Chapter One
Navier – Stokes Equations

Contents

1- Navier-Stokes equations.
2- Steady laminar flow between parallel flat plates.
3- Hydrodynamic lubrication.
4- Laminar flow between concentric rotating cylinders.
5- Example.
6- Problems; sheet No. 1

1- Navier-Stokes equations:

The general equations of motion for viscous incompressible, Newtonian fluids may be written in the following form:

x- direction:

$$\rho\left[\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right]=\rho g_x-\frac{\partial p}{\partial x}+\mu\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right]\text{ ------(1)}$$

y- direction:

$$\rho\left[\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right]=\rho g_y-\frac{\partial p}{\partial y}+\mu\left[\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right]\text{ ------(2)}$$

Equations (1) and (2) are called: Navier Stokes equations.

2- Steady laminar flow between parallel flat plates:

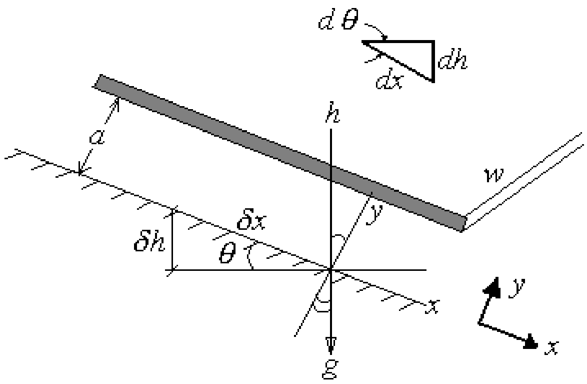


Fig.25

The fluid moves in the x- direction without acceleration.

v= 0 , w= 0 , $\frac{\partial}{\partial t}=0$

the Navier-Stokes equation in the x- direction (eq. 1) reduces to:

$$-\rho g_x + \frac{dp}{dx} = \mu \frac{d^2u}{dy^2} \quad \text{---------(3)}$$

$$g_x = g.\sin\theta = -g\frac{dh}{dx}$$

eq. 3 will be

$$\frac{d^2u}{dy^2} = \frac{1}{\mu}\frac{d(p+\gamma h)}{dx} \quad \text{---------(4)}$$

Integration of eq. 4:

$$\frac{du}{dy} = \frac{1}{\mu}\frac{d(p+\gamma h)}{dx}y + A$$

$$u = \frac{1}{2\mu}\frac{d(p+\gamma h)}{dx}y^2 + Ay + B \quad \text{-----------(5)}$$

<u>B.C</u> (Two fixed parallel plates)

$y = 0 \qquad u = 0 \quad \Rightarrow \quad B = 0$

$y = a \qquad u = 0 \quad \Rightarrow \quad A = \frac{-a}{2\mu}\frac{d(p+\gamma h)}{dx}$

eq. 5 will be

$$u = \frac{1}{2\mu}\frac{d(p+\gamma h)}{dx}\left(y^2 - ay\right) \quad \text{-----------(6)}$$

<u>B.C</u> (One plate is fixed and the other plate moves with a constant velocity U) (Couette flow)

$y = 0 \qquad u = 0 \quad \Rightarrow \quad B = 0$

$y = a \qquad u = U \quad \Rightarrow \quad A = \frac{U}{a} - \frac{a}{2\mu}\frac{d(p+\gamma h)}{dx}$

eq. 5 will be

$$u = \frac{1}{2\mu}\frac{d(p+\gamma h)}{dx}\left(y^2 - ay\right) + \frac{\text{Uy}}{\text{a}} \quad \text{-----------(7)}$$

For the case of horizontal parallel plates:

$$\frac{dh}{dx} = 0$$

eq. 7 will be

$$u = \frac{1}{2\mu}\frac{dp}{dx}\left(y^2 - ay\right) + \frac{\text{Uy}}{\text{a}} \quad \text{-----------(8)}$$

The location of maximum velocity u_{max} may be found by evaluating $\frac{du}{dy}$ and setting it to zero.

The volume flow rate is

$$Q = w\int_0^a u.dy \text{ ------------(9)}$$

3- Hydrodynamic lubrication:

Sliding bearing

Large forces are developed in small clearance when the surfaces are slightly inclined and one is in motion so that fluid is wedged into the decreasing space. Usually the oils employed for lubrication are highly viscous and the flow is of laminar nature.

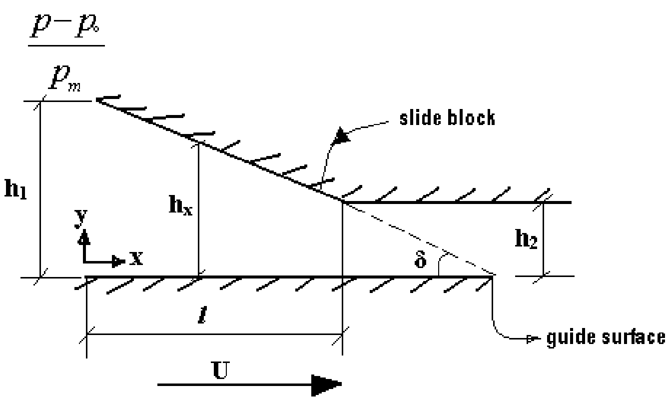


Fig.32

Assumptions:

The acceleration is zero.

The body force is small and can be neglected.

Also $\frac{\partial^2 u}{\partial y^2} \gg \frac{\partial^2 u}{\partial x^2}$ and $\frac{\partial^2 u}{\partial y^2} \gg \frac{\partial^2 u}{\partial z^2}$

The Navier-Stokes equation in the x-direction (eq. 1) reduces to:

$$\frac{d^2u}{dy^2} = \frac{1}{\mu}\frac{dp}{dx}$$

Integration:

$$u = \frac{1}{2\mu}\left(\frac{dp}{dx}\right)y^2 + Ay + B$$

B.C

$$y = 0 \qquad u = U \quad \Rightarrow \quad B = U$$

$$y = h_x \qquad u = 0 \quad \Rightarrow \quad A = \frac{-h_x}{2\mu}\frac{dp}{dx} - \frac{U}{h_x}$$

$$u = \frac{1}{2\mu}\frac{dp}{dx}\left(y^2 - h_x y\right) + U\left(1 - \frac{y}{h_x}\right)$$

The volume flow rate in every section will be constant.

$$Q = w\int_0^{h_x} u.dy \qquad \text{assume w} = 1$$

$$\therefore\; Q = \frac{Uh_x}{2} - \frac{h_x^3}{12\mu}\frac{dp}{dx} \quad \text{--------(*)}$$

** For a constant taper bearing:

$$\delta = \frac{h_1 - h_2}{l}$$

$$\therefore\; h_x = \left(h_1 - \delta x\right)$$

Sub in eq.(*) and solving for $\frac{dp}{dx}$ produces:

$$\frac{dp}{dx} = \frac{6\mu U}{\left(h_1 - \delta x\right)^2} - \frac{12\mu Q}{\left(h_1 - \delta x\right)^3}$$

Integration gives:

$$p(x) = \frac{6\mu U}{\delta\left(h_1 - \delta x\right)} - \frac{6\mu Q}{\delta\left(h_1 - \delta x\right)^2} + C \quad \text{---------(**)}$$

B.C

$$x = 0 \qquad p = p_o = 0$$

$$x = l \qquad p = p_o = 0$$

$$\Rightarrow\; Q = \frac{Uh_1 h_2}{h_1 + h_2} \quad \text{and} \quad C = \frac{-6\mu U}{\delta\left(h_1 + h_2\right)}$$

With these values inserted in eq.(**) we obtain the pressure distribution inside the bearing.

$$p(x) = \frac{6\mu Ux\left(h_x - h_2\right)}{h_x^2\left(h_1 + h_2\right)}$$

The load that the bearing will support per unit width is:

$$F = \int_0^l p(x).dx$$

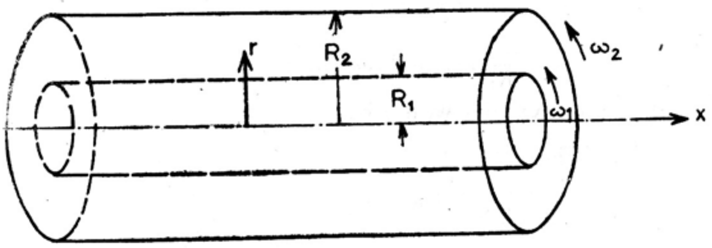
$$F = \frac{6\mu Ul^2}{\left(h_1 - h_2\right)^2}\left[\ln k - \frac{2(k-1)}{k+1}\right]$$

where

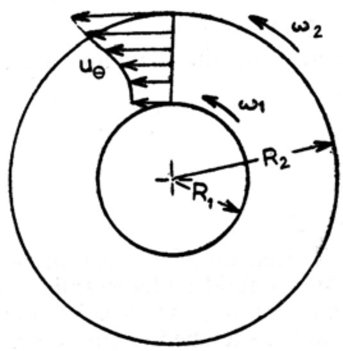
$$k = \frac{h_1}{h_2}$$

4- Laminar flow between concentric rotating cylinders:

Consider the purely circulatory flow of a fluid contained between two long concentric rotating cylinders of radius R_1 and R_2 at angular velocities ω_1 and ω_2.



(a) PICTORIAL REPRESENTATION



(b) VELOCITY PROFILE

In this case the Navier-Stokes equations in cylindrical coordinates are used.

r- direction:

$$\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}-\frac{u_\theta^2}{r}+w\frac{\partial u_r}{\partial z}=\frac{-1}{\rho}\frac{\partial p}{\partial r}+\frac{\mu}{\rho}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_r}{\partial r}\right)-\frac{u_r}{r^2}+\frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}+\frac{\partial^2 u_r}{\partial z^2}\right]+g_r$$

θ- direction:

$$\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+\frac{u_r u_\theta}{r}+w\frac{\partial u_\theta}{\partial z}=\frac{-1}{\rho r}\frac{\partial p}{\partial \theta}+\frac{\mu}{\rho}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_\theta}{\partial r}\right)-\frac{u_\theta}{r^2}+\frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}+\frac{\partial^2 u_\theta}{\partial z^2}\right]+g_\theta$$

In the above equations:

$u_r = 0$

$w = 0$

$$\frac{\partial}{\partial t}=0\ ,\quad \frac{\partial u_\theta}{\partial \theta}=0\ ,\quad \frac{\partial p}{\partial \theta}=0$$

body force = 0

The equation in θ- direction reduces to:

$$\frac{d^2 u_\theta}{dr^2}+\frac{d}{dr}\left(\frac{u_\theta}{r}\right)=0$$

Integration:

$$\frac{1}{r}\frac{d}{dr}\left(ru_\theta\right)=A$$

$$u_\theta = Ar+\frac{B}{r} \quad \text{---------(i)}$$

B.C

$r=R_1 \qquad u_\theta = R_1\omega_1$

$r=R_2 \qquad u_\theta = R_2\omega_2$

$$\Rightarrow \quad A=\omega_1+\frac{R_2^2}{R_2^2-R_1^2}\left(\omega_2-\omega_1\right)$$

$$B=-\frac{R_1^2R_2^2}{R_2^2-R_1^2}\left(\omega_2-\omega_1\right)$$

Sub. in eq.(i) yields:

$$u_\theta=\frac{1}{R_2^2-R_1^2}\left[\left(\omega_2R_2^2-\omega_1R_1^2\right)r-\frac{R_1^2R_2^2}{r}\left(\omega_2-\omega_1\right)\right] \quad \text{-------(ii)}$$

The shear stress may be evaluated by the equation:

$$\tau=\mu\left[r\frac{d}{dr}\left(\frac{u_\theta}{r}\right)\right]$$

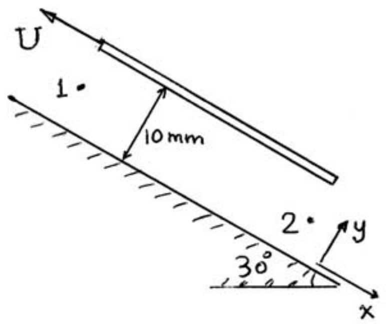
By using eq.(ii):

$$\tau=\frac{2\mu}{R_2^2-R_1^2}\frac{R_1^2R_2^2}{r^2}\left(\omega_2-\omega_1\right)$$

5- Example:

1- Using the Navier-Stokes equation in the flow direction, calculate the power required to pull (1m × 1m) flat plate at speed (1 m/s) over an inclined surface. The oil between the surfaces has (ρ = 900 kg/m^3 , μ = 0.06 Pa.s).The pressure difference between points 1 and 2 is (100 kN/m^2) .

