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Kinematics Modeling and Simulation of Holonomic Wheeled Mobile Robot with Mecanum Wheels

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ABSTRACT: The wheeled mobile robot WMR has been considered as a planner mechanism with linear and angular movement over a horizontal plane. In particular, the movement of WMR is assumed resulting by pure rolling of wheels without slipping. In this paper inverse and forward kinematics models for a wheeled mobile robot have been obtained. The WMR type proposed in this approach is a holonomic mobile robot with three mecanum wheels. It is known in literature, that there are many types of wheels are used in robots design but we prefer mecanum wheels to used in this study because this type of wheels makes the mobile robot movement in lateral and longitudinal directions smooth and improve the tracking ability to travel in every direction under any orientation.. The kinematic modeling technique has been examined by simulation examples presented in this paper. The first example is to evaluate the angular velocities of the wheels of the robot resulting by input linear and angular velocities of the WMR. The second example is to evaluate the linear and angular velocities of the WMR resulting by input angular velocities of the wheels of the robot

KEYWORDS: kinematics, trajectory tracking, wheeled mobile robot, Jacobian, PI controller

INTRODUCTION

In the last decades, many researches studying the modeling of the kinematics problems for the wheeled mobile robot MWR have been carried out using different types of mobile robots, structures and different types of wheels design. In developed an innovative method of modeling and kinematics simulation in RecurDyn for omnidirectional wheeled chair WMR with four mecanum wheels [1-3]. In order to study the motion characteristics and performance of mobile robot, a virtual prototype simulation model was established using SolidWorks software and carried out in RecurDyn before the manufacturing of the robot. In an implementation of inverse kinematic model was applied for mobile robot using four wheels mecanum driver [4]. The inverse kinematics is conducted to control the mobile robot movement and to convert the robot velocity components of V_x, V_y and ω toward angular velocity each wheel of ω_1, ω_2, ω_3, ω_4 and wheel turn direction.

Proposed WMR used in this research is a holonomic type because this type of mobile robots has a good tracking ability to implement its tasks by travelling in every direction and under any orientation. The robot consists of a platform (robot body) and three identical mecanum wheels. The mecanum wheel body consists of a fixed wheel with radius R as well as many passive rollers. The angle γ occurring between the axis of the wheels rotation and the axis of roller rotation is 45°. The location of the wheeled WMR coordinate frame origin assumed to locate at its center of gravity [5-8]. The structure of the WMR body with the mecanum wheels and its dimensions are as shown in Fig.1. This paper is organized as follows. In section 1, the introduction briefly explained the different types of WMR and the advantage of each type and the proposed mobile robot explanation. Section 2, presented the derivation of the mathematical models for the inverse kinematics of the proposed WMR. The obtaining of forward kinematic has been explained in Section 3. By using PI kinematic controller in section 4, the position and orientation error of the WMR trajectory has been expressed. The paper contains in section 5 two simulation examples of inverse and forward kinematics.

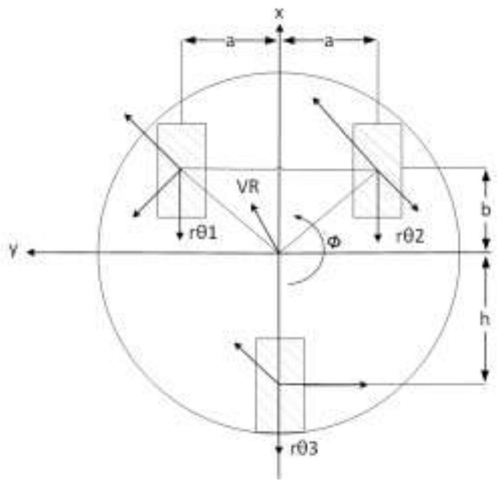

Figure 1. The structure of the proposed mobile robot

INVERSE KINEMATIC MODEL

To obtain the WMR inverse kinematic model, the contact between the mobile robot wheels and the floor has been assumed a pure rolling (without slipping). Also in this approach the WMR has three mecanum wheels each of them has been controlled on velocity independently by a separate actuator [9-13]. The position and orientation of mobile robot center described by a vector of generalized coordinates $q = [x \quad y \quad \varphi]^T$ in the WMR local coordinate frame. Inverse kinematic model of the WMR can be obtained from the movement vector of its three mecanum wheels in terms of its center linear and angular movements. Firstly, it has been known that, the two components of the WMR linear velocity are V_{rx} . V_{ry} . Their directions are parallel with the mobile robot local frame x and y axis directions respectively. Secondly, the WMR angular velocity according to its local frame coordinates center (origin of xyz coordinate frame) is $\dot{\varphi}$. Angular velocity of WMR mecanum wheels is ω_i, i= 1,2,3 and the mecanum tilted angle for all wheels rollers is $\gamma = 45^{\circ}$.

