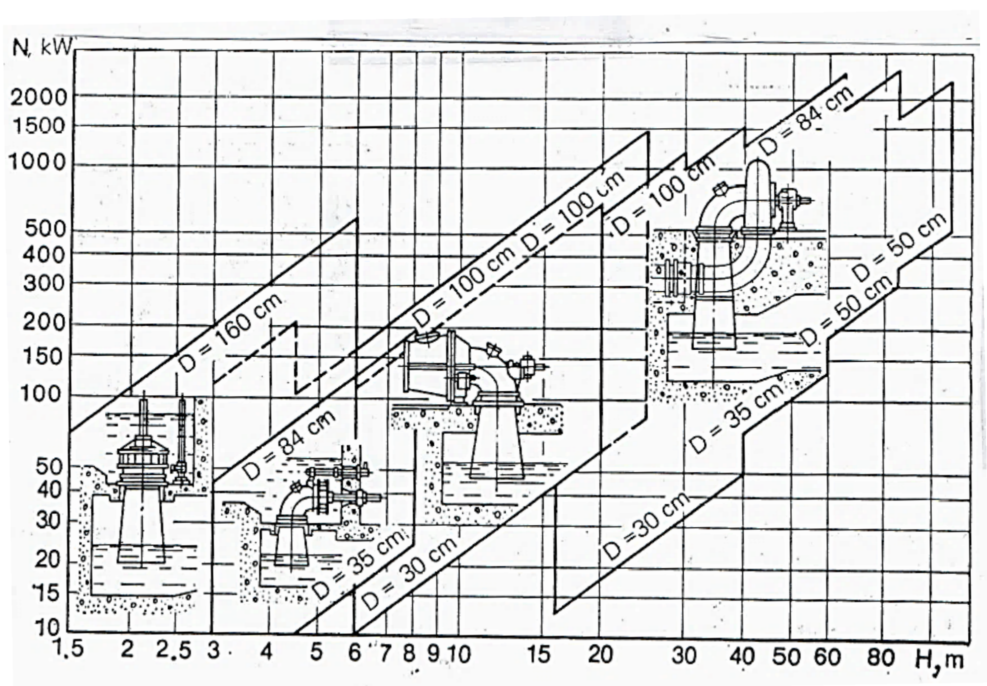


Fig. (4) Nomenclature of small turbines.

In the turbine marking (sometimes as a fraction: the numerator is the maximum head, and the denominator the type of turbine.

3. The layout is determined by the position of the shaft of the turbine-generator set and it can be vertical B or horizontal Γ.

4. Nominal turbine diameter D_1 (Fig. 5, 6). For diagonal turbine the runner blade angle Ө and for Pelton turbine the nozzle diameter and the number of jets are given.

Example of turbine markings:

II JT 20/811-B-800 is the marking of an axial-flow adjustable – blade turbine, maximum head is 20 m, type of water passage No. 811, B-vertical D_1 =8 m.

II 120/45-2556-B-600 mixed flow adjustable – blade Turbine, maximum head is 120 m, Ө=45^o type No. 2556, B-vertical, nominal dia. D_1 =6 m.

II JT K 15/548- -600 is the marking of an axial-flow adjustable – blade Bulb-type turbine, maximum head is 15 m, the type of water Passage No. 548, -horizontal Γ diameter=6 m.

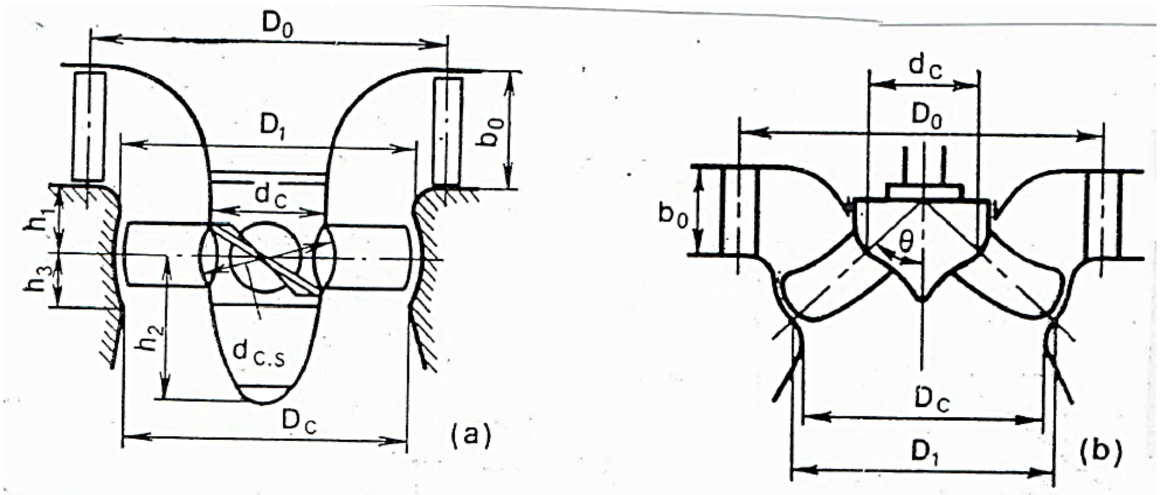


Fig. (5) Basic dimensions of adjustable- turbine:

(a) axial- flow; (b) mixed-flow.

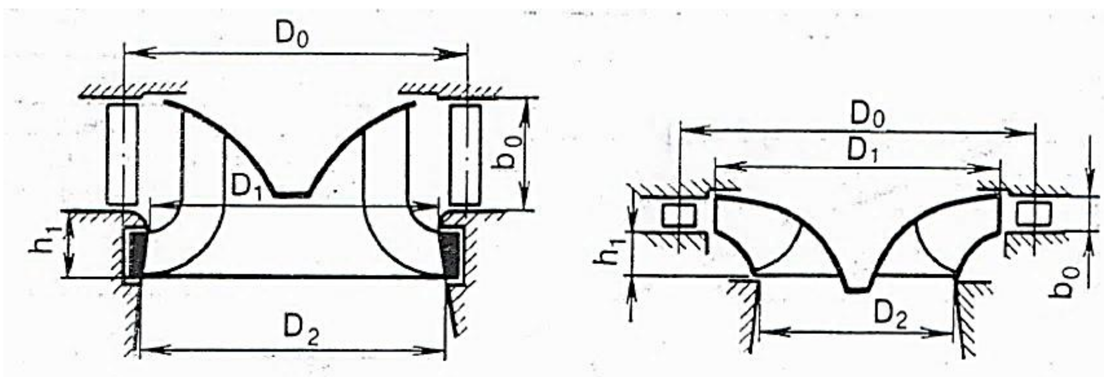


Fig. (6) Basic dimensions of radial –axial flow turbines.

5.5 Hydraulic turbines classification and selection Fig (7)

The following table gives an overview of reference values of specific speed and stage reaction for different hydraulic turbines.

The specific speed increases as flow rate increases and hydraulic head decreases.

Therefore, turbines with high specific speed have also high values of stage reaction, because work exchange between fluid and runner decreases if R increases. Where R is the ratio of piezometric head charge in the runner and draft tube and the total piezometric head charge.

	k	n_s	R
Pelton 1 jet	0.05 ÷ 0.2	5 ÷ 10	0
Pelton 2 jets	0.1 ÷ 0.3	7 ÷ 14	0
Pelton(>2 jets)	0.3 ÷ 0.4	14 ÷ 20	0
Francis ("slow")	0.3 ÷ 0.6	15 ÷ 33	0.30
Francis ("medium")	0.6 ÷ 1.0	33 ÷ 55	0.40
Francis ("fast")	1.0 ÷ 1.6	55 ÷ 80	0.50
Francis ("ultrafast")	1.6 ÷ 2.3	80 ÷ 120	0.60
Propeller, Kaplan	1.4 ÷ 5.7	75 ÷ 300	0.70

Where

$$K = w\frac{Q^{1/2}}{(gH)^{3/4}} \qquad N_s = N\frac{\sqrt{P_t}}{H^{5/4}} \qquad R = \frac{\Delta H_{p.r}}{\Delta H_p}$$

Reference value of working parameters and output for the main types of hydraulic turbines shown as follows.

- Pelton:
  - Flow rate ~ 0.5 ÷20 m^3/s
  - Head ~ 300 ÷1500 m
  - Net power up to ~ 200 MW
- Francis:
  - Flow rate ~ 2 ÷800 m^3/s
  - Head ~ 500 ÷400 m
  - Net power up to ~ 800 MW
- Kaplan:
  - Flow rate ~ 1000 ÷20 m^3/s
  - Head ~ 40 m
  - Net power up to ~ 200 MW

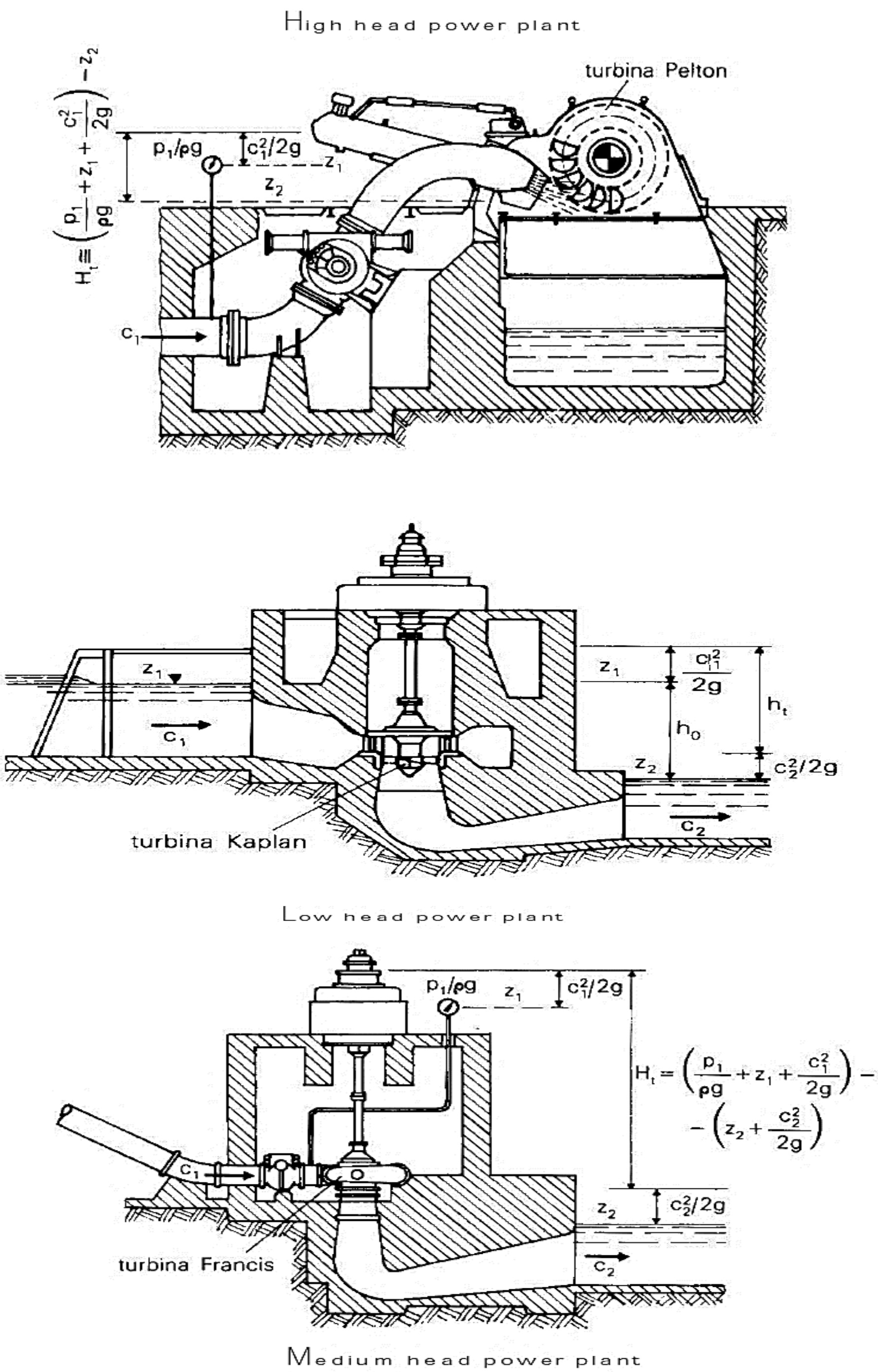


Fig. (7) Hydraulic Turbine Classification.

The relation between the net head of a turbines and the power generated show in Fig (8).

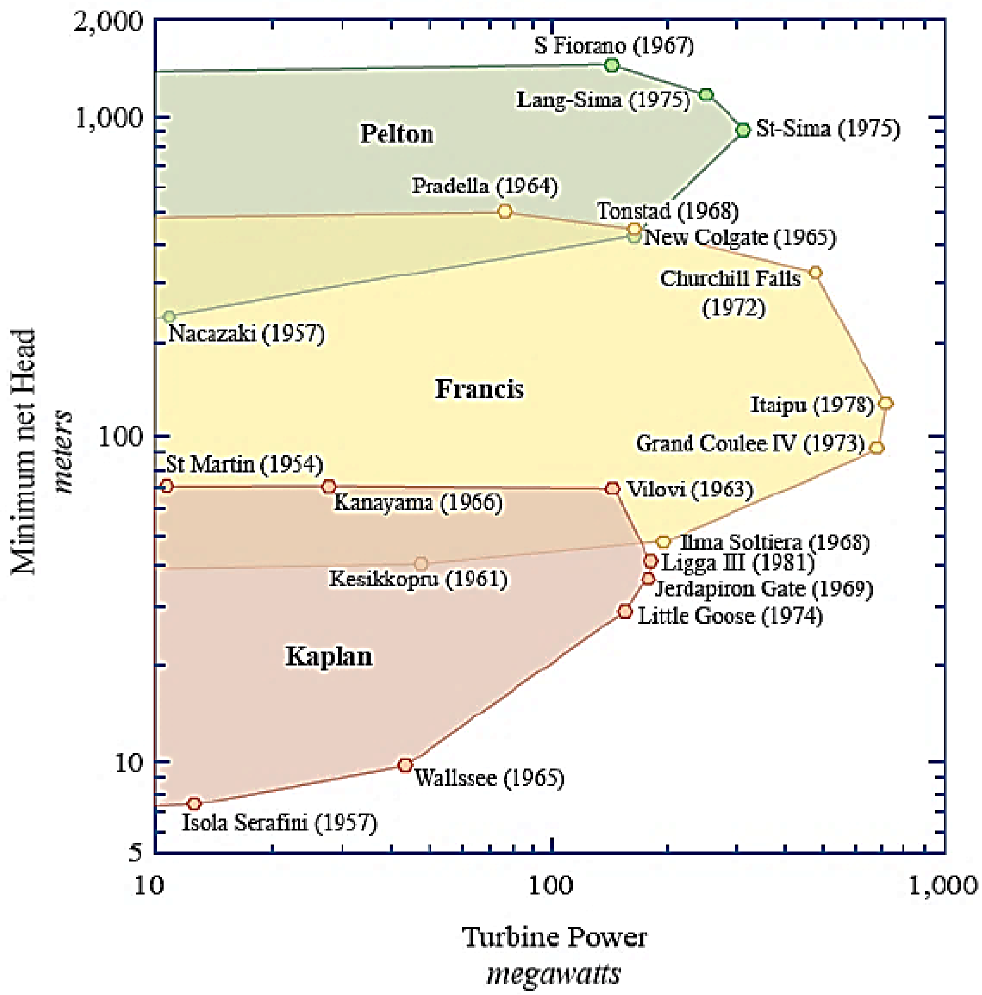


Fig. (8) Relation between the minimum net head and the power of turbines.

The most common and preferred type of turbine is the Francis turbine. It is used to generate about 60% of the global hydropower in the world making them the most widely used type of turbine. The following charts shows a comparison between all the turbine and how to select each one according to the head and flow rate also the power consumed by the turbine.

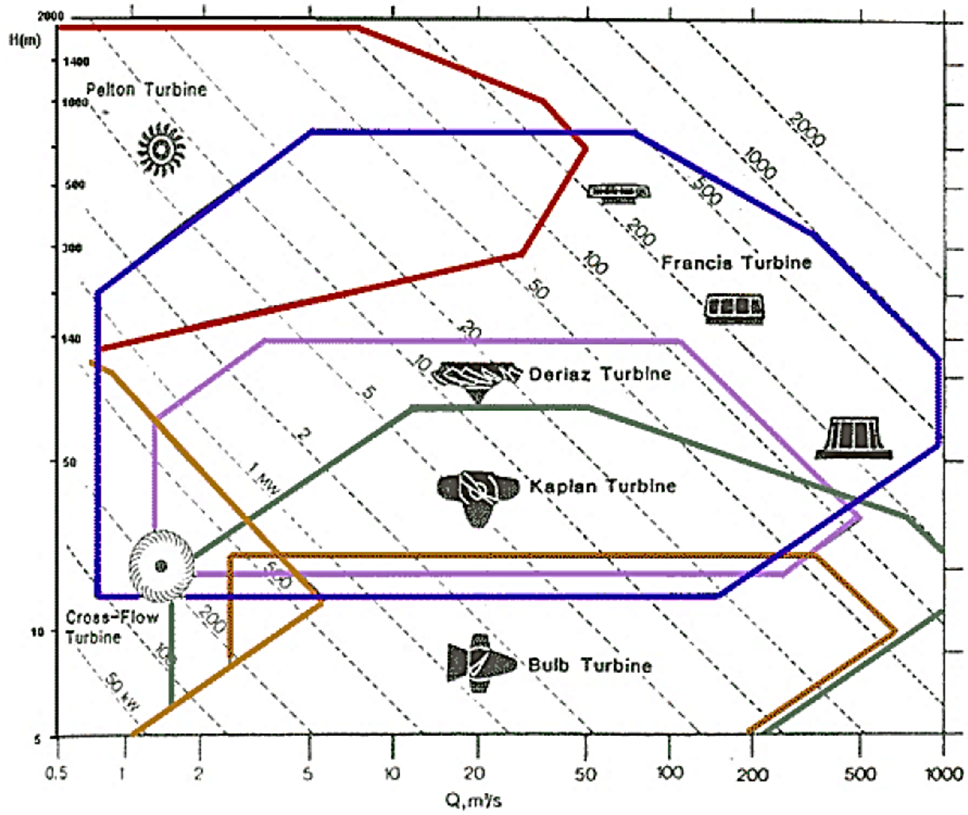
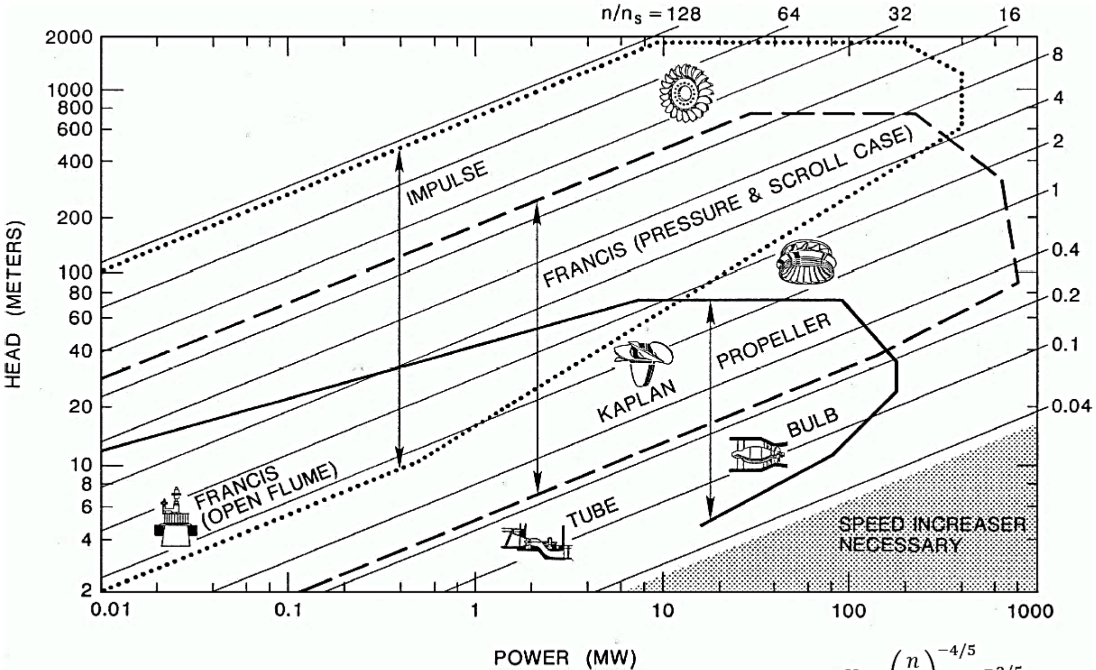


Chart -1-

Turbine selection chart of turbine types.

Figure from Heinzmann Hydro Tech Private Limited India.

Hydraulic Turbine Classification.



$$H = \left(\frac{n}{n_s'}\right)^{-4/5} P^{2/5}$$

Chart -2-

Specific speed expressed as $nP^{1/2}H^{-5/4}$

Source: John S. Gulliver, Roger E.A. Arndt, *Hydroelectric Power Stations, In: Encyclopedia of Physical Science and Technology (Third Edition)*, Academic Press, New York, 2003, Pages 489-504, ISBN 9780122274107, 10.1016/B0-12-227410-5/00321-5. (http://www.sciencedirect.com/science/article/pii/B0122274105003215)

Energy Systems - Hydraulic turbines and hydroelectric power plants

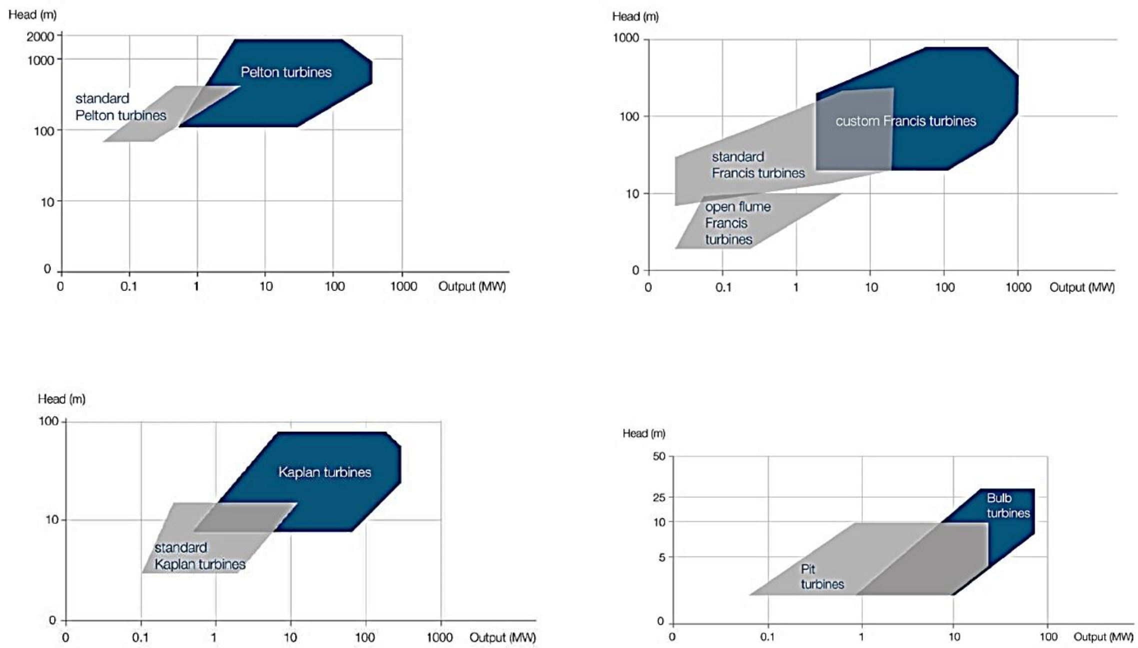


Chart -3-
Hydraulic Turbines Classification

Source: Voith-Siemens

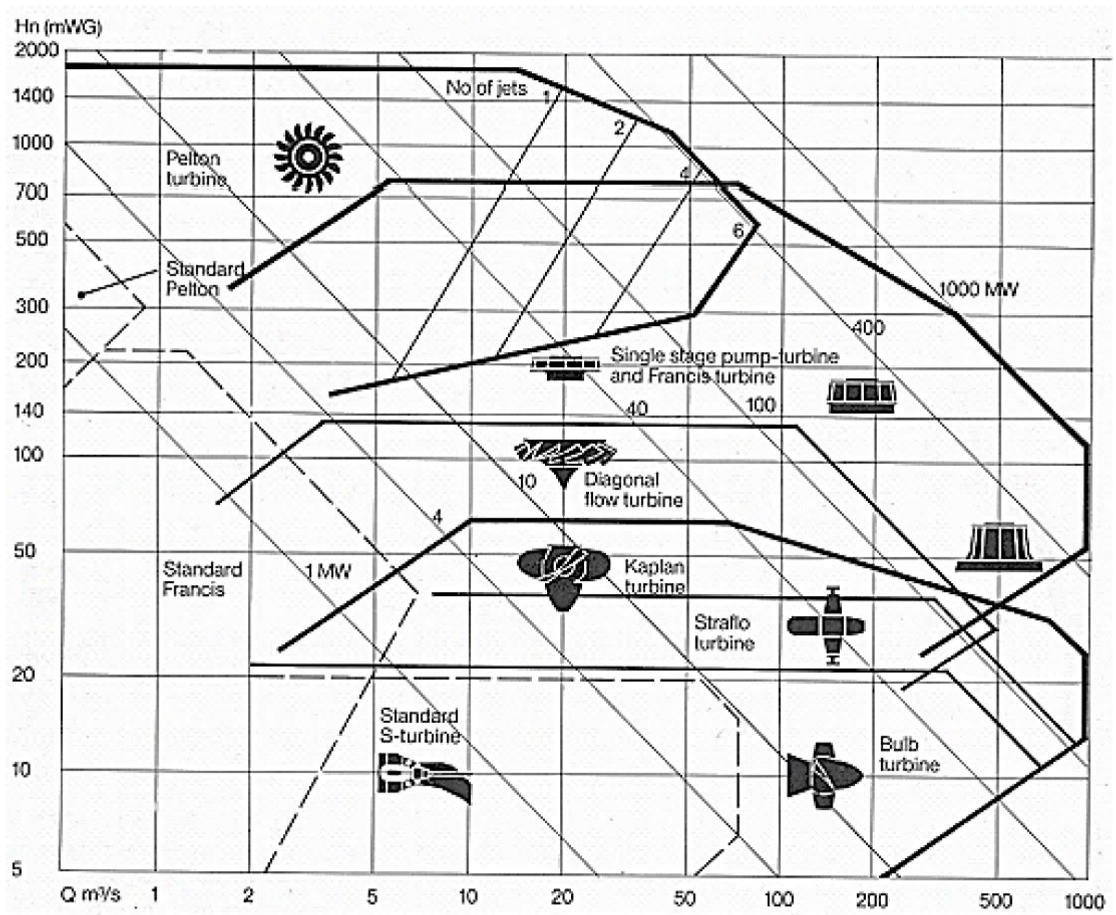


Chart -4-

Hydraulic Turbine Selection

Solved Turbine Problems

Q.1- A power house is equipped with two water turbine each developing 2100 kW when running at 250 rpm under a maximum head of 23.4 m

a) Calculate the specific speed of the turbine.

b) state suitable type of the turbine and its runner

c) Determine approximately the inlet diameter of the runner if the speed coefficient Ө=0.8

d) Draw the typical inlet outlet triangles velocities.

