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Dynamic Analysis Modeling of a Holonomic Wheeled Mobile Robot with Mecanum Wheels Using Virtual Work Method

Hassan M. Alwan

Mechanical Engineering Department, University of Technology, Baghdad, Iraq.

*Corresponding Author E-mail: 20071@uotechnology.edu.iq

ABSTRACT: In order to study the kinematics and dynamics of a wheeled mobile robot WMR, an innovative method of kinematic and dynamic modeling using a principle of virtual work is proposed in this paper. The fundamental essence of this research is to use the principle of virtual work in dynamic model equations to control the actuators torques to follow the desired trajectory during implementation of the mobile robot task. This approach complements existing work on dynamic modeling and trajectory tracking control of holonomic mobile robots. Proposed method has been successfully examined and shows small errors in the trajectory tracking which confirmed the correctness of the theoretical expression of the dynamic model for the wheeled mobile robot. Simulation testing of the proposed method to perform the motion process of WMR in specific workspace during implementation of its task has been carried out. Two simulation examples presented in this work to show the effect of wheels torques on the robot body velocities and its trajectory tracking.

KEYWORDS: dynamic, wheeled mobile robot WMR, virtual work, trajectory tracking

INTRODUCTION

At the last decades in various fields including industry, wheeled mobile robots are used. The essential aspects to be considered for building these types of robots are autonomy, mobility, position and control. First of all kinematic and dynamic models would be considered to carry out the purpose of trajectory tracking control of WMR. Several researches studying the modeling of kinematics and dynamic problems for wheeled mobile robot WMR using Lagrangian formulation to obtain dynamic equations of motion are discussed in many resaerchs. In a study, dynamic equations of motion of WMR have been presented with use of Lagrange equations of second kind. In order to derive dynamic model, the kinetic energy of the system is determined as well as the generalized forces affecting the system [1,2]. For the holonomic mobile robot the dynamic equations of motion are derived using Euler – Lagrangian method based on the Lagrange function and assuming the potential energy equal to zero because the mobile robot followed a path on a planer surface [3-5]. In other study, kinematics equations modeling for a WMR with mecanum wheels under the same assumption in this approach have been expressed and examined [6].

Proposed WMR used in this paper is a holonomic type to improve the tracking ability to travel in every direction under any orientation [7-10]. Each mecanum wheel includes a fixed wheel as well as passive rollers. The angle γ occurring between the axis of the wheel rotation and the axis of roller rotation is 45°. The robot consists of a platform (robot body) and three identical mecanum wheels. The wheeled mobile robot coordinate frame origin is located at its center of gravity. The mecanum wheels with radius R moved the WMR are fixed with the WMR body as shown in Figure 1. Structure of this paper is as follows. In introduction, we discussed the existing work and contribution of this approach. Kinematics models including inverse and forward equations have been presented in section 2. Our version of dynamic model consideration based on virtual work method has been introduced in section 3 [11]. By using PI controller in section 4, the wheels driving torques, the position and orientation error of the WMR trajectory has been expressed. The paper contains in section 5 two simulation examples to examine the dynamic models.

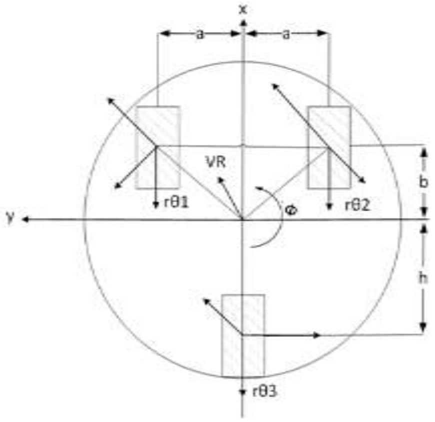

Figure 1. The structure of the proposed mobile robot

MOBILE ROBOT KINEMATICS MODEL

The WMR kinematic model has been assumed under the condition that the contact between the wheels and the floor is pure rolling (without slipping). Inverse kinematic model of the WMR can be obtained from the movement vector of its three mecanum wheels [12]. The two components of the WMR linear velocity are v_x and v_y which have parallel direction with x and y axis respectively. WMR angular velocity according to its center of gravity (origin of xyz coordinate frame) is $\dot{\vartheta}$. Angular velocity of WMR mecanum wheels is ω_i, i= 1,2,3 and the mecanum tilted angle for all wheels rollers is $\gamma = 45^{\circ}$.