First step in the inverse kinematic modeling of the mobile robot is to obtain the expression of motion constraints of its individual wheels. Then the expression of the motions of individual wheels can be later combined to compute the motion of the robot as a whole. The WMR mecanum wheel (i) affected four velocities moving it in various directions [14]. The first one is a linear velocity $(R\omega_i)$ along x-axies occurring by the wheel rotation. The second velocity is the mobile robot linear velocitie along x, y axis. The third one is a linear velocity resulting by the WMR rotation around its center. The magnitude of this velocity is equal to$\left(\sqrt[2]{a^2 + b^2}\ \dot{\varphi}\right)$ and acting perpendicularly to the line connecting between the robot center and the wheel center. The last velocity is roller linear velocity acting by (γ_i) with robot movement along x axis of the robot local frame.

Velocity vector of (i) wheel of the WMR in the x and y directions can be written as follow:

$$R\omega_i + R\omega_i \cos(\gamma_i) = V_{rx} - b\dot{\varphi} \tag{1}$$

$$R\omega_i \sin(\gamma_i) = V_{ry} + a\dot{\varphi} \tag{2}$$

It has been assumed that the tilted angle for all wheels rollers is $\gamma = 45^{\circ}$, and by taking in the account the direction of the tilted angles for each wheel, the constrain of the velocities of the three wheels are:

$$V_{rx} + V_{ry} - \dot{\varphi}(a - b) - R\omega_1 = 0 \tag{3}$$

$$V_{rx} - V_{ry} - \dot{\varphi}(a + b) - R\omega_2 = 0 \tag{4}$$

$$V_{rx} - V_{ry} + \dot{\varphi}(h) - R\omega_3 = 0 \tag{5}$$

The inverse kinematic problem of the whole mobile robot can be derived from the relationships between the wheels angular velocities and the WMR translation and rotation velocities as follow:

$$\omega_1 = \frac{1}{R}\left(V_{rx} + V_{ry} - \dot{\varphi}(a-b)\right) \tag{6}$$

$$\omega_2 = \frac{1}{R}\left(V_{rx} - V_{ry} - \dot{\varphi}(a+b)\right) \tag{7}$$

$$\omega_3 = \frac{1}{R}\left(V_{rx} - V_{ry} + \dot{\varphi}(h)\right) \tag{8}$$

A wheels jacobian matrix will be formulated to explain the relationship between the wheels angular velocities (ω) and the linear, angular velocities of the mobile robot center point (V_r). Jacobian matrix can be presented in the following form:

$$J = \left(\frac{\partial \omega}{\partial V_r}\right) \tag{9}$$

Where, $\omega = (\omega_1 \quad \omega_2 \quad \omega_3)^T$ the wheels angular velocities and $V_r = (V_{rx} \quad V_{ry} \quad \dot{\varphi})^T$ the robot center point linear and angular velocities.

From equations (6), (7) and (8), the jacobian matrix can be written as follow:

$$J = \frac{1}{R}\begin{bmatrix} 1 & 1 & (-a+b) \\ 1 & -1 & (-a-b) \\ 1 & -1 & H \end{bmatrix} \tag{10}$$

Equation (6), (7), and (8) can be presented the inverse kinematics equation for the mobile robot as in the form of following equation:

$$\omega = J V_r \tag{11}$$

FORWARD KINEMATIC MODEL

To measure the WMR position, there is no direct way to fix the values of its linear and angular displacements but we can evaluate the values by integration of the robot linear and angular velocities over time. In this case it will be needed to express a relationship described the mobile robot velocities as a function of the mecanum wheels angular velocities [15]. This relationship leads to obtain the forward kinematics. To obtain the forward kinematics problem, the inverse jacobian matrix should be performed as follow using the Moore-Penrose Theorem on inversion of rectangular matrices. The forward jacobian matrix can be written as follow:

$$J_F = (J^T J)^{-1} J^T \tag{12}$$

Forward kinematics described the relationship between linear and angular velocities of the mobile robot center point in terms of mecanum wheels angular velocities. It can be expressed by the following equation:

$$V_r = J_F \omega \tag{13}$$

By derivative of the two sides of equation (13), the mobile robot center linear and angular accelerations can be expressed by the following expression:

$$\dot{V}_r = J_F \dot{\omega} \tag{14}$$

Where $\dot{V}_r = \left(\dot{V}_{rx} \quad \dot{V}_{ry} \quad \ddot{\varphi}\right)^T$ components of linear and angular acceleration of the mobile robot center point $\dot{\omega} = (\dot{\omega}_1 \quad \dot{\omega}_2 \quad \dot{\omega}_3)^T$ is the vector of the mecanum wheels angular acceleration.

PI KINEMATIC CONTROLLER

PI controllers will be used to achieve the mobile robot trajectory tracking. The main goal is to find the mobile robot center controlled velocities vector (linear and angular velocities) $V_r = (V_{rx} \quad V_{ry} \quad \dot{\varphi})^T$by controlling the vector of the three wheels angular velocities $\omega = (\omega_1 \quad \omega_2 \quad \omega_3)^T$. For implementation its task, the mobile robot will start from posture$(x_0 \quad y_0 \quad \varphi_o)^T$ and follow a desire posture $(x_d \quad y_d \quad \varphi_d)^T$. Definition of the post error vector can be used to design the controller to find the controlled velocity vector. It was known that, at any time, the error vector can be written as:

$$e_q = \begin{pmatrix} e_x \\ e_y \\ e_\emptyset \end{pmatrix} = \begin{pmatrix} x(t) - x_d \\ y(t) - y_d \\ \varphi(t) - \varphi_d \end{pmatrix} \tag{15}$$

From equation (15), we can write [5]:

$$\dot{e}_q = \begin{pmatrix} \dot{e}_x \\ \dot{e}_y \\ \dot{e}_\varphi \end{pmatrix} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\varphi} \end{pmatrix} = J^{-1}\omega \tag{16}$$

To asymptotically stabilize the system, we propose the PI control law where the gain matrices K_P and K_I are symmetric and positive definite [5]:

$$\omega = J\left[-K_P\begin{pmatrix} e_x \\ e_y \\ e_\varphi \end{pmatrix} - K_I\begin{pmatrix} \int e_x \\ \int e_y \\ \int e_\varphi \end{pmatrix}\right] \tag{17}$$

Taking (17) in (16) the following expression can be obtained:

$$\dot{e}_q = J^{-1}J\left[-K_P\begin{pmatrix} e_x \\ e_y \\ e_\varphi \end{pmatrix} - K_I\begin{pmatrix} \int e_x \\ \int e_y \\ \int e_\varphi \end{pmatrix}\right] \tag{18}$$

The proposed kinematics controller allows the mobile robot to follow a trajectory where position and orientation error goes to zero. For the stability of the of the error system, a Lyabunov function is chosen as follow:

$$V = R\omega = \frac{1}{2}\begin{bmatrix} e_x & e_y & e_\varphi \end{bmatrix}\begin{bmatrix} e_x \\ e_y \\ e_\varphi \end{bmatrix} + \frac{1}{2}\begin{bmatrix} \int e_x & \int e_y & \int e_\varphi \end{bmatrix}\begin{bmatrix} \int e_x \\ \int e_y \\ \int e_\varphi \end{bmatrix} \tag{19}$$

SIMULATION RESULTS AND DISCUSSION

Inverse Kinematic Simulation Results

The proposed mobile robot dimensions are as described in table 1. Mobile robot assumed to move in different cases including four cases in straight lines along x-axis, y-axis (front and rear) and four cases in oblique directions each 30° as shown in fig.2 [16]. In each case the linear and angular velocities of the mobile robot center have been assumed to evaluate the mobile wheels angular velocity according to the inverse kinematic model equation (2.11). Table (1) shows the evaluated wheels angular velocities resulting from WMR linear, angular velocities and the variation of its direction [17].