(22.26, Francis, fast runner, 1.3)

Solution:-

$$P_t = 2100\ kW\ \ ; N = 250\ rpm\ \ ; H = 23.4\ m$$

a) $N_s = \frac{N\sqrt{P_t}}{H^{\frac{5}{4}}} = \frac{250\sqrt{2100}}{(23.4)^{\frac{5}{4}}} = 222.57\ m - kWunit$

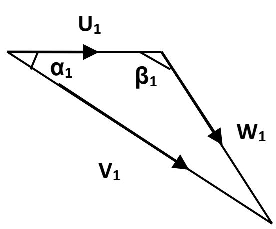
b) $For\ N_s\ 50 \rightarrow 250\ \ Francis\ turbine\ is\ employed, Fast\ runner$

c) $Speed\ ratio\ \emptyset = 0.8$

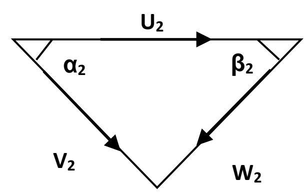
$$\therefore\ \ U_1 = \emptyset\sqrt{2gH}\ \ and\ \ \ U_1 = \frac{\pi DN}{60}$$

$$\therefore\ D_1 = \frac{60 \times 0.8\sqrt{2 \times 9.81 \times 23.4}}{\pi \times 250} = 1.3\ m$$

d)



Inlet



Outlet

Q.2- A Kaplan turbine having an overall efficiency of 75% required to give 130 kW. The head is 6 m, velocity of periphery of runner is $0.8\sqrt{2gH}$ and the radial velocity of flow is $0.35\sqrt{2gH}$. the runner is to make 250 rpm and the hydraulic losses in the turbine 20% of the available energy. Determine

a) The angle of the guide blade at inlet.

b) The runner vane angle at inlet.
c) The diameter of the runner
d) The width of the runner at inlet.
Assuming radial discharge $\alpha_2 = 90°$

Solution:-

$$P_t = 310\ kW\ \ ; N = 250\ rpm\ \ ; H = 6\ m$$

$$\eta_t = 0.95 \quad peripherel\ velocity$$

$$U_1 = 0.8\sqrt{2gH} = 0.8\sqrt{2 \times 9.81 \times 6}$$

$$\therefore U_1 = 8.68\ m/s$$

$$Velocity\ of\ flow\ \ V_{f1} = V_{f2} = V_2 \quad radial\ discharge$$

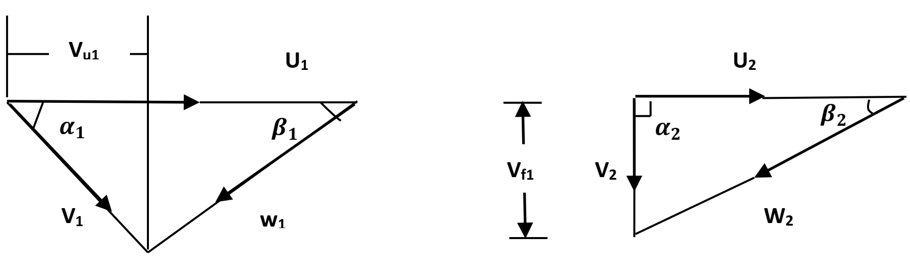
$$= 0.35\sqrt{2gH} = 0.35\sqrt{2 \times 9.81 \times 6} = 3.8\ \ m/s$$

$$losses = 0.2\ \ \text{Hydraulic energy}$$

$$As\ the\ available\ energy\ per\ unit\ weight\ of\ water = H\ m$$

$$and\ \eta_h = \frac{V_{u1}U_1}{gH} = 0.8$$

$$\therefore\ V_{u1} = \frac{9.81 \times 6 \times 0.8}{8.68} = 5.42\ m/s$$



Inlet velocity triangle **Outlet velocity triangle**

a) angle of guide vane at inlet α_1

$$\alpha_1 = \tan^{-1}\frac{V_{f1}}{V_{u1}} = \tan^{-1}\frac{3.8}{5.42} = 35.03°$$

b) runner vane angle at inlet β_1

$$\tan\beta_1 = \frac{V_{f1}}{U_1 - V_{u1}} = \frac{3.8}{8.68 - 5.42} = 1.165$$

$$\therefore\ \beta_1 = 49.36°$$

c) $U_1 = \frac{\pi D_1 N}{60}$ $\quad or \quad 8.68 = \frac{\pi \times D_1 \times 250}{60}$

$$D_1 = 0.663$$

d) with of the runner at inlet β_1

$$P_t = \gamma Q H \eta_t \qquad \therefore Q = \frac{130}{9.81 \times 6 \times 0.75} = 2.94\, m^3/s$$

$$and \qquad Q = \pi D_1 B_1 V_{f1}$$

$$\therefore\ B_1 = \frac{2.94}{\pi \times 0.663 \times 3.8} = 0.371\, m$$

Q.3- a reaction turbine runner is required to operate under a head of 10 m at a speed of 175 rpm and to develop 175 kW. Find the diameter of the runner at inlet and outlet, the discharge, the guide vane angle and the runner vane angle at inlet and outlet, assuming the following data:

Outlet diameter = 0.66× inlet diameter

Discharge radial $\alpha_2 = 0$; $U_1 = 0.75\sqrt{2gH}$; $V_{f1} = 0.16\sqrt{2gH}$; $\eta_h = 0.86$; $\eta_t = 0.81$

(1.146 m, 0.75 m, 1.885 m^3/s, 42.75^0, 18.6^0)

<u>Solution:-</u>

$$P_t = 150\, kW \ ;\ N = 175\, rpm \ ; H = 10\, m$$

$$D_2 = 0.66 D_1 \quad ;\ V_{f1} = V_{f2} = 0.16\sqrt{2gH}$$

$$U_1 = 0.75\sqrt{2gH} \quad ;\ \eta_h = 0.86 \ ;\ \eta_t = 0.81$$

a) diameter of the runner D_1

$$U_1 = 0.75\sqrt{2 \times 9.81 \times 10} = \frac{\pi D_1 N}{60} = 10.5\, m/s$$

$$and \quad \therefore D_1 = \frac{60 \times 10.5}{\pi \times 175} = 1.146\, m$$

b) $D_2 = 0.66 D_1 = 0.756\, m$

c) $Q = \frac{P_t}{\gamma H \eta_t} = \frac{150}{9.81 \times 10 \times 0.81} = 1.887\, m^3/s$

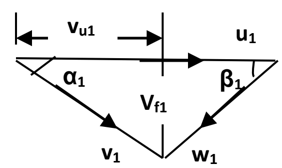
d) $\eta_h = \frac{V_{u1} U_1}{gH} \quad for \quad \alpha_2 = 90°$

$$\therefore \; V_{u1} = \frac{0.86 \times 9.81 \times 10}{10.5} = 8.03\, m/s$$

$$V_{f1} = 0.16\sqrt{2gH} = 0.16\sqrt{2 \times 9.81 \times 10} = 2.24\, m/s$$

e)

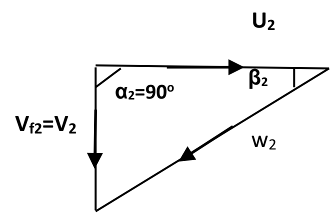
$$\tan \beta_1 = \frac{V_{f1}}{U_1 - V_{u1}} = \frac{2.24}{10.5 - 8.03}$$



$\therefore \; \beta_1 = 42.2°$

f) Since

$V_{f1} = V_{f2} = 2.24\, m/s$



$$and \quad U_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.756}{60} = 6.93\, m/s$$

$$\tan \beta_2 = \frac{V_{f1}}{U_2} = \frac{2.24}{6.93} = 0.323$$

$$\therefore \; \beta_2 = 179°$$

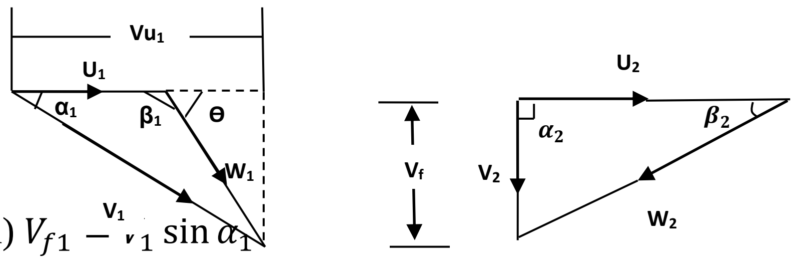
Q.4 A turbine works under a head of 20 m and makes 380 rpm. The outer and inner diameters are 60 cm and 30 cm respectively. The velocity of water entering the runner at 63 m/s and makes an angle of 10^0 with tangent. Assuming radial discharge. Determine the direction of the tangent to the vane of runner at inlet and outlet, also the hydraulic efficiency.

(110.5^0, 21^0, 77.5 %)

<u>**Solution:-**</u>

$$H = 20\, m \; ; N = 380\, rpm \; ; \; D_1 = 0.6\, m \; ; \; D_2 = 0.3$$

$$V_1 = 13 \frac{m}{s} \; ; \; \alpha_1 = 10° \; ; \; V_f = constant \; ; \; \alpha_2 = 90°$$



a) $V_{f1} = V_1 \sin\alpha_1$

$$= 13 \sin 10° = 2.26\ m/s$$

$$V_{u1} = V_1 \cos\alpha_1 = 13 \cos 10° = 12.8\ m/s$$

$$U_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.6 \times 380}{60} = 11.94\ m/s$$

$$\tan\theta = \frac{V_{f1}}{V_{u1} - U_1} = \frac{2.26}{12.8 - 11.94} = 2.628$$

$$\therefore\ \theta = 69.2°$$

b) $\tan\beta_2 = \frac{V_{f1}}{U_2}$ $\qquad V_{f2} = V_{f1} = 2.628\ m/s$

$$U_2 = \frac{\pi D_2 N}{60} = \frac{U_1}{2} = 5.97\ m/s$$

$$\therefore\ \tan\beta_2 = \frac{2.628}{5.97}$$

$$\beta_2 = 20.73°$$

$$\eta_h = \frac{V_{u1} U_1}{gH} = \frac{12.8 \times 11.94}{9.81 \times 20} = 77.9\%$$

Q.5- Find the leading dimensions of the runner of a Francis turbine to develop 625 kW at 1000 rpm under a head of 100 m assuming a guide vane angle of 16^0, axial length of the vane at inlet 0.1 times the outer diameter, inner radius 0.6 of outer radius, radial velocity of flow constant, Find discharge radial, hydraulic efficiency 0.88, overall efficiency 0.86, allowance thickness for vane thickness 5%(D_1 =0.527 m, D_2 = 0.316 m, B_1=5.27 cm, β_1 = 112.0^0, β_2 =31.62^0)

Solution:-

$$P_t = 625\ kW\ ;\ N = 1000\ rpm\ ; H = 100\ m$$

$$D_2 = 0.6 D_1\quad ;\ V_{f1} = V_{f2}\ ; K = 0.95$$

$$B_1 = 0.1 D_1\quad ;\ \eta_h = 0.88\ ;\ \eta_t = 0.86$$

$$\alpha_1 = 16°\ ;\quad \alpha_2 = 90°$$

$$\therefore \quad P_t = \gamma Q H \eta_t \quad or \quad 625 = 9.81 \times Q \times 100 \times 0.86$$

$$Q = 0.741 \frac{m^3}{s} = \pi D_1 B_1 V_{f1} K$$

$$= \pi D_1 \times 0.1 D_1 \times V_{f1} \times 0.95$$

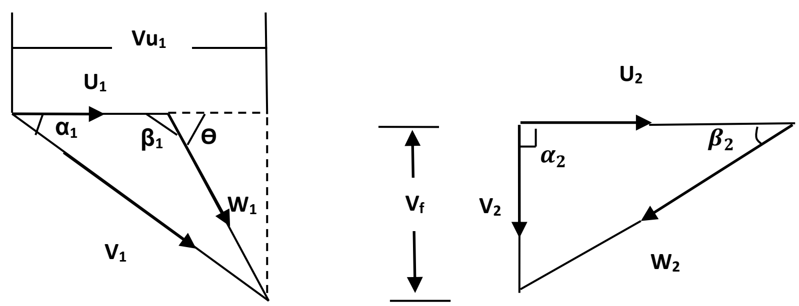
$$V_{f1} = \frac{2.486}{{D_1}^2} \ \ldots\ldots\ldots (1)$$

$$\eta_h = \frac{V_{u1} U_1}{gH} = 0.8 \quad for \quad \alpha_2 = 90° \ldots\ldots\ldots (2)$$

$$and \quad U_1 = \frac{\pi D_1 N}{60} = \frac{\pi D_1 \times 1000}{60} = 52.36 D_1 \, m/s$$

And substituting in (2)

$$0.8 = \frac{V_{u1} \times 52.36 D_1}{9.81 \times 100} \qquad \therefore \quad V_{u1} = \frac{16.49}{D_1} \, m/s$$



From inlet velocity triangle $\tan \alpha_1 = \frac{V_{f1}}{V_{u2}}$

$$\tan 16° = \frac{2.486}{{D_1}^2} \qquad \therefore \quad D_1 = 0.526 \, m$$

$$\therefore D_2 = 0.6 D_1 = 0.316 \, m$$

And Brealth of runner at inlet

$$B_1 = 0.1 D_1 = 0.0526 \, m$$

Inlet

$$and \quad \tan \theta = \frac{V_{f1}}{V_{u1} - U_1} \qquad V_{u1} = \frac{16.49}{D_1} = \frac{16.49}{0.526}$$

$$\therefore V_{u1} = 31.35 \, m/s$$

$$V_{f1} = \frac{2.486}{{D_1}^2} = 8.985\ m/s$$
$$U_1 = 52.36D_1 = 27.54\ m/s$$
$$\therefore \tan\theta = \frac{8.985}{31.35 - 27.54} = 2.358$$
$$\theta = 67°$$
$$\therefore \beta_1 = 180 - 67 = 113°$$

Outlet

$\tan\beta_2 = \frac{V_{f2}}{U_2}$ $\quad$ $since\ V_{f1} = V_{f2} = 8.985\ m/s$

$U_2 = 0.6U_1 = 0.6 \times 27.54 = 16.52\ m/s$

$\therefore\ \beta_2 = \tan^{-1}\frac{8.985}{16.52} = 28.5°$

$\therefore\ Dimension\quad D_1 = 0.526\ m \quad D_2 = 0.316\ m$

$B_1 = 0.0526\ m \quad \beta_1 = 113°\ ;\ \beta_2 = 28.5°$

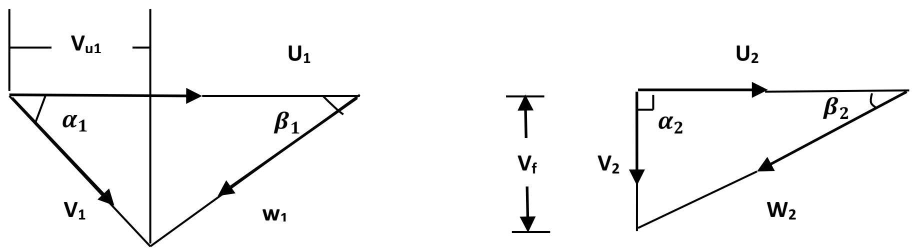
Q.6- the following data refers to reaction turbine Q = 1 m^3/s; H = 25 m; D_1 =75 m; D_2 =50 m; V_{f2}=2.5 m/s; β_1 = 35^0, Calculate the power and rpm of the turbine. Assuming the width of the runner as constant and turbine efficiency 80%, hydraulic efficiency 82%.
(196.3 kW, 349.5 rpm)

Solution:-

$$Q = 1\ \frac{m^3}{s}\ ; H = 25\ m\ ;\ D_1 = 75\ cm\ ; D_2 = 50\ cm$$
$$V_2 = 2.5\frac{m}{s}\ ;\ \alpha_2 = 90°\ ;\ \beta_1 = 35°$$
$$B_1 = B_2\ ;\ \eta_h = 80\%\ ;\ \eta_t = 82\%$$

a) $\eta_t = \frac{P_t}{\gamma Q H}$ $\quad \therefore\ P_t = 0.8 \times 9.81 \times 25 \times 1 = 196.3\ kW$

b)



Inlet velocity triangle **Outlet velocity triangle**

$$sine\ \ B_1 = B_2$$

$$and\ \ \therefore Q = \pi D_1 B_1 V_{f1} = \pi D_2 B_2 V_{f2}$$

$$\therefore\ D_1 V_{f1} = D_2 V_{f2}$$

$$and\ \ V_{f2} = V_2 \sin\alpha_2 = 2.5\ m/s$$

$$V_{f1} = \frac{0.5 \times 2.5}{0.75} = 1.67\ m/s$$

$$\eta_h = \frac{V_{u1} U_1}{gH}$$

$$\therefore V_{u1} U_1 = 0.82 \times 9.81 \times 25 = 201.11 \ldots\ldots\ldots (1)$$

And from velocity triangle at inlet

$$\tan\beta_1 = \frac{V_{f1}}{U_1 - V_{u1}}$$

$$\therefore U_1 - V_{u1} = \frac{1.67}{\tan 35°} = 2.385 \ldots\ldots\ldots (2)$$

From 1, 2

$$U_1 - \frac{201.11}{U_1} = 2.385$$

$$or\ \ {U_1}^2 - 2.385 U_1 - 201.11 = 0$$

$$U_1 = \frac{-2.385 \pm \sqrt{(-2.385)^2 - 4(-201.11 \times 1)}}{2}$$

$$= \frac{-2.385 \pm \sqrt{5.69 + 884.44}}{2}$$

$$U_1 = 13.725 = \frac{\pi D_1 N}{60}$$

$$N = 349.5\ rpm$$

Q.7- A 233 l/s are supplied to a reaction turbine. The head available is 11 m. the wheel vanes are radial at inlet and the inlet diameter is twice the outlet diameter. The velocity of flow is constant and equal to 1.83 m/s. The runner makes 370 rpm Find:

a) guide vane angle

b) runner vane angle

c) inlet and outlet diameters of the runner. And

d) the width of the runner at inlet and outlet.

Neglect the thickness of the vanes (K=1). Assuming that the discharge is radial and that there are no losses in the runner.

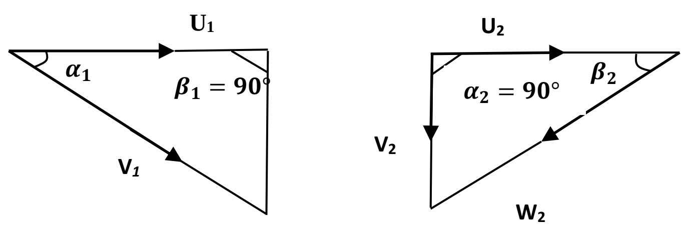
Solution:-

$$Q = 0.233\ \frac{m^3}{s}\ \ ; H = 11\, m\ \ ;\ D_1 = 2D_2$$

$$\alpha_2 = 90°\, ;\ \beta_1 = 90°$$

$$V_{f1} = V_{f2} = 1.83\frac{m}{s}\ ; N = 370\, rpm$$

$$No\ lossos\ \ in\, runner. \quad V_{f1} = V_{f2}$$



Inlet velocity triangle **Outlet velocity triangle**

a) Work done by the runner per unit weight

$$\frac{V_{u1}U_1}{g} = H - \frac{{V_2}^2}{2g} \qquad\qquad V_2 = V_{f2}$$

$$\therefore\ \ \frac{{U_1}^2}{g}11 - \frac{(1.83)^2}{2g}$$

$$U_1 = 10.3\, m/s$$

Now

$$\tan\alpha_1 = \frac{V_{f1}}{U_1} = \frac{1.83}{10.3}$$

$$\alpha_1 = 10°$$

b) $since\ \ \beta_1 = 90°\ \ radial\ at\ inlet$

c) $U_1 = \frac{\pi D_1 N}{60} \quad \therefore \quad D_1 = \frac{10.3 \times 60}{\pi \times 370} = 0.532\,m$

d) $D_2 = \frac{1}{D_1} = 0.266\,m$

e) $Q = \pi D_1 B_1 V_f$

$$0.233 = \pi \times 0.532 \times B_1 \times 1.83$$

$$B_1 = 0.0762\,m = \frac{1}{2}\,B_2$$

$$or\ B_2 = 2B_1 = 0.1524\,m$$

Q.8- A turbine runner has an exit velocity of 10 m/s. the loss of head due to friction and other causes in the draft tube should not exceed 1.5 m. what maximum height of setting will you recommend for the turbine if the cavitation is to be avoided?

Assuming:

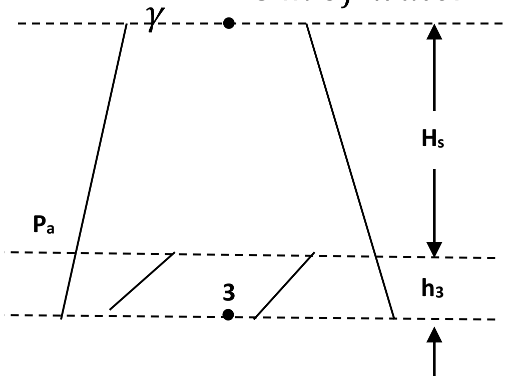
i) The velocity of water at the outlet of draft tube as 2.5 m/s.

ii) The cavitation commences when the pressure is 2.5 m of water absolute. (4.52 m)

Solution:-

$$V_2 = 10\frac{m}{s}\ ;\ H_{L_{2-3}} = 1.5\,m\ ;\ V_3 = 2.5\frac{m}{s}$$

$$\frac{P_V}{\gamma} = 2\ \ 5\,m\,of\,water$$



Apply B.E between 2, 3

$$\frac{P_2}{\gamma}+\frac{{V_2}^2}{2g}+Z_2=\frac{P_3}{\gamma}+\frac{{V_3}^2}{2g}+Z_3+H_{L_{2-3}}$$

$$\frac{P_2}{\gamma}=\frac{P_3}{\gamma}-(Z_2-Z_3)-\frac{\left({V_2}^2-{V_3}^2\right)}{2g}+H_{L_{2-3}}$$

$$\frac{P_3}{\gamma}=\frac{P_a}{\gamma}+h_3 \quad and\ Z_2-Z_3-h_3=H_s$$

$$\frac{P_2}{\gamma}=\frac{P_a}{\gamma}-H_s-\frac{(10^2-2.5^2)}{2g}+1.5 \quad since\ \frac{P_2}{\gamma}=\frac{P_V}{\gamma}$$

$$2.5=10.3-H_s-4.778+1.5$$

$$H_s=4.52\ m$$

Q.9 the following data were obtained from an efficiency test of Kaplan turbine:-

Diameter of the boss of runner =0.35 times external diameter

Speed ratio Ø =2 flow ratio Ψ =0.6

Power output = 13250 kW Head = 8 m

Find the efficiency of the turbine (81%)

Solution:-

$$D_2=0.35D_1\ ;\ \emptyset=2\ ;\ \varphi=0.6$$

$$N=75\ rpm\ ; H=8\ m\ :\ P_t=13250\ kW$$

$$since\ V_f=0.6\sqrt{2gH}=0.6\sqrt{2\times 9.81\times 8}$$

$$=7.5\ m/s$$

$$U_1=\emptyset\sqrt{2gH}=2\sqrt{2\times 9.81\times 8}$$

$$=25.06\ m/s$$

$$U_1=\frac{\pi D_1 N}{60} \quad \therefore\ D_1=\frac{25.06\times 60}{\pi\times 75}$$

$$D_1=6.38\ m$$

$$\therefore Q=\frac{\pi}{4}\left({D_1}^2-{D_2}^2\right)V_f$$

$$=\frac{\pi}{4}((6.38)^2-(0.32\times 6.38)^2)\times 7.5$$

$$=210\ m^3/s$$

$$\eta_t=\frac{P_t}{\gamma QH}=\frac{13250}{2\times 9.81\times 8}=80.3\%$$

Q.10- Kaplan turbine installed at power house develops 1300 kW at average head of 29 m. the speed and flow ratios are 2.1 and 0.62 respectively. d = 0.35 D_1. Overall efficiency 0.89 calculate the diameter and the speed of the runner (0.705 m, 1350 rpm)

Solution:-

$$D_2 = 0.34D_1 \ ; \ \emptyset = 2.1 \ ; \ \varphi = 0.62$$

$$\eta_t = 0.81 \ ; \ H = 29\,m \ : \ P_t = 1300\,kW$$

$$\therefore Q = \frac{P_t}{\gamma H \eta_t} = \frac{1300}{9.81 \times 29 \times 0.81} = 5.13\,m^3/s$$

$$and \quad V_f = 0.62\sqrt{2gH} = 0.6\sqrt{2 \times 9.81 \times 29}$$

$$= 14.787\,m/s$$

$$and \quad Q = \frac{\pi}{4}\left(D_1{}^2 - D_2{}^2\right)V_f$$

$$5.13 = \frac{\pi}{4}\left(D_1{}^2 - (0.34D_1)^2\right) \times 14.787$$

$$D = 0.707\,m$$

$$U_1 = \emptyset\sqrt{2gH} = 2.1\sqrt{2 \times 9.81 \times 29}$$

$$= 50\,m/s$$

$$U_1 = \frac{\pi D_1 N}{60} \qquad \therefore \ N = \frac{50 \times 60}{\pi \times 0.707} = 1350\,rpm$$

Q.11- Each Kaplan turbine at Vargon (Sweden) in rated at 11350 kW when working under an average head of 4.3 m. the diameter of the hub is 0.3 times the external diameter of runner. Overall efficiency of turbine is 0.91. Find the speed and diameter of runner. Take the values of Ø = 2 and Ψ = 0.65 respectively. (42.2 rpm, 8.38 m).

Solution:-

$$D_2 = 0.3D_1 \ ; \ \emptyset = 2 \ ; \ \varphi = 0.65$$

$$\eta_t = 0.91 \ ; \ H = 4.3\,m \ : \ P_t = 11250\,kW$$

a) $$Q = \frac{P_t}{\gamma H \eta_t} = \frac{11250}{9.81 \times 4.3 \times 0.91} = 296\,m^3/s$$

$$V_f = \varphi\sqrt{2gH} = 0.65\sqrt{2 \times 9.81 \times 4.3}$$

$$= 5.97\,m/s$$

$$and \quad Q = \frac{\pi}{4}\left(D_1{}^2 - D_2{}^2\right)V_f$$

$$296 = \frac{\pi}{4}\left({D_1}^2 - (0.3D_1)^2\right) \times V_f$$

$$D_1 = 8.33\ m$$

b) $$U_1 = \emptyset\sqrt{2gH} = U_1 = \frac{\pi D_1 N}{60}$$

$$= 2\sqrt{2 \times 9.81 \times 4.3} = \frac{\pi \times 8.33N}{60}$$

$$\therefore \quad N = 42.1\ rpm$$

Chapter six
Pumps

6.1 Centrifugal pumps

All types of pumps depend on the change of momentum during the flow over the impeller across the blades which called C.P.

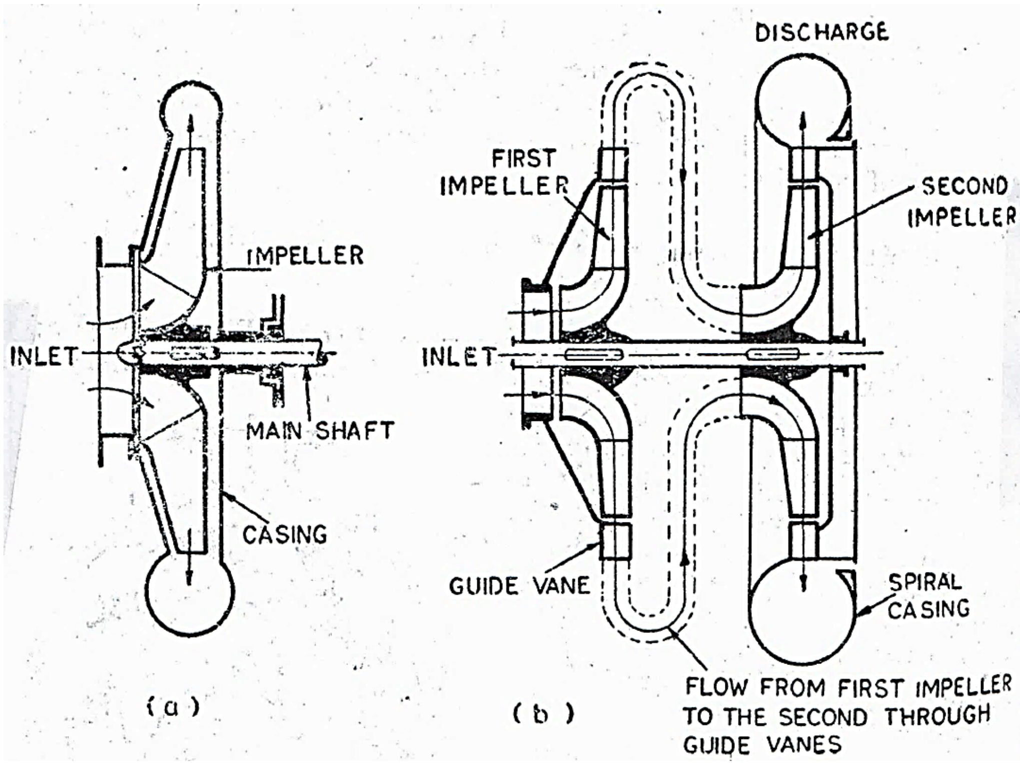
The basic principle of the C.P. is the blades or impellers rotating inside a closed fitting housing draw the liquid into the pump through a central inlet opening and by means of centrifugal force or change in momentum in the liquid outward through a discharge outlet at the periphery of the housing, which means that changes the kinetic head to pressure head.

6.2 Classification of C.P.

1. Working head

It is the head at which water is delivered by the pump depending on the number of stages Fig (1).

types of pumps	Working head m	
Low lift C.P.	Up to 15 m	Single stage
Medium lift C.P.	$15 \rightarrow 40$	
High lift C.P.	>40 m	Multistage



a) Single stage centrifugal pump

b) Multi stage (two stage centrifugal pump)

Fig. (1) Single and multistage diagram.

