Vibration Analysis of Uniform and Tapered Composite Beams with Embedded Shape Memory Alloy

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Abstract

In this study, laminated composite materials were hybridized with E-glass fiber and Nitinol (Nickel-Titanium) wires. Hand lay-up technique was used to prepare the samples, epoxy resin type (Sikadur 52 N) was used as matrix reinforced by one fiber from E-glass fiber woven roving with embedded nitinol wires with a diameter 0.5 mm for samples and number of wires such as 0, 1, 3, 5 and 9 to find the effect of the number of wires on the natural frequency. The samples were fixed as a cantilever beam. The effects of increasing the number of nitinol wires, the diameter of nitinol wires, the length of the cantilever beam and the thickness of beam on the natural frequencies of the beam were studied. Also, the effects of the tapered in width side and thickness side on the natural frequencies of cantilever beam were studied. The results showed that the increasing in the number of nitinol wires and the diameter of nitinol wires lead to decrease the natural frequency in martensite phase and increase the natural frequency in austenite phase. Also, the increasing in thickness of beam and width ratio of the beam lead to increase the natural frequency. As well as, the increasing in the thickness ratio leads to increase the first natural frequency and decrease the second and third ones. In addition, the increasing in the length of the beam decreases the natural frequency.

Key words: Shape Memory Alloy (SMA), Cantilever beam, Natural frequency.

Symbol	Definition	Units
A	Cross section area of the specimen	mm^2
b_{in}	Width of root side of cantilever beam	mm
b_{out}	Width of free side of cantilever beam	mm
E_c, E_m, E_r	Modulus of elasticity of composite material, epoxy and fiber	MPa
E_b	Modulus of elasticity of the beam	MPa
E_n	The modules of elasticity of the nitinol wire	MPa
h_{in}	Thickness of root side of cantilever beam	mm
h_{out}	Thickness of free side of cantilever beam	mm
I	Cross section moment of inertia	mm^4
ξ_c, ξ_m, ξ_r	Volume fraction of composite material, epoxy and fiber	
ξ_n	The ratio of the total cross sectional area of wires to the cross sectional area of the beam.	
ρ_c, ρ_m, ρ_r	The density of the composite material, epoxy and fiber	Kg/m^3
ρ_n	The density of the nitinol	Kg/m^3
ρ_b	The density of the specimen	Kg/m^3
ω_n	The natural frequency	rad/sec
Width ratio	The ratio width of root side of cantilever beam to width of free side of cantilever beam (b_{in}/b_{out})	
Thickness ratio	The ratio thickness of root side of cantilever beam to thickness of free side of cantilever beam (h_{in}/h_{out})	

1. Introduction

Recently, Shape Memory Alloys (SMAs) have been on the front of research due to that the SMAs are unique alloys having ability to remember an original shape after being deformed. The natural frequencies have a big influence on the design of system that exposed to vibrations because when the system worked at a frequency near to the natural frequencies, the acceleration will be at the highest value, which lead to the failure of the system. Lau et al. [1] estimated the natural frequency of glass fiber composite beams with embedded shape memory alloy (SMA) wires. The results showed the natural frequencies of all the beams decrease with increasing number of SMA wires in martensitic phase. Dezfuli et al. [2] used the nitinol wires as reinforcement, into plate made up Aluminum as a matrix. The results showed the phase transformation starting from martensite to austenite phase and that led to increasing in the stiffness of the plate, which results in increasing in the natural frequency. M. Yuvaraja and M. Senthilkumar [3] compared between utilizing piezoelectric (PZT) based and SMA wires composites on the vibration characteristics. The smart composite cantilever beam contains a glass fiber reinforced polymer (GFRP) with attached SMAs externally and with surface bonded PZT. The results demonstrated using SMA wires is more effective than using PZT. Cem and Mustafa [4] investigated the free vibration analysis on a blade of Air X 4140W horizontal axis wind turbine. 4140 steel, shape memory alloys (Ni-Ti, Cu-Zn-Al and Cu-Al-Ni) in blade root connection was used. The results showed that the maximum total deformation was observed in the fifth mode of the natural frequency for the blade root connection from Cu-Zn-Al alloy. Gupta et al. [5] studied the effect of shape memory alloys to increase the damping of glass fiber reinforced plastic (GFRP) composites. The results exhibited the damping ratio of SMA hybrid composite beam was found to be higher as compared to the pristine and steel hybrid GFRP composite beam.

The main target of this work is to study the effect of the number and diameter of nitinol wires and the length, thickness, width ratio and thickness ratio of cantilever beams on the natural frequency.