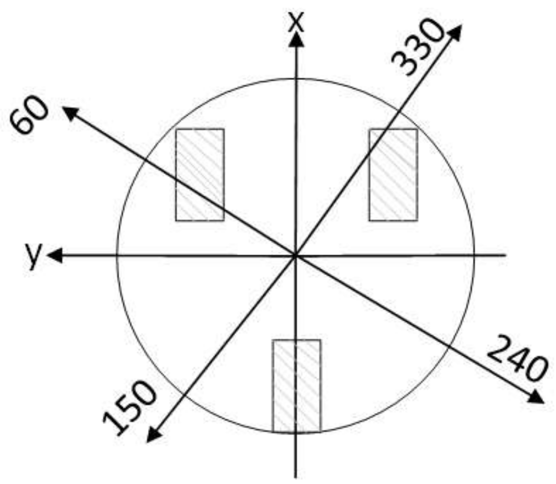

Figure 2. Robot movement direction

Table 1. Robot dimensions

Symbol		value	unit
Wheels radius	R	0.05	m
	a	0.08	m
	b	0.06	m
	h	0.12	m

Table 2. Wheels angular velocities resulting by solving inverse kinematics

Line of WMR movement	Robot center Velocities m/s m/s rad/s			Wheels angular velocities rad/s		
	V_{rx}	V_{ry}	$\dot{\varphi}$	ω_1	ω_2	ω_3
1	0.2	0	0	4	4	4
2	0.1	0.17	0	5.46	-1.46	-1.46
3	0	0.2	0	4	-4	-4
4	-0.17	0.1	0	-1.46	-5.46	-5.46
5	-0.2	0	0	-4	-4	-4
6	-0.1	-0.17	0	-5.46	1.46	1.46
7	0	-0.2	0	-4	4	4
8	0.17	-0.1	0	1.46	5.46	5.46

Forward Kinematic Simulation Results

To examine the forward kinematics equation, mobile robot assumed to move in the same movements as in inverse kinematics example, four cases in straight lines along x-axis, y-axis (front and rear) and four cases in oblique directions depending on the wheels angular velocities [18]. In each case the angular velocities of the mobile robot wheels are known to evaluate the mobile robot center linear and angular velocities according to the forward kinematic model equation (3.2).

Table (3) shows the evaluated mobile robot linear and angular velocities resulting from forward kinematics of mobile robot.

Table 3. WMR center velocities resulting by solving forward kinematics

Line of WMR movement	Wheels angular velocities rad/s			WMR linear and angular velocities m/s m/s rad / s		
	ω_1	ω_2	ω_3	V_{rx}	V_{ry}	$\dot{\varphi}$
1	12	12	12	0.6	0	0
2	-12	-12	-12	-0.6	0	0
3	-12	12	12	0	-0.6	0
4	12	-12	-12	0	0.6	0
5	12	12	0	0.415	0.1385	-2.307
6	-12	-12	0	-0.415	-0.1385	2.307
7	0	12	12	0.3	-0.3	0
8	0	-12	-12	-0.3	0.3	0
9	12	0	12	0.4847	0.1615	2.307
10	-12	0	-12	-0.4847	-0.1615	-2.307
11	12	0	0	0.3	0.3	0
12	-12	0	0	-0.3	-0.3	0

Controlling Of Wmr Velocities

To examine the forward kinematics model of the mobile robot by assuming the wheels angular velocities and evaluating the WMR center translation and orientation velocities, two cases of trajectory tracking have been implemented. In the he first case, the mobile robot follows a desired circular path [19]. The desired path described by the functions of $= 1\cos(0.4\,t)$, $y = 1\sin(0.4\,t)$. The estimated robot wheels angular velocities are $\omega_1 = 0.4\frac{rad}{s}$, $\omega_2 = 0.5\frac{rad}{sec}$ $and\ \omega_3 = 0.6\,rad/sec$. Figure 3 shows the mobile robot center linear velocity in x-axis direction. Figure 4 shows the mobile robot center linear velocity in y-axis direction. Figure 5 shows the mobile robot angular velocity.