2. Type of casing

Pump casing should be so designed as to minimize the loss of kinetic head through the eddy formation etc. Efficiency of the pump Longley depends on the type of casing Fig (2).

a) Volute casing (spiral casing):-

In the kind a gradual increase in the area of flow, then decreasing the velocity of water and corresponding increasing the pressure

A considerable loss takes place due to the formation of eddies. Fig. (2-a)

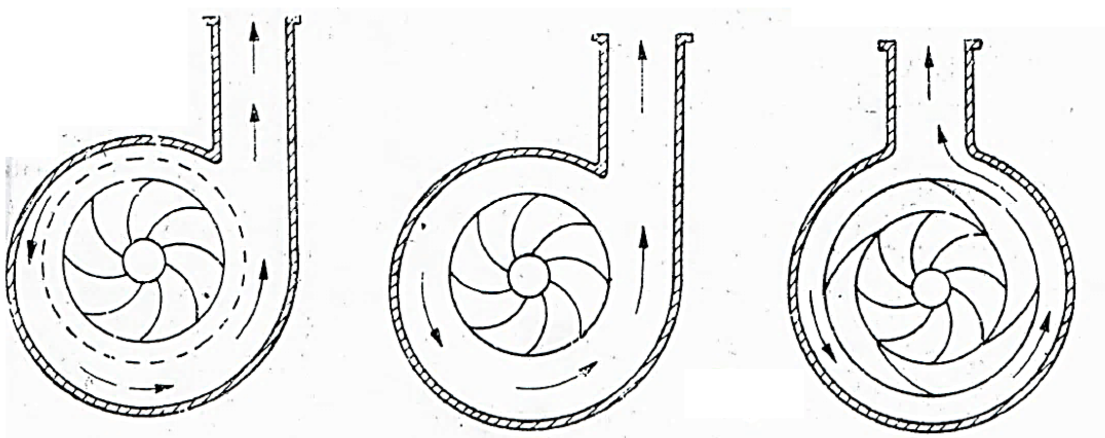
b) Vortex casing:-

it is an improved type of a volute casing in which the spiral casing is combined with circular chamber as shown in Fig(2-b). In vortex casing, the eddies are reduced to a considerable extent and an increased efficiency.

c) Volute casing with guide blades:-

There are guides surrounding the impeller as in Fig (2-c). These guide blades are arranged at such an angle, that the water enters without shock and forms a passage of increasing area, through which the water passes and reaches the delivery pipe.

The ring of the guide blades is called diffuser and is very efficient.



b) Vortex casing a) Volute casing c) Volute casing with guide blades

Fig. (2) Types of casing.

3. Number of impeller per shaft:-

a) Single stage C.P. it has one impeller keyed to the shaft.

b) Multi-stage C.P. it has two or more impellers keyed to single shaft enclosed in the same case.

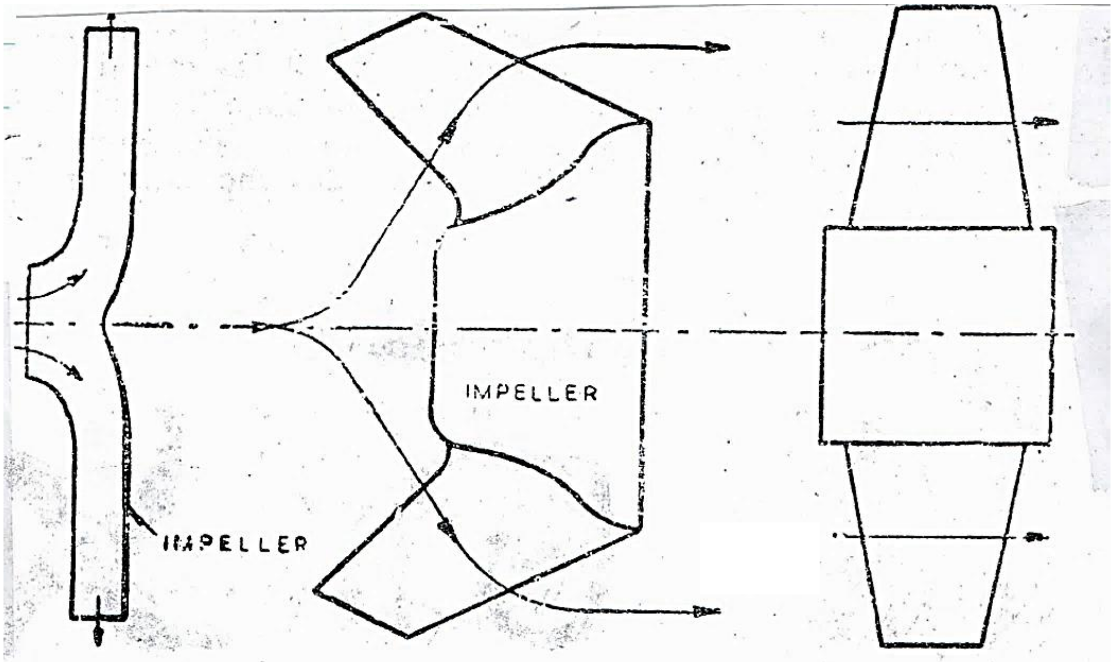
4. Relative direction of flow through impeller:-

a) Radial flow pump.

b) Mixed flow pump.

c) Axial flow pump.

Fig. (3) show the flow direction through the impellers.



a) Radial flow b) Mixed flow c) Axial flow

Fig. (3) Flow direction through the impeller.

5. Number of entrance to the impeller:-

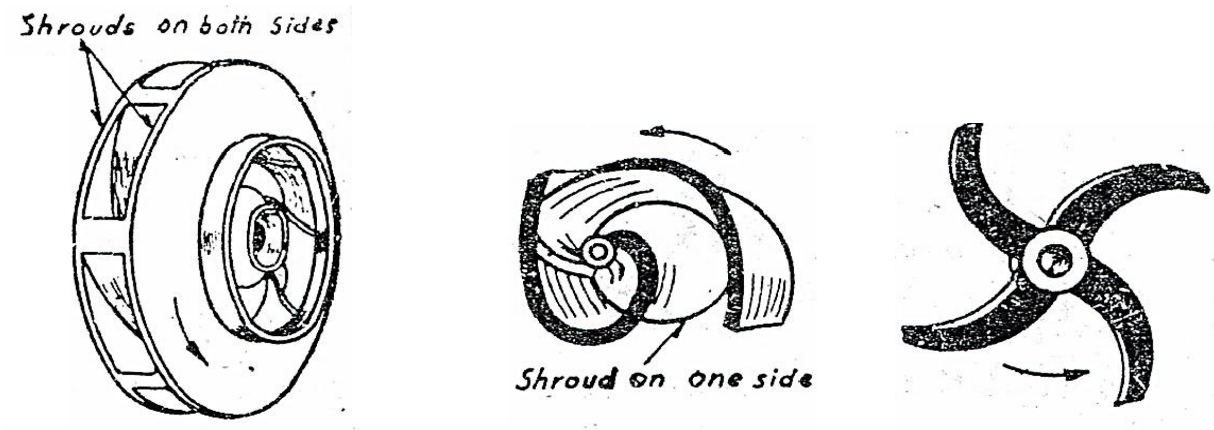
The pump may be single or double entry type. In single entry the water inters the impeller from suction pipe on one side, while for double entry admit from both side.

6. Disposition of the shaft:-

The shaft may be disposed horizontally or vertically.

7. Liquid handled:-

Depending on the type and viscosity of liquid to be pumped, the pump may have a closed or opined impeller Fig (4). Each types may have ferrous, nonferrous or stone coated impeller to resist chemical attack of liquid being pumped.



a) Closed type impeller b) Semi-open type impeller c) open type impeller

Fig. (4) Type of impellers.

8. Specific speed:- N_s

Is defined as the speed of a geometrically similar pump when delivering one m^3/s against a head of one meter then

$$N_s = \frac{N\sqrt{Q}}{H^{3/4}}$$

$$N = speed\ rpm$$

$$H = \ head\ m$$

$$Q = flow\ rate\ m^3/s$$

Note:- the value of H used in this equation as a single stage, i.e. for multistage should be divided by the No. of stages.
Also flow rate used as single suction i.e. for double suction should be divided by (2).

6.3 Theory of centrifugal pump

- Fundamental Equation of centrifugal pump:-
  Let point on the liquids path at suction inlet, impeller inlet, impeller outlet denoted by i, 1, 2, d as shown in Fig (5) below:-

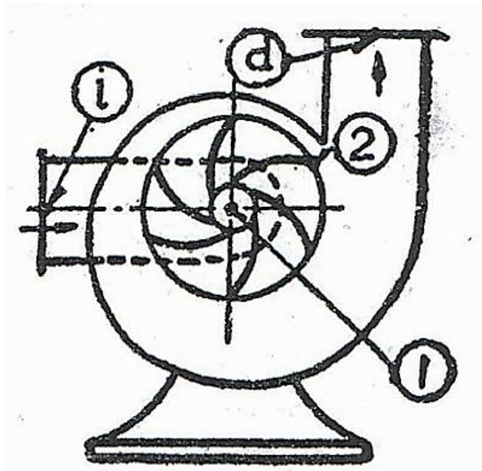


Fig. (5) Liquid flow through a centrifugal pump.

a) Between i & 1, through the stationary suction pipe V_i, V_1 the absolute velocities.

$$\frac{{V_i}^2}{2g}+\frac{P_i}{\gamma}+Z_i=\frac{{V_1}^2}{2g}+\frac{P_1}{\gamma}+Z_1+H_{L_{i-1}} \dots\dots\dots (1)$$

b) Between 1 & 2 through the movable impeller and W_1+W_2 the relative velocities.

$$\frac{{W_1}^2}{2g}+\frac{{U_1}^2}{2g}+\frac{P_1}{\gamma}+Z_1=\frac{{W_2}^2}{2g}+\frac{{U_2}^2}{2g}+\frac{P_2}{\gamma}+Z_2+H_{L_{1-2}} \dots\dots\dots (2)$$

c) Between 2 & d also

$$\frac{{V_2}^2}{2g}+\frac{P_2}{\gamma}+Z_2=\frac{{V_d}^2}{2g}+\frac{P_d}{\gamma}+Z_d+H_{L_{2-d}} \dots\dots\dots (3)$$

From 1, 2, 3 we get

$$\frac{V_2{}^2 - V_3{}^2}{2g} + \frac{W_1{}^2 - W_2{}^2}{2g} + \frac{U_2{}^2 - U_1{}^2}{2g} = \left[\left(\frac{V_d{}^2}{2g} + \frac{P_d}{\gamma} + Z_d\right) - \left(\frac{V_i{}^2}{2g} + \frac{P_i}{\gamma} + Z_i\right)\right] + \left(H_{L_{i-1}} + H_{L_{1-2}} + H_{L_{2-d}}\right) \dots\dots\dots (4)$$

The first terms on the right hand side by definition is the total manometric head and equal to H_{mono}, and the second term is the total losses due to fluid resituate inside the pup only [(2)]
i.e.

$$\frac{V_2{}^2 - V_1{}^2}{2g} + \frac{W_1{}^2 - W_2{}^2}{2g} + \frac{U_2{}^2 - U_1{}^2}{2g} = H_{mono} + \Delta H_{mono} \dots (5)$$

This is known as the fundamental equation of C.P.
Considering the losses of head in the pump, its efficiency

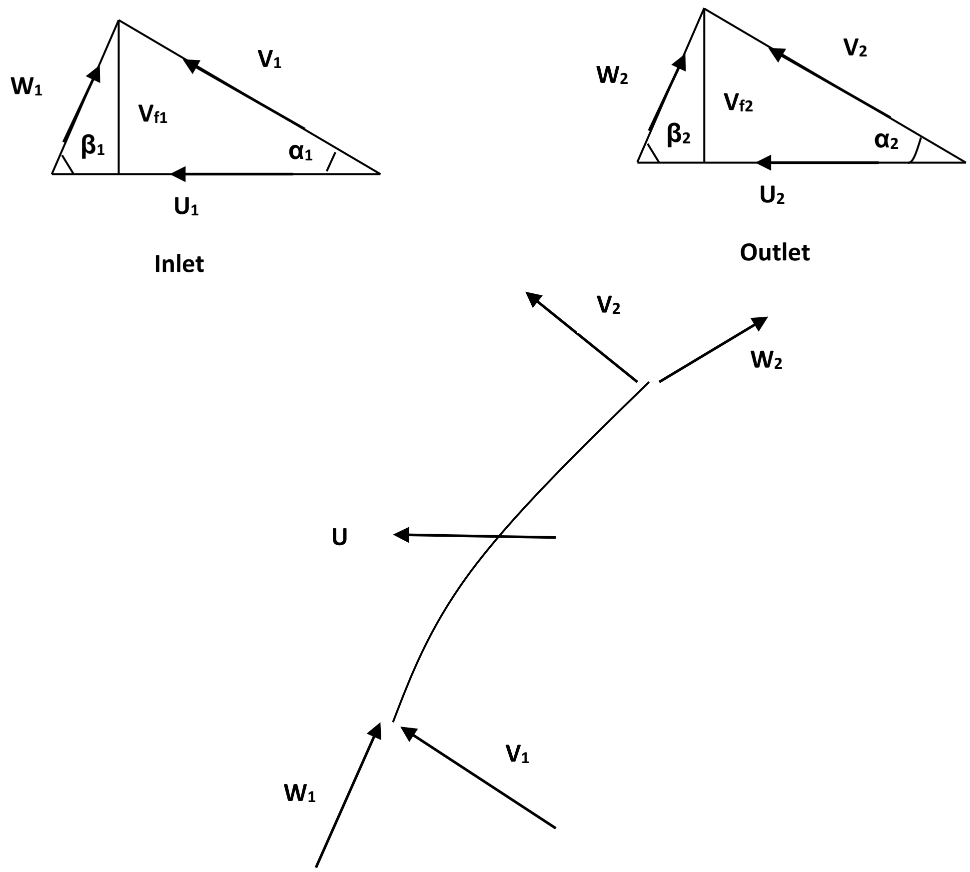
$$\eta_{mono} = \frac{H_{mono}}{H_{mono} + \Delta H_{mono}} = monometric\ efficiency$$

$$or \quad H_{mono} + \Delta H_{mono} = \frac{H_{mono}}{\eta_{mono}}$$

Therefore

$$\frac{V_2{}^2 - V_1{}^2}{2g} + \frac{W_1{}^2 - W_2{}^2}{2g} + \frac{U_2{}^2 - U_1{}^2}{2g} = \frac{H_{mono}}{\eta_{mono}} \dots (6)$$

This equation can be simplified by substituting for W_1+W_2 from velocity triangle at inlet and outlet.



$$W_1^{\ 2} = U_1^{\ 2} + V_1^{\ 2} - 2U_1V_1\cos\alpha_1$$

$$W_2^{\ 2} = U_2^{\ 2} + V_2^{\ 2} - 2U_2V_2\cos\alpha_2$$

$$W_1^{\ 2} - W_2^{\ 2} = U_1^{\ 2} - U_2^{\ 2} + V_1^{\ 2} - V_2^{\ 2} - 2U_1V_1\cos\alpha_1$$

$$+2U_2V_2\cos\alpha_2 \ldots\ldots\ldots(7)$$

From equation 6, 7 we get

$$\frac{H_{mono}}{\eta_{mono}} = \frac{2U_2V_2\cos\alpha_2 - 2U_1V_1\cos\alpha_1}{2g}$$

And $V \cos \alpha = V_u$ also in general $\alpha_1 = 90°$

Radial flow at inlet i.e $\cos \alpha_1 = 0$

$$\therefore \eta_{mono} = \frac{gH_{mono}}{U_2 V u_2} \dots \dots \dots (8)$$

6.4 Head of the pump

The term head of a pump stands for the following: - Fig. (6)

a) Static head: - it is difference between elevation of upper and lower reservoirs, or the sum of the suction and delivery head.

$$H_{static} = H_{\text{suction}} + H_{\text{delivery}}$$

b) Manometric head:- it is the head measured across the pump inlet and outlet flanges. It expresses the increase in pressure energy per unit weight of liquid handled by the impeller.

$$\eta_{mono} = \frac{P_d - P_s}{\gamma} + h_g$$
$$= H_{mono(d)} - H_{mono(s)} + h_g$$

$h_g =$ The vertical distance between the pressure tapping for the suction and delivery gauge.

Not: in general, the suction pressure is negative (-)

The manometric head including all losses against which the pump has to work except the kinetic head.

c) Total or gross or effective head:- it is the actual head against which the pump has to work

$$H_p = \frac{P_d - P_s}{\gamma} + h_g + \frac{{V_d}^2 - {V_s}^2}{2g}$$

$V_d =$ Velocity of flow at delivery pipe

$V_s =$ Velocity of flow at suction pipe

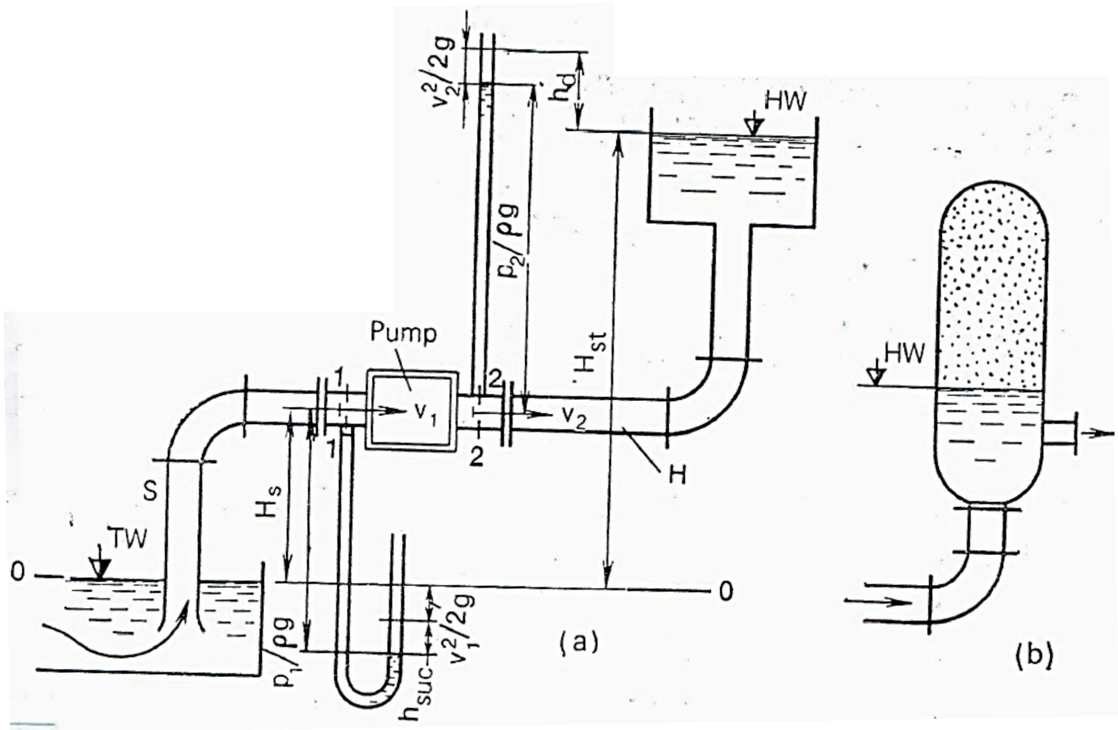


Fig. (6) Schematic diagram of pump installation.

i.e. the defiance between the manometric head and the total head is the kinetic head at it is comparatively much less than in practical, then we take the manometric head is equal to the total head.

$$\therefore\ H_{mono} = H_{total} = H_{static} + H_L$$

Where
$H_L =$ The total losses of head

6.5 Force and Power

The force due to momentum change in the impeller is

$$F = \dot{m}(V_{u2} - V_{u1}) \qquad \dot{m} = \rho Q$$

$\therefore Power = F.U\ \ power$ delivered by the impeller

The output power of the pump $= \gamma Q H_p$

$$\therefore\ \eta_{mono} = \frac{\rho\, Q(V_{u2}U_2 - V_{u1}U_1)}{\gamma QH}$$

For radial inlet flow $\alpha_1 = 90°$ & $\cos\alpha_1 = 0$

$$\therefore\ \eta_{mono} = \frac{V_{u2}U_2}{gH}$$

If these is no losses (theoretically) then

$$\frac{V_{u2}U_2}{g} = H_{mono} \text{ The work done per unit mass}$$

6.6 Efficiencies of C.P.

a) Overall efficiency:

$$\eta_{overall} = \frac{Fluid\ power\ outlet}{Power\ input\ by\ the\ shaft}$$

$$\eta_{o.} = \frac{\gamma QH_{mono}}{SKW}$$

$$P_{shaft} = P_{input\ to\ the\ impeller} + P_{leakage} + P_{mech.loss}$$

$$P_{input\ to\ the\ impeller} = energy\ given\ to\ impeller\ per\ kW$$
$$= \frac{V_{u2}U_2}{g}$$

b) Mechanical efficiency: it is the ratio of power delivered by the impeller to the fluid to the power input to the shaft.

$$\eta_{mech} = \frac{SKW - P_{mech}}{SKW}$$

$$P_{mech.loss}: Power\ required\ to\ overcome\ all\ mechanical\ losses\ i.e\ fraction\ losses, Bearing\ and\ glands\ losses.$$

c) Volumetric efficiency:

$$\eta_Q = \frac{Q}{Q + \Delta Q}$$

$Q = discharge\ developed\ by\ the\ pump$

$\Delta Q = amount\ of\ leakage$

d) Manometric efficiency:

$$\eta_{mono} = \frac{actual\ measured\ head\ or\ gross\ lift}{head\ impeller\ to\ the\ fluid\ by\ impeller}$$

$$\eta_{mono} = \frac{H_{static} + \sum H_L}{\frac{V_{u2}U_2}{g}} \quad or \quad = \frac{gH}{V_{u2}U_2}$$

This also known as hydraulic efficiency

$$\eta_o = \eta_{mech} . \eta_Q . \eta_{mono}$$

6.7 Flow rate throughout the pump

Let Q be the quantity of flow m^3/s flowing through the impeller, then

$$Q = \pi D_1 B_1 V_{f1} = \pi D_2 B_2 V_{f2}$$

Where D & B the diameter and the width or breadth of the impeller. For constant radial flow velocity V_f then

$$D_1 B_1 = D_2 B_2$$

6.8 Net positive suction head (NPSH)

It is a term describing conditions related to cavitation, which is undesired and harmful.

In analyzing a pump operating in a system to determine if cavitation is likely, there are two aspects of NPSH to consider, $NPSH_a$ and $NPSH_r$.

a) Net positive suction head available ($NPSH_a$):-

it is the suction head present head at the pump suction over and above the vapor pressure of the liquid. $NPSH_a$ is a function of the suction system and is independent of the type of pump is system. It should be calculated by the engineer of pump user as follows

$$\mathrm{NPSH_a = H_a \mp H_s - H_f - H_v} \dots \dots \dots$$

Where

H_a = Atmospheric pressure (m)
H_s = Suction head (m) + reservoir above the pump
 - reservoir below the pump
H_f = Losses due to friction in suction line (m)
H_v = Vapor pressure of liquid as a function of temp (m)

b) Net positive suction head required ($NPSH_r$):-
it is the suction head required at the impeller centerline over and above the vapor pressure of the liquid. $NPSH_r$ is strictly a function of the pump inlet design, and is independent of the suction piping system. $NPSH_r$ is given by the pump manufacturer.

6.8.1 Negative suction lift

NPSH$_a$: Net **P**ositive **S**uction **H**ead **a**vailable, depends on the installation.

NPSH$_r$: Net **P**ositive **S**uction **H**ead **r**equested, depends on the pump.
(given by the pump manufacturer).

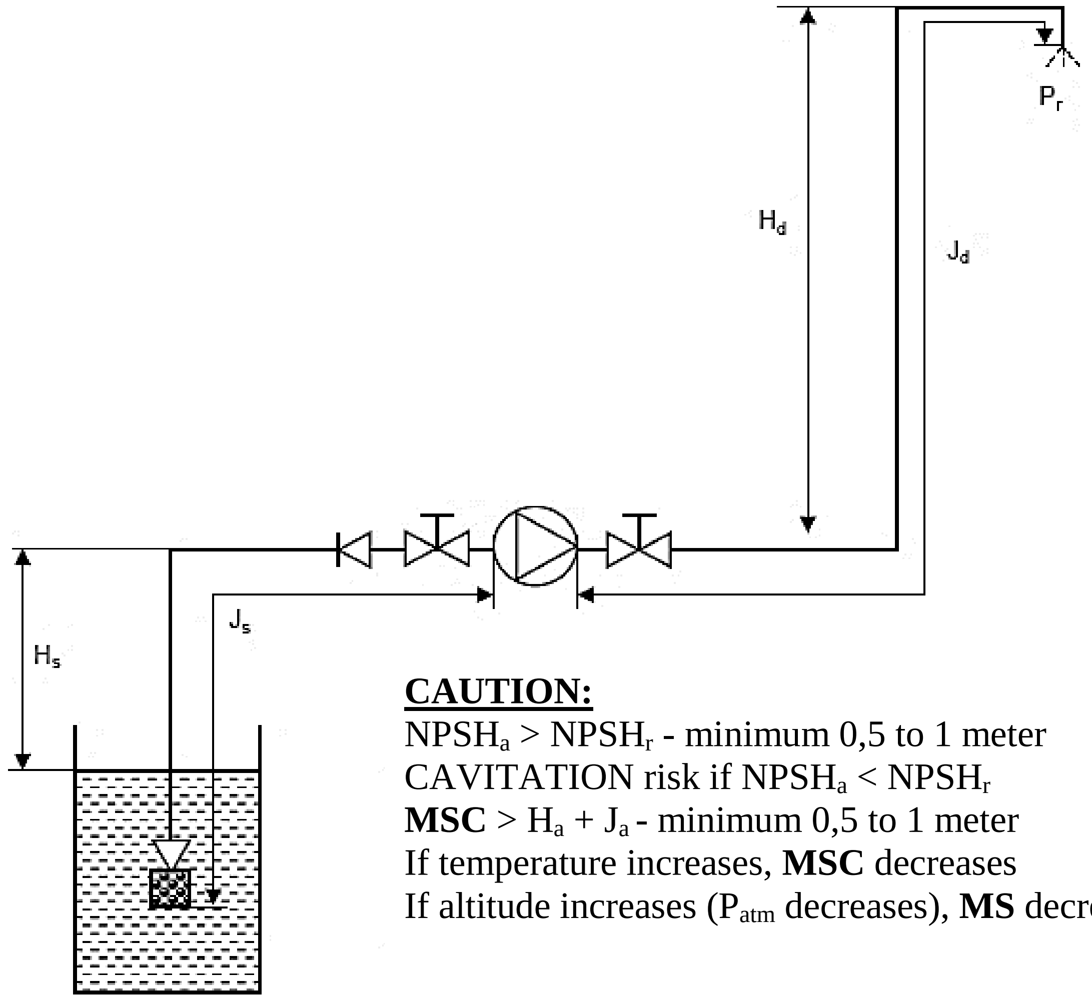
MSC: Maximum **S**uction **C**apacity

TH: Total **H**ead

=> **NPSH$_a$** = $(P_{atm} - P_v) / SG - H_s – J_s$

=> **TH** = $H_s + J_s + H_d + J_d + P_r$

=> **MSC** = $P_{atm} – NPSH_r$



CAUTION:

$NPSH_a > NPSH_r$ - minimum 0,5 to 1 meter

CAVITATION risk if $NPSH_a < NPSH_r$

MSC $> H_a + J_a$ - minimum 0,5 to 1 meter

If temperature increases, **MSC** decreases

If altitude increases (P_{atm} decreases), **MS** decreases

6.8.2 Positive suction lift

NPSH$_a$: Net **P**ositive **S**uction **H**ead **a**vailable, depends on the installation.

NPSH$_r$: Net **P**ositive **S**uction **H**ead **r**equested, depends on the pump.
(given by the pump manufacturer).