2. Experimental work

2.1 Used Materials

The materials used to prepare the samples consist of epoxy resin as a matrix (type Sikadur 52 N), Table (4.1) lists the properties of the epoxy used in this work [6]. And two reinforced composite materials, which are shape memory alloy wires (Nitinol), a near ideal 50/50 combination of nickel and titanium. The diameters used of nitinol wires are 0.5 mm. Table (4.2) contains the physical properties of Nitinol [7] and E-glass fiber woven roving. Table (4.3) lists the properties of the fibers type E-glass [8].

Table (1): The properties of epoxy [6]

Properties	Values
Density (kg/m^3)	1100
Modulus of Elasticity (MPa)	1800
Poisson's Ratio	0.35

Table (2): Properties of the nitinol wire [7]

Properties	Values
Melting point (°C)	1240-1310
Density (kg/m^3)	6450
Modulus of Elasticity (GPa)	28 (martensite)
	70 (austenite)
Poisson's Ratio	0.33
Austenite start temperature (A_s) °C	32-36°C
Austenite finish temperature (A_f) °C	45 °C

Table (3): The properties of the E-glass [8].

Properties	Values
Density (kg/m^3)	2580
Modulus of Elasticity (MPa)	72000
Poisson's Ratio	0.22

2.2 Samples

The technique used to form the samples is hand lay-up. The uniform samples have dimensions (220 mm x 50 mm x 4 mm), as shown in figure (1), while the tapered samples are designed as follows: the width of root side is constant 50 mm and the width of free side is variable (20 mm, 30 mm and 40 mm), the thickness is 4 mm and the length is 220 mm, in all samples the length decreases 20 mm for fixation.

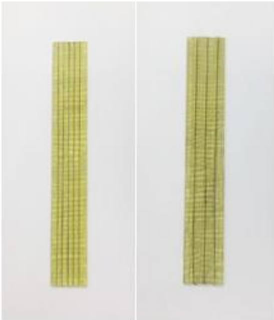

Figure (1): Samples of composite materials with embedded nitinol wires

2.3 Vibration Test

The vibration system was used to find the natural frequency, see figure (2).

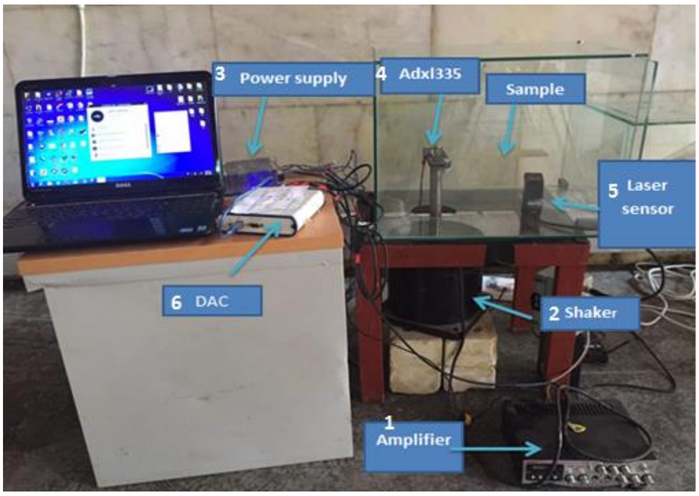

Figure (2): Vibration system

The experimental frequency-amplitude curve was found from Fast Fourier Transformation (FFT) screen by using LABVIEW program in vibration system test, see figure (3).

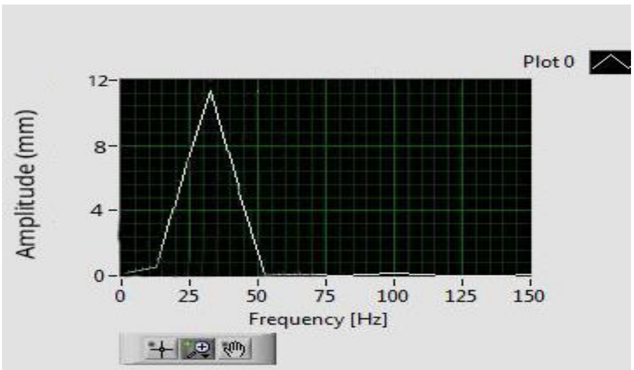

Figure (3): Output of Fast Fourier Transformation (FFT) analyze.

3. Theoretical Analysis

3.1 Rule of Mixture

This rule was used to estimate the density and modulus of elasticity.