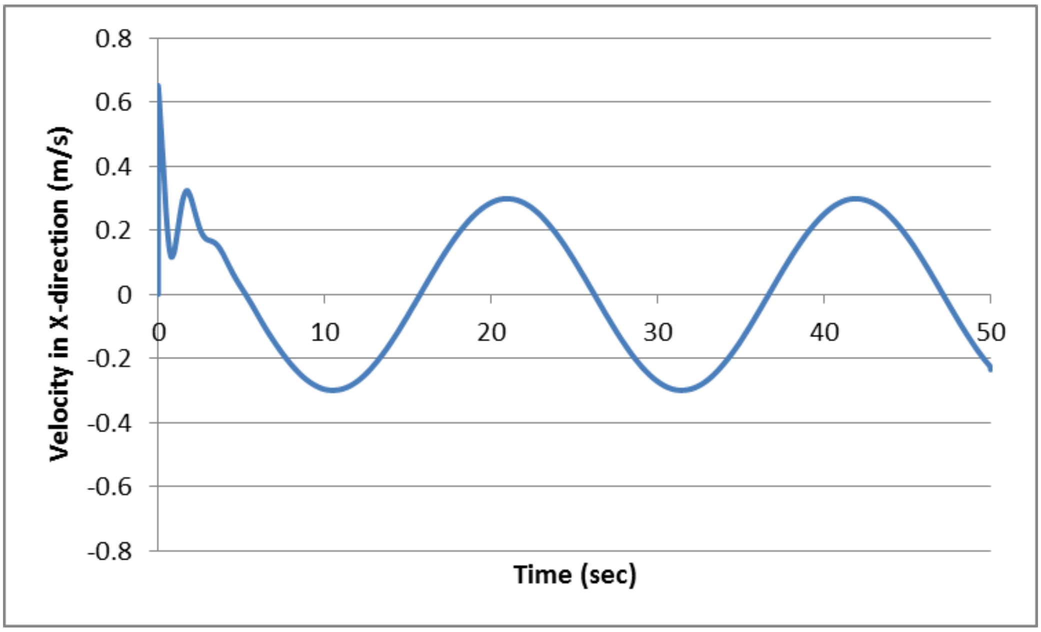

Figure 3. WMR velocity Vx

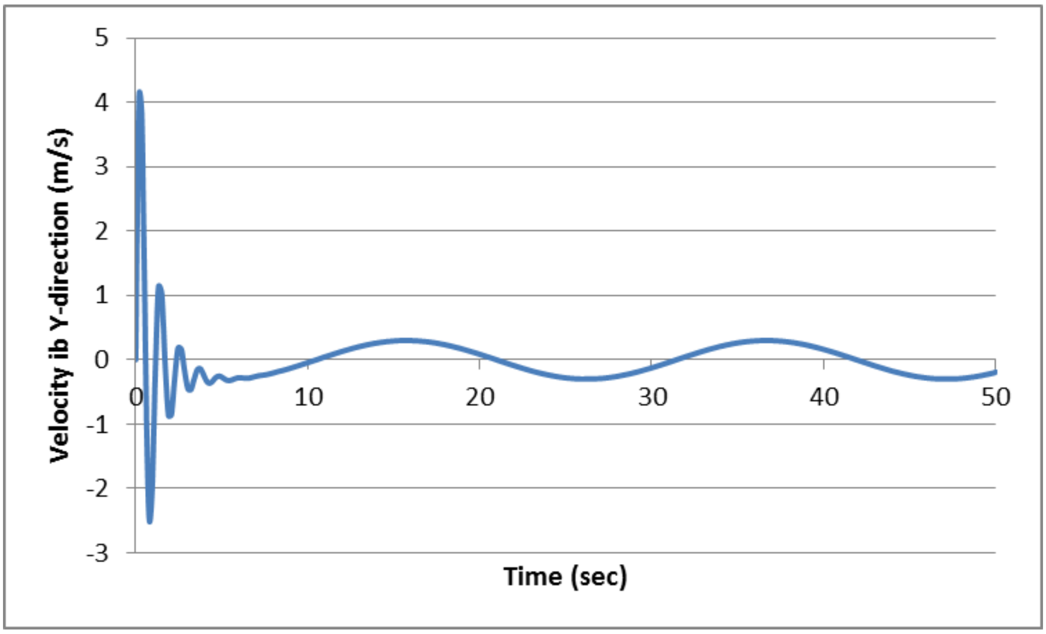

Figure 4. WMR velocity V_y

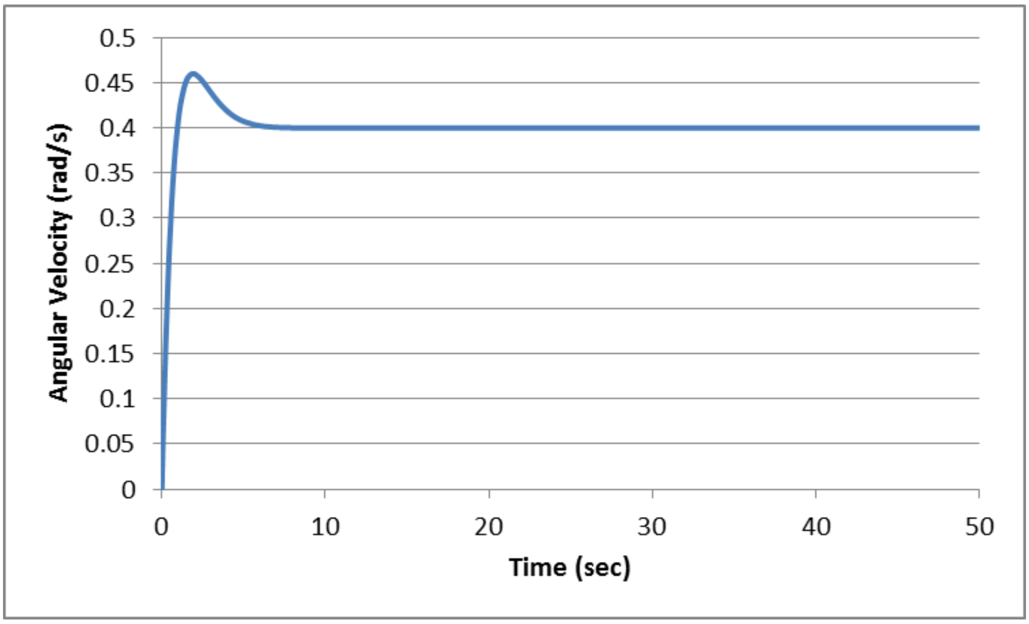

Figure 5. WMR angular velocity

In the second case, the mobile robot follows a desired infinity shape path. The desired path described by the functions of $x = 1 \sin(0.2\,t)$, $y = 1 \sin(0.4\,t)$.The estimated robot wheels angular velocities are $\omega_1 = 0.4\frac{rad}{s}$, $\omega_2 = 0.5\frac{rad}{sec}$ and $\omega_3 = 0.6\, rad/sec$. Figure 6 shows the mobile robot center linear velocity in x-axis direction. Figure 7 shows the mobile robot angular velocity. Figure 8 shows the mobile robot center linear velocity in y-axis direction.