Solution:



The Navier-Stokes equation in x- direction

$$\rho\left[\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right]=\rho g_x-\frac{\partial p}{\partial x}+\mu\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right]$$

We have: Acceleration =0 , v=0 , w=0 , $\frac{\partial^2 u}{\partial x^2}=0$, $\frac{\partial^2 u}{\partial z^2}$

The equation reduces to:

$$\frac{d^2u}{dy^2}=\frac{1}{\mu}\frac{dp}{dx}-\frac{\rho}{\mu}g_x$$

Integration

$$\frac{du}{dy}=\frac{1}{\mu}\frac{dp}{dx}y-\frac{\rho}{\mu}g_x y+A$$

$$u=\frac{1}{2\mu}\frac{dp}{dx}y^2-\frac{\rho}{2\mu}g_x y^2+Ay+B$$

B.C (b=10 mm)

$$y=0 \qquad u=0 \quad \Rightarrow \quad B=0$$

$$y=b \qquad u=-U \quad \Rightarrow \quad A=\frac{-U}{b}-\frac{b}{2\mu}\frac{dp}{dx}+\frac{\rho b}{2\mu}g_x$$

$$\therefore \quad \frac{du}{dy}=\frac{1}{\mu}\frac{dp}{dx}y-\frac{\rho}{\mu}g_x y-\frac{U}{b}-\frac{b}{2\mu}\frac{dp}{dx}+\frac{\rho b}{2\mu}g_x$$

The shearing force on the moving plate:

$$F=\tau_o\times area$$

$$F=\mu\cdot\left.\frac{du}{dy}\right|_{y=b}\times area$$

area=1 m^2

$$F=-\frac{\mu U}{b}+\frac{b}{2}\frac{dp}{dx}-\frac{b}{2}\rho g_x$$

We have $g_x=g\cdot\sin\theta$, $\frac{dp}{dx}=\frac{-\Delta p}{l}$

$$F=\frac{-0.06\times 1}{0.01}-\frac{0.01}{2}\left(\frac{100\times 10^3}{1}\right)-\frac{0.01}{2}\times 900\times 9.81\times\sin 30$$

$$F=-528\,N$$

$$Power=F\cdot U$$

$$Power=528\times 1=528\,W \qquad \text{(Ans)}$$

University of Technology	Sheet No. 1
Mechanical Engineering Dep.	Navier-Stokes Equations
Aerodynamics (3 rd year)	2011/2012

1- Using the Navier-Stokes equations, determine the pressure gradient along flow, the average velocity, and the discharge for an oil of viscosity 0.02 N.s/m^2 flowing between two stationary parallel plates 1 m wide maintained 10 mm apart. The velocity midway between the plates is 2 m/s. [-3200 N/m^2 per m ; 1.33 m/s ; 0.0133 m^3/s]

2- An incompressible, viscous fluid is placed between horizontal, infinite, parallel plates as shown in figure. The two plates move in opposite directions with constant velocities U_1 and U_2. The pressure gradient in the x-direction is zero. Use the Navier-Stokes equations to derive expression for the velocity distribution between the plates. Assume laminar flow.

[$u = \frac{y}{b}(U_1 + U_2) - U_2$]

3- Two parallel plates are spaced 2 mm apart, and oil (μ = 0.1 N.s/m^2 , S = 0.8) flows at a rate of 24×10^{-4} m^3/s per m of width between the plates. What is the pressure gradient in the direction of flow if the plates are inclined at 60^o with the horizontal and if the flow is downward between the plates? [-353.2 kPa/m]

4- Using the Navier-Stokes equations, find the velocity profile for fully developed flow of water (μ = 1.14×10^{-3} Pa.s) between parallel plates with the upper plate moving as shown in figure. Assume the volume flow rate per unit depth for zero pressure gradient between the plates is 3.75×10^{-3} m^3/s. Determine:
a- the velocity of the moving plate.
b- the shear stress on the lower plate.
c- the pressure gradient that will give zero shear stress at y = 0.25b. (b = 2.5 mm)
d- the adverse pressure gradient that will give zero volume flow rate between the plates.
[3 m/s ; 1.37 N/m^2 ; 2.19 kN/m^2 per m ; -3.28 kN/m^2 per m]

5- A vertical shaft passes through a bearing and is lubricated with an oil (μ = 0.2 Pa.s) as shown in figure. Estimate the torque required to overcome viscous resistance when the shaft is turning at 80 rpm. (Hint: The flow between the shaft and bearing can be treated as laminar flow between two flat plates with zero pressure gradient). [0.355 N.m]

6- Determine the force on the piston of the figure due to shear, and the leakage from the pressure chamber for U = 0. [295.1 N ; 1.636×10^{-8} m^3/s]

7- A layer of viscous liquid of thickness b flows steadily down an inclined plane. Show that, by using the Navier-Stokes equations that velocity distribution is: $u = \frac{\gamma}{2\mu}\left(2by - y^2\right)\sin\theta$ and that the discharge per unit width is: $Q = \frac{\gamma}{3\mu} b^3 \sin\theta$

8- A wide moving belt passes through a container of a viscous liquid. The belt moving vertically upward with a constant velocity V_o, as illustrated in figure. Because of viscous forces the belt picks up a film of fluid of thickness h. Gravity tends to make the fluid drain down the belt. Use the Navier-Stokes equations to determine an expression for the average velocity v_{av} of the fluid film as it is dragged up the belt. Assume the flow is laminar, steady, and uniform.

[$v_{av} = V_o - \frac{\gamma h^2}{3\mu}$]

9- Determine the formulas for shear stress on each plate and for the velocity distribution for flow in the figure when an adverse pressure gradient exists such that Q = 0.

[$\tau_{y=0} = \frac{-2\mu U}{b}$; $\tau_{y=b} = \frac{4\mu U}{b}$; $u = 3U\frac{y^2}{b^2} - 2U\frac{y}{b}$]

10- A plate 2 mm thick and 1 m wide is pulled between the walls shown in figure at speed of 0.4 m/s. The space over and below the plate is filled with glycerin ($\mu = 0.62$ N.s/m^2). The plate is positioned midway between the walls. Using the Navier-Stokes equations, determine the force required to pull the plate at the speed given for zero pressure gradient; and the pressure gradient that will give zero volume flow rate.

[496 N ; 372 kN/m^2.m]

11- A slider plate 0.5 m wide constitutes a bearing as shown in figure. Estimate:

a- the load carrying capacity.

b- the drag.

c- the power lost in the bearing.

d- the maximum pressure in the oil and its location.

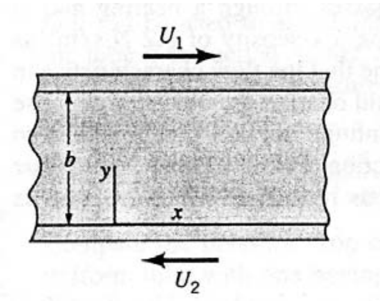
[739.6 kN ; 348.6 N ; 348.6 W ; 12500 kN/m^2 ; 150 mm]

12- Consider a shaft that turns inside a stationary cylinder, with a lubricating fluid in the annular region. Using the Navier-Stokes equation in θ-direction, show that the torque per unit length acting on the shaft is given by: $T = \frac{4\pi\mu\omega R_1^2}{\left(\frac{R_1}{R_2}\right)^2 - 1}$

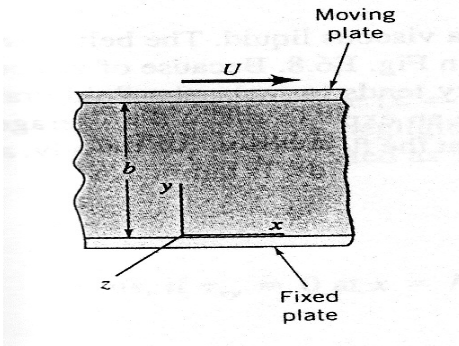
Where: ω = angular velocity of the shaft.

R_1 = radius of the shaft.

R_2 = radius of the cylinder.