The density and the modulus of elasticity of the composite beam (epoxy and E-glass fiber) is calculated from the these equation [9]

$$\rho_c = \sum \rho * \xi = \rho_m * \xi_m + \rho_r * \xi_r \quad \text{.......... (1)}$$

$$E_c = E_m \ \xi_m + E_r \ \xi_r \quad \text{.......... (2)}$$

The density and the modulus of elasticity of beam, which consists of composite materials and nitinol wires is calculated from these equations [1]

$$\rho_b = \rho_c + (\rho_n - \rho_c) * \xi_n \quad \text{.......... (3)}$$

$$E_b = E_c + (E_n - E_c) * \xi_n \quad \text{.......... (4)}$$

3.2 Exact solution of natural frequency of cantilever beam

When the beam is uniform and assumed cantilever beam as Euler-Bernoulli beam as shown in figure (4).

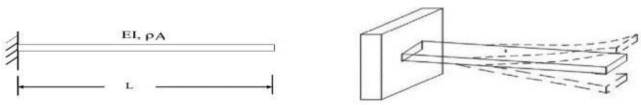

Figure (4): Cantilever beam

The natural frequencies can be found from the equation [10]:

$$\omega_n = \left(\frac{2n-1}{2}\pi + e_n\right)^2 * \frac{1}{l^2} * \sqrt{\frac{E\,I}{\rho\,A}} \ \text{(rad/sec)} \quad \text{.......... (5)}$$

n = 1, 2 . . . ∞

Where e_n is small correction terms and obtained $e_1 = 0.3042$, $e_2 = -0.018$, $e_3 = 0.001$

The unit of the natural frequency is converted from (rad/sec) to Hz by the following equation:

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi} * \left(\frac{2n-1}{2}\pi + e_n\right)^2 * \frac{1}{l^2} * \sqrt{\frac{E\,I}{\rho\,A}} \quad \text{.......... (6)}$$

3.3 Finite Element Analysis (FEA)

ANSYS Workbench software, version 17, is for the finite element analysis to investigate the natural frequencies and its vibration mode shapes, see figure (5).

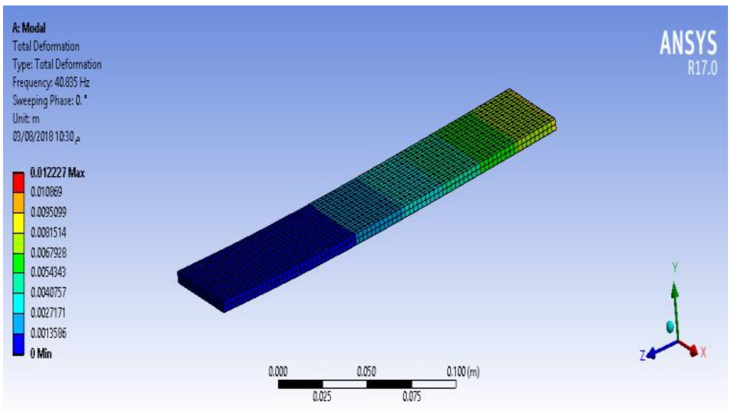

Figure (5): First mode shape, modal response of the uniform cantilever composite beam

4. Results and Discussions

4.1 The Density and Modulus of Elasticity

The rule of mixtures was used to calculate the density and the modulus of elasticity for uniform beam.

In figures (6 and 7), the samples contain epoxy reinforced by E-glass fiber layer with embedded nitinol wires. In figure (6) the density increased by 7.74% if the number of embedded nitinol wires rose from 1 to 20 wires. And in figure (7) the modulus of elasticity increased by 5.02% if the number of embedded nitinol wires rose from 1 to 20 in the phase of martensite, also the modulus of elasticity increased by 13.914% the number of embedded nitinol wires raised from 1 to 20 in the phase of austenite.

