Subject : Fluid Mechanics (I) موضوع : موائع 1
Weekly Hours : Theoretical: UNITS:5 الساعات الأسبوعية : نظري : 2 الوحدات : 5
Tutorial : 1 مناقشة : 1
Experimental : 1 عملي : 1

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Fluid Mech. ميكانيك الموائع

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   1.1 Definitions
   1.2 Newton Law of viscosity
   1.3 Bulk Modulus of Elastity
   1.4 surface tension

2. Fluid static
   2.1 Definitions
   2.2 pressure at a point
   2.3 Hydrostatic law
   2.4 Units and scales of pressure Mesurment
   2.5 Manometers (pressure Mesurement).
   2.6 Force on plane surface
   2.7 Force on Curved surface.
   2.8 Buoyant Force.
   2.9 Stability of Floating and submerged Bodies.
   2.10 Relative Equilibrium.

3. Fluid flow Concept and Basic Equations.
   3.1 Definitions
   3.2 Continuity equation
   3.3 Euler Equation of motion along streamline.
   3.4 Bernoulli Equation (Energy equation)
   3.5 Flow mesurement.
   pitot tube, orifice meter, Venturi meter, Nozzle
   3.6 Resistance to Flow in open and closed Conduits
   3.7 Linear Momentum Equation and Its

Application

3.8 Introduction for pumping and Turbines application

4. Dimensional analysis and dynamic similitude.
   4.1 The π - Theorem.
   4.2 Discu of Dimensionless parameters
   Reynols No., Froude No., Euler No.
   Weber No., Mach No.
   4.3 Similitude ; Model studies.

Referance.

1. Fluid Mechanics , Vector L. Streeter
   E. Benjamin Wylie.

2. Fluid Mechanics and Engineering application
   Robert L. Dogerti and Joshef B. Frinzieng

-1- CH-1

Fluid mechanics

Definisions

1. Fluid; It is a substance that deforms continuosly when subjected to a shear stress. It is either gas or liquid.

2. shear stress :- $\tau = \frac{F}{A} = \frac{\text{shear force}}{\text{surface area}}$

3- shear force :- It is the force components tangents to a surface of liquid.

4. Viscosity :- μ اللزوجة :- It is the property of fluid by virtue of which it offers resistance to shear.

- Molasses (دبس) and tar (قير) are example for highly viscous liquids.
- water and air have very small resistance

- The viscosity of gas increase with temperature
- " " " liquid decrease " "

units $\mu = N.s/m^2$ or $kg/m.s$

A common unit is Poise (P) =

$1 \text{ poise } (g/cm.s) = 0.1 \ N.s/m^2 \quad (Pa.s)$

$= 0.1 \ kg/m.s$

$10P = 1 \ kg/m.s.$

5. Kinematic viscosity :- ν : It is the ratio of viscosity to mass density.

$$\nu = \frac{\mu}{\rho} = 1 \ m^2/s$$
$$= 1 \ cm^2/s \ (stoke)$$

6. Density : ρ الكثافة : It is the mass per unit volume

$$\rho = \frac{m}{V} = kg/m^3$$

كثافة الماء $\rho_{water} = 1000 \ kg/m^3$

7- Specific weight γ : (unit gravity force) The force per unit volume. It change الكثافة الوزنية with location.

الكثافة الوزنية للماء $\gamma_\omega = \rho g = 9.81 \times 1000 = 9810 \ N/m^3$

8- Specific gravity S :- (relative density)

الوزن النوعي

$$S = \frac{\gamma_s}{\gamma_\omega} = \frac{\text{Specific weight of substance}}{\text{" " " water}}$$

المادة Substance : تعني الاجسام الصلبة والموائع

2

9. pressure : P : the normal force pushing against a plane area divided by the area.
units : N/m^2 or Pascal (P_a)

10. Vapor pressure :- The vapor molecules exert a partial pressure in the space known as vapor pressure.

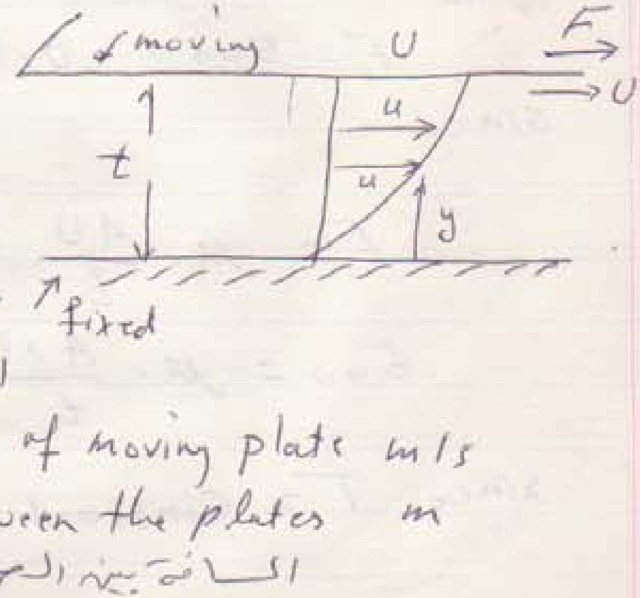
ان نو تقاطات البخار يؤثر في على الكائن الهيدروليكي

11. Perfect gas : It is a substance that satisfies the Perfect gas Law $PV_s = RT$ or $P = \rho RT$

Newtons Law of Viscosity

experimentaly shown that

$$F \propto \frac{AU}{t}$$



A = the area of the moving plate m^2

المساحة السطحية الصفيحة التي كم

U = Steady Velocity of moving plate m/s

t = the distance between the plates m

المسافة بين الصفيحتين.

i.e $$\boxed{F = \mu \cdot \frac{AU}{t}}$$ ~~Newton Law of Viscosity~~ —— (1)

since $\tau = \frac{F}{A}$

$$\therefore \tau = \mu \cdot \frac{U}{t} \quad \text{——— (2)}$$

$\frac{U}{t}$ = the angular deformation of fluid.
التشوه الزاوي للمائع

$$= \frac{du}{dy}$$

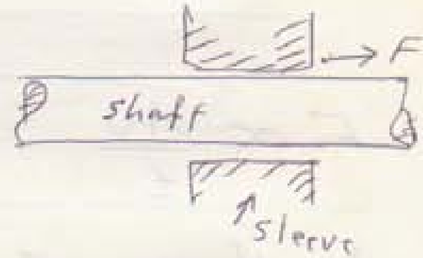
$$\boxed{\tau = \mu \frac{du}{dy}} \qquad \mu = \text{اللزوجة} \quad \text{—— (3)}$$

Newtons low of viscosity

newtonian fluid :- هو المائع الذي يتبع قانون
نيوتن للزوجة .

Q.1.5

$F_1 = 500N \quad U = 1 m/s$



since

$$F = \mu \cdot \frac{AU}{t}$$

$$500 = \mu \cdot \frac{A \times 1}{t} \qquad \therefore \mu = \frac{500t}{A}$$

since T = Constant درجة الحرارة ثابتة
اي ان اللزوجة ثابتة

$\therefore$

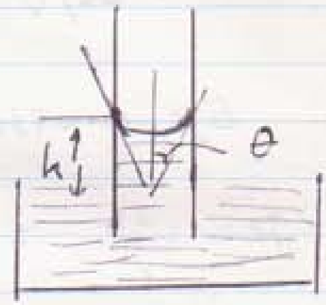
$$F_2 = \mu \cdot \frac{AU}{t} \qquad 1500 = \frac{500\cancel{t}}{\cancel{A}} \times \frac{\cancel{A}U}{\cancel{t}}$$

$U = 3 m/s$

12. Specific Volume : v_s : It is the reciprocal of density

$$v_s = \frac{1}{\rho} = m^3/kg$$

13. Surface tension الشد السطحي



$$\gamma h \pi r^2 = 2\pi r \sigma \cos\theta$$

σ = Surface tension Constant
معامل الشد السطحي .

$$h = \frac{2\sigma\cos\theta}{\gamma r}$$

القوة الى الاعلى (قوة الشد) = الوزن (الثقل الى الاسفل)

h = ارتفاع عمود السائل .

pressure at droplet الضغط داخل نقطه ارتفاعه $\quad P = \frac{2\sigma}{r}$ N/m^2

14. Bulk modulus of Elasticity :- K

$$K = -\frac{dP}{dV/V} = N/m^2$$

$$\text{or } K = -\frac{\Delta P}{\Delta V/V} = -\frac{P_2 - P_1}{\frac{V_2 - V_1}{V_1}}$$

K : The ~~Volumetric~~ Compressive stress per Unit Volumetric strain.

Ex: A liquid compressed in a cylinder has a volume of 1 liter (1000 cm³) at 1 MN/m² and volume of 995 cm³ at 2 MN/m². What is its bulk modulus of elasticity?

$$K = -\frac{\Delta P}{\Delta V / V} = \frac{(2-1)\ MN/m^2}{\frac{(995-1000)}{1000}} = 200\ MPa.$$

Q.142 $\sigma_w = 0.0736$ N/m

$$P = \frac{2\sigma}{r} = \frac{2 \times 0.0736}{\frac{0.05}{2}} \times 1000 = 5.89\ kPa$$

gage.

1.18 $d_s = 50$ mm $d_c = 50.1$ mm

$$\therefore t = \frac{50.1 - 50}{2} \qquad \therefore t = 0.05\ mm$$

μ at $0^\circ C = 1.6 \times 10^{-2}$ Pa.s

μ at $120^\circ C = 2 \times 10^{-3}$ Pa.s

$$F_1 = \mu \frac{AU}{t} = 1.6 \times 10^{-2} \frac{AU}{0.05 \times 10^{-3}} = 320\ AU$$

$$F_2 = 2 \times 10^{-3} \times \frac{1}{0.05 \times 10^{-3}} \times AU = 40\ AU$$

$$\therefore \frac{F_1}{F_2} = 8 \qquad \& \quad \frac{F_1 - F_2}{F_1} = \frac{320\,AU - 40\,AU}{320\,AU}$$

$$= 87\%$$

upper surface is in contact with air, which offers almost no resistance to the flow. Using Newton's law of viscosity, decide what the value of du/dy, y measured normal to the inclined plane, must be at the upper surface. Would a linear variation of u with y be expected?

1.4 What kinds of rheological materials are paint and grease?

1.5 A Newtonian fluid is in the clearance between a shaft and a concentric sleeve. When a force of 500 N is applied to the sleeve parallel to the shaft, the sleeve attains a speed of 1 m/s. If a 1500-N force is applied, what speed will the sleeve attain? The temperature of the sleeve remains constant.

1.6 Determine the gravity force in newtons of 3 kg mass at a place where $g = 9.7\ m/s^2$.

1.7 When standard scale masses and a balance are used, a body is found to be equivalent in force of gravity to two of the 1-kg scale masses at a location where $g = 9.7\ m/s^2$. Calculate the gravity force on a correctly calibrated spring balance (for sea level) at this location.

1.8 Determine the unit gravity force γ for water at 25° C and $g = 9.75\ m/s^2$.

1.9 On another planet, where gravity is 3 m/s^2, find the force of gravity on 400 L of material $\rho = 800\ kg/m^3$.

1.10 A correctly calibrated spring scale records the gravity force of a 2-kg body as 17.0 N at a location away from the earth. What is the value of g at this location?

1.11 The gravity force on a bag of flour at sea level is 20 N. What is its mass at a location where $g = 9.6\ m/s^2$?

1.12 What is the kinematic viscosity of liquid of viscosity 0.002 Pa·s and a relative density of 0.8?

1.13 A shear stress of 4 mPa causes a Newtonian fluid to have an angular deformation of 1 rad/s. What is its viscosity?

1.14 A plate, 0.5 mm distant from a fixed plate, moves at 0.25 m/s and requires a force per unit area of 2 Pa to maintain this speed. Determine the viscosity of the substance between the plates.

1.15 Determine the viscosity of fluid between shaft and sleeve in Fig. 1.6.

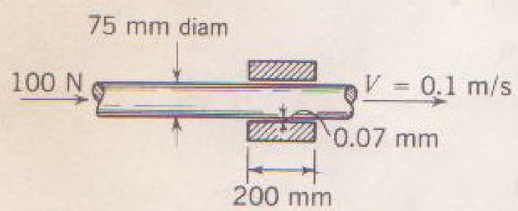


Figure 1.6 Problem 1.15.

1.16 A flywheel weighing 600 N has a radius of gyration of 300 mm. When it is rotating 600 rpm, its speed reduces 1 rpm/s owing to fluid viscosity between sleeve and shaft. The sleeve length is 50 mm; shaft diameter is 20 mm; and radial clearance is 0.05 mm. Determine the fluid viscosity.

1.17 A 25mm diameter steel cylinder 300 mm long falls, because of its own gravity force at a uniform rate of 0.1 m/s inside a tube of slightly larger diameter. A castor-oil film of constant thickness is between the cylinder and the tube. Determine the clearance between the tube and the cylinder. The temperature is 38°C. Relative density of steel = 7.85.

1.18 A piston of diameter 50.00 mm moves within a cylinder of 50.10 mm. Determine the percent decrease in force necessary to move the piston when the lubricant warms up from 0 to 120°C. Use crude-oil viscosity from Fig. C.1, Appendix C.

1.19 How much greater is the viscosity of water at 0°C than at 100°C? How much greater is its kinematic viscosity for the same temperature range?

1.20 A fluid has a viscosity of 0.6 Pa·s and a relative density of 0.7. Determine its kinematic viscosity.

1.21 A fluid has a relative density of 0.78. For a kinematic viscosity of 1.0×10^{-6} m^2/s determine the viscosity.

1.22 A body with gravity force of 500 N with a flat surface area of 0.2 m^2 slides down a lubricated inclined plane making a 30° angle with the horizontal. For viscosity of 0.1 Pa·s and body speed of 1 m/s determine the lubricant film thickness.

1.23 What is the viscosity of gasoline at 25°C?

1.24 Determine the kinematic viscosity of benzene at 27°C.

1.25 Calculate the value of the gas constant R for relative molecular mass of 44.

1.26 What is the specific volume of a substance of relative density 0.75?

1.27 What is the relation between specific volume and unit gravity force?

1.28 The density of a substance is 2900 kg/m^3. What is its (a) relative density, (b) specific volume, and (c) unit gravity force?

1.29 A force, expressed by $\mathbf{F} = 4\mathbf{i} + 3\mathbf{j} + 9\mathbf{k}$, acts upon a square area, 2 by 2 cm, in the xy plane. Resolve this force into a normal-force and a shear-force component. What are the pressure and the shear stress? Repeat the calculations for $\mathbf{F} = -4\mathbf{i} + 3\mathbf{j} - 9\mathbf{k}$.

1.30 A gas at 20°C and 0.2 MPa abs has a volume of 40 L and a gas constant $R = 210$ m·N/kg·K. Determine the density and mass of the gas.

1.31 What is the density of air at 400 kPa abs and 30°C?

1.32 What is the density of water vapor at 0.3 kPa abs and 30°C?

1.33 A gas with relative molecular mass 28 has a volume of 100 L and a pressure and temperature of 80 kPa abs and 330 K, respectively. What are its specific volume and density?

1.34 One kilogram of hydrogen is confined in a volume of 150 L at −40°C. What is the pressure?

1.35 Express the bulk modulus of elasticity in terms of density change rather than volume change.

1.36 For constant bulk modulus of elasticity, how does the density of a liquid vary with the pressure?

1.37 What is the bulk modulus of a liquid that has a density increase of 0.02 percent for a pressure increase of 0.6 MPa?

1.38 For $K = 2.2$ GPa for bulk modulus of elasticity of water what pressure is required to reduce its volume by 0.5 percent?

1.39 A steel container expands in volume 1 percent when the pressure within it is increased by 70 MPa. At standard pressure, P = 101.3 kPa it holds 450 kg water, $\rho = 1000$ kg/m^3. For $K = 2.06$ GPa when it is filled, how many kilograms mass water need be added to increase the pressure to 70 MPa?

1.40 What is the isothermal bulk modulus for air at 0.4 MPa abs?

1.41 At what pressure can cavitation be expected at the inlet of a pump that is handling water at 20°C?

1.42 What is the pressure within a droplet of water of 0.05 mm diameter at 20°C if the pressure outside the droplet is standard atmospheric pressure of 101.3 kPa?

1.43 A small circular jet of mercury 0.1 mm in diameter issues from an opening. What is the pressure difference between the inside and outside of the jet when at 20°C?

1.44 Determine the capillary rise for distilled water at 40°C in a circular 6 mm diameter glass tube.

1.45 What diameter of glass tube is required if the capillary effects on the water within are not to exceed 0.5 mm?

1.46 Using the data given in Fig. 1.4, estimate the capillary rise of tap water between two parallel glass plates 5 mm apart.

1.47 A method of determining the surface tension of a liquid is to find the force needed to pull a

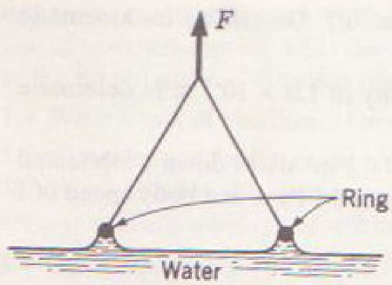
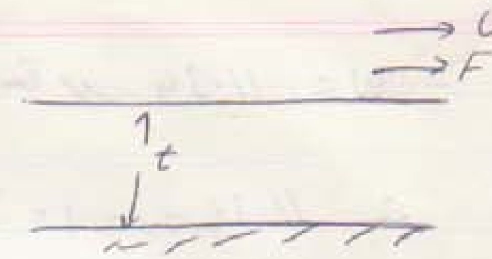


Figure 1.7 Problem 1.47.

platinum wire ring from the surface (Fig. 1.7). Estimate the force necessary to remove a 20-mm-diameter ring from the surface of water at 20°C. Why is platinum used as the material for the ring?

Q.1.14 $\frac{F}{A} = 2\ Pa\ (N/m^2)$



$t = 0.5\ mm$

$U = 0.25\ m/s$

since

$$F = \mu \frac{AU}{t} \quad \text{or} \quad \frac{F}{A} = \mu \frac{U}{t} = 2\ Pa$$

$$\therefore 2 = \mu \cdot \frac{0.25}{0.5 \times 10^{-3}} \qquad \therefore \mu = 0.004\ Pa.s$$

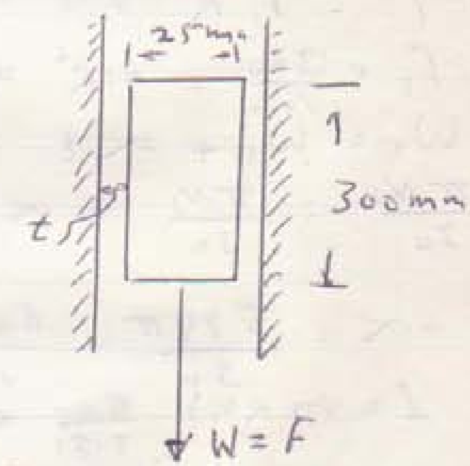
Q. 1.17.

$U = 0.1\ m/s \qquad T = 38\ ^\circ C$

Castor-oil $\quad \therefore \mu = 3 \times 10^{-1}$

$= 0.3\ Pa.s$

$S_{steel} = 7.85 \qquad t = ?$



$$F = \mu . \frac{AU}{t} = W$$

$$A = \pi D L = \pi \times 0.025 \times 0.3$$
$$= 0.023562\ m^2$$

$$W = mg = \rho V g = \gamma_s V$$

since $S = \frac{\gamma_s}{\gamma_w} \qquad \therefore \gamma_s = 7.85 \times 9810 = 77008.5\ N/m^3$

$$V = \frac{\pi}{4} D^2 L = \frac{\pi}{4} \times (0.025)^2 \times 0.3 = 1.47 \times 10^{-4}\ m^3$$

$\therefore W = 11.34$ N

$$\therefore 11.34 = 0.3 \times \frac{0.023562 \times 0.1}{t}$$

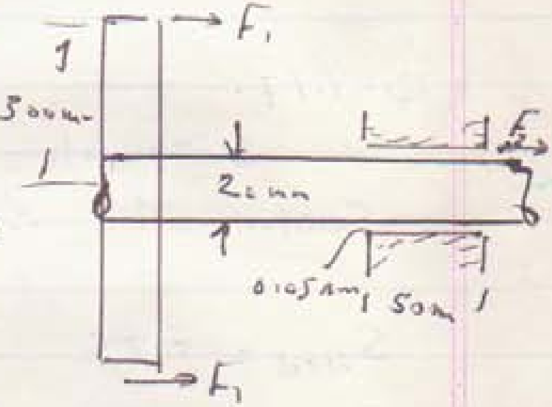
$$\therefore t = \frac{0.3 \times 0.023562 \times 0.1}{11.34} = 6.233 \times 10^{-5} \text{ m}$$

$$= 0.06233 \text{ mm.}$$

1.16 $W = 600$ N $\quad R = 300$ mm $\quad D_s = 20$ mm

$N = 600$ rpm reduced by 1 rpm/s

find $\mu = ?$



العزم $T = F_1 R = I \alpha$

$$F_1 \times \frac{300}{1000} = mR^2 \alpha \quad \text{--- (1)}$$

$$\omega_2 = \omega_1 + \alpha t$$

$$\frac{\pi N_2}{30} = \frac{\pi N_1}{30} + \alpha \times 1$$

$$\therefore \alpha = \frac{599\pi}{30} - \frac{600\pi}{30} = -\frac{\pi}{30} \quad \text{deceleration}$$

$$I = mR^2 = \frac{600}{9.81} \times 0.3^2$$

$$\therefore F_1 = 1.92 \text{ kN}$$

ΣM أخذ عزوم حول ال shaft.

$$F_1 R_1 = F_2 R_2$$

$$1.92 \times 0.3 = \mu \times \frac{\pi D N}{60} \times \frac{\pi \times 20 \times 50}{10^6} \times \frac{10^3}{0.05} \times 0.01$$

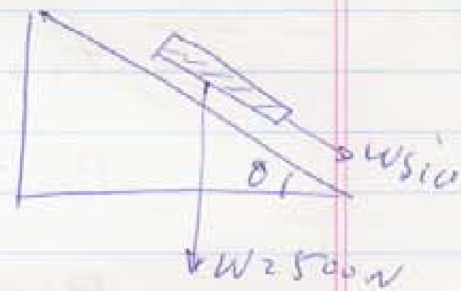
$$\mu = 1.46 \text{ N.s/m}^2$$

1.6 $W = mg = 3 \times 9.7 = 29.1\ N$

1.22 $W = 500\ N \quad A = 0.2\ m^2 \quad \theta = 30°$

$\mu = 0.1\ Pa.s \quad U = 1\ m/s$

$F = W\sin\theta = \mu \dfrac{AU}{t}$



$500 \sin 30 = 0.1 \times \dfrac{0.2 \times 1}{t}$

$t = \dfrac{0.1 \times 0.2 \times 1}{250}$

$= 0.08 \times 10^{-3}\ m$

$= 0.08\ mm$

1.15

$F = \mu \dfrac{AU}{t}$

$100 = \mu \times \dfrac{\pi \times 0.075 \times 0.2 \times 0.1}{0.07 \times 10^{-3}}$

$\mu = \dfrac{100 \times 0.07 \times 10^{-3}}{\pi \times 0.075 \times 0.2 \times 0.1} = 1.485\ Pa.s$

1.25

$R = \dfrac{8312}{M} = \dfrac{8312}{44} = 188.9\ m.N/kg.K$

1.29 a) $F = F_x + F_y + F_z$

$= 4i + 3j + 9k$

$F_z = F_k = 9\ N \quad F_s = \sqrt{F_x^2 + F_y^2} = \sqrt{16 + 9} = 5$ ✓

$P_N = \frac{9}{4} = 2.25\, P_a \quad P_s = \frac{5}{4} = 1.25\, P_a$

b) $F = -4i + 3j + 9k$

$F_z = F_k = -9 \qquad F_s = 5$

$P_N = \frac{-9}{4} = -2.25\, P_a$

$P_s = \frac{5}{4} = 1.25\, P_a$

1.30 $PV = mRT \qquad P = \rho RT$

$0.2 \times 10^6 \times 0.04 = m \times 210 \times 293$

$m = 0.13\ kg$

$0.2 \times 10^6 = \rho \times 210 \times 293$

$\rho = 3.25\ kg/m^3$

1.37 $k = \frac{\Delta P}{\frac{\Delta \rho}{\rho}} = \frac{0.6 \times 10^6}{0.02 \times 10^{-2}} = 3 \times 10^9\ Pa$

1.38 $k = -\frac{\Delta P}{\frac{\Delta V}{V}} = -\frac{\Delta P}{0.5 \times 10^{-2}} = 2.2 \times 10^9$

$\Delta P = -0.5 \times 2.2 \times 10^7 = -1.1 \times 10^7\ Pa$

$= 11\ MPa$

1.39 $V_2 = 1.01\, V_1 \qquad V_1 = \frac{m}{\rho} = \frac{450}{1000} = 0.45\, m^3$

$\therefore V_2 = 1.01 \times 0.45 = 0.4545$

$\Delta \rho = \rho \frac{\Delta P}{k} = 1000 \times \frac{70 \times 10^6}{2.06 \times 10^9} = 33.98\ kg/m^3$

$\therefore \rho_2 = 1000 + 33.98 = 1033.98 \ kg/m^3$

$\therefore m_2 = \rho_2 V_2 = 1033.98 \times 0.4545$

$m_2 = 469.94 \ kg$

$\therefore \Delta m = 469.94 - 450 = 19.94 \ kg$ be added.