Problem No. 2



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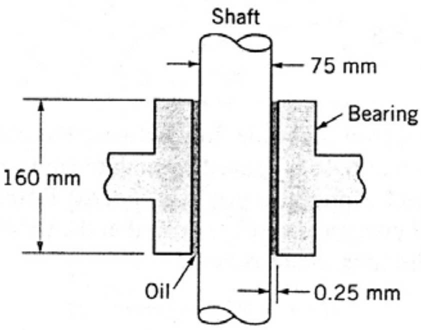

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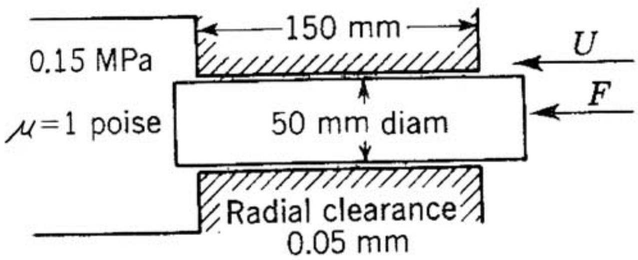

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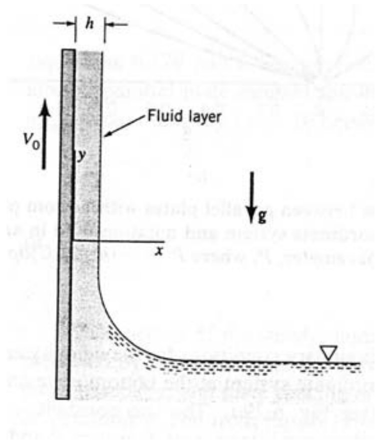

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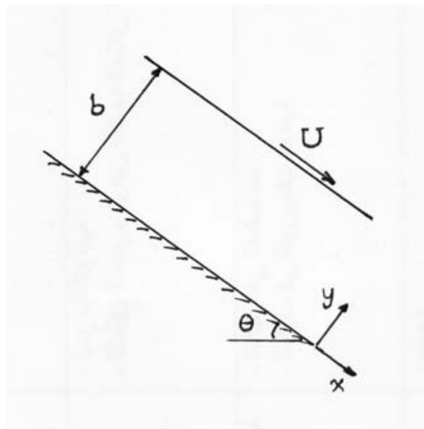
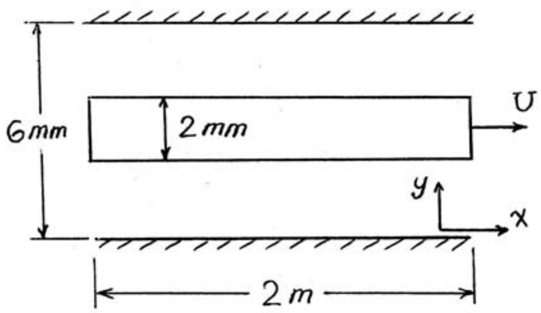


Problem No. 9



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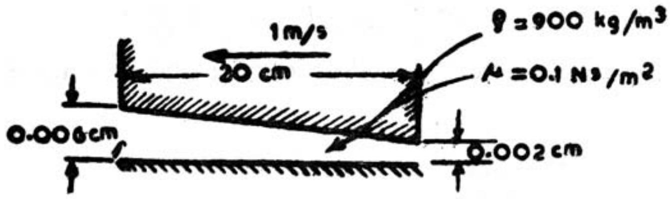


Problem No. 11

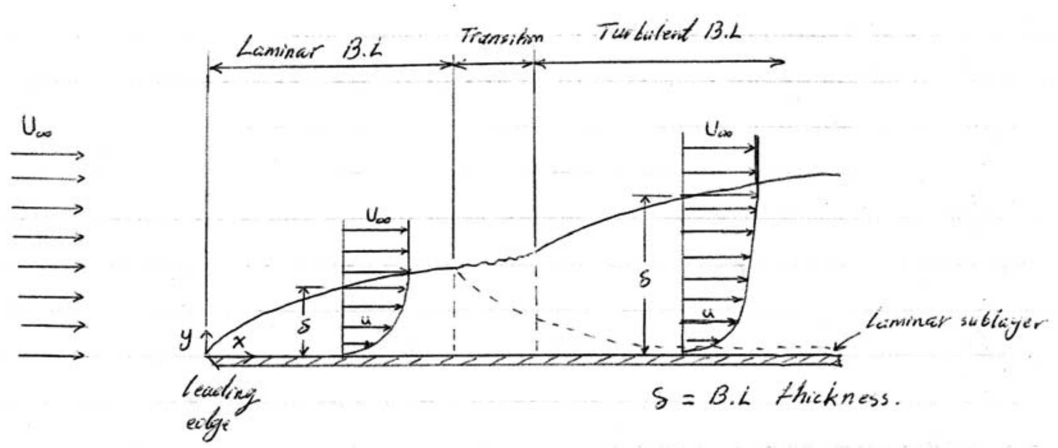
Chapter Two
Boundary Layer Theory

Contents

1- Introduction:

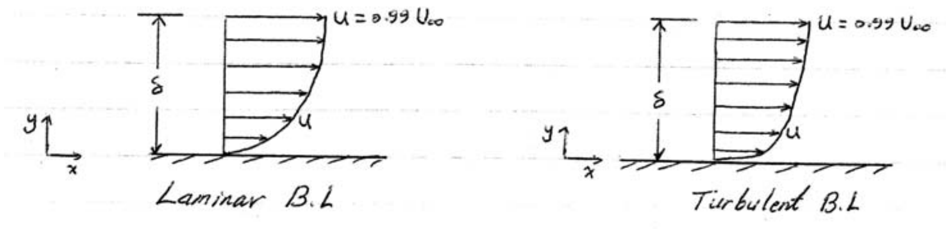
Development of boundary layer on a flat plate

The flow of a viscous fluid on a solid surface represents a region in which velocity increases from zero at the surface and approaches the velocity of the main stream. This region is known as *the boundary layer*.



The figure shows the development of a boundary layer on one side of a long flat plate held parallel to the flow direction.

Velocity distribution in boundary layer



The velocity gradient will give rise to a large shear stress at the wall τ_o (or τ_W).

$$\tau_o = \mu \left(\frac{du}{dy} \right)_{y=0} \qquad \tau_o \equiv \tau_w$$

As shown in figure:
the velocity gradient in the turbulent boundary layer is larger than that in the laminar boundary layer.

$$\left(\frac{du}{dy} \right)_{y=0} \text{(in turbulent BL)} \rangle \left(\frac{du}{dy} \right)_{y=0} \text{(in laminar BL)}$$

$$\therefore \quad \tau_o \ \text{(in turbulent BL)} \ \rangle \tau_o \ \text{(in laminar BL)}$$

The shear stress for a turbulent boundary layer is greater than the shear stress for a laminar boundary layer.

Boundary layer thickness (δ)

Boundary layer thickness is the distance from the solid surface to the point in the flow where $u = 0.99U_\infty$.

Displacement thickness (δ^*)

Displacement thickness represents the outward displacement of the streamlines caused by the viscous effects on the solid surface.

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U_\infty} \right) dy$$

Or

$$\delta^* = \delta \int_0^1 (1 - f(\eta)) d\eta \qquad \text{Where } \eta = \frac{y}{\delta} \quad \text{and} \quad f(\eta) = \frac{u}{U_\infty}$$

Momentum thickness (θ)

Momentum thickness, is defined as the thickness of a layer of fluid , with velocity U_∞ , for which the momentum flux is equal to the deficit of momentum flux through the boundary layer.

$$\theta = \int_0^\delta \frac{u}{U_\infty}\left(1-\frac{u}{U_\infty}\right)dy$$

Or

$$\theta = \delta\int_0^1 f(\eta)\left(1-f(\eta)\right)d\eta$$

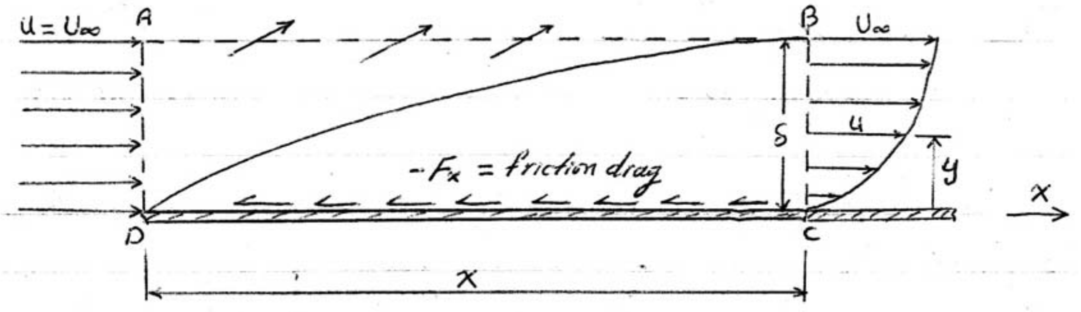
Shape factor (H)

H is a velocity profile shape factor.

$$H = \frac{\delta^*}{\theta}$$

2- Momentum equation for boundary layer:

Consider the control volume for flow over one side of a flat plate of width b.



$$-F_x = \rho b\int_0^\delta u^2 dy + \rho\left(U_\infty b\delta - b\int_0^\delta u dy\right)U_\infty - \rho\left(U_\infty b\delta\right)U_\infty$$

$$F_x = \rho b\int_0^\delta u\left(U_\infty - u\right)dy \qquad \text{------(*)}$$

F_x is the total friction drag on the plate from the leading edge up to x.

Assuming that the velocity profiles at various distances along the plate are similar to each other.

$$\frac{u}{U_\infty} = f\left(\frac{y}{\delta}\right) = f(\eta)$$

where

$$\eta = \frac{y}{\delta}$$

Equation (*) may be written as:

$$F_x = \rho b U_\infty^2 \delta \alpha \qquad \text{------(1)}$$

where

$$\alpha = \int_0^1 f(\eta)\left(1 - f(\eta)\right) d\eta$$

The local wall shear stress is :

$$\tau_o = \rho U_\infty^2 \alpha \frac{d\delta}{dx} \qquad \text{------(2)}$$

Equations (1) and (2) are valid for either laminar or turbulent flow in the boundary layer.

<u>3- Laminar boundary layer:</u>

The wall shear stress:

$$\tau_o = \mu \left(\frac{du}{dy}\right)_{y=0}$$

Let $\beta = \left[\frac{df(\eta)}{d\eta}\right]_{\eta=0}$

$$\Rightarrow \quad \tau_o = \frac{\mu U_\infty \beta}{\delta}$$

Another expression for shear stress:

$$\tau_o = \frac{1}{2} \rho U_\infty^2 c_f$$

Where c_f = local friction coefficient.

The total friction drag is:

$$F_f = b\int_0^L \tau_o dx$$

also

$$F_f = \frac{1}{2}\rho U_\infty^2 (b.L) C_f$$

Where C_f = total friction coefficient

The boundary layer thickness:

$$\delta = \sqrt{\frac{2\beta}{\alpha}}\frac{x}{\sqrt{\mathrm{Re}_x}}$$

Where Re_x = local Reynolds number

$$\mathrm{Re}_x = \frac{\rho U_\infty x}{\mu}$$

Blasius solution:

($\alpha = 0.135$; $\beta = 1.63$)

$$\frac{\delta}{x} = \frac{4.91}{\sqrt{\mathrm{Re}_x}}$$

$$\frac{\delta^*}{x} = \frac{1.721}{\sqrt{\mathrm{Re}_x}}$$

$$c_f = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$$

$$C_f = \frac{1.328}{\sqrt{\mathrm{Re}_L}}$$

Where Re_L = total Reynolds number

$$\mathrm{Re}_L = \frac{\rho U_\infty L}{\mu}$$

* The laminar boundary layer will remain laminar up to a value of $\mathrm{Re}_x = 500000$

4- Turbulent boundary layer:

Velocity profile in turbulent boundary layer:

$$\frac{u}{U_\infty} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}} = \eta^{\frac{1}{7}}$$

$$\therefore \quad \alpha = \int_0^1 f(\eta)(1 - f(\eta))d\eta = \int_0^1 \eta^{\frac{1}{7}}\left(1 - \eta^{\frac{1}{7}}\right)d\eta = \frac{7}{72}$$