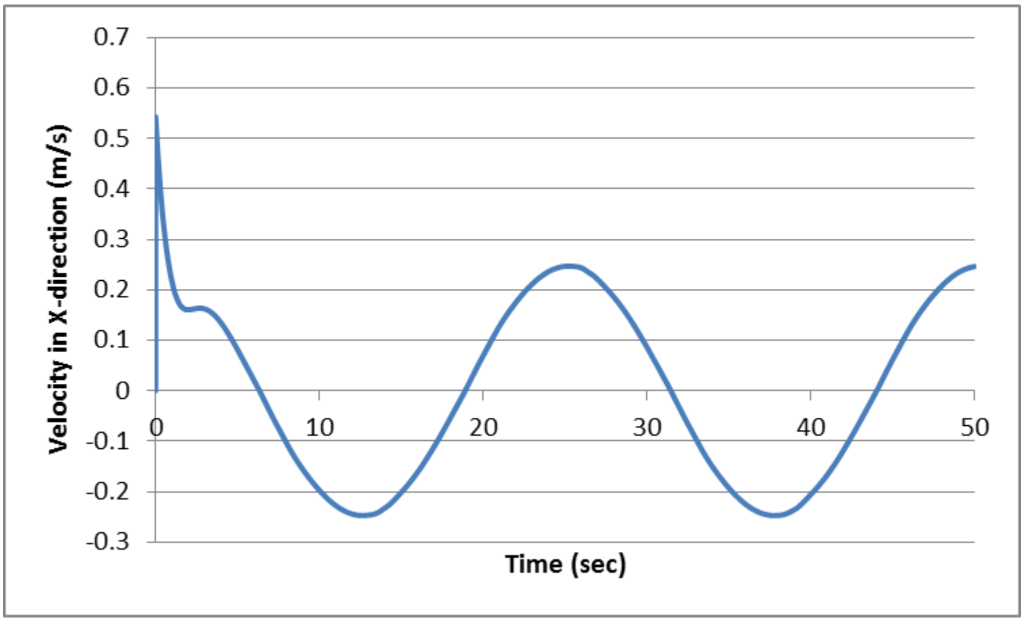

Figure 6. WMR velocity VX

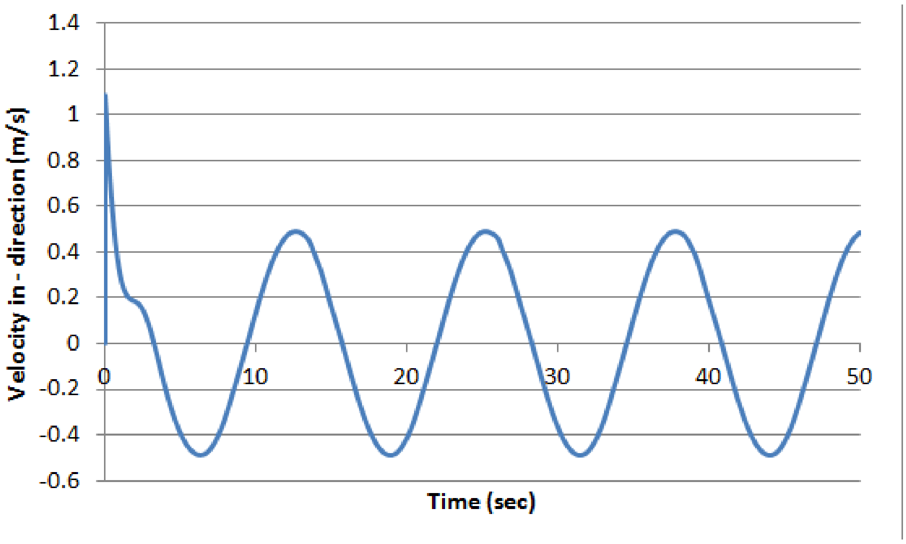

Figure 7. WMR velocity V_y

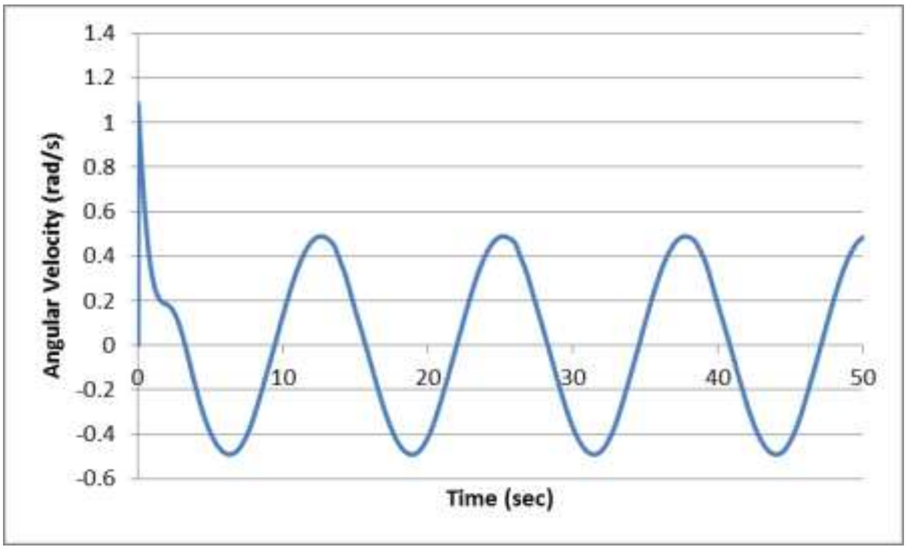

Figure 8. WMR angular velocity

CONCLUSION

This paper presents a holonomic WMR with three mecanum wheels design. Inverse and forward kinematic models have been successfully obtained. A virtual prototype simulation model has been established to study the kinematics and path tracking for this type of WMR. The results of the inverse kinematics simulation showed that the WMR movement direction affected to the variation of wheels angular velocities values and directions. Wheel angular velocity with sign (+) means that the mecanum wheel rotates in forward direction, value with (-) sign means that mecanum wheel rotates in vise versa direction. Also the inverse kinematics simulation results showed that, the usage of mecanum wheels is equivalent to the ordinary wheels when the WMR moves forward along a straight line because the angular velocities of all mecanum wheels were equal in values and directions. The forward kinematics simulation results show the confirmation of the models. MATLAB has been used to simulate the WMR mobile robot velocities controlling. Results showed a good matching between expected inverse, forward kinematics models mobile robot wheels angular velocities and its translation and orientation velocities. All results are in acceptable values.

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