1.40 $\quad K = -\dfrac{dP}{dV/V}$ and $PV = mRT$

$\therefore \dfrac{dP}{dV} = -\dfrac{mRT}{V^2}$

$\therefore K = \cancel{-} V \times \cancel{-} \dfrac{mRT}{V^2} = \dfrac{mRT}{V} = P$

$\therefore K = 0.4 \ MPa$

1.41 ^{534} of C-1 الجدول

1.43 $\quad F_s = F_p$ or $2\sigma L = P_1 D L$

$P_1 = \dfrac{2\sigma}{D} = \dfrac{2 \times 0.0736}{0.1 \times 10^{-3}} = 1472 \ Pa$

تقريبا 1.44 $\quad h = \dfrac{2\sigma}{\gamma r} = \dfrac{2 \times 0.0701}{9\cancel{8}\cancel{3}\cancel{0} \times 3 \times 10^{-3}} = 4.8 \ mm$

1.45 } fig 1.4
1.46 }

1.47 $\quad F = 2(\pi D \sigma) = 2 \times \pi \times 0.02 \times 0.0736$
$= 9.25 \times 10^{-3} \ N$

يستخدم البلاتين لكونه لا يتآكل ~~[illegible]~~
Corrosion

1

CH-2 Fluid Statics

It can be divided in two parts

1. Study of pressure and its variation throughout a fluid.

دراسة الضغط وتغيره خلال المائع .

2. Study of pressure forces on a finite surface

دراسة قوة الضغط على سطوح غير محدد .

عندما يكون السائل ساكن فان القوة الوحيدة المؤثرة على السائل هي قوة الضغط على السطح وان اجهاد القص (shear stress) يساوي صفر .

Pressure at a point

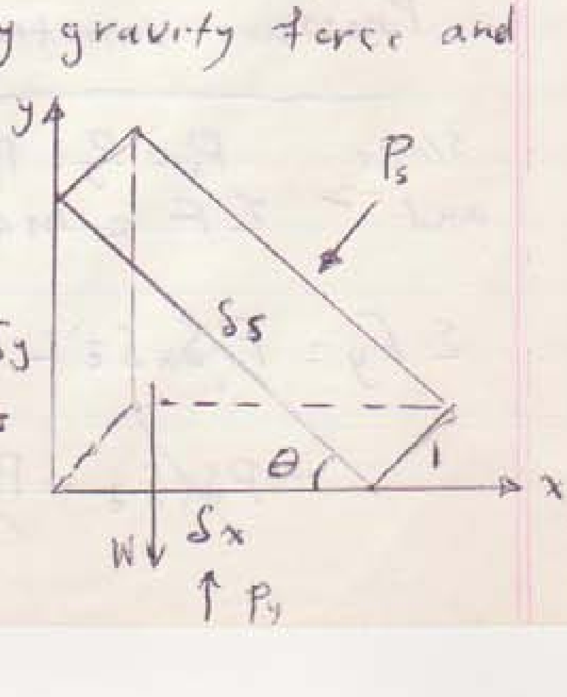
At a point a fluid at rest has the same pressure in all direction.

To determine this assume an element its coordenats dx, dy, ds, and unit depth.

since there is only gravity force and pressure force.

i.e $\Sigma F = ma = 0$

(static fluid)



$\xrightarrow{P_x}$

$$\Sigma F_x = P_x \delta y \times 1 - P_s \delta s \sin\theta = \rho \frac{\delta x \delta y}{2} \times 1 \; a_x = 0$$

$$\Sigma F_y = P_y \delta_x \times 1 - P_s \delta_s \cos\theta - \gamma \frac{\delta_x \delta_y \times 1}{2} = \rho \frac{\delta_x \delta_y \times 1}{2} \times a_y = 0$$

Since P_x, P_y & P_s is the average pressure on each phase., a_x & $a_y = 0$ static fluid.
and

$$\delta s \sin\theta = \delta y \quad , \quad \delta s \cos\theta = \delta x$$

$$\therefore \quad \Sigma F_x = P_x \cancel{\delta y} - P_s \cancel{\delta y} = 0 \qquad \therefore \boxed{P_x = P_s}$$

also

$$\Sigma F_y = P_y \delta x - P_s \delta x - \gamma \frac{\delta x \delta y}{2} = 0$$

since

$$\delta x \delta y \longrightarrow 0 \text{ with respect to } \delta x \text{ or } \delta y$$

$$\therefore$$

$$P_y = P_s \quad \text{and} \quad \boxed{P_x = P_y = P_s = P}$$

Pressure Variation in static fluid

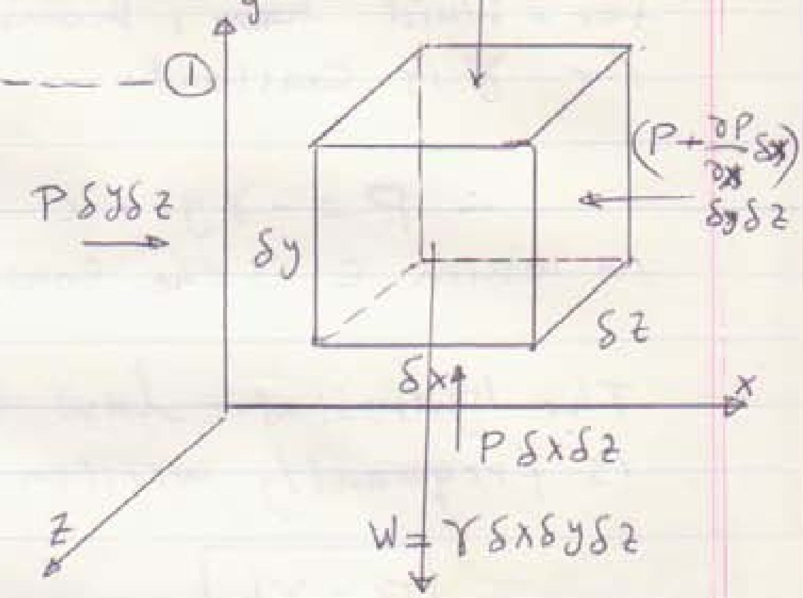
since $P_x = P_y = P_z = P$
and $\Sigma F = ma = 0$ static fluid

$$\Sigma F_y = P \delta_x \delta z - \left(P + \frac{\partial P}{\partial y} \delta y\right) \delta x \delta z - \gamma \delta_x \delta_y \delta_z = 0$$

$$= \cancel{P \delta_x \delta z} - \cancel{P \delta_x \delta z} - \frac{\partial P}{\partial y} \delta y \delta_x \delta z - \gamma \delta x \delta y \delta z = 0$$

$$= -\frac{\partial P}{\partial y}\,\delta y \cancel{\delta x \delta z} - \gamma\, \delta x \cancel{\delta y \delta z} = 0$$

$$\frac{\partial P}{\partial y} = -\gamma \qquad \text{------} \ 1$$



$$\Sigma F_x = P\,\delta y \delta z - \left(P + \frac{\partial P}{\delta x}\delta x\right)\delta y \delta z = 0$$

$$= -\frac{\partial P}{\partial x}\,\delta x \delta y \delta z = 0$$

since $\delta x \delta y \delta z \neq 0$ (Volume حجم العنصر)

$$\therefore \frac{\partial P}{\partial x} = 0 \qquad \text{------} \ (2) \qquad \therefore P = c \ \text{ثابت}$$

also

$$\Sigma F_z = -\frac{\partial P}{\partial z}\,\delta x \delta y \delta z \qquad \text{i.e}$$

i.e

$$\frac{\partial P}{\partial z} = 0 \qquad \text{---} \ 3 \qquad \therefore P = c \ \text{ثابت}$$

$\therefore$ P in x & z direction is constant and P is a function of y in y-dir. only.

$\therefore \; dp = -\gamma \, dy$

for a fluid ~~h~~, homogeneous and incompressible
i.e γ is constant.

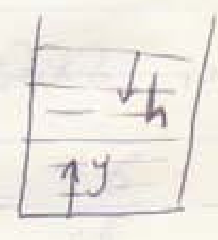
$$\therefore \; P = -\gamma y + c$$

in which c is the constant of integration

The hydrostatic law of pressure veriation
is frequantly written in the form:

$$\boxed{P = \gamma h} \qquad \text{when} \quad h = -y$$

h = the depth of fluid from the surface
to a certain point.



3.

Pressure Variation in a compressible fluid:

When the fluid is a perfact gas at rest and Constant temperature:

i.e $P = \rho R T$, T = Const. + R = Const.

or $\frac{P}{\rho}$ = Const. i.e $\frac{P}{\rho} = \frac{P_o}{\rho_o} \Rightarrow \rho = \rho_o \frac{P}{P_o}$ -- ①

and $dP = -\gamma dy = -\rho g dy$ -- ②

from equ. 1, 2

$$dP = -\rho_o \frac{P}{P_o} g\, dy$$

$$\therefore \int_{y_o}^{y} dy = -\frac{P_o}{\rho_o g} \int_{P_o}^{P} \frac{dP}{P}$$

$$\therefore y - y_o = -\frac{P_o}{\rho_o g} \ln \frac{P}{P_o}$$

$$\boxed{\therefore P = P_o \exp\left(-\frac{(y - y_o)}{P_o/\rho_o g}\right)} \quad \text{-- ③}$$

which is the equation of variation of pressure for isothermal gas, T = const.

Since the atmospheric frequently is assumd to have a temperture gradiant

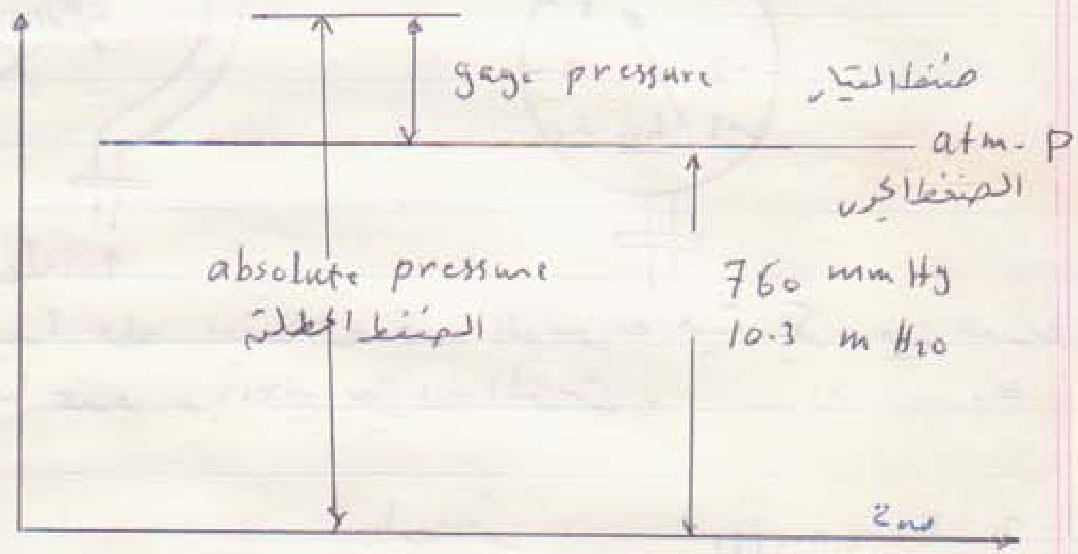
$$T = T_0 + \beta y \qquad \beta = -0.00651 \quad {}^\circ C/m$$

$$\therefore \rho = \frac{P}{RT} = \frac{P}{R(T_0 + \beta y)}$$

$$\therefore \boxed{dP = -\frac{P}{R(T_0 + \beta y)} g \, dy} \quad \text{--- } (4)$$

B) Units and Scales of pressure measurment :

Pressure may be expressed with respect to any aribitrary datum - The usual datum are absolute zero and local atmospheric pressure.



absolute pressure = pressure gage + barometer reading

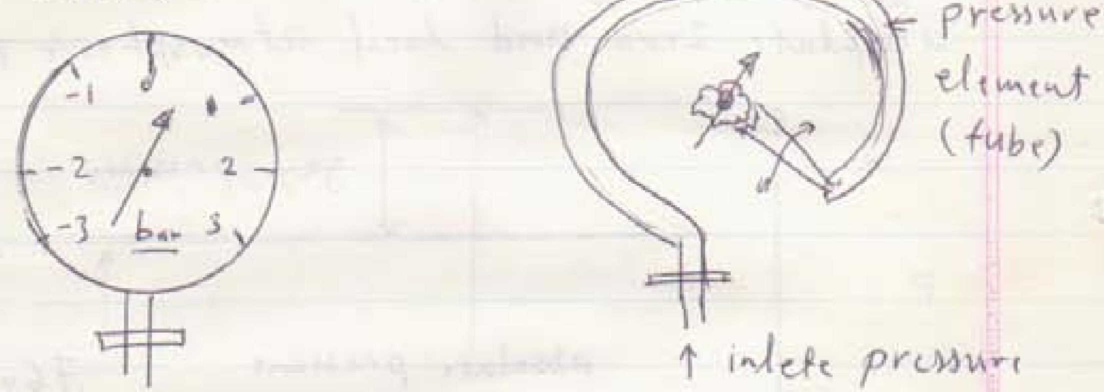
الضغط المطلق = ضغط المقياس + قراءة البارومتر (الضغط الجوي)

علماً ان الضغط الجوي يقاس بواسطة البارومتر ويتغير من منطقة الى آخر ويسمى (local barometer P).

$$\therefore P_{abs} = P_g + P_{atm}$$

1- Bourdon gage مقياس بوردن

Typical devices used for measuring gage pressure.

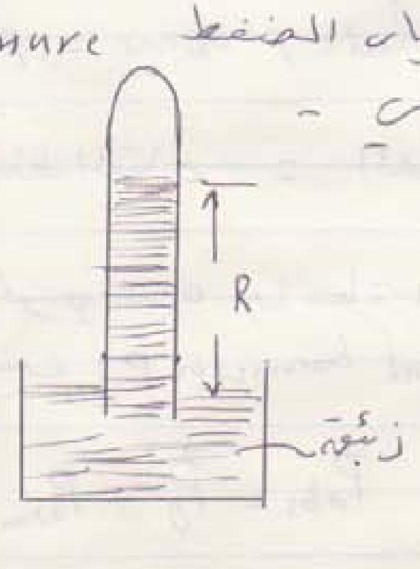


مبدأ عمله يعتمد على تمدد الانبوب حيث يتكون من معدن ذو قابلية تمدد وبالتالي يتحرك المؤشر.

2- Barometer بارومتر

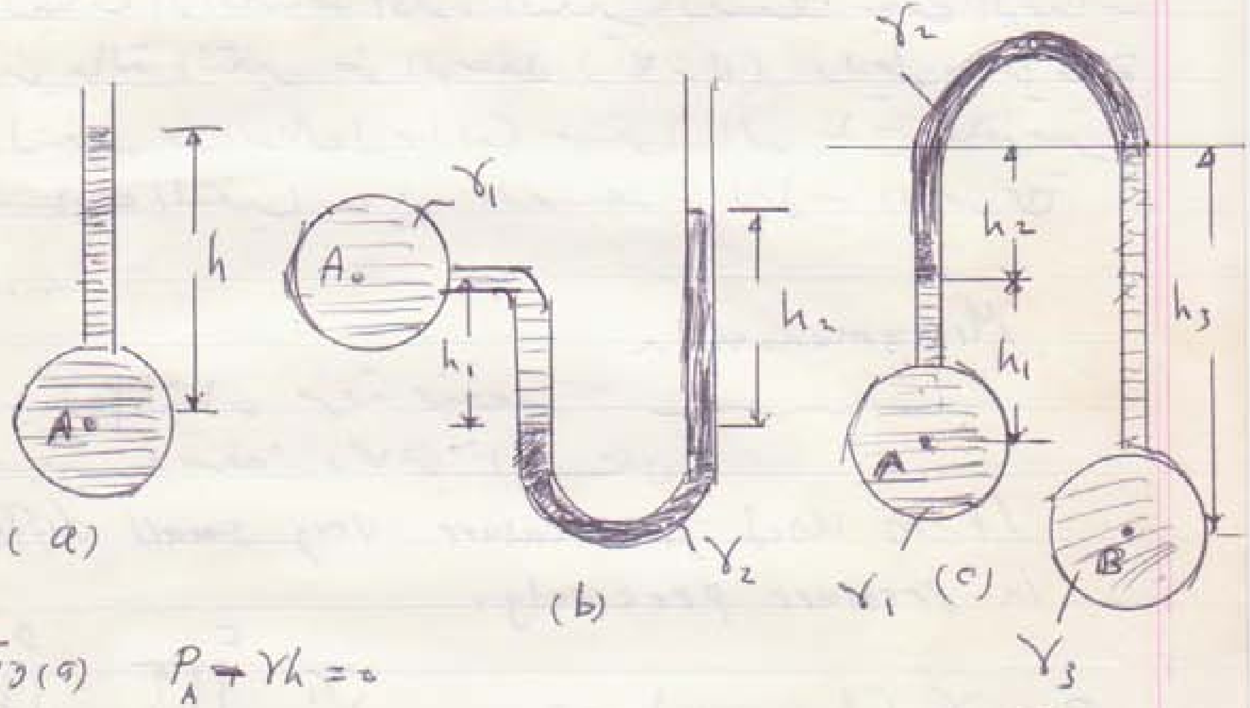
devices used to measure the local atmospheric pressure جهاز يستخدم لقياس الضغط الجوي الموقعي.

$$P = \gamma h = \gamma R$$

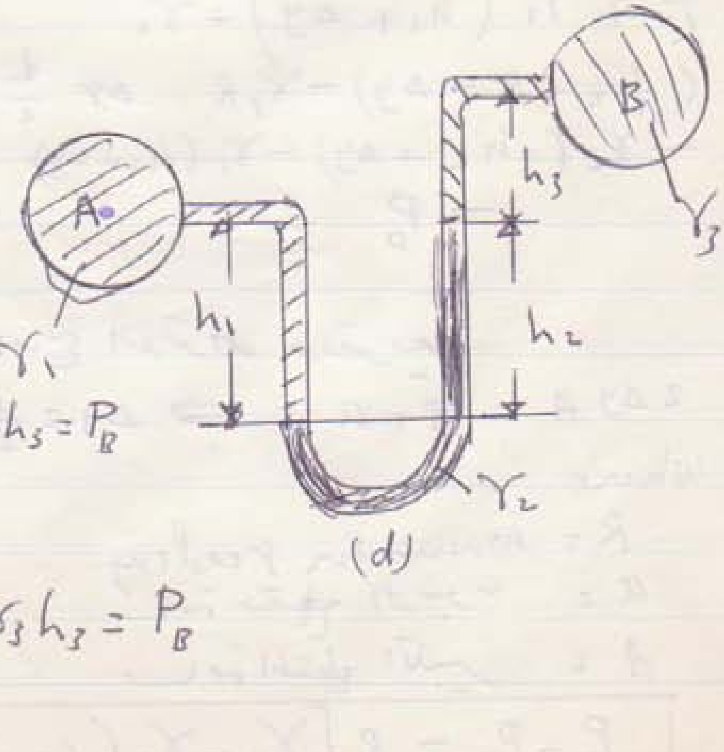


(5) 3- Manometers :-

It is used to determine the differance in pressure



(a) (b) (c)



(d)

Fig (a) $P_A - \gamma h = 0$

$\therefore P_A = \gamma h$

Fig (b)

$P_A + \gamma_1 h_1 - \gamma_2 h_2 = 0$

Fig (c)

$P_A - \gamma_1 h_1 - \gamma_2 h_2 + \gamma_3 h_3 = P_B$

Fig (d)

$P_A + \gamma_1 h_1 - \gamma_2 h_2 - \gamma_3 h_3 = P_B$

يمكن حساب الضغط في جميع المانوميترات وفقاً لما يلي

١. نبدأ بضغط احدى جهتي المانوميتر مثلا P_A على ان نلاحظ الوحدات.

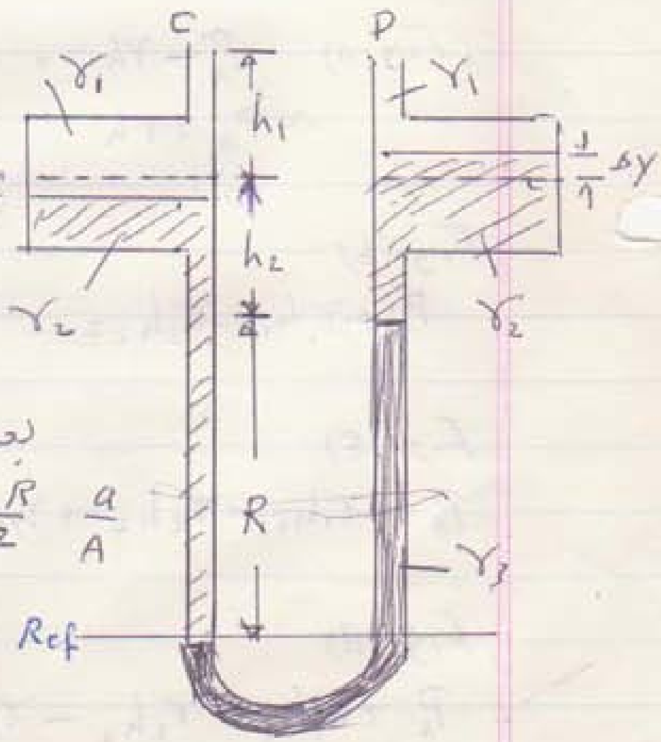
٢. نضيف اليه الضغط الناتج عن التغير بالضغط بنفس الوحدات في حالة التغير نحو الاسفل (نازل) ونطرح في حالة التغير نحو الاعلى (صاعد) حتى نصل الى نهاية المانوميتر

٣. نساوي المتغيرات مع الضغط في الجهة الاخرى

Micromanometer

يستخدم لقياس فرق ضغط صغير وبدقة عالية.