The wall shear stress for the turbulent boundary layer on smooth plate is:

$$\tau_o = 0.023\rho U_\infty^2 \left(\frac{\mu}{\rho U_\infty \delta}\right)^{\frac{1}{4}} \quad \text{--------(**)}$$

$$\frac{\delta}{x} = \frac{0.377}{(\mathrm{Re}_x)^{\frac{1}{5}}}$$

$$c_f = \frac{0.0587}{(\mathrm{Re}_x)^{\frac{1}{5}}}$$

The total friction coefficient is calculated form the following relations:

$$C_f = \frac{0.0735}{(\mathrm{Re}_L)^{\frac{1}{5}}} \qquad \text{for } \mathrm{Re}_L \langle 10^7$$

$$C_f = \frac{0.455}{(\log_{10} \mathrm{Re}_L)^{2.58}} \qquad \text{for } \mathrm{Re}_L \rangle 10^7$$

and

$$F_f = \frac{1}{2}\rho U_\infty^2 (b.L) C_f$$

Note: equation (**) was obtained from the following pipe equations:

$$\tau_o = \frac{1}{8}\rho f U_{av}^2 \qquad (f = 4c_f)$$

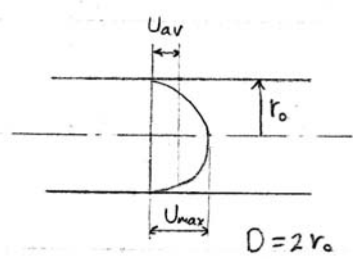
$$f = \frac{0.316}{(\text{Re})^{\frac{1}{4}}} \quad ; \quad \text{Re} = \frac{\rho D U_{av}}{\mu}$$

$$U_{av} = \frac{U_{max}}{1.235}$$

To transfer to the flat plate:

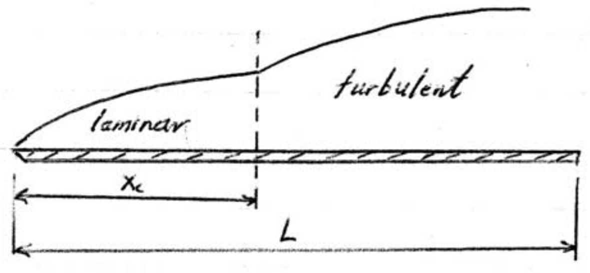
$$r_o \approx \delta \quad , \quad U_{max} \approx U_\infty$$

$$\therefore \quad \tau_o = \frac{1}{8}\rho \frac{0.316}{\left[\frac{\rho(2\delta)\frac{U_\infty}{1.235}}{\mu}\right]^{\frac{1}{4}}}\left(\frac{U_\infty}{1.235}\right)^2$$



$$\Rightarrow \quad \tau_o = 0.023\rho U_\infty^2 \left(\frac{\mu}{\rho U_\infty \delta}\right)^{\frac{1}{4}}$$

5- Friction drag in transition region:

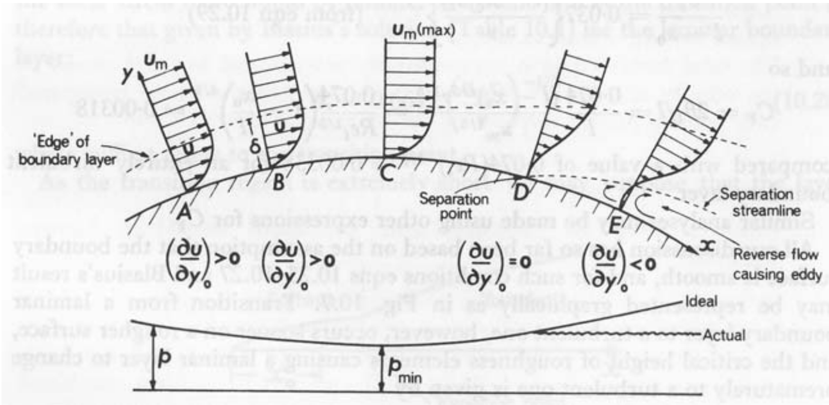


$F_f = F_{laminar} + F_{turbulent}$

$$F_f = \frac{1}{2}\rho U_\infty^2 (b.L) C_f$$

$$C_f = \left[1.328\frac{\sqrt{\text{Re}_{xc}}}{\text{Re}_L} + \frac{0.455}{(\log_{10}\text{Re}_L)^{2.58}} - 0.0735\frac{\text{Re}_{xc}^{\frac{4}{5}}}{\text{Re}_L}\right]$$

6- Effect of pressure gradient:



Consider the flow over a curved surface as shown in figure. As the fluid is deflected round the surface it is accelerated over the left-hand section (points A and B) until at point C, the velocity just outside the boundary layer is a maximum and the pressure is a minimum.

Beyond C, the velocity outside the boundary layer decreases, resulting in an increase in pressure. The velocity of the layer close to the wall is reduced and finally brought to a stop at D. Now the increasing pressure calls for further retardation so the boundary layer separates from the wall. At E there is a backflow (reverse flow) next to the wall, driven in the direction of decreasing pressure.

Down stream from the separation point the flow is characterized by irregular turbulent eddies. This disturbed region is called the wake of the body. The pressure within the wake remains close to that at the separation point. The pressure is always less than the pressure at the forward stagnation point.

An additional drag force is resulted from differences of pressure. This force is known as *the pressure drag (or form drag)*

** The total drag on a body is the sum of the friction drag and the pressure drag.

$$F_D = F_f + F_P$$

$$F_D = \frac{1}{2}\rho U_\infty^2 A C_D$$

Where A = projected area of the body perpendicular to the oncoming flow.

C_D = total drag coefficient.

Values of C_D for two- and three-dimensional bodies are given in figures (1) and (2) respectively.

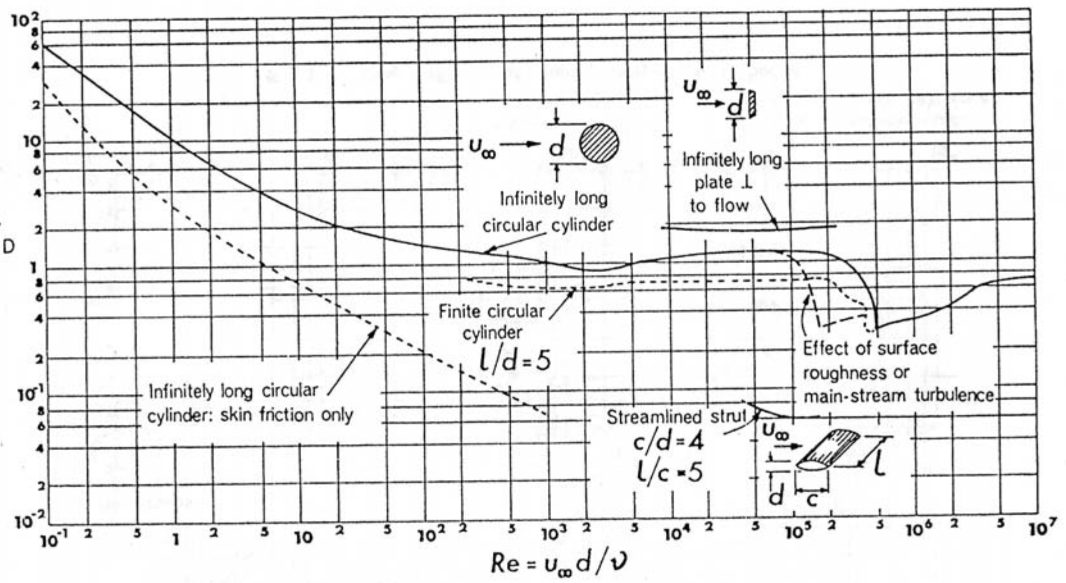


FIG. 1 Drag coefficient for two-dimensional bodies

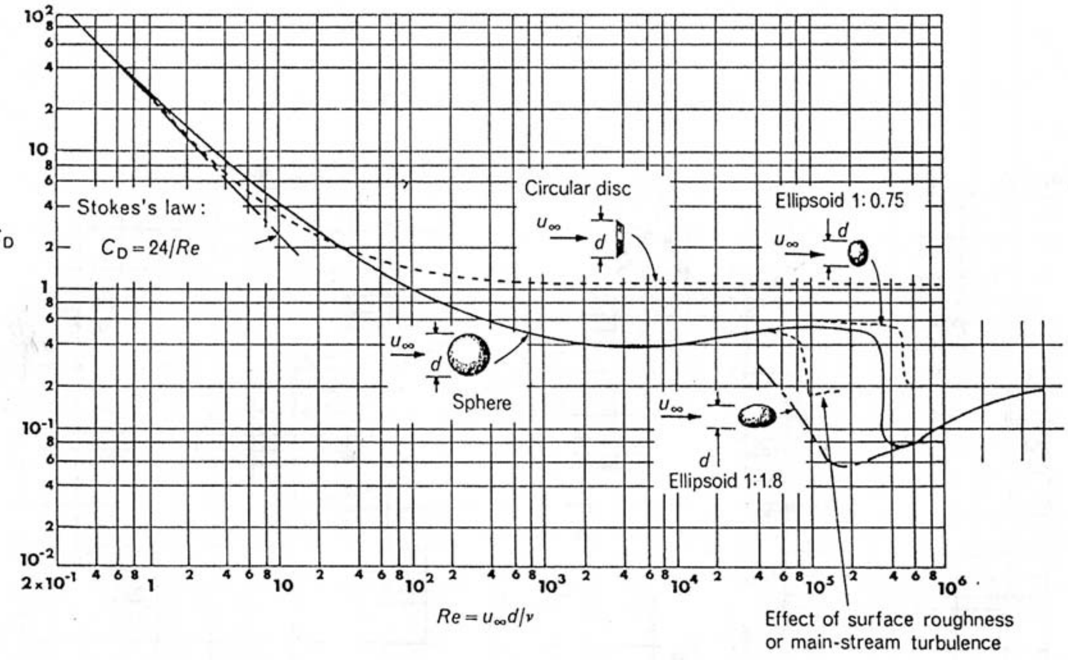


FIG. 2 Drag coefficients of smooth, axially-symmetric bodies

Typical drag coefficients for objects of interest are given in table (1).

