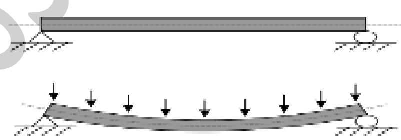
Element of structures

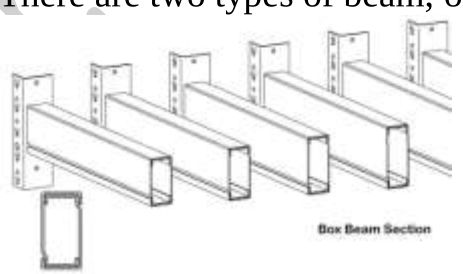
Structural elements are used in structural analysis to split a complex structure into simple elements. Within a structure, an element cannot be broken down (decomposed) into parts of different kinds (e.g., beam or column). Structural elements can be linear, surfaces or volumes. Linear elements:

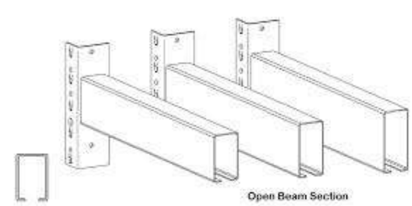
- **Rod - axial loads:**
  Is one of the most basic curvilinear geometric shapes, the surface formed by the points at a fixed distance from a given line segment, the axis of the cylinder. The solid enclosed by this surface and by two planes perpendicular to the axis is also called a cylinder.

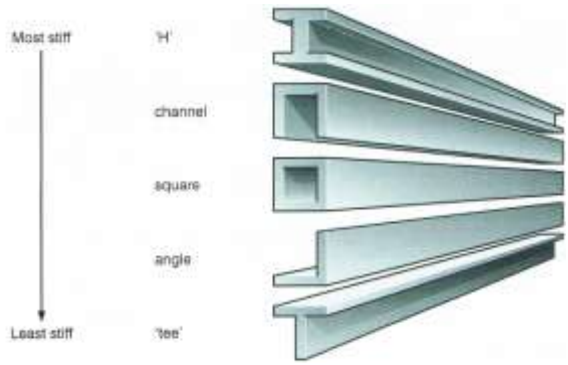
- **Beam - axial and bending loads:**
  A beam is a structural element that is capable of withstanding load primarily by resisting bending. The bending force induced into the material of the beam as a result of the external loads, own weight, span and external reactions to these loads is called a bending moment.



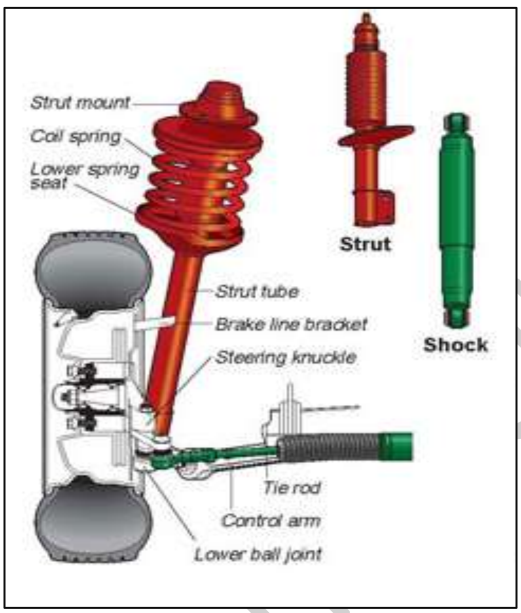
There are two types of beam; open and close beam

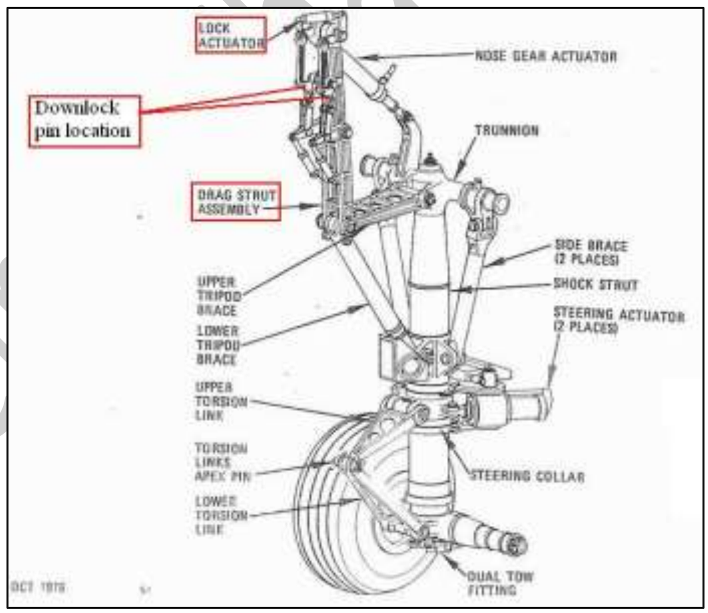






- **Struts or Compression members- compressive loads:**
  A strut is a structural component designed to resist longitudinal compression. Use in landing gear.

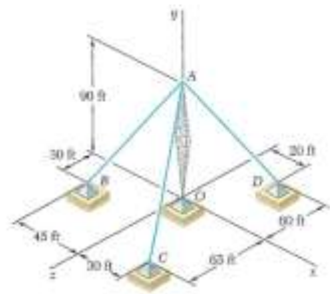




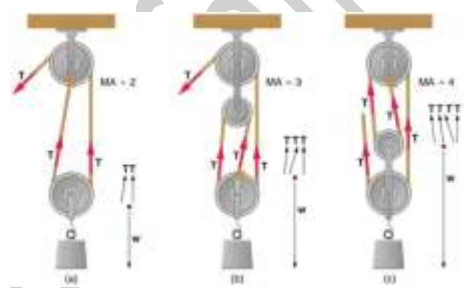
- **Column :**
  A large proportion of an aircraft's structure comprises thin webs stiffened by slender longerons or stringers; both are susceptible to failure by buckling at a buckling stress or critical stress, which is frequently below the limit of proportionality and seldom appreciably above the yield stress of the material.

- **Ties, Tie rods, eyebars, guy-wires, suspension cables, or wire ropes-tension loads:**
  A tie, structural tie, or strap, is a structural component designed to resist tension. It is the opposite of a strut or column, which is designed to resist compression.
  Eyebar (is a straight bar, usually of metal, with a hole ("eye") at each end for fixing to other components).
  Guy-wires (also known as simply a guy, is a tensioned cable designed to add stability to a free-standing structure).

Eyebar



Guy wires



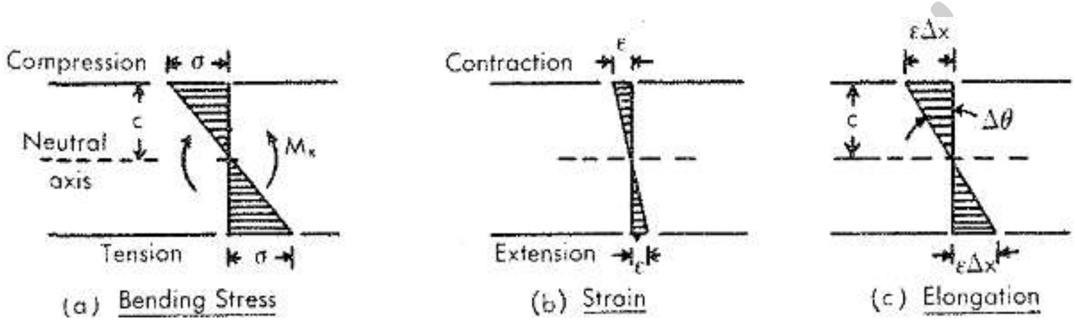
Ropes- Tension

- Surface elements:
  - **Membrane** – has load in-plane loads only.
  - **Shell** – has load in plane and bending moments:
    are light weight constructions using shell elements. These elements are typically curved and are assembled to large structures (skin in aircraft).

General type of load applied on structure and stresses

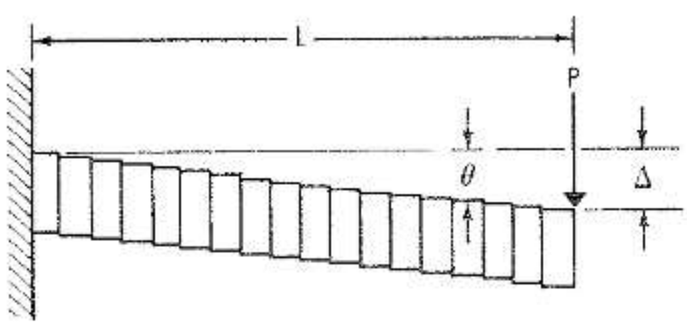
- Tensile stress
  The simplest type of loading on a member is tension. A tensile load applied (axially) in line with the center of gravity of the section will result in tensile stress (tension stress) distributed uniformly across the plane of the cross section lying at right angles to the line of loading.
- Bending stress
  Any force applied transversely to the structural axis of a partially supported member sets up bending moments along the length of

the member. These in turn stress the cross sections in bending. As shown in figure below, the bending stresses are zero at the neutral axis, and are assumed to increase linearly to a maximum at the outer fiber of the section. The fibers stressed in tension elongate, the fibers stressed in compression contract. This cause each section so stressed to rotate. The cumulative effect of this movement is an overall deflection (or bending) of the member.



- Shear stress

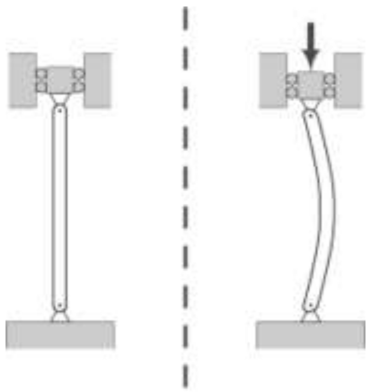
  Shear stress is presented when the load applied parallel to the area, or due to bending, (A shear load is a force that tends to produce a sliding failure on a material along a plane that is parallel to the direction of the force).

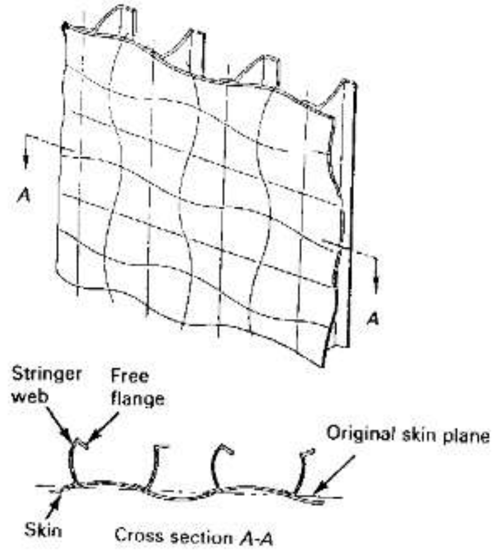


Shear deflection of cantilever beam with concentrated load

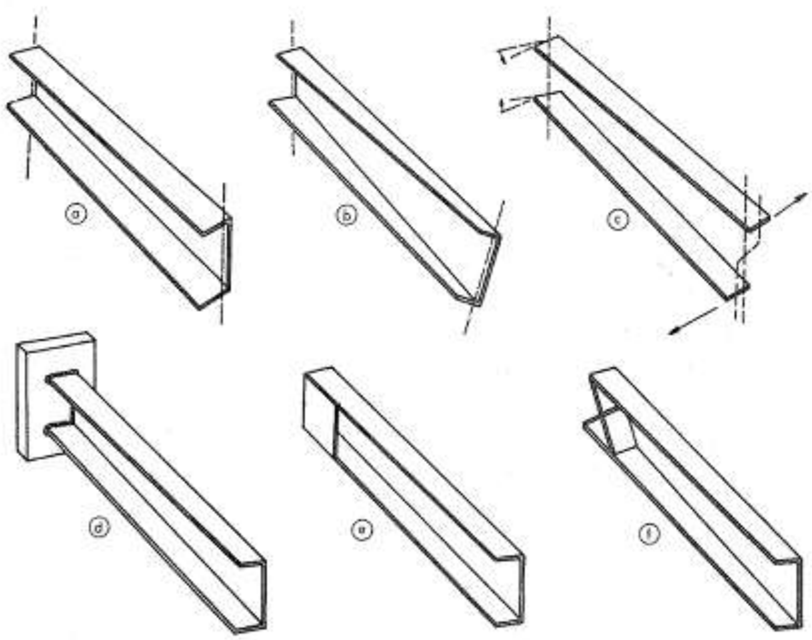
- Buckling and stability of structures

  Buckling is characterized by a sudden failure of a structural member subjected to high compressive stress, where the actual compressive stress at the point of failure is less than the ultimate compressive stresses that the material is capable of withstanding





- Torsion (twist or warp)
  Torsional loading is the application of a force that tends to cause the member to twist about its structural axis. The principal deflection caused by torsion is measured by the angle of twist, or by the vertical movement of one corner of the frame.



- Fatigue

  Fatigue is the weakening of a material caused by repeatedly applied loads. It is the progressive and localized structural damage that occurs when a material is subjected to cyclic loading. The nominal maximum stress values that cause such damage may be much less than the strength of the material typically quoted as the ultimate tensile stress limit, or the yield stress limit.

Type of load on aircraft

The structure of an aircraft is required to support two distinct classes of load: the first, termed ***ground loads***, includes all loads encountered by the aircraft during movement or transportation on the ground such as taxiing and landing loads, towing and hoisting loads as shown in Figure 1; while the second, ***air loads***, comprises loads imposed on the structure during flight by maneuvers and gusts. In addition, aircraft designed for a particular role encounter loads peculiar to their sphere of operation. Carrier born aircraft, for instance, are subjected to catapult take-off and arrested landing loads: most large civil and practically all military aircraft have pressurized cabins for high altitude flying; amphibious aircraft must be capable of landing on water and aircraft designed to fly at high speed at low altitude, e.g. the Tornado, require a structure of above average strength to withstand the effects of flight in extremely turbulent air.