It is used to measure very small differeance in pressure precisely.



$$P_c + \gamma_1 (h_1 + \Delta y) + \gamma_2 (h_2 + R - \Delta y) - \gamma_3 R - \gamma_2 (h_2 + \Delta y) - \gamma_1 (h_1 - \Delta y) = P_D$$

بحذف الاقواس وتعويض

$$2 \Delta y A = R \cdot a \Rightarrow \Delta y = \frac{R}{2} \frac{a}{A}$$

where

R = manometer reading

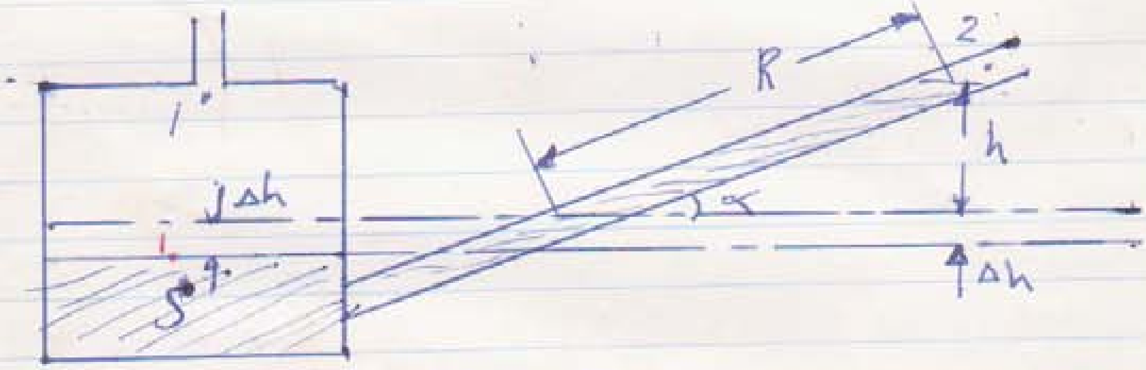
a = مساحة مقطع الانبوب

A = مساحة المقطع الكبير

$$\boxed{P_c - P_D = R \left\{ \gamma_3 - \gamma_2 \left(1 - \frac{a}{A}\right) - \gamma_1 \frac{a}{A} \right\}}$$

Inclined Manometer

يستخدم لقياس ضغط الغاز المحصور في الخزان. يتم التصفير عندما تكون الفتحات A ، B مفتوحة ، ويحدد ارتفاع السائل في الأنبوب المائل حيث يرتفع بسبب فرق المساحة ، ثم نغلق A فيتحرك السائل في الأنبوب ويقرأ الارتفاع الجديد



$$P_1 - S\gamma_w (h + \Delta h) = P_2$$

$$P_1 - P_2 = S\gamma_w (h + \Delta h) \quad \text{——} ①$$

since

$$h = R \sin\alpha \quad \text{and} \quad \Delta h \, A = R \cdot a \quad \text{——} ②$$

$a =$ مساحة مقطع الأنبوب m^2

$A =$ " " الخزان

$R =$ قراءة المانومتر

$\alpha =$ زاوية الميل

$$\therefore P_1 - P_2 = S\gamma_w \left(R \sin\alpha + \frac{a}{A} R\right)$$

or

$$P_1 - P_2 = S\gamma_w \left(\sin\alpha + \frac{a}{A}\right) R \quad \text{——} ③$$

or

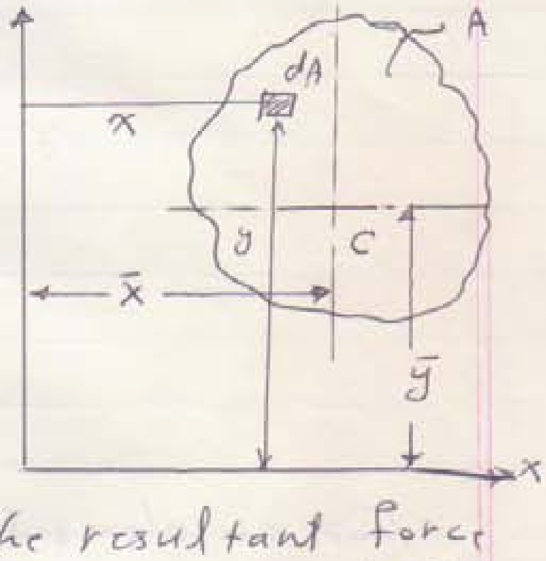
$$\boxed{P_1 - P_2 = C R} \quad \text{——} 4 \qquad \text{Where } C = S\gamma_w \left(\sin\alpha + \frac{a}{A}\right)$$

Force on plane area

The distributed forces resulting from the action of fluid on finite area may be conveniently replaced by a resultant force

ان توزيع القوة على السطوح يمكن ان يتحول الى محصلة قوى

1- Horizontal surface سطح افقي



The magnitude of the force acting on one side of the surface is

قيمة القوة مؤثرة على سطح افقي

$$F = \int P\,dA = P\int dA = PA$$

حيث P ثابت على السطح

$$\therefore \boxed{F = PA}$$

To find the action of the resultant force take an element and take the moment of the distribution force about any axis say xy plane

ان نقطة او مركز تأثير القوة في مركز المساحة وابعاده $\bar{x}$ ، $\bar{y}$

The moment about y-axis

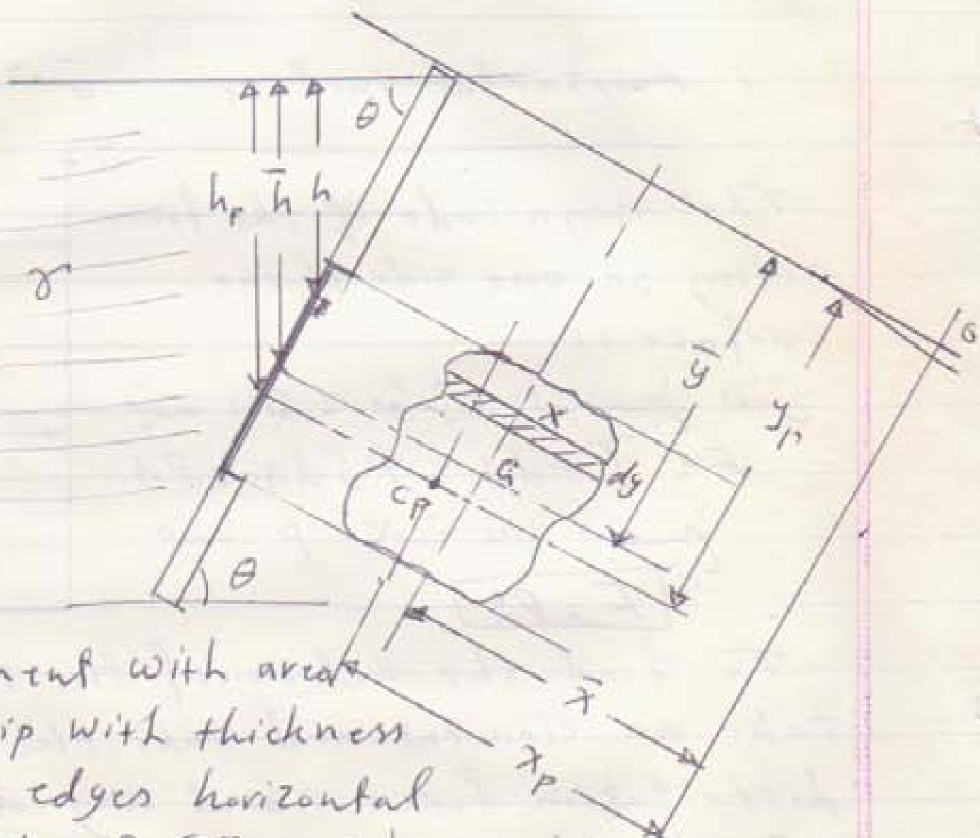
$$PA\bar{x} = \int_A xP\,dA \qquad \text{since } P = \text{constant}$$

$$\therefore \bar{x} = \frac{1}{A}\int_A x\,dA$$

$\bar{x}$ = the distance from the y-axis to the Centroid of the area بعد مركز المساحة عن المحور y also $\bar{y} = \frac{1}{A}\int_A y\,dA$

2- Inclined surface سطح مائل

$\bar{y}$ = بعد مركز المساحة المائل عن مستوى سطح السائل



For an element with area δA as A a strip with thickness dy with long edges horizontal the magnitude of δF acting on it is

$$\delta F = P\,\delta A = \gamma h\,\delta A = \gamma y \sin\theta\ \delta A$$

since all elemental forces are parallel

$$\therefore F = \int P\,dA = \gamma \sin\theta \int y\,dA$$

from appendix (A) $\int y\,dA = \bar{y} A$

$\therefore F = \gamma \sin\theta \, \bar{y} A = \gamma \bar{h} A = \boxed{P_G A = F}$

$P_G =$ الضغط في مركز المساحة المغمورة

$\therefore F =$ pressure at the centroid x area

القوة = الضغط في مركز المساحة المغمورة x المساحة المغمورة من السطح

Center of Pressure

ان مركز تأثير القوة في نقطة تسمى مركز الضغط واحداثيا x_p ، y_p

To find the pressure Center: take a moment about the axises i.e

$$x_p F = \int x \, p \, dA \quad , \quad y_p F = \int_A y p \, dA$$

$\therefore$

$$x_p = \frac{1}{F} \int_A x p \, dA \quad , \quad y_p = \frac{1}{F} \int_A y p \, dA$$

$\therefore$ from fig

$$x_p = \frac{1}{\gamma \bar{y} A \sin\theta} \int_A x \gamma y \sin\theta \, dA$$

$$= \frac{1}{\bar{y} A} \int_A x y \, dA$$

from Appendix A $\int_A x y \, dA = I_{xy} =$ عزم القصور الذاتي حول المحاور

$$\therefore x_p = \frac{I_{xy}}{\bar{y} A} \quad \text{also} \quad I_{xy} = \bar{I}_{yx} + \bar{x} \bar{y} A$$

$$\therefore x_p = \frac{\bar{x} \bar{y} A + \bar{I}_{xy}}{\bar{y} A} = \boxed{\therefore x_p = \bar{x} + \frac{\bar{I}_{xy}}{\bar{y} A}}$$

also $\qquad y_p = \dfrac{1}{\bar{y} A} \int_A y^2 \, dA$

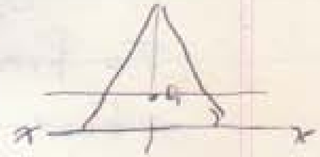
and

$$\left.\begin{aligned} \int y^2 \, dA &= I_x \\ \& \ I_x &= I_G + \bar{y}^2 A \end{aligned}\right\} \quad \text{(Appendix A)}$$

$$\boxed{\therefore \ y_p = \bar{y} + \frac{I_G}{\bar{y} A}}$$

$I_x =$ عزم القصور الذاتي حول المحور $x-x$

$I_G =$ " " حول المركز.



Force components on curved surface :-

1- Horizontal Component ; المركبة الافقية

~~Take~~ It is equal to the pressure force exerted on a projection of the curved surface - The vertical plane of projection is normal to the direction of the component.

$\delta F_x = P \, \delta A \, \cos\theta$

$F_x = \int_A P \cos\theta \, dA$

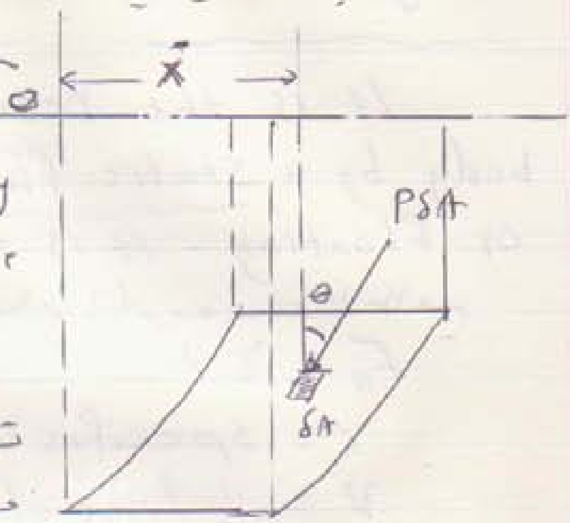
$\cos\theta \, \delta A$ مسقط مساحة الشريحة

δA على المحور العمودي على المحور $x-x$.

$F_H =$ الضغط في مركز المساحة × مسقط المساحة المعرض من الجسم على المحور العمودي.

$$\boxed{F_H = P_G \times A}$$

2- Vertical Component المركبة العمودية



It is equal to the weight of liquid vertically above the curved surface and extending up to the free surface.

تساوي الوزن السائل فوق المنحني مستمراً الى سطح السائل .

$$F_v = \int_A P \cos\theta \, dA \qquad p = \gamma h$$

$$\therefore F_v = \gamma \int_A h \cos\theta \, dA$$

$\cos\theta \, \delta A =$ مسقط الشريحة δA على المحور $x\text{-}x$

$$\therefore F_v = \gamma \int_V dV \qquad \text{where } h\delta A = V$$

حجم السائل فوق الشريحة

$$\boxed{\therefore F_v = \gamma V}$$

$\gamma =$ معامل الكثافة الوزنية للسائل

$V =$ حجم السائل فوق المنحني

$$\bar{x} = \frac{1}{V} \int_V x \, dV$$

لايجاد بعد احداثيات الحجم

= The distance from o to the line of action

δV = the volume of the prism of heigh h and base $\cos\theta \, \delta A$ or the volume of liquid vertically above the area element

Buoyant Force القوة الدافعة

It is the resultant force exerted on a body by a static fluid in which it is submerged or floating. It is always acts vertically upward.
هي محصلة القوة المؤثرة على الأجسام المغمورة او الطافية ويكون اتجاهها دائماً الى الاعلى

$F_B = \gamma V$

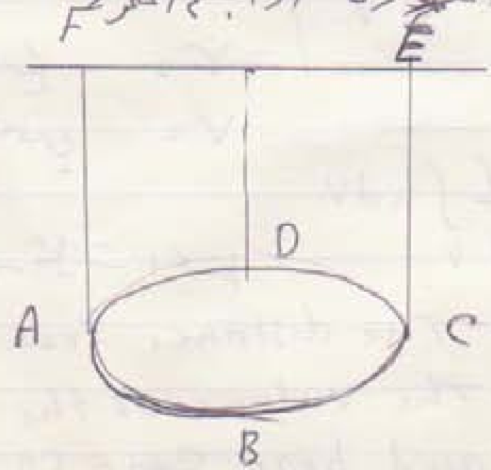
γ = specific weight of the liquid N/m^3

V = volume of fluid displaced

$$\bar{x} = \frac{1}{V}\int_v x \, dV$$

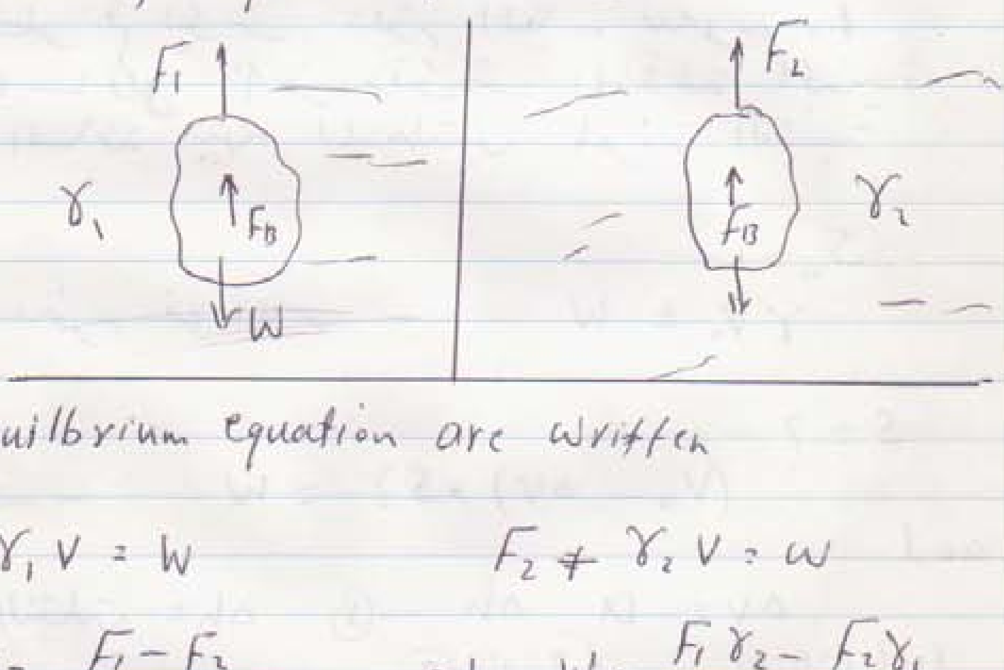
$$F_B = \underset{ABCEFA}{F_v} - \underset{ADCEFA}{F_v}$$

The buoyant force acts through the centroid of the displaced volume of fluid. القوة الدافعة تؤثر في مركز المساحة المغمورة او الحجم المزاح



In solving a static problem involving submerged or floating objects. weighing an odd-shaped object syspended in two different fluids yields sufficent data, To determine its weight, volume, unit gravity force and relative density as shown in fig.

F_1 = قوة شد الخيط
W = gravity force.
V = volume of liquid dispaced



The equilbrium equation are written

$$F_1 + \gamma_1 V = W \qquad F_2 + \gamma_2 V = W$$

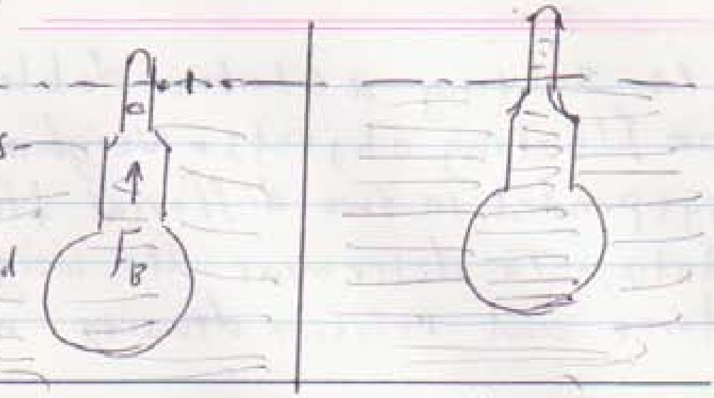
$$\therefore V = \frac{F_1 - F_2}{\gamma_2 - \gamma_1} \quad \text{and} \quad W = \frac{F_1 \gamma_2 - F_2 \gamma_1}{\gamma_2 - \gamma_1}$$

من العلاقات اعلاه. يمكن استنتاج طريقة تدريج
مقياس الوزن النوعي للسوائل.

Hydrometer

It is uses the principle of bouyant force to detemine the relative density of liquids.

assume
a = the upper cross-section
V_0 = volume displaced
γ = specific weight of destial water



S = relative density of liquid to be determined.

Hydrometer ~~النوعي~~ بنفس الطريقة اعلاه نغمر مقياس الوزن
في ماء مقطر ثم نؤشر علامة المقياس بعلامة 1.0 .
ثم نغمر في سائل آخر ونؤشر النقطة الجديدة
ونقسم المسافة بين النقطتين للوزن النسبي .

$S_w = 1$

$\gamma V_0 = W$ القياس مغمور في الماء

$S = ?$

$$(V_0 - \Delta V) \cdot S\gamma = W \quad --(2)$$

and

$$\Delta V = a \, \Delta h \quad --(3)$$ $\Delta h =$ المسافة بين النقطتين

substituting (3) in (2)

$$\therefore \cancel{\gamma} V_0 = (V_0 - a \Delta h) \cancel{\gamma} \cdot S$$

$$\therefore V_0 = S V_0 - a S \Delta h$$

$$\boxed{\therefore \Delta h = \frac{V_0}{a}\left(\frac{S-1}{S}\right)}$$

Ex. 2.12

$W = 1.5\,N$ وزن الجسم في الهواء

قوة شد الخيط $F_B = 1.1\,N$ " " " الماء

$\gamma V =$ وزن حجم السائل المزاح

$\therefore W = F + \gamma V$

$1.5 = 1.1 + 9806\,V \qquad \therefore V = 0.0000408\,m^3$

$\therefore V = 40.8\,cm^3$

$W = S\gamma_w V \qquad \therefore S = \dfrac{1.5}{9806 \times 0.0000408} = 3.75$

Stability of floating and submerged bodies

A body floating in a static liquid has vertical stability

Stable Unstable Neutral.

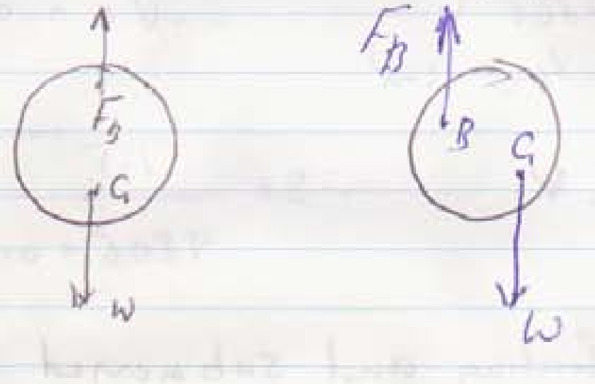
A body has linear stability when a small linear displacement in any direction sets up restoring force tending to return it to its original position.

يكون الجسم مستقر عند التأثير عليه بعزم ويعود الى حالته الاصلية
اما في حالة عدم العودة الى حالته الاصلية يعتبر الجسم غير مستقر اما
اذا دار حول نفسه فيسمى (Neutral)

Determination of Rotational stability of floating objects :-

Any floating object with center of gravity below its center of buoyancy (center of displaced volum) floats in stable equilibrium.

اي جسم يطفو مركز ثقله اسفل مركز تأثير القوة الدافعة (مركز الجزء المغمور) يكون الجسم مستقر.



Stable Unstable

When a body submerged or floating in a liquid as shown below

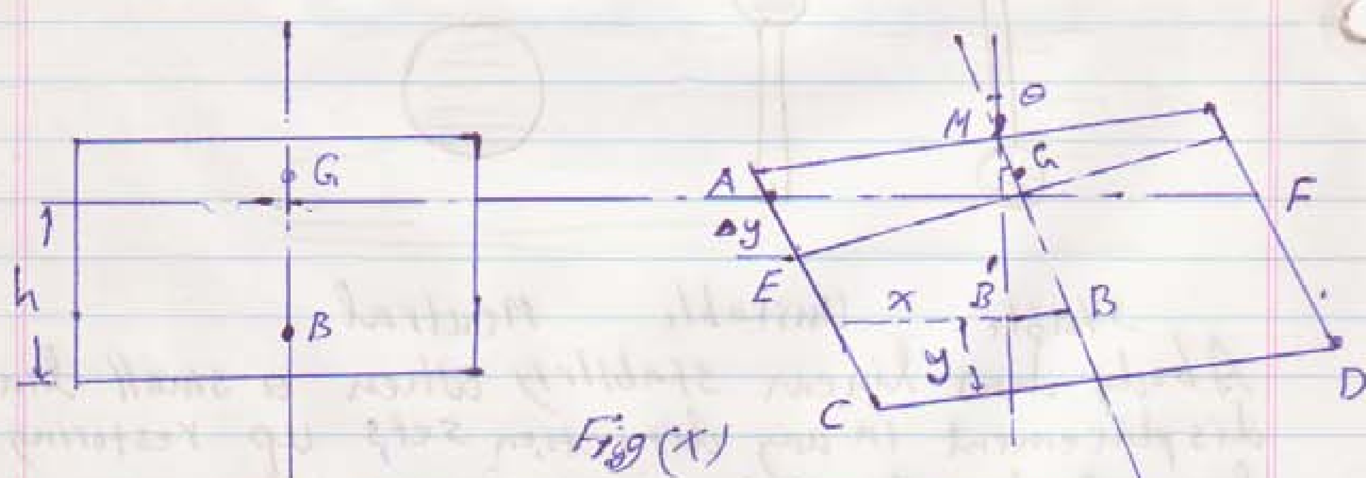


Fig (X)

B' the center of buoyant force acts upward and G the center of gravity of the body

The Intersection of buoyant force and the Centerline is called the metacentric, designated M.

M above G	the body is stable
M at G	" " " neutral
M below G	" " " Unstable

The distance MG is called the metacentric height.
Then the restoring Coupling is $= W\, MG \sin\theta$

in which θ is the angular displacement and ~~the~~ W the weight of the body.

Ex: As shown in fig (x) above a block 6m wide and 20 m long has a gross mass of 200 Mg. Its Center of gravity is 30 cm above the water surface.
Find the Metacentric hight and restoring Couple when $\Delta y = 30$ cm.