Velocity vector of each wheel of the WMR wheels in the forward direction can be obtained as follow:

$$v_x - b\dot{\vartheta} = r\omega_i + r\omega_i \, cos(\gamma_i) \quad (1)$$

$$v_y - a\dot{\vartheta} = r\omega_i \, sin(\gamma_i) \quad (2)$$

Because of the tilted angle between the wheel axis and the rollers rotation axis for all wheels is $\gamma = 45^{\circ}$, the mecanum wheels velocities can be obtained from equations (1) and (2) and take in the account the direction of the tilted angles as follow:

$$v_x + v_y - \dot{\vartheta}\,(a - b) - r\omega_1 = 0 \quad (3)$$

$$v_x - v_y - \dot{\vartheta}\,(a + b) - r\omega_2 = 0 \quad (4)$$

$$v_x - v_y + \dot{\vartheta}\,(h) - r\omega_3 = 0 \quad (5)$$

The inverse kinematics problem is given a position vector and orientation of the mobile robot center $[x \quad y \quad \vartheta]^T$ and possibility at any time (δt) to find the robot wheels angular velocities that get the WMR there. An inverse kinematic model of mobile robot is proposed by implementation of obtaining of mathematical model regarding the following relationships between the angular velocities of the mecanum wheels and the WMR translation and rotation velocities as follow:

$$\omega_1 = \frac{1}{r}\left(v_x + v_y - \dot{\vartheta}(a - b)\right) \quad (6)$$

$$\omega_2 = \frac{1}{r}\left(v_x - v_y - \dot{\vartheta}(a + b)\right) \quad (7)$$

$$\omega_3 = \frac{1}{r}\left(v_x - v_y + \dot{\vartheta}(h)\right) \quad (8)$$

Jacobian matrix can be presented in the form below:

$$J = \frac{1}{r}\begin{bmatrix} 1 & 1 & (a+b) \\ 1 & -1 & (-a-b) \\ 1 & -1 & (h) \end{bmatrix} \quad (9)$$

Inverse kinematics model can be obtained from equations (6), (7), and (8) can be presented in the form of following equation:

$$\omega = Jv_r \tag{10}$$

Where, $v_r = \begin{pmatrix} v_x & v_y & \dot{\vartheta} \end{pmatrix}^T$ the mobile robot center linear velocity and angular rotation of the WMR around its center of gravity,

$\omega = (\omega_1 \quad \omega_2 \quad \omega_3)^T$ is the vector of the angular velocities of the mobile robot mecanum wheels

To solve the forward kinematics problem, a jacobian matrix should be performed as follow using the Moore-Penrose Theorem on inversion of rectangular matrices. The following relationship can be received as follow:

$$J_F = (J^T J)^{-1} J^T \tag{11}$$

Forward kinematics problem solution can be obtained by the following relationship:

$$v_r = J_F \omega \tag{12}$$

The WMR acceleration can be evaluated by derivative of the above equation (12) as follow:

$$\dot{v}_r = J_F \dot{\omega} \tag{13}$$

Where $\dot{v}_r = \begin{bmatrix} \dot{v}_x & \dot{v}_y & \ddot{\vartheta} \end{bmatrix}^T$ is the vector of the mobile robot center linear and angular acceleration
$\dot{\omega} = [\dot{\omega}_1 \quad \dot{\omega}_2 \quad \dot{\omega}_3]^T$ is the vector of the mecanum wheels angular acceleration.

DYNAMIC MODEL USING VIRTUAL WORK METHOD

The WMR body affected from one side under the total force which consists of its weight and inertia forces and torques. From another side WMR affected under the mecanum wheels controlled torques. The total force consists of robot weight (mg) and inertia force which can be calculated by the following formulation:

$$F_I = -m\dot{V}_R \tag{14}$$

Where F_I is the inertia force, $\dot{V}_R = \begin{pmatrix} \dot{v}_r & 0 & 0 \\ 0 & \dot{v}_r & 0 \\ 0 & 0 & \ddot{\vartheta} \end{pmatrix}^T$ is the vector projection of robot center acceleration.
The virtual work caused by the weight, inertia force and torque can be obtained by the equation below:

$$\delta W_1 = (mg + F_I)\delta X + M_I \delta\emptyset \tag{15}$$

Where:

$\delta X = [\delta x \quad \delta y \quad 0]^T$: a little translation of the mobile robot center along x, y axis directions, M_I : inertia moment of the mobile robot body, $\delta\emptyset = [0 \quad 0 \quad \delta\vartheta]^T$: little rotation of the WMR about its center.