The two classes of loads may be further divided into surface forces which act upon the surface of the structure, e.g. aerodynamic and hydrostatic pressure, and body forces which act over the volume of the structure and are produced by gravitational and inertial effects.

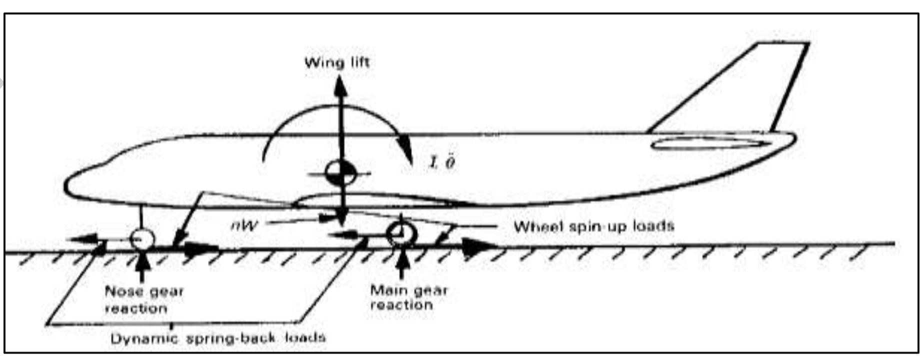
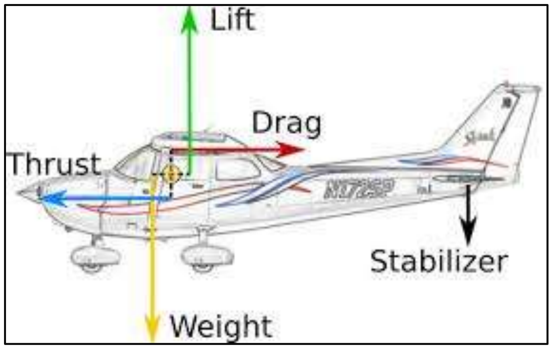


Fig. 1 Ground load



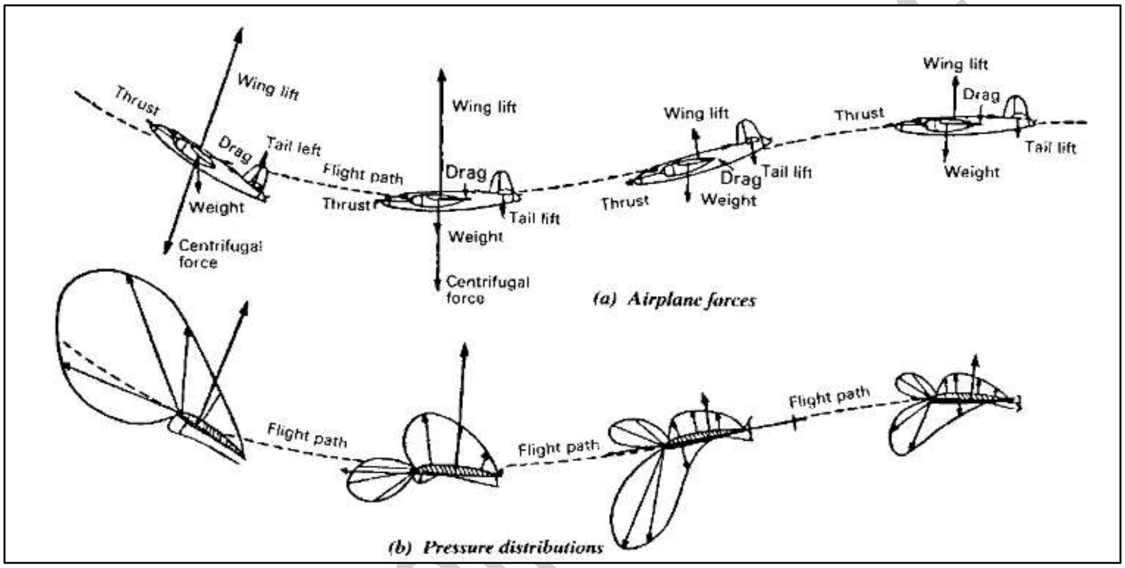


Fig. 2 Air load

Load factor

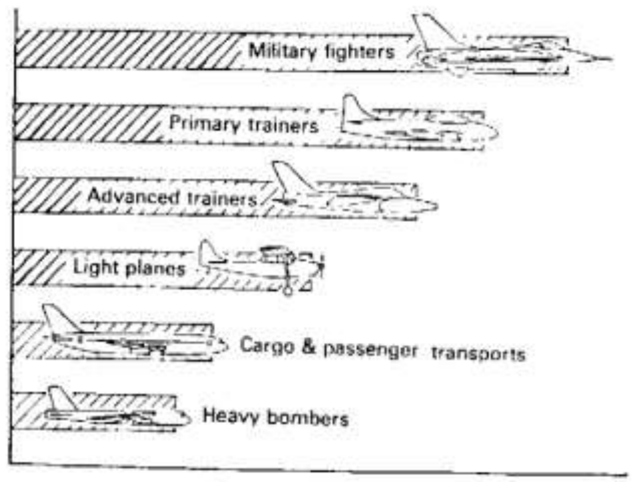
The airworthiness of an aircraft is concerned with the standards of safety incorporated in all aspects of its construction. These range from structural strength to the provision of certain safeguards in the event of crash landings, and include design requirements relating to aerodynamics, performance and electrical and hydraulic systems. The selection of minimum standards of safety is largely the concern of “national and international” airworthiness authorities who prepare handbooks of official requirements. The handbooks include operational requirements, minimum safety requirements, recommended practices and design data, etc.

In normal straight and level flight the wing lift supports the weight of the airplane. During maneuvers or flight through turbulent (gusty) air, however, additional loads are imposed which will increase or decrease the net loads on the airplane structure. The amount of additional load depends on the severity of the maneuvers or the turbulence, and its magnitude is measured terms of load factor. Load factor is a multiplying factor which defines a load in terms of weight.

$$\text{Lift} = nW = \frac{1}{2}\rho V^2 S\, C_{L.max}$$

n=load factor , W=gross weight , $\rho =$ density, V= air velocity, S=area of wing, $C_{L.max} = lift$ coefficient

The maximum maneuvering load factor to which an airplane is designed on its intended usage. Fighters, which are expected to execute violent maneuvers, are designed to withstand loads commensurate with acceleration a pilot can physically withstand. Long range, heavily loaded bombers, on the other hand, are designed to low load factors and must be handled accordingly. The magnitude to be used for design are specified by the licensing or procuring agencies in their specifications. Transports have low values of load factor (2 to 3). While fighters, Are designed to higher values (6 to 8).



Design load factor.

Materials

Several factors influence the selection of the structural material for an aircraft, but amongst this strength allied to lightness is probably the most important. Other properties having varying, though sometimes critical significance are stiffness, toughness, resistance to corrosion, fatigue and the effects of environmental heating, ease of fabrication, availability and consistency of supply and, not least important, cost. The main groups of materials used in aircraft construction have been wood, steel, aluminum alloys with, more recently, titanium alloys, and fiber-reinforced composites.

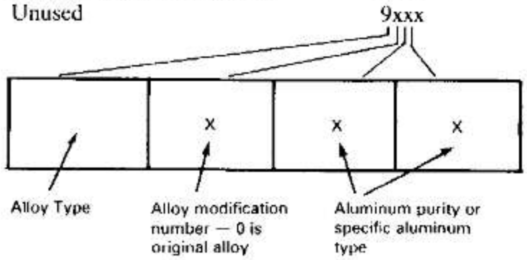
Aluminum

Pure aluminum is a relatively low strength extremely flexible metal with virtually no structural applications. However, when alloyed with other metals its properties are improved significantly. Three groups of aluminum alloy have been used in the aircraft industry for many years and still play a major role in aircraft construction.

For pressurized cabins and lower wing skins two areas particulary prone to fatigue through the long continued application and relaxiation of tension stresses. The standard material (for commercial transports , entire fuselages of which are pressurized) is an aluminium alloy designated 2024-T3. For upper wing skins, which have to withstand mainly compression stresses the wing flexes upwards during flight, 7075-T6 is used. This alloy is also used extensively for military aircraft structures, which generally have stiffer wings and expect for the cockpit area un pressurized fuselages.

Designations For Alloying Elements

Aluminum 99.00% Min.	1xxx
Aluminum-Copper	2xxx
Aluminum-Manganese	3xxx
Aluminum-Silicon	4xxx
Aluminum-Magnesium	5xxx
Aluminum-Magnesium-Silicon	6xxx
Aluminum-Zinc	7xxx
Aluminum-Other Elements	8xxx
Unused	9xxx



Heat Treat Conditions

F: As fabricated, usually forgings where machining will be done prior to heat treatment. Castings are also included. There is no guarantee on properties.

O: Annealed — softest condition for forming.

W: Solution heat treated, unstable condition occuring after quenching, forming possible in this condition similar to O.

T: Heat treated to stable tempers other than O or F.

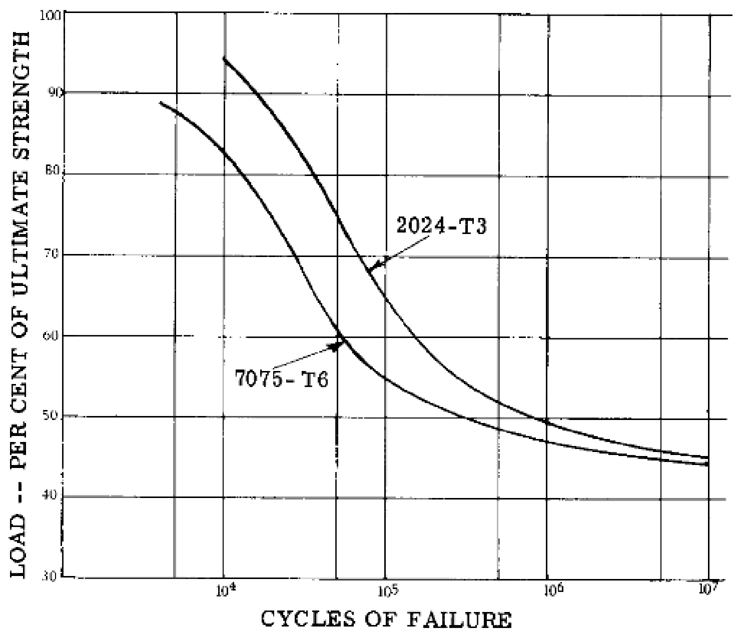
- T3 — Solution heat treated, cold worked and naturally aged (usually sheet or plate)
- T4 — Solution heat treated and naturally aged
- T6 — Solution heat treated and artificially aged
- T7 — Solution heat treated and overaged
- T8 — Solution heat treated, cold worked and artificially aged

One or more numbers may appear after the heat treat number — T7xx. These show a stress relieving operation or the extent of aging or overaging.

For example, 7075 — T7351 is overaged aluminum-zinc alloy that has been stress relieved by stretching.

Fig. 4.3.3—Fig. 4.3.10 show mechanical properties of a few selected aluminum alloys.

Material	Recommended Application
2024-T3, T42, T351, T81	Use for high strength tension application; has best fracture toughness and slow crack growth rate and good fatigue life. -T42 has lower strength than -T3. Thick plate has low short transverse properties and low stress corrosion resistance. Use -T81 for high temperature applications.
2224-T3 2324-T3	8% improvement strength over 2024-T3; fatigue and toughness better than 2024-T3.
7075-T6, T651, T 7351	Has higher strength than 2024, lower fracture toughness, use for tension applications where fatigue is not critical. Thick plate has low short transverse properties and low stress corrosion resistance (-T6). -T7351 has excellent stress corrosion resistance and better fracture toughness.
7079-T6	Similar to 7075 but has better thick section (> 3 in) properties than 7075. Fracture toughness, between 7075 and 2024. Thick plate has low stress corrosion resistance.
7150-T6	11% improvement strength over 7075-T6. Fatigue and toughness better than 7075-T6.
7178-T6, T651	Use for compression application. Has higher strength than 7075, lower fracture toughness and fatigue life.
Aluminum-Lithium	Compared to conventional aluminum alloys: 10% lighter, 10% stiffer, and superior fatigue performance.
PM Aluminum	Compared to conventional aluminum alloys: Higher strength, good fatigue life, good toughness, higher temperature capability and superior corrosion resistance.



Steel

The use of steel for the manufacture of thin-walled, box-section spars in the 1930s has been superseded by the aluminum alloys.

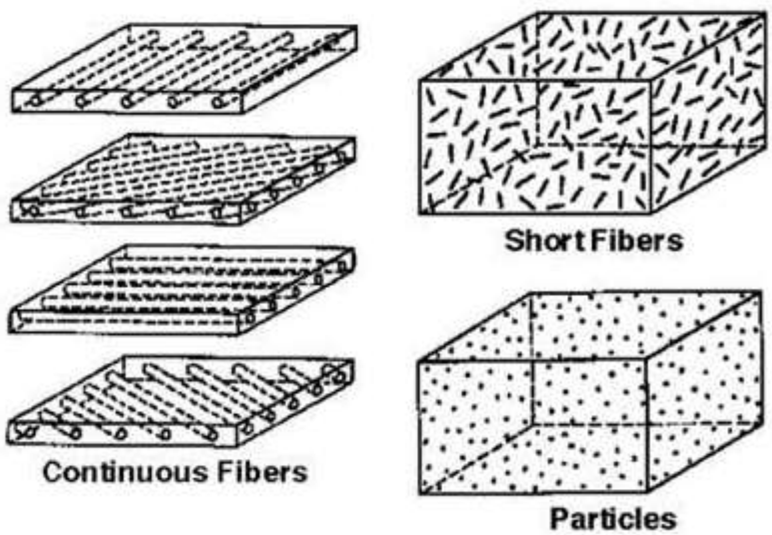
Clearly, its high specific gravity prevents its widespread use in aircraft construction, but it has retained some value as a material for castings for small components demanding high tensile strengths, high stiffness and high resistance to wear. Such components include undercarriage pivot brackets, wing-root attachments, fasteners and tracks.