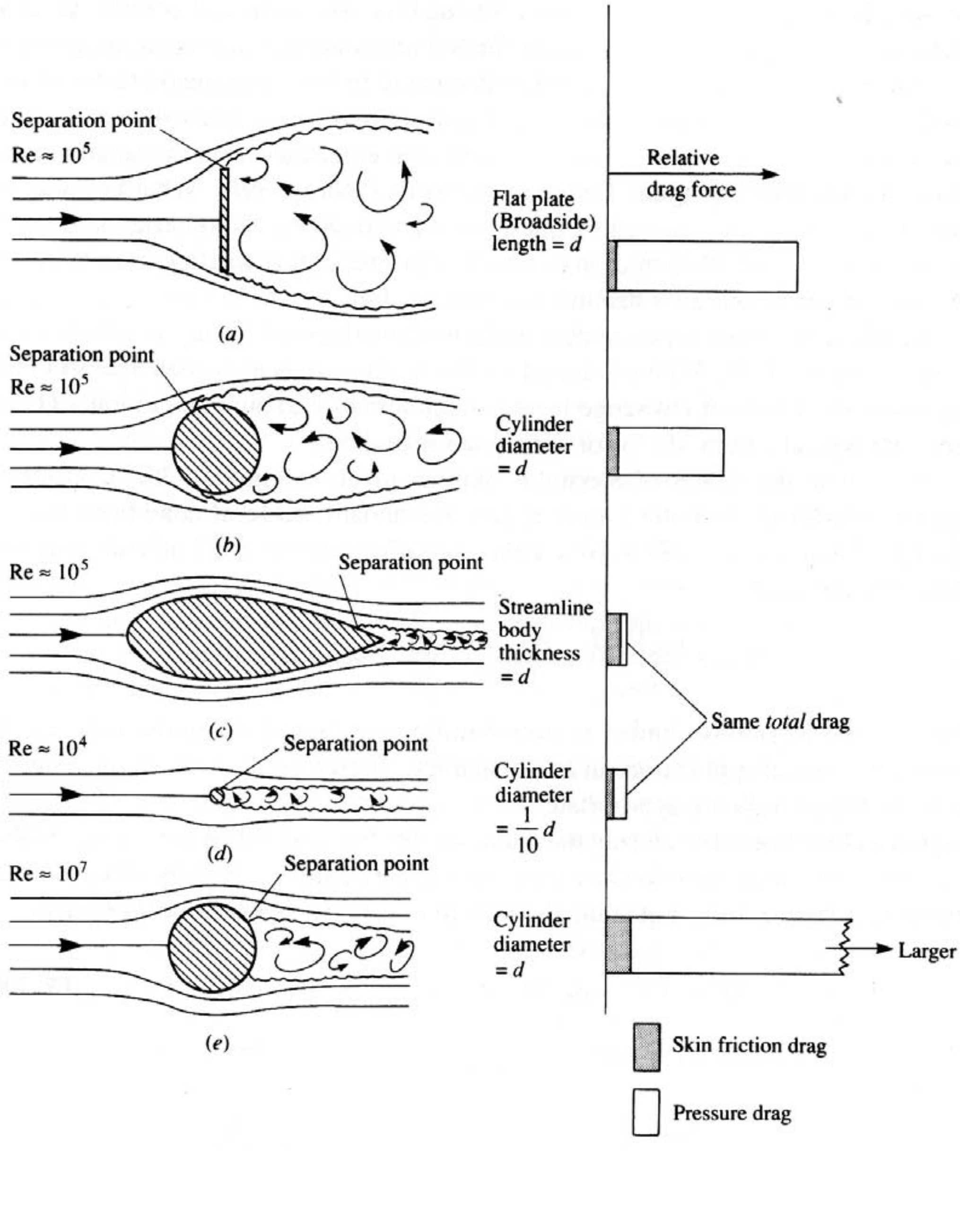
Shape	Reference area	Drag coefficient C_D
Parachute	Frontal area $A = \frac{\pi}{4}D^2$	1.4
Porous parabolic dish	Frontal area $A = \frac{\pi}{4}D^2$	Porosity: 0 / 0.2 / 0.5; → 1.42 / 1.20 / 0.82; ← 0.95 / 0.90 / 0.80; Porosity = open area/total area
Average person	Standing	$C_DA = 9\ ft^2$
	Sitting	$C_DA = 6\ ft^2$
	Crouching	$C_DA = 2.5\ ft^2$
Fluttering flag	$A = \ell D$	ℓ/D = 1: C_D = 0.07; ℓ/D = 2: C_D = 0.12; ℓ/D = 3: C_D = 0.15
Empire State Building	Frontal area	1.4
Six-car passenger train	Frontal area	1.8
Bikes: Upright commuter	$A = 5.5\ ft^2$	1.1
Racing	$A = 3.9\ ft^2$	0.88
Drafting	$A = 3.9\ ft^2$	0.50
Streamlined	$A = 5.0\ ft^2$	0.12
Tractor-trailor tucks: Standard	Frontal area	0.96
With fairing	Frontal area	0.76
With fairing and gap seal	Frontal area	0.70
Tree: $U = 10\ m/s$	Frontal area	0.43
$U = 20\ m/s$		0.26
$U = 30\ m/s$		0.20
Dolphin	Wetted area	0.0036 at $Re = 6 \times 10^6$ (flat plate has $C_{Df} = 0.0031$)
Large birds	Frontal area	0.40

Table (1)

- **Streamlined body:** the pressure drag on streamlined body is small and the friction drag is the major part of the total drag.

- **Bluff body:** the pressure drag on bluff body is much greater than the friction drag.

Relative comparison between skin friction drag and pressure drag for various aerodynamic shapes



7- Examples:

1- A smooth flat plate 3 m wide and 30 m long is towed through still water ($\rho = 998$ kg/m^3 , $\nu = 1.007\times10^{-6}$ m^2/s) with speed of 6 m/s. Determine the friction drag on one side of the plate and on the first 3 m of the plate.

Solution:

For the whole plate:-

$$Re_L = \frac{U_\infty L}{\nu} = \frac{6 \times 30}{1.007 \times 10^{-6}} = 1.787 \times 10^{8} > 5 \times 10^{5}$$

the B.L is turbulent.

$$C_f = \frac{0.455}{(\log_{10} Re_L)^{2.58}} = 0.00196$$

The drag on one side is:-

$$F_f = \frac{1}{2} \rho U_\infty^2 A C_f$$

$$F_f = \frac{1}{2} * 998 * 6^2 * (3*30) * 0.00196 = \underline{3169\ N} \quad (Ans)$$

For the first 3m of the plate: $Re_x = 1.787 \times 10^7 > 10^7$

$$F_f = \frac{1}{2} * 998 * 6^2 * (3*3) * \frac{0.455}{[\log_{10}(1.787 * 10^7)]^{2.58}}$$

$$F_f = \underline{443\ N} \quad (Ans)$$

2- Calculate the diameter of a parachute (in the form of a hemispherical shell) to be used for dropping a small object of mass 90 kg so that it touches the earth at a velocity no greater than 6 m/s. The drag coefficient for a hemispherical shell with its concave side upstream is approximately 1.32 for Re > 10^3, ($\rho = 1.22$ kg/m^3).

Solution:

$$F_D = \frac{1}{2} \rho U_\infty^2 A C_D \quad —①$$

also

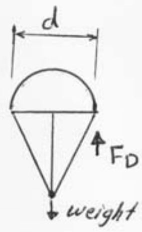
$$F_D = weight = m.g \quad —②$$

$$= 90 * 9.81 = 882.9 N$$

eqn ① = eqn ②

$$882.9 = \frac{1}{2} * 1.22 * 6^2 \frac{\pi d^2}{4} * 1.32$$

$$\Rightarrow \underline{d = 6.23\ m} \quad (Ans)$$



3- If $\frac{u}{U_\infty} = 2\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2$, find the thickness of the boundary layer, the shear stress at the trailing edge, and the drag force on one side of the plate 1 m long, if it is immersed in water flowing with a velocity of 0.3 m/s (ρ = 1000 kg/m^3 , μ = 0.001 Pa.s)

Solution:

$$Re_L = \frac{\rho L U_\infty}{\mu} = \frac{1000 * 1 * 0.3}{0.001} = 3 \times 10^5 < 5 \times 10^5$$

The flow is laminar ; assume the width of the plate = 1m

Velocity profile: $f(\eta) = 2\eta - \eta^2$

$$\alpha = \int_0^1 f(\eta)(1 - f(\eta))\, d\eta = \int_0^1 2\eta - \eta^2 (1 - 2\eta + \eta^2)\, d\eta = \frac{2}{15} = 0.133$$

$$\beta = \left[\frac{d f(\eta)}{d\eta}\right]_{\eta=0} = 2$$

Boundary layer thickness at the trailing edge :- $(x = L)$

$$\delta = \sqrt{\frac{2\beta}{\alpha}} \frac{L}{\sqrt{Re_L}} = 0.01\, m = \underline{10\, mm} \quad (Ans)$$

Shear stress at the trailing edge :-

$$\tau_o = \frac{\mu U_\infty \beta}{\delta} = \frac{0.001 * 0.3 * 2}{0.01} = \underline{0.06\ N/m^2} \quad (Ans)$$

$$F_f = \rho b U_\infty^2 \delta \alpha$$

$$F_f = 1000 * 1 * (0.3)^2 * 0.01 * 0.133$$

$$= \underline{0.12\ N} \quad (Ans)$$

University of Technology	Sheet No. 2
Mechanical Engineering Dep.	Boundary Layer Theory
Aerodynamics (3 rd year)	2011/2012

1- Calculate the displacement thickness and momentum thickness for the following velocity profiles in the boundary layer:

a- $\frac{u}{U_\infty} = 2\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2$; b- $\frac{u}{U_\infty} = \left(\frac{y}{\delta}\right)^{\frac{1}{9}}$ [$\frac{1}{3}\delta$; $\frac{2}{15}\delta$; 0.1δ ; $\frac{9}{110}\delta$]

2- Air ($\nu = 1.8\times10^{-5}$ m^2/s) flows along a flat plate with a velocity of 150 km/hr. How long does the plate have to be to obtain a laminar boundary layer thickness of 8 mm.

[6.146 m]

3- Assuming that the velocity distribution in the laminar boundary layer: $\frac{u}{U_\infty} = \sin\left(\frac{\pi y}{2\delta}\right)$.

Determine the total friction coefficient in terms of the Reynolds number. [$1.31/\sqrt{\mathrm{Re}_L}$]

4- A thin plate 2 m wide is placed in a uniform air stream of velocity 100 m/s, ($\rho = 1.2$ kg/m^3). If the skin friction drag force is 60 N, calculate the displacement thickness of the boundary layer at trailing edge of the plate. Assume that the velocity profile at all points in the boundary layer is: $f(\eta) = \eta^{1/6}$. [3.3 mm]

5- A river barge which is 50 m long and 12 m wide has flat bottom; therefore, its resistance is similar to one side of a flat plate. If the barge is towed at speed of 3 m/s through still water, what towing force is required to overcome viscous resistance and what is the boundary layer thickness at mid length? Assume the boundary layer is turbulent for the entire length. ($\rho = 1000$ kg/m^3 ; $\nu = 1.21\times10^{-6}$ m^2/s) [5.57 kN ; 0.26 m]

6- A uniform free stream of air at 0.8 m/s flows over a flat plate (4 m long × 1 m wide). Assuming the boundary layer to be laminar on the plate and the velocity profile is:

$\frac{u}{U_\infty} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$. Find the ratio of the drag force on the front half portion to the drag force on the rear half portion of the plate. ($\rho = 1.2$ kg/m^3 ; $\nu = 1.51\times10^{-5}$ m^2/s) [2.42]

7- Air flows over a horizontal smooth flat plate at speed 14.5 m/s. The plate length is 1.5 m and its width is 0.8 m. The boundary layer is turbulent from the leading edge. The velocity profile is: $\frac{u}{U_\infty} = \eta^{\frac{1}{6}}$ where $\eta = \frac{y}{\delta}$. Evaluate the boundary layer thickness and the wall shear stress at the trailing edge of the plate. ($\rho = 1.21$ kg/m^3 ; $\nu = 1.5\times10^{-5}$ m^2/s)

[30.75 mm ; 0.447 N/m^2]

8- Air (ρ = 1.21 kg/m^3) flows over a thin flat plate 1 m long and 0.3 m wide. The flow is uniform at the leading edge of the plate. Assume the velocity profile in the boundary layer is linear, and the free stream velocity is 2.7 m/s. Using control volume (abcd) shown in figure, compute the mass flow rate across surface (ab). Determine the magnitude and direction of the x- component of the force required to hold plate stationary.