Equation (15) can be expressed in a matrix form as follow:

$$\delta W_1 = \left(\begin{pmatrix} mg & 0 & 0 \\ 0 & mg & 0 \\ 0 & 0 & I \end{pmatrix} + \begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I \end{pmatrix} (J_F \dot{v}_r) \right) \begin{pmatrix} \delta x & 0 & 0 \\ 0 & \delta y & 0 \\ 0 & 0 & \delta\vartheta \end{pmatrix} \tag{16}$$

The virtual work done by the controlled torques from the actuators in the mecanum wheels can be expressed as follow:

$$\delta W_2 = \begin{pmatrix} \tau_1 & 0 & 0 \\ 0 & \tau_2 & 0 \\ 0 & 0 & \tau_3 \end{pmatrix} \begin{pmatrix} \delta\omega_1 & 0 & 0 \\ 0 & \delta\omega_2 & 0 \\ 0 & 0 & \delta\omega_3 \end{pmatrix} \tag{17}$$

In static equilibrium condition the virtual work caused by the torques of the mecanum wheels δW_2 shall equal to the virtual work caused by WMR weight and inertia forces δW_1. The controlled wheels torques can be obtained by the equation:

$$(\tau) = \left(\begin{pmatrix} mg & 0 & 0 \\ 0 & mg & 0 \\ 0 & 0 & I \end{pmatrix} + \begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I \end{pmatrix} (J_F \dot{v}_r) \right) \begin{pmatrix} \frac{r}{2} & 0 & 0 \\ 0 & \frac{(-a+b-h)r}{2(a+b+h)} & 0 \\ 0 & 0 & \frac{r}{a+b+h} \end{pmatrix} \tag{18}$$

TRAJECTORY TRACKING CONTROL

The aim of a dynamic controller is to follow a robot trajectory described by its position or velocity profiles as a function of time. In general, the posture of the WMR can be defined as a three parameters position and orientation of its center of gravity. The position vector of the proposed WMR center at every time (t) can be calculated by integration of the forward kinematic equation (14) and can be written as follow:

$$q = J_F \dot{\theta}\, t \tag{19}$$

Where: $q = (x \quad y \quad \vartheta)^T$ presented the mobile robot center of gravity point position and orientation vector in the global coordinate system.

During the WMR movement to follow the desired trajectory, its center of gravity actual position and orientation vector is $q_d = (x_d \quad y_d \quad \vartheta_d)^T$, and the theoretical position and orientation vector of the same center is $q = (x \quad y \quad \vartheta)^T$.

The position and orientation error of the WMR center e_q in the global coordinate frame represented the difference between the desired and the theoretical results and can be evaluated as follow:

$$e_q = \begin{pmatrix} e_x \\ e_y \\ e_\emptyset \end{pmatrix} = \begin{pmatrix} x(t) - x_d \\ y(t) - y_d \\ \vartheta(t) - \vartheta_d \end{pmatrix} \tag{20}$$

To transform this error to the WMR local coordinate frame by multiplying the error vector by the rotation matrix of the local coordinate frame according to the global coordinate frame as follow:

$$e_G = (R) \begin{pmatrix} e_x \\ e_y \\ e_\emptyset \end{pmatrix} \tag{21}$$

Where$e_G = (e_{Gx} \quad e_{Gy} \quad e_{G\emptyset})^T$, R denotes the rotation matrix of mobile robot local coordinate frame with respect to global coordinate frame with $\emptyset$ angle (rotation around Z- axis).

By differentiation of equation (21) the following relationships will be received: $\dot{e}_{Gx} = \dot{e}_x cos\emptyset - e_x sin\emptyset \dot{\emptyset} + \dot{e}_y sin\emptyset + e_y cos\emptyset \dot{\emptyset}$ $\dot{e}_{Gy} = -\dot{e}_x sin\emptyset + e_x cos\emptyset \dot{\emptyset} + \dot{e}_y cos\emptyset + e_y sin\emptyset \dot{\emptyset}$ $\dot{e}_{G\emptyset} = \dot{e}_\emptyset$