Titanium

The use of titanium alloys increased significantly in the 1980s, particularly in the construction of combat aircraft as opposed to transport aircraft. This increase continued in the 1990s to the stage where, for combat aircraft, the percentage of titanium alloy as a fraction of structural weight is of the same order as that of aluminum alloy. Titanium alloys possess high specific properties, have a good fatigue strength/tensile strength ratio with a distinct fatigue limit, and some retain considerable strength at temperatures up to 400–500 Co . Generally, there is also a good resistance to corrosion and corrosion fatigue although properties are adversely affected by exposure to temperature and stress in a salt environment. The latter poses particular problems in the engines of carrier operated aircraft. Further disadvantages are a relatively high density so that weight penalties are imposed if the alloy is extensively used, coupled with high primary and high fabrication costs, approximately seven times those of aluminum and steel.

Composite materials

Composite materials consist of strong fibres such as glass or carbon set in a matrix of plastic or epoxy resin, which is mechanically and chemically protective. The fibres may be continuous or discontinuous but possess a strength very much greater than that of the same bulk materials. A sheet of fibre-reinforced material is anisotropic, i.e. its properties depend on the direction of the fibres. Generally, therefore, in structural form two or more sheets are sandwiched together to form a lay-up so that the fibre directions match those of the major loads.



Wing construction

The wing is intended to create lift force. Besides the lateral stability and controllability as well as perform additional functions: it carries the engines, landing gear accommodates the fuel tanks, armament, etc. the wing amount 7-16% of the aircraft weight and it affords a considerable of its total drag.

Wing layout

Selection of the external shape of the wing (profile, view, front view, angle of wing setting, etc.) is mainly designed according to the aerodynamics needs. The modern of aircraft use the swept wings and short wings with thin and symmetrical profiles which permit to increase considerably the critical Mach number of flight and to decrease the drag of the wing at high speeds.

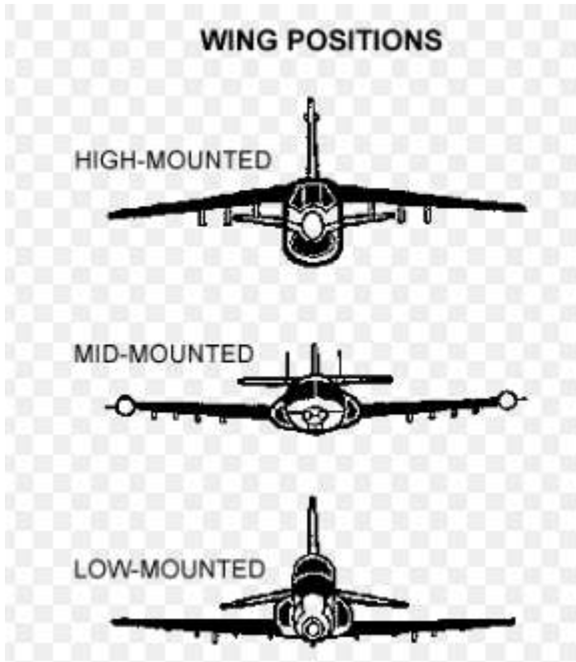
Wing (mounted)

The wing locations height wise of the fuselage are; (high- mounted , mid- mounted, low- mounted). These type of mounted determined by the design considerations.

The mid wing has the minimum interference between the wing and the fuselage and is widely used in modern aircraft.

The low-wing is less suitable owing to harmfully effect of interference. However, such arrangement of the wing is convenient for accommodation of the landing gear and safer in the sliding crash for passenger aircraft.

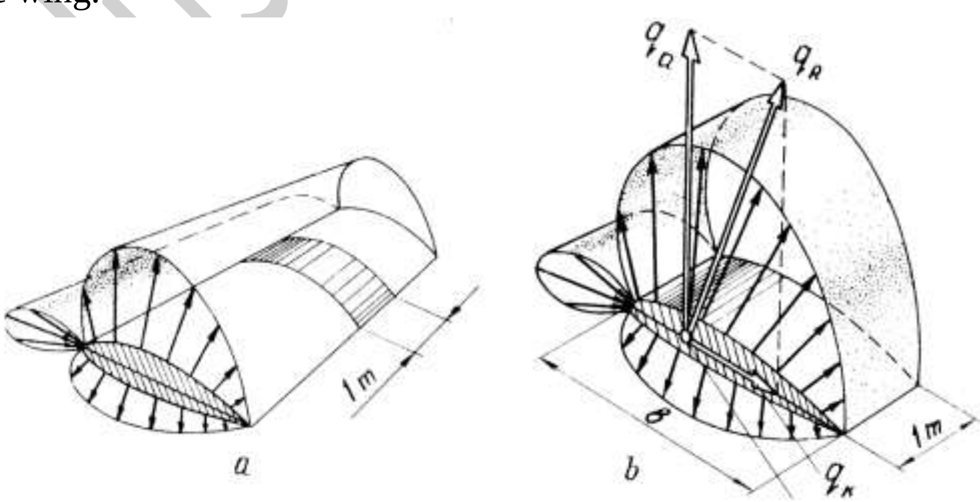
The high- wing is used usually for Para-trooping and cargo planes. The advantages of such configuration; the power plant located on the wing are considerably separated from the ground, which is very important for turboprop engine with large diameter propellers; high-wing is more advantageous than the low-wing position as regards interference. The disadvantage of high mounted are; impossibility to accommodate the landing gear and increased lateral stability of the aircraft which may cause oscillation instability.



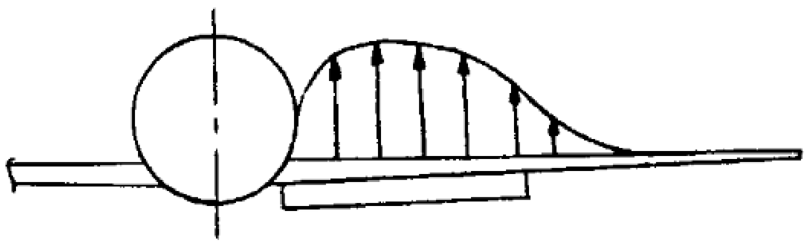
Loads acting on wing

Flight load

The aerodynamic (air) load (P) is distributed along the wing according to a fairly complex pattern. Its direction and value in flight vary and depend on the flight conditions. The required strength of the wing is determined by not absolute value of the lift force but also by the nature of its distribution. The distribution load (pressure) converted to concentrated load with two components applied along the centers of pressures of the wing.

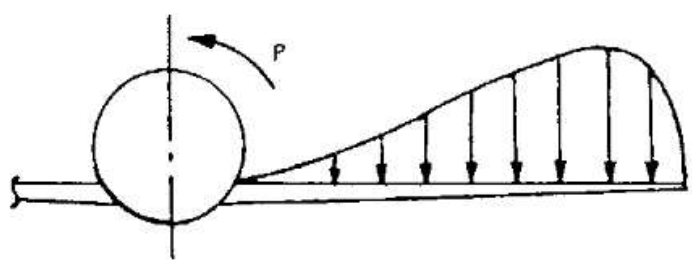


The wing flaps, slats, ailerons, spoilers, or other high lift devices, additional flight loading conditions must be investigated for these control surfaces extended. These, conditions are usually not critical for wing bending stresses but may be critical for wing torsion, shear in the rear spar, or down tail loads.



Load distribution from flap

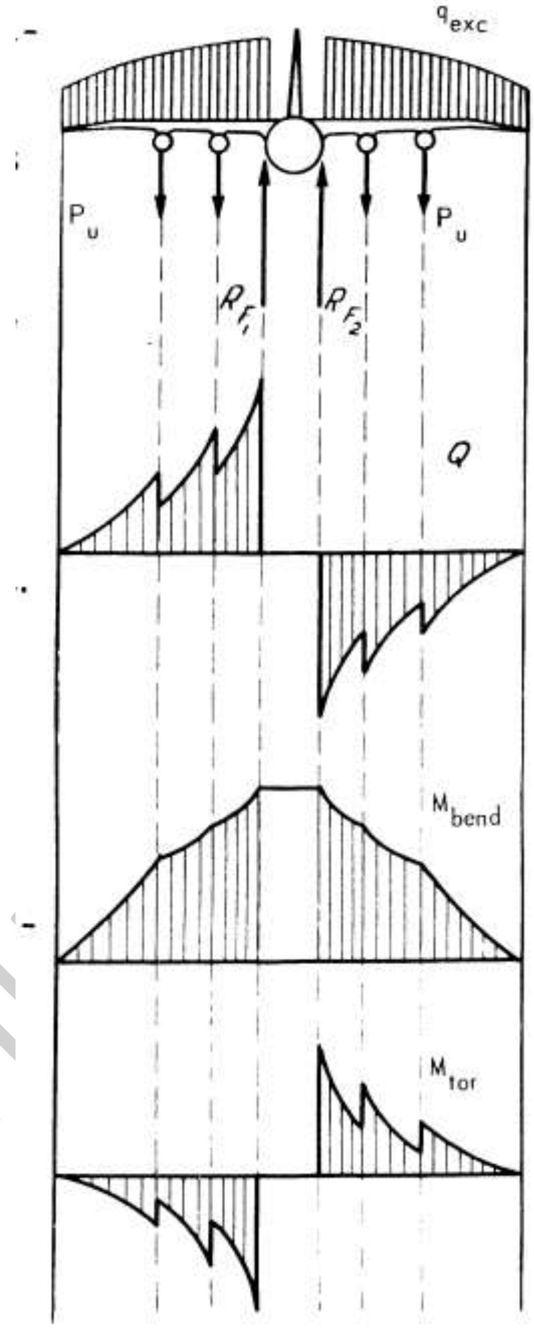
Load distribution from aileron



Damping load distribution on wing span due to rolling.

Concentrated mass forces

From the units located on the wing , these forces are applied to the center of gravity of the units and are considered to be directed perpendicularly to the plane of the chords.

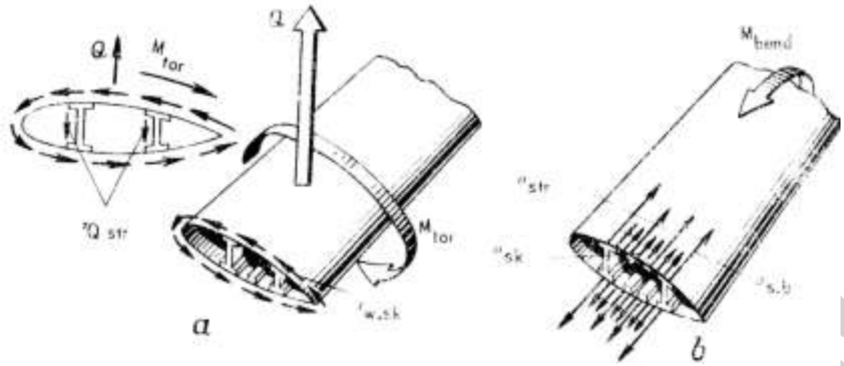


Shear

Bending moment

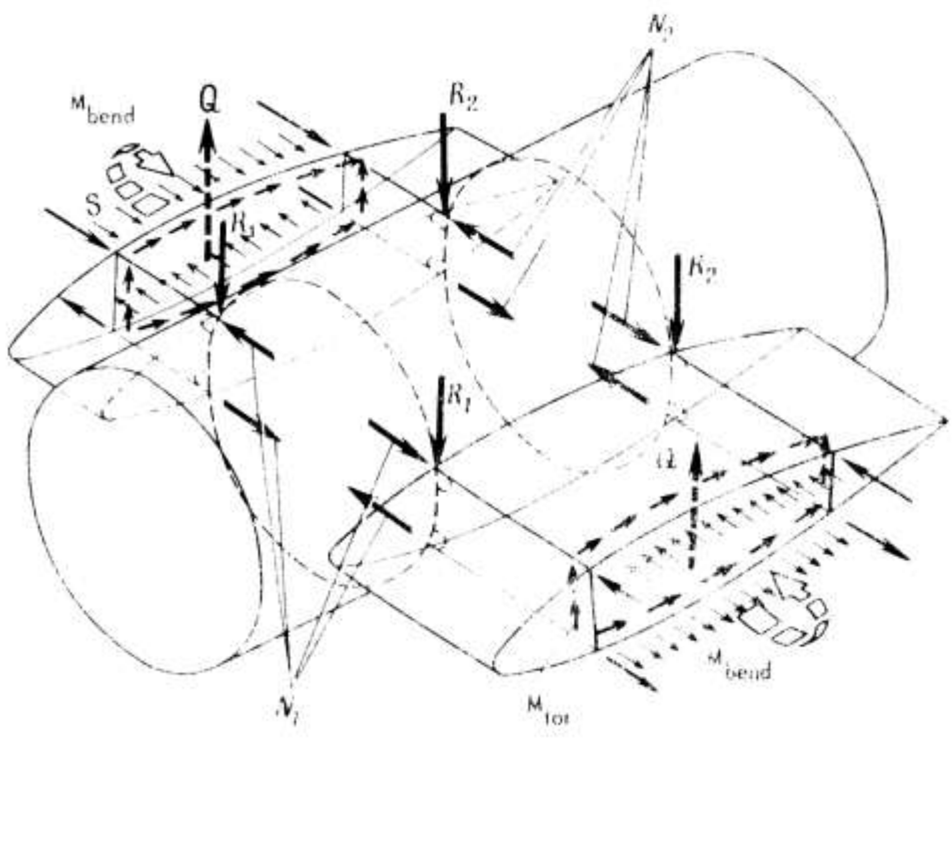
Torsion

These two types of loads caused bending, torsion and shear force in the wing as shown below