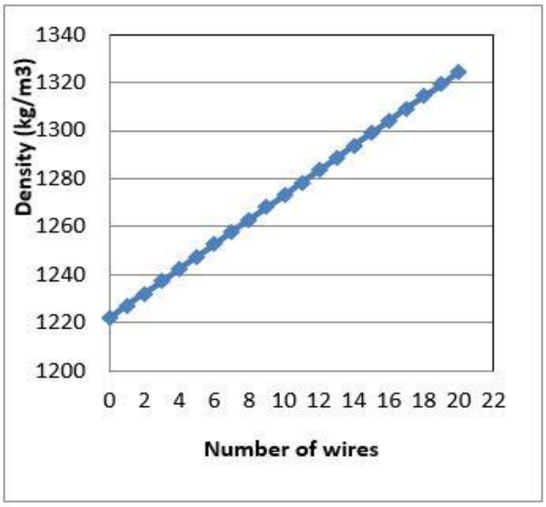

Figure (6): The variation of density with the number of nitinol wires

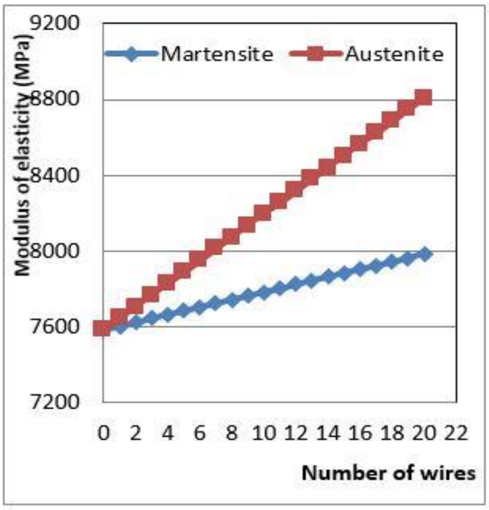

Figure (7): The variation of modulus of elasticity with number of nitinol wires

4.2 The natural frequency of uniform cantilever beam

Table (4) reveals the verification of the results, which include the theoretical results of the density, the theoretical results of the modulus, and the theoretical, numerical and experimental results of the vibration natural frequency modes. Theoretical results of natural frequency were found from the equation (6).

Table (4): Composite materials containing epoxy reinforced by E-glass fiber layer with embedded nitinol wires and its diameter 0.5 mm (martensite phase).

Numbers of wires	Density (kg/m^3)	E_{beam} (MPa)	Modes	Natural frequency (Hz)				
				Experimental	Theoretical	ANSYS	Error% With Theo.	Error% With ANSYS
0	1221.89	7581.9	1	33.3	40.234	40.835	17.23	18.45
			2	222.5	252.206	254.99	11.7	12.7
			3	662.5	706.1349	713.86	6.17	7.19
1	1227.022	7601.94	1	33.4	40.203	40.803	16.9	18.14
			2	222.5	252.010	254.79	11.7	12.67
			3	662.4	705.044	713.3	6.04	7.13
3	1237.287	7642.03	1	33.2	40.141	40.741	17.29	18.5
			2	221.9	251.623	254.4	11.81	12.77
			3	660.9	704.504	713.21	6.18	7.33
5	1247.55	7682.12	1	33.2	40.080	40.67	17.16	18.36
			2	221.7	251.242	254.01	11.75	12.71
			3	660	703.438	711.13	6.17	7.18
9	1268.082	7762.3	1	33	39.961	40.559	17.41	18.63
			2	220.8	250.497	253.26	11.85	12.81
			3	657.1	701.352	709.02	6.3	7.32

4.2.1 The effect of the numbers of nitinol wires

The samples of the figures (8), (9) and (10) are containing epoxy and one E-glass fiber layer with changing the numbers of wires (diameter is 0.5 mm) that embedded in the samples. The results of these figures were calculated theoretically by equation (6). Figures (8), (9) and (10) illustrate the behavior of changing the beam natural frequencies which embedded with SMA wires in two phases, the first is nitinol wires totally martensite, and the second is nitinol wires totally austenite, for martensite case, the initial 3 natural frequencies decreased by 1.44% if embedded nitinol wires raised from 1 to 20 wires. And in austenite phase, the initial 3 natural frequencies raised by 3.39% if embedded nitinol wires raised from 1 to 20.

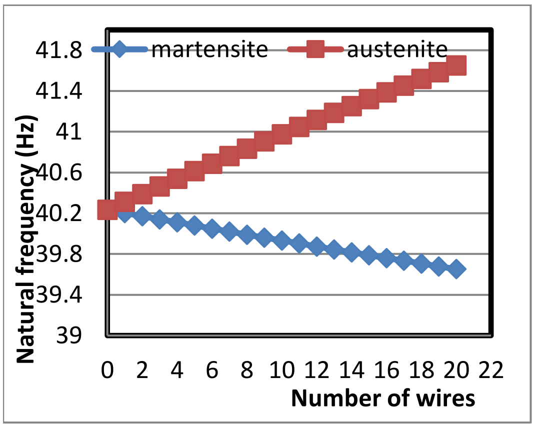

Figure (8): The variation of first natural frequencies with numbers of embedded nitinol wires

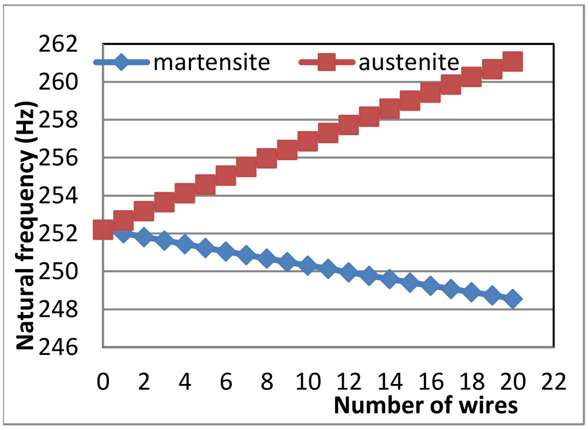

Figure (9): The variation of second natural frequencies with numbers of embedded nitinol wires