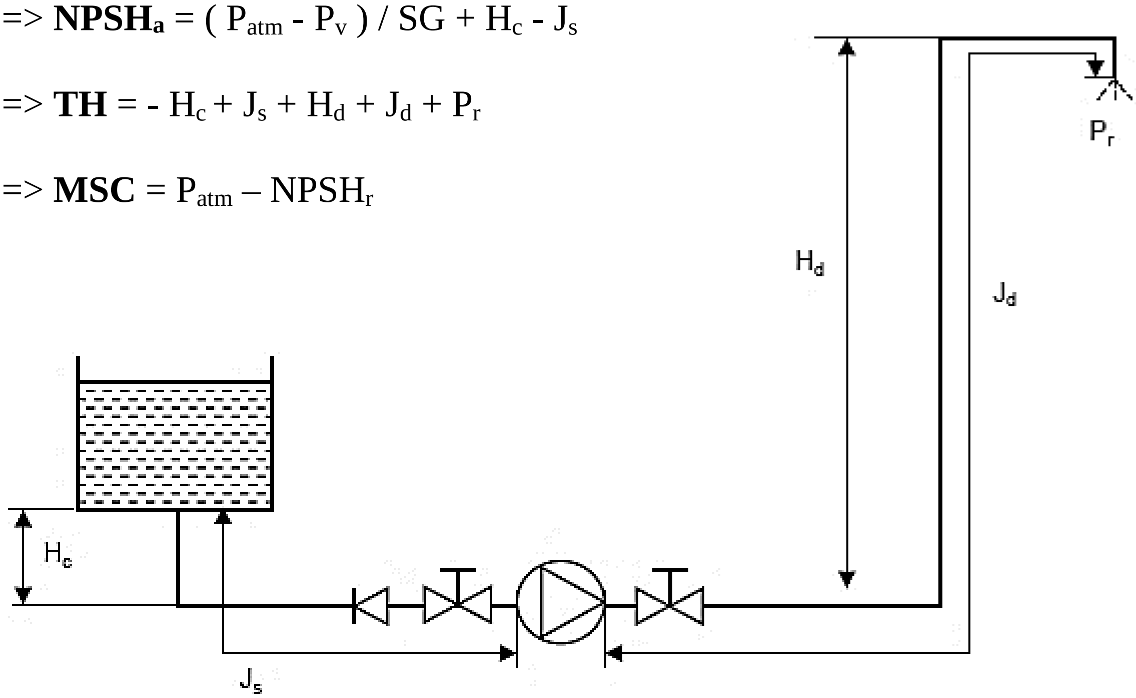
MSC: Maximum **S**uction **C**apacity

TH: Total **H**ead

=> **NPSH$_a$** $= (P_{atm} - P_v) / SG + H_c - J_s$

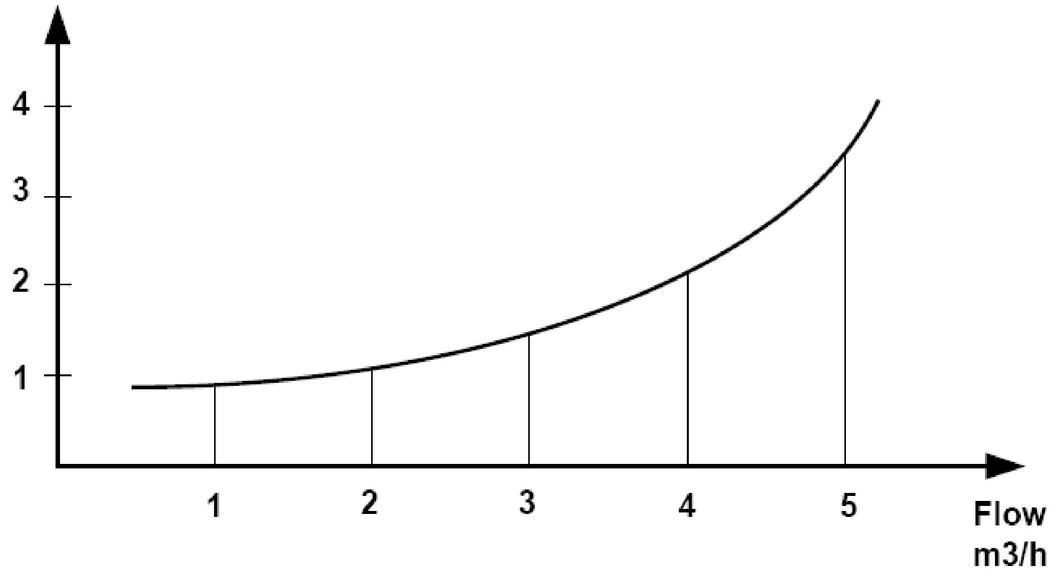
=> **TH** $= - H_c + J_s + H_d + J_d + P_r$

=> **MSC** $= P_{atm} - NPSH_r$



CAUTION:
$NPSH_a > NPSH_r$ - minimum 0,5 to 1 meter
CAVITATION risk if $NPSH_a < NPSH_r$
MSC $> H_a + J_a$ - minimum 0,5 to 1 meter
If temperature increases, **MSC** decreases
If altitude increases (P_{atm} decreases), **MSC** decreases

6.8.3 NPSH curse according flow



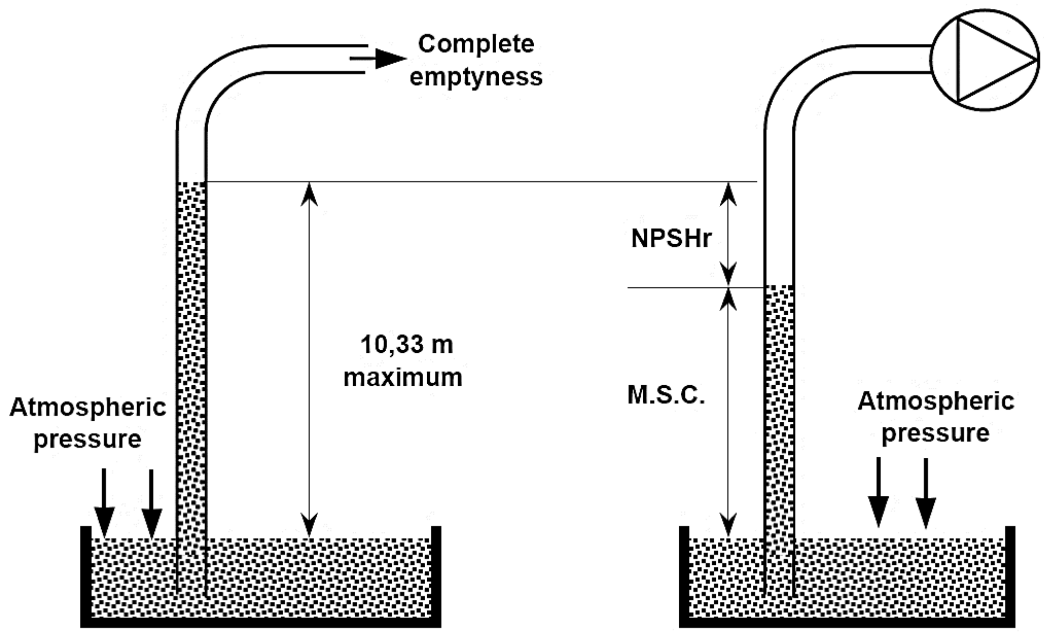
The pump's suction capacity is defined by the NPSH (Net Positive Suction Head).
NPSH is a measurement of the difference between the local net pressure and pumped liquid's
Vapor pressure (Hva)
It is one of the essential parameters to take into account when selecting a centrifugal pump in the
following cases:
a/ When the pumped water level is below the pump shaft.
b/ When the pumped fluid reaching the pump entrance is close to vapor point.

In each installation it is necessary to ensure NPSHa is greater than NPSHr.

6.8.4 NPSH required

Is determinate by the pump manufacturer; it depends of the pump type, the impeller diameter,
the flow and the pump speed.
The NPSHr (requested) of a pump, given in meters of liquid, indicates the minimum absolute
pressure necessary at the pump's suction for correct running.

NPSHr enables the pump's **MSC** (Maximum suction capacity) to be calculated. The MSC is given
in meters of water and indicates the height above which a pump is able to draw water and pump normally.



a/ Pump drawing cold water from a tank at atmospheric pressure:

M.S.C. = Patm - NPSHr

b/ Pump sucking from a pressurized tank

$$\mathbf{M.S.C.} = \mathbf{10}\left(\frac{\mathbf{Po} - \mathbf{Pv}}{\boldsymbol{SG}}\right)$$

M.S.C.: Maximum Suction Capacity
Po: Absolute pressure in the suction tank (in bar)
Pv: Vapor tension of the pumped liquid at the pumped temperature (in bar)
SG: Specific Gravity - pure water SG is 1 g/cm^3

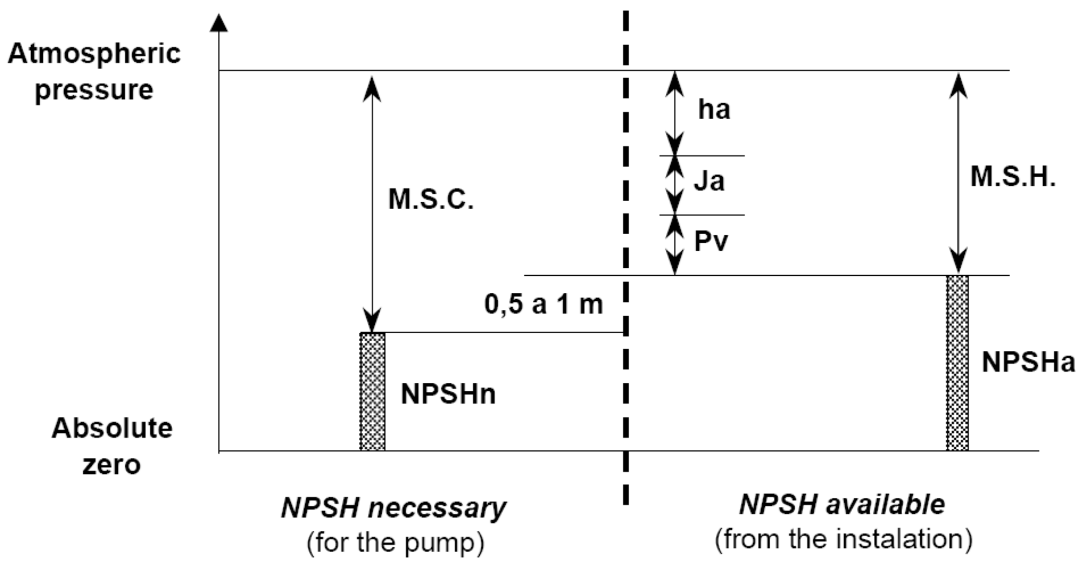
6.8.5 NPSH available

NPSHa at pump suction depends on individual installation (fluid's nature and pressure,
temperature, vapor tension, altitude, diameter and shape of the pipes, etc.). It is totally
independent of the pump.

The NPSH available is equal to the absolute pressure at the suction flange over the vaporization
pressure of the fluid.
IN order to have the installation running properly, it is mandatory to have the NPSH
available the pump's suction higher than the NPSH necessary for the same pump.
NPSH available > NPSH required
The security range is to be between 0,5 to 1 m head depending on the pump.

$$M.S.C. = 10\left(\frac{\mathrm{Pa}-\mathrm{Pv}}{SG}\right)+\frac{Va^2}{2g}$$

NPSHa in meters
Pa: Absolute pressure at the pump's suction.
Pv: Vapor tension of the pumped fluid.
SG: Specific Gravity - pure water SG is 1 g/cm^3
Va: Speed at the pump suction
g: gravity acceleration



ha: suction geometric head
Ja: Suction piing network's total friction looses

6.9 Cavitation in pump

When the liquid is flowing in the pump, it is possible that the pressure at any part of the pump may fall below the vapor pressure, the liquid will vaporize and the flow will no longer continuous. The vaporization will appear in the form of bubbles. These bubbles when passing a region of high pressure will collapse on a metallic surface such as tips of impellers blades, the cavities are formed. This phenomenon known as cavitation

To prevent cavitation

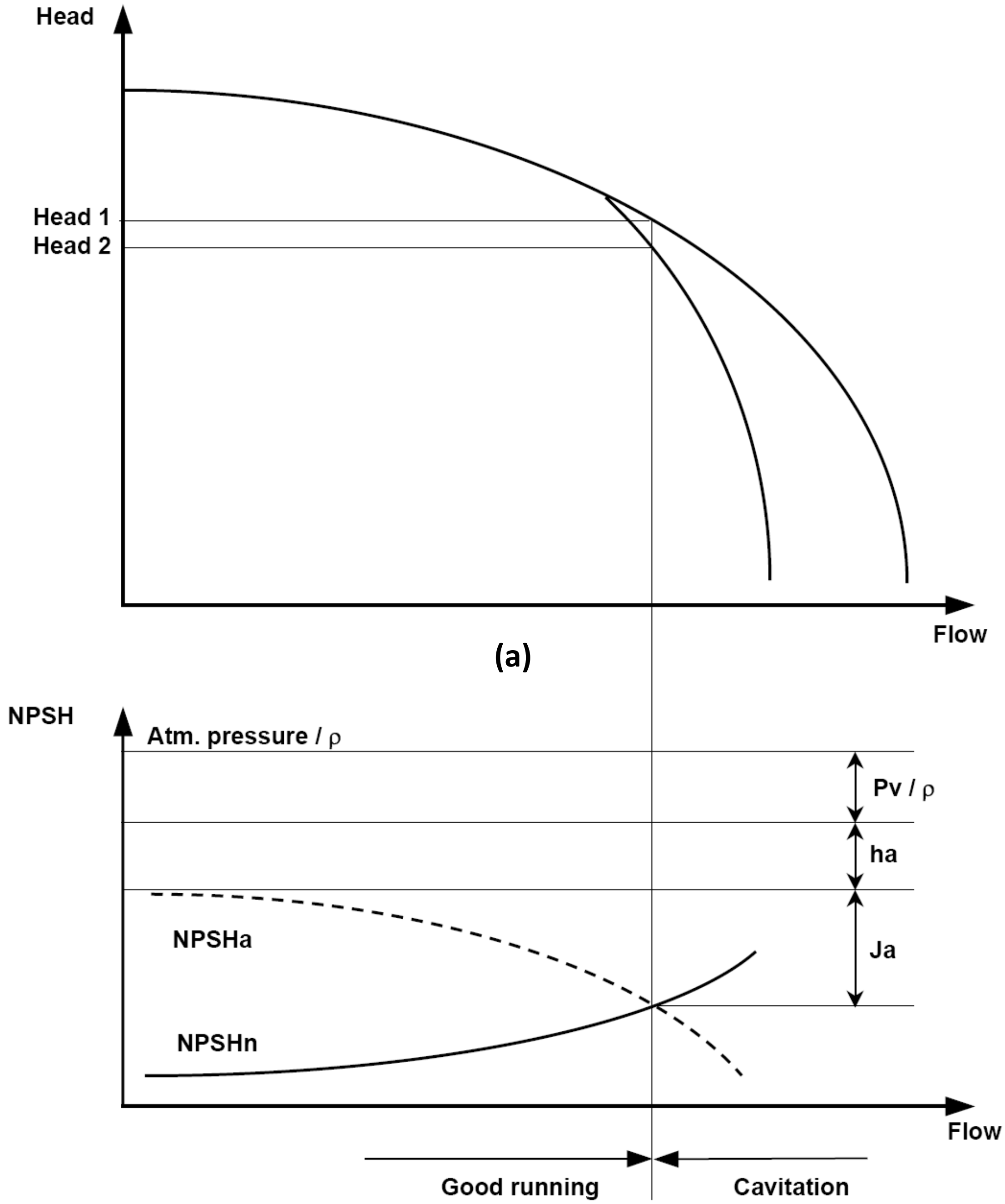
$$NPSH_a > NPSH_r$$

The risk of cavitation in systems can be reduced by:

- Lowering the pump compared to the water level- open system.
- Increasing the system pressure- closed system.
- Shortening the suction line to reduce the friction loss.
- Increasing the suction lines cross-section area reduce the fluid velocity and thereby reduce friction.
- Avoiding pressure drops coming from bends and other obstacles in the suction line.
- Lowering fluid temperature to reduce vapor pressure.

The relation between the head, NPSH, and the flow rate shown in Fig (7).

Pump cavitation



(b)

Fig. (7) (a) the relation between the head and flow rate.
(b) the relation between the NPSH and flow rate.

6.10 Similarity low, specific speed and cavitation

1- Similarity low: After pumps manufactured if needs to change the speed and diameter of the impeller on both the following conditions used if we need a minor change in head on capacity.

(i) Effect of speed variation

$$\frac{Q}{Q'} = \frac{N}{N'} \dots \dots \dots (1)$$

$$\frac{H}{H'} = \left(\frac{N}{N'}\right)^2 \dots \dots \dots (2)$$

$$\frac{P}{P'} = \left(\frac{N}{N'}\right)^3 \dots \dots \dots (3)$$

b) Efficiency of diameter variation

$$\frac{Q}{Q'} = \left(\frac{D}{D'}\right)^2 \dots \dots \dots (1)$$

$$\frac{H}{H'} = \left(\frac{D}{D'}\right)^2 \dots \dots \dots (2)$$

$$\frac{P}{P'} = \left(\frac{D}{D'}\right)^4 \dots \dots \dots (3)$$

2- Pump modeling: for a large size of pumps the model should have complete geometrical similarity with the prototype and they should have the same specific speed.

i.e

$$N_s = \frac{N\sqrt{Q}}{H^{\frac{3}{4}}} \dots \dots \dots (10$$

$$or\ (N_s)_m = (N_s)_p \dots \dots \dots (2)$$

For the same condition the speed, flow rate and the power called be found as follows:

$$\frac{N_m}{N_p} = \frac{D_m}{D_p}\sqrt{\frac{H_m}{H_p}} \ldots\ldots\ldots(3)$$

$$\frac{Q_m}{Q_p} = \left(\frac{D_m}{D_p}\right)^2\sqrt{\frac{H_m}{H_p}} \ldots\ldots\ldots(4)$$

$$\frac{P_m}{P_p} = \left(\frac{D_m}{D_p}\right)^2\left(\frac{H_m}{H_p}\right)^{3/4}$$

Where $P = \gamma Q H$

3- Minimum starting speed of pump: N:AC.P.

Will start delivering the liquid, only when the head developed by it is equal to the manometric head at the time of start, the liquid velocities are zero therefore the reassure head caused by the centrifugal force.

$$\frac{{U_2}^2 - {U_1}^2}{2g} \geq H_{mono} \qquad U = \frac{\pi D N}{60}$$

$$\therefore \frac{{U_2}^2 - {U_1}^2}{2g} = H_{mono} = \frac{V_{u2}U_2}{g}\eta_{mono}$$

4- Minimum diameter of impeller: Normally at the starting of design an impeller the assumption

$$D_2 = 2D_1$$

$$or \ \frac{\left(\frac{\pi D_2 N}{60}\right)^2 - \left(\frac{\pi D_1 N}{60}\right)^2}{2g} = H_{mono}$$

$$D_2 = \frac{97.7\sqrt{H_{mono}}}{N}$$

$$D_2 = outlet\ diameter\ (m)$$

$$D_1 = inlet\ diameter\ (m)$$

The following Fig. (8, 9, 10, 11, 12, 13, 14, and 15) describes the characteristics and the working principles of the pump

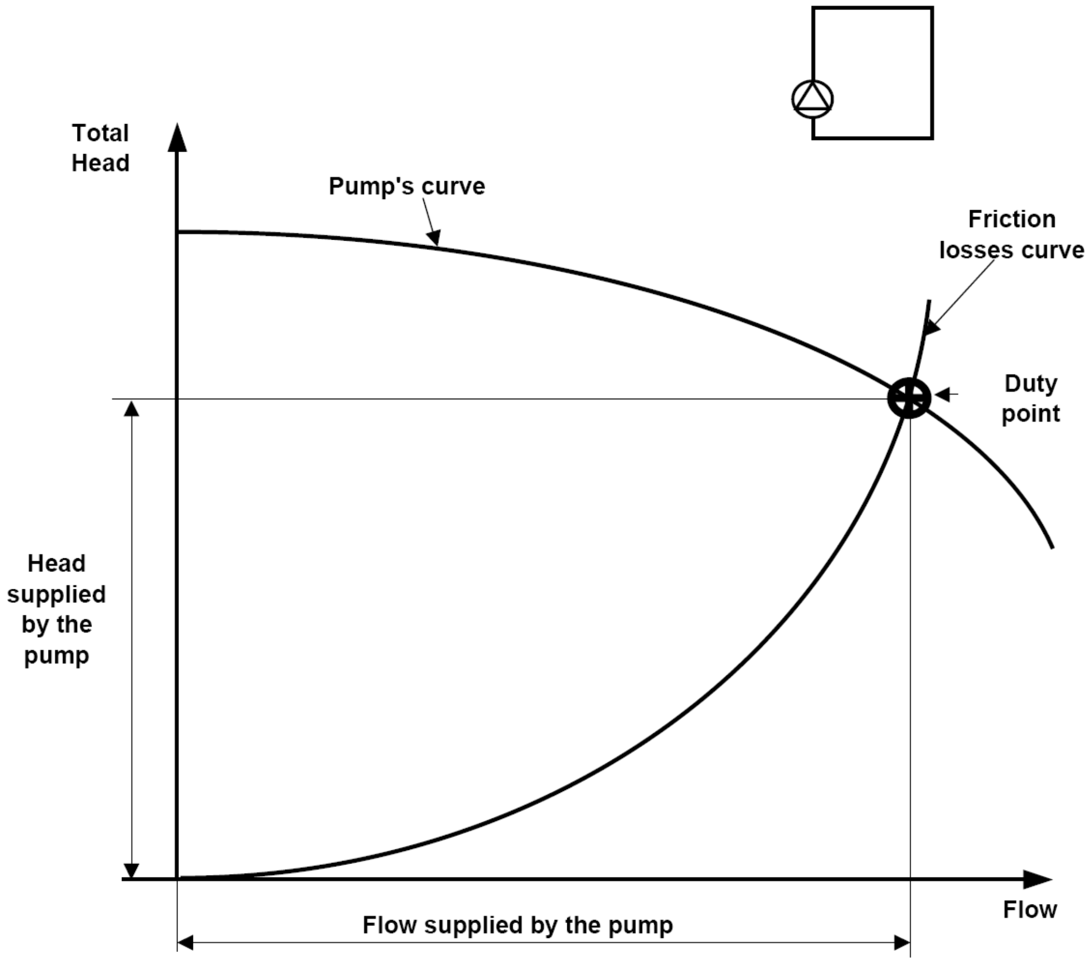


Fig. (8) Pump's hydraulic characteristic: closed loop.

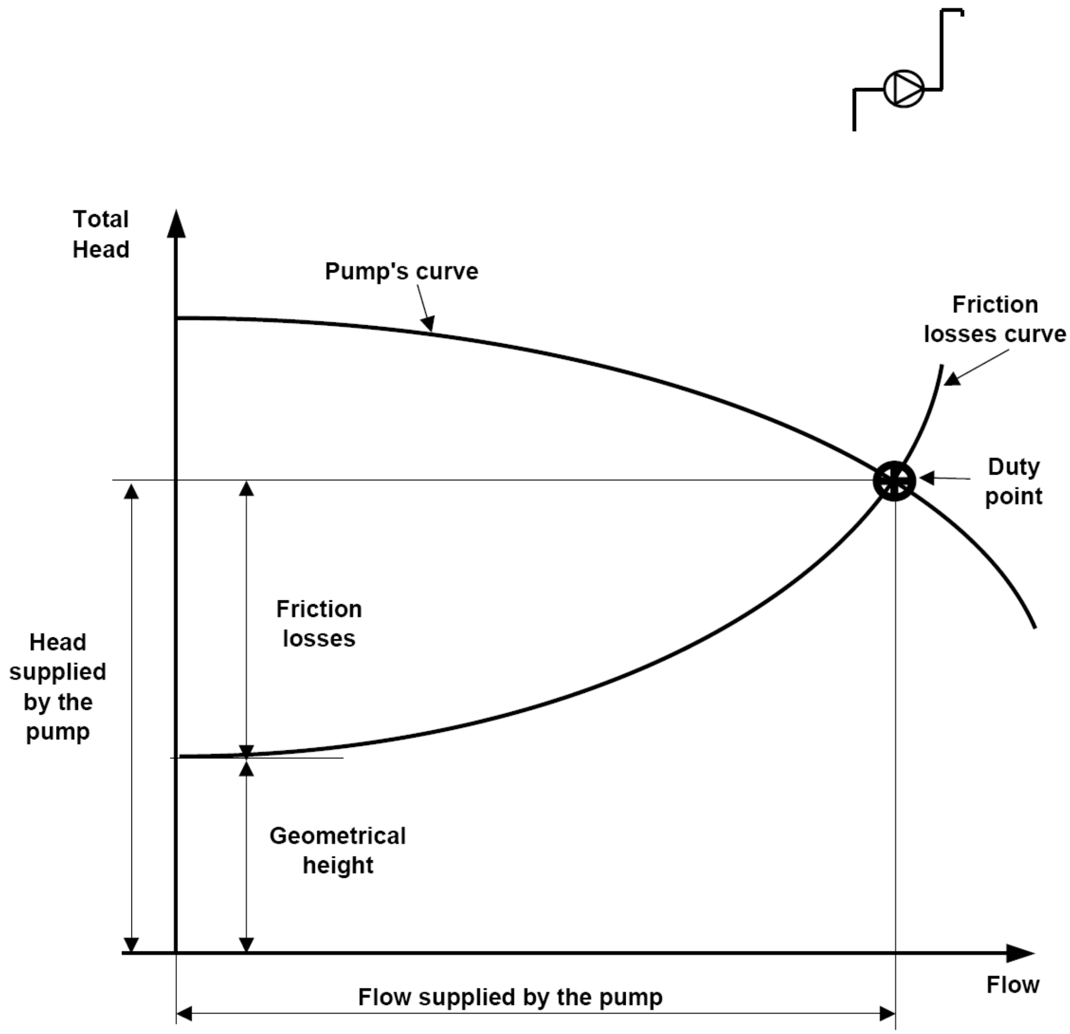


Fig. (9) Pump's hydraulic characteristic: open loop.

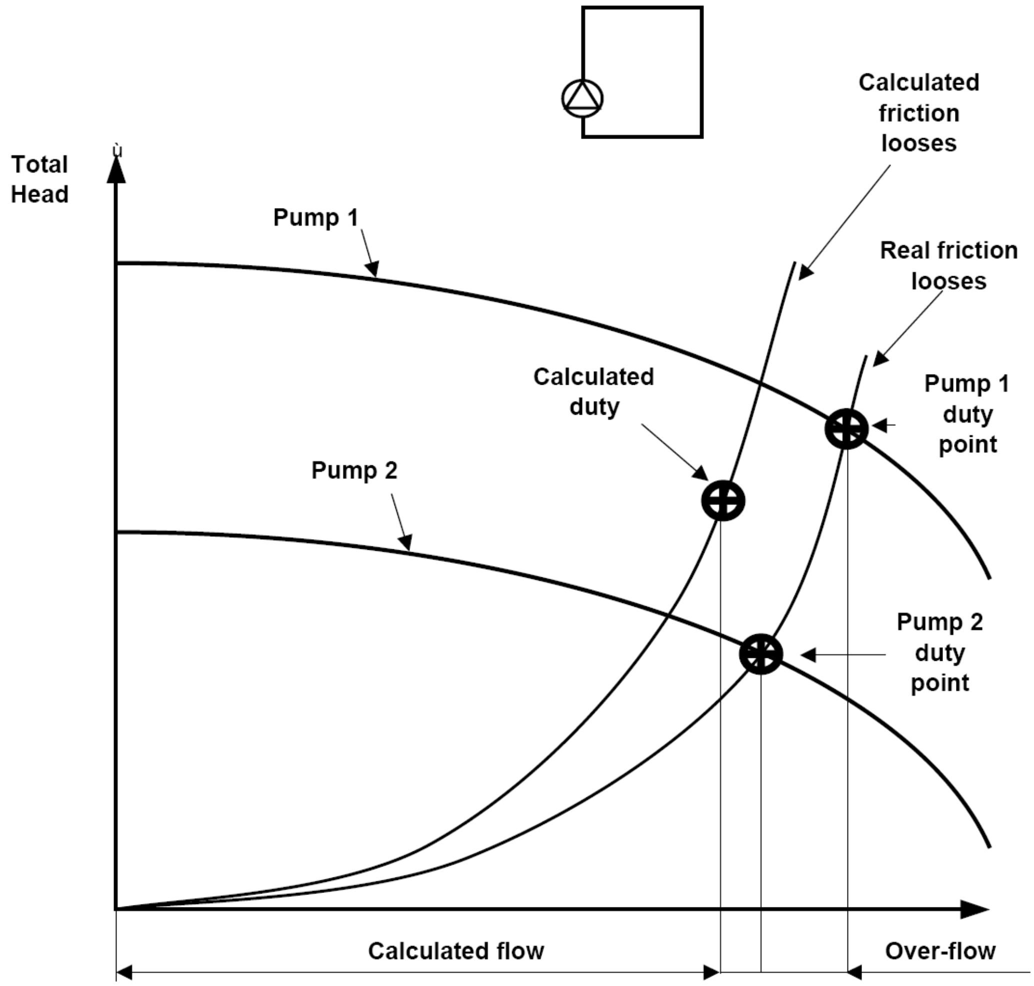


Fig. (10) Over estimation of friction losses - Closed loop.