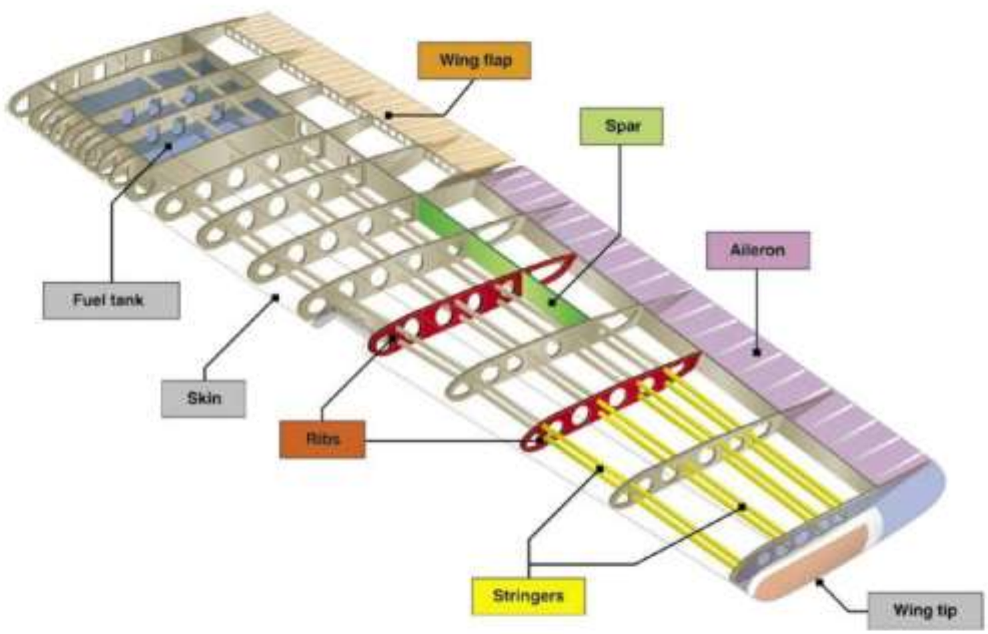
a- Shearing and torsion b- bending

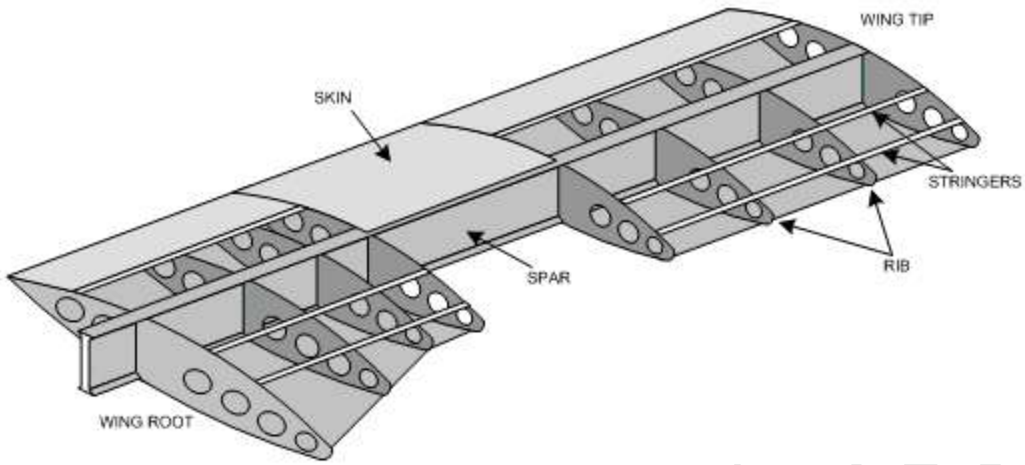
All the loads acting on the wing (air, mass and mass caused by the units) are balanced on the wing to fuselage attachment units.



The wing construct from the following main parts:

1- Spar.
2- Stringers.
3- Ribs.
4- Skin.
5- Moving surfaces.





1-Spar

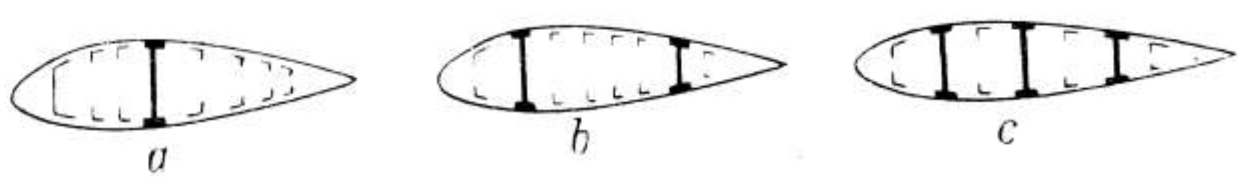
It is used to transmit loads from wing structure to fuselage structure, and it has other secondary jobs:

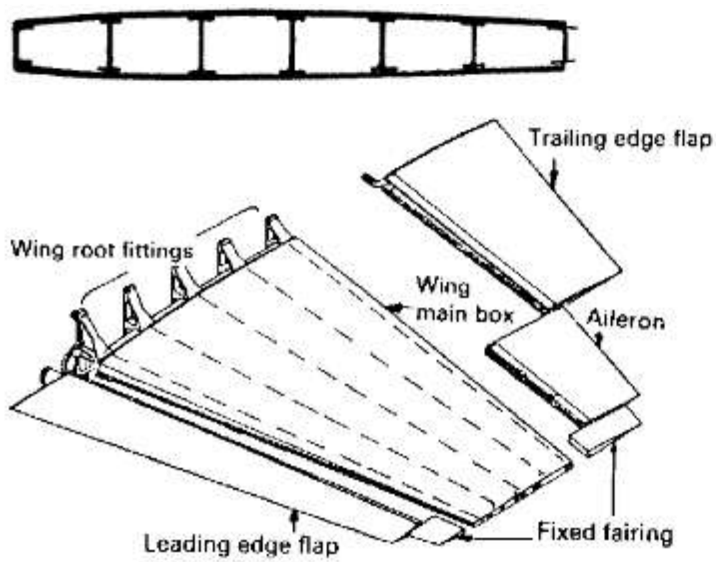
1. Support wing skin.
2. Spar webs hold shear and torsion loads.
3. Spar flanges (cops) hold axial and bending loads.
4. To hold loads due to engine, fuel tanks …etc. that attached to the wing.

Usually spars are made of Aluminum Alloys.

According to the aspect- ratio, there are several types of box wing structure for modern high speed airplanes:

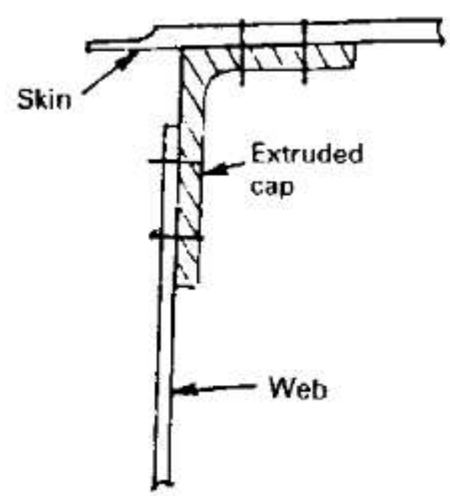
a- Single spar (high aspect ratio).
b- Two spar (one cell) (high aspect ratio).
c- Three spars (multi cell) (high aspect ratio).
d- Multi spars (lower aspect ratio)





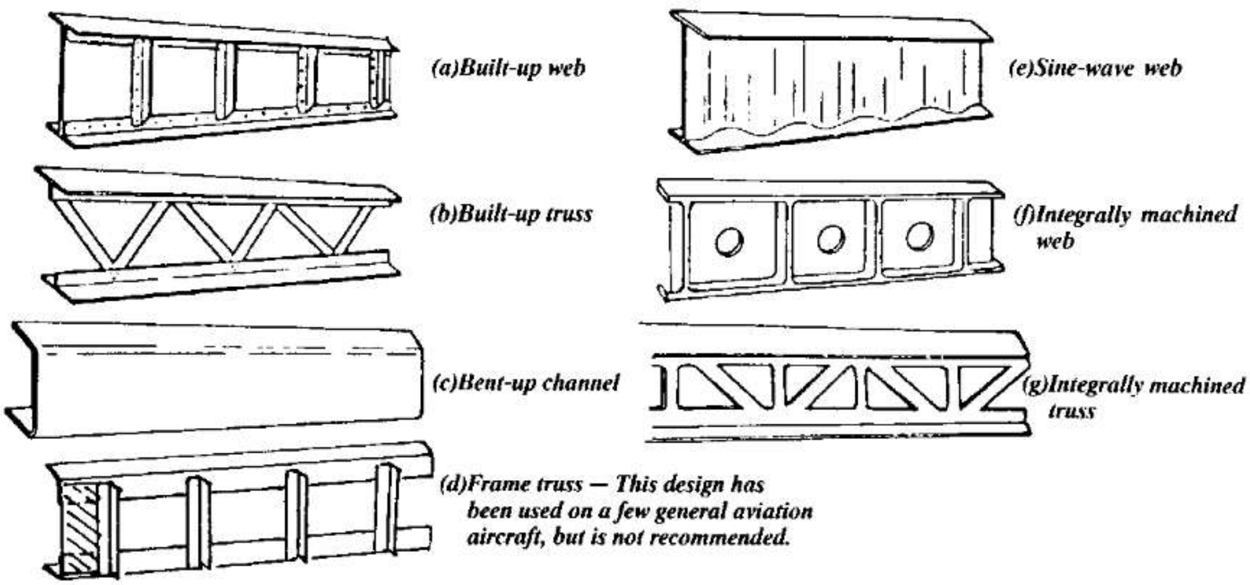
d- Multi spars

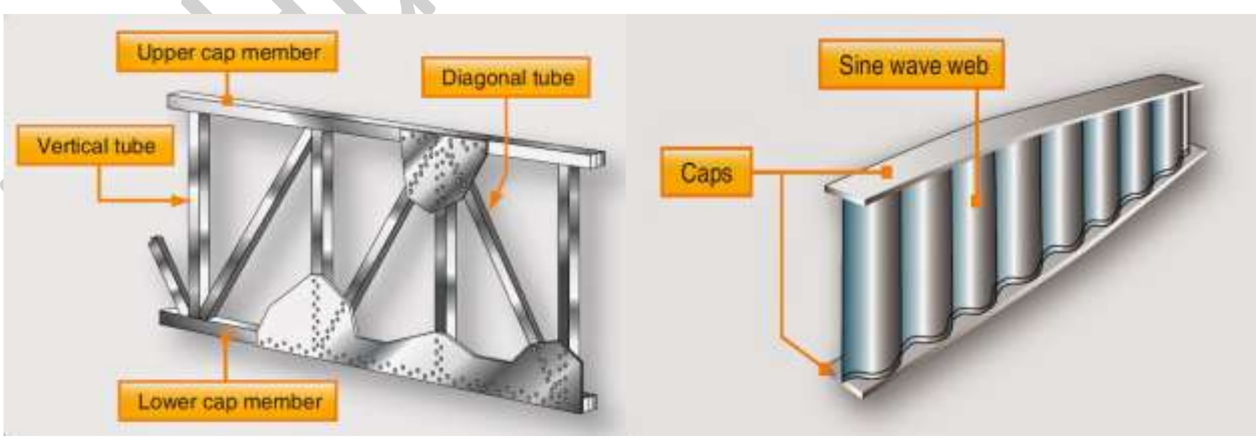
For strength / weight efficiency, the beam (or spar) cap should be designed to make the radius of gyration (second moment of area)of the beam section as large as possible and at the same time maintain a cap section which will have a high local crippling stress. The cap sections for large cantilever beams which are frequently used in wing design should be of such shape as to permit efficient tapering or reducing of the section as the beam extend outboard. Figure below shows typical beam cap sections for cantilever metal wing cover constructions where additional stringers and skins are also used to provide bending resistance.



The air load act directly on the wing skin which transmits the load s to the ribs. The ribs transmit the load in shear to the spar webs. The using of several spars permit a reduction in rib stresses and also provides a better support for the spanwise bending material.

Different types of spar beam construction are shown below . the spars can be divided into two basic types; shear web type and truss type. The shear type is widely used than the truss type .

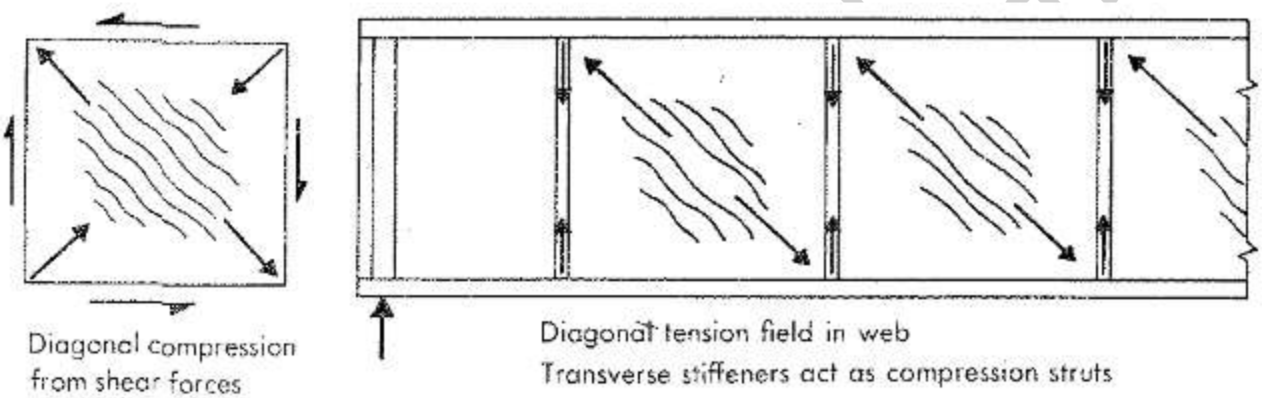




Typical spar configurations

Transverse intermediate stiffeners:

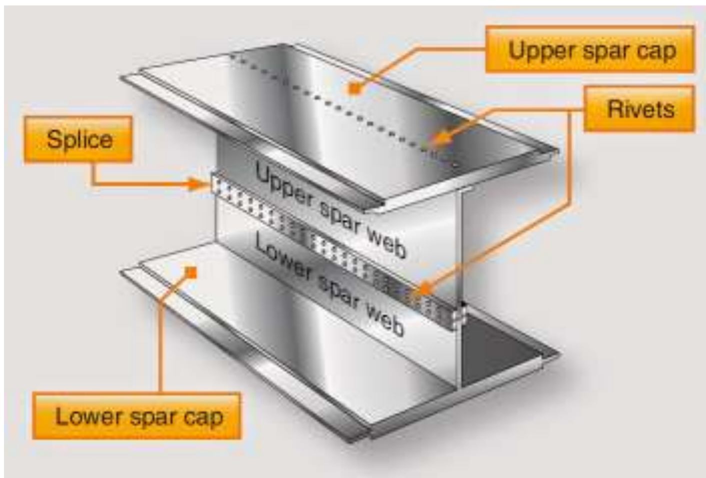
load applied to beams cause bending moments along the length of the member. When these moments are non-uniform along the length of the member. both horizontal and vertical shear stress are set up because shear is equal to the rate of change of moment. The horizontal shear forces would cause the flange (cap) of the spar (beam) to slide past the web. These horizontal and vertical shear stresses combined and produce both diagonal tension and compression, each at 45^o to the shear stresses. Generally in metal structure, tension is not the problem; however, the diagonal compression could be high enough to cause the web to buckle. Stiffeners are used to prevent the web from buckling in regions of high shear stress.