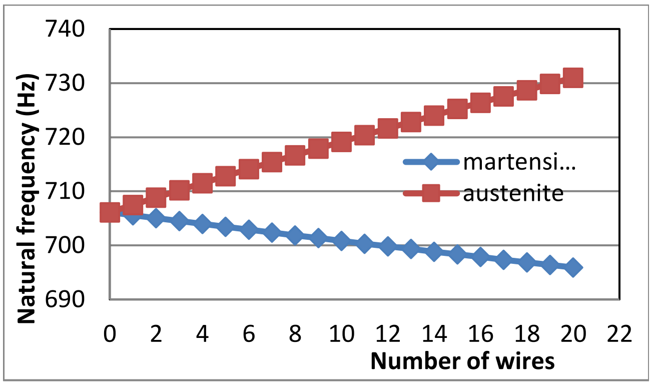

Figure (10): The variation of third natural frequencies with numbers of embedded nitinol wires

4.2.2 The effect of the diameter of nitinol wires

Figures (11), (12) and (13) elucidate the behavior of changing the natural frequencies of epoxy reinforced by one E-glass fiber layer and embedded by three nitinol wires with increased diameters of these three wires at martensite and austenite phases. The results of these figures were calculated theoretically by equation (6).

for martensite case, the initial 3 natural frequencies decreased by 6.79% if diameter of the embedded nitinol wires was raised from 0.5 mm to 3.3 mm. And in austenite phase, the initial 3 natural frequencies raised by 13.20% if diameter of the embedded nitinol wires was increased from 0.5 mm to 3.3 mm.

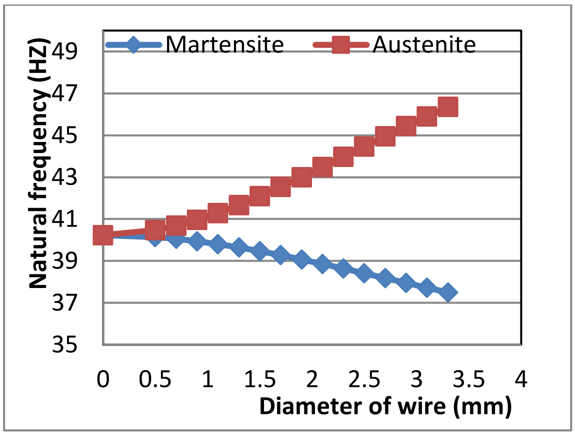

Figure (11): The variation of first natural frequencies vs diameter of embedded nitinol wires

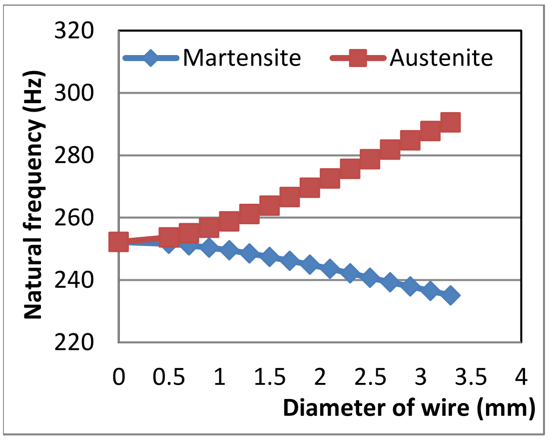

Figure (12): The variation of second natural frequencies with diameter of embedded nitinol wires

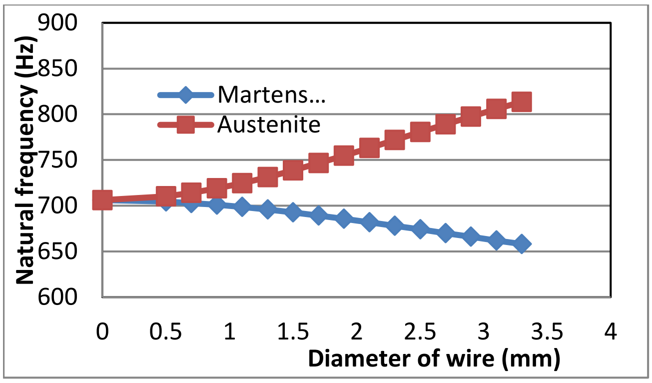

Figure (13): The variation of third natural frequencies with diameter of embedded nitinol wires

4.2.3 The effect of the length of the beam

Figure (14) displays the trend of changing the cantilever natural frequencies of composite of epoxy reinforced via one E-glass fiber layer with increased length of beam. The results of this figure were calculated theoretically by equation (6). In this figure, if the beam length is increased, the natural frequencies will decrease in all modes. The initial 3 natural frequencies reduced by 75% if increasing length of beam from 200 mm to 400 mm.