[3.9×10^{-3} kg/s ; -3.5×10^{-3} N]

9- Estimate the power required to move a flat plate 9 m long and 3 m wide in oil (ρ = 920 kg/m^3 ; μ = 0.067 Pa.s) at 8 m/s. For the following cases:
a- the boundary layer is laminar over the surface of the plate.
b- the boundary layer is turbulent over the surface of the plate from the leading edge.
c- transition from laminar to turbulent at $Re_c = 5\times10^5$.
(Assume the velocity profile for the turbulent boundary layer is $f(\eta) = \eta^{1/9}$).

[8.5 kW ; 28.55 kW ; 18.05 kW]

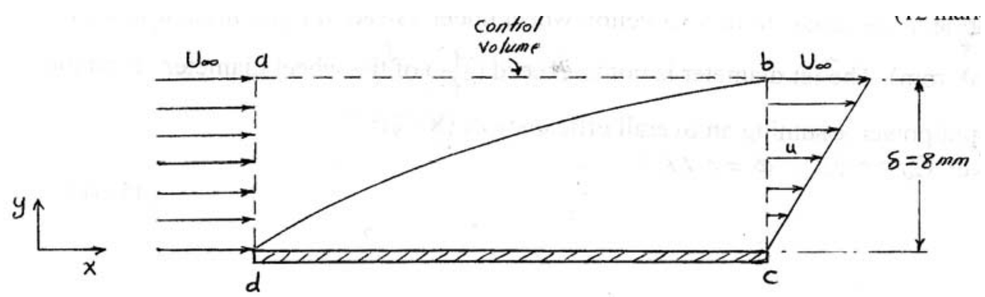
10- For the velocity profile: $\frac{u}{U_\infty} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$, determine whether the flow has separated or not separated or will attach with the surface after separation.

11- A honeycomb type of flow straightener is formed from perpendicular flat metal strips to give 25 mm square passages, 150 mm long. Water of kinematic viscosity 1.21 mm^2/s approaches the straightener at 1.8 m/s. Calculate the displacement thickness of the boundary layer and the velocity of the main stream at the outlet end of the straightener. Applying Bernoulli's equation to the main stream, deduce the pressure drop through the straightener. [0.546 mm ; 1.968 m/s ; 316.5 Pa]

12- Air of kinematic viscosity 15 mm^2/s and density 1.21 kg/m^3 flows past a smooth 150 mm diameter sphere at 60 m/s. Determine the drag force. What would be the drag force on a 150 mm diameter circular disc held perpendicular to this air stream.

[3 N ; 42 N]

13- The chimney of a boiler house is 50 m tall and has an outside diameter of 3 m. Compute the overturning moment about the base if a 30 m/s wind blows past it at the standard atmospheric conditions. (ρ = 1.21 kg/m^3 ; ν = 15×10^{-6} m^2/s) [1430 kN.m]



Problem No. 8

Chapter Three
Potential Flow Theory (Ideal Fluid)

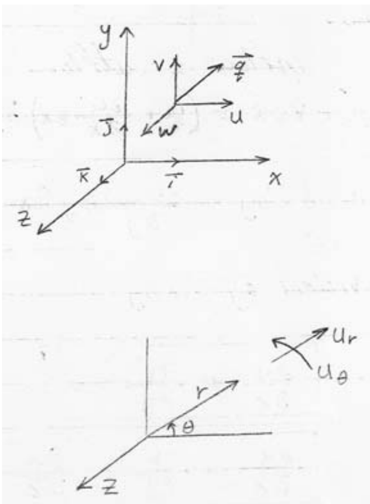
Contents

1- Introduction

Velocity vector

$\vec{q} = u\vec{i} + v\vec{j} + w\vec{k}$ In Cartesian coordinates

$\vec{q} = u_r\vec{r} + u_\theta\vec{\theta} + w\vec{k}$ In Polar coordinates



Divergence of $\vec{q} = \nabla \cdot \vec{q}$

$$\nabla \cdot \vec{q} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

Continuity equation

$\nabla \cdot \vec{q} = 0$

Or

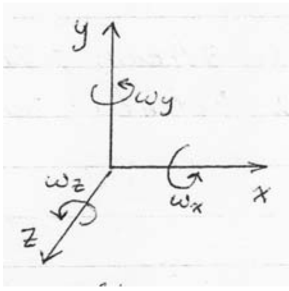
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Curl of $\vec{q} = \nabla \times \vec{q}$

Vorticity equation

$$\nabla \times \vec{q} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\vec{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\vec{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k}$$

$$\nabla \times \vec{q} = \omega_x \vec{i} + \omega_y \vec{j} + \omega_z \vec{k}$$



If $\nabla \times \vec{q} \neq 0$ the flow is called rotational

If $\nabla \times \vec{q} = 0$ the flow is called irrotational

2- Requirements for ideal- fluid flow

1- non viscous.
2- incompressible.
3- $\nabla \cdot \vec{q} = 0$
4- $\nabla \times \vec{q} = 0$

3- Relationships between stream function (ψ), potential function (φ) and velocity component

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial \phi}{\partial x}$$

$$v = -\frac{\partial \psi}{\partial x} = \frac{\partial \phi}{\partial y}$$

In cylindrical coordinates :

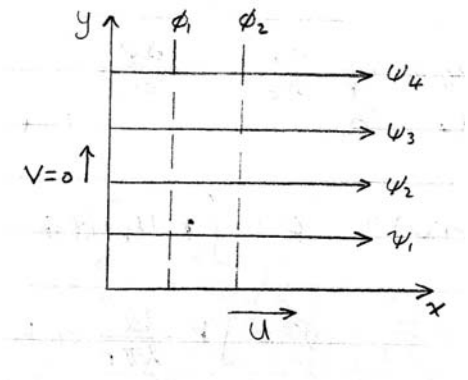
$$u_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = \frac{\partial \phi}{\partial r}$$

$$u_\theta = -\frac{\partial \psi}{\partial r} = \frac{1}{r}\frac{\partial \phi}{\partial \theta}$$

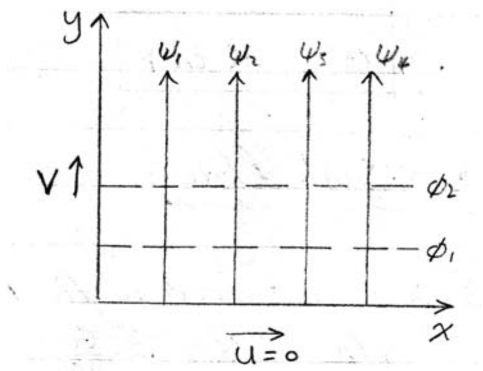
4- Basic flow patterns:

1- Uniform flow

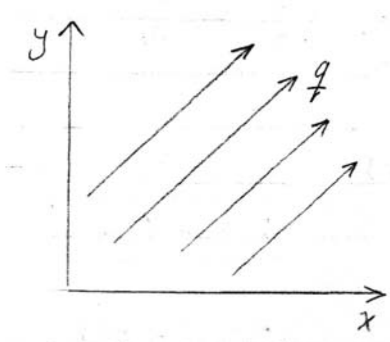
a- Uniform flow in the x- direction



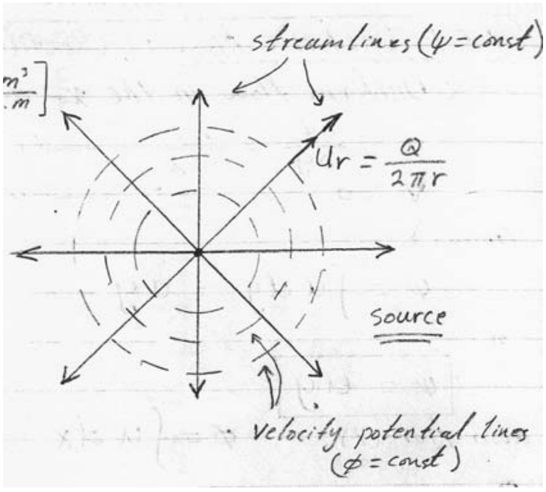
b- Uniform flow in the y- direction



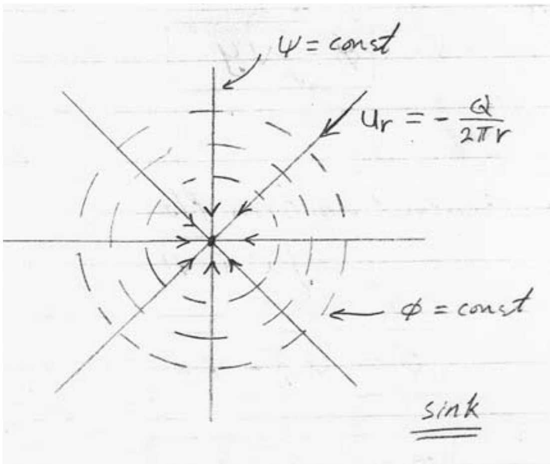
c- General uniform flow



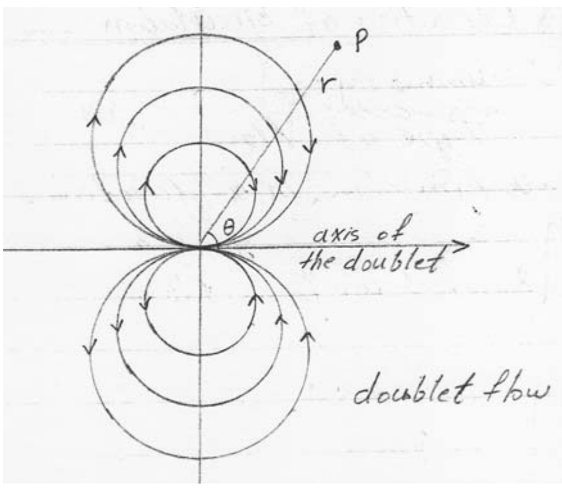
2- Source flow



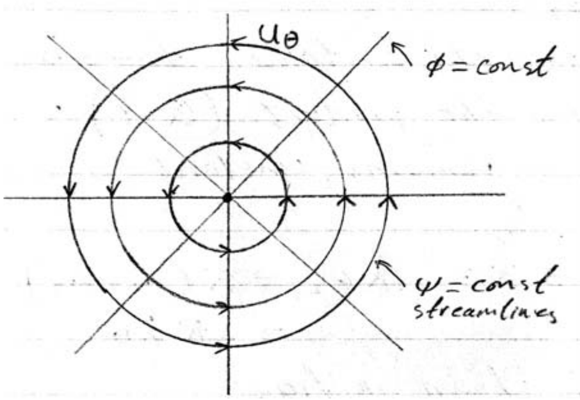
3- Sink flow



4- Doublet flow



5- Free vortex flow



Stream function and Potential function for Basic flow patterns:

Type of flow	ψ	φ
Uniform flow in the x- direction Uniform flow in the y- direction General uniform flow	uy -vx uy-vx	ux vy ux+vy
Source	kθ	k ln r
Sink	-kθ	-k ln r
Doublet	$\frac{-\mu}{2\pi}\frac{\sin\theta}{r}$	$\frac{\mu}{2\pi}\frac{\cos\theta}{r}$
Free vortex	$-\frac{\Gamma}{2\pi}\ln r$	$\frac{\Gamma}{2\pi}\theta$

Note: k = strength of the source = $\frac{Q}{2\pi}$ or (= $\frac{m}{2\pi}$)

Definition of circulation (Γ):

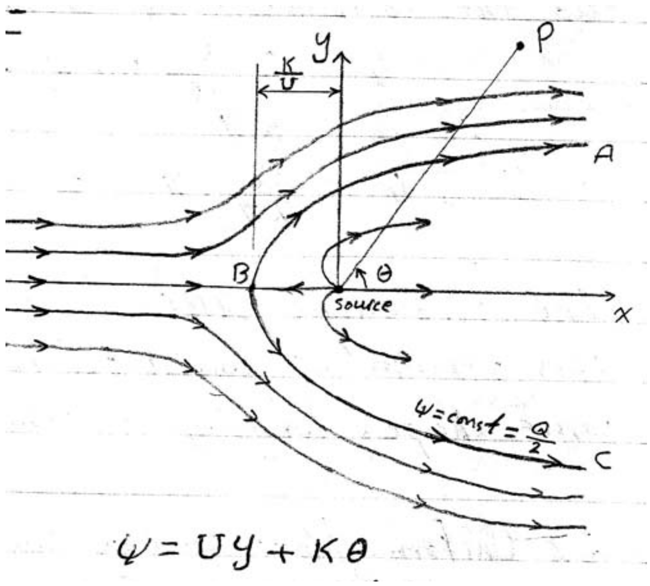
$$\Gamma = \oint_c q_s ds$$

Circulation = vorticity×area

$$\Gamma = \omega_z \times A$$

5- Combination of basic flows:

1- Uniform flow and a source.



The stream function:

$$\psi = Uy + k\theta$$

$$\psi = U.r\sin\theta + k\theta$$

The velocity components:

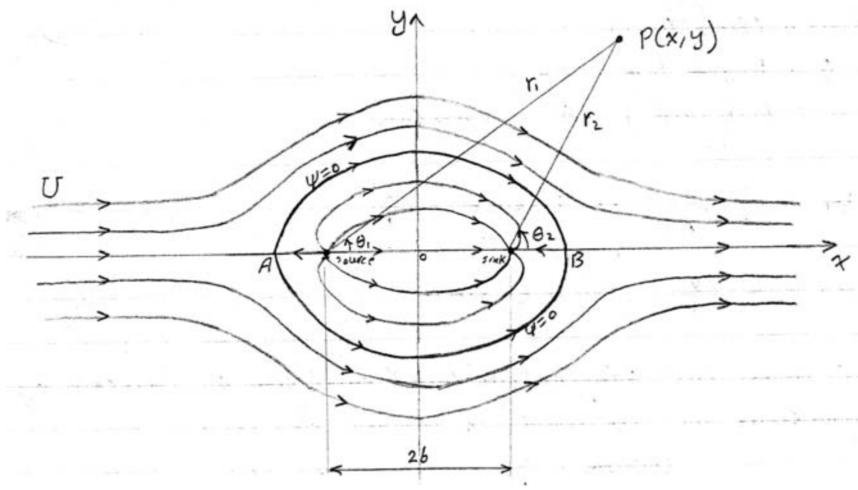
$$u_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = U.\cos\theta + \frac{k}{r}$$

and

$$u_\theta = -\frac{\partial\psi}{\partial r} = -U.\sin\theta$$

The dividing streamline ($\psi = \frac{Q}{2}$) could be replaced by a solid surface of the same shape, forming a semi-infinite body (half-body).

2- Uniform flow and a source-sink pair.



The stream function:

$$\psi = Uy + k\theta_1 - k\theta_2$$

$$\psi = Uy + k\tan^{-1}\frac{y}{x+b} - k\tan^{-1}\frac{y}{x-b}$$

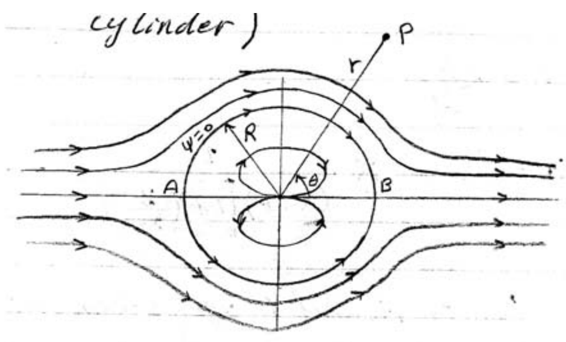
The velocity component:

$$u = \frac{\partial\psi}{\partial y} = U + \frac{k}{(x+b)\left[1+\left(\frac{y}{x+b}\right)^2\right]} - \frac{k}{(x-b)\left[1+\left(\frac{y}{x-b}\right)^2\right]}$$

The dividing streamline ($\psi = 0$) could be replaced by a solid surface of the same shape, forming an oval called a Rankine oval.

3- Uniform flow and a doublet:

(Non lifting flow over a cylinder)



The stream function:

$$\psi = Uy - \frac{\mu}{2\pi}\frac{\sin\theta}{r}$$

$$\psi = U.r\sin\theta\left(1 - \frac{R^2}{r^2}\right)$$

where

$$R^2 = \frac{\mu}{2\pi U}$$

The velocity components:

$$u_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = U.\cos\theta\left(1 - \frac{R^2}{r^2}\right)$$

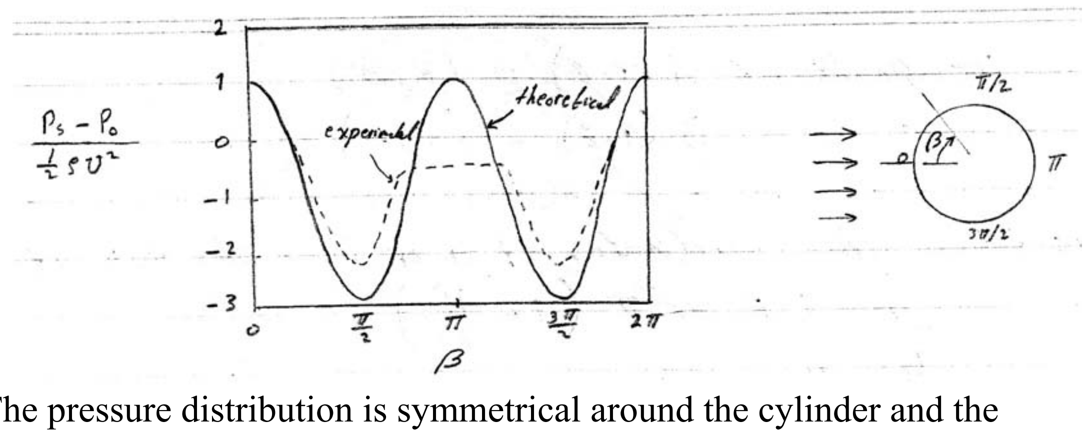
and

$$u_\theta = -\frac{\partial\psi}{\partial r} = -U.\sin\theta\left(1 + \frac{R^2}{r^2}\right)$$

The dividing streamline ($\psi = 0$) could be replaced by a solid surface of the same shape, forming a circular cylinder with radius $R = \sqrt{\frac{\mu}{2\pi U}}$.

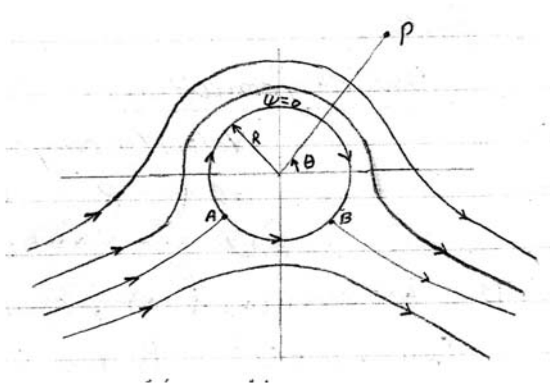
The pressure distribution on the cylinder surface is obtained from:

$$P_s = P_o + \frac{1}{2}\rho U^2\left(1 - 4\sin^2\theta\right)$$



The pressure distribution is symmetrical around the cylinder and the resultant force developed on the cylinder = zero.

4- Doublet and free vortex in a uniform flow:
(Lifting flow over a cylinder)



The stream function:

$$\psi = U.r\sin\theta\left(1-\frac{R^2}{r^2}\right)+\frac{\Gamma}{2\pi}\ln r$$

The velocity components:

$$u_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = U.\cos\theta\left(1-\frac{R^2}{r^2}\right)$$

and

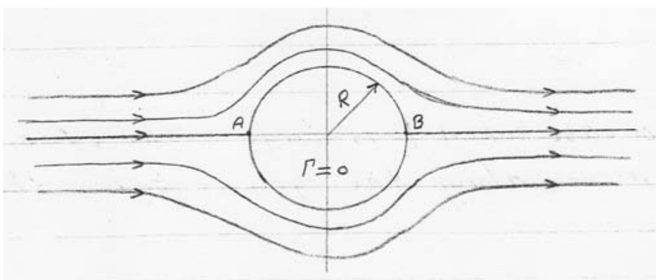
$$u_\theta = -\frac{\partial\psi}{\partial r} = -U.\sin\theta\left(1+\frac{R^2}{r^2}\right)-\frac{\Gamma}{2\pi r}$$

The location of the stagnation points is given by:

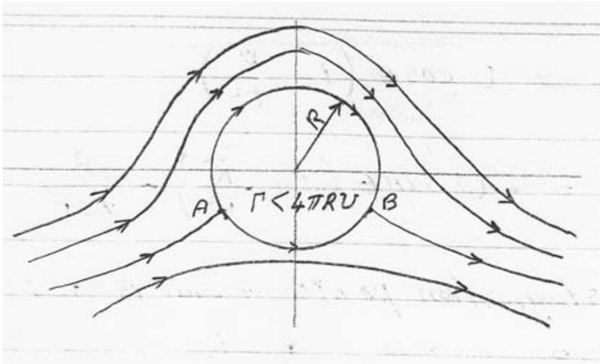
$$r = R \;\; ; \;\; \sin\theta = \left(\frac{-\Gamma}{4\pi RU}\right)$$

There are four possible cases:

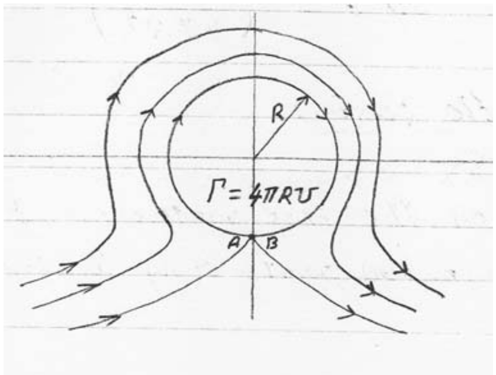
a- ($\Gamma = 0$)