Splice:

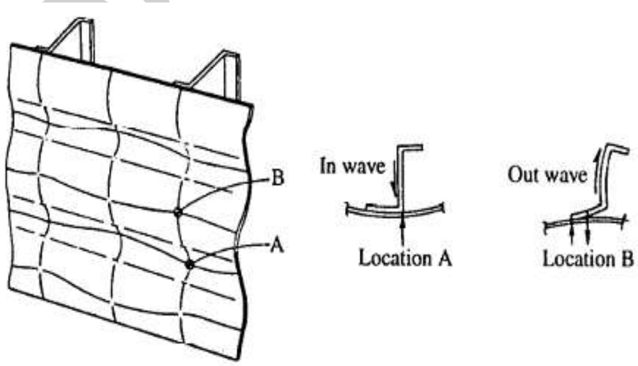
Splice is a type of connection which is used for:

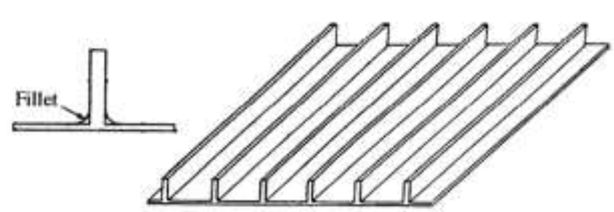
- Limitations on sheet width and length (manufacturing consideration).
- To obtain desired span wise taper of section area (cost consideration).
- For fail safe design (safety consideration). Prevent the crack and fracture in the web of spar.



2-The stringers

Serve for increasing stability of the skin and transmission of the air load from the skin to the ribs. The stringers works simultaneously for compression and lateral bending.

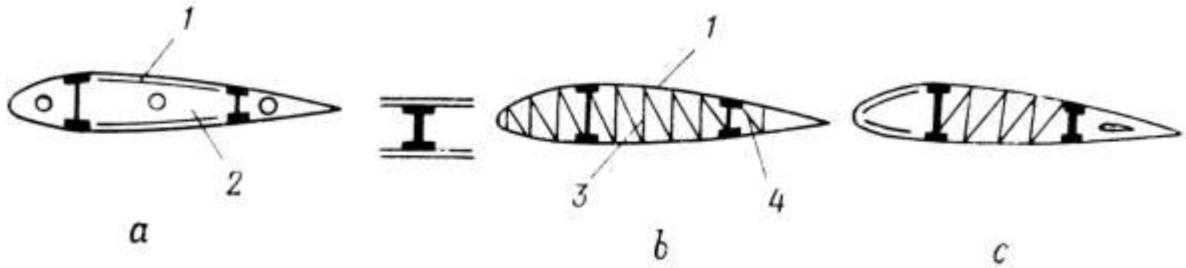




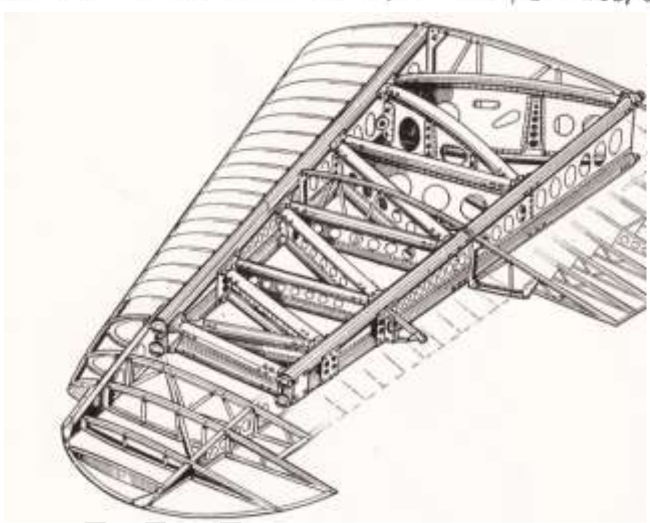
Unflanged integrally stiffened panel

3- The ribs

Serve to preserve the shape of the wing profile bending and action of the air load and to increases the skin to the webs of the spars and to the aerodynamic load from the wing. Besides, the ribs taking up the concentrated loads the wing serve for uniform distribution of the above loads along the thin skin of the wing.
As a construction, the ribs are divided into the: **a- beam, b- truss, c- beam/truss.**



a – beam; b – truss; c – beam/truss; 1 – band; 2 – web; 3 – strut; 4 – brace



4-The Skin

The skin forms the outer surface of the wing, takes up the air load and transmits it to the stringers and ribs. It's also serves for making a closed contour of the wing section and for taking up the torsional moment.

Fuselage

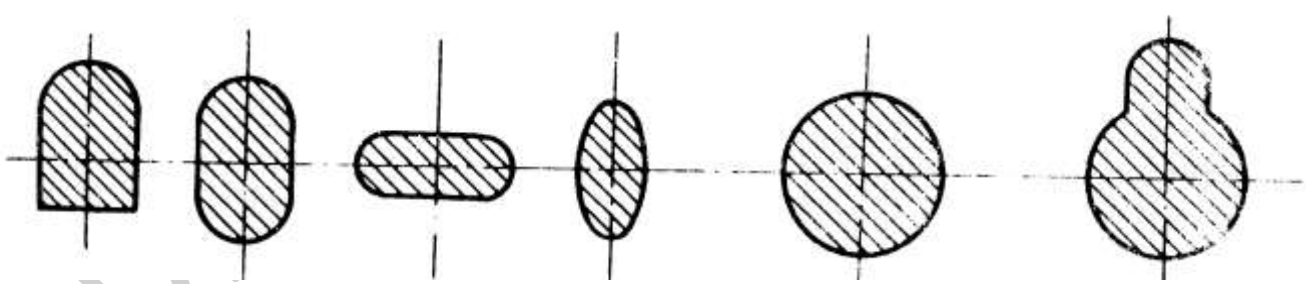
The fuselage serves for accommodation of the crew, passengers, cargoes, equipment and for attachment to it of main components such as the wing, tail unit, and sometimes landing gear.

The fuselage should meet the following requirements:

- Convenience for accommodating the crew, passengers, equipment and cargoes, convenience of loading and unloading of cargoes.
- Sufficient bending and torsional stiffness on which the tail unit angles of attack depend, and hence, tail unit efficiency
- The minimum possible drag under the characteristic flight conditions which is ensured by the shape and position of the fuselage relative to the wing
- Ensuring of reliable and safe bailout.
- Ensuring of good observation for the pilot, especially at landing.
- Maximum use of internal volumes of the fuselage.

Fuselage layout

The shape and dimensions of the fuselages are determined by the overall dimensions of the cargoes, equipment and units arranged therein, as well as by the aerodynamic requirements.

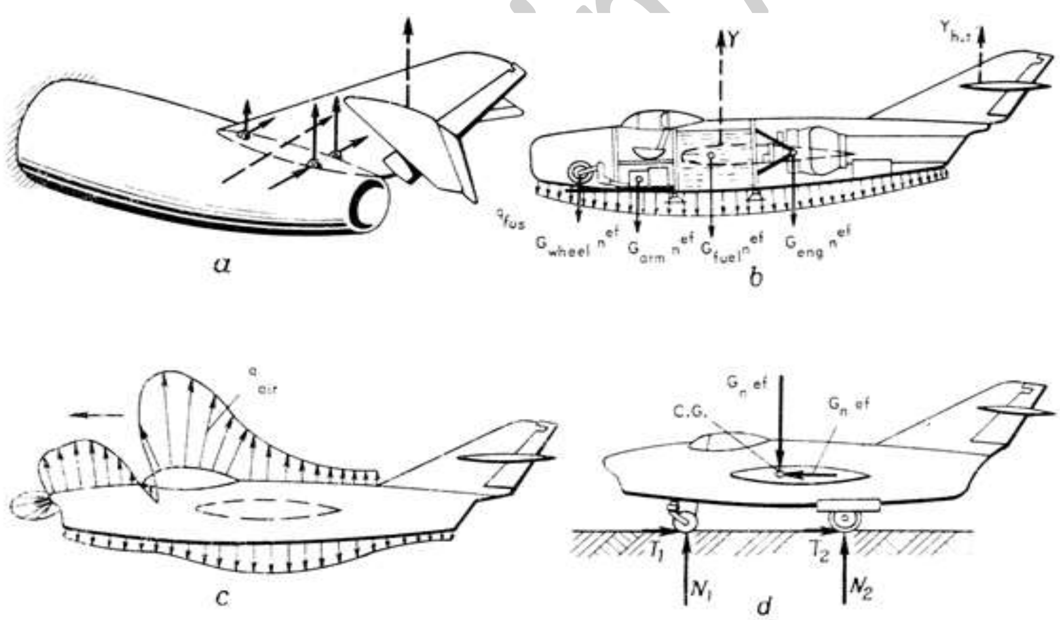


Loads acting on fuselage

The fuselage experiences the loads transmitted from the aircraft main components secured on it, the fuselage is main construction base of the entire aircraft.

The main loads acting on the fuselage are :

- Concentrated loads from the tail unit, figure –a.
- Mass loads from the units and cargoes located in the fuselage as shown in figure –b; they are transmitted through the respective attachment units to the fuselage structure.
- Distributed mass loads from the construction of the fuselage, figure-b.
- Surface distributed air loads figure –c ; these loads influence only local strength of the skin and glazing of the canopy .
- Load transmitted from the landing gear if the latter is mounted on the fuselage, figure –d .
- Excessive pressure forces for the pressurized cabin fuselages.



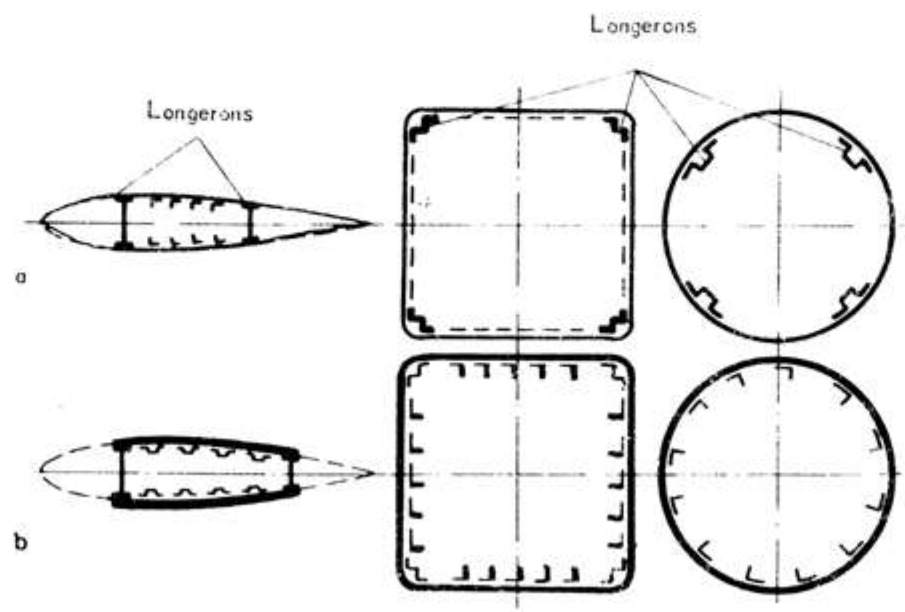
Load acting on fuselage
a- concentrated loads from tail unit; b- mass loads from units and cargoes arranged in fuselage; c- distributed air loads ; d- loads caused by landing gear

All the forces acting on the fuselage are balanced by the reactions in the wing attachment units, when analyzing work of the fuselage, it considered as a two support cantilever.

Load Bearing patterns of fuselages

From the loading pattern discussed above, it is evident that the fuselage being a load-bearing base of the aircraft is the cross section of which experience the lateral forces, bending and torsional moments. Thus, the fuselage should be constructionally similar to the wing.

Similarly to the wing, the fuselage may be of a longitudinal framework (longer on) design as shown in fig-a, or of mnoblock as in figure –b.