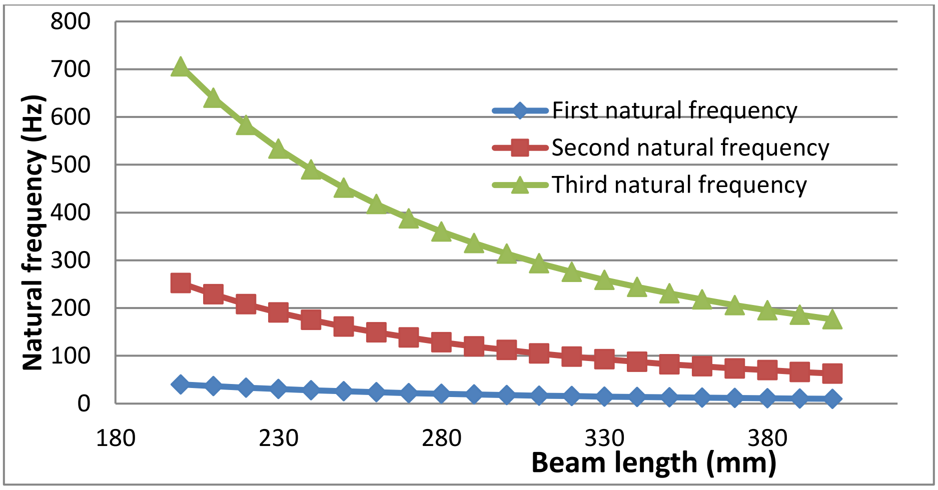

Figure (14): The variation of the first three natural frequencies with length of beam

4.2.4 The effect of the thickness of the beam

Figure (15) clarifies the relation between the thickness of beam containing epoxy and one E-glass fiber layer with the first three natural frequencies. The results of this figure were calculated theoretically by equation (6).

In this figure the increasing in thickness leads to increasing in the natural frequencies. The first three natural frequencies rose by 50% if the thickness was raised from (2-4) mm.

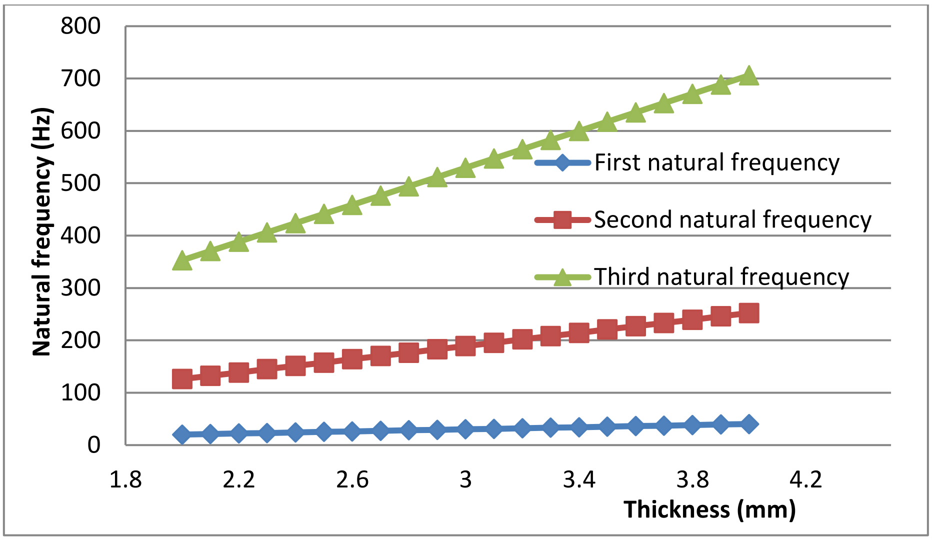

Figure (15): The variation of first three natural frequencies with thickness of beam

4.3 The results and discussion of the tapered cantilever beam

Table (5) show of the theoretical results of the density, the theoretical results of the modulus of elasticity, the verification from the experimental natural frequency, and with numerical methods by using ANSYS program. And, in all these tables, the cantilever beam is containing epoxy reinforced by E-glass fiber layer.