The depth of submegence in water is

$$F_B = W$$
$$\gamma V = 200\,000 \times 9.81$$
$$9810 \times 6 \times 20 \times h = 200\,000 \times 9.81$$
$$h = 1.667 \text{ m}$$

The centroid in the tipped position is located with moment about AC and CD

$$\bar{x} = \frac{1.367 \times 6 \times 3 + 0.6 \times 6 \times \frac{1}{2} \times 2}{1.667 \times 6} = 2.82 \text{ m}$$

$$\bar{y} = \frac{1.367 \times 6 \times \frac{1.367}{2} + 0.6 \times 6 \times \frac{1}{2}(0.2 + 1.367)}{1.667 \times 6} = 0.842$$

By similar triangle AEO and B'BM

$$\frac{\Delta y}{\frac{b}{2}} = \frac{B'B}{MB} \qquad \Delta y = 0.3 \qquad \frac{b}{2} = 3 \text{ m}$$

$$B'P = 3 - 2.82 = 0.18 \text{ m}$$

$$\therefore MB = 1.8 \text{ m}$$

G = 1.967 from the bottom (CD)

$$\therefore GB = 1.967 - 0.842 = 1.125 \text{ m}$$

$$\therefore MG = MB - GB = 1.8 - 1.125$$
$$= 0.675 \text{ m}$$

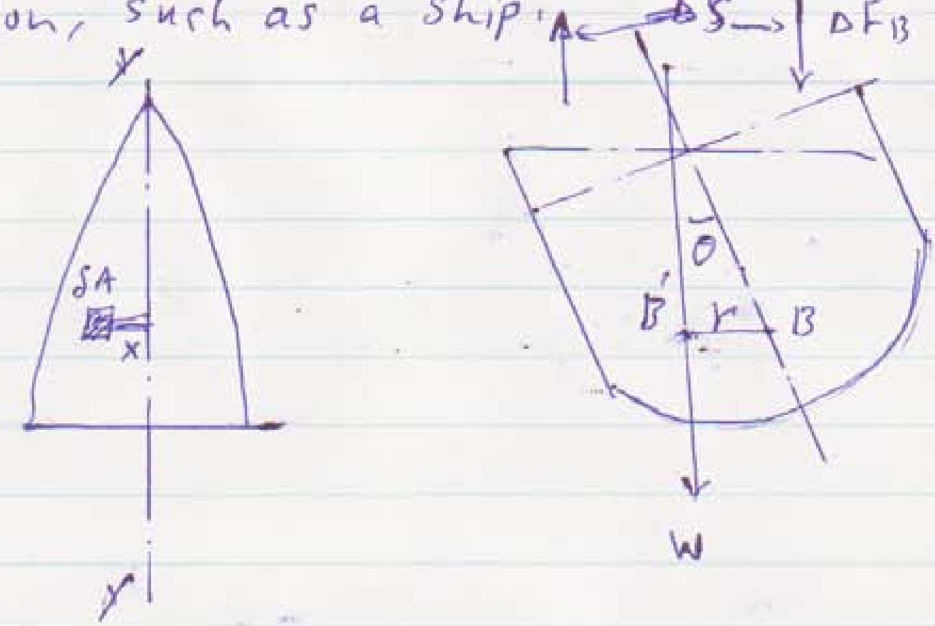
∴ the body is stable where MG is positive.

The restoring coupling = W MG sin θ

$$= 200\,000 \times 9.806 \times 0.675 \times \frac{0.3}{\sqrt{3^2 + 0.3^2}}$$
$$\therefore = 131 \text{ kN.m.}$$

Nonprismatic Cross-section body

For a floating object of variable cross-section, such as a ship:



The restoring coupling $F_B \cdot S$ should be equal to Wr where W is the weight of the body and r the distance shift.

$$\Delta F_B \cdot S = W \cdot r$$

if we take an element δA on horizontal section through the body at liquid surface.

The volume $x \theta \delta A$

Force $= \gamma V = \gamma x \theta \delta A$

moment about $O = \gamma x^2 \theta \delta A$ for small θ

$$\therefore \Delta F_B \cdot S = \gamma \theta \int_A x^2 \, dA = \gamma \theta I$$

where $\int_A x^2 \, dA = I$ (moment of Inertia)

$\therefore \gamma \theta I = W r = \gamma V r$
whir V is the total volume of liquid displaced
since θ is very small

$$\therefore MB \sin\theta = MB\,\theta = r \quad \text{or} \quad MB = \frac{r}{\theta} = \frac{I}{V}$$

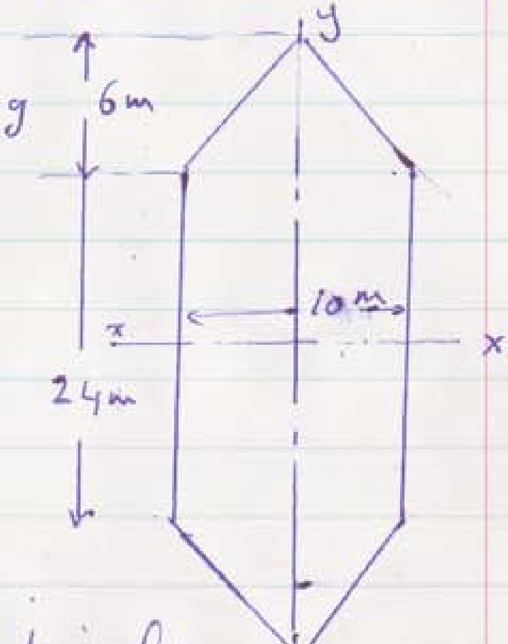
The ~~moment~~ metacentric height is thin

$$MG = MB \mp GB$$

$$\text{or } MG = \frac{I}{V} \mp GB$$

The minus sign is used if G is above B $-$
and the plus ~~singn~~ sign when G is below B $+$

Ex. A body displacing 1 Gg has the horizontal cross-section at the waterline shown in fig. Its center of ~~bu~~buoyancy is 2 m below the water surface and its center of gravity is 0.5 m ~~a~~ below the water surface. Determine the metacentric height for rolling about ~~y-y~~ axisis and pitching about ~~X-X~~ axisis.



mass displaced = 1 Gg = 1 000 000 kg

Center of buoyancy = 2 m below water surface
" " gravity = 0.5 m " " "

$\therefore GB = 2 - 0.5 = 1.5\ m$

as for rolling about y-y axis's

$$V = \frac{\text{mass displaced}}{\text{density}} = \frac{1000\,000}{1000} = 1000\ m^3$$

$$I_{yy} = \frac{bh^3}{12} + 4\left(\frac{1}{12} b h^3\right) = \frac{1}{12} \times 24 \times 10^3 + 4 \times \frac{1}{12} \times 6 \times 5^3$$

$$= 2250\ m^4$$

$$I_{xx} = \frac{1}{12} \times 10 \times 24^3 + 2 \times \frac{1}{36} \times 10 \times 6^3$$

$$= 23400\ m^4$$

For rolling $MG = \frac{I}{V} - GB = \frac{2250}{1000} - 1.5 = 0.75\ m$

For pitching $MG = \frac{23400}{1000} - 1.5 = 21.9\ m$

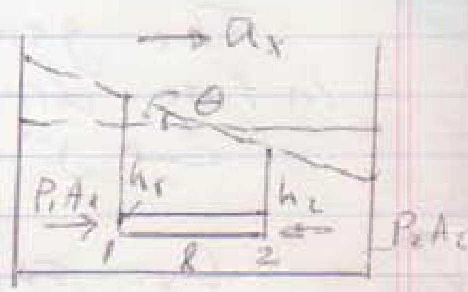
$\therefore$ the body is stable.

Relative Equilibrium

Fluid masses in Relative equilibrium- for steady flow mass in motion no shear stress will occure if there is no relative motion between adjacent Layer of the fluid.

1- Uniform linear acceleration:

a. horizontal acceleration:



$$\Sigma F = ma$$

$$P_1\,dA - P_2\,dA_2 = \frac{\gamma}{g}\, l\, dA\, a_x$$

or $$\frac{P_1}{\gamma} - \frac{P_2}{\gamma} = \frac{l\,a_x}{g}$$ dA [illegible] γ

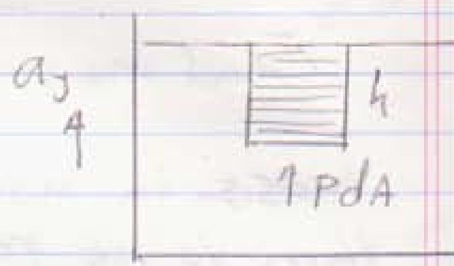
$$h_1 - h_2 = \frac{l\,a_x}{g}$$

or $$\frac{h_1 - h_2}{l} = \frac{a_x}{g}$$

from fig. the left side is the slope

$$\boxed{\therefore \tan\theta = \frac{a_x}{g}}$$

b- Vertical acceleration :-



$$\Sigma F_y = m\, a_y$$

$$\therefore PdA - \gamma h dA = \frac{\gamma h\, dA}{g} a_y$$

$$\therefore P = \gamma h \left(1 + \frac{a_y}{g}\right) \quad \text{upward}$$

$$P = \gamma h \left(1 - \frac{a_y}{g}\right) \quad \text{down ward.}$$

The general equation for a tank moved in two direction x & y

عندما يتحرك جسم او خزان على سطح مائل حيث

a_x = The acceleration in x-dir.
a_y = " " " y-dir.
P_o = The intial pressure and equal to atmospheric pressure when the tank is opend.

$$\boxed{P = P_o - \gamma \frac{a_x}{g} x - \gamma \left(1 + \frac{a_y}{g}\right) y} \quad \text{—— (1)}$$

and $$\boxed{\tan \theta = - \frac{a_x}{a_y + g}}$$ ميل السطح —— (2)

(−) لتعني الزاوية في الربع
الثاني.

2- Uniform Rotational Vortex flow

Consider liquid rotating about the central axises with angular velocity (ω) rad/sec.

The slope of water caused by normal acceleration (a_n) and the gravitational acceleration (g).

$$\text{slope} = \frac{dh}{dr} = \frac{a_n}{g}$$

$$\therefore dh = \frac{a_n}{g}\, dr$$

since $a_n = \omega^2 r$

$$\therefore dh = \frac{\omega^2 r^2}{g}\, dr$$

or $h = \dfrac{\omega^2 r^2}{2g} + c$ at $r = 0$ $h = 0$

$\therefore c = 0$

$$\therefore \boxed{h = \frac{\omega^2 r^2}{2g}} \quad \text{---} \; 1$$

$$\boxed{P = P_0 + \gamma \frac{\omega^2 r^2}{2g} - \gamma y} \quad \text{---} \; 2$$

$\gamma \dfrac{\omega^2 r^2}{2g}$ تمثل التغير بالضغط مع تغير r

$\gamma y =$ التغير بالضغط مع تغير y-dir

$y = -h$

PROBLEMS

2.1 Prove that the pressure is the same in all directions at a point in a static fluid for the three-dimensional case.

2.2 The container of Fig. 2.37 holds water and air as shown. What is the pressure at A, B, C, and D in pascals?

2.3 The tube in Fig. 2.38 is filled with oil. Determine the pressure at A and B in metres of water.

2.4 Calculate the pressure at A, B, C, and D of Fig. 2.39 in pascals.

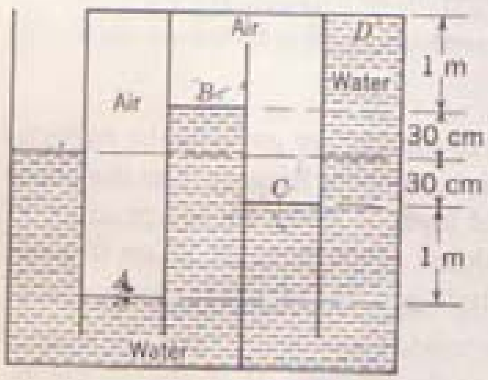


Figure 2.37 Problem 2.2.

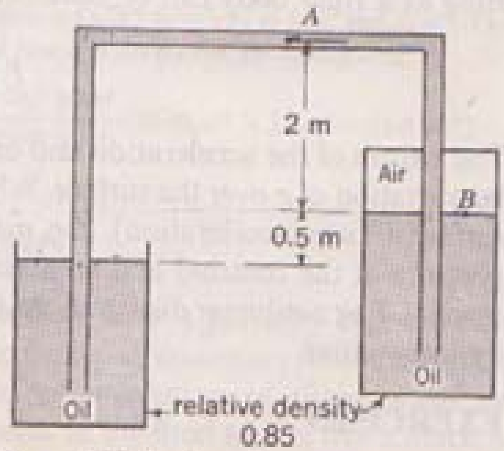


Figure 2.38 Problem 2.3.

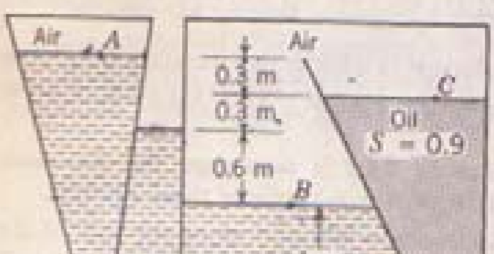


Figure 2.39 Problem 2.4.

2.5 Derive the equations that give the pressure and density at any elevation in a static gas when conditions are known at one elevation and the temperature gradient β is known.

2.6 By a limiting process as $\beta \to 0$, derive the isothermal case from the results of Prob. 2.5.

2.7 By use of the results of Prob. 2.5, determine the pressure and density at 3000 m elevation when $P = 100$ kPa, $t = 20°C$, at elevation 300 m for air and $\beta = -0.005°C/m$.

2.8 For isothermal air at 0°C, determine the pressure and density at 3000 m when the pressure is 0.1 MPa abs at sea level.

2.9 In isothermal air at 25°C what is the vertical distance for reduction of density by 10 percent?

2.10 Express a pressure of 50 kPa in (*a*) millimetres of mercury, (*b*) metres of water, (*c*) metres of acetylene tetrabromide, S = 2.94.

2.11 A bourdon gage reads 15 kPa suction, and the barometer is 750 mm Hg. Express the pressure in two other ways.

2.12 Express 300 kPa abs in metres of water gage, barometer reading 750 mm.

2.13 Bourdon gage *A* inside a pressure tank (Fig. 2.40) reads 80 kPa. Another bourdon gage *B* outside the pressure tank and connected with it reads 120 kPa, and an aneroid barometer reads 750 mm Hg. What is the absolute pressure measured by *A* in centimetres of mercury?

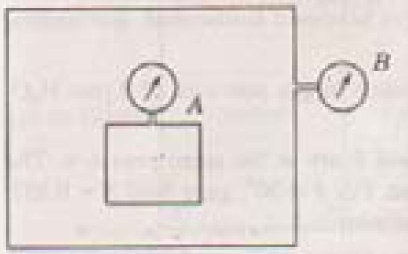


Figure 2.40 Problem 2.13.

2.14 Determine the heights of columns of water; kerosene, $S = 0.83$; and acetylene tetrabromide, $S = 2.94$, equivalent to 200 mm Hg.

2.15 In Fig. 2.6*a*, for a reading $h = 50$ cm, determine the pressure at *A* in pascals. The liquid has a relative density of 1.90.

2.16 Determine the reading *h* in Fig. 2.6*b* for $p_A = 30$ kPa suction if the liquid is kerosene, $S = 0.83$.

2.17 In Fig. 2.6*b*, for $h = 15$ cm and barometer reading 750 mm Hg, with water the liquid, find P_A in metres of water absolute.

2.18 In Fig. 2.6*c* $S_1 = 0.86$, $S_2 = 1.0$, $h_2 = 90$ mm, $h_1 = 150$ mm. Find P_A in millimetres of mercury gage. If the barometer reading is 720 mm, what is P_A in metres of water absolute?

2.19 Gas is contained in vessel *A* of Fig. 2.6*c*. With water the manometer fluid and $h_1 = 75$ mm, determine the pressure at *A* in millimetres of mercury.

2.20 In Fig. 2.7*a* $S_1 = 1.0$, $S_2 = 0.95$, $S_3 = 1.0$, $h_1 = h_2 = 280$ mm, and $h_3 = 1$ m. Compute $p_A - p_B$ in millimetres of water.

2.21 In Prob. 2.20 find the gage difference h_2 for $p_A - p_B = -350$ mm H_2O.

2.22 In Fig. 2.7*b* $S_1 = S_3 = 0.83$, $S_2 = 13.6$, $h_1 = 150$ mm, $h_2 = 70$ mm, and $h_3 = 120$ mm. (*a*) Find p_A if $p_B = 70$ kPa gage. (*b*) For $p_A = 140$ kPa gage abs and a barometer reading of 720 mm, find p_B in metres of water gage.

2.23 Find the gage difference h_2 in Prob. 2.22 for $p_A = p_B$.

2.24 In Fig. 2.41, *A* contains water, and the manometer fluid has a relative density of 2.94. When the left meniscus is at zero on the scale, $p_A = 90$ mm H_2O. Find the reading of the right meniscus for $p_A = 8$ kPa with no adjustment of the U tube or scale.

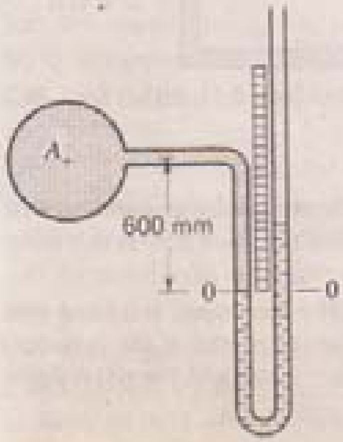


Figure 2.41 Problem 2.24.

2.25 The Empire State Building is 381 m high. What is the pressure difference in pascals of a water column of the same height?

2.26 What is the pressure at a point 10 m below the free surface in a fluid that has a variable density in kilograms per cubic metre given by $\rho = 450 + ah$, in which $a = 12$ kg/m^4 and h is the distance in metres measured from the free surface?

2.27 A vertical gas pipe in a building contains gas, $\rho = 0.72$ kg/m^3 and $p = 8$ cm H_2O gage in the basement. At the top of the building 250 m higher, determine the gas pressure in centimetres water gage for two cases: (a) gas assumed incompressible and (b) gas assumed isothermal. Barometric pressure 10.34 m H_2O; $t = 20$°C.

2.28 In Fig. 2.8 determine R, the gage difference, for a difference in gas pressure of 9 mm H_2O. $\gamma_2 = 9.8$ kN/m^3; $\gamma_3 = 10.5$ kN/m^3; $a/A = 0.01$.

2.29 The inclined manometer of Fig. 2.9 reads zero when A and B are at the same pressure. The diameter of reservoir is 5 cm, and that of the inclined tube 6 mm. For $\theta = 30°$, gage fluid $S = 0.832$, find $p_A - p_B$ in pascals as a function of gage reading R in centimetres.

2.30 Determine the gravity force W that can be sustained by the force acting on the piston of Fig. 2.42.

2.31 Neglecting the mass of the container (Fig. 2.43), find (a) the force tending to lift the circular top CD and (b) the compressive load on the pipe wall at A-A.

2.32 Find the force of oil on the top surface CD of Fig. 2.43 if the liquid level in the open pipe is reduced by 1.3 m.

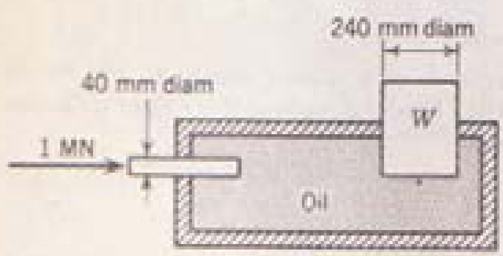


Figure 2.42 Problem 2.30.

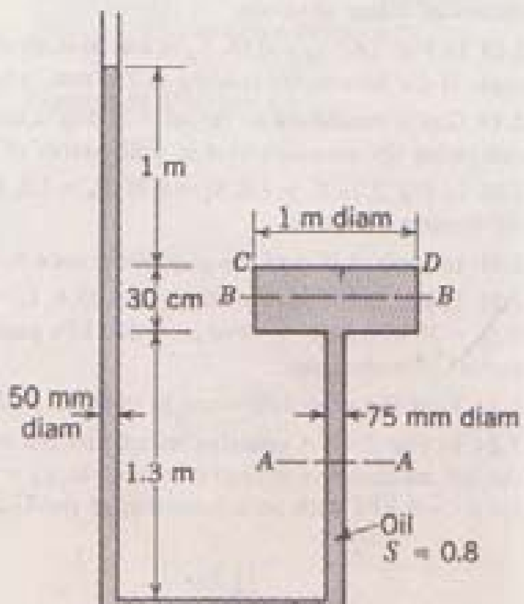


Figure 2.43 Problems 2.31, 2.32.

2.33 The container shown in Fig. 2.44 has a circular cross section. Determine the upward force on the surface of the cone frustum $ABCD$. What is the downward force on the plane EF? Is this force equal to the gravity force of the fluid? Explain.

2.34 The cylindrical container of Fig. 2.45 has a gravity force of 400 N when empty. It is filled with water and supported on the piston. (a) What force is exerted on the upper end of the cylinder? (b) If an additional 600-N gravity force is placed on the cylinder, how much will the water force against the top of the cylinder be increased?

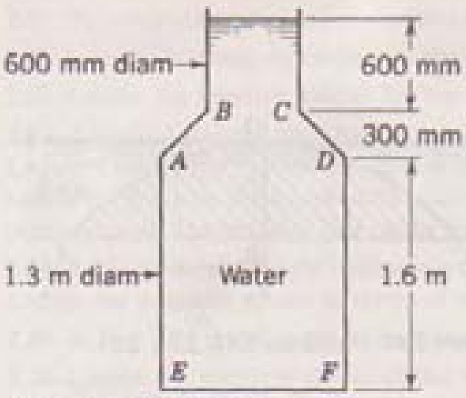


Figure 2.44 Problem 2.33.

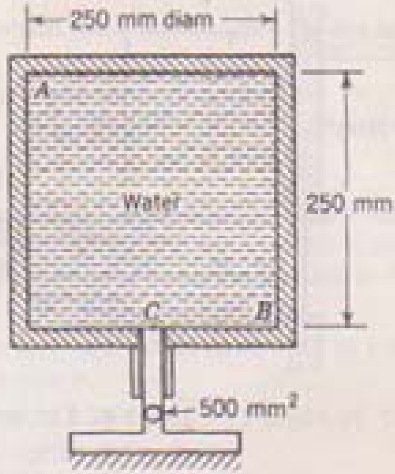


Figure 2.45 Problem 2.34.

2.35 A barrel 600 mm in diameter filled with water has a vertical pipe of 12 mm diameter attached to the top. Neglecting compressibility, how many kilograms of water must be added to the pipe to exert a force of 4 kN on the top of the barrel?

2.36 A vertical right-angled triangular surface has a vertex in the free surface of a liquid (Fig. 2.46). Find the force on one side (*a*) by integration and (*b*) by formula.

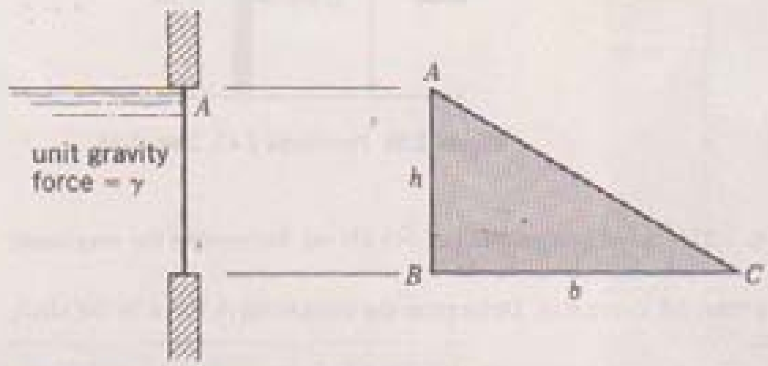


Figure 2.46 Problems 2.36, 2.38, 2.49, 2.50.

2.37 Determine the magnitude of the force acting on one side of the vertical triangle *ABC* of Fig. 2.47 (*a*) by integration and (*b*) by formula.