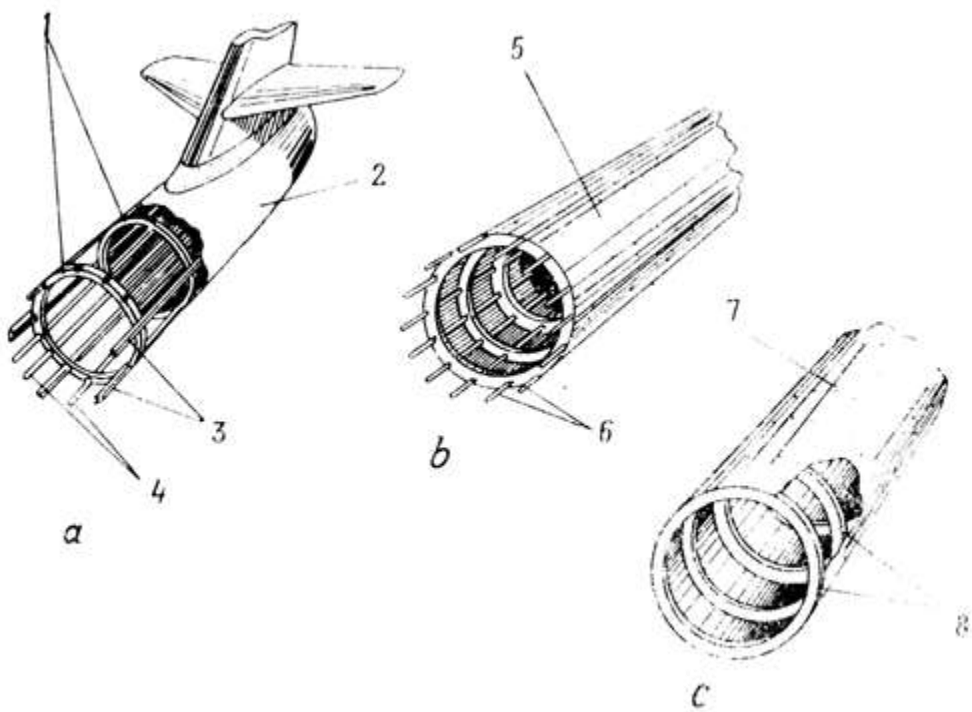


Fuselage load bearing patterns
a- longeron pattern b- monblock pattern

There are three types of the beam fuselages depending on the longitudinal framework:

- **Longeron fuselage (figure –a):** consist of powerful longerons, weak stringers, thin skin and set of frames. In contrast of the wing, the fuselage longerons have no webs and they take up the axial bending forces of the fuselage. The skin of such a fuselage works to resist shearing forces caused by lateral force and torsional moment.
- **Stringer fuselage (figure-b):** including a thicker skin and thick network of stringers and frames. Similarly to the monoblock wing, the bending moment is taken up in such a fuselage by compression/ tension of the skin and stringers.
- **Skin pattern (figure –c)** : formed by a thick skin strengthened with frames . such a fuselage which is a logic development of the stringer fuselage has a thick skin which takes up all types of strain (shearing , bending and torsion) . However, such a pattern is seldom used .

The stringer fuselage is most widely used on the modern aircraft.



a- longeron; b- stringer; c- skin; 1 and 8 –frames; 2-5 thin skin; 3- longerons
4-6- stringers; 7- thick skin

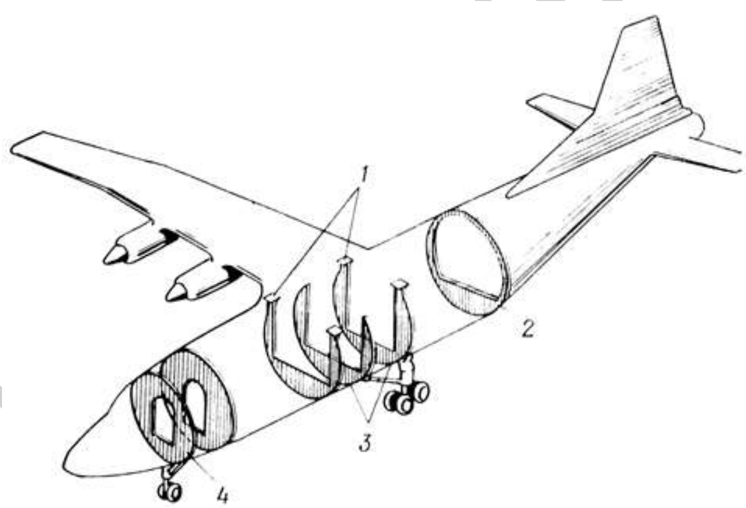
Purpose and works of fuselage load-bearing members

The skin : similary to the wing , serves for creating the exeternal shell of the fuselage; it is a main load bearing member which takes up lateral forces and torsional moments by tangential stress.

The stringers: serve to take up (together with the skin) the compression and tension forces created at bending of fuselage , besides, the stringers with frames strengthen the skin, thus increasing its critical compression and shearing stresses.

Longerons: serve to take up the axial forces due bending moment.

The frames: are similar to the ribs in their purpose; they determine the cross-section shape of the fuselage and preclude loss of stability of the shell, making it work as a beam, without distortions of the lateral cross sections. But constructionally, the frames differ from ribs. The normal frames serve for preserving the shape of a thin walled shell of the fuselage at bending (similarly to normal ribs).



1-Wing attachment frames; 2- jointing frame;
3,4- landing gear attachment frames

Aircraft structures

Calculation of section properties and defections

Center of gravity (centroid of):

It is a point in a body where the resultant gravity force is acted through. This resultant force is equal to the sum of weights (w_i) of all elements of the body, then:

$$\bar{x} = \frac{\sum x.w_i}{\sum w_i} \qquad \bar{y} = \frac{\sum y.w_i}{\sum w_i} \qquad \bar{z} = \frac{\sum z.w_i}{\sum w_i}$$

Centroid of area:

For plate of uniform thickness and density lies in xy plane:

$$\bar{x} = \frac{\sum x.w.A_i}{\sum w.A_i} = \frac{\sum x.A_i}{\sum A_i} \qquad \bar{y} = \frac{\sum y.w.A_i}{\sum w.A_i} = \frac{\sum y.A_i}{\sum A_i}$$

Where w is weight unit area, A_i is the area of each element.

Centroid of wing cross section

It is the centroid of this cross section area and it is evaluated after accomplishment of structural idealization for this cross section.

Second moments of area of standard sections

- Second Moment of area

$\mathrm{I}_{xx} = \int y^2\, dA$ Moment of inertia of cross section about x- axis.

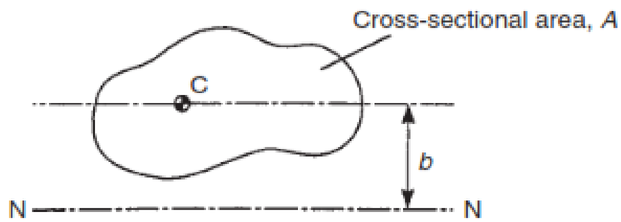
$\mathrm{I}_{yy} = \int x^2\, dA$ Moment of inertia of cross section about y-axis.

$\mathrm{I}_{xy} = \int x.y\, dA$ Product moment of inertia of cross section.

- Parallel axes theorem:

Suppose that the second moment of area, I_c , about an axis through its centroid C is known. The second moment of area, I_N , about a parallel axis, NN, a distance b from the centroidal axis is then given by

$$I_N = I_c + Ab^2$$



Neutral axis:

It is an axis passing through the centroid of cross section. The bending moments Mx and M_y about it produce zero direct stress in all the points on it.

Principle axes:

Two perpendicular axes a bout which product moment of area is zero ($I_{xy} = 0$), and the two other moments, I_{xx} and I_{yy} , are either maximum or minimum for example it I_{xx} is maximum about one axis , I_{yy} is minimum about this axis, and I_{xx} will be minimum about the other axis, while I_{yy} become maximum. If either x-axis or y-axis is coincided with principle axis then $I_{xy} = 0$ also.

Elastic axis:

It is an axis for the wing about which rotation will occur when the wing is loaded in pure torsion. It is important for flutter analysis of the wing.

Shear center:

It is a point in the wing cross section at which the resultant shear load must act to produce a wing deflection with no rotation (twist).

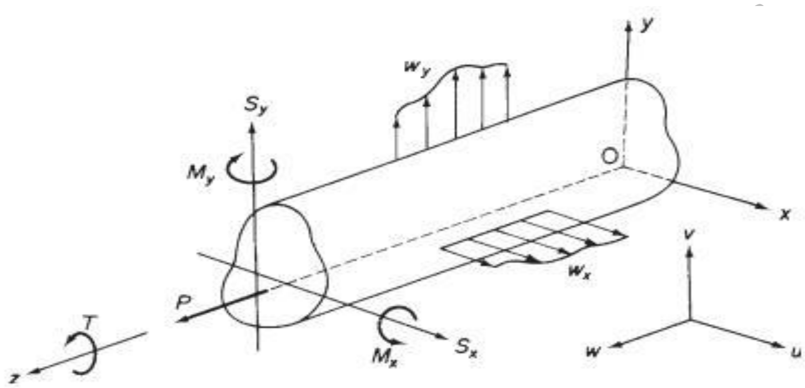
Shear centers for all wing sections must lie on the elastic axis of the wing, about this is not true because skin wrinkles and becomes ineffective in resisting compression loads. For practical purpose the elastic axis may be assumed to coincide with line joining the shear centers for all wing sections.

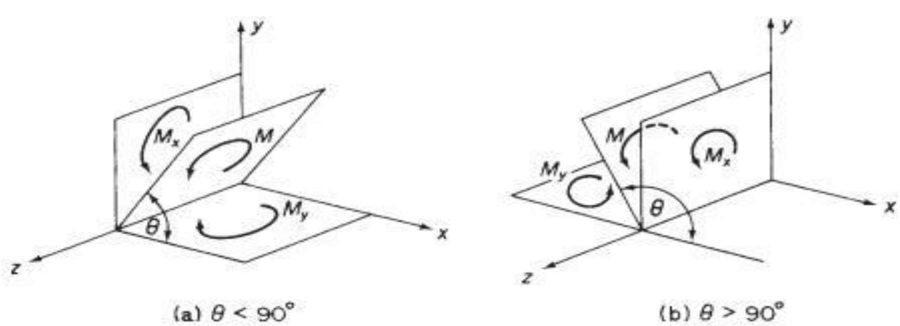
Bending moments

Resolution of bending moments:

A bending moment M applied in any longitudinal plane parallel to the z- axis may be resolved into components M_x and M_y by the normal rules of vectors.

M_x and M_y are positive when the induced tension in the positive xy quadrant of the beam cross-section.





(a) $\theta < 90°$ (b) $\theta > 90°$

Bending moment M applied in any longitudinal plane parallel to the z-axis may be resolved into components M_x and M_y.

$$M_x = M \sin\theta$$

$$M_y = M \cos\theta$$

For positive M,

if $\theta < \pi/2$ M_x is +ve and M_y is +ve

If $\theta > \pi/2$ $\quad$ M_x is +ve and M_y is +ve

Direct stress distribution due to bending:

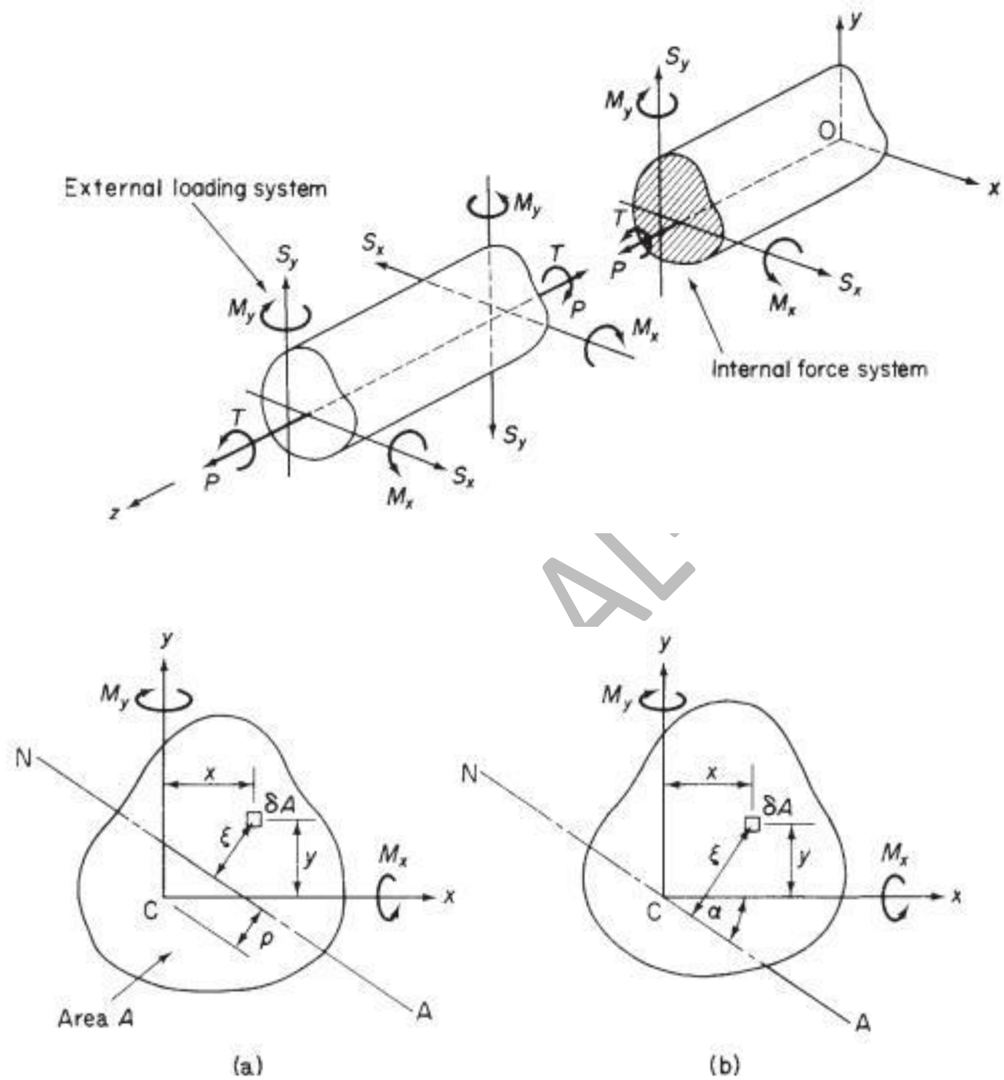


Figure 1

C is the centroid, N.A. is neutral axes.

Origin of axes is coincide with the centroid G, (ξ)is the distance from element (δA) to N.A.