Risks:

Noise in the pipe work
Higher NPSHn requested
More power consumption
Faster wear

Solutions:

Trim the impeller diameter
Increase the friction losses (diaphragm, valves)
Reduce the speed

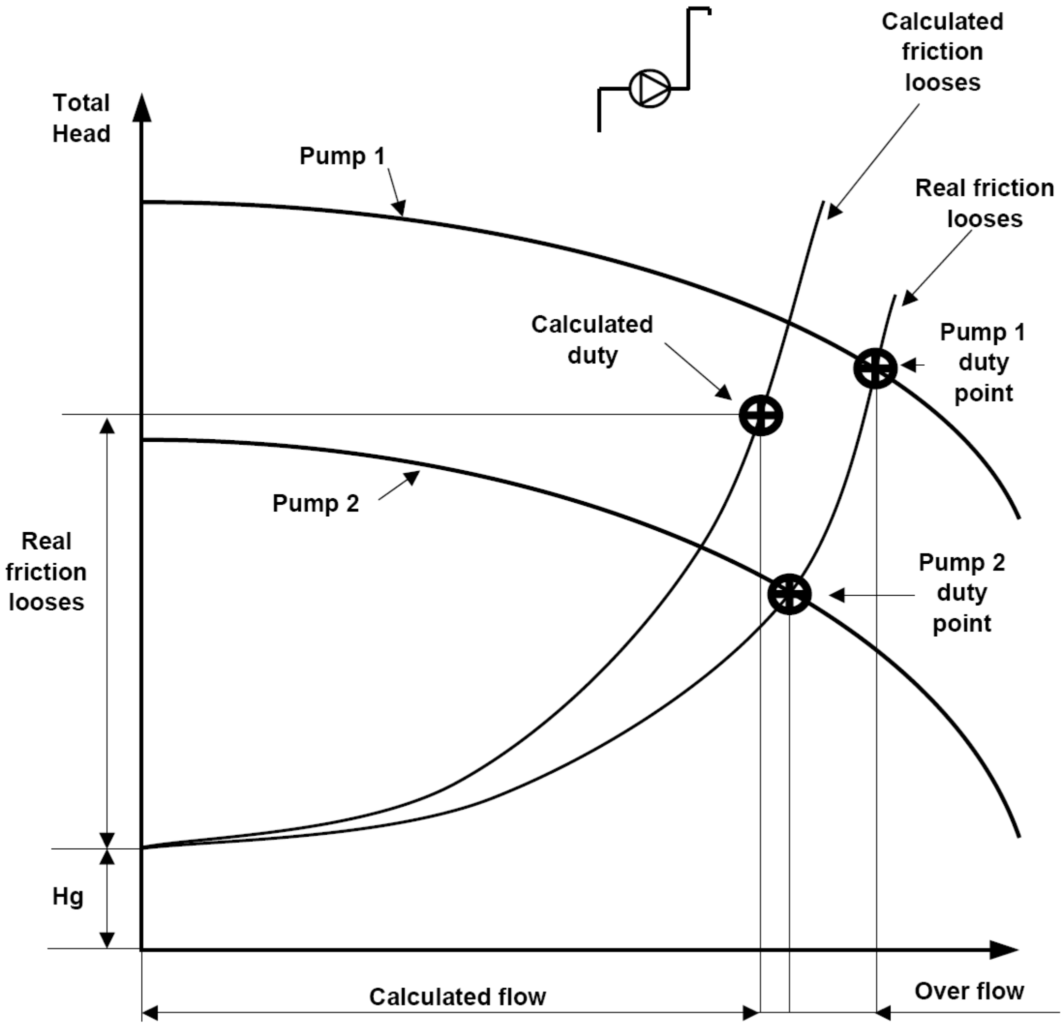


Fig. (11) Over estimation of friction losses - Open loop.

Risks:
Noise in the pipe work
Higher NPSHn requested
More power consumption
Faster wear
Solutions:
Trim the impeller diameter
Increase the friction losses (diaphragm, valves)
Reduce the speed

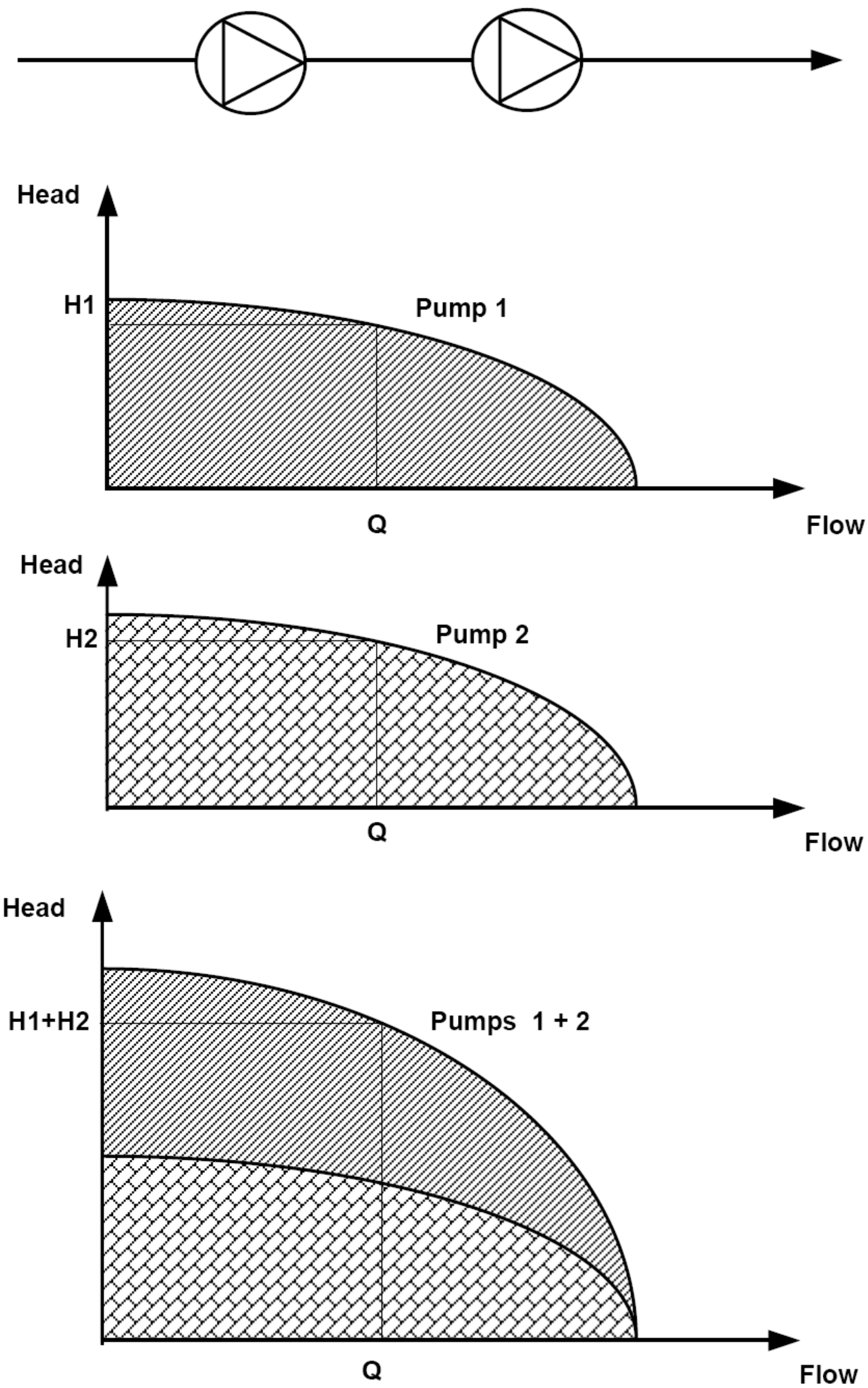


Fig. (12) Same pumps in series.

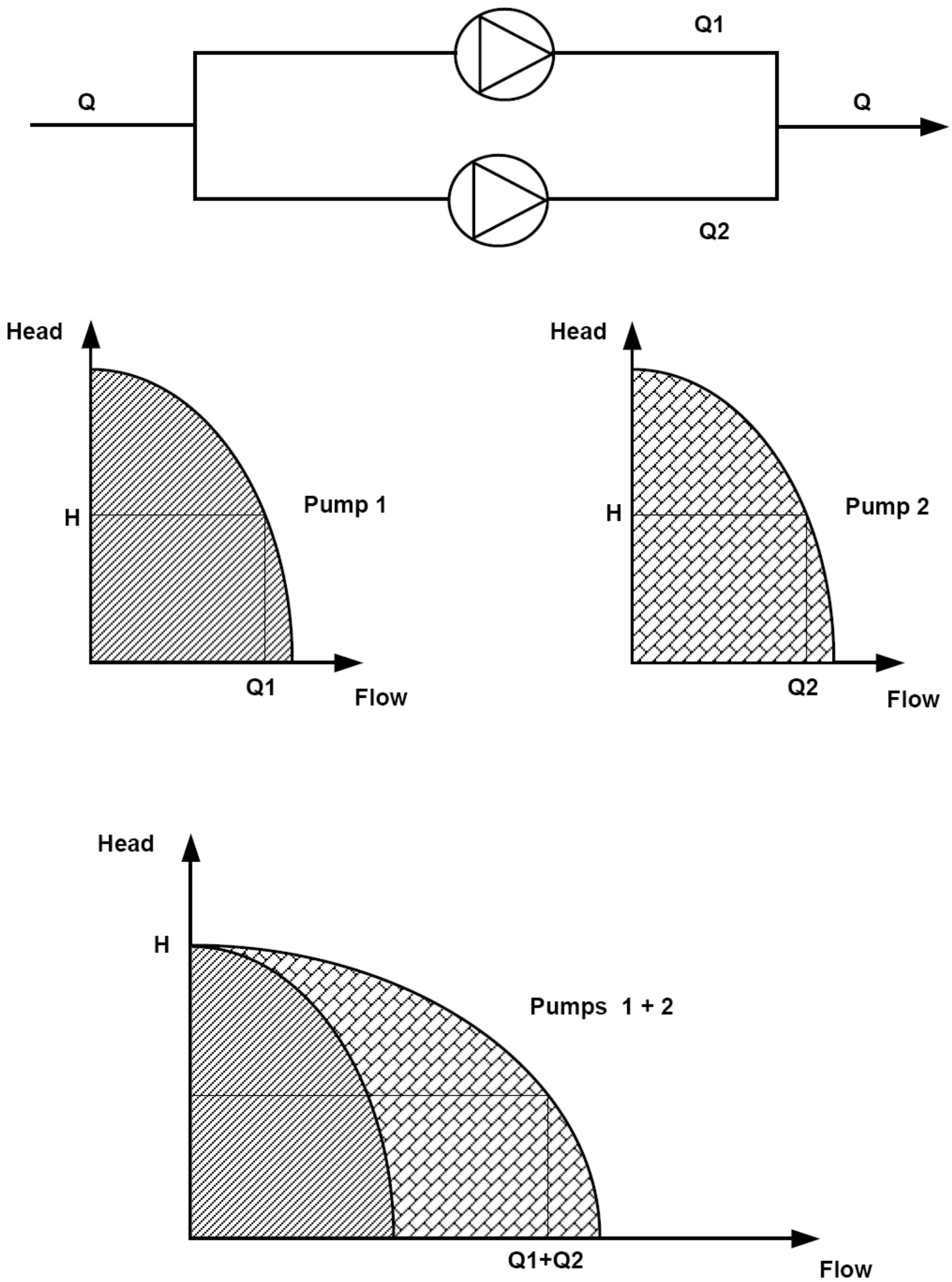


Fig. (13) Same pumps in parallel.

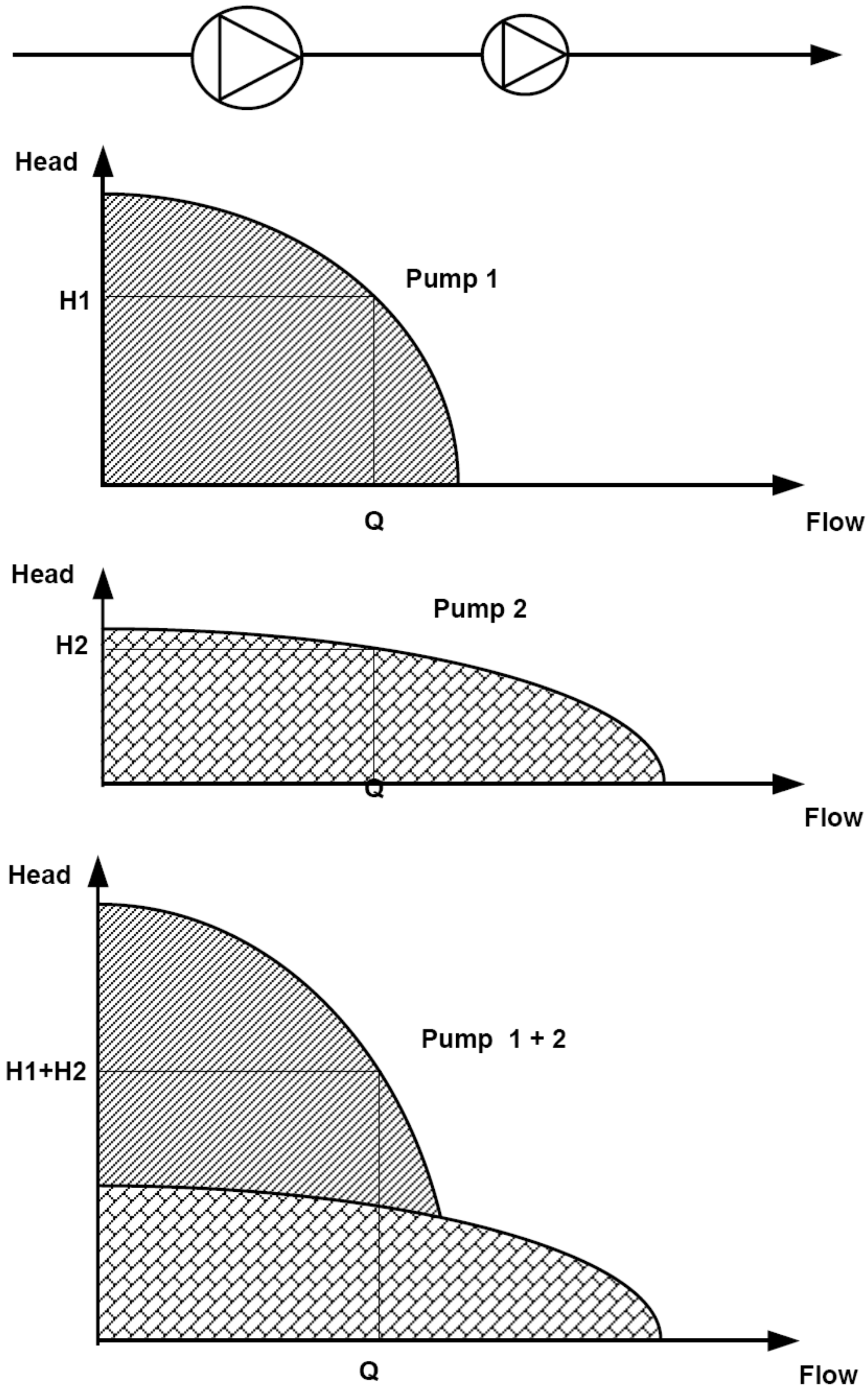


Fig. (14) Different pumps in series.

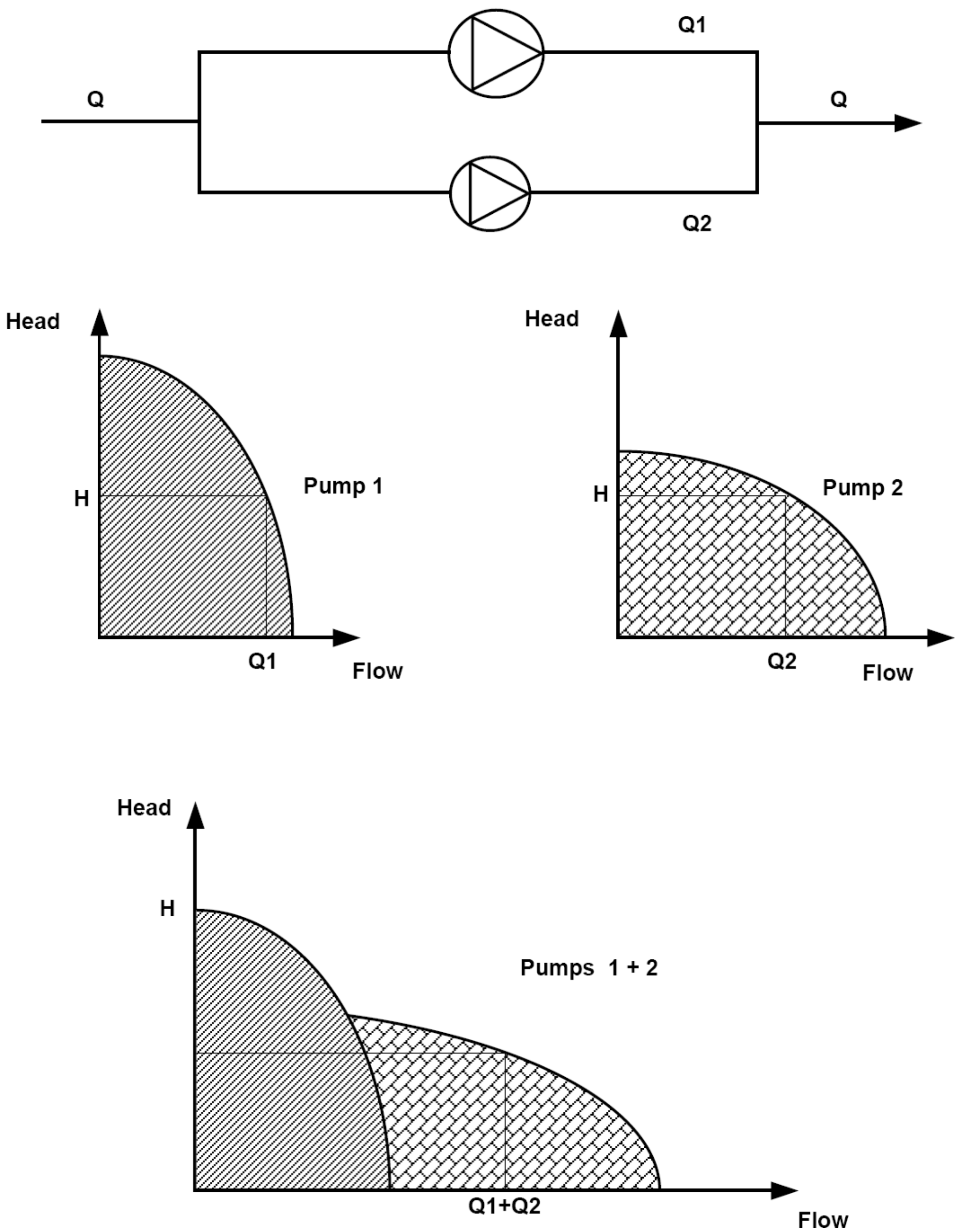


Fig. (15) Different pumps in parallel.

6.11 Pump selection and performance charts

The data required for selection a pump size i.e the flow rate and the head (Q, H) of the desired operating point are assumed to be known from the system characteristic curve. The electric main frequency is also given. With these values it is possible and, if necessary, the number of stages from the suction chart in the sales literature Fig (16, 17).

Further details of the chosen pump P. the required $NPSH_r$ Fig (8). And reduced impeller diameter can then be determined.

The complete characteristics curves of a centrifugal pump shown in Fig (8).

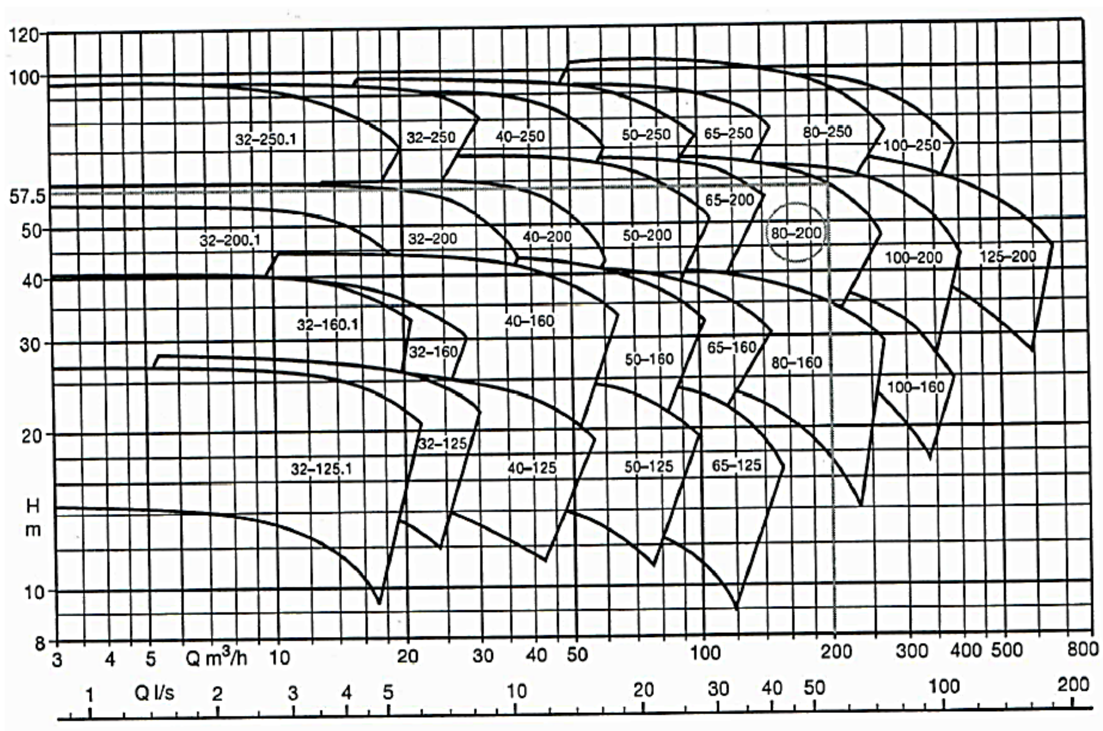


Fig. (16) Selection chart for a volute casing pump series for n=2900 rpm (First number=nominal diameter of the discharge nozzle, second number =nominal impeller diameter).

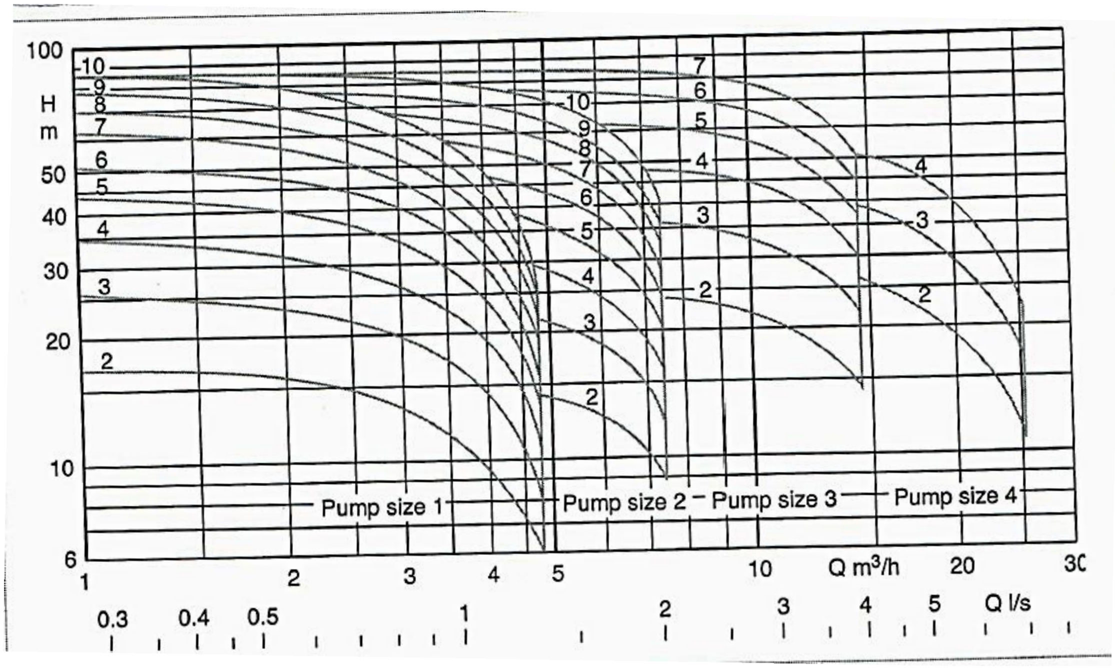


Fig. (17) Selection chart for a multistage pump series for n = 2900 rpm.

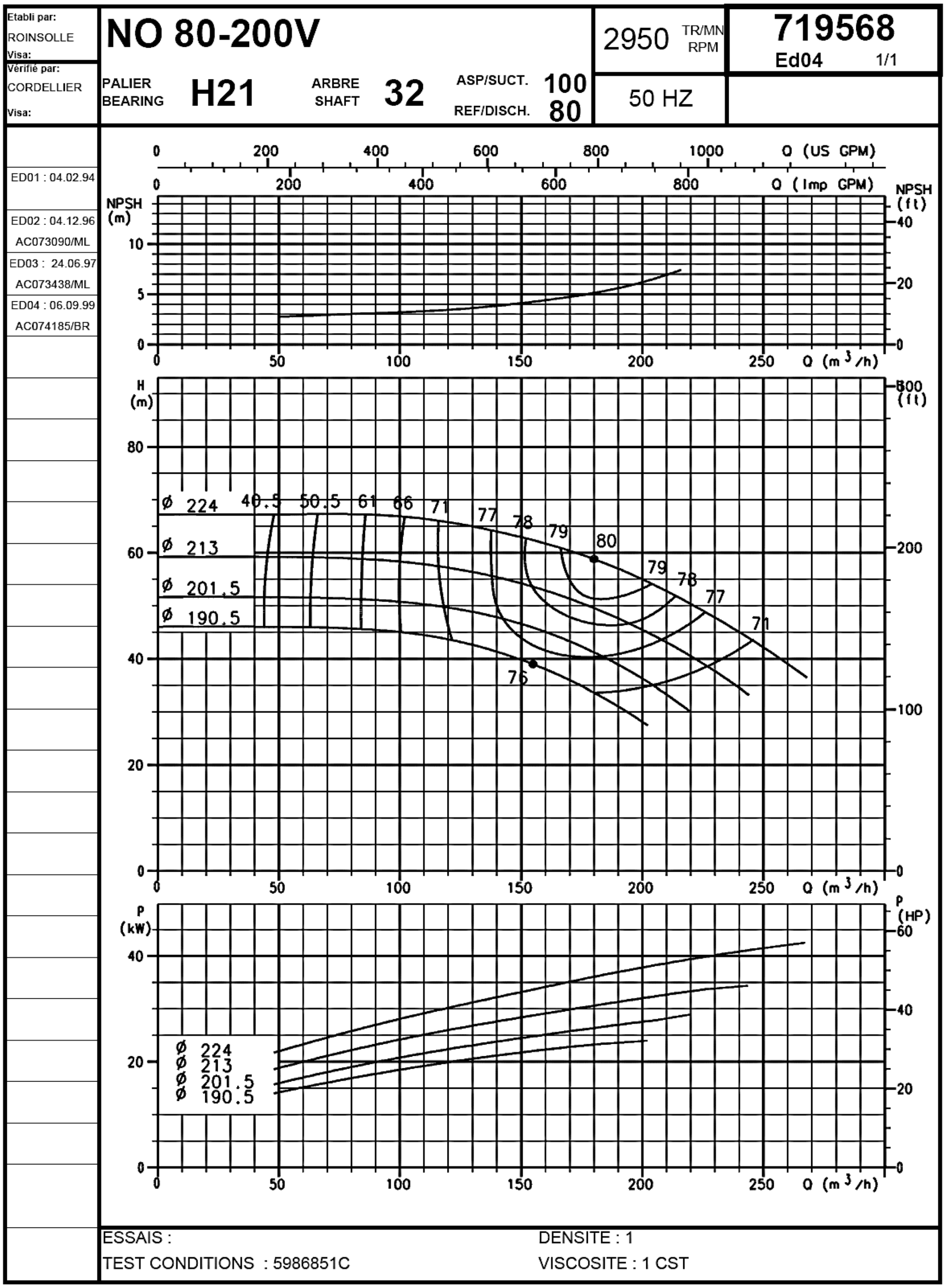


Fig. (18) Characteristics curves of C.P.

6.12 Positive Displacement Pumps

6.12.1 Reciprocating pump: - In its simples from, consists of the following parts as shown in Fig (19)

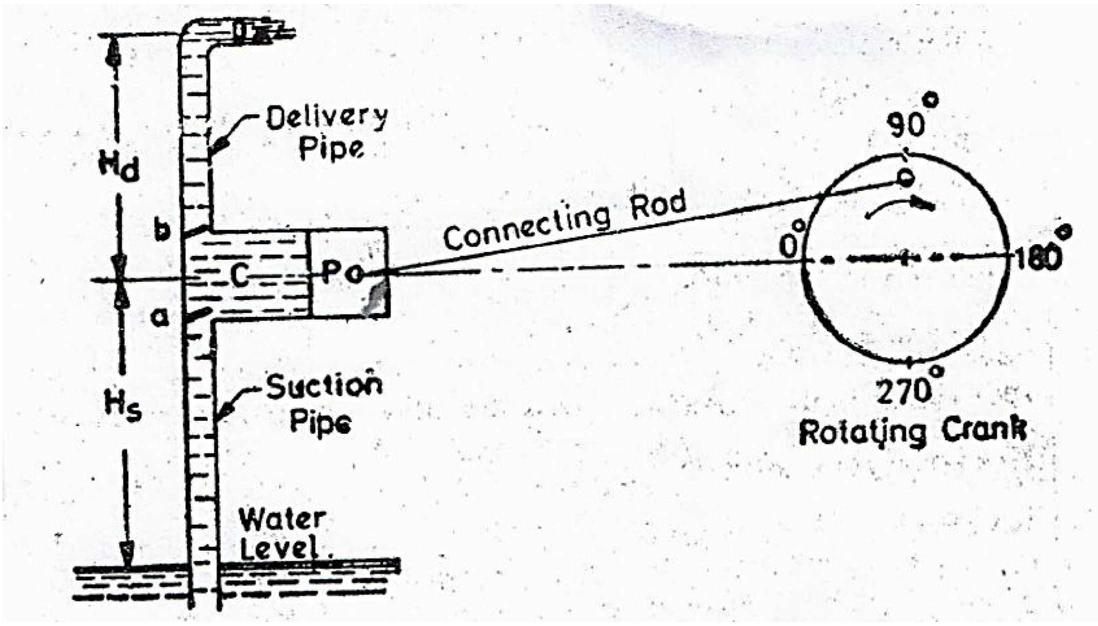


Fig. (19) Part of a reciprocating pump.

a- A cylinder C, in which a piston P works, the movement of the piston is obtained by a connecting rod, which connects the piston and the routing crank.
b- A suction pipe, connecting the source of water and the cylinder.
c- A delivery pipe, into which the water is discharged from the cylinder.
d- A valve a, which admits the flow from the suction pipe into the cylinder.
e- A valve b, which admits the flow from the cylinder into the delivery pipe.