2.38 Find the moment about *AB* of the force acting on one side of the vertical surface *ABC* of Fig. 2.46. $\gamma = 9000\ N/m^3$.

2.39 Find the moment about *AB* of the force acting on one side of the vertical surface *ABC* of Fig. 2.47.

2.40 Locate a horizontal line below *AB* of Fig. 2.47 such that the magnitude of pressure force on the vertical surface *ABC* is equal above and below the line.

2.41 Determine the force acting on one side of the vertical surface *OABCO* of Fig. 2.48. $\gamma = 9\ kN/m^3$.

2.42 Calculate the force exerted by water on one side of the vertical annular area shown in Fig. 2.49.

2.43 Determine the moment at *A* required to hold the gate as shown in Fig. 2.50.

2.44 If there is water on the other side of the gate (Fig. 2.50) up to *A*, determine the resultant force due to water on both sides of the gate, including its line of action.

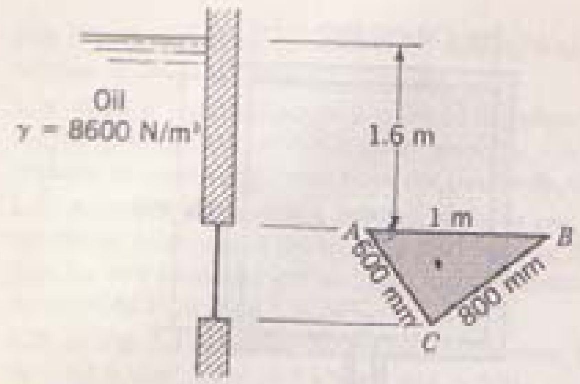


Figure 2.47 Problems 2.37, 2.39, 2.40, 2.47, 2.48.

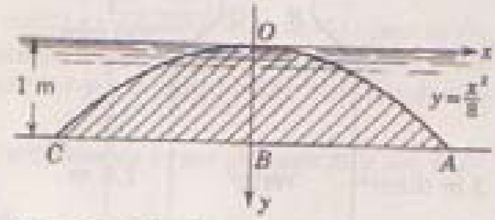


Figure 2.48 Problems 2.41, 2.56, 2.83.

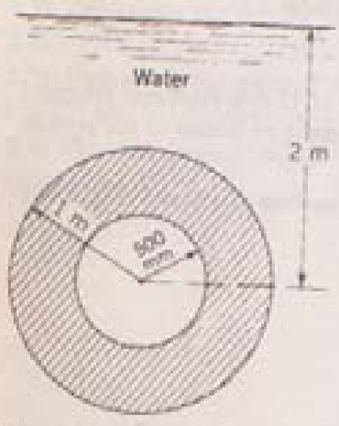


Figure 2.49 Problems 2.42, 2.51.

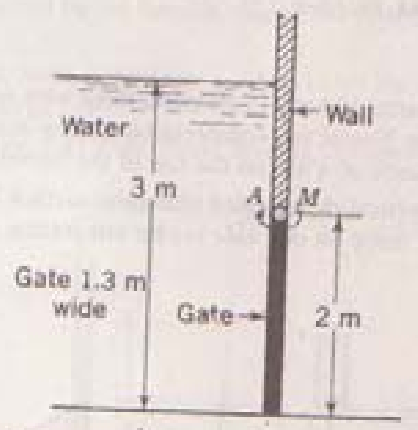


Figure 2.50 Problems 2.43, 2.44, 2.52.

2.45 The shaft of the gate in Fig. 2.51 will fail at a moment of 145 kN · m. Determine the maximum value of liquid depth h.

2.46 The dam of Fig. 2.52 has a strut AB every 6 m. Determine the compressive force in the strut, neglecting the mass of the dam.

2.47 Locate the distance of the pressure center below the liquid surface in the triangular area ABC of Fig. 2.47 by integration and by formula.

2.48 By integration locate the pressure center horizontally in the triangular area ABC of Fig. 2.47.

Figure 2.51 Problems 2.45, 2.55.

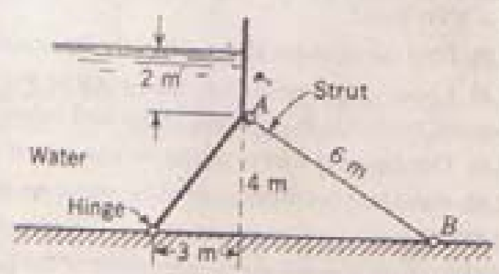


Figure 2.52 Problem 2.46.

2.49 By using the pressure prism, determine the resultant force and location for the triangle of Fig. 2.46.

2.50 By integration, determine the pressure center for Fig. 2.46.

2.51 Locate the pressure center for the annular area of Fig. 2.49.

2.52 Locate the pressure center for the gate of Fig. 2.50.

2.53 A vertical square area 2 by 2 m is submerged in water with upper edge 1 m below the surface. Locate a horizontal line on the surface of the square such that (*a*) the force on the upper portion equals the force on the lower portion and (*b*) the moment of force about the line due to the upper portion equals the moment due to the lower portion.

2.54 An equilateral triangle with one edge in a water surface extends downward at a 45° angle. Locate the pressure center in terms of the length of a side *b*.

2.55 In Fig. 2.51 develop the expression for y_p in terms of h.

2.56 Locate the pressure center of the vertical area *OABCO* of Fig. 2.48.

2.57 Locate the pressure center for the vertical area of Fig. 2.53.

2.58 Demonstrate the fact that the magnitude of the resultant force on a totally submerged plane area is unchanged if the area is rotated about an axis through its centroid.

2.59 The gate of Fig. 2.54 weighs 450 kg/m normal to the paper. Its center of gravity is 45 cm from the left face and 60 cm above the lower face. It is hinged at *O*. Determine the water-surface position for the gate just to start to come up. (Water surface is below the hinge.)

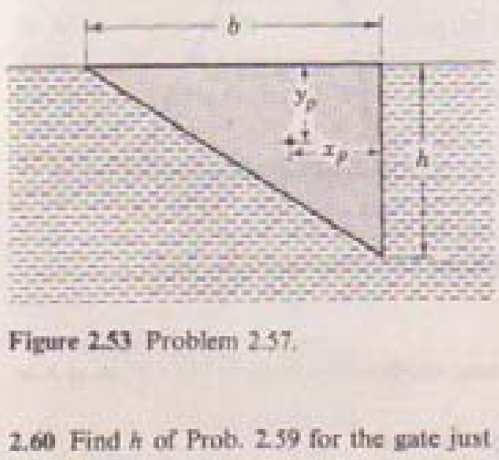


Figure 2.53 Problem 2.57.

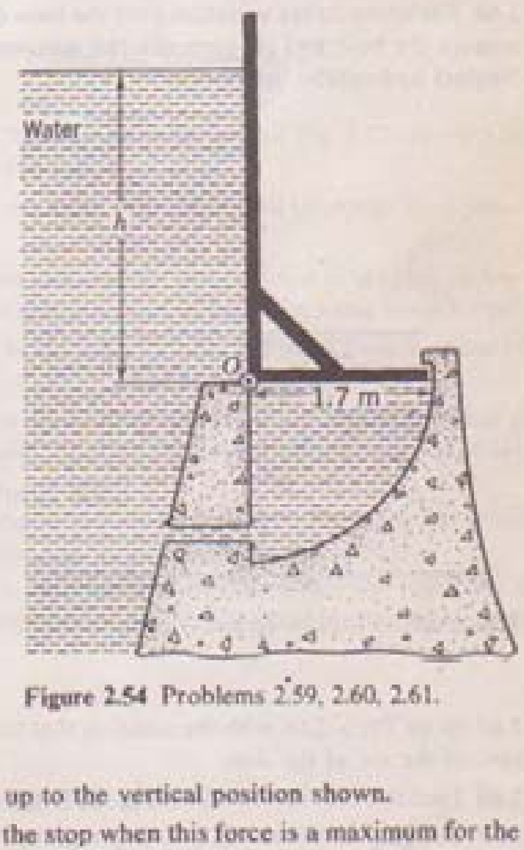


Figure 2.54 Problems 2.59, 2.60, 2.61.

2.60 Find h of Prob. 2.59 for the gate just to come up to the vertical position shown.

2.61 Determine the value of h and the force against the stop when this force is a maximum for the gate of Prob. 2.59.

2.62 Determine y of Fig. 2.55 so that the flashboards will tumble when water reaches their top.

2.63 Determine the pivot location y of the rectangular gate of Fig. 2.56 so that it will open when the liquid surface is as shown.

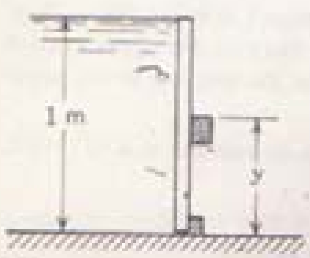


Figure 2.55 Problem 2.62.

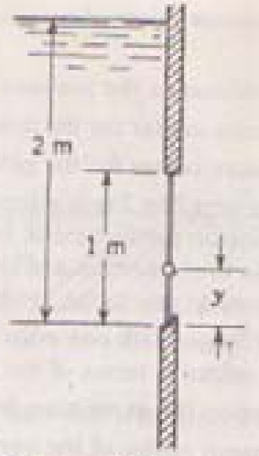


Figure 2.56 Problem 2.63.

2.64 By use of the pressure prism, show that the pressure center approaches the centroid of an area as its depth of submergence is increased.

2.65 (*a*) Find the magnitude and line of action of force on each side of the gate of Fig. 2.57. (*b*) Find the resultant force due to the liquid on both sides of the gate. (*c*) Determine F to open the gate if it is uniform and has a mass of 2 Mg.

2.66 For linear stress variation over the base of the dam of Fig. 2.58, (*a*) locate where the resultant crosses the base and (*b*) compute the maximum and minimum compressive stresses at the base. Neglect hydrostatic uplift.

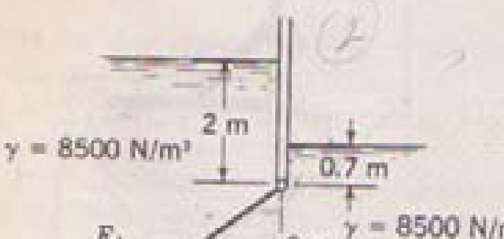


Figure 2.57 Problem 2.65.

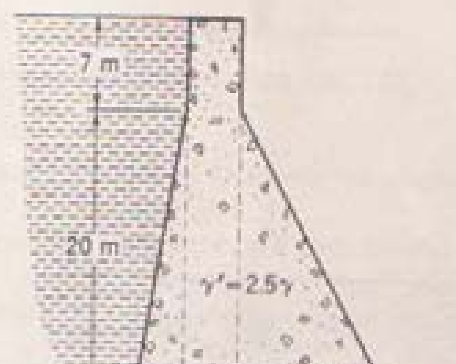


Figure 2.58 Problems 2.66, 2.67.

2.67 Work Prob. 2.66 with the addition that the hydrostatic uplift varies linearly from 20 m at A to zero at the toe of the dam.

2.68 Find the moment M at O (Fig. 2.59) to hold the gate closed.

2.69 The gate shown in Fig. 2.60 is in equilibrium. Compute W, the gravity force of counterbalance per metre of width, neglecting the mass of the gate. Is the gate in stable equilibrium?

2.70 How high (h) will the water on the right have to rise to open the gate shown in Fig. 2.61? The gate is 2 m wide, and it is constructed of material with relative density $S = 2.5$. Use the pressure prism method.

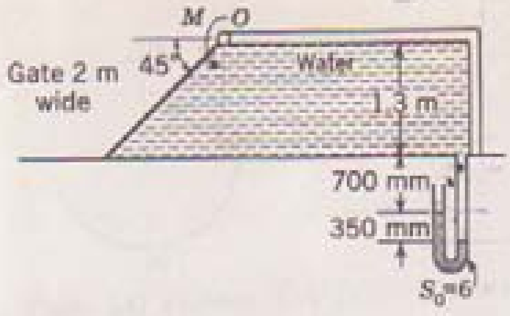


Figure 2.59 Problem 2.68.

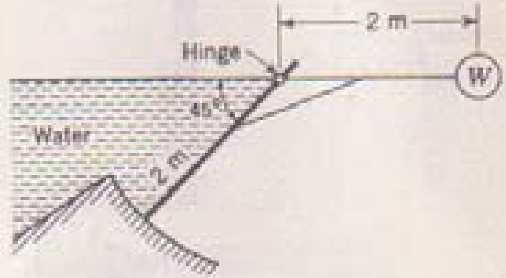


Figure 2.60 Problem 2.69.

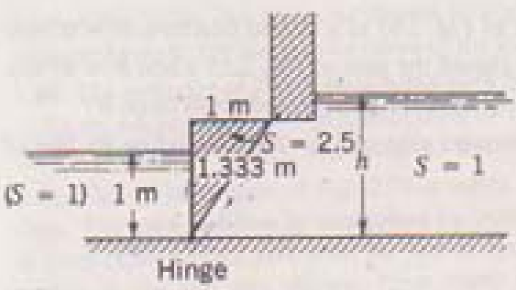


Figure 2.61 Problem 2.70.

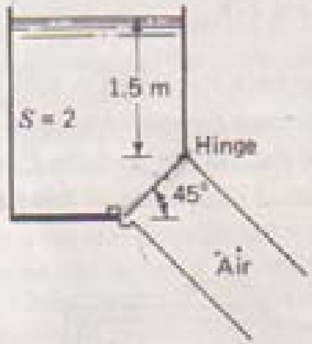


Figure 2.62 Problem 2.71.

2.71 Compute the air pressure required to keep the 700-mm-diameter gate of Fig. 2.62 closed. The gate is a circular plate that has a gravity force of 1800 N.

2.72 A 5 m-diameter pressure pipe carries liquid at 1.4 MPa. What pipe-wall thickness is required for maximum stress of 70 MPa.

2.73 To obtain the same flow area, which pipe system requires the least steel, a single pipe or four pipes having half the diameter? The maximum allowable pipe-wall stress is the same in each case.

2.74 A thin-walled hollow sphere 3 m in diameter holds gas at 1.5 MPa. For allowable stress of 60 MPa determine the minimum wall thickness.

2.75 A cylindrical container 2.3 m high and 1.3 m in diameter provides for pipe tension with two hoops 30 cm from each end. When it is filled with water, what is the tension in each hoop due to the water?

2.76 A 20-mm-diameter steel ball covers a 10-mm-diameter hole in a pressure chamber where the pressure is 30 MPa. What force is required to lift the ball from the opening?

2.77 If the horizontal component of force on a curved surface did *not* equal the force on a projection of the surface onto a vertical plane, what conclusions could you draw regarding the propulsion of a boat (Fig. 2.63)?

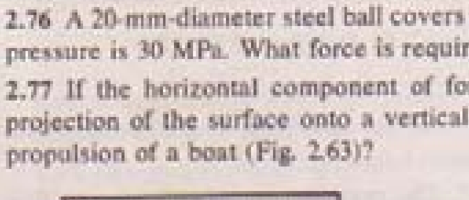


Figure 2.63 Problem 2.77.

X **2.78** (*a*) Determine the horizontal component of force acting on the radial gate (Fig. 2.64) and its line of action. (*b*) Determine the vertical component of force and its line of action. (*c*) What force *F* is required to open the gate, neglecting its mass? (*d*) What is the moment about an axis normal to the paper and through point *O*?

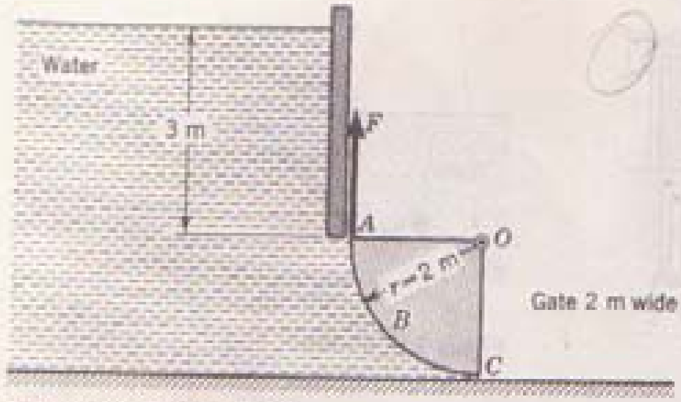


Figure 2.64 Problem 2.78.

2.79 Calculate the force F required to hold the gate of Fig. 2.65 in a closed position, $R = 60$ cm.

2.80 Calculate the force F required to open or hold closed the gate of Fig. 2.65 when $R = 45$ cm.

2.81 What is R of Fig. 2.65 for no force F required to hold the gate closed or to open it?

2.82 Find the vertical component of force on the curved gate of Fig. 2.66, including its line of action.

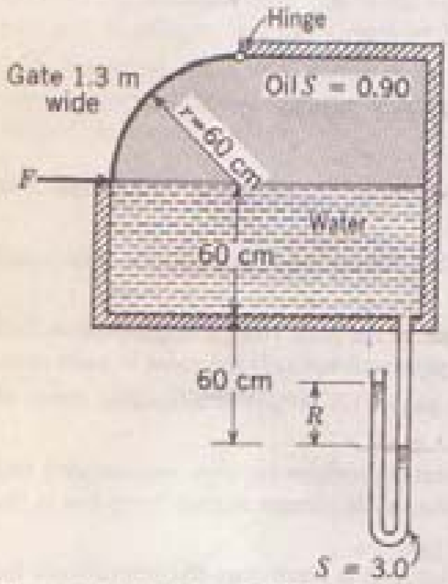


Figure 2.65 Problems 2.79, 2.80, 2.81.

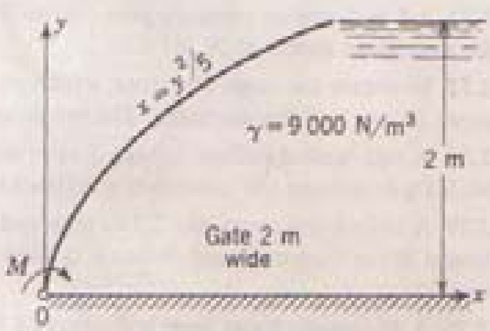


Figure 2.66 Problems 2.82, 2.86.

2.83 What is the force on the surface whose trace is OA of Fig. 2.48? The length normal to the paper is 3 m, $\gamma = 9\text{kN/m}^3$.

2.84 A right-circular cylinder is illustrated in Fig. 2.67. The pressure, in pascals, due to flow around the cylinder varies over the segment ABC as $p = 2\rho(1 - 4\sin^2\theta) + 500$. Calculate the force on ABC.

2.85 If the pressure variation on the cylinder in Fig. 2.67 is $p = 2\rho \times [1 - 4(1 + \sin\theta)^2] + 500$, determine the force on the cylinder.

2.86 Determine the moment M to hold the gate of Fig. 2.66, neglecting its mass.

2.87 Find the resultant force, including its line of action, acting on the outer surface of the first quadrant of a spherical shell of radius 600 mm with center at the origin. Its center is 1.2 m below the water surface.

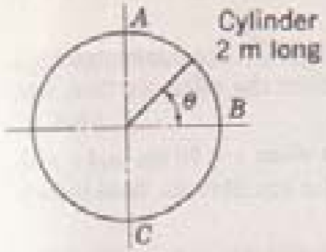


Figure 2.67 Problems 2.84, 2.85.

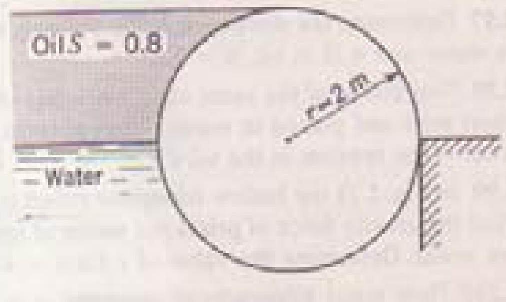


Figure 2.68 Problem 2.89.

2.88 The volume of the ellipsoid given by $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$ is $4\pi abc/3$, and the area of the ellipse $x^2/a^2 + z^2/c^2 = 1$ is πac. Determine the vertical force on the surface given in Example 2.9.

2.89 A log holds the water as shown in Fig. 2.68. Determine (*a*) the force per metre pushing it against the dam, (*b*) the gravity force of the cylinder per metre of length, and (*c*) its relative density.

2.90 The cylinder of Fig. 2.69 is filled with liquid as shown. Find (*a*) the horizontal component of force on *AB* per unit of length, including its line of action, and (*b*) the vertical component of force on *AB* per unit of length, including its line of action.

2.91 The cylinder gate of Fig. 2.70 is made up from a circular cylinder and a plate hinged at the dam. The gate position is controlled by pumping water into or out of the cylinder. The center of gravity of the empty gate is on the line of symmetry 1.3 m from the hinge. It is in equilibrium when empty in the position shown. How many cubic metres of water must be added per metre of cylinder to hold the gate in its position when the water surface is raised 1 m?

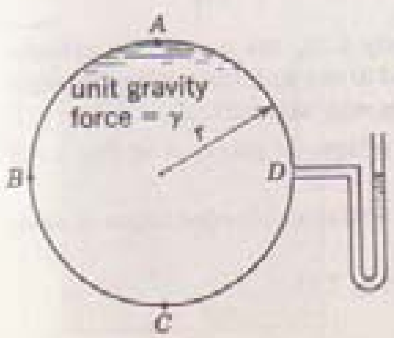


Figure 2.69 Problem 2.90.

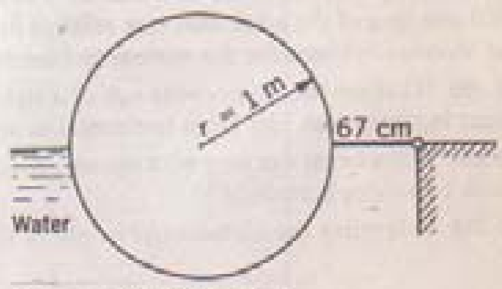


Figure 2.70 Problem 2.91.

2.92 A hydrometer with gravity force of 0.035 N and has a stem 5 mm in diameter. Compute the distance between relative density markings 1.0 and 1.1.

2.93 Design a hydrometer to read relative densities in the range from 0.80 to 1.10 when the scale is to be 75 mm long.

2.94 A sphere 250 mm in diameter, relative density 1.4, is submerged in a liquid having a density varying with the depth y below the surface given by $\rho = 1000 + 0.03y$ kg/m³. Determine the equilibrium position of the sphere in the liquid.

2.95 Repeat the calculations for Prob. 2.94 for a horizontal circular cylinder with a relative density of 1.4 and a diameter of 250 mm.

2.96 A cube 60 cm on an edge, has its lower half of S = 1.4 and upper half of S = 0.6. It is submerged into a two-layered fluid, the lower S = 1.2 and the upper S = 0.9. Determine the height of the top of the cube above the interface.

2.97 Determine the density, specific volume, and volume of an object that has a gravity force 3 N in water and 4 N in oil, S = 0.83.

2.98 Two cubes of the same size, 1 m^3, one of S = 0.80, the other of S = 1.1, are connected by a short wire and placed in water. What portion of the lighter cube is above the water surface, and what is the tension in the wire?

2.99 In Fig. 2.71 the hollow triangular prism is in equilibrium as shown when z = 30 cm and y = 0. Find the gravity force of prism per metre of length and z in terms of y for equilibrium. Both liquids are water. Determine the value of y for z = 45 cm.

2.100 How many kilograms of concrete, ρ = 2.55 Mg/m^3, must be attached to a beam having a volume of 0.1 m^3 and S = 0.65 to cause both to sink in water?

2.101 The gate of Fig. 2.72 has a mass of 225 kg/m normal to the page. It is in equilibrium as shown. Neglecting the mass of the arm and brace supporting the counterbalance, (*a*) find W and (*b*) determine whether the gate is in stable equilibrium. The mass W is made of concrete, S = 2.50.