Table (5): Effect of width ratio on the natural frequency of composite beam

Width ratio (b_{in}/ b_{out})	Density (kg/m^3)	E_{beam} (MPa)	Mode	Natural frequency (Hz)		
				Experimental	ANSYS	Error with %
50/50	1221.89	7581.9	1	33.3	40.835	18.45
			2	222.5	254.99	12.7
			3	662.5	713.86	7.19
50/40	1221.89	7581.9	1	35.4	43.644	18.88
			2	227.6	260.15	12.51
			3	665.7	718.12	7.29
50/30	1221.89	7581.9	1	38.3	47.451	19.28
			2	234.5	266.96	12.15
			3	669.5	724.05	7.53
50/20	1221.89	7581.9	1	42.6	53.028	19.66
			2	240.6	276.99	13.13
			3	672.3	733.78	8.37

4.3.1 The effect of variation of tapered cantilever beam width ratio on the natural frequency

Figure (16) shows the relation between first three natural frequencies with width ratio of cantilever beam contain epoxy and E-glass fiber layer by ANSYS program. In this figure the first three natural frequencies increased by 22.9%, 7.9% and 2.7%, respectively if the ratio width was raised from (1-2.5) mm.

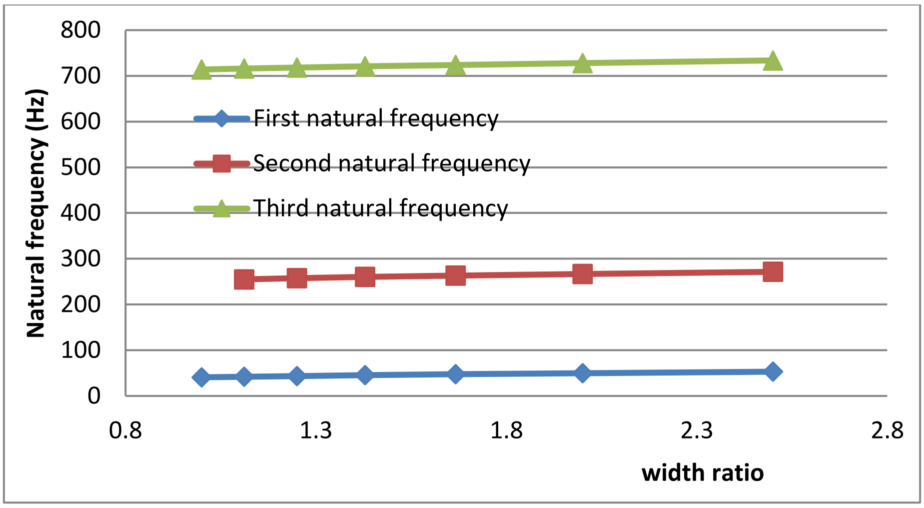

Figure (16): The variation of first three natural frequencies with width ratio (bin/bout)

4.3.2 The effect of variation of tapered cantilever beam thickness ratio on the natural frequency

Figure (17) elucidate the relation between first three natural frequencies and width ratio of cantilever beam containing epoxy and E-glass fiber layer by ANSYS program. The first natural frequency increased by 11%, and the second natural frequency decreased by 21.4% the third natural frequency decreased by 29.7% when the thickness ratio was increased from 1 to 2.666.

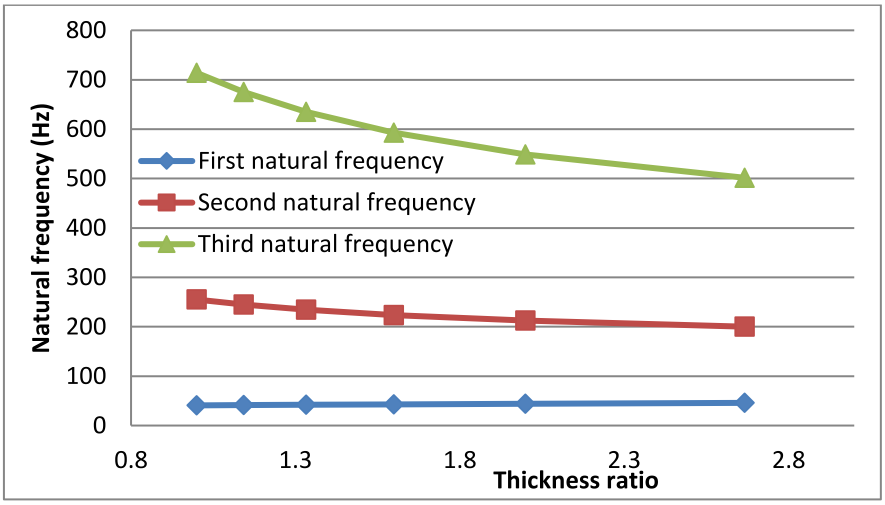

Figure (17): The variation of first three natural frequencies with thickness ratio (h_{in}/h_{out})

5. Conclusions

The major goals of this work are to study the effect of the geometrical and mechanical properties on the natural frequency of uniform and tapered cantilever beam:

1. Increasing the number and the diameter of nitinol wires which embedded in the composite beam leads to decrease the natural frequency in martensite phase and leads to increase the natural frequency in austenite phase.
2. Increasing the thickness of the cantilever composite beam leads to increase the natural frequencies. Also, increasing the length of the beam leads to decrease the natural frequencies.
3. Increasing width ratio of the tapered cantilever beam embedded with shape memory alloy leads to increase the natural frequencies.