6.12.2 Type of reciprocating pumps

May be classified as follows

a- According to acting of water
1. single acting pump.
2. double acting pump.

b- According to number of cylinders
1. single cylinder pump.
2. double cylinder pump and

c- According to existence of air vessels

1. with air vessel, and
2. without air vessel

Following table gives the comparison of a centrifugal pump and a reciprocating pump.

S.No.	Centrifugal pump	Reciprocating pump
1.	Simple in construction, because Of less number of parts.	Complicated in construction because of more number of parts.
2.	Total weight of the pump is less for a given discharge.	Total weight of the pump is more for a given discharge.
3.	Suitable for large discharge and Smaller heads.	Suitable for less discharge and higher heads.
4.	Requires less floor area and Simple foundation.	Requires more floor area and Comparatively heavy foundation.
5.	Less wear and tear.	More wear and tear.
6.	Maintenance cost is less.	Maintenance cost is More.
7.	Can handle dirty water.	Cannot handle dirty water.
8.	Can run at higher speeds.	Cannot run at higher speeds.
9.	Its delivery is continuous.	Its delivery is pulsating.
10.	No air vessels are required.	Air vessels are required.
11.	Thrust on the crankshaft is uniform.	Thrust on the crankshaft is not uniform.
12.	Operation is quite simple.	Much care is required in Operation.
13.	Needs priming.	Does not need priming.
14.	It has less efficiency.	It has more efficiency.

6.12.3 Discharge of a reciprocating pump

For single acting pump

$$Q = \frac{LAN}{60}$$

$L: length\ of\ the\ stroke\ or\ piston.$
$A: cross - sectional\ area\ of\ the\ piston.$
$N: No.of\ revolutions, per\ minute\ of\ the\ crank.$

For double acting

$$Q = \frac{2LAN}{60}$$

$$Slip\ of\ the\ pump = Q_{th} - Q_a$$

$$Percentage\ of\ the\ slip = \frac{Q_{th} - Q_a}{Q_{th}}$$

$$Discharge\ coefficient\ C_d = \frac{Q_a}{Q_{th}}$$

6.12.4 Power of the pump

$$P_{out} = \gamma QH \qquad kW$$

$$H = H_s + H_d$$

$H_s = section\ head\ m$
$H_d = delivery\ head\ m$

Ex: A single acting reciprocating pump has a plunger of diameter 30 cm and stroke of 20 cm. if the speed of the pump is 30 rpm and it delivers 6.5 l/s of water. Find the coefficient of discharge and the percentage slip of the pump.

Solution:-

$$D = 30\ cm \quad \therefore\ A = \frac{\pi}{4} D^2 = 706.86\ cm^2$$

$$L = 20\ cm\ ; N = 30\ rpm$$

$$Q_a = 6.5\ l/s = 6500\ cm^3/s$$

Then

$$Q_{th} = \frac{LAN}{60} = \frac{20 \times 706.86 \times 30}{60}$$
$$= 7068.6\ cm^3/s$$
$$\therefore C_d = \frac{Q_a}{Q_{th}} = \frac{6500}{7068.6} = 0.92$$
$$Percentage\ of\ the\ slip = \frac{Q_{th} - Q_a}{Q_{th}}$$
$$= \frac{7068.6 - 6500}{7068.6}$$

$$= 8.04\%$$

Ex: a single acting R.P. having a bore of 150 mm diameter and a stroke of 300 mm length discharge 200 l/min at 40 rpm. Neglecting losses find:

1- Theoretical discharge.

2- Coefficient of discharge.

3- Slip of the pump.

4- The power of the pump for H_d =26, H_s =4 m.

Solution:-

$$D = 150\ cm \qquad \therefore A = \frac{\pi}{4} D^2 = 706.86\ cm^2$$

(1)

$$Q_{th} = \frac{LAN}{60} = \frac{0.3 \times 0.0177 \times 40}{60} = 0.00354\ cm^3/s$$

$$= 0.00354\ cm^3/min$$

(2)

$$C_d = \frac{Q_a}{Q_{th}} = \frac{200}{212} = 0.94$$
$$Slip\ of\ the\ pump = Q_{th} - Q_a = 12\ l/min$$

(3)

$$slip\ of\ thePercentage\ = \frac{Q_{th} - Q_a}{Q_{th}} = 5.66\%$$

(4)

$The\ Power\ of\ the\ pump$ (theoretical) $= \gamma Q H$
$$= 9.81 \times 0.00354 \times 30 = 883\ watt$$
$$P_{therotical} = 1.042\ kW$$
$$P_{actual} = 9.81 \times \frac{0.200}{60} \times 30 = 0.981\ kW$$

6.12.5 Air Vessel

Is a closed chamber fitted on the suction as well as on the delivery side, near the pump cylinder to reduce the accelerating head as shown in Fig Function:

a- suction side:

(1) Reduces the possibility of separation.

(2) Pump can be run at a higher speed.

(3) Length of the suction pipe below.

The air vessel can be increased.

b- Delivery side:

(1) A large amount of power consumed in supplying accelerating head can be saved.

(2) Constant rate of discharge can be ensured.

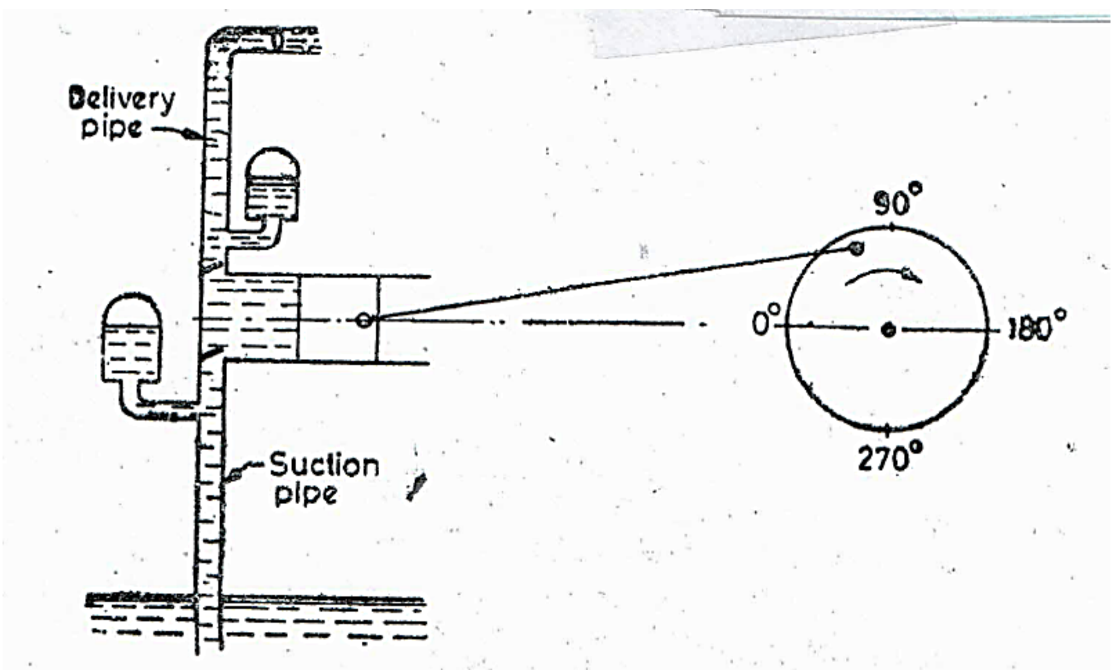
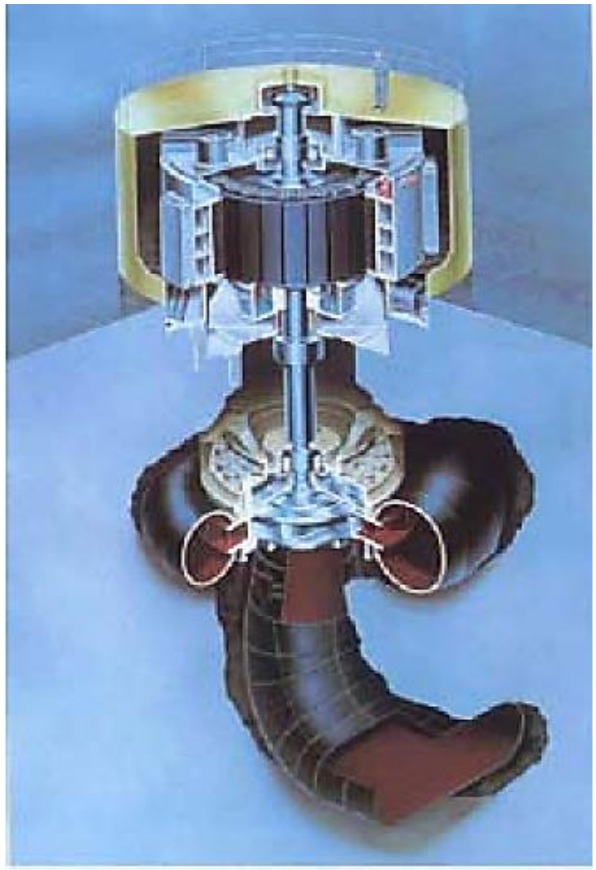


Fig. (20) Air vessels fitted to the suction and delivery pipes.

6.13 Pump Turbine



When water enters the rotor at the periphery and flows inward the machine acts as a turbine

With water entering at the center and flowing outward, the machine acts as a pump

The pump turbine is connected to a motor generator, which acts as either a motor or generator depending on the direction of rotation.

The pump turbine is used at pumped storage hydroelectric plants, which pump water from a lower reservoir to an upper reservoir during off-peak load periods so that water is

available to drive the machine as a turbine
during the peak power generation needs.

Pump Turbine

Pump turbines are classified into three principal types analogous to reaction turbines and pumps.

Radial flow – Francis	23-800 m
Mixed flow or diagonal flow	11-76 m
Axial flow or propeller	1-14 m

⇒ As a turbine
– Develops 240 MW at a maximum head of 220 m.
– Develops 177 at minimum net head of 185 m.

⇒ As a Pump
– Delivers 110 m^3/s at a minimum net head of 198 m.
– Delivers 86 m^3/s at minimum net head of 185 m.

⇒ To reduce the head loss at submerged discharge and thereby to increase the net head available to the turbine runner. This is accomplished by using a gradually diverging tube whose cross sectional area at discharge is cross-considerably larger than the cross-sectional area at entrance to the tube.

Pump Turbine Specification

	Turbine	Pump
Type	: Vertical Francis	Centrifugal
Rated horse power	: 59656 kW	76061 kW
Rated head	: 58 m	62.5 m
Rated discharge	: 118.3 m^3/s	110 m^3/s
Rated speed	: 106 rpm	106 rpm
Maximum runaway speed	: 161 rpm	121 rpm
Direction of rotation	: clockwise	counterclockwise
Specific speed at rating	: 42.1	121

Solved Problems

Q.1- A centrifugal pump lifts water under a static head of 36 m of which 4 m/s is suction lift. Suction lift and delivery pipes are both 150 mm in diameter. The head loss in suction pipe is 1.8 m and in delivery pipe 7 m. the impeller 380 mm in diameter and 25 mm wide at the mouth and revolves at 1200 rpm. Its exit angle is 145^0. If the manometric efficiency. Of the pump is 82%. Determine the discharge and the pressure at the suction and delivery branches of the pump. The flow at the inlet is radial i.e. $\alpha_1 = 90^0$.

Solution:-

$$H_{static} = 36\,m\ ;\ D_2 = 380\,mm\ ; H_s = 4\,m$$
$$B_2 = 25\,mm\ ;\ H_d = 36 - 4 = 32\,m\ ; N = 1200\,rpm$$
$$H_{Ls} = 1.8\,m\,;\ H_{Ld} = 7\,m\ ;\ d_d = d_s = 150\,mm$$
$$\eta_{mano} = 82\%\ ;\ \beta_2 = 35°$$

Total head to be supplied by the pump

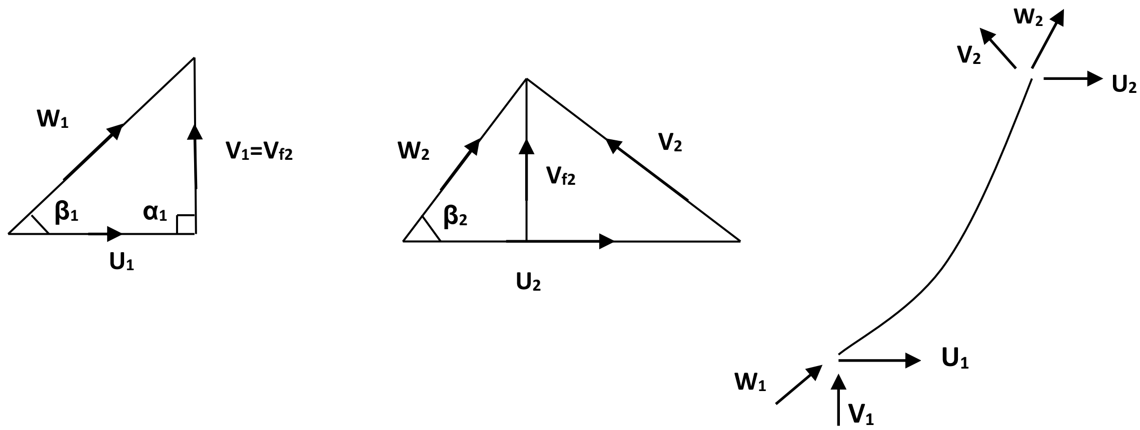
$$H_{mano} = H_{static} + \sum H_L$$
$$= 36 + 1.8 + 7 = 44.8\,m$$

Peripheral velocity of the impeller at outlet

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.38 \times 1200}{60}$$
$$= 23.9\,m/s$$

Since the flow radial at inlet $\alpha_1 = 90^0$

$$\therefore \eta_{mano} = \frac{H_{mano}}{\frac{V_{u2}U_2}{g}} = \frac{44.8}{\frac{V_{u2} \times 23.9}{9.81}} = 0.82$$
$$\therefore\ V_{u2} = 22.4\,m/s$$
$$\tan\beta_2 = \frac{V_{f2}}{U_2 - V_{u2}}\ \ from\ velocity\ triangle$$
$$= \frac{V_{f2}}{23.9 - 22.4} \qquad \therefore\ V_{f2} = 1.05\,m/s$$



$$and\ Q = \pi D_2 B_2 V_{f2} = \pi \times 0.38 \times 0.025 \times 1.05$$
$$= 0.0314\ m^3/s$$

Velocity in suction or delivery pipe

$$V_d = V_s = \frac{Q}{a_p} = \frac{0.0314}{\frac{\pi}{4} \times (0.15)^2} = 1.78\ m/s$$

$$\text{Velocity head} = \frac{V^2}{2g} = \frac{(1.78)^2}{2 \times 9.81} = 0.161\ m$$

Total efficiency pressure head on the delivery

$$side = H_d + H_{Ld} + \frac{{V_d}^2}{2g} = 32 + 7 + 0.161$$
$$= 39.161\ m$$
$$P_d = \gamma H_d = 9.81 \times 39.161 = 384\ kPa.$$
$$Total\ efficiency\ pressure\ head\ on\ the\ suction\ side$$
$$= 4 + 1.8 + 0.161 = 5.961\ m$$
$$P_s = \gamma H_s = 58.5\ kPa.$$

Q.2- A radial single stage, double suction, C.P. is manufactured for the following data:

$$Q = 35\ \frac{l}{s}\ ;\ D_1 = 100\ mm\ ;\ D_2 = 270\ mm$$
$$N = 1750\ rpm\ ;\ B_1 = 25\ mm\ perside\ ;\ B_2 = 23\ mm\ intotal$$
$$\eta_{overall} = 55\%\ ;\eta_{mech.losses} = 1.04\ kW\ :\ \alpha_1 = 90°\ ;\ \beta_2 = 27°$$
$$contraction\ coefficient\ due\ to\ blde\ thicknees = 0.87$$

Determine: -

(a) The inlet blade angle β_1

(b) The angle at which the water leaves α_2

(c) The speed ratio Ø
(d) The absolute velocity of water leaving Impeller V_2
(e) The manometric efficiency
(f) The volumetric and mechanical efficiency

Solution:-

Total quantity of water handled by the pump

$$Q_t = Q_d + Q_{leaky} = 75 + 2.25 = 77.25\ l/s$$

$$Q_{per\ side} = \frac{77.25}{2} = 38.625\ l/s$$

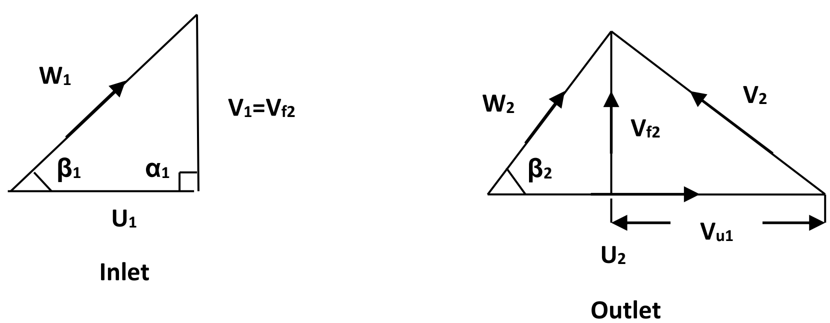
a) Peripheral speed at outlet $U_1 = \frac{\pi D_1 N}{60}$

$$U_1 = \frac{\pi \times 0.1 \times 17500}{60} = 9.15\ m/s$$

$$\text{Area of flow at inlet} = \pi D_1 B_1 \times 0.87$$
$$= \pi \times 0.025 \times 0.1 \times 0.87 = 0.00683\ m^3$$

$$V_{f1} = \frac{Q}{area\ of\ flow} = \frac{38.625 \times 10^{-3}}{0.00683} = 5.66\ m/s$$

$$\alpha_1 = 90°$$



From inlet velocity triangle

$$\tan\beta_1 = \frac{V_{f1}}{U_2} = \frac{5.66}{9.15} = 0.62$$

$$\beta_1 = 31.8°$$

b) $$V_{f2} = \frac{Q}{area\ of\ flow} = \frac{38.625 \times 10^{-3}}{\pi \times 0.29 \times \left(\frac{23}{2} \times 10^{-3}\right) \times 0.87}$$

$$V_{f2} = 4.25\ m/s$$

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.29 \times 1750}{60}$$
$$= 26.55\ m/s$$

$$Now\ \ \beta_2 = 27°$$

From velocity triangle at outlet

$$\tan \beta_2 = \frac{V_{f2}}{U_2 - V_{u2}} = \frac{4.25}{26.55 - V_{u2}}$$

$$\therefore \; V_{u2} = 18.21\, m/s$$

$$or \quad \tan \alpha_2 = \frac{V_{f2}}{V_{u2}} = \frac{4.25}{18.21}$$

$$\alpha_2 = 13.13°$$

$$c) \quad Speed\, ratio \quad \emptyset = \frac{U_2}{\sqrt{2gH_{mano}}} = \frac{26.55}{\sqrt{2 \times 9.81 \times 30}}$$

$$\emptyset = 1.0905$$

d) Absolute velocity of water leaving the impeller

$$V_2 = \frac{V_{u2}}{\cos \alpha_2} = \frac{18.21}{0.971} = 18.75\, m/s$$

$$e) \; Manometric\, efficiency \;\; \eta_{mano} = \frac{gH_{mano}}{V_{u2}U_2}$$

$$\eta_{mano} = \frac{9.81 \times 30}{26.55 \times 18.21} = 61\%$$

$$Volumetric\, efficiency \;\; \eta_Q = \frac{Q}{Q_{total}} = \frac{75}{77.25} = 97.1\%$$

$$Shaft\, power\, SKW = \frac{\gamma QH}{\eta_{overall}} = \frac{9.81 \times 75 \times 10^{-30} \times 30}{0.55}$$

$$= 20.65\, kW$$

$$\eta_{mech} = \frac{SKW - P_{mech\, loss}}{SKW} = \frac{20.65 - 1.04}{20.65} = 94.9\%$$

Q.3- A six stage C.P. delivers 120 l/s against a head pressure rise of 5000 kN/m^3. Determine its specific speed if it rotates at 1450 rpm.

Solution:-

$$N_s = \frac{N\sqrt{Q}}{H^{3/4}}$$

$$N_s = \frac{1450\sqrt{0.12}}{\left(\frac{5000}{9.81 \times 6}\right)^{3/4}} = 17.9 \approx 18$$

Q.4- A C.P. heaving an overall efficiency of 75% delivers 1220 l/min to a height of 12 m through a pipe of 100 m diameter and 90 m length. If f=0.0048, calculate the power required to drive the pump.

Solution:-

$$Q = \frac{1.820}{60} = 0.0303\, m^3/s$$

$$\eta_o = 0.75 \;;\; H_{st} = 18\, m \;; d = 0.1\, m \;; l = 90\, m$$

$$Friction\ head\ loss\ h_f = f.\frac{L}{D}.\frac{V^2}{2g}$$

$$V = \frac{Q}{A} = \frac{0.0303}{\frac{\pi}{4}(0.1)^2} = 3.86\, m/s$$

$$\therefore h_f = 0.0048 \times \frac{90}{0.1}.\frac{(3.86)^2}{2 \times 9.81} = 3.28\, m$$

$$Total\ head\ H_{mano} = H_{st} + H_f = 18 + 3.28$$

$$= 21.28\, m$$

$$P = \frac{\gamma Q H}{\eta_o} = \frac{9.81 \times 0.0303 \times 21.28}{0.75}$$

$$= 8.43\ kW$$

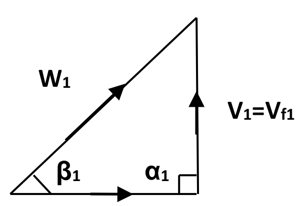
Q.5- Calculate the vane angle at inlet of a C.P. impeller having 300 mm diameter at inlet and 600 mm diameter at outlet. The outlet angle of the vane is 135^0 and the entry of the pump is radial, the pump runs at 1000 rpm and the velocity of flow through the impeller is constant at 3 m/s. also calculate the power per unit mass developed by the pump and the velocity and direction of water at outlet.

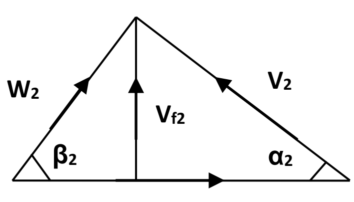
$$\alpha_1 = 90° \;;\; \beta_2 = 45° \;;\; N = 1000\, rpm$$

$$D_1 = 300\, mm \;; D_2 = 600\, mm \;;\; V_{f1} = V_{f2} = 3\, m/s$$

$$Find\ \ \beta_1\ \ , V_2\ \ , \alpha_2 ?$$

Solution:-





$$U_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.3 \times 1000}{60} = 15.7\, m/s$$

$$U_2 = \frac{\pi D_2 N}{60} = 31.4\, m/s$$

$$\tan\beta_1 = \frac{V_{f1}}{U_2} = \frac{3}{15.7} = 10.8°$$

$$\tan\beta_2 = \frac{V_{f2}}{U_2 - V_{u2}} = \tan 45$$

$$= \frac{3}{31.4 - V_{u2}} \qquad \therefore \; V_{u2} = 28.4\, m/s$$

$$V_{u2} = V_2 \cos\alpha_2$$

$$\tan\alpha_2 = \frac{V_{f2}}{V_{u2}} = \frac{3}{28.4}$$

$$\therefore \alpha_2 = 6°$$

$$\therefore V_2 = \frac{V_{u2}}{\cos\alpha_2} = \frac{28.4}{\cos 6°} = 28.56\, m/s$$

$$or \;\; V_2 = \sqrt{{V_{f2}}^2 + {V_{u2}}^2} = 28.56\, m/s$$

Q.6- A C.P. delivers 30 l/s of water against a head of 12 m and running at 1450 rpm. requires 4.5 kW. Determine the discharge, head of the pump and power required, if the pump runs at 1800 rpm for the same size.

Solution:-

$$N = 1450\, rpm \;\; ; Q = 30\frac{l}{s} \;\; ; H = 12\, m \;\; ; \; Power = 4.5\, kW$$

$$Find \;\; H \;\; , Q \;\; , P \;\; for\; N = 1800\, rpm D_1 = D_2 \quad same\; size$$

$$N_{sm} = N_{sp}$$

$$\frac{N_m}{N_p} = \frac{D_p}{D_m}\sqrt{\frac{H_m}{H_p}}$$

$$\frac{1450}{1800} = 1 \times \sqrt{\frac{12}{H_p}} \qquad \therefore \;\; H_p = 18.5\, m$$

Also

$$\frac{Q_m}{Q_p} = \left(\frac{D_m}{D_p}\right)^2 \sqrt{\frac{H_m}{H_p}}$$

$$\frac{30}{Q_p} = 1 \times \sqrt{\frac{12}{18.5}} \qquad Q_p = 37.3\, l/s$$

$$\frac{P_m}{P_p} = \left(\frac{D_m}{D_p}\right)^2 \times \left(\frac{H_m}{H_p}\right)^{3/2}$$

$$\frac{4.5}{P_p} = 1 \times \left(\frac{12}{18.5}\right)^{3/2}$$

$$P_p = 8.6\ kW$$

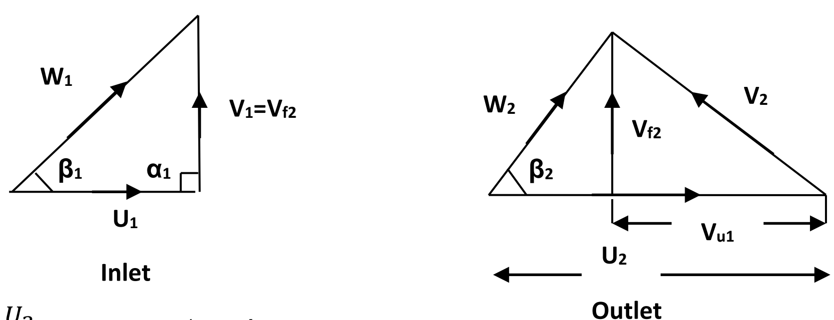
Q.7- A C.P. impeller runs at 950 rpm. Its external and internal diameters are 500 mm and 250 mm. The blade is set back at an angle of 35^0 to the water rim. If the radial velocity of water through the impeller be maintained constant at 2 m/s. find the angle of the blade at inlet, the velocity and direction of water at outlet and the work done by the impeller per kN of water.

Solution:-

$$\beta_2 = 35° \ ;\ N = 950\ rpm$$

$$D_1 = 0.25\ m\ ; D_2 = 0.5\ m\ ;\ V_{f1} = V_{f2} = 2\ m/s$$

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.5 \times 950}{60} = 24.87\ m/s$$



(i) $U_1 = \frac{U_2}{2} = 12.435\ m/s$ $\qquad \alpha_1 = 90°$

$$\therefore V_{u1} = 0 \quad radial\ flow$$

$$\tan\beta_1 = \frac{V_{f1}}{U_1} = \frac{2}{12.435} \quad \therefore\ \beta_1 = 9.146°$$

(ii) $$V_{u2} = U_2 - \frac{V_{f2}}{\tan\beta_2} = 24.87 - \frac{2}{\tan 35}$$

$$V_{u2} = 22.1\ m/s$$

$$V_2 = \sqrt{{V_{f2}}^2 + {V_{u2}}^2} = \sqrt{2^2 + (22.1)^2} = 22.2\ m/s$$

(iii) $\alpha_2 = ?$

$$\tan\alpha_2 = \frac{V_{f2}}{V_{u2}} = \frac{2}{22.1} = 0.09$$

$$\therefore\ \alpha_2 = 5.142°$$

(iv) $$Work\ done\ per\ unit\ weight = \frac{V_{u2}U_2}{g}$$

$$= \frac{22.1 \times 24.87}{9.81} = 56\ kN$$

Q.8- A C.P. of 1.2 m diameter. Runs at 200 rpm and pumps 1880 l/s, the average lift being 6 m. the angle which the blade make at exit with the tangent to the impeller is 26^0. And the radial velocity of flow is 2.5 m/s. Determine the useful power and the efficiency.