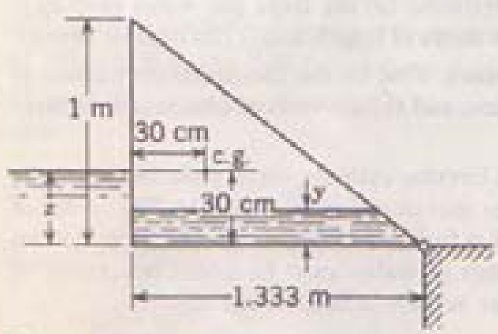


Figure 2.71 Problem 2.99.

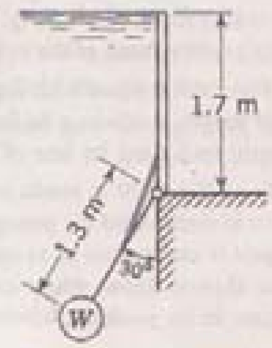


Figure 2.72 Problem 2.101.

2.102 A wooden cylinder 600 mm in diameter, relative density 0.50, has a concrete cylinder 600 mm long of the same diameter, relative density 2.50, attached to one end. Determine the length of wooden cylinder for the system to float in stable equilibrium with axis vertical.

2.103 What are the proportions r_0/h of a right-circular cylinder of specific gravity S so that it will float in water with end faces horizontal in stable equilibrium?

2.104 Will a beam 4 m long with square cross section, S = 0.75, float in stable equilibrium in water with two sides horizontal?

2.105 Determine the metacentric height of the torus shown in Fig. 2.73.

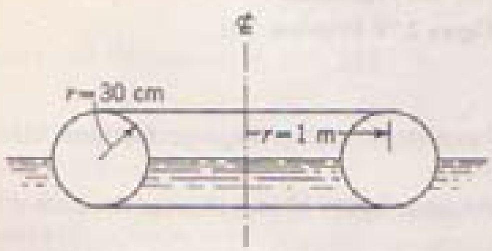


Figure 2.73 Problem 2.105.

2.106 The plane gate (Fig. 2.74) weighs 2000 N/m normal to the paper, and its center of gravity is 2 m from the hinge at O. (*a*) Find h as a function of θ for equilibrium of the gate. (*b*) Is the gate in stable equilibrium for any values of θ?

2.107 A spherical balloon 15 m in diameter is open at the bottom and filled with hydrogen. For barometer reading of 710 mm Hg and 20°C, what is the total gravity force of the balloon and the load to hold it stationary?

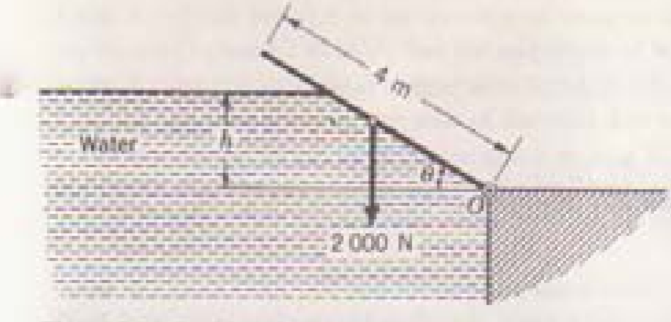


Figure 2.74 Problem 2.106.

2.108 A tank of liquid $S = 0.86$ is accelerated uniformly in a horizontal direction so that the pressure decreases within the liquid 20 kPa/m in the direction of motion. Determine the acceleration.

2.109 The free surface of a liquid makes an angle of 20° with the horizontal when accelerated uniformly in a horizontal direction. What is the acceleration?

2.110 In Fig. 2.75, $a_x = 3.9$ m/s², $a_y = 0$. Find the imaginary free liquid surface and the pressure at B, C, D, and E.

2.111 In Fig. 2.75, $a_x = 0$, $a_y = 2.45$ m/s². Find the pressure at B, C, D, and E.

2.112 In Fig. 2.75, $a_x = 2.45$ m/s², $a_y = 4.902$ m/s². Find the imaginary free surface and the pressure at B, C, D, and E.

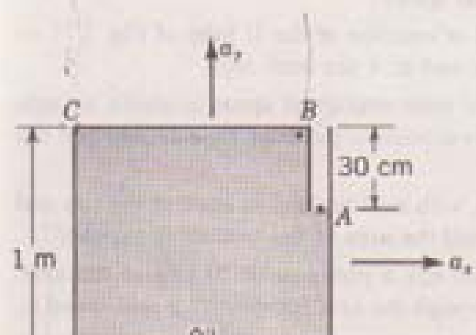


Figure 2.75 Problems 2.110, 2.111, 2.112, 2.116.

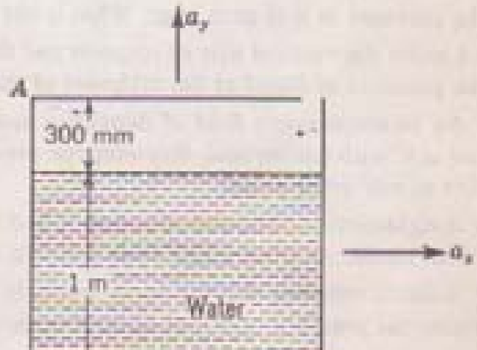


Figure 2.76 Problems 2.113, 2.114.

2.113 In Fig. 2.76, $a_x = 9.806$ m/s², $a_y = 0$. Find the pressure at A, B, and C.

2.114 In Fig. 2.76, $a_x = 4.903$ m/s², $a_y = 9.806$ m/s². Find the pressure at A, B, and C.

2.115 A circular cross-sectional tank of 2 m depth and 1.3 m diameter is filled with liquid and accelerated uniformly in a horizontal direction. If one-third of the liquid spills out, determine the acceleration.

2.116 Determine a_x and a_y in Fig. 2.75 for pressure at A, B, and C to be the same.

2.117 The tube of Fig. 2.77 is filled with liquid, S = 2.40. When it is accelerated to the right 2.45 m/s², draw the imaginary free surface and determine the pressure at A. For $p_A = 56$ kPa vacuum determine a_x.

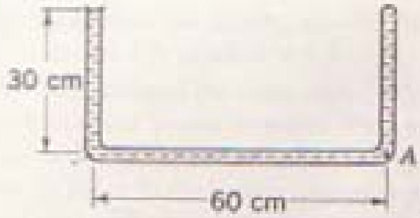


Figure 2.77 Problems 2.117, 2.123, 2.124, 2.134.

2.118 A cubical box 1 m on an edge, open at the top and half filled with water, is placed on an inclined plane making a 30° angle with the horizontal. The box alone has a gravity force of 500 N and has a coefficient of friction with the plane of 0.30. Determine the acceleration of the box and the angle the free-water surface makes with the horizontal.

2.119 Show that the pressure is the same in all directions at a point in a liquid moving as a solid.

2.120 A closed box contains two immiscible liquids. Prove that, when it is accelerated uniformly in the x direction, the interface and zero-pressure surface are parallel.

2.121 Verify the statement made in Sec. 2.9 on uniform rotation about a vertical axis that, when a fluis rotates in the manner of a solid body, no shear stresses exist in the fluid.

2.122 A vessel containing liquid, $S = 1.2$, is rotated about a vertical axis. The pressure at one point 0.6 m radially from the axis is the same as at another point 1.2 m from the axis and with elevation 0.6 m higher. Calculate the rotational speed.

2.123 The U tube of Fig. 2.77 is rotated about a vertical axis 15 cm to the right of A at such a speed that the pressure at A is zero gage. What is the rotational speed?

2.124 Locate the vertical axis of rotation and the speed of rotation of the U tube of Fig. 2.77 so that the pressure of liquid at the midpoint of the U tube and at A are both zero.

2.125 An incompressible fluid of density ρ moving as a solid rotates at speed ω about an axis inclined at $\theta°$ with the vertical. Knowing the pressure at one point in the fluid, how do you find the pressure at any other point?

2.126 A right-circular cylinder of radius r_0 and height h_0 with axis vertical is open at the top and filled with liquid. At what speed must it rotate so that half the area of the bottom is exposed?

2.127 A liquid rotating about a *horizontal* axis as a solid has a pressure of 70 kPa at the axis. Determine the pressure variation along a vertical line through the axis for density ρ and speed ω.

2.128 Determine the equation for the surfaces of constant pressure for the situation described in Prob. 2.127.

2.129 Prove by integration that a paraboloid of revolution has a volume equal to half its circumscribing cylinder.

2.130 A tank containing two immiscible liquids is rotated about a vertical axis. Prove that the interface has the same shape as the zero-pressure surface.

2.131 A hollow sphere of radius r_0 is filled with liquid and rotated about its vertical axis at speed ω. Locate the circular line of maximum pressure.

2.132 A gas following the law $p\rho^{-n}$ = const is rotated about a vertical axis as a solid. Derive an expression for pressure in a radial direction for speed ω, pressure p_0, and density ρ_0 at a point on the axis.

2.133 A vessel containing water is rotated about a vertical axis with an angular velocity of 50 rad/s. At the same time the container has a downward acceleration of 4.903 m/s². What is the equation for a surface of constant pressure?

2.134 The U tube of Fig. 2.77 is rotated about a vertical axis through A at such a speed that the water in the tube begins to vaporize at the closed end above A, which is at 20°C. hat is the angular velocity? What would happen if the angular velocity were increased?

2.135 A cubical box 1.3 m on an edge is open at the top and filled with water. When it is accelerated upward 2.45 m/s^2, find the magnitude of water force on one side of the box.

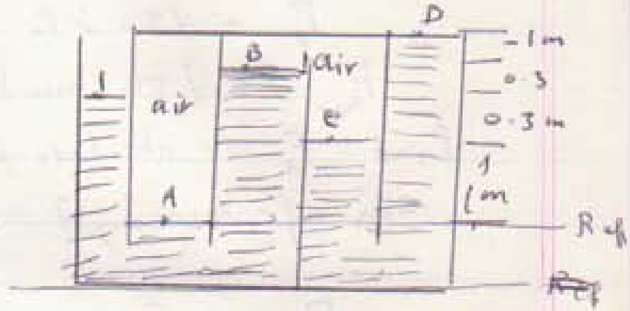
2.136 A cube 1 m on an edge is filled with liquid, $S = 0.65$, and is accelerated downward 2.45 m/s^2. Find the resultant force on one side of the cube due to liquid pressure.

2.137 A cylinder 60 cm in diameter and 2 m long is accelerated uniformly along its axis in a horizontal direction 4.903 m/s^2. It is filled with liquid, $\gamma = 7850$ N/m^3, and it has a pressure along its axis of 70 kPa before acceleration starts. Find the horizontal net force exerted against the liquid in the cylinder.

2.138 A closed cube, 300 mm on an edge, has a small opening at the center of its top. When it is filled with water and rotated uniformly about a vertical axis through its center at ω rad/s, find the force on a side due to the water in terms of ω.

Problems

2.2



$~~P_1~~ \gamma h_1 = P_A$

$0 + 9810 \times 1.3 = P_A$

$\therefore P_A = 12.753\ kPa$

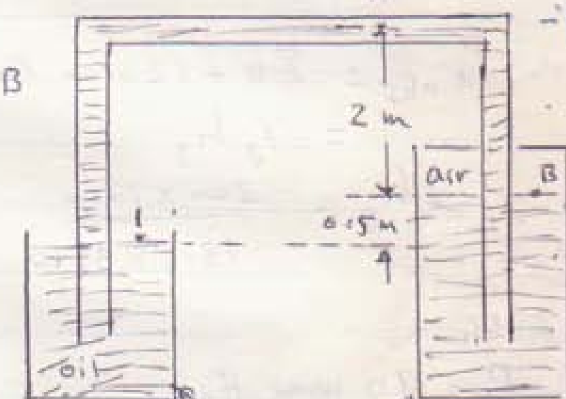
$P_1 + \gamma_1 h_1 - \gamma_2 h_2 = P_B \quad \therefore 0 + 9810 \times 1.3 - 9810 \times 1.6 = P_B$

$\therefore P_B = -2.943\ kPa = P_C \quad P_C + \gamma_1 h_1 - \gamma_2 h_2 = P_D$

$\therefore P_D = -18.639\ kPa.$

2.3

find the pressure at A, B in meter of water?



$P_1 - \gamma h_1 + \gamma h_2 = P_B$

$0 - 0.85 \times 2.5 \times 9810$

$+ 0.85 \times 9810 \times 2 = P_B$

$P_B = 0.85 \times 9810 \times (2 - 2.5)$

$= -0.425 \times 9810 = -4169.25\ N/m^2$

$P_B = \gamma_{oil} h_{oil} = \gamma_w h_w$

$$h_w = \frac{\gamma_{oil} h_{oil}}{\gamma_w} = \frac{-0.425 \times \cancel{9810}}{\cancel{9810}} = -0.425\ m \text{ of } H_2O$$

also

$0 - \gamma h_1 = P_A = -20846\ N/m^2$

$\therefore P_A = -\gamma_{oil} h_{oil} = \gamma_w h_w \quad \therefore h_w = \frac{-0.85 \times \cancel{9810} \times 2.5}{\cancel{9810}}$

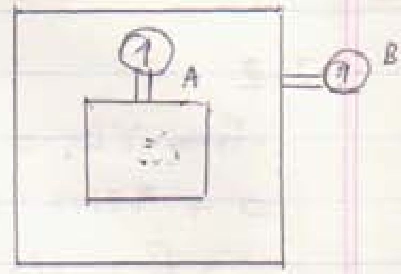
$h_w = -2.125$ m of water.

Q.13 $P_A = 80$ kPa gage

$P_B = 120$ kPa gage

$P_{bar} = 750$ mm Hg

find P_A in absolute pressure in cm of mercury?



$$P_{abs} = P_{gage} + P_{bar}$$

$$P_{bar} = 13.6 \times 9810 \times 0.75 = 100 \text{ KPa}$$

$$\therefore P_{A\,abs} = 80 + 120 + 100 = 300 \text{ KPa}$$

$$= \gamma_g h_g$$

$$\therefore h_g = \frac{300 \times 1000}{13.6 \times 9810} = 2.25 \text{ m of } Hg$$

$$= 225 \text{ cm of } Hg$$

Q.2.24

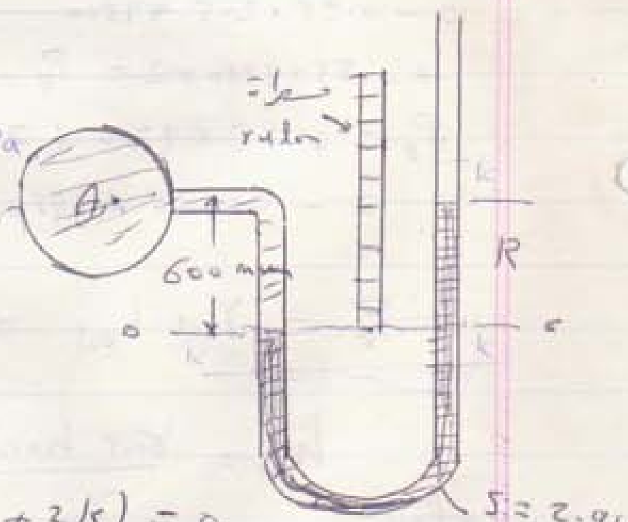
a) $P_A = 90$ mm H_2O

$$= \gamma_w h_w = 9810 \times 0.09 = \text{Pa}$$

$$P_A + \gamma_w h_1 - \gamma_s R = 0$$

$$9810 \times 0.09 + 9810 \times 0.6 - 2.94 \times 9810\, R = 0$$

$$R = 0.2347 \text{ m}$$



b) $P_A = 8$ kPa

$$P_A + \gamma_w(h_1 + k) - \gamma_s(R + 2k) = 0$$

$$8000 + 9810(0.6 + k) - 2.94 \times 9810(0.2347 + 2k) = 0$$

$$\therefore k = 0.148 \text{ m}$$

The total Reading without adjustment $= k + R = 0.148 + 0.23$

$= 0.383$ m.

Q 2-4 $\quad P_a + \gamma h_1 - \gamma h_2 = 0$

$$\therefore P_a = \gamma h_2 - \gamma h_1 = 9810\,(1.6 - 2.2)$$
$$= -5.88 \text{ kPa.}$$

$$P_b + 9810 \times 1 - 1.6 \times 9810 = 0$$
$$P_b = 5.88 \text{ kPa}$$

$P_c = P_b = 5.88$ $\qquad$ [إهمال ضغط الهواء

$$P_D = P_c + 1.4 \times 0.9 \times 9810 = 22.54 \text{ KPa.}$$

Q.2.14

a) Water $\quad \gamma_w h_w = \gamma_g \cdot h_g$

$$9810 \times h_w = 13.6 \times 9810 \times 0.2$$
$$h_w = 2.72 \text{ m}$$

b) Kerosene $\quad \gamma_K h_K = 13.6 \times 9810 \times 0.2$

$$h_K = \frac{13.6 \times 9810 \times 0.2}{0.83 \times 9810} = 3.277 \text{ m}$$

c) acetylene

$$h_{ac} = \frac{13.6 \times 9810 \times 0.2}{2.94 \times 9810} = 0.925 \text{ m}$$

Q.2.16

$P_a + \gamma h = 0 \qquad P_a = -9810 \times 0.83 \cdot h = 30000$

$$\therefore h = 3.688 \text{ m}$$

Q.2.19 $\quad P_A = \gamma h_w$

$\gamma_g h_g = \gamma_w h_w \qquad \therefore h_g = \dfrac{9810 \times 0.075}{9810 \times 13.6} \times 1000$

$$= 5.5 \text{ mm Hg.}$$

Q. 2-20 $\quad S_1 = 1.0 \quad S_2 = 0.95 \quad S_3 = 1.0$

$h_1 = h_2 = 280\ mm \quad h_3 = 1m$

$P_A - P_B = ?$ in metre of water.

$$P_A - \gamma_1 h_1 - \gamma_2 h_2 + \gamma_3 h_3 = P_B$$

$$P_A - P_B = \gamma_1 h_1 + \gamma_2 h_2 - \gamma_3 h_3$$

$$= 9810 \times 0.28 + 9810 \times 0.95 \times 0.28 - 9810 \times 1 \times 1$$

$$= -4.45\ kN/m^2$$

$$\therefore h_A - h_B = \frac{-4.45 \times 1000}{9810} = 0.454\ m\ H_2O$$

$$= 454\ mm\ H_2O$$

Q. 2-22

a) $P_A + S_1 \gamma_w h_1 - S_2 \gamma_w h_2 - S_3 \gamma_3 h_3 = P_B$

$$P_A + 0.83 \times 9810 \times 0.15 - 13.6 \times 9810 \times 0.07 - 0.83 \times 9810 \times 0.12 = 70000$$

$$P_A = 79.09\ kPa.$$

b)

$$P_{abs} = P_{gay} + P_{atm} \qquad \therefore P_g = 140000 - 13.6 \times 9810 \times 0.72$$

$$= 43.9\ kPa$$

$\therefore$

$$43900 + 0.83 \times 9810 \times 0.15 - 13.6 \times 9810 \times 0.07 - 0.83 \times 9810 \times 0.12 = P_B$$

$$\therefore P_B = 34.82\ kPa$$

$$\therefore h_B = \frac{P_B}{\gamma_w} = 3.55\ m\ of\ water.$$

2.42 $F = P_G A = \gamma \bar{h} A$

$= 9810 \times 2 \left(\pi r_1^2 - \pi r_2^2 \right)$

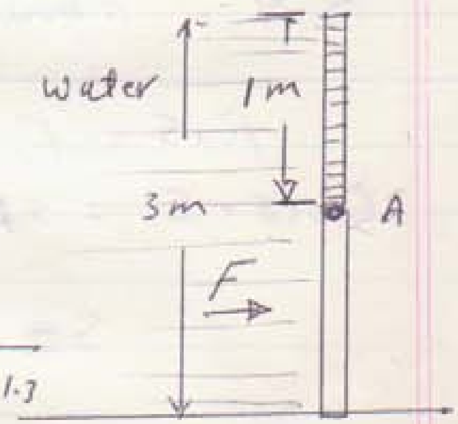
$= 46.2\ kN$

· Q.43

$F = P_G A = \gamma \bar{h} A$

$= 9810 \times 2 \times 2 \times 1.3$

$= 51.012\ kN$



$y_p = \bar{y} + \dfrac{I_G}{\bar{y} A} = 2 + \dfrac{1.3 \times 2^3}{12 \times 2 \times 2 \times 1.3}$

$= 2.166\ m$

1.3 m wide

The moment about A

$M_A = F \times (y_p - 1) = 51.012 \times (2.166 - 1)$

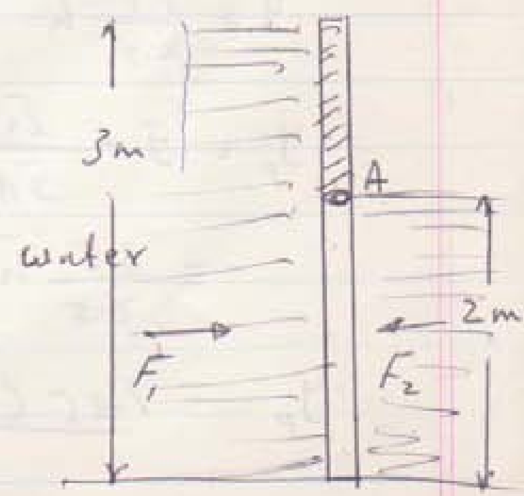
$= 59.5\ kN.m$

· Q.44

$F_1 = P_G A = \gamma \bar{h} A$

$= 9810 \times 2 \times 2 \times 1.3 =$

$= 51.012\ kN$



$y_{p_1} = 2 + \dfrac{1.3 \times 2^3}{12 \times 2 \times 2 \times 1.3} = 2.166\ m$

$F_2 = 9810 \times 1 \times 2 \times 1.3 =$

$= 25.506\ kN$

$$y_{p_2} = 1 + \frac{1.3\times 2^3}{12\times 1\times 2\times 1.3} = 1.333 \text{ m}$$

$$R = F_1 - F_2 = 25.506 \text{ kN} \rightarrow$$

$\Sigma M = 0$ about A جداً

$$R\times S = F_1(y_{p_1} - 1) - F_2\times y_{p_2}$$

$$25.066\times S = 51.012(1.166) - 25.506\times 1.333$$

$S \cong 1$ m the line of action of the resultant force from A

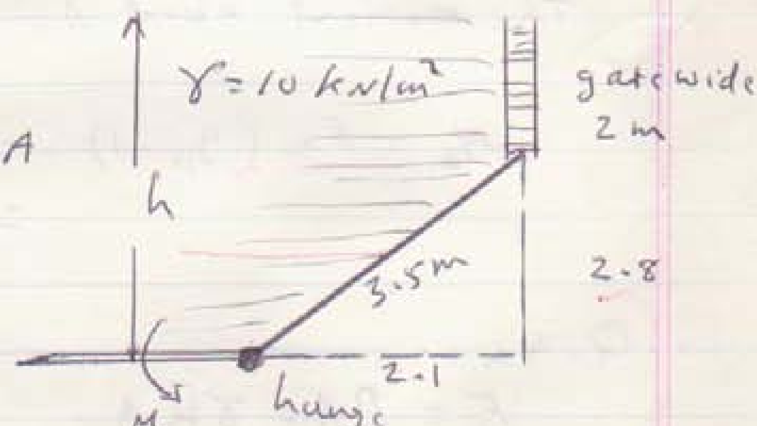
Q. 2.45

$$F = \gamma \bar{h} A$$
$$= \gamma(h - 1.4)A$$

when $\bar{h} = h - 1.4$

$$\bar{h} = \bar{y}\sin\theta$$

$$\bar{y} = \frac{3.5}{2.8}\bar{h}$$



$$y_p = \bar{y} + \frac{I_G}{\bar{y}A} = \frac{3.5}{2.8}\bar{h} + \frac{2\times 3.5^3}{12\times\frac{3.5}{2.8}\bar{h}\times 3.5\times 2}$$

$$= \frac{3.5}{2.8}\bar{h} + \frac{3.5^3\times 2.8}{12\,\bar{h}} = 1.25\bar{h} + \frac{0.82}{\bar{h}}$$

$$y_p = \frac{1.25(h-1.4)^2 + 0.82}{(h-1.4)}$$

$\Sigma M = 145 \, kN.m \quad F(L - y_p)$

$L \sin\theta = h \qquad \therefore L = \frac{3.5}{2.8} h$

$\therefore 14{,}5000 = \gamma A (h - 1.4) \left(\frac{3.5}{28} h - \left(1.25 (h - 1.4) + \frac{0.8^2}{h - 1.4} \right) \right)$

$14{,}5000 = 10000 \times 2 \times 3.5 (h - 1.4) \left(\cancel{1.25h} - \cancel{1.25h} + 1.75 - \frac{0.8^2}{h - 1.4} \right)$

$= 70000 (h - 1.4) \left(1.75 - \frac{0.82}{h - 1.4} \right)$

$\frac{145000}{70000} = 1.75 (h - 1.4) - 0.82$

$= 1.75 h - 2.45 - 0.82$

$\therefore h = 3.05 \, m$

Q. 46

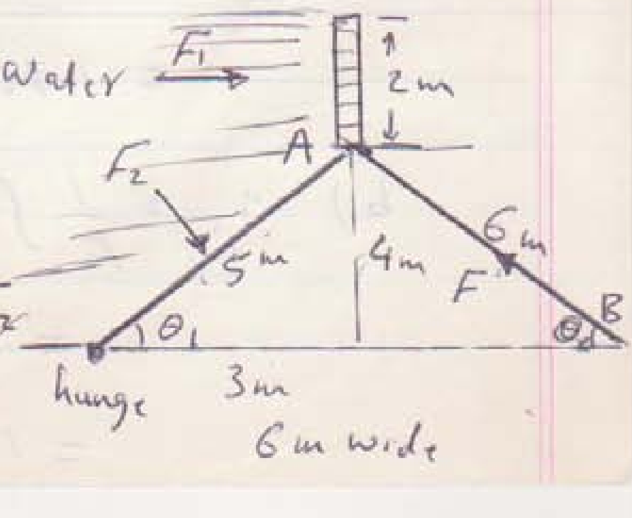
$F_1 = \gamma \bar{h} A$

$= 9810 \times 1 \times 2 \times 6$

$= 117.72 \, kN$

$y_{p_1} = \bar{y} + \frac{I_G}{\bar{y} A} = 1 + \frac{6 \times 2^3}{12 \times 1 \times 6 \times 2}$

$= 1.333 \, m$



$F_2 = \gamma \bar{h} . A = 9810 \times 4 \times 5 \times 6$
$= 1177.2$ kN

$y_{p_2} = \bar{y} + \frac{I_G}{\bar{y} A}$ $\bar{h} = \bar{y} \sin\theta_1$ $\therefore \bar{y} = \frac{4}{\sin\theta} = 5$ m

$= 5 + \frac{6 \times 5^3}{12 \times 5 \times 6 \times 5} = 5.41$ m

$\Sigma M = 0$ about the hinge.