In matrix form, the above equations can be written as follow:

$$\dot{e}_G = \begin{pmatrix} cos\emptyset & sin\emptyset & -e_x sin\emptyset + e_y cos\emptyset \\ -sin\emptyset & cos\emptyset & e_x cos\emptyset + e_y sin\emptyset \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \dot{e}_x \\ \dot{e}_y \\ \dot{\emptyset} \end{pmatrix} \tag{22}$$

The WMR path tracking based on geometrical consideration, position and orientation error evaluated in equation (22) can be computed at any time. A small error is needed in order to achieve a smooth trajectory tracking. The dynamic controller proposed in this approach is able to make a better tracking of the desired trajectory and reduce the tracking error [13]. From the velocity error$\dot{e}_G$, the dynamical controller evaluates a control signal which is added to the reference acceleration signal. The revised reference velocities and accelerations are used in inverse dynamic solution to evaluate the controlled wheels torques.

SIMULATION STUDY AND RESULTS

To examine the dynamics model of the mobile robot by assuming the wheels angular velocities and evaluating the WMR center translation and orientation velocities, two cases of trajectory tracking have been implemented. In the he first case, the mobile robot follows a desired circular trajectory [14]. The desired trajectory described by the functions of $= 1\cos(0.4\,t)$, $y = 1\sin(0.4\,t)$. The estimated robot angular velocity is $\dot{\vartheta} = 0.4\frac{rad}{s}$. Figure 2 shows the mobile robot motion trajectory in x and y-axles direction. Figure 3 shows the mobile robot trajectory error in y – axis direction. Figure 4 shows the mobile robot trajectory error in x – axis direction. Figure 5 show the mobile robot wheels No.1 driving torque [15]. Figure 6 show the mobile robot wheels No.2 driving torque. Figure 7 shows the mobile robot wheels No.3 driving torque.