Solution:-

$$D_2 = 1.2\,m\ ; N = 200\,rpm\ ; Q = 1.88\,m^3/s$$

$$H = 6\,m\ ;\ \beta_2 = 26°\ ;\ V_{f2} = V_{f1} = 2.5\,m/s$$

$$D_1 = 0.6\,m$$

(i) $P = \gamma QH = 9.81 \times 1000 \times 1.88 \times 6 = 110\ kW$

(ii) $Theoretical\ head = \frac{V_{u2}U_2}{g}$

To manometric head (H_{mano}) if losses is neglected $\alpha_1 = 90°$

$$\therefore\ H_{mano} = \frac{V_{u2}U_2}{g}$$

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 1.2 \times 200}{60} = 12.56\,m/s$$

$$V_{u2} = U_2 - \frac{V_{f2}}{\tan\beta_2} = 12.56 - \frac{2.5}{\tan 26°}$$

$$= 7.44\,m/s$$

$$\therefore\ H_{mano} = \frac{7.44 \times 12.56}{9.81} = 9.526\,m$$

$$\therefore\ \eta_{mano} = \frac{6}{9.526} \times 100 = 63\%$$

Chapter seven

Hydraulic system

7.1 Types of Hydraulic system

It is a circuit in which force and pressure are transmitted through a fluid generally an oil, and divided into two groups.

a) Hydro static system: in this system the transmission of force and power by pressure, Ex. Pumping unit in hydraulic press, hydraulic cran …. etc.
b) Hydro kinematic system: in this system the trans mission of power by changes in velocity of flow of working media. Ex. Hydraulic coupling, torque convertor.

7.2 Structure of hydraulic system

Bulldozer, Forklift, Power shovel and grader … etc. operated by an execute various works including excavation, loading and transportation … etc. by moving the heavy blade bucket or fork by simply operating a single lever by means of hydraulic pressure.
The hydraulic system contents of five elements

1. Hydraulic tank
2. Hydraulic pump
3. Hydraulic valve (control valve)
4. Hydraulic pressure actuator (operation section)
5. Piping and other accessories.

Fig. (1) shows the hydraulic structure of all hydraulic system.

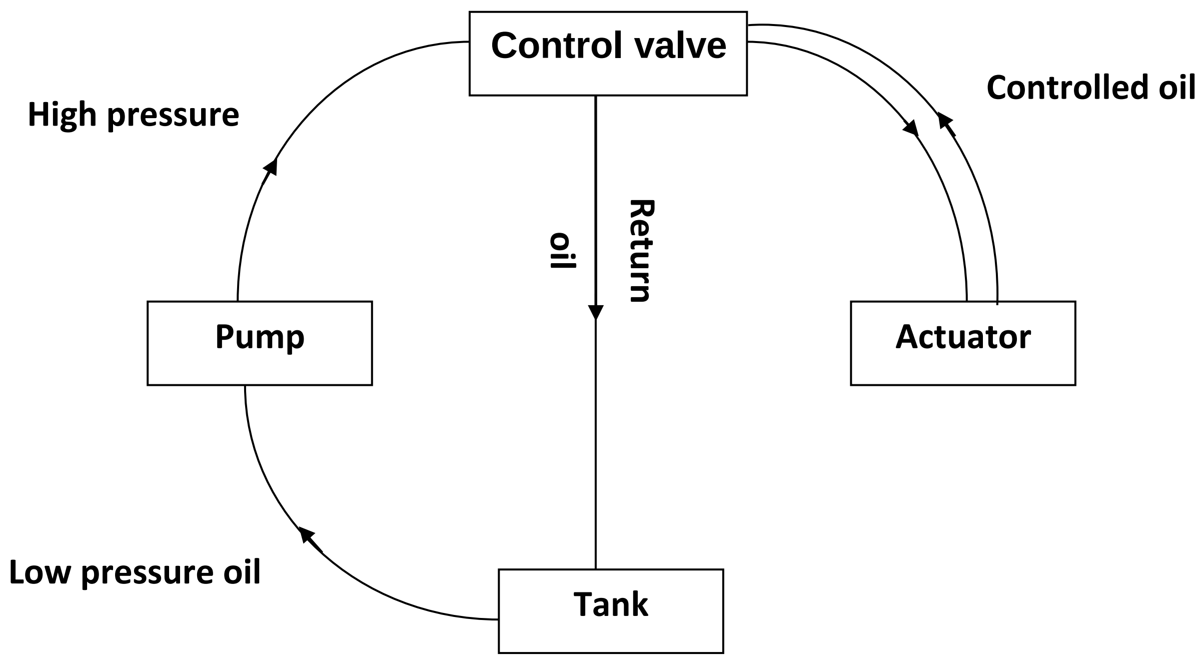


Fig. (1) Hydraulic Structure.

1. Hydraulic tank: A hydraulic device requires to supply the system by the oil. And required an equipment for
- reducing the temperature raised by application (cooling the oil)

The dissipation of temperature or heat depend on

a) Tank size

b) Oil size in the tank

c) Temperature difference between in and out the tank (ambient)

d) The tank site.

- precipitating lust or rust included in the oil mixed while oil is circulated.
- Ventilated the air may included.
- The power unit fixed on it and other indicators.

The volume of the tank depends on

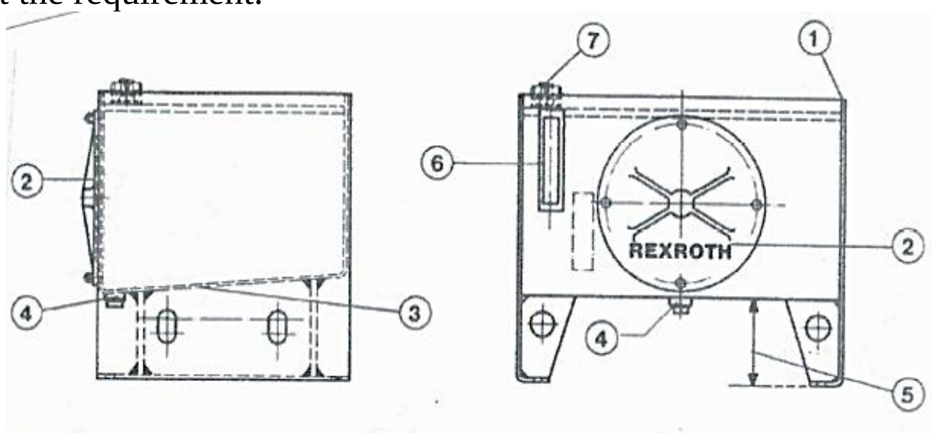
a) The flow rate of the pump 3Q →5Q

b) Damping of air 10% →15%

c) Size of equipment fixed on it
d) Cooling the oil.
Fig. (2) shows types of tanks.

2- Hydraulic pump:-
Its sucks oil from the tank and generates maximum pressure and maximum flow (i.e maximum torque and speed required by the work) required by the hydraulic valve.

3- Hydraulic valve (control valve):-
The hydraulic valve plays the role of feeding the actuator with oil regulated to meet the requirement.



(a)

(b)

Fig. (2) (a) Hydraulic tank.
(b) Hydraulic tank with power unit and gauges.

Hydraulic valves comprise pressure control, flow control and direction control.

4- Hydraulic actuator:-

It oil flow into the speed of work and the force of oil into the size of work. There are two types of work

- Reciprocated motion (hydraulic cylinders)
- Rotary motion (hydraulic motors)

5- Piping and other accessories:-

Pipes, high pressure rubber hose, strainer, oil filter, oil cooler, heater, packing gasket, liquid gasket, oil seal oil pressure gage, oil temperature gage.

7.3 Advantages and Disadvantages of hydraulic system

Advantages:-

1- Strong force or torque is obtained by a device of small size.
2- Prevention of overload may be available correctly difficulty.
3- Control of force is simple and correct.
4- Stepless speed conversion is simple and the operation is smooth.
5-Generation of vibration is slight and action is smooth.
6-Motive power may be transmitted to optional location by means of hydraulic line so that remote control is available.
7-Durability is provided.

Disadvantages:-

1. Oil leakage may be generated.
2. Easily influenced by temperature.
3. Big noise is generated from pump.
4. Sometimes, the speed may fluctuate.
5. The work of piping is not so simple as electric wiring.

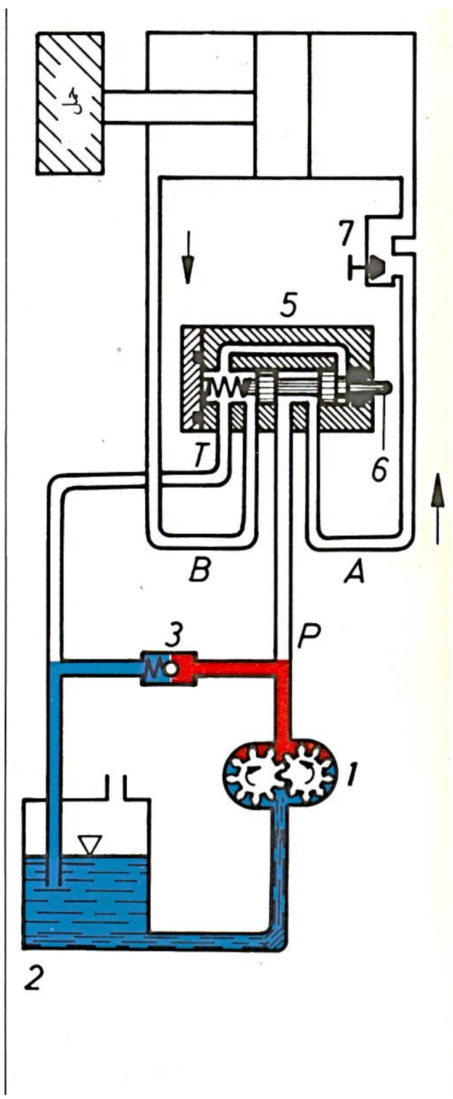
7.4 Function of hydrostatic system

1. Actuation of mechanical control.
2. Force multiplication.
3. Transmission and control of power.

7.5 Simple hydraulic circuit

Fig. (3) show a simple hydraulic circuit contents of a pump (1) driven by a mechanical or an electrical motor. The pump drawn the liquid from the tank (2) and pushed to high pressure line (red) which include a control valve (5) to the cylinder (4) (actuator).

4.1 piston of the cylinder
4.2 connecting rod
3 non-return valve maintain the pressure.
6 spool of the valve
P pressure line (pump)
T return linn (Tank) (Blue)
A, B working lines from or to the actuator
Fig. (4) shows the neutral position of hydraulic circuit.
Fig. (5) shows the reverse diagram.
Fig. (6) shows the reverse diagram including flow control valve (7).
Fig. (7) shows the line diagram of the circuit.



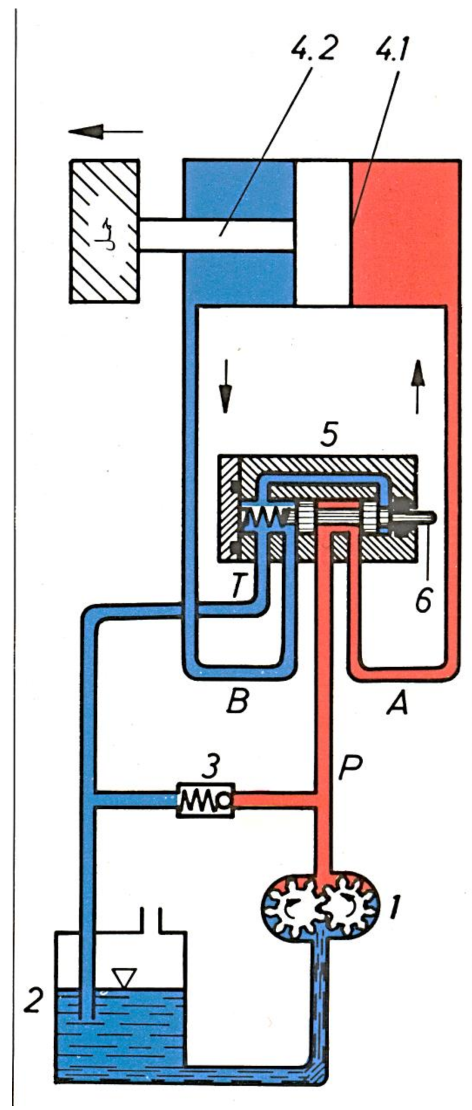


Fig. (4)
Simple Hydraulic Circuit
At neutral position.

Fig. (3)
Simple Hydraulic
Circuit.

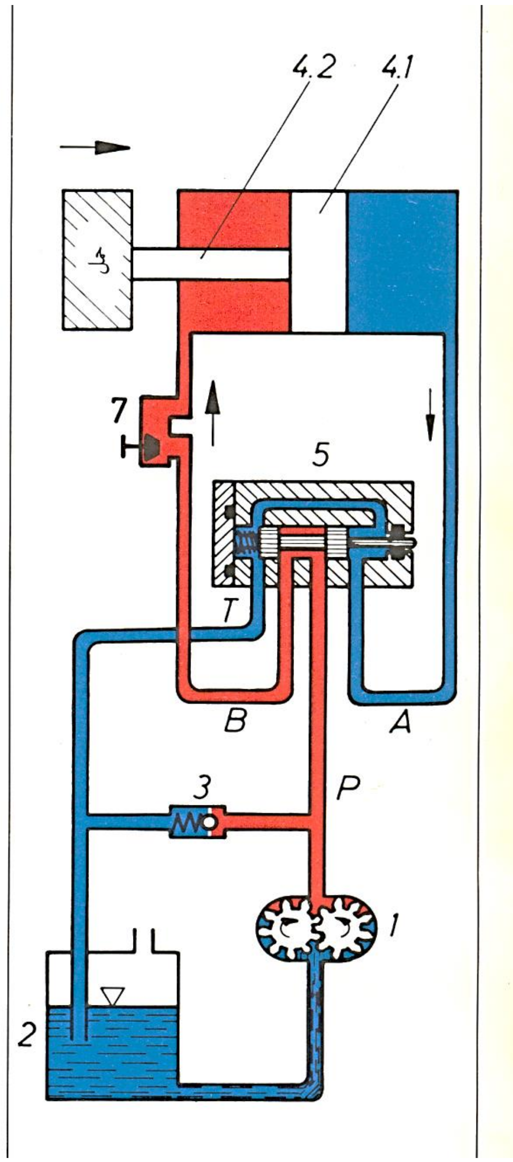


Fig. (6)
Hydraulic Circuit with flow C.V.

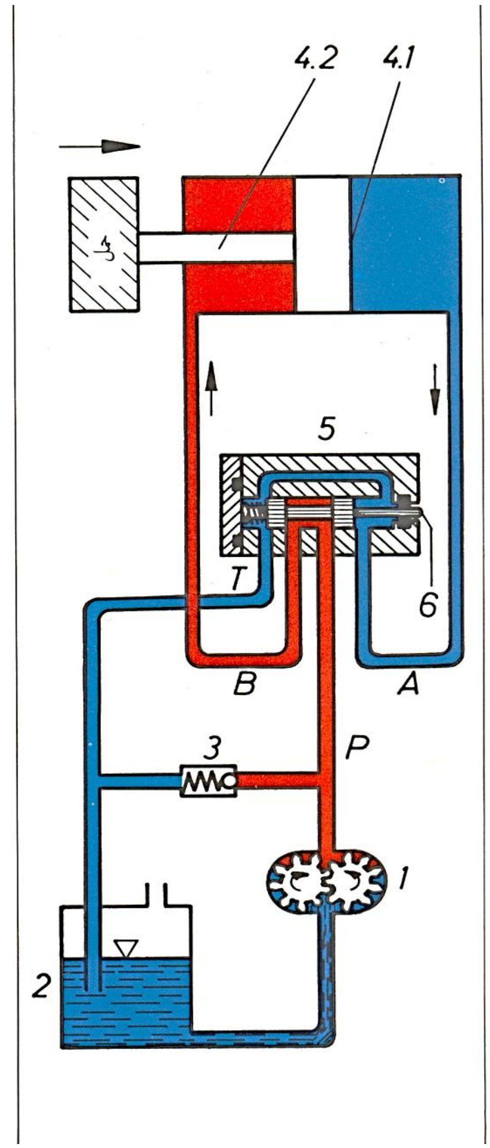


Fig. (5)
Reverse Position.

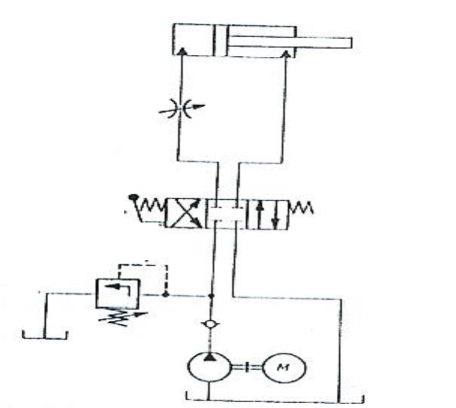


Fig. (7) Diagram of Hydraulic Circuit.

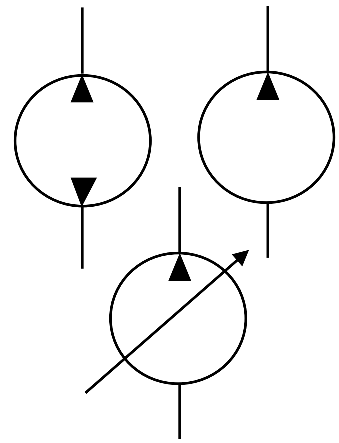
7.6 Hydraulic symbols

(1) Pumps

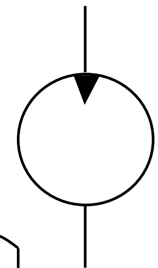
a. fixed displacement

b. variable

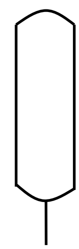
Note: direction of flow (liquid)
for compressor (gas)



(2) Motor

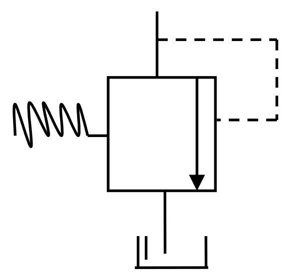


(3) Accumulator

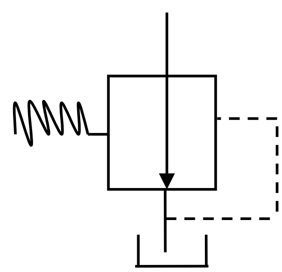


(4) Pressure C.V.

a. Pressure relief valve

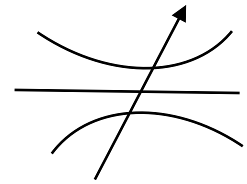


b. Pressure reducing

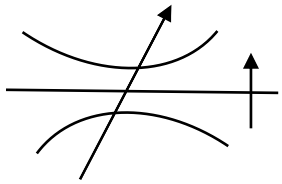


(5) Flow C.V.

a. Non - compensated adjustable

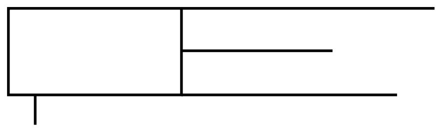


b. pressure - compensated adjustable

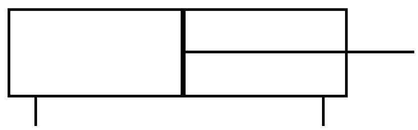


(6) Cylinders

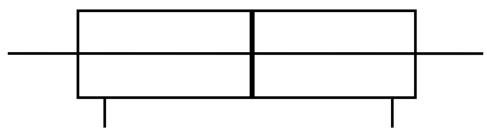
a. Single acting



b. Double acting

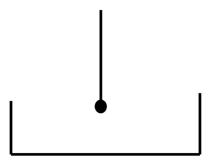


c. Double acting
and rod

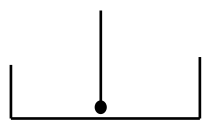


(7) Tanks

a. The connection of the pipe
at the upper side



b. The connection of the pipe
at the lower side



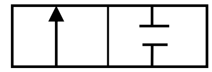
(8) Pipes

a. Any kinds of pipes

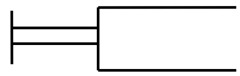
b. Draining pipes

c. Flexible pipes

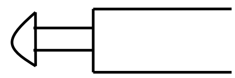
(9) Controls of valves



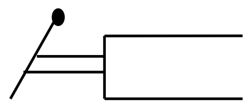
a. Manual



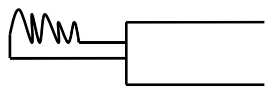
b. Push button



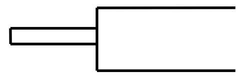
c. Push – pull lever



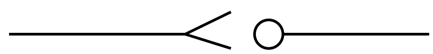
d. Detent



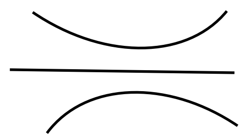
e. Mechanical



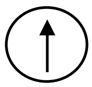
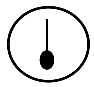
(10) Check valve



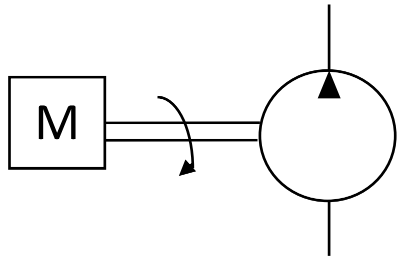
(11) Chock



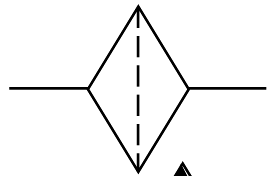
(12) Guage

a. Pressure  b. Temp 

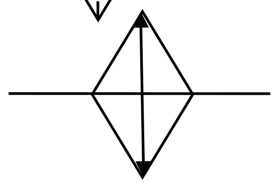
(13) Pump with electrical motor



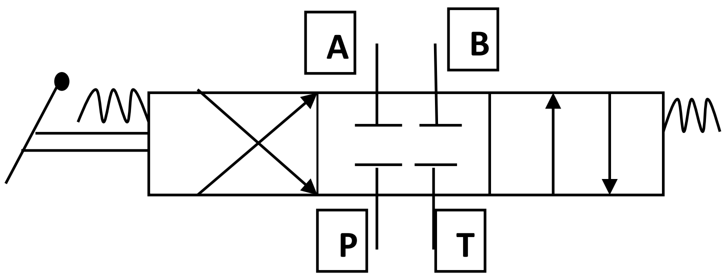
(14) Filter

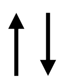


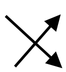
(15) Radiator



(16) Directional control valve symbol



 The flow from P → A and return to the tank B → T

 The flow from P → B and return to the tank A → T

$\frac{3}{4}$ (3) Three position of supply

P → A , B → T

P → B , A → T

P → T neutral

(4) Four opening P, T, A, B

7.7 Hydraulic Pumps

It is used primarily as a source of hydraulic fluid power. They are classified to:
a) Constant delivery pumps: Q = Constant
b) Variable delivery pumps: Q = Variable

a. Constant delivery pumps:
They are classified with respect to the impeller element.
1- Gear pump.
   a. External type.
   b. Internal type.
2- Screw pump.
3- Vane pump.
4- Axial piston pump.
5- Radial piston pump.

7.7.1 Gear pump

a. External type gear pump:- it consists of two identical intermeshing gears Fig (8) (spur, Helical, Herring bore) working with fine clearance inside a suitable shape casing. One gear is keyed to the driving shaft af a motor and the other revolves idly oil is entrain in the spaces between the gears from suction port to the discharge port. It cannot slip back into the into the inlet side due to the meshing of gears.

They are available for
1- Continuous pressure up to 250 bars
2- Minimum speed 400 → 600 rpm
3- Minimum speed 3000 → 6000 rpm
4- Minimum deliveries up to 10 l/s.

b. Internal type gear pump: Fig (9)
in the type the idlel is an internal meshes with the diving gear. The space between the outside diameter of idter is seald by a crescent shape projection which from a part of the cover.
The pump can be reversible if a provision is made for swinging thr crescent through 180^0.

They are available for:
1- Continuous pressure of 130 bar.
2- Maximum speed 2000 →3600 rpm
3- Maximum deliveries of 4
Advantages
1- Structure is simple.
2- Light in weight.
3- Size is small.
4- Price is low.
5- Ratio of trouble generation is small.
6- Maintenance cost is small.

Disadvantages
1- Noise is great.
2- Generates large pulsation.

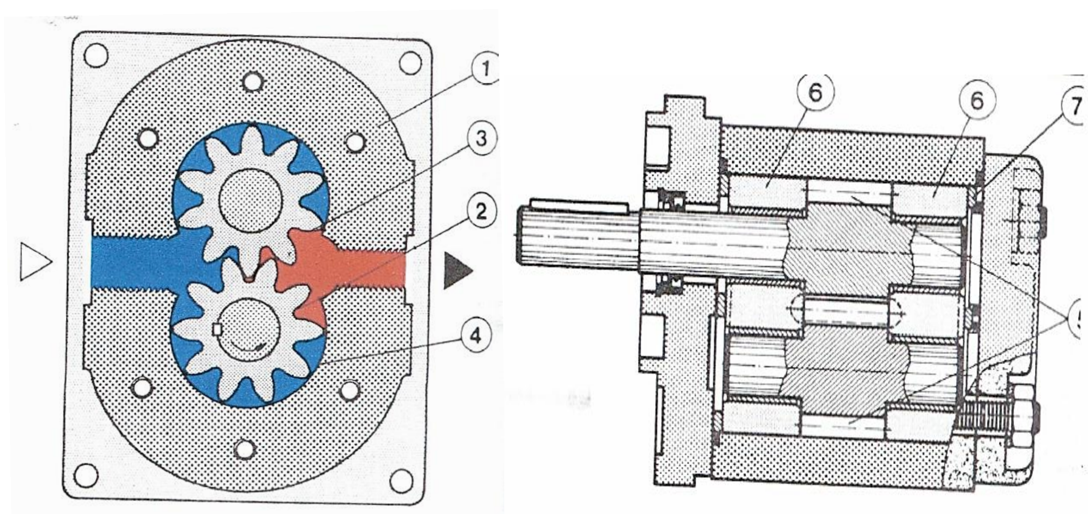


Fig. (8)
1- Pump body 2- Idler gear.
3- Driven gear 4- Gear house.

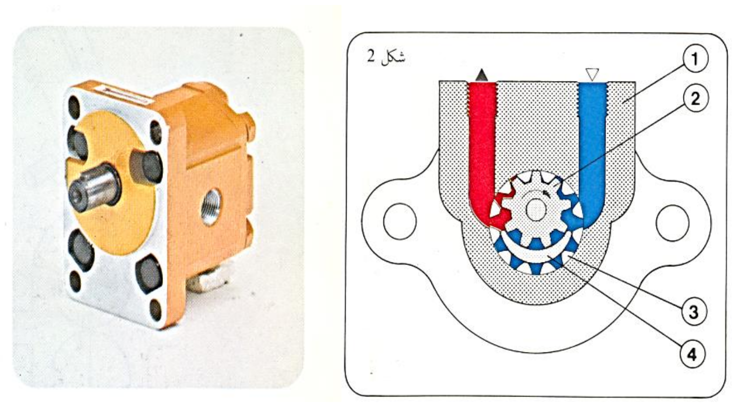
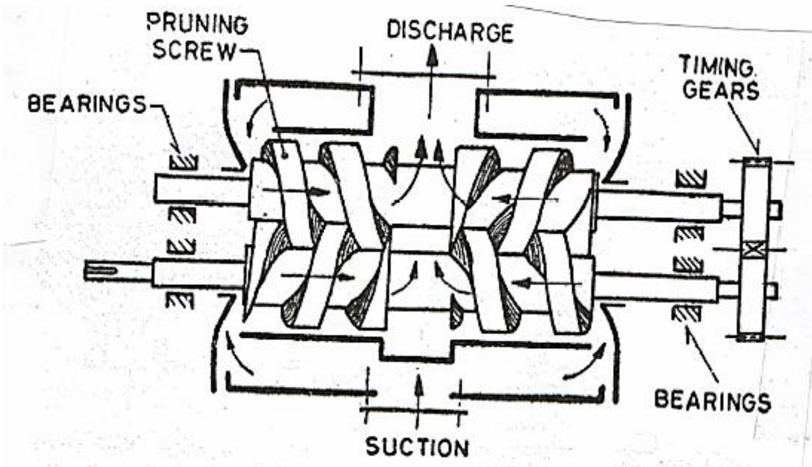


Fig. (9)
1- Pump body 2- Internal.
3- External gear 4- Crescent.

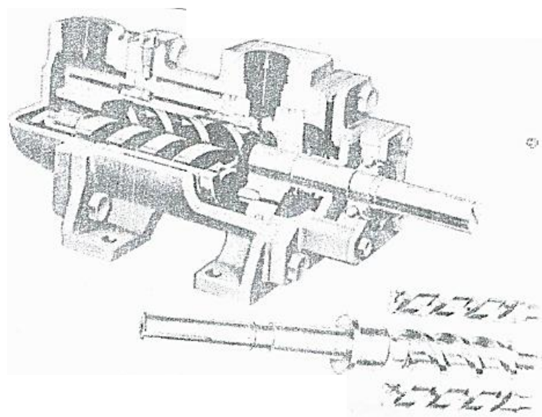
7.7.2 Screw pump

Fig (10) it consists of a rotor directly connected to the source of power. The rotor may be double helical 9right and left hand) or single helical (one hand only). The fluid is carried forward to the discharge along the rotor in pockets formed between teeth and the casing.
There may be one, two or three screws in a 2- screw pump, one is a rotor and the other is idler. In a 3- screw pump there are two idlers on either side.