$F_1 (6 - y_{p_1}) + F_2 (L - y_{p_2}) - F \cos\theta_2 \times 4 - F \sin\theta_2 \times 3 = 0$

$L \sin\theta_1 = h = 6$ $\therefore L = 7.5$

$\sin\theta_2 = \frac{4}{6}$ $\cos\theta_2 = \frac{4.47}{6}$

$\therefore$ $F = 602.37$ kN.

Q. 47

a) $y_p = \bar{y} + \frac{I_G}{\bar{y} A} = \left(\frac{0.48}{3} + 1.6 \right) + \frac{b h^3}{36 \times 1.76 \times \frac{1 \times 0.48}{2}}$

$= 1.76 + \frac{1 \times 0.48^3 \times 2}{36 \times 1.76 \times 0.48} = 1.76727$ m

b) $y_p = \frac{1}{F} \int y P \, dA = \frac{\int \bar{y}^2 dA}{\bar{y} A}$ $dA = l \, dy$

$= \frac{\int_{1.6}^{2.08} (2.08 y^2 - y^3) \, dy}{\bar{y} A \times 0.48} = \frac{\left. \frac{2.08}{3} y^3 - \frac{y^4}{4} \right]_{1.6}^{2.08}}{0.48 \times 1.76 \times \frac{0.48}{2}}$

$= 1.76727$ m

Q.62 $y_p = \bar{y} + \frac{I_G}{\bar{y}A} = 0.5 + \frac{1\times1^3}{12\times0.5\times1\times1} = 0.666\ m$

$\therefore y = 1 - y_p = 0.333$ The flashbord will tumble.

Q.63 The gate opened when y at pressure center
i.e

$y_p = \bar{y} + \frac{I_G}{\bar{y}A} = 1.5 + \frac{1\times1^3}{12\times1.5\times1} = 1.55\ m$

$\therefore y = 2 - 1.55 = 0.45\ m$

Q.65 mass = 2 Mg

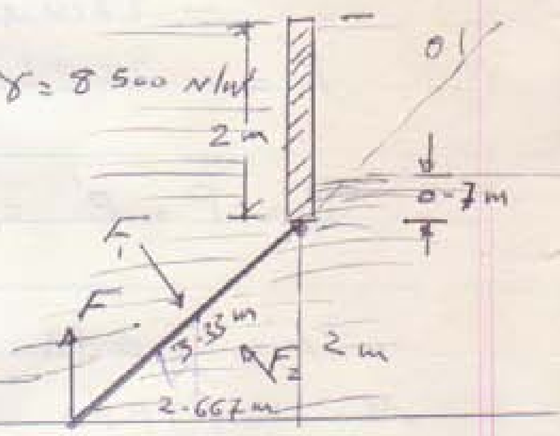
$F_1 = \gamma \bar{h}_1 A_1$

$= 8500\times3\times3.33\times2$

$= 170.0136\ kN$

$F_2 = 8500\times1.7\times3.33\times2$

$= 96.32\ kN$



Gate wide 2m.

W = 2000×9.81

$\bar{y}_1 = 3 \times \frac{1}{\sin\theta} = 3\times\frac{3.333}{2} = 5\ m$

$y_{p_1} = \bar{y} + \frac{I_G}{\bar{y}A}$

$= 5 + \frac{2\times3.33^3}{12\times5\times2\times3.33} = 5.185\ m$

$\bar{y}_2 = 1.7\times\frac{3.333}{2} = 2.83\ m$

$y_{p_2} = 2.83 + \frac{2\times3.33^3}{12\times2.83\times2\times3.33} = 3.157\ m$

$\Sigma M = 0$

$F_1\left(y_{p_1} - \frac{2}{\sin\theta}\right) - F_2\left(y_{p_2} - \frac{0.7}{\sin\theta}\right) - F\times2.667\ m + W\times\frac{2.667}{2} = 0$

$R = 74\ kN$

$\cancel{a_j = 1.667\ m}$

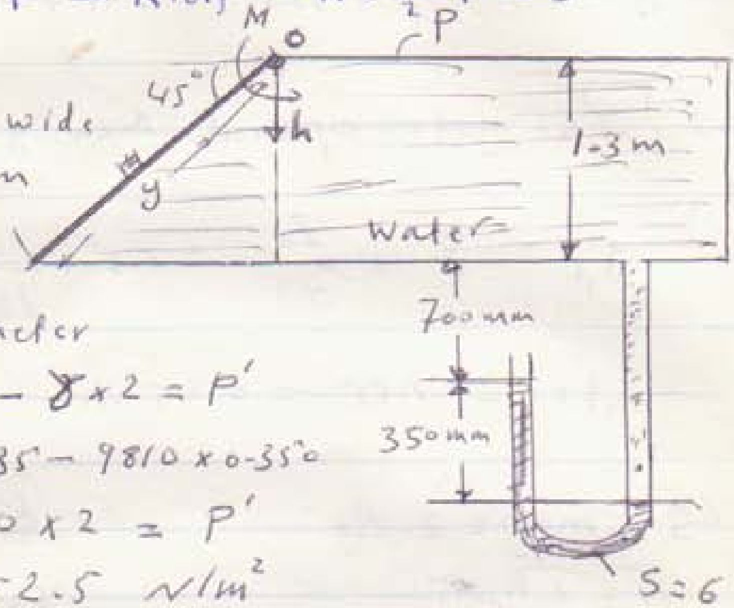
$-\cancel{170} \times 1.855 + \cancel{96} \times 1.166 + R \cdot a_j = 0 \qquad a_j = 2.749$

$170.0136 \times 1.852 - 96.32 \times \cancel{1.166} - F \times 2.667 =$

$+ W \frac{2.667}{2} = \qquad \therefore F = \cancel{89}\ kN.\ 85.84\ kN$

or $F \cdot 2.667 - R \cdot a_j - W \cdot \frac{2.667}{2} = 0$

2-68



from the manometer

$0 + \gamma_s R - \gamma R - \gamma \times 2 = P'$

$0 + 6 \times 9810 \times 0.35 - 9810 \times 0.350$

$- 9810 \times 2 = P'$

$P' = -2452.5\ N/m^2$

$P_a = P + P' = \gamma h + P' = \gamma \frac{y}{\sqrt{2}} - 2452.5$

$=$

$F = \int p\, dA$

$M = \int_A y p\, dA = \int y \left(\gamma \frac{y}{\sqrt{2}} - 2452.5\right) dA$

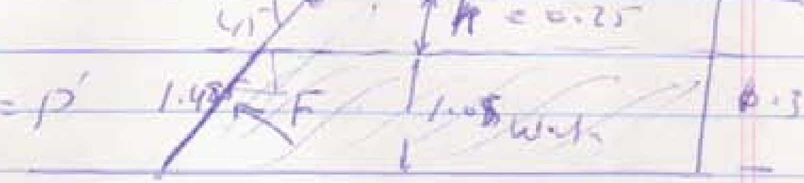
$dA = 2\, dy$

$\therefore M = \int_0^{1.3\sqrt{2}} 2y \left(\gamma \frac{y}{\sqrt{2}} - 2452.5\right) dy$

$= \int_0^{1.3\sqrt{2}} \left(\frac{2 \times 9810}{\sqrt{2}} y^2 - 4905 y\right) dy$

$= \left[\frac{2 \times 9810}{3\sqrt{2}} y^3 - \frac{4905}{2} y^2\right]_0^{1.3\sqrt{2}} = 20.447\ kN \cdot m$

Q.68



$$0 + \gamma_s R - \gamma R - \gamma \times 2 = p'$$

$$p' = -2452.5 \ N/m^2$$
$$h' = -0.25 \, m \quad \text{vacuum}$$

$$F = \gamma \bar{h} A = 9810 \times \frac{1.05}{2} \times (1.485 \times 2)$$
$$= 15.3 \ kN$$

$$y_p = \bar{y} + \frac{I_G}{\bar{y}A}$$

$$\bar{y} \sin\theta = \bar{h}$$
$$\therefore \bar{y} = \frac{1.05}{2\sin\theta} = 0.742 \, m$$

$$\therefore y_p = 0.742 + \frac{2 \times (1.485)^3}{12 \times 0.742 \times 1.485 \times 2} = 0.99 \, m$$

$$M = F\left(y_p + \frac{0.25}{\sin 45}\right)$$
$$= 15.3 \times \left(0.99 + \frac{0.25}{\sin 45}\right)$$

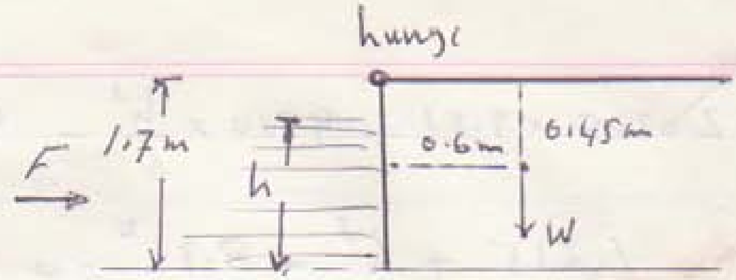
$a = 1.34 \, m$

$$= 20.5 \ kN$$

2-59

$m = 450\ kg$

$W = 450 \times 9.81\ N$



$$F = \gamma \bar{h} A = 9810 \times \frac{h}{2} \times h \times 1 = \frac{9810}{2} h^2$$

$$y_p = \bar{y} + \frac{I_G}{\bar{y}A} = \frac{h}{2} + \frac{1 \times h^3}{\frac{h}{2} \times h \times 1 \times 12} = \frac{2}{3} h$$

$\Sigma M = 0$ about the hinge:

$F \times arm = W \times 0.6$

$arm = 1.7 - \frac{1}{3} h$

$$\therefore\ 9810 \times \frac{h^2}{2} \left(1.7 - \frac{1}{3} h\right) = 450 \times 9.81 \times 0.6$$

$$h^3 - 5.1 h^2 + 1.62 = 0$$

$$h = 0.6\ m$$

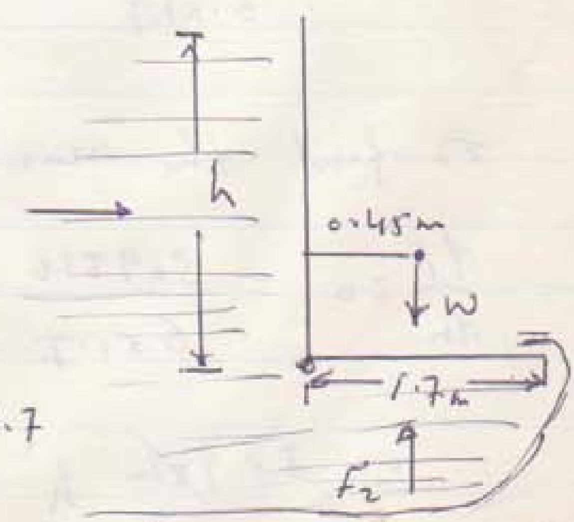
2-60

$W = 450 \times 9.81$

$F_1 = \gamma \bar{h} A = \gamma \frac{h}{2} \times h \times 1$

$= \gamma \frac{h^2}{2}$

$y_{p_1} = \frac{2}{3} h$



$F_2 = \gamma h \times 1.7 \times 1 = \gamma h \times 1.7$

$\Sigma M = 0$ about the hinge

$$W \times 0.45 + F_1 (h - y_p) - F_2 \times \frac{1.7}{2} = 0$$

$$450 \times 9.81 \times 0.45 + \gamma \frac{h^2}{2} \left(h - \frac{2}{3} h\right) - \gamma h \times \frac{1.7^2}{2} = 0$$

$$202.5\times 9.81 + 9810\times\frac{h^3}{6} - 9810\,h^2\times \cancel{1.445} = 0$$

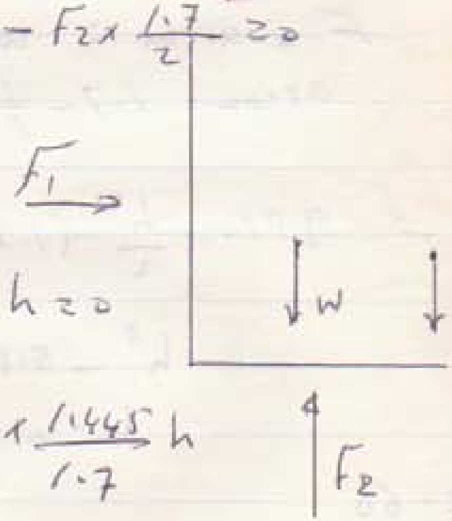
$$1.215 + h^3 - 8.7h^2 = 0$$

$$h^3 - 8.7h^2 + 1.215 = 0$$

$$\therefore\ h = \overset{0.44}{\cancel{2.88}}\ m$$ [illegible] 2.85 m [illegible]

2-61

$$F_1\times\frac{1}{3}h + W\times 0.45 + F\times 1.7 - F_2\times\frac{1.7}{2} = 0$$



$$\frac{9810\,h^3}{6} + 202.5\times 9.81$$

$$+ F\times 1.7 - 9810\times 1.445\,h = 0$$

$$\therefore\ F = \frac{9810}{6\times 1.7}h^3 - 9810\times\frac{1.445}{1.7}h$$

$$+ 202.5\times 9.81$$

To find the max. h for max. F

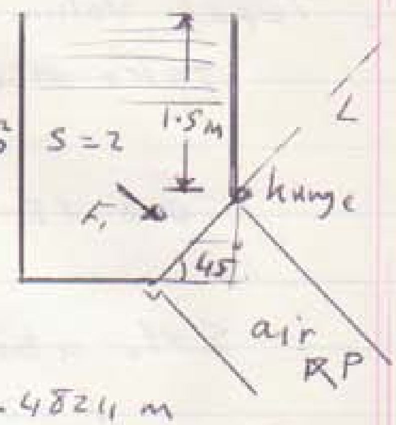
$$\frac{dF}{dh} = 0 = \frac{3\times 9810}{6\times 1.7}h^2 - \frac{9810\times 1.445}{1.7} + 0 = 0$$

$$\therefore\ \frac{\cancel{3}\times \cancel{9810}}{\cancel{6}2\times \cancel{1.7}}h^2 = \frac{\cancel{9810}\times 1.445}{\cancel{1.7}}$$

$$\boxed{h = 1.7\ m}$$

2-71 $D = 700$ mm

$W_{gate} = 1800$ N



$F_1 = \gamma \bar{h} A = 2 \times 9810 \times \left(\frac{0.7 \sin 45}{2} + 1.5 \right) \times \frac{\pi}{4} D^2$

$= 13.195$ kN

$y_p = \bar{y} + \frac{I_c}{\bar{y} A}$ $\quad \bar{h} = \bar{y} \sin\theta$

$\bar{y} = 2.47$ m

$= 2.47 + \frac{\pi r^4}{4 \times 2.47 \times \frac{\pi}{4} \times 0.7^2} = 2.4824$ m

$F_1 \times (y_p - L) + W \times \frac{0.7}{2} \cos 45 - PA \times 0.35 = 0$

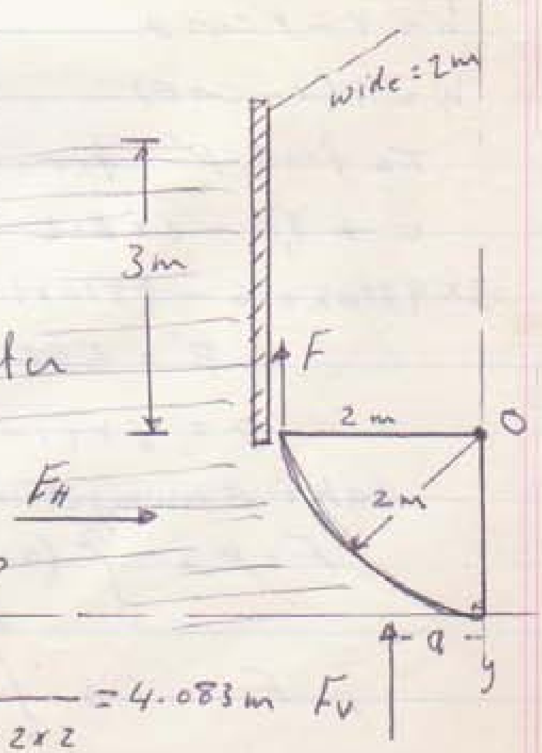
$L \sin\theta = 1.5 \quad \therefore L = 2.12$

$\therefore 13.195 \times 0.3624 + 1.8 \times 0.247$

$5.23 = P \times \frac{\pi}{4} \times 0.7^2 \times 0.35$

$\therefore P = 38.82$ kPa

? 2-78



$F_H = P_a A = \gamma \bar{h} A$

$= 9810 \times 4 \times 2 \times 2$

$= 156.96$ kN

$F_v = \gamma V$

$= 9.81 \left[2 \times \frac{\pi r^2}{4} + r \times 2 \times 3 \right]$

$= 179.358$ kN

To calculate F it need the c.p of the forces.

$y_p = \bar{y} + \frac{I_c}{\bar{y} A} = 4 + \frac{2 \times 2^3}{12 \times 4 \times 2 \times 2} = 4.083$ m

Total volume $= 2\times3\times2 + \pi r^2 \times 2 \times \frac{1}{4} = 18.28\,m^3$

take moments about y-y

$$a \times 18.28 = 12\times1 + 2\pi \times \frac{4r}{3\pi}$$

$$a = 0.948\ m$$

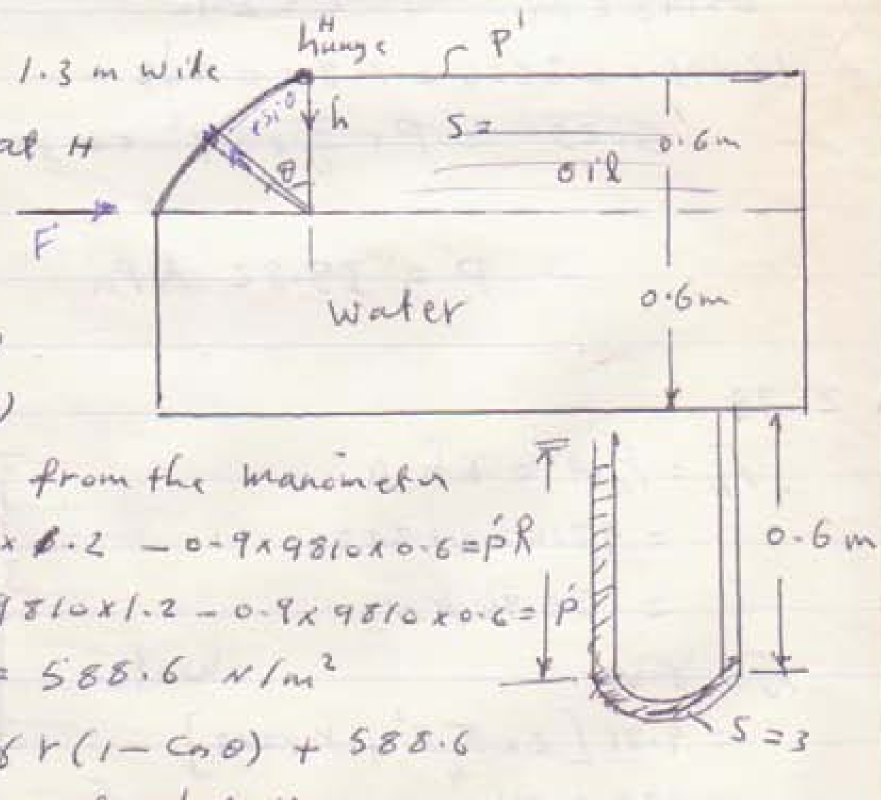
ΣM_o about o

$$F\times2 + F_v \times a = F_H\,(y_p - 3)$$

$$F \approx 0$$

Q.79

1.3 m wide



P' = pressure at H

$dA = r\,d\theta \cdot L$

$p = \gamma h + p'$

$h = r - r\cos\theta$

$h = r(1-\cos\theta)$

To find p' from the manometer

$0 + \gamma_s R - \gamma \times 1.2 - 0.9\times9810\times0.6 = p'$

$3\times9810\times0.6 - 9810\times1.2 - 0.9\times9810\times0.6 = p'$

$\therefore p' = 588.6\ N/m^2$

$\therefore p = \gamma r(1-\cos\theta) + 588.6$

take a moment about H

$$F\times X = \int_0^{\pi/2} (0.9\times9810\times0.6\,(1-\cos\theta) + 588.6)\,r\sin\theta\, r\,d\theta \cdot L$$

$$F = 1.3 \int_0^{\pi/2} (0.9\times9810\times0.6\,(1-\cos\theta) + 588.6)\,0.6\sin\theta\,d\theta$$

since $\sin\theta\cos\theta = \frac{\sin 2\theta}{2}$

$\therefore F = 1.3 \times 0.6 \left[\frac{0.9 \times 9810 \times 0.6}{2} + 588.6\right]$

$F = 2.5251$ kN.