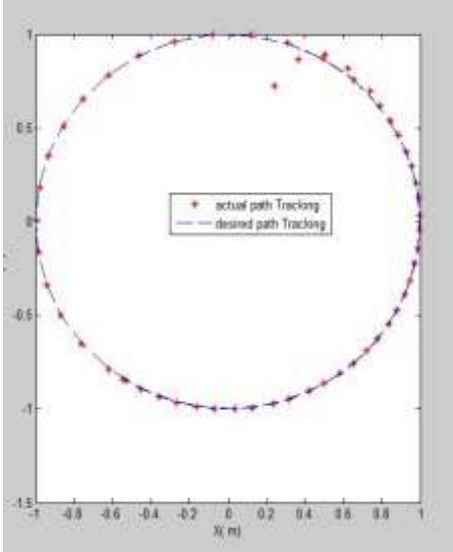

Figure 2. Robot motion trajectory with circle path

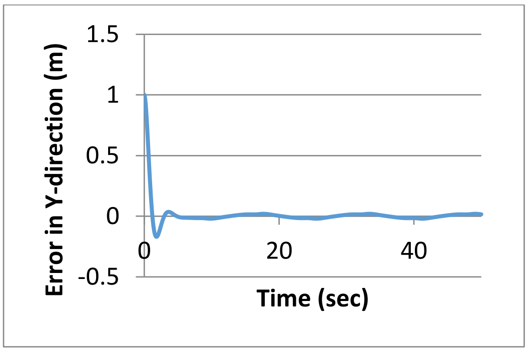

Figure 3. Trajectory error in y – direction

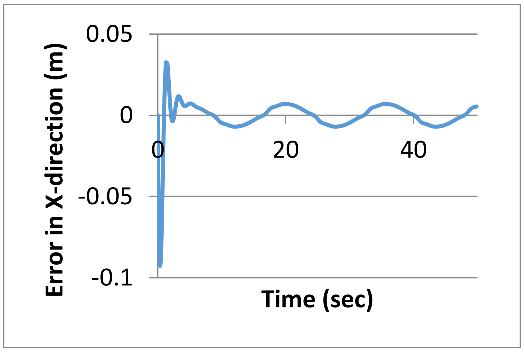

Figure 4. Trajectory error in x – direction

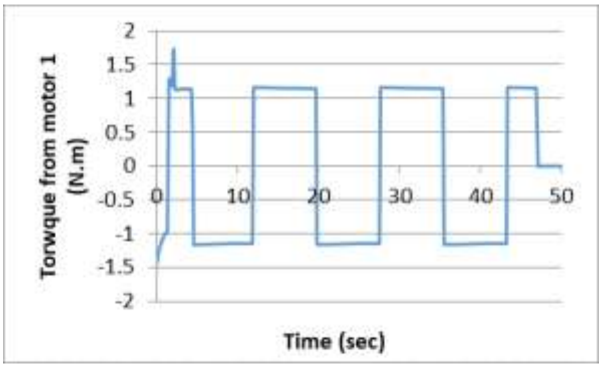

Figure 5. Driving Torque of Wheel No.1

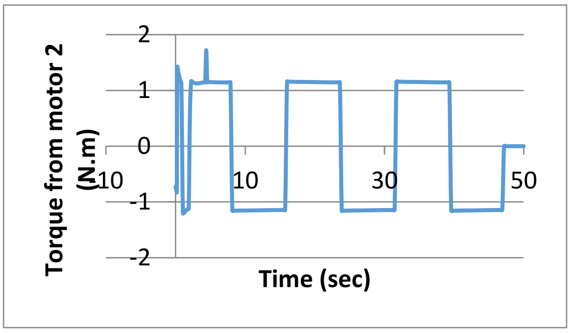

Figure 6. Driving Torque of Wheel No.3

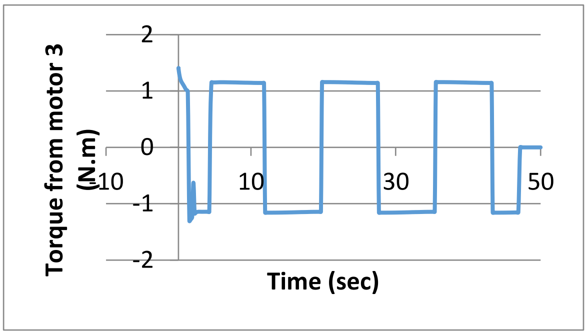

Figure 7. Driving Torque of Wheel No.3

In the he second case, the mobile robot follows a desired infinity shape trajectory. The desired trajectory described by the functions of $= 1\,\mathrm{cossin}(0.2\,t)$, $y = 1\,\sin(0.4\,t)$. The estimated robot angular velocity is $\dot{\vartheta} = 0.4\frac{rad}{s}$. Figure 8 shows the mobile robot motion trajectory in x and y-axles direction. Figure 9 shows the mobile robot trajectory error in y – axis direction [16-20]. Figure 10 shows the mobile robot trajectory error in x – axis direction. Figure 11 show the mobile robot wheels No.1 driving torque. Figure 12 show the mobile robot wheels No.2 driving torque. Figure 13 shows the mobile robot wheels No.3 driving torque.