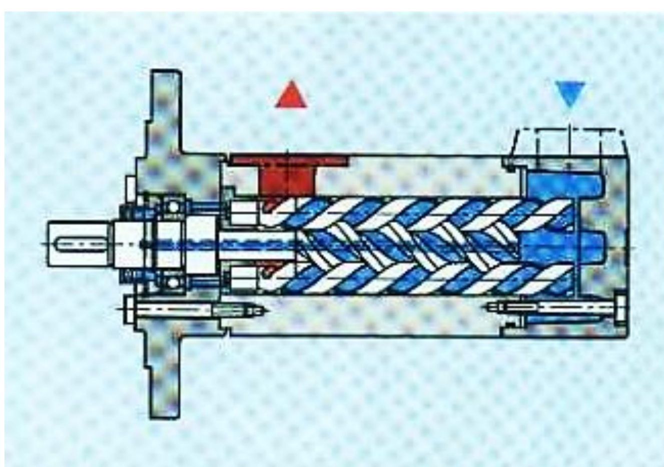
There are available for.
1- Continuous pressure up to 200 bar.
2- Maximum speed $3000 \rightarrow 4500$ rpm.
3- Maximum delivery up to 300 l/s.
Advantages
1- It is the quietest pump and free from vibration while running.
2- It is free from turbulence and pulsation.
Disadvantages
Fluid of higher viscosity may require considerable derating, similarly efficiency e.g. the efficiency at 29 C^0 is 67% while at 50 C^0 is 54%



(a) Schematic diagram two- Screw.



(b) Three- Screw.



(c) Two- Screw.

Fig. (10) Screw Pump.

7.7.3 Vane pumps

Fig. (11) there are two types.

1- Fixed displacement unbalanced pump.

2- Fixed displacement balanced pump.

Both types consist of a rotor dies having a number of slots into which fit sliding blades or vanes (generally 4 → 8 vanes).

The vanes are free to slide radially with help of spring or hydraulic oil, thus from the required seal between the suction and discharge connections.

They are available for.

1- Continuous pressure up to 160 bar.

2- speed 200 → 2500 rpm.

3- Maximum delivery to 10 l/s.

Advantages

1- Pulsation of discharge pressure is small.

2- The shape and dimension are smaller in relation to the pump oil and power.

3- Trouble is seldom generated and main trance is made without difficulty.

4- Repair is easy.

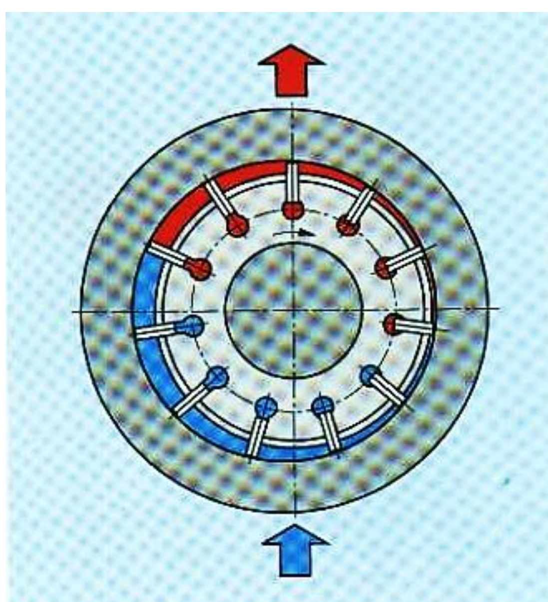
5- The time required for attaining the maximum pressure is shore.

Disadvantages

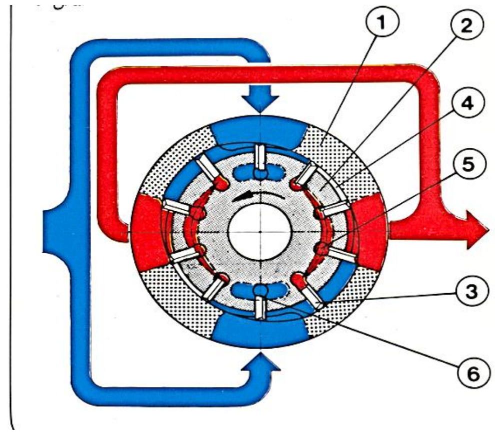
1- Working precision is high (cost is high).

2- The viscosity of oil to be employed is restricted.

3- Attention is required for the cleanness of oil.



(a) Single stroke vane pump.



1- Pump body

2- Rotor

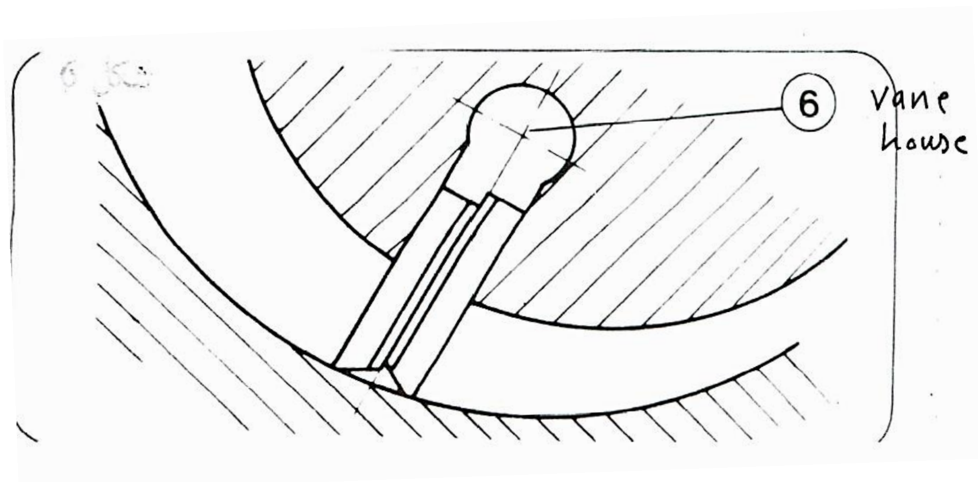
3-Vane

4- Pump house

5- Outlet

6- Inlet

(b) Double chamber vane pum



(c) Vane design.

Fig. (11) Vane pump.

7.7.4 Axial piston pumps

The rotary piston- types are either radial or axial in designs.

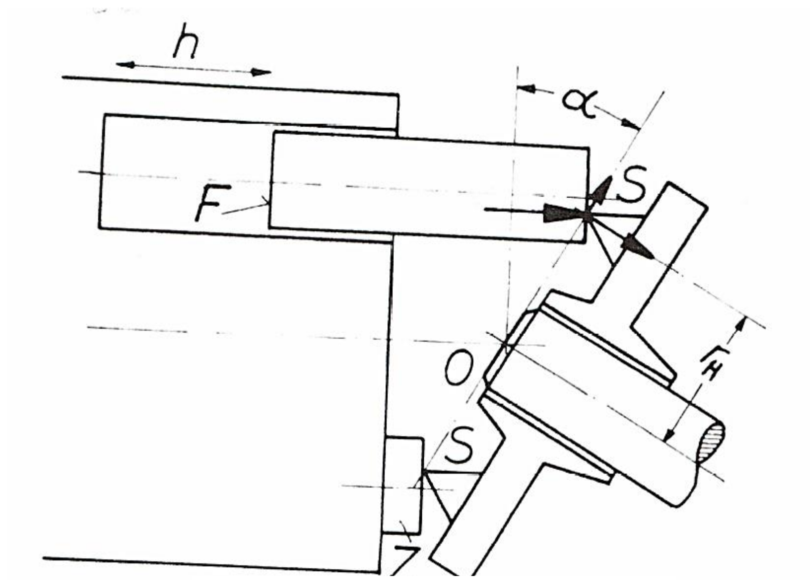
The axial type Fig. (12) the piston is arranged parallel to the shaft of the pump rotor. The driving means of the pump rotates the cylinder barrel. The axial reciprocation of the pumping pistons that are confined in the cylinder is caused by shoe retainer which is spring – load toward the swash plate (cam plate). The piston stroke and the quantity of oil delivered are limited by the angle of the swash plate.

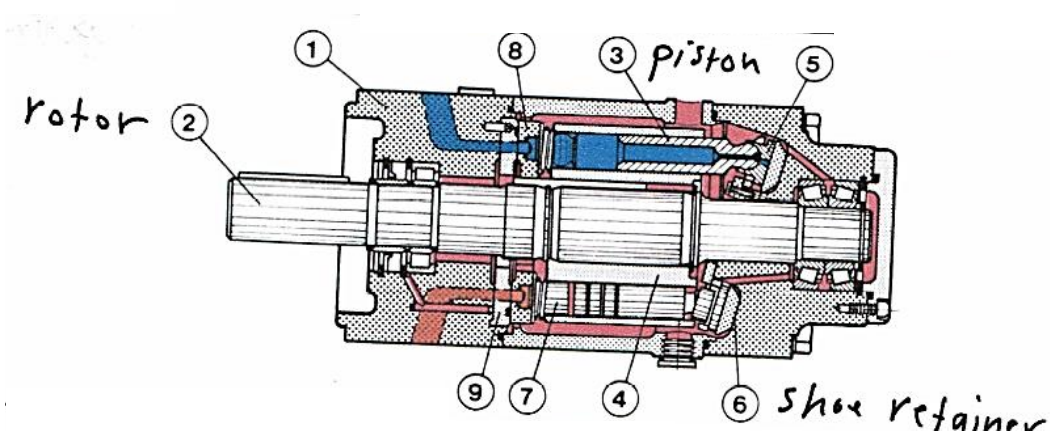
They are available for.

1- Continuous pressure 70 → 700 bar.
2- Speed 900 → 1800 rpm.
3- Maximum delivery to 1-500 l/s.

Advantages

1- The flow rate and the pressure can be controlled and depends on piston diameter.
2- The pump can be work in two direction which useful for mechanic tool.





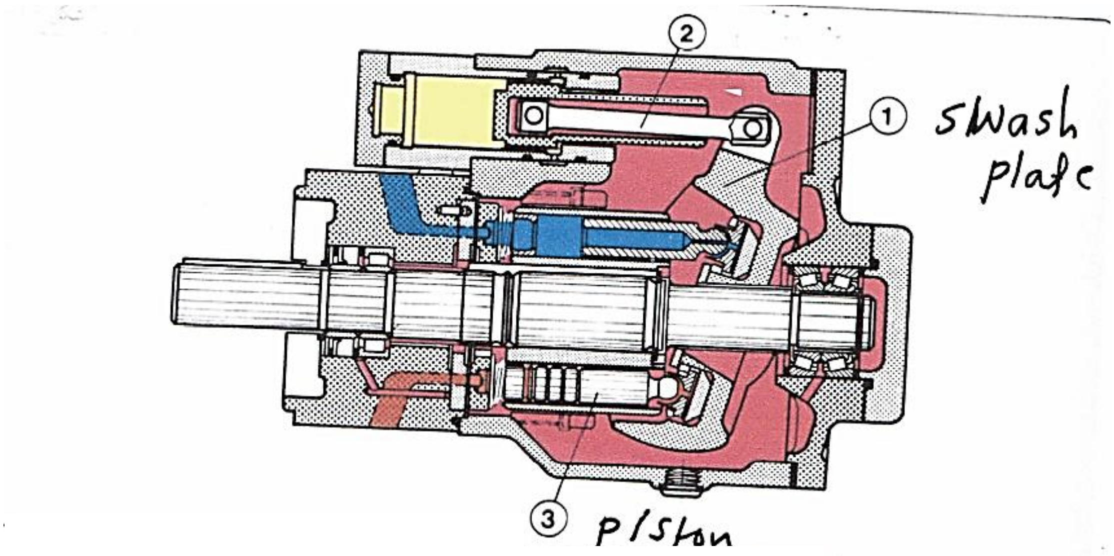


Fig. (12)

7.7.5 Radial piston pump

Fig (13) the radial piston pump consists of a number of cylinder displaced radially about an eccentric on the driving shaft. The rotation of the cam shaft causes reciprocation of the piston. Number if piston (2- 24) piston which is a function of flow rate and pressure.

They are available for.

1- Continuous pressure up to 650 bar.
2- Speed up to 3200 rpm.
3- Maximum delivery up to 200 l/min.

Variable delivery pumps.

In general, all the pumps are used as a variable delivery pump by controlling the flow rate and pressure. As an example, the swash plate could be radial type at a variable angle, the rotor of radial type at a variable speed …. Etc.

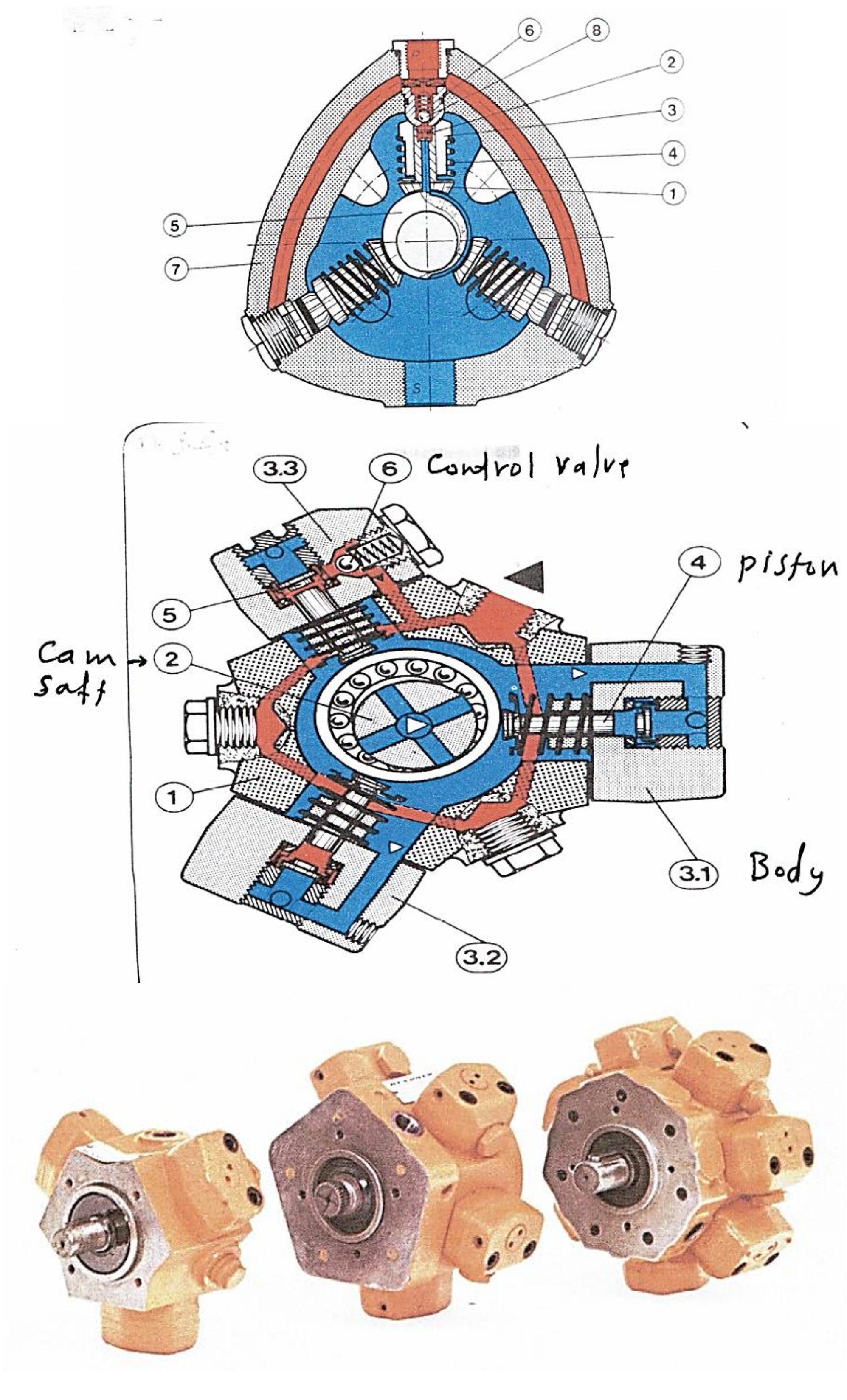
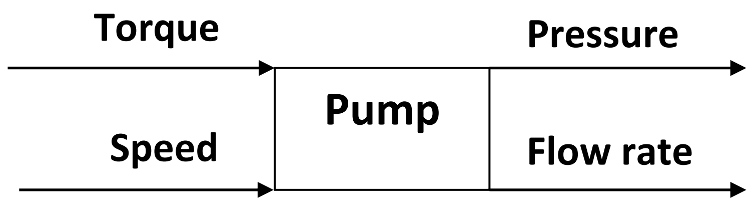
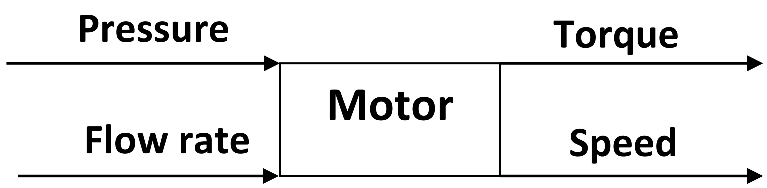


Fig. (13)





$$Power = T\omega$$

- Pump design as a function of volumetor efficiency.
- Motor design a function of mechanical efficiency 0.9 → 0.98

7.8 Pumping Theory

a- the ratio of the output from a pump to the input, describing overall efficiency

$$\eta_{overall} = \frac{\text{Pump output(Fluid power)}}{\text{Pump input(Brake power)}}$$

$$\eta_o = \frac{P \times Q}{T \times \omega}$$

Ex: A pump operatives at 174 bar and 22× 10^{-5} m^3/s. the input power 4 kW compute the overall efficiency and the input Torque to the pump when it run at 1725 rpm

<u>Solution :-</u>

$$\eta_o = \frac{P \times Q}{T \times \omega} = \frac{174 \times 10^5 \times 22 \times 10^{-5}}{4 \times 1000} = 96\%$$

$$also\ \eta_o = \frac{P \times Q}{T \times \omega} \quad \therefore \quad T = \frac{174 \times 10^5 \times 22 \times 10^{-5}}{0.96 \times 1725 \times \frac{1}{60} \times 2}$$

$$T = \frac{138.7}{2\pi}\ N.m = 22.1\ N.m$$

b- Theoretical Pump Torque is expressed in term of pump displacement and specified operating pressure:

$$T_{th} = \frac{Displacement(V_p) \times Pressure(P)}{2\pi}$$

$$T_{th} = \frac{\mathrm{V_p} \times P}{2\pi}$$

$$\mathrm{V_p} = \text{Volume displaced per revolute} = \text{CC/rev or l/rev}$$

$$Torque\ efficiency = \frac{Tactual}{T_{th}} \times 100$$

Which is the same as mechanical efficiency.

c- Leakage or pump cavity flow in positive displacement hydraulic pumps is computed in liter/min

$$Volumetric\ efficiency\ \eta_Q = \frac{Pump\ volume\ output}{Pump\ displacement \times Speed}$$

$$\eta_Q = \frac{Q_o}{V_p.N} \times 100$$

Then

$Pump\ overall\ efficiency$

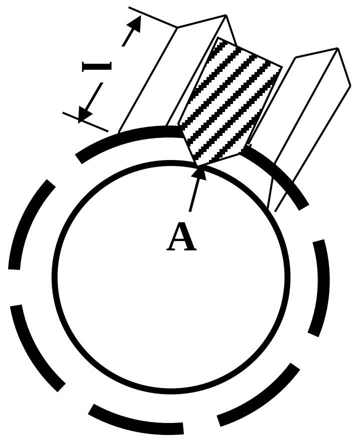
$$= Pump\ volumetric\ eff. \times Pump\ Mechanical\ eff.$$

$$\eta_{overall} = \eta_Q.\eta_T$$

d- Frictional power losses are computed as a potion of the brake power at pump drive shaft.

$$Pump\ Frictional\ Power = Pump\ Brake\ Power[1 - Power\ Mech.efficiency]$$

$$FP = BP[1 - \eta_T]$$



For gear pump
Let
A- The area enclosed between
Two adjacent teeth and casing.
l- The axial length of teeth.
n- Number of teeth in each pinion.
N- The speed in rpm.

$$\therefore \text{The volume of liquid pumped in one revolution} = 2Al.n$$

$$Or\ V_p = 2Al.n \text{ for external type}$$

$$\therefore \text{Total or ideal discharge} = \frac{2AlN}{60}$$

$$\text{If } \eta_Q \text{ be the Volumetric efficiency}$$

$$\therefore Q_{act} = \frac{2AlN}{60}.\eta_Q$$

$$or\ Q_{act} = V_p.N.\eta_Q$$

$$Note: \text{for internal type } V_p = Al.n$$

$$n = \text{Number of teeth}(total)$$

The theoretical displacement of gear pump is

$$V_p = 2(A - A_m)l.n$$

A_m = The area between the meshing teeth

It is difficult to determine A_m the following empirical relation could be used.

$$V_p = 0.95\,\pi.C.(D - C)l$$

$C -$ *The center to center distance between gear axis's.*
$D -$ *Outside diameter of gears.*
$n -$ *Number of teeth related to the ratio* $\frac{D}{C}$

n	7	10	13	18
D/C	1.28	1.21	1.13	1.12

For Piston Pump
A- The cross- section area of the piston or cylinder
S- Stroke

$$\therefore Cylinder\ Volume = A.S$$
$$\therefore V_p = A.S$$
$$or\ Q_{act} = V_p.N.\eta_Q$$

$N - Speed\ rpm.$

Q.1- Compute the volumetric efficiency of a pump that has a positive displacement of 61.5 cm^3 and delivers 3.28 l/s of fluid while operating at 3300 rpm.

Solution:-

$$\eta_Q = \frac{Q}{V_p \times N} = \frac{3.28}{61.5 \times 10^{-3} \times \frac{3300}{60}}$$
$$= 0.97\ or\ 97\%$$

Q.2- A hydraulic pump is observed to have an overall efficiency of 87% while consuming 7.457 kW Brake power measured at the pump drive shaft. Compute the mechanical efficiency and the frictional power loss from the system when the volumetric efficiency 94%.

Solution:-

$$\eta_T = \frac{\eta_{overall}}{\eta_Q} = \frac{0.87}{0.94} = 0.9255$$
$$FP = 7.457[1 - 0.9255] = 0.555\ kW$$

Q.3- A) Determine the output torque of a hydraulic motor with a displacement of 60 cm^3/rev when the pressure drop across the motor is 100 bar (assume 100% torque efficiency). B) Determine the SIKW power delivered to the load of the motor is 600 rev/min, and the overall efficiency of the motor is 100%.

Solution:-

A)

$$Torque\ (N.m) = \frac{P\left(\frac{N}{m^2}\right).V_p\left(\frac{m^3}{rev}\right)}{2\pi\left(\frac{rev}{rod}\right)}$$

$$= \frac{100 \times 10^5 \times 60}{2\pi \times 10^6}$$
$$= 95.5\ N.m$$

B)

$$Power = T.W \qquad W = \frac{2\pi N}{60}$$
$$= 95.5\ \times \frac{2\pi N}{60}$$
$$= 6\ kW$$

Q.4- What are the input power and the overall efficiency of internal gear type pump when the speed 900 rpm, torque 40 N.m, the flow rate 10 l/min and the pressure 200 bar?

Solution:-

$$(Mech\ rotary) or\ input\ power = T.W$$
$$P_{in} = 40 \times \frac{2\pi N}{60}$$
$$= 40 \times \frac{2\pi \times 900}{60} = 3.77\ kW$$
$$\eta_{overall} = \frac{P_{out}}{P_{in}} = \frac{P \times Q}{3.77}$$
$$= \frac{200 \times 10^5 \times 10 \times \left(\frac{1}{1000 \times 60}\right)}{3.77}$$
$$= \frac{3.33}{3.77} \times 100 = 88\%$$

Q.5- Calculate the following efficiency for an assumed pumping system of internal pump.

a) Electric drive motor efficiency.

b) Hydraulic pump efficiency.

c) System overall efficiency.

Use the following input and outputs.

1- Motor input power = 3.73 kW

2- Motor output power = 4.5 hp × 0.745 = 3.357 kW

3- Pump input power = 4.5 hp× 0.746 =3.357 kW

4- Pump output flow rate = 441.6 cm^3/s

5- Pressure output = 68 bar

Solution:-

a)

$$Electric\ motor\ efficiency = \frac{P_{out}}{P_{in}}$$

$$= \frac{3.357}{3.73} = 90\%$$

b)

$$Pump\ overall\ efficiency = \frac{P_{out}}{P_{in}}$$

$$= \frac{Pressure \times Flow\ rate}{Pump\ input\ power(Electrical\ motor\ output)}$$

$$= \frac{68 \times 10^5 \times 441.6 \times 10^{-6}}{3.357 \times 1000} = 89.5\%$$

c)

$$System\ overall\ efficiency = \frac{Pump\ output}{Electrical\ motor\ input}$$

$$Pump\ output = P.Q = 68 \times 10^5 \times 441.6 \times 10^{-6} \times \frac{1}{1000}$$

$$= 3\ kW$$

$$\therefore\ System\ overall\ efficiency\ \eta_o = \frac{3}{3.73} \times 100$$

$$= 80.4\%$$

$$or\ \ \eta_{overall} = \eta_{motor} \times \eta_{pump}$$

$$= 0.9 \times 0.895$$

$$= 0.8055\ \ or\ \ 80.55\%$$

Q.6- What will be the hydraulic power delivered by external gear pump if it supplies 315.5 cm^3/s and P= 117 bar. It the overall efficiency of the pump is 78%. What mechanical power output of motor to drive it.

Q.7- What will be the overall efficiency an internal type gear pump for N= 1200 rpm, T = 100 N.m, Q = 28 l/min and P = 240 bars.

Ex.3: A hydraulic motor produces 7.5 kW at a speed of 150 rod/s with a pressure loss of 10 bar from the pump to motor. Determine the input power to the pump, pump flow rate, output torque when the efficiency of the pump and the motor are.

efficiency	pump	motor
η_V	0.97	0.9
η_T	0.95	0.85

System pressure 90 bar
Leakage in system 6 l/min

Solution:-

$$P_{out} = T.W$$

$$The\ motor\ torque\ output = \frac{Power\ out}{Speed}$$

$$or\ = \frac{7.5 \times 10^3}{150} = 50\ N.m$$

$$Theoretical\ output\ Torque = \frac{actual\ Torque}{\eta_T}$$

$$= \frac{50}{0.85} = 59\ N.m$$

$$Pressure\ available\ at\ the\ motor = Pressure\ at\ the\ pump - \ losse$$

$$= \ 90 - 10 = 80\ bar$$

$$\therefore Theoretical\ flow\ rate\ at\ the\ motor = \frac{T_{th} \times W}{Pressure}$$

$$= \frac{59 \times 150}{80 \times 10^5} = 1.1 \times 10^{-3}\ \ m^3/s$$

$$\therefore \text{Actual flow rate at the motor} = \frac{1.1 \times 10^{-3}}{\eta_V}$$

$$= \frac{1.1 \times 10^{-3}}{0.9} = 1.23 \times 10^{-3}\ m^3/s$$

$$Flow\ out\ of\ the\ pump = Actual\ flow\ at\ the\ motor + System\ leakage$$

$$= 1.23 \times 10^{-3} + \frac{6 \times 10^{-3}}{60} = 1.33 \times 10^{-3}\ m^3/s$$

$$P_{out}\text{of the pump} = \text{PQ}$$

$$= 90 \times 10^5 \times 1.33 \times 10^{-3}$$

$$= 12\ \text{kW}$$

$$P_{in}\text{of the pump} = \frac{12}{\eta_o} = \frac{12}{0.47 \times 0.95} = 13\ \text{kW}$$

$$\text{Theoratical flow of the pump} = \frac{1.33 \times 10^{-3}}{0.97}$$

$$= 1.38\ \text{l/s}$$

$$or\ The\ pump\ input\ (neglecting\ the\ losses) = \frac{motor\ output}{\eta_{Vm}\eta_{Tm}\eta_{Vp}\eta_{Tp}}$$

$$= \frac{7.5 \times 10^3}{0.9 \times 0.85 \times 0.97 \times 0.95} = 10.64\ kW$$

Ex.2: A hydraulic pump is operating at 10.3 MPa pressure and 37.85 l/min flow rate. The pump is driven by an electric motor at 1725 rpm. The electric motor produce 3.95 kN-m of torque. Find the overall efficiency of the pump.

Solution:-

$$\eta_{overall} = \frac{P_{out}}{P_{in}}$$

$$P_{out} = \gamma Q H = P \times Q$$

$$= 10.3 \times 10^6 \times 37.85 \times \frac{1}{1000 \times 60}$$

$$P_{in} = T.W = 3.95 \times 1000 \times \frac{2\pi N}{60}$$

$$= \frac{3950 \times 2\pi \times 1725}{60}$$

$$\eta_o = \frac{10.3 \times 10^6 \times 37.85 \times \frac{1}{60000}}{3.95 \times 1000 \times 2\pi \times 1725 \times \frac{1}{60}}$$

$$= 0.91 \quad or\ \ 91\%$$

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