4. Increasing thickness ratio of the tapered cantilever beam embedded with shape memory alloy leads to increase the first natural frequency and decrease the second and third natural frequencies.

CONFLICT OF INTERESTS.

- There are no conflicts of interest.

6. References

[1] Kin-tak Lau, Li-min Zhou, and Xiao-ming Tao, "Control of Natural Frequencies of a Clamped–Clamped Composite Beam with Embedded Shape Memory Alloy Wires", Elsevier, Composite Structures, Vol. 58, pp. 39–47, 2002.

[2] F. Hedayati Dezfuli, S. Khalilian, A. Abedian, "Dynamic control of plate with embedded shape memory alloy wires", 27th International Congress of the Aeronautical Sciences, 2010.

[3] Yuvaraja and M. Senthilkumar, "Comparative Study on Vibration Characteristics of a Flexible GFRP Composite Beam Using SMA and PZT Actuators", Manuf. and Ind. Eng., Vol. 11, pp. 28-33, 2012.

[4] Cem Emekiz and Mustafa Tufan Altunok, "Free Vibration Analysis of Shape Memory Alloys Used In Wind Turbine Blade Root Connection", International Refereed Journal of Engineering and Science (IRJES), Vol. 5, pp. 11-17, 2016.

[5] Amit Kumar Gupta, R. Velmurugan and Makarand Joshi, "Comparative Study of Damping in Pristine, Steel, and Shape Memory Alloy Hybrid Glass Fiber Reinforced Plastic Composite Beams of Equivalent Stiffness", Defence Science Journal, Vol. 68, pp. 91-97,2018.

[6] Product Data Sheet, Sikadur 52 N. https://irn.sika.com/dms/getdocument.get/bfb30323-c047-331a-a5f1- 29ce266bde3f/Sikadur%2052.

[7] Product Data Sheet, Kellogg's Research Labs. http://www.kelloggsresearchlabs.com/

[8] Product Data Sheet, Glass Fiber Technology Co. Ltd. http://frptechnology.com/

[9] Robert M. Jones, "Mechanics of Composite Materials", Second Edition' Book, Taylor and Francis, 1999.

[10] P. Hagedorn and A. Das Gupta, 'Vibration and Waves in Continuous Mechanical Systems', John Wiley and Sons Ltd, First edition, 2007.

تحليل الاهتزاز لعتبات مركبة منتظمة ومستدقة ومدعمة بسبيكة ذاكرة الشكل

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الخلاصة:

في هذا البحث تم تهجين مواد مركبة باستخدام اسلاك النتنول (نيكل–تيتانيوم). تم تصنيع العينات بتقنية القولبة اليدوية، وتم استخدام راتنج الايبوكسي نوع (Sikadur 52 N) وطبقة من ليف الزجاج ذات الحصيرة المنتظمة، وتم استخدام اسلاك النتنول لتدعيم العينات وبقطر (٠,٥ملم) وبعدد (٠،١،٣، ٥ و٩) سلك لايجاد تاثير عدد الاسلاك على التردد الطبيعي. العينات تكون على شكل عتبة كابولية مثبتة من طرف وحرة من الطرف الاخر. تمت دراسة تاثير زيادة كل من عدد اسلاك النتنول وقطراسلاك النتنول وطول وسمك العتبة الكابولية على الترددات الطبيعية للعتبة. وكذلك تمت دراسة تاثير ميلان عرض وسمك العتبة الكابولية على التردد الطبيعي. وضحت النتائج زيادة كل من عدد اسلاك النتنول وقطر اسلاك النتنول يؤديان لتقليل التردد الطبيعي في طور المارتنسايت ويؤديان الى زيادة التردد الطبيعي في طور الاوستنايت. كذلك اوضحت النتائج ان زيادة سمك العينة وزيادة نسبة العرض العرض يؤدي الى زيادة التردد الطبيعي. وكذلك زيادة نسبة السمك للعينة يؤدي الى زيادة التردد الطبيعي الاول وتقليل التردد الطبيعي الثاني والثالث. وكذلك زيادة طول العينة يقلل من التردد الطبيعي.

الكلمات الداله: سبائك ذاكرة الشكل، عتبة كابولية، التردد الطبيعي.