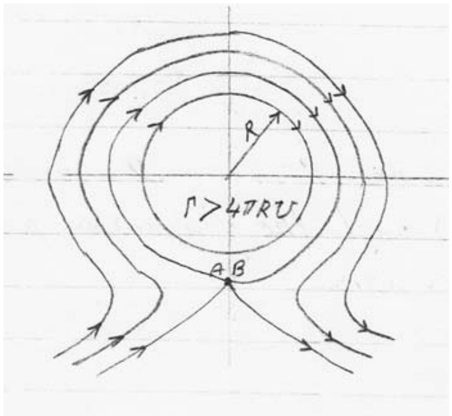
b- (Γ < 4πRU)



c- (Γ = 4πRU)



d- (Γ > 4πRU)

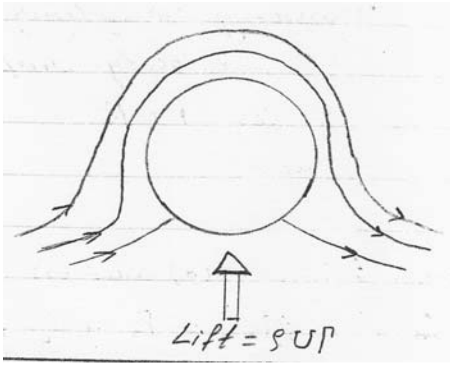


The pressure distribution on the cylinder surface is obtained from:

$$P_s = P_o + \frac{1}{2}\rho U^2 - \frac{1}{2}\rho\left(-2U.\sin\theta - \frac{\Gamma}{2\pi R}\right)^2$$

The lift force on the cylinder is

Lift = $\rho U \Gamma L$ where L = length of the cylinder



6- Examples:

1- Does the stream function ($\psi = xy$) represent a physically possible flow? If so, determine the velocity at a point (2,3).

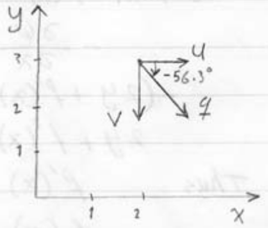
Solution:

To prove that the flow is possible, the continuity equation must be satisfied.

$$\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial y \partial x} = 0$$

$$1 - 1 = 0 \checkmark$$

$$u = \frac{\partial \psi}{\partial y} = x \quad ; \quad v = -\frac{\partial \psi}{\partial x} = -y$$



at point (2, 3)

$$u = x = 2$$

$$v = -y = -3$$

$$q = \sqrt{u^2 + v^2} = \sqrt{4 + 9} = 3.6 \text{ units} \quad \text{(Ans)}$$

$$\theta = \tan^{-1} \frac{v}{u} = \frac{-3}{2} \implies \theta = -56.3^\circ \quad \text{(Ans)}$$

2- A velocity potential in two-dimensional flow is given by ($\varphi = y+x^2-y^2$); find the stream function for this flow.

Solution:

a)- $\phi = y + x^2 - y^2$

$$\frac{\partial \psi}{\partial y} = \frac{\partial \phi}{\partial x} = 2x$$

$$\partial \psi = 2x\, \partial y$$

$$\therefore \quad \psi = 2xy + f(x) \quad \text{———} (*) \qquad f(x) = \text{constant of integration.}$$

To find $f(x)$

$$-\frac{\partial \psi}{\partial x} = \frac{\partial \phi}{\partial y}$$

$$-(2y + f'(x)) = 1 - 2y$$

$$2y + f'(x) = -1 + 2y$$

Thus $\quad f'(x) = -1$

$$f(x) = -x + C$$

sub in eqn $(*)$ $\Rightarrow$ $\psi = 2xy - x + C$ (Ans)

3- A stream function in two-dimensional flow is ($\psi = 9+6x-4y+7xy$); find the velocity potential for this flow.

Solution:

$\psi = 9 + 6x - 4y + 7xy$

$$\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y} = -4 + 7x$$

$$\therefore \quad \phi = -4x + \frac{7}{2}x^2 + f(y) \qquad \text{where } f(y) = \text{constant of integration.}$$

to find $f(y)$

$$\frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}$$

$$f'(y) = -(6 + 7y) = -6 - 7y$$

$$\therefore \quad f(y) = -6y - \frac{7}{2}y^2 + C$$

$$\therefore \quad \phi = -4x - 6y + \frac{7}{2}(x^2 - y^2) + C \qquad \text{(Ans)}$$

University of Technology	Sheet No. 3
Mechanical Engineering Dep.	Ideal Fluid Flow
Aerodynamics (3 rd year)	2011/2012

1- Show that the two-dimensional flow described by the equation $\psi = x + 2x^2 - 2y^2$ is irrotational. Find the velocity potential for this flow. [$\phi = -y - 4xy + c$]

2- A certain flow field is described by the velocity potential $\phi = A\ln r + Br\cos\theta$ where A and B are positive constants. Determine the corresponding stream function and locate any stagnation points in this flow field. [$\psi = A\theta + Br\sin\theta + c$; $\left(\frac{A}{B},0\right)$ $\left(\frac{A}{B},\pi\right)$]

3- The velocity components in a two-dimensional flow field for an incompressible fluid are expressed as: $u = \frac{y^3}{3} + 2x - x^2 y$; $v = xy^2 - 2y - \frac{x^3}{3}$.

a) show that these functions represents a possible case of irrotational flow.

b) obtain expressions for the stream function and velocity potential.

4- The formula $\phi = 0.04x^3 + axy^2 + by^3$ represent the velocity potential of a two-dimensional ideal flow. Evaluate the constants a and b, and calculate the pressure difference between the points (0,0) and (3,4)m, if the fluid has density of 1300 kg/m^3.

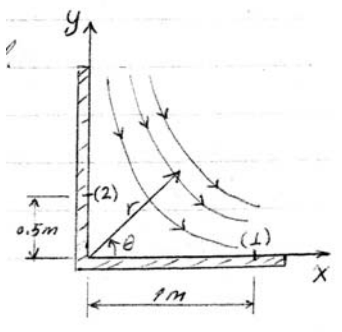
[a = -0.12, b = 0 ; 5.85 kN/m^2]

5- The two-dimensional flow of a non-viscous, incompressible fluid in the vicinity of the 90^o corner of figure is described by the stream function $\psi = 2r^2 \sin 2\theta$.

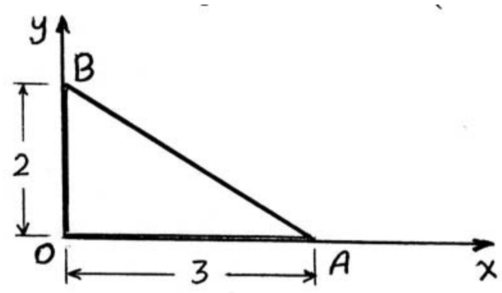
a) determine the corresponding velocity potential.

b) if the pressure at point(1,0) on the wall is 30kPa, what is the pressure at point (0,0.5) , assume ρ = 1000 kg/m^3 , and x-y plane is horizontal. [$\phi = 2r^2 \cos 2\theta + c$; 36 kPa]

6- The stream function for an incompressible flow filed is given by the equation $\psi = 3tx^2 y - ty^3$. Find the potential function and determine the flow rates across the faces of the triangular prism OAB shown in figure having a thickness of 5 units in the z-direction at time t = 1. [$\phi = tx^3 - 3txy^2 + c$; 40; 0; 40]



Problem No. 5



Problem No.6

7- Prove that for a two-dimensional flow, the vorticity at a point is twice the rotation (angular velocity).

8- The pressure far from an irrotational vortex in the atmosphere is zero gage. If the velocity at r = 20 m is 20 m/s, find the velocity and pressure at r = 2 m. ($\rho = 1.2$ kg/m^3)

[200 m/s ; -23.76 kPa]

9- A non viscous incompressible fluid flow between wedge shaped-wall into small opening as shown in figure. The velocity potential which described the flow is $\phi = -2\ln r$. Determine the volume rate of flow (per unit length) in the opening. [-π/3 m^3/s per m]

10- A source with strength 0.2/2π m^3/s.m and a vortex with strength 1/2π m^2/s are located at the origin. Determine the equations for velocity potential and stream function. What are the velocity components at x = 1 m , y = 0.5 m? [0.0285 m/s ; 0.143 m/s]

11- In an infinite two-dimensional flow filed, a sink of strength 3/2π m^3/s.m is located at the origin, and another of strength 4/2π m^3/s.m at (2 , 0). What is the magnitude and direction of the velocity at point (0 , 2). [0.429 m/s ; -68.22^o]

12- Flow over a plane half-body is studied by utilizing a free-stream at 5 m/s superimposed on a source at the origin. The body has a maximum width 2 m. Calculate:

a) the coordinates of the stagnation point.

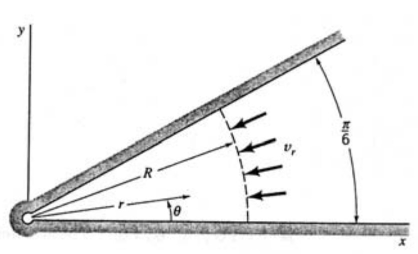
b) the width of the body at the origin.

c) the velocity at a point (0.5 , π/2). [(0.32 , π) ; 1 m ; 5.93 m/s]

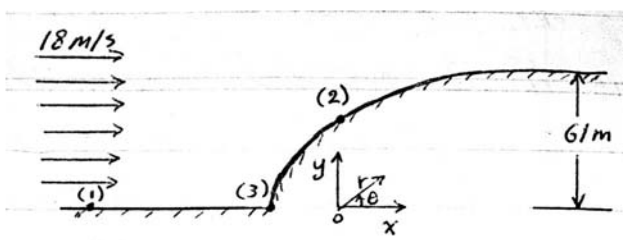
13- The shape of a hill arising from a plain can be approximated with the top section of a half-body as is shown in figure. The height of the hill approaches 61 m. When a 18 m/s wind blows toward the hill, what is the magnitude of the air velocity at point (2) above the origin. What is the elevation of point (2) and what is the difference in pressure between point (1) and point (2). ($\rho_{air} = 1.23$ kg/m^3) [21.34 m/s ; 30.5 m ; 448.83 Pa]

14- A circular cylinder 0.5 m diameter rotates at 600 rpm in a uniform stream of 15 m/s. Locate the stagnation points. Calculate the minimum rotational speed for detached stagnation point in the same uniform flow. [-31.6^o and -148.4^o ; 1146 rpm]

15- A circular cylinder 20 m long is placed in a uniform stream of 100 m/s (ρ= 0.7 kg/m^3). The lift force generated by the cylinder is 2100 kN. The stagnation points are at (-60^o and -120^o). Derive a relationship between the locations of the stagnation points and the circulation around the cylinder. Calculate the diameter of the cylinder. [2.75 m]



Problem No. 9



Problem No. 13