From Hooke's low

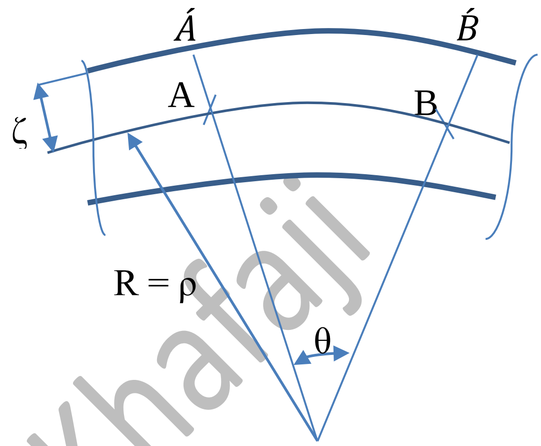
$$\sigma_z = E\varepsilon_z$$

If the beam is bent to a radius of curvature ρ about neutral axis at this particular section and since plane sections are assumed to remain plane after bending, then:

$$\varepsilon_z = \frac{\xi}{\rho}$$

$$\sigma_z = E\varepsilon_z$$

$$\sigma_z = \frac{E\xi}{\rho}$$



The beam supports pure bending moments so that the resultant end load on any section must be zero.

$$\int_A \sigma_z \, dA = 0$$

$$\int_A \frac{E\xi}{\rho} \, dA = 0 \quad \Longrightarrow \quad \int_A \xi \, dA = 0$$

(E/ ρ) is constant and has been canceled. It follows that the N.A. passes through the centroid see figure b.

Suppose that N.A. is inclined to C_x by angle α, then

$$\xi = x \sin\alpha + y \cos\alpha$$

$$\sigma_z = \frac{E\xi}{\rho} \quad \Longrightarrow \quad \sigma_z = \frac{E}{\rho}(x \sin\alpha + y\cos\alpha) \quad \text{———} \; ✸$$

The moment resultants of the internal direct stress distributions have the same sense as the applied moments (M_x) and (M_y). Thus

$$M_x = \int_A P_z \,.\, y \quad \Longrightarrow \quad M_x = \int_A \sigma_z y \, dA,$$

$$M_y = \int_A P_z \,.\, x \quad \Longrightarrow \quad M_y = \int_A \sigma_z x \, dA$$

Substitute for σ_z from Equation ✸

$$M_x = \int_A \frac{E}{\rho}(x\, y\, sinx + y^2 cosx)\, dA$$

$$I_{xx} = \int_A y^2 \, dA, \quad I_{yy} = \int_A x^2 \, dA, \quad I_{xy} = \int_A xy \, dA$$

$$M_x = \frac{E}{\rho} \sin\alpha \;\, I_{xy} + \frac{E}{\rho}\cos\alpha \;\, I_{xx} \qquad M_y = \frac{E}{\rho}\sin\alpha \;\, I_{yy} + \frac{E}{\rho}\cos\alpha \;\, I_{xy}$$

Now let $\quad A = \dfrac{E}{\rho}\sin\alpha \quad$ and $\quad B = \dfrac{E}{\rho}\cos\alpha$

$$M_x = A\, I_{xy} + B\, I_{xx} \qquad \text{———————— ✸✸}$$

$$M_y = A\, I_{yy} + B\, I_{xy} \qquad \text{———————— ✸✸✸}$$

Solving these two equations (✸✸ and ✸✸✸) gives:

$$A = \frac{M_y\, I_{xx} - M_x\, I_{xy}}{I_{xx}\, I_{yy} - I^2{}_{xx}} \qquad B = \frac{M_x\, I_{yy} - M_y\, I_{xy}}{I_{xx}\, I_{yy} - I^2{}_{xx}}$$

Then $\quad \sigma = A.x + B.y$

$$\sigma = \left(\frac{M_y\, I_{xx} - M_x\, I_{xy}}{I_{xx}\, I_{yy} - I^2{}_{xx}}\right).x + \left(\frac{M_x\, I_{yy} - M_y\, I_{xy}}{I_{xx}\, I_{yy} - I^2{}_{xx}}\right).y$$

$$\sigma = \frac{\overline{M_x}}{I_{xx}} y + \frac{\overline{M_y}}{I_{yy}} x$$

Where $\overline{M_x}$ and $\overline{M_y}$ are effective bending moment,

$$\overline{M_x} = \left(\frac{M_x - M_y \, I_{xy} / \, I_{yy}}{1 - \, I^2{}_{xx} / \, I_{xx} \, I_{yy}} \right) \qquad \overline{M_y} = \left(\frac{M_y - M_x \, I_{xy} / \, I_{xx}}{1 - \, I^2{}_{xx} / \, I_{xx} \, I_{yy}} \right)$$

If either C_x or C_y is axis of symmetry or both, then I_{xy} is zero and that axis is principal axis and

$$\overline{M_x} = M_x \qquad \overline{M_y} = M_y$$

$$\sigma = \frac{M_x}{I_{xx}} . y + \frac{M_y}{I_{yy}} . x$$

Further if either M_x or M_y is zero then

$$\sigma = \frac{M_x}{I_{xx}} . y \qquad \text{or} \qquad \sigma = \frac{M_y}{I_{yy}} . x$$

Which is the result of simple engineering theory of bending for beams having at least singly symmetrical cross section.

M_x and M_y are pure bending moment about x-axis and y-axis respectively.{if shear loads S_x and S_y are acted on any section then they will produce bending moment on the next section.

<u>Position of the neutral axis</u>

At all points on the neutral axis the direct stress is zero by definition.

Therefore:

$$\sigma = \frac{\overline{M_x}}{I_{xx}} y + \frac{\overline{M_y}}{I_{vv}} x \quad \Longrightarrow \quad 0 = \frac{\overline{M_x}}{I_{xx}} y + \frac{\overline{M_y}}{I_{vv}} x$$

$$0 = \left(\frac{M_y I_{xx} - M_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} \right) x_{\mathrm{NA}} + \left(\frac{M_x I_{yy} - M_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} \right) y_{\mathrm{NA}}$$

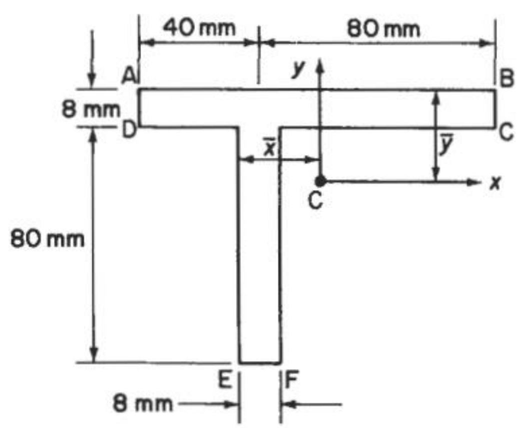
Where $x_{N.A.}$ and $y_{N.A.}$ are coordinates of points on the neutral axis.Hence

$$\frac{y_{\mathrm{NA}}}{x_{\mathrm{NA}}} = -\frac{M_y I_{xx} - M_x I_{xy}}{M_x I_{yy} - M_y I_{xy}}$$

Or from figure 1 (b)

Example:

A beam having the cross-section is subjected to a bending moment of 1500 N m in a vertical plane. Calculate the maximum direct stress due to bending stating the point at which it acts.



The position of the centroid of the section may be found by taking moments of areas about some convenient point. Thus

$(120 \times 8 + 80 \times 8)\bar{y} = 120 \times 8 \times 4 + 80 \times 8 \times 48$

Giving $\bar{y} = 21.6\, mm$

And

$(120 \times 8 + 80 \times 8)\bar{x} = 80 \times 8 \times 4 + 120 \times 8 \times 24$

Giving $\bar{x} = 16\, mm$

The next step is to calculate the section properties referred to axes C_{xy}.

$$I_{xx} = \frac{120 \times (8)^3}{12} + 120 \times 8 \times (17.6)^2 + \frac{8 \times (80)^3}{12} + 80 \times 8 \times (26.4)^2$$
$$= 1.09 \times 10^6 \text{ mm}^4$$
$$I_{yy} = \frac{8 \times (120)^3}{12} + 120 \times 8 \times (8)^2 + \frac{80 \times (8)^3}{12} + 80 \times 8 \times (12)^2$$
$$= 1.31 \times 10^6 \text{ mm}^4$$
$$I_{xy} = 120 \times 8 \times 8 \times 17.6 + 80 \times 8 \times (-12) \times (-26.4)$$
$$= 0.34 \times 10^6 \text{ mm}^4$$

Hence

Since $M_x = 1500$ N m and $M_y = 0$

$$\sigma_z = 1.5y - 0.39x \qquad \text{(i)}$$

By inspection of Eq. (i) the σ_z will be a maximum at F where

$$x = -8mm \quad , y = -66.4\, mm$$

Thus $\sigma_{z,max} = -96\ N/mm^2$ (compressive)

General case of loading

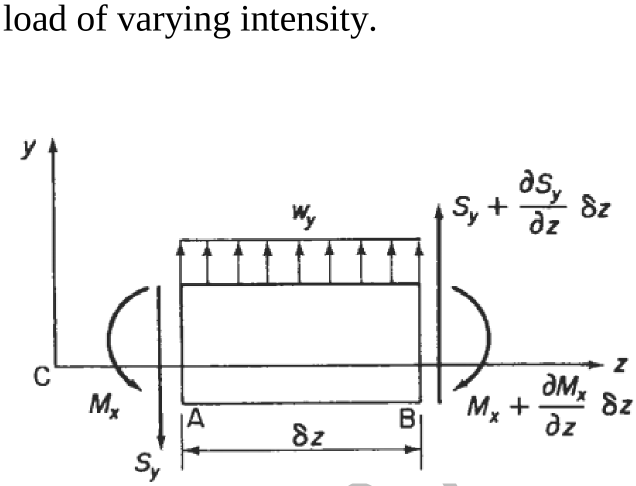
Consider the element shown , where

δz : element of length in z-direction.

S_y : shear load in y-direction.

M_x : bending moment about x-axis.

w_y : distributed load of varying intensity.



As the element length is small, then the intensity is assumed constant.

(M_x) and S_y are +ve in the direction shown.

There for, for equilibrium of the element in the y direction

$$\left(S_y + \frac{\partial S_y}{\partial z}\delta z\right) + w_y \delta z - S_y = 0$$

From which

$$w_y = -\frac{\partial S_y}{\partial z}$$

Taking moments about A

$$\left(M_x + \frac{\partial M_x}{\partial z}\delta z\right) - \left(S_y + \frac{\partial S_y}{\partial z}\delta z\right)\delta z - w_y \frac{(\delta z)^2}{2} - M_x = 0$$

Or , when second order terms are neglected,

$$S_y = \frac{\partial M_x}{\partial z}$$

Combine these results into a single expression

$$-w_y = \frac{\partial S_y}{\partial z} = \frac{\partial^2 M_x}{\partial z^2}$$

Similarly for loads in the xz plane

$$-w_x = \frac{\partial S_x}{\partial z} = \frac{\partial^2 M_y}{\partial z^2}$$

If it assumed that a parameter $\bar{S}_y$ bears the same relationship to $\bar{M}_x$ as S_y does to M_x then

$$\overline{S_y} = \frac{\partial \overline{M_x}}{\partial z} = \frac{\partial M_x/\partial z - (\partial M_y/\partial z) I_{xy}/I_{yy}}{1 - I_{xy}^2/I_{xx}I_{yy}}$$

Or $$\overline{S_y} = \frac{S_y - S_x I_{xy}/I_{yy}}{1 - I_{xy}^2/I_{xx}I_{yy}}$$

In similar fashion

$$\overline{S_x} = \frac{S_x - S_y I_{xy}/I_{xx}}{1 - I_{xy}^2/I_{xx}I_{yy}}$$

Parameters $\bar{w}_y$ and $\bar{w}_x$ are related to load intensities $\bar{w}_y$ and $\bar{w}_x$.

$$\overline{w_x} = \frac{w_x - w_y I_{xy}/I_{xx}}{1 - I_{xy}^2/I_{xx}I_{yy}}$$ and $$\overline{w_y} = \frac{w_y - w_x I_{xy}/I_{yy}}{1 - I_{xy}^2/I_{xx}I_{yy}}$$

The parameters $\bar{M}_x$, $\bar{M}_y$, $\bar{S}_x$, $\bar{S}_y$, $\bar{w}_x$, $\bar{w}_y$ are often termed “effective” bending moments, shear forces and load intensities.

Approximations for thin-walled sections

We may exploit the thin-walled nature of aircraft structures to make simplifying assumptions in the determination of stresses and deflections produced by bending. Thus, the thickness t of thin-walled sections is assumed to be small compared with their cross-sectional dimensions so that stresses may be regarded as being constant across the thickness. Furthermore, neglect squares and higher powers of **t** in the computation of sectional properties and take the section to be represented by the mid-line of its wall. As an illustration of the procedure, consider the channel section of figure (a). The section is singly symmetric about the x axis so that

Ixy = 0.