2.80 $R = 45$ cm

$\therefore P' = -3825.9$ N/m²

from qu. 79

$F = Lr\left[\frac{\gamma r}{2} + p'\right)$

$= -918.2$ N

2-81 from Q. 79

$F = Lr\left[\frac{\gamma r}{2} + p'\right]$

$p' = 0 + 3 \times 9810 R - 9810 \times 1.2 - 0.9 \times 9810 \times 0.6$

$= 29430R - 17069.4$

$\therefore F = 1.3 \times 0.6 \left[\frac{0.9 \times 9810 \times 0.6}{2} + 29430R - 17069.4\right]$

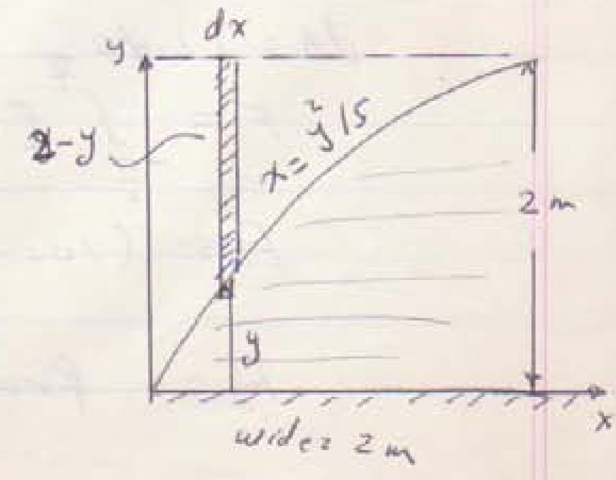
$= 0$

$\therefore R = 0.49$ m

Q.2.82 $x = \frac{y^2}{5}$ $L = 2$

for $y = 2$ $x = 0.8$

$F_v = \gamma L \int_0^{x_0} (2-y)\,dx$



$$F_v = 9 \times 2 \int_0^{0.8} (2 - \sqrt{5}\, x^{1/2})\, dx$$

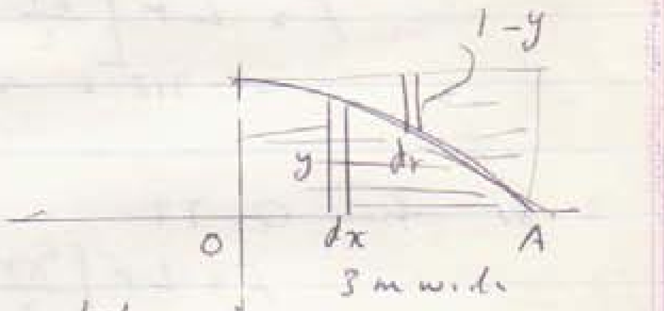
$$F_v = 9.8\ kN$$

$$F_v \bar{x} = \gamma L \int_0^{0.8} (2 - \sqrt{5}\, x^{1/2})\, x\, dx$$

$$\bar{x} = \frac{1}{9.8} \times 9 \times 2 \left[x^2 - \sqrt{5} \times \frac{2}{5} x^{5/2} \right]_0^{0.8}$$

$$\bar{x} = 0.248\ m$$

X Q. 2.83



$$F = \gamma L \int_0^A (1-y)\, dx$$

$$= 9 \times L \int_0^{2\sqrt{2}} \left(1 - \frac{x^2}{8}\right) dx$$

$$F = 9 \times L \left[x - \frac{x^3}{24} \right]_3^{2\sqrt{2}} = 50.92\ kN \qquad y = \frac{x^2}{8}$$

3 m wide

at $y = 1$ $x = 2\sqrt{2}$

$$\cancel{F = 9 \times 3 \left[\frac{x^3}{24} \right]_0 = 25.45\ kN.}$$

2.84

$$F_x = \int_A p \cos\theta\, dA$$

$$dA = r L\, d\theta$$

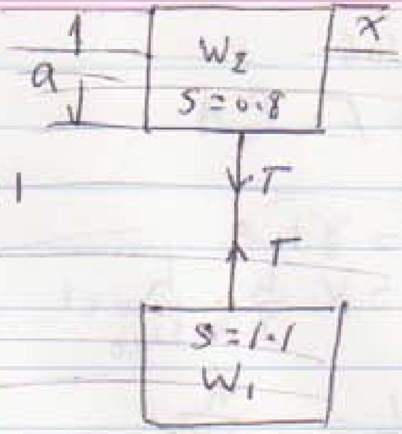
$$F = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[2P(1 - 4\sin^2\theta) + 10 \right] \cos\theta\, r L\, d\theta$$

$$F_x = \left(100 - \frac{20}{3} P\right) r$$

$F_y = 0$ from symetrical عند التعويض في

الدالة ينتج = ٠

2.98



$T + F_{V_1} = W_1$

$T + 1 \times 9810 \times 1 = 1.1 \times 9810 \times 1$

$T = 980\,N$

als. $T + W_2 = F_{V_2}$

$980 + 0.8 \times 9810 \times 1 = 9810 \times a^2 (a - x)$

since $a = 1$

$\therefore x = 0.1\,m = 100\,mm$

2.105

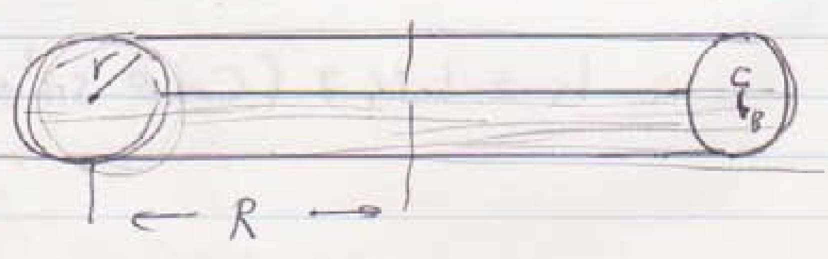
$GB = \frac{4r}{3\pi} = \frac{4 \times 0.3}{3\pi} = 0.127\,m$

$MB = \frac{I}{V} = \frac{\pi (R+r)^4/4 - \pi (R-r)^4/4}{\frac{1}{2}\pi r^2 \cdot 2\pi R}$

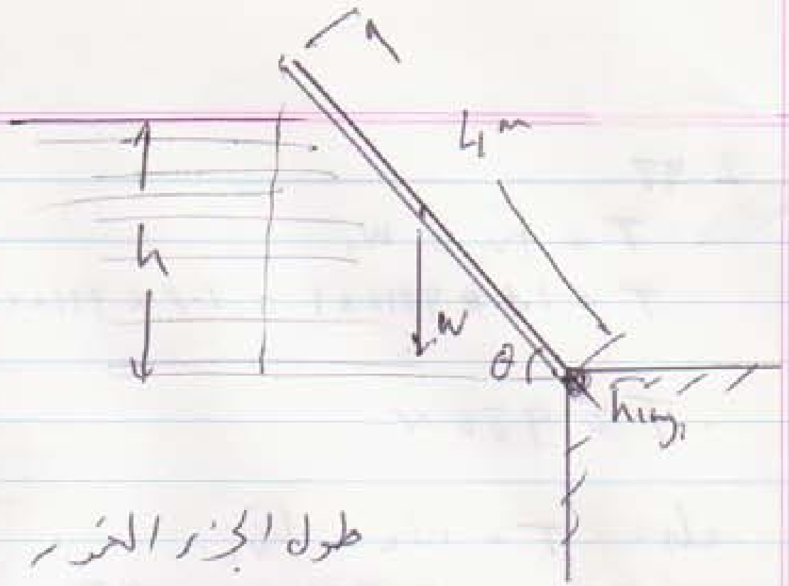
$= \frac{\cancel{(R+r)^2 [(R+r)^2 -}}{\cancel{4\pi R r^2}}$

$= \frac{(1.3)^4 - (0.7)^4}{4 \times \pi \times 1 \times (0.3)^2} = 2.313$

$\therefore MG = MB - GB = 2.186\,m$



2.106



a) $F = P_G A$

$= \gamma \bar{h} A$

$= \gamma \frac{h}{2} \cdot \frac{h}{\sin\theta} \times 1$

$L = \frac{h}{\sin\theta}$ طول الجزء المغمور

$y_p = \bar{y} + \frac{I_G}{\bar{y}A}$ $\bar{y} = \frac{1}{2} \frac{h}{\sin\theta}$ $I_G = \frac{bh^3}{12}$

$\therefore y_p = \frac{h}{2\sin\theta} + \frac{1 \times (h/\sin\theta)^3}{\cancel{12}\,6 \times \frac{h}{\cancel{2}\sin\theta} \times \frac{h}{\sin\theta} \times 1}$

$= \frac{2}{3} \frac{h}{\sin\theta}$

The arm take from $O = \frac{1}{3} \frac{h}{\sin\theta}$

$\therefore \Sigma M$ about $O = 0$

$F \times \frac{1}{3} \frac{h}{\sin\theta} = 2000 \times 2 \times \cos\theta$

$\frac{\gamma h^2}{2\sin\theta} \times \frac{1}{3} \frac{h}{\sin\theta} = 4000 \cos\theta$

$\therefore h = 1.347 (\cos\theta \cdot \sin^2\theta)^{1/3}$

b) from (a) $\Sigma M = \dfrac{\gamma h^3}{6\sin^2\theta} - 4000\cos\theta$

$$\frac{\partial M}{\partial \theta} = -\frac{\gamma h^3}{3}\csc^3\cos\theta + 4000\sin\theta$$

Substituting for h

$$\frac{\partial M}{\partial \theta} = -\frac{\gamma}{3}(2.45\cos\theta\sin^2\theta)\csc^3\theta\cos\theta + 4000\sin\theta$$

$$= -\frac{\gamma}{3}\times 2.45\cos\theta\sin^2\theta \times \frac{1}{\sin^3\theta}\cos\theta + 4000\sin\theta$$

$$= -8000\frac{\cos^2\theta}{\sin\theta} + 4000\sin\theta$$

$$= 4000\left(\sin\theta - \frac{2\cos^2\theta}{\sin\theta}\right)$$

$\dfrac{\partial M}{\partial \theta} < 0$ stable

$$\therefore \sin\theta < \frac{2\cos^2\theta}{\sin\theta}$$

or $\tan^{-1}\sqrt{2} > \theta$ $\quad \therefore \theta = 54.4^\circ$

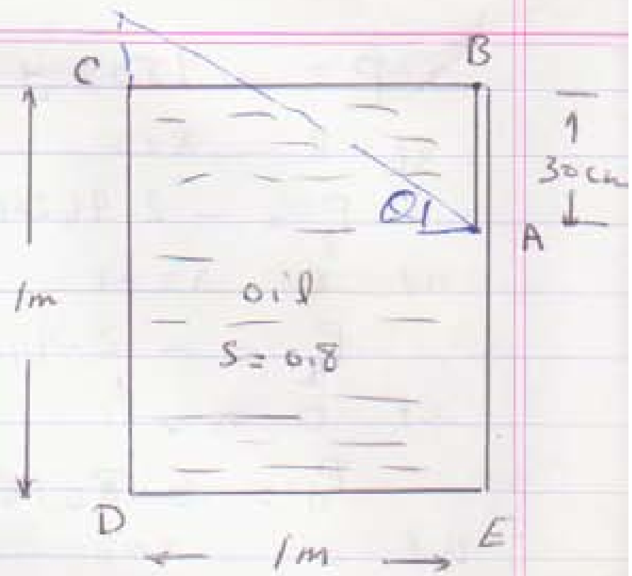
3

imaginary line $\theta = 21.7^\circ$

Q.110

find P_B, P_C, P_D, P_E

$a_x = 3.9\ m/s^2 \quad a_y = 0$



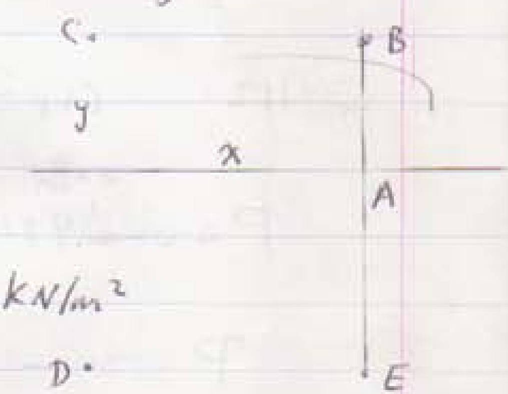
since The tanke filled and

$P_A = P_{atm}$

$$P = P_o - \gamma \frac{a_x}{g} x - \gamma \left(1 + \frac{a_y}{g}\right) y$$

$$\therefore P = 0 - 0.8 \times 9810 \times \frac{3.9}{9.81} x - 0.8 \times 9810 \left(1 + \frac{0}{g}\right) y$$

$$P = -3120\, x - 7848\, y$$



at point B $x = 0$, $y = 0.3$

$$\therefore P_B = 0 - 7848 \times 0.3 = -2.354\ kN/m^2$$

at C $\quad x = -1 \quad y = 0.3$

$\therefore P_C = 0.7656\ kPa$

at D $\quad x = -1 \quad y = -0.7$

$P_D = 8.614\ kPa$

at E $\quad x = 0 \quad y = -0.7$

$P_E = 5.494\ kPa$

Q 2.111 $\quad a_x = 0 \quad a_y = 2.45\ m/s^2$ find P_B, P_C, P_D, P_E

$$\therefore P = 0 - 0.8 \times 9810 \times \frac{0}{9.81} x - 0.8 \times 9810 \left(1 + \frac{2.45}{9.81}\right) y$$

$\therefore P = -9808\,y$

at B $x = 0$ $y = 0.3$

$P_B = -2.9424$ KPa

at C $x = -1$ $y = 0.3$

$P_C = -2.9424$ kPa

at D $x = -1$ $y = -0.7$

$P_D = 6.8656$ KPa

at E $x = 0$ $y = -0.7$

$P_E = 6.8656$ KPa.

Q.112 $a_x = 2.45\,m/s^2$ $a_y = 4.902\,m/s^2$

$$P = 0 - 0.8 \times 9810 \times \frac{2.45}{9.81} \times X - 0.8 \times 9810 \left(1 + \frac{4.902}{9.81}\right) y$$

$$P = -1960\,X - 11772\,y$$

at B $x = 0$ $y = 0.3$

$P_B = -3.532$ KPa

at C $x = -1$ $y = 0.3$ $P_c = -1.572$ kPa

at D $x = -1$ $y = -0.7$ $P_D = 10.2$ kPa

at E $x = 0$ $y = -0.7$ $P_E = 8.24$ kPa

Q. 2.113 find P_A, P_B, P_C

let the origin at E

$\tan\theta = \dfrac{a_x}{a_y+g} = \dfrac{9.806}{0+9.806} = 1$ the water pass point A $\theta > 25°$

$\theta = 45°$ $\tan\theta = \dfrac{h}{x}$ $\therefore x\tan\theta = h$

Volume of space initial = Volume of new space final

$0.3 \times 1.3 \times 1 = x \cdot \dfrac{h}{2} = x \cdot \dfrac{x\tan\theta}{2}$

$\therefore x = 0.883176$ m

$h = 0.883176$ m

also $\tan\theta = \dfrac{h+s}{1.3} = 1$

$\therefore s = 1.3 - 0.883176 = 0.416824$ m

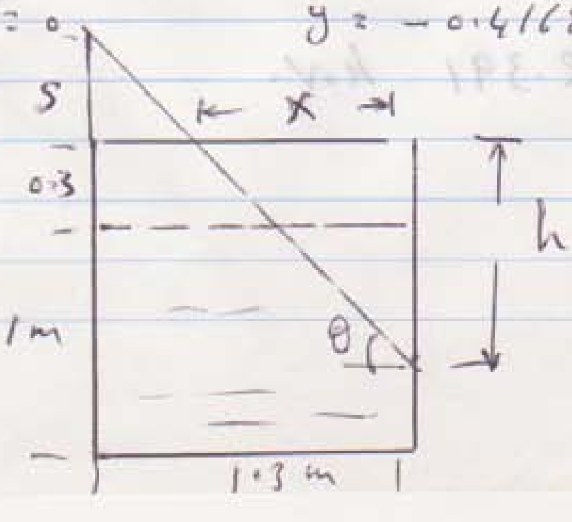
$\therefore P = P_0 - \gamma\dfrac{a_x}{g}x - \gamma\left(1+\dfrac{a_y}{g}\right)y$

$P = -9810x - 9810y$

at A $x = -1.3$ $y = 0.883176$ $\therefore P_A = 4.09$ kPa

at B $x = -1.3$ $y = -0.416824$ $\therefore P_B = 16.842$ kPa

at C $x = 0$ $y = -0.416824$ $P_C = 4.09$ kPa.



2.135

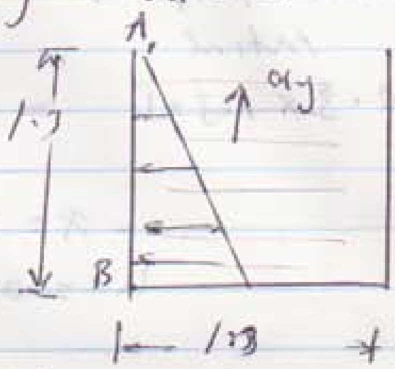
$$P = \gamma\left(1 + \frac{a_y}{g}\right) h \qquad h = -y$$

$$P = 9810\left(1 + \frac{2.45}{9.81}\right) 1.3 = 15.94 \text{ kpa.}$$

$F = \gamma \bar{h} . A$

$F = \left(\frac{P_B + P_A}{2}\right) 1.3 \times 1.3$

$= 13.47$ kN.



2.136 $a_y = 2.45$ m/s² ↓

$$P = 0.65 \times 9810\left(1 - \frac{2.45}{9.81}\right) \times 1$$

$P = 4.7823$ kPa

$$F = \left(\frac{P_A + P_B}{2}\right) 1 \times 1$$

or

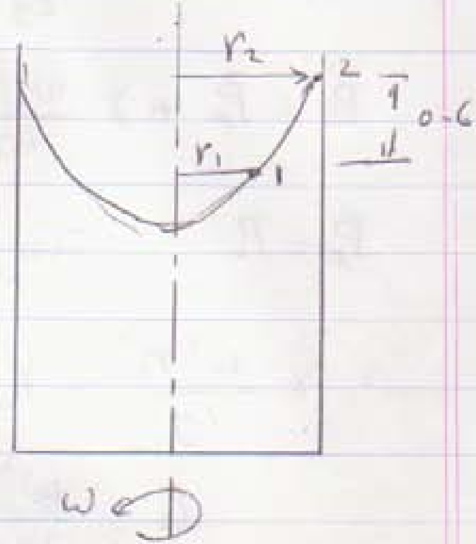
$$F = \gamma \bar{h} A \qquad \bar{h} = \frac{P}{\gamma} + \frac{1}{2}$$

$F = 2.391$ kN.

Q.12.119 $P_x = P_y = P_z$ استقلالية الضغط عن نقطة

120, 121 ش 2

Q.122



$r_1 = 0.6$ m $\quad r_2 = 1.2$ m

$$P_1 = P_o + \gamma \frac{\omega^2 r_1^2}{2g} - \gamma y$$

$$P_2 = P_o + \gamma \frac{\omega^2 r_2^2}{2g} - \gamma (y + 0.6)$$

since $P_1 = P_2$

$$\therefore \gamma \frac{\omega^2 r_1^2}{2g} - \cancel{\gamma y} = \gamma \frac{\omega^2 r_2^2}{2g} - \cancel{\gamma y} - 0.6\gamma$$

$$0.6 = \frac{\omega^2}{2g}\left(r_2^2 - r_1^2\right) = \frac{\omega^2}{2 \times 9.81}\left((1.2)^2 - (0.6)^2\right)$$

$$\omega = 3.3 \text{ rad/s} = \frac{2\pi N}{60}$$

$$\therefore N = 31.5 \text{ rpm}$$

also طريقة

$$h = \frac{\omega^2 r^2}{2g} \qquad \therefore h_1 = \frac{\omega^2 r_1^2}{2g} \qquad h_2 = \frac{\omega^2 r_2^2}{2g}$$

$$h_2 = h_1 + 0.6$$

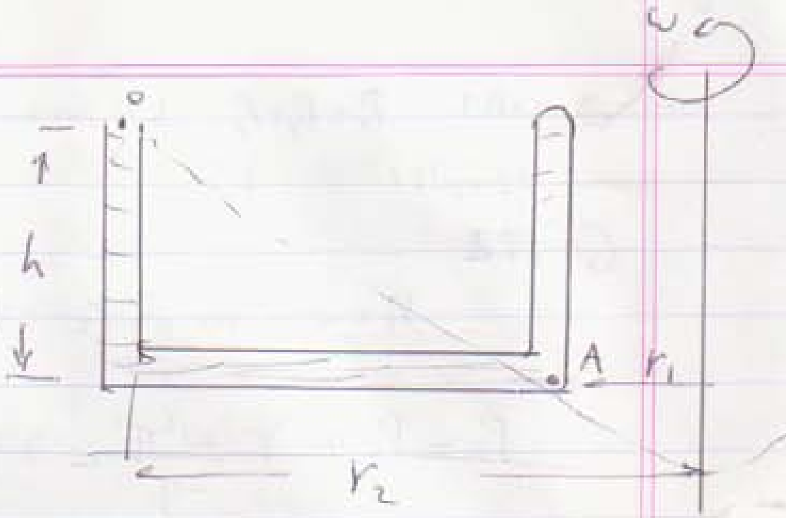
$$\therefore 0.6 = \frac{\omega^2}{2g}\left(r_2^2 - r_1^2\right) \Rightarrow \omega = 3.3 \text{ rad/s}$$

$N = 31.5$ rpm

2.123

$$P_A = P_o + \gamma \frac{\omega^2 r_1^2}{2g} - \gamma y$$

$$P_o = P_o + \gamma \frac{\omega^2 r_2^2}{2g} - \gamma(y+h)$$



$P_A = P_o$ zero gage للاثنين

$$\therefore \cancel{\gamma} \frac{\omega^2 r_1^2}{2g} - \cancel{\gamma y} = \cancel{\gamma} \frac{\omega^2 r_2^2}{2g} - \cancel{\gamma y} - \gamma h$$

$$h = \frac{\omega^2}{2g}\left(r_2^2 - r_1^2\right) = \frac{\omega^2}{2 \times 9.81}\left((0.75)^2 - (0.15)^2\right)$$

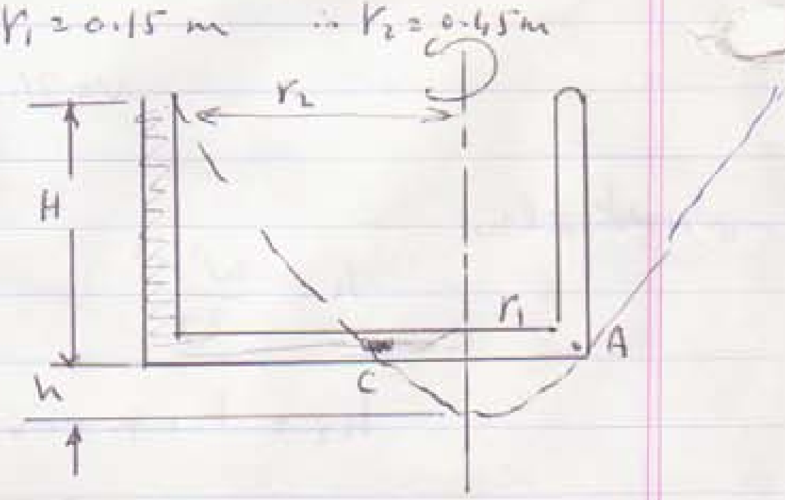
$$\therefore \omega = \sqrt{\frac{0.3 \times 2 \times 9.81}{0.54}} = 3.3 \text{ rad/s}$$

$$= \frac{2\pi N}{60} \qquad \therefore N = 31.5 \text{ rpm}$$

2.124 from symetrical $r_1 = 0.15$ m $\therefore r_2 = 0.45$ m

$$H + h = \frac{\omega^2 r_2^2}{2g} \quad \text{--- (1)}$$

$$h = \frac{\omega^2 r_1^2}{2g} \quad \text{--- (2)}$$



بالطرح

$$H = \frac{\omega^2 (r_2^2 - r_1^2)}{2g}$$

$$\therefore \omega = \sqrt{\frac{0.3 \times 2 \times 9.81}{(0.45)^2 - (0.15)^2}} = 5.7 \text{ rad/s}$$