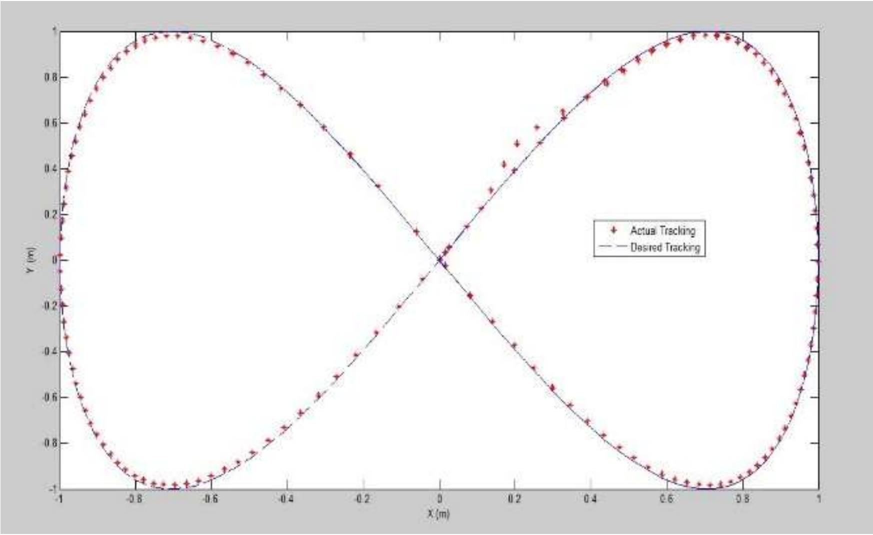

Figure 8. Robot motion trajectory with infinity shape

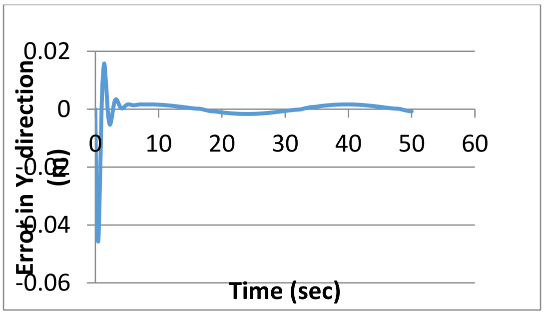

Figure 9. Trajectory error in y – direction

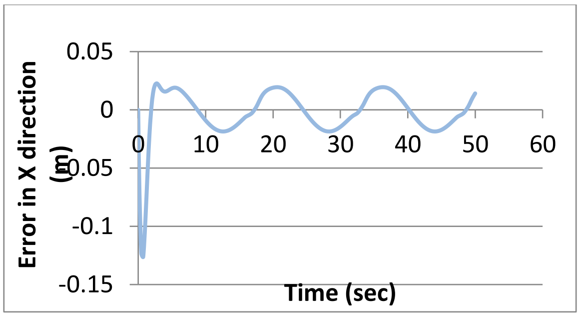

Figure 10. Trajectory error in x – direction

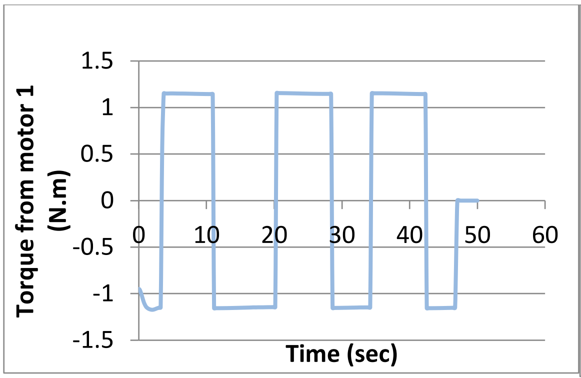

Figure 11. Driving Torque of Wheel No.1

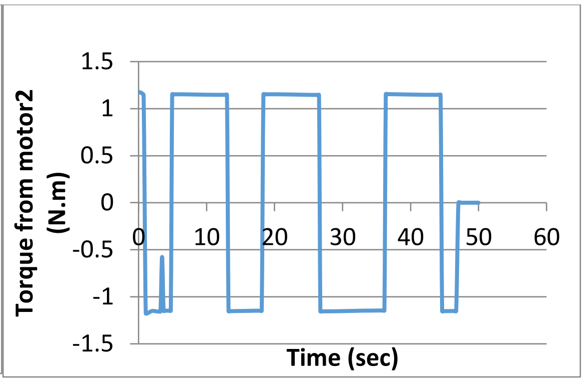

Figure 12. Driving Torque of Wheel No.2

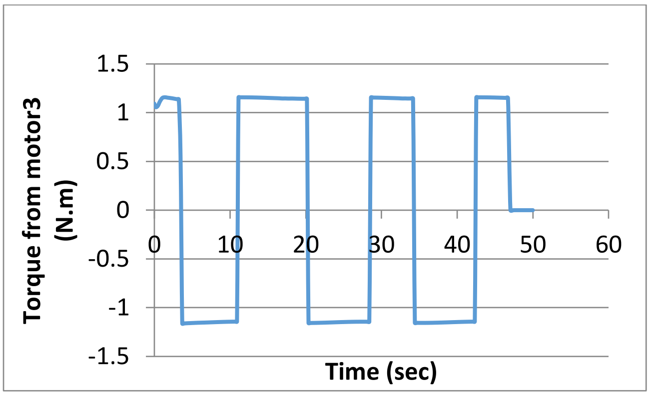

Figure 13. Driving Torque of Wheel No.3

CONCLUSION

This paper presents a novel method for obtaining a dynamic model for WMR with mecanum wheels based on a virtual work. This paper shares the usage of virtual work method in dynamic modeling of WMR. According to the WMR design, inverse and forward kinematic models have been successfully obtained for a holonomic mobile robot type with three mecanum wheels drive. The explanation enables the expression of the dynamic model in the formulation that allows its application in the building of the trajectory tracking control and simulation study. MATLAB and Simulink programmers were used in the simulation study of the two examples. The results show that during the desired trajectory following, the trajectory errors are very small values and in expected range. Using of dynamic controller to reach the desired trajectory showed a good matching between desired and actual trajectories and ensured that the errors in very small and acceptable values which confirmed the correctness of the derived dynamic model for the proposed WMR.

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