The second moment of area Ixy is then given by

$$I_{xx} = 2\left[\frac{(b+t/2)t^3}{12} + \left(b+\frac{t}{2}\right)th^2\right] + t\frac{[2(h-t/2)]^3}{12}$$

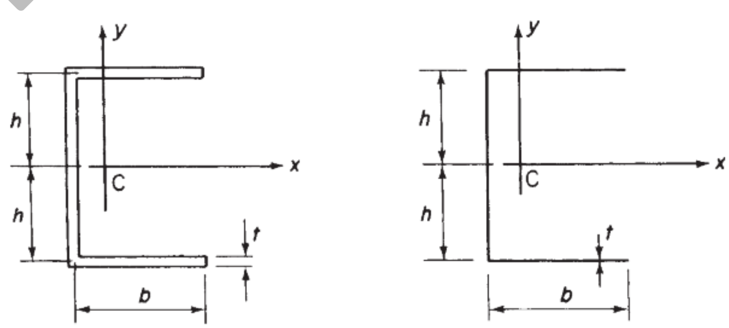
Expanding the cubed term to be

$$I_{xx} = 2\left[\frac{(b+t/2)t^3}{12} + \left(b+\frac{t}{2}\right)th^2\right] + \frac{t}{12}\left[(2)^3\left(h^3 - 3h^2\frac{t}{2} + 3h\frac{t^2}{4} - \frac{t^3}{8}\right)\right]$$

Which reduces, after powers of $\mathbf{t^2}$ and upwards are **ignored** to

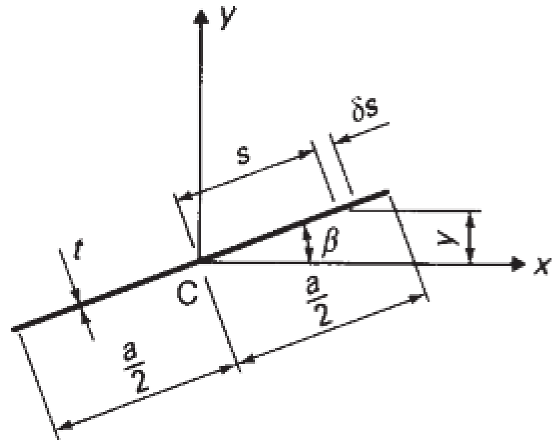
$$I_{xx} = 2bth^2 + t\frac{(2h)^3}{12}$$

The second moment of area of the section about G_y is obtained in a similar manner.



(a) (b)

Thin- walled sections frequently have inclined or curved walls which complicated the calculation of section properties.



Consider the inclined thin section of figure below. Its second moment of area about a horizontal axis through its centroid is given by

$$I_{xx} = 2\int_0^{a/2} ty^2\,\mathrm{d}s = 2\int_0^{a/2} t(s\sin\beta)^2\,\mathrm{d}s$$

From which $$I_{xx} = \frac{a^3 t \sin^2\beta}{12}$$

Similarly $$I_{yy} = \frac{a^3 t \cos^2\beta}{12}$$

The product second moment of area is

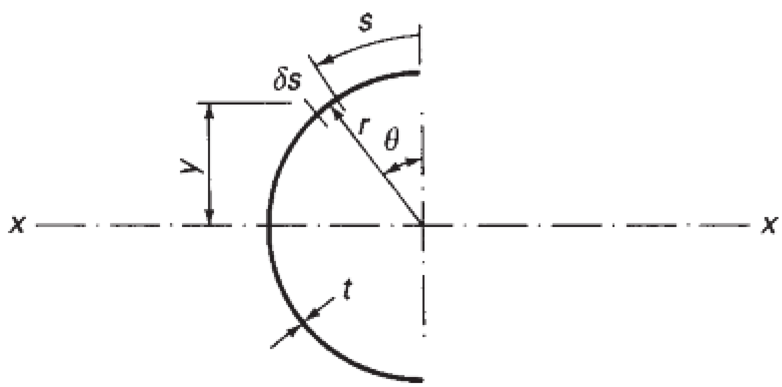
$$I_{xy} = 2\int_0^{a/2} txy\,\mathrm{d}s$$

$$= 2\int_0^{a/2} t(s\cos\beta)(s\sin\beta)\,\mathrm{d}s$$

Which gives $$I_{xy} = \frac{a^3 t \sin 2\beta}{24}$$

It could be note here that expressions are approximate in that their derivation neglects power of t^2 and upwards by ignoring the second moments of area of the element δs about axes through its own centroid.

Properties of thin-walled curved sections are found in a similar manner. Thus, I_{xx} for semicircular section of figure below.



$$I_{xx} = \int_0^{\pi r} t y^2 \, \mathrm{d}s$$

Expressing y and s I terms of a single variable θ simplifies the integration, hence

$$I_{xx} = \int_0^{\pi} t(r\cos\theta)^2 r \, \mathrm{d}\theta$$

From which

$$I_{xx} = \frac{\pi r^3 t}{2}$$

Example:

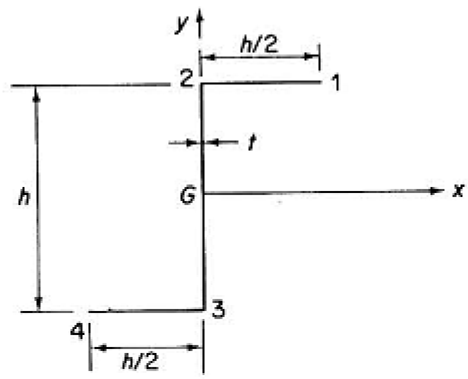
Determine the direct stress distribution in the thin-walled Z-section shown produced by a positive bending moment M_x.

M_x = 1000 kg.mm

h = 40 mm

t = 1mm

M_y = 0



Sol./

The section is antisymmetrical with its centroid at the mid-point of the vertical web. Therefore, the direct stress distribution is given by

$$\sigma_z = \frac{M_x(I_{yy}y - I_{xy}x)}{I_{xx}I_{yy} - I_{xy}^2} \qquad \text{(i)}$$

Where, for this problem

$$\overline{M_x} = \frac{M_x}{1 - I_{xy}^2/I_{xx}I_{yy}}, \quad \overline{M_y} = \frac{-M_x I_{xy}/I_{xx}}{1 - I_{xy}^2/I_{xx}I_{yy}} \qquad \textbf{(ii)}$$

The section properties are calculated as follows

$$I_{xx} = 2\frac{ht}{2}\left(\frac{h}{2}\right)^2 + \frac{th^3}{12} = \frac{h^3t}{3}$$

$$I_{yy} = 2\frac{t}{3}\left(\frac{h}{2}\right)^3 = \frac{h^3t}{12}$$

$$I_{xy} = \frac{ht}{2}\left(\frac{h}{4}\right)\left(\frac{h}{2}\right) + \frac{ht}{2}\left(-\frac{h}{4}\right)\left(-\frac{h}{2}\right) = \frac{h^3t}{8}$$

Substituting these values in Eqs. (ii) gives

$$\overline{M_x} = 2{\cdot}29 M_x, \quad \overline{M_y} = -0{\cdot}86 M_x$$

$$\bar{M}_x = 2286 \text{ kg mm} \quad , \quad \bar{M}_y = -857 \text{ kg mm}$$

These expression when substituted in Eq. (i) give

$$\sigma_z = \frac{M_x}{h^3 t}(6{\cdot}86y - 10{\cdot}30x) \qquad \text{(iii)}$$

$$\sigma_z = 0.107y - 0.161x$$

On the top flange y= 20 , $0 \le x \le 20$ (y=h/2 $0 \le x \le h/2$) and the distribution of direct stress is given by

$$\sigma_z = \frac{M_x}{h^3 t}(3{\cdot}43h - 10{\cdot}30x)$$

$$\sigma_{z1} = 1.08 \text{ kg/mm}^2$$

At point (1) x = 20 mm, y= 20 mm

$$\sigma_{z,1} = -\frac{1{\cdot}72 M_x}{h^3 t} \text{ (compressive)} \qquad \sigma_z = 0.107 * 20 - 0.161 * 20$$

$$\sigma_{z1} = -1.08 \text{ kg/mm}^2 \quad \text{(Compressive)}$$

At point (2) x = 0 mm, y=20 mm

$$\sigma_z = 0.107 * 20 - 0.161 * 0$$

$$\sigma_{z2} = +2.14 \text{ kg/mm}^2 \quad \text{(Tensile)}$$

In the web $h/2 \leqslant y \leqslant -h/2$ and $x = 0$. Again the distribution is of linear form and is given by the equation

$$\sigma_z = \frac{M_x}{h^3 t} 6{\cdot}86y \qquad \sigma_z = 0.107 * y$$

At point (3) $x = 0$ mm, y= -20 mm

$$\sigma_{z3} = 0.107 * (-20)$$

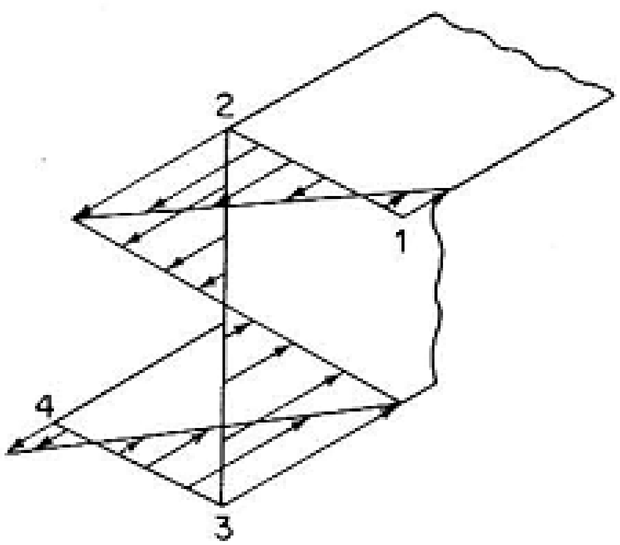
$$\sigma_{z3} = -2.14 \ \text{kg/mm}^2 \ \text{(Compression)}$$

At point (4) x= -20 mm, y= -20 mm

$$\sigma_{z4} = 0.107 * (-20) - 0.161 * (-20)$$

$$\sigma_{z4} = +1.08 \ kg/mm^2 \quad \text{(Tensile)}$$

The distribution in the lower flange may be deduced from antisymmetry , the complete distribution is then as shown.



Shear force and stress

For the analysis of open section beams supporting shear loads and closed section beams carrying shear and torsional loads, there is some assumptions will be taken in consideration.

- Plane stress where the normal stress to the surface of the wall equal to zero
- Plane strain where the strain in the direction normal to the surface of wall is equal to zero.

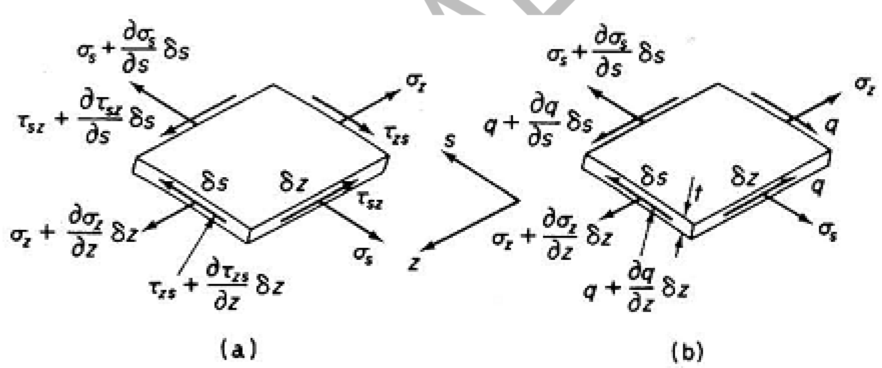
- Generally in the analysis, the axial constraint effects are negligible.
- Shear stress normal to the tube surface may be neglected since they are zero at each surface and the wall is thin.

- The wall is thin that the direct and shear stress on plane normal to the tube surface are constant across the thickness.

- The thickness and finally that the tube is of uniform section so that the thickness may vary with distance round each section but is constant a long tube.

- Ignore squares and higher powers of the thickness t. in the calculation of section constants.
- The hoop stress (σ_s) is usually zero for open tube, while for closed tube for closed tube, if there is an internal pressure, the hoop stress (σ_s) will be not equal to zero.

An element ($\delta s \times \delta z \times t$) of the tube wall is maintained in equilibrium by a system of direct and shear stress as shown in fig. (a). The direct stress σ_z is produced by bending moments or by bending action of shear loads while the shear stresses are due to shear and/or torsion of a closed tube or shear of an open tube.



It is assumed that the thickness is constant in the direction of s (round the section).

$\tau_{zs} = \tau_{sz} = \tau$

It will be use *shear flow* (q), which is the shear force per unit length rather than terms of shear stress.

$$q = \tau\, t$$

Shear flow (q) is positive in the direction of increasing s.

For equilibrium of the element in the z direction and neglecting body forces:

Which reduces to

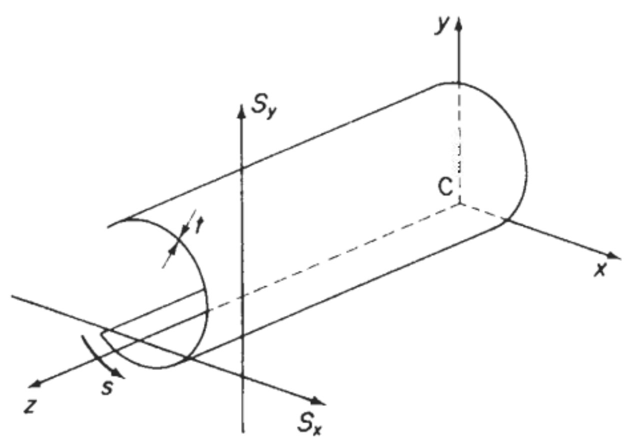
……….(1)

Similarly for equilibrium in the s direction

………..(2)

Shear of open tubes

The open section beam of arbitrary section shown in the below figure supports shear loads S_x, and S_y such that there is no twisting of the beam cross-section. For this condition to be valid the shear loads must both pass through a particular point in the cross-section known as the shear center.
Since there are no hoop stresses in the beam the shear flows and direct stresses acting on an element of the beam wall are related by Equation.



Assume that the direct stresses are obtained with reasonable accuracy from the Engineer's theory of bending. Thus

Substituting this expression in equation (1) gives

And

Hence

………..(3)

Integrating Eq. (3) with respect to s from some origin for s to any point round the cross-section

………...(4)

If the origin for s is taken at the open edge of the tube then
$q=0$ when s=0 Eq (4) becomes

…………(5)

Example:

Determine the shear flow distribution in thin-walled Z- section as shown . due to a shear load S_y applied through the shear center of the section.

Sol./

Since S_y is applied through the shear center then no torsion exists and the shear flow distribution is given by:

In which

The second moments of area of the section are :

Substituting these values in Eqs. of $\overline{S_y}$ and $\overline{S_x}$ to obtain

Whence from Eq. (5) (shear flow distribution):

On